

2019-05-01 OFFICE HOUR

- singular homology; face and boundary maps; categories Top , Ab , GrAb , Ch ; functors Map , Sing_n , Δ_* , H_*
- group theory review; Hurewicz homomorphism; examples with 0-, 1-, and 2-dim manifolds; functors H_0 , H_1 , π_1 , $(-)^{\text{ab}}$
- relative homology; s.e.s.'s; l.e.s. of a pair; snake lemma; categories Top_pairs , (hTop_pairs) , (triangulated Ch)
- Eilenberg--Steenrod axioms; definition of a co/homology theory with coefficients
- derivation of properties in an ordinary homology theory; definition of degree; homotopy classes of maps of spheres; example with antipodal map
- CW-complexes; cellular homology computes singular homology; examples with $\mathbb{R}P^n$, $\mathbb{C}P^n$, surfaces; (equivariant CW-complexes); (cellular maps); (cellular approximation)
- product CW-complexes; a cross product exists; acyclic models; verification of the homotopy axiom
- verification of excision; Euler characteristic; examples with 2-dim manifolds; (computations with simplicial homology); (proof of Meyer--Vietoris)
- transition to cohomology; free resolutions; (structure theorems for Ab , modules over PIDs); Ext and Tor ; chain homotopies; (homotopy category of chain complexes)
- universal coefficients; Künneth theorem; examples with Euler characteristic, surfaces, (finite fields), non-natural splitting
- Eilenberg--Zilber theorem; Alexander--Whitney diagonal approximation; (Krönecker pairings); cup products; cap products
- topological manifolds; orientations; (maps of fiber bundles); Poincaré duality