Topology. *Topological spaces, fundamental group, covering spaces, principal bundles, vector bundles, frame bundles, tangent and cotangent bundles, classifying spaces.*

1. Topological spaces. Examples: spheres, projective spaces, surfaces (as identification spaces of polyhedra, and as group quotients). Homotopy equivalence. More generally, spaces constructed as cell complexes, yielding further examples: projective spaces, Grassmannians, flag varieties. Topological manifolds.
2. Fundamental group; Van Kampen, and computations with it. Examples: Riemann surfaces, real projective space, etc.
3. Covering spaces; examples drawn from Riemann surfaces. Galois theory of covering spaces.
4. Topological groups, with examples from classical groups.
5. Principal bundles as generalizations of covering spaces. Definition of vector bundles. Frame bundle, tangent bundle, and cotangent bundle of embedded submanifolds of $\mathbf{R}^n$, Grassmannians and their tautological bundles.
6. Classifying spaces.

Differentiable manifolds. *Differentiable manifolds; implicit function theorem and statement of Sard's theorem; smooth vector bundles, tangent vectors, tensors, vector fields and flows. Lie derivatives, Lie groups and Lie algebras. Integral manifolds, Frobenius's theorem. Differential forms and the de Rham complex (without cohomology). Orientation, integration, Riemannian metrics, geodesics, exponential map.*

1. Differentiable manifolds, definition and examples. Examples: Manifolds defined from regular values, projective spaces
2. Implicit function theorem and statement of Sard's theorem
3. Smooth vector bundles; tangent vectors as derivations and the tangent, cotangent bundles. Examples: $\mathbf{RP}^1$, Riemann surfaces, projective spaces and Grassmannians
4. Higher rank tensors (associated bundles via transition functions), vector fields and flows. Examples: top forms, calculus ODE's, vector fields from symmetries
5. Lie derivatives, Lie bracket, Lie groups and Lie algebras. Examples: Cartan's formula, Heisenberg group and algebra, $SL_2(\mathbf{R})$, $SU(2)$
6. Integral manifolds and the Frobenius theorem. Examples: integrability for PDE's; when's a vector field a grad?
7. Differential forms, and the de Rham complex (no cohomology yet). Examples: div-grad-curl.
8. Orientability, integration, Stokes's theorem. Examples: $\mathbf{RP}^2$, calculus, Gauss's law.
9. Riemannian metrics; geodesics as minimizers. Examples: submanifolds of Euclidean space.
10. Exponential map; tubular neighborhood theorem; second variation and Jacobi fields. Examples: Lie groups, S^2.

Cohomology. *de Rham cohomology, Mayer-Vietoris, Poincaré duality, singular homology and cohomology. Cohomolgy of cell complexes, simplicial cohomology, Cech cohomology, equivalences between cohomology theories. Cup product; sheaves.*

1. de Rham cohomology; integration as map from H^n to scalars.
2. Mayer-Vietoris; computation of many examples.
3. Poincaré duality.
4. Singular homology (abrupt change of gears) and pairing with de Rham cohomology on a smooth manifold.
5. Singular cohomology. Mayer-Vietoris for singular homology and cohomology.
6. Cohomology of cell complexes, simplicial cohomology, some notions of equivalence of all these flavors.
7. Cech cohomology, Weil's proof of Cech-de-Rham theorem; sketch of proof that singular homology is dual to de Rham.
8. Cup product in singular and de Rham theories; relationship to intersection theory.
9. Sheaves.