Predictive Coding

Computational Psychiatry Course

15.09.2021

Lilian A. Weber

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Laplace approximation

free energy

Bayesian brain

model inversion

predictive processing

predictive coding

active inference

redundancy reduction

neural networks

sampling

variational Bayes

Bayesian inference

generative model

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Levels of analysis

computational

algorithmic

implementational

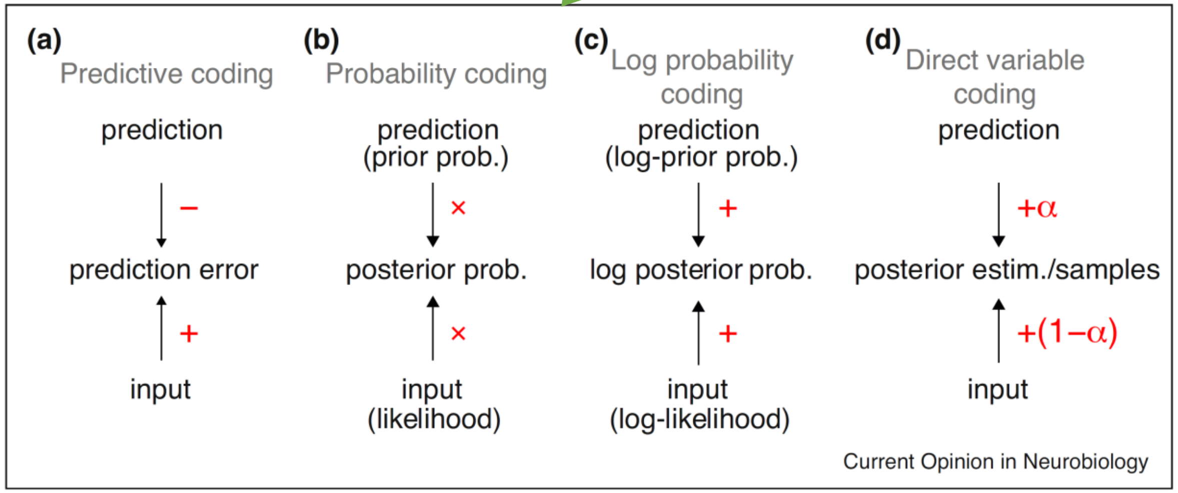
(approximate)
Bayesian inference

predictive coding

(predictive coding
in the brain)

redundancy
reduction

...



"Bayesian predictive coding"

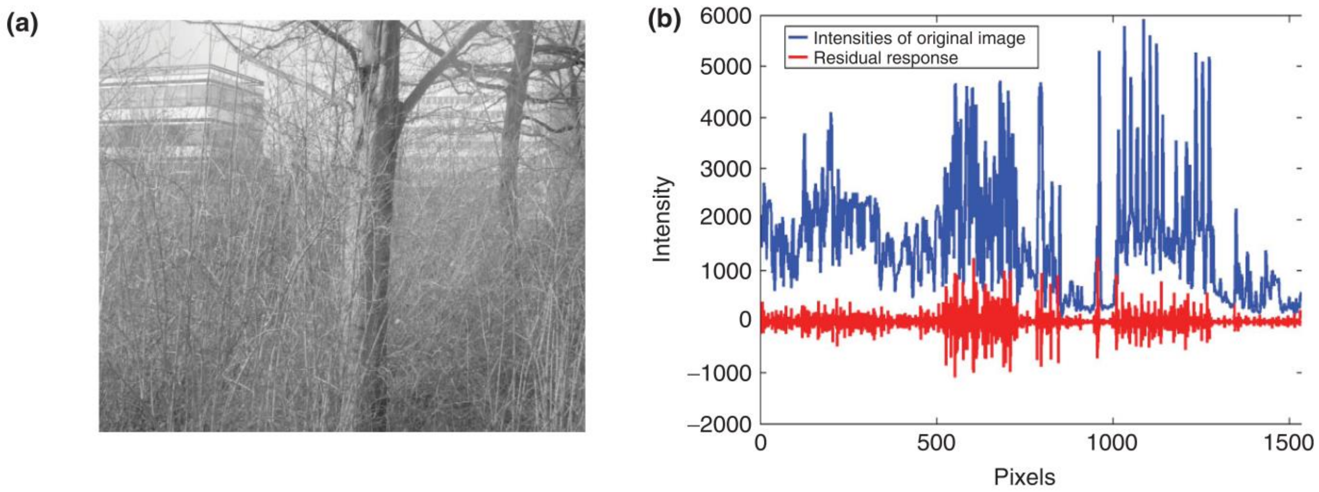
Aitchison & Lengyel 2017 *Curr Op Neurobiol*

