- I just read another proof online and hated it (indices, yuck).
- But I also found (and admire) this comment. Has all the ammo I needed.
  - *Yes, that's true, by the (i) universal coefficient theorem and (ii) the fact that for any finitely generated abelian group A and any prime p, we have*

    $$\dim_{\mathbb{Q}} A \otimes_{\mathbb{Z}} \mathbb{Q} = \operatorname{rank}_{\mathbb{Z}} A = \dim_{\mathbb{F}_p} A \otimes_{\mathbb{Z}} \mathbb{Z}/p\mathbb{Z} - \dim_{\mathbb{F}_p} \operatorname{Tor}^1_{\mathbb{Z}}(A, \mathbb{Z}/p\mathbb{Z}).$$

  - `https://math.stackexchange.com/posts/996525/`