$$\int_\Omega \frac{\partial \underline{\underline{K}}^{-1}(\psi)}{\partial \psi} \underline{v} \cdot \underline{q}_k^{n+1} \psi_{k+1}^{n+1}$$

$$\int_\Omega \frac{\partial \underline{\underline{K}}^{-1}(\psi)}{\partial \psi} \underline{v} \cdot \underline{q}_k^{n+1} \psi_{k+1}^{n+1} =$$

$$= \int_\Omega \frac{\partial \underline{\underline{K}}^{-1}(\psi)}{\partial \psi} \phi_i \cdot \Big(\sum_j q_j \phi_j\Big) \Big(\sum_s \psi_s \Psi_s(x)\Big) =$$

$$= \int_\Omega \frac{\partial \underline{\underline{K}}^{-1}(\psi)}{\partial \psi} \phi_i \cdot \Big(\sum_j q_j \phi_j\Big) \Big(\sum_s \psi_s \cdot \alpha_s I_{\Omega_s}\Big) =$$

$$\frac{\partial \underline{\underline{K}}^{-1}}{\partial \psi}$$

$$= \int_\Omega \Big(\phi_i \cdot \sum_j q_j \phi_j\Big) \Big(\sum_s \psi_s \alpha_s \beta_s I_{\Omega_s}\Big) =$$

If we reason locally, then:

$$\int_{\Omega_s} \Big(\phi_i \cdot \sum_j q_j \phi_j\Big) \psi_s \alpha_s \beta_s =$$

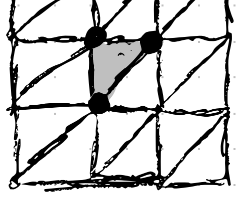
$$= \psi_s \Big[\alpha_s \beta_s \sum_j q_j \int_{\Omega_s} \phi_i \cdot \phi_j\Big] \quad \forall i$$

Steps:

column   rows

1) for each Ω_s, find ϕ_i, ϕ_j, ϕ_k q basis
2) compute local matrix:

$$\begin{bmatrix} \Big(\sum_{l=1}^{3} q_l \int_{\Omega_s} \phi_l \cdot \phi_1\Big) \alpha_s \beta_s \\ \Big(\sum_{l=1}^{3} q_l \int_{\Omega_s} \phi_l \cdot \phi_2\Big) \alpha_s \beta_s \\ \Big(\sum_{l=1}^{3} q_l \int_{\Omega_s} \phi_l \cdot \phi_3\Big) \alpha_s \beta_s \end{bmatrix} = \alpha_s \beta_s M_{Loc} q_{Loc}$$



where $[M_{Loc}]_{ij} = \int_{\Omega_s} \phi_i \cdot \phi_j$

Now:

1) Porepy makes pp.RT0.massHdiv available to compute the local mass matrix scaled by K:

2) $$\underline{\underline{K}} = \chi(\psi) \underline{\underline{I}} \Rightarrow \underline{\underline{K}}^{-1} = \frac{\underline{\underline{I}}}{\chi(\psi)} \Rightarrow$$

$$\Rightarrow \frac{\partial \underline{\underline{K}}^{-1}}{\partial \psi} = \frac{\partial}{\partial \psi}\Big(\frac{1}{\chi(\psi)}\Big) \underline{\underline{I}} =$$

$$= -\frac{1}{\chi(\psi)^2} \frac{\partial \chi}{\partial \psi} \underline{\underline{I}}$$

3) Now, after the assembly, we have that:

$$C_{Loc,s} = \alpha_s \beta_s M_{Loc} q_{loc}$$

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alternatively: No. Mass considers . Porepy?

1) compute mass matrix (global), no $\underline{\underline{K}}$
2) manually assemble c:

$$c_{ij} = \begin{cases} \Big(\sum_l q_l \int_{\Omega_j} \phi_i \cdot \phi_l\Big) \frac{\partial \underline{\underline{K}}^{-1}(\psi)}{\partial \psi}\Big|_j \cdot \frac{1}{\sqrt{Vol(\Omega_j)}} \\ 0 \quad \text{otherwise} \end{cases}$$

Local mass:

$$M^{Loc} = \begin{bmatrix} 1/3 & 0 & -1/6 \\ 0 & 1/6 & 0 \\ -1/6 & 0 & 1/3 \end{bmatrix}$$

$$\int_{\Omega_i} \psi_i \cdot \psi_j = \delta_{ij} \frac{1}{|\Omega_i|}$$

$$\alpha_i^2 |\Omega_i| = \frac{1}{|\Omega_i|} \Rightarrow \boxed{\alpha_i = \frac{1}{|\Omega_i|}}$$

$$\psi_i = \frac{I_{\Omega i}}{|\Omega i|}$$