Structure of this talk

1. What is predictive coding? ← neural networks
2. What is Bayesian predictive coding? ← maths, Bayesian inference
3. Can and does the brain use predictive coding? ← neurobiology/neuroscience
4. Why is it useful for computational psychiatry? ← psychiatry

What is predictive coding?

prediction error = $\boxed{\text{prediction}}$ – input



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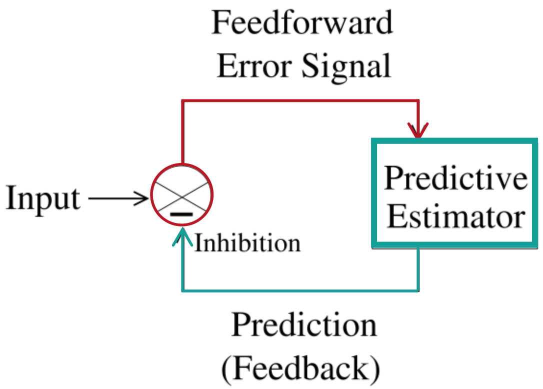
Huang & Rao 2011 *WIREs Cogn Sci*

What is predictive coding?

prediction error = **prediction** – **input**

$$\begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ \vdots \\ i_K \end{bmatrix}$$



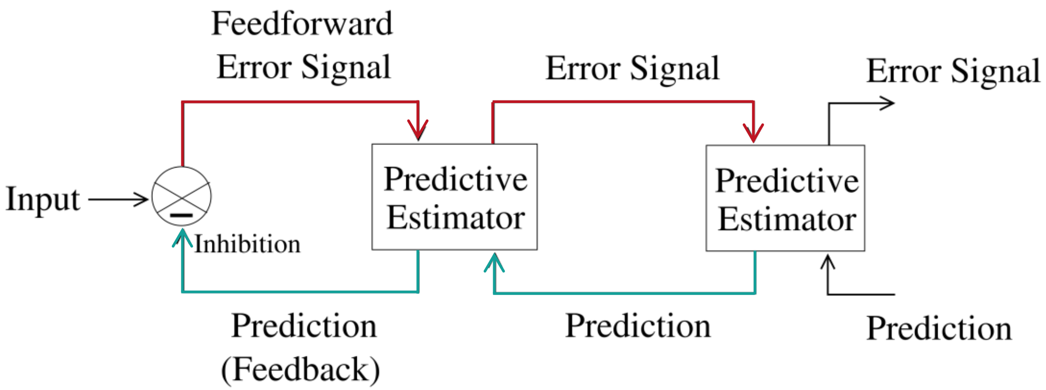
$$\begin{bmatrix} r_1 \\ r_2 \\ \vdots \\ r_N \end{bmatrix}$$

$$\mathbf{I} = \boxed{f(U\boxed{\mathbf{r}}) + \mathbf{n}}$$

Rao & Ballard 1999 *Nat Neurosci*

What is predictive coding?

