INTRODUCTION TO HOMOLOGY

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1. Problems due 2019-01-30

1.1. Let $X = \bigcup_{i \in I} X_i$ where the X_i are the (disjoint) path components of X. Prove that

$$H_*(X) \cong \bigoplus_{i \in I} H_*(X_i).$$

1.2. Let X and Y be path connected spaces.

(a) Prove that any map $f\colon X \to Y$ induces an isomorphism $f_*\colon H_0(X) \xrightarrow{\cong} H_0(Y)$.

(b) Prove that any map $f\colon X \to X$ induces the identity on $H_0(X)$.

1.3. Use the Hurewicz theorem to solve the following problems:

(a) Compute $H_1(K)$ for the Klein bottle K.

(b) Compute H_1 of $X = \prod_{j \in J} X_j$ for topological spaces X_j in terms of $H_1(X_j)$.

(c) Let X_i with base points $x_i \in X_i$. Suppose that there are open sets $x_i \in U_i \subseteq X_i$ such that x_i is a deformation retract of U_i. Show that

$$H_1\left(\bigvee_{i \in I} X_i\right) \cong \bigoplus_{i \in I} H_1(X_i).$$

1.4. Let $f\colon X \to Y$ be a covering space of path connected spaces with $f(x_0) = y_0$. By the fundamental theorem of covering spaces, $f_\#\colon \pi_1(X, x_0) \to \pi_1(Y, y_0)$ is a monomorphism. Is $f_*\colon H_*(X) \to H_*(Y)$ also a monomorphism. Prove that it is, or give a counterexample.

Date: 2019-01-14.
Git Repo: `https://github.com/coltongrainger/fy19top2`.