prediction error = prediction – input

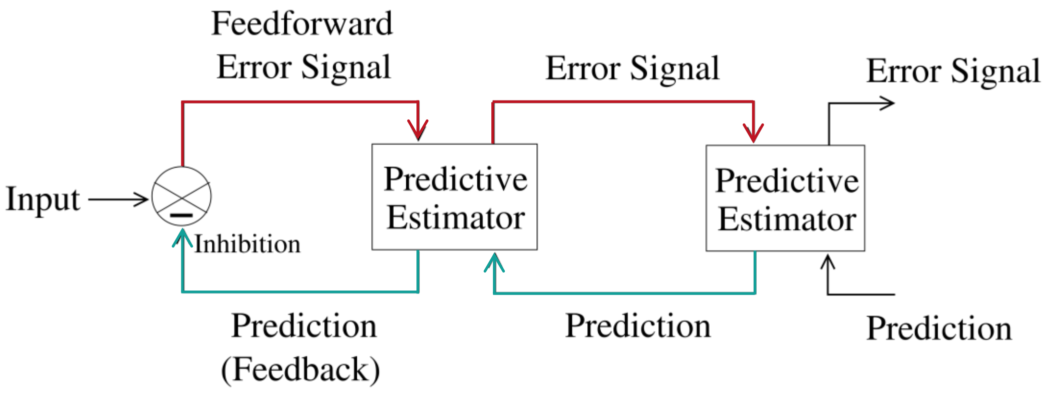


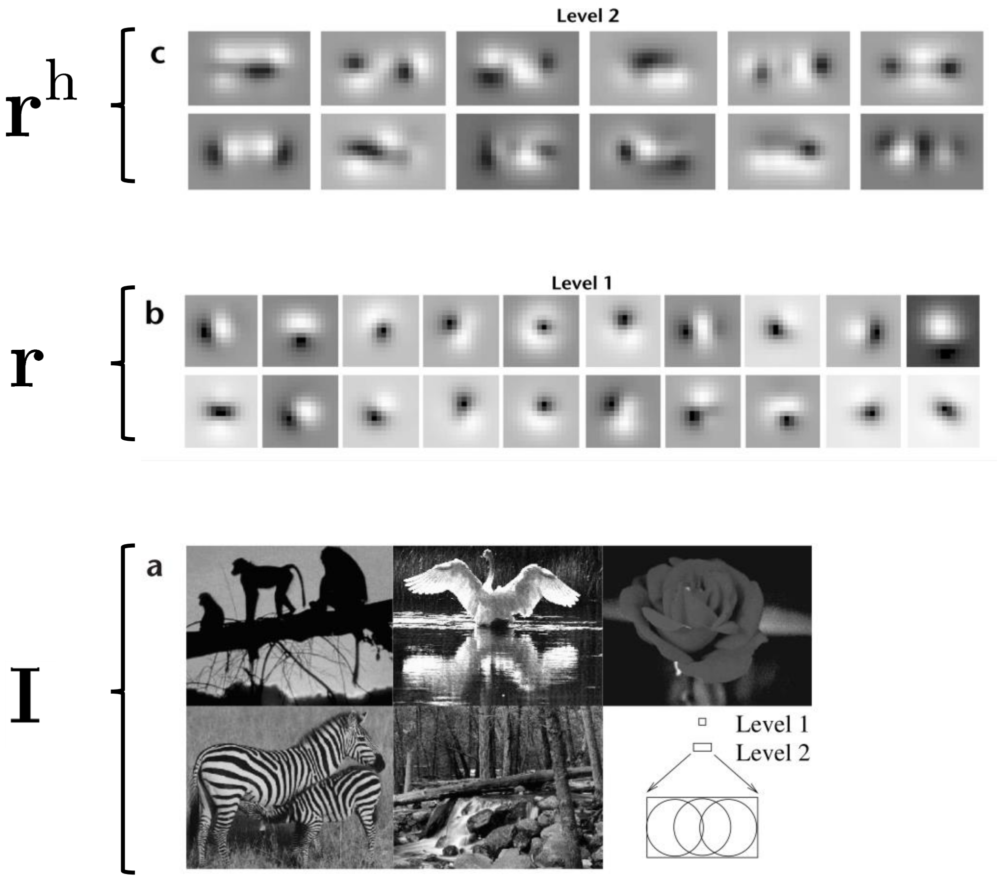
$$\mathbf{I} = f(U\mathbf{r}) + \mathbf{n} \qquad \mathbf{r} = f(U^{\mathrm{h}}\mathbf{r}^{\mathrm{h}}) + \mathbf{n}^{\mathrm{td}}$$

Rao & Ballard 1999 *Nat Neurosci*

What is predictive coding?

prediction error = **prediction** – **input**

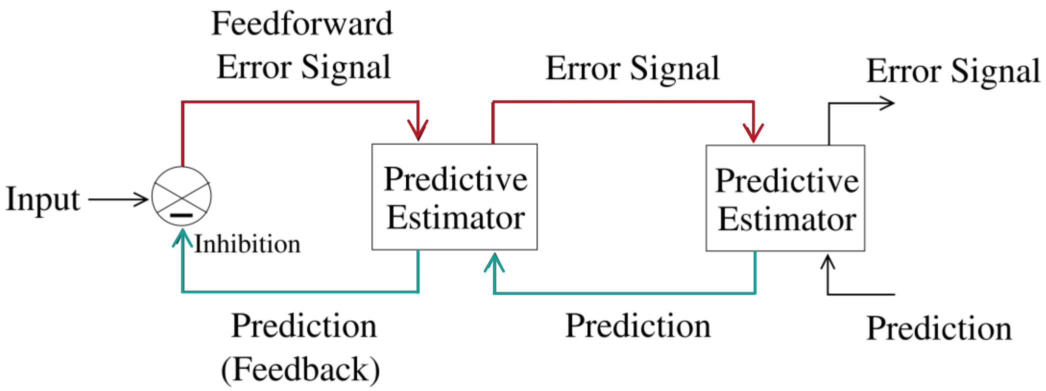


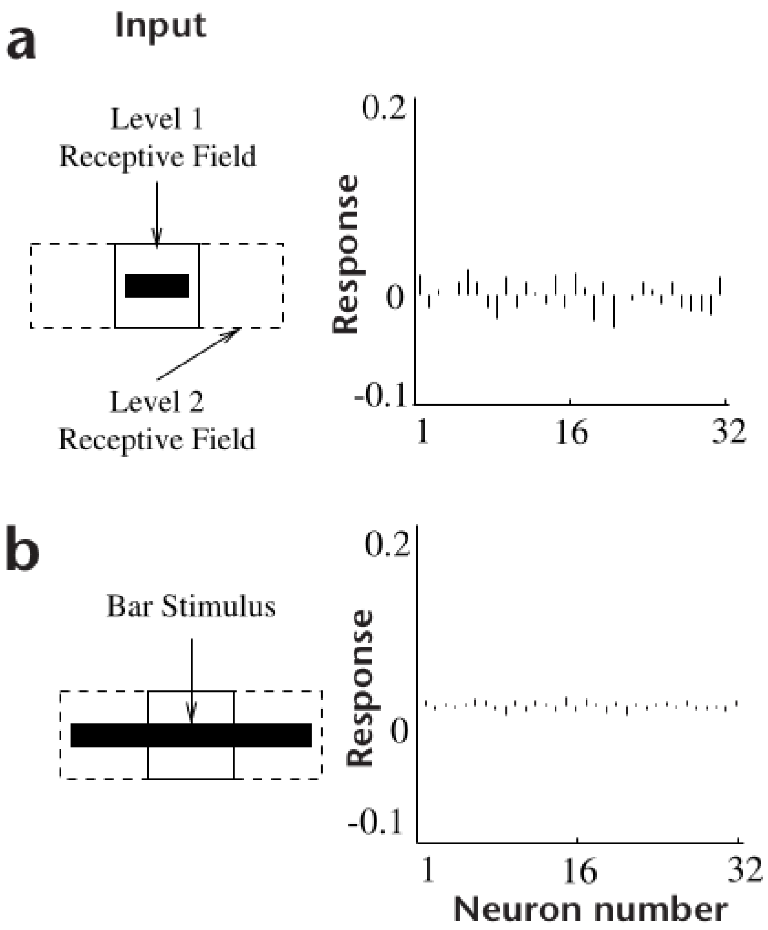


Rao & Ballard 1999 *Nat Neurosci*

What is predictive coding?

prediction error = **prediction** – **input**





- Recurrent neural networks outperform purely feedforward networks in the presence of noise (e.g., **Alamia et al. 2021 arXiv**)
- Predictive coding networks can approximate the backpropagation of error algorithm (**Whittington & Bogacz 2017** ***Neural Comput***; **Song et al. 2020** ***Adv Neural Inf Process Syst***)

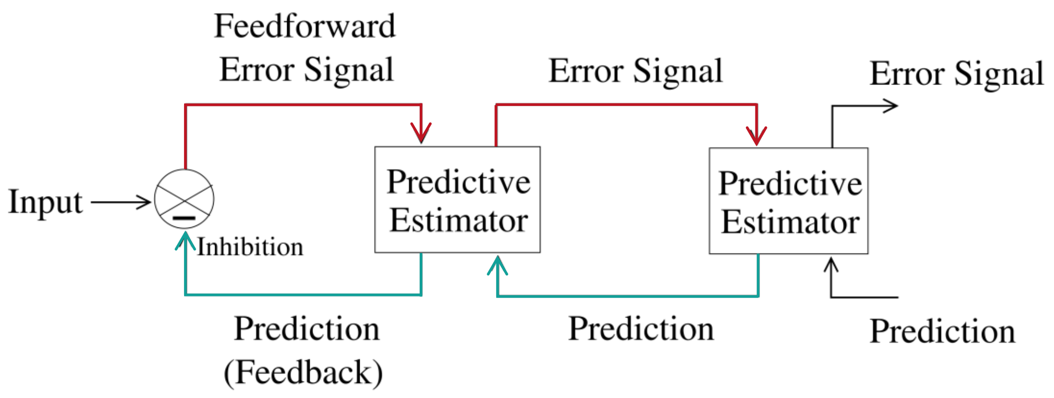
Rao & Ballard 1999 ***Nat Neurosci***

What is Bayesian (about) predictive coding?

1. The architecture represents a hierarchical generative model of how sensory inputs are caused.

$$\mathbf{I} = f(U\mathbf{r}) + \mathbf{n}$$

$$\rightarrow \; p(\mathbf{I}|\mathbf{r}, U)$$



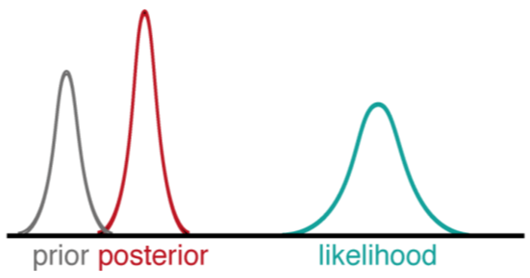
2. The cost function is the negative log joint: maximising the joint means finding the MAP estimates in Bayesian inference.

$$\begin{aligned} E &= -\log p(\mathbf{I}|\mathbf{r}, U) - \log p(\mathbf{r}) - \log p(U) \\ &= -\log \boxed{\left(p(\mathbf{I}|\mathbf{r}, U)\, p(\mathbf{r})\, p(U)\right)} \end{aligned}$$

Bayes' Rule

posterior~ likelihood · prior

p(x|y,m)~ p(y|x,m)p(x|m)



What is Bayesian (about) predictive coding?

3. The prediction errors that update the estimate causes are weighted by their inverse variance.

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = -\frac{k_1}{2}\frac{\partial E}{\partial \mathbf{r}}$$

Inputs

Expectations

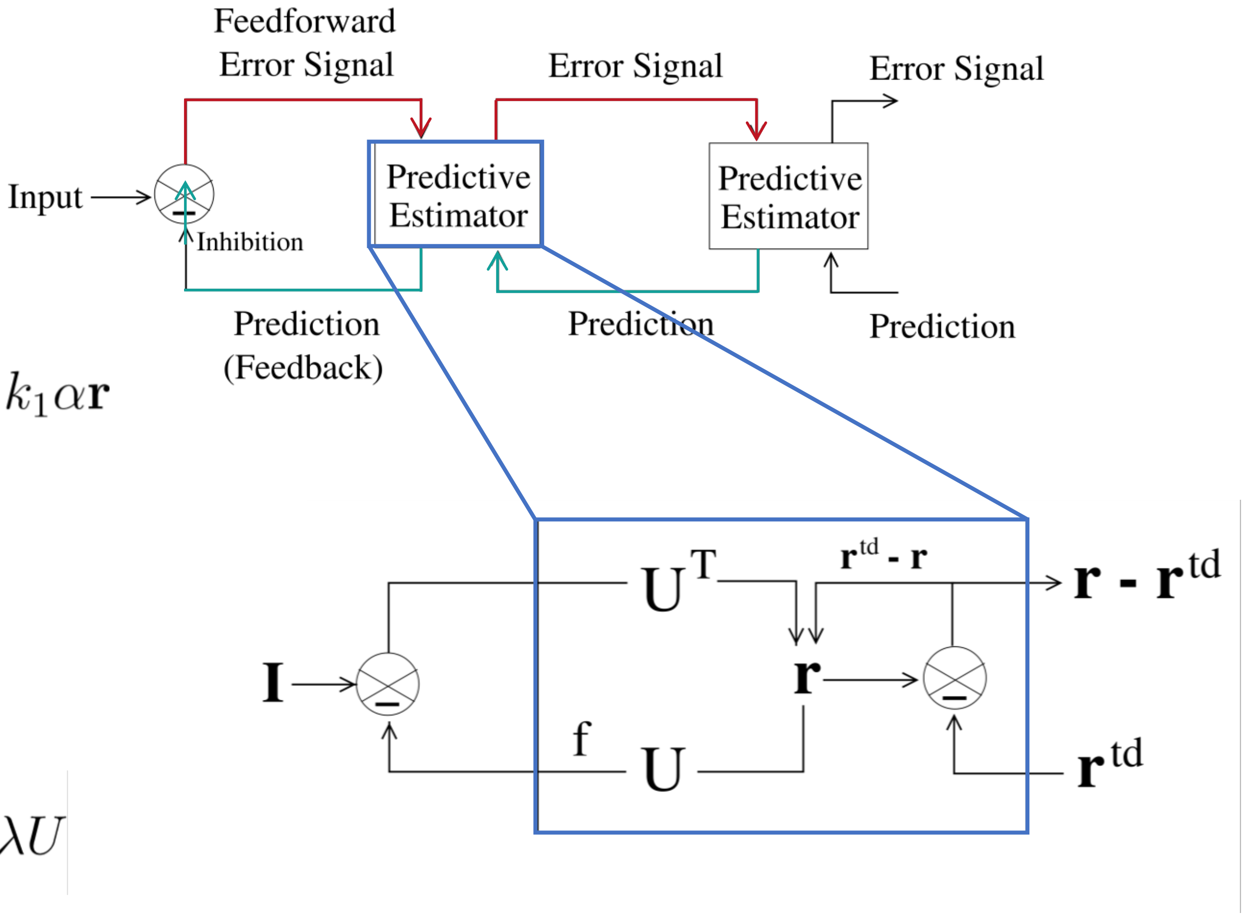
$$= \frac{k_1}{\sigma^2}U^{\mathsf{T}}\frac{\partial f}{\partial U\mathbf{r}}^{\mathsf{T}}(\mathbf{I} - f(U\mathbf{r})) + \frac{k_1}{\sigma_{\mathrm{td}}^2}(\mathbf{r}^{\mathrm{td}} - \mathbf{r}) - k_1\alpha\mathbf{r}$$

precision-weighting

4. A single cost function accounts for inference (udating **r**) and learning (updating U)

$$\frac{\mathrm{d}U}{\mathrm{d}t} = -\frac{k_2}{2}\frac{\partial E}{\partial U} = \frac{k_2}{\sigma^2}\frac{\partial f}{\partial U\mathbf{r}}^{\mathsf{T}}(\mathbf{I} - f(U\mathbf{r}))\mathbf{r} - \frac{k_2}{2}\lambda U$$

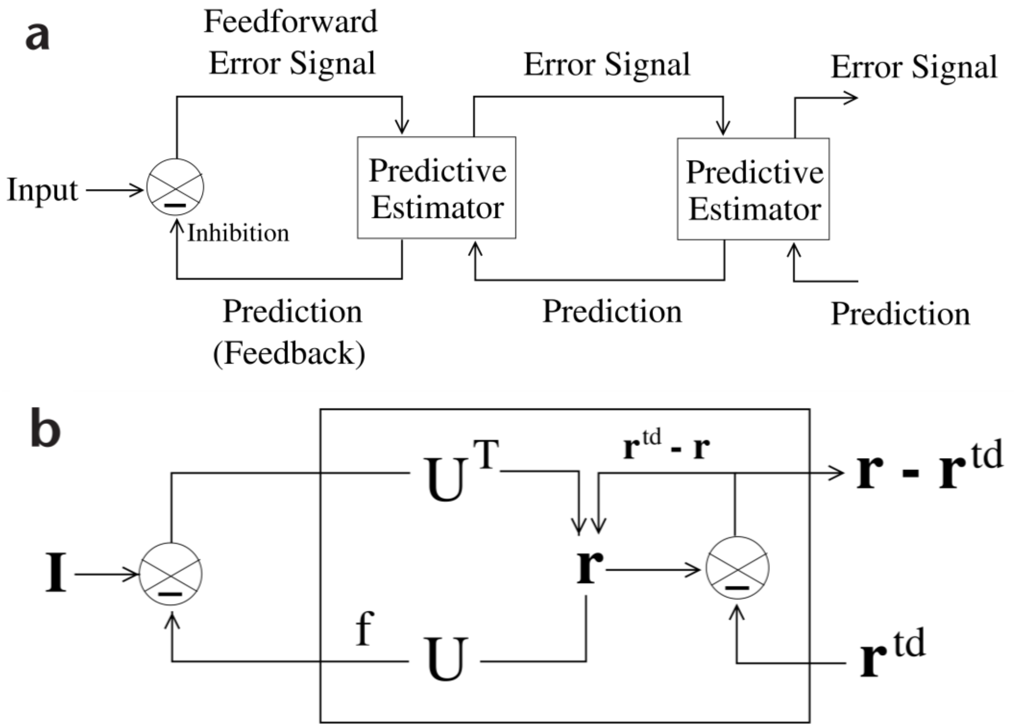
Hebbian learning



Predictive coding as approximate inference

Summary

- A hierarchy of causes (predictive estimators and PE units)
- **Recurrent message passing:** PEs are signaled upwards (bottom-up) and predictions are signaled top-down
- **Inference:** finding the most appropriate causes to describe the data
- **Learning:** finding the most appropriate model to describe the data
- Both involve **minimizing precision-weighted prediction errors**
- This looks like **Bayesian inference** for finding the most likely causes of sensory inputs (**MAP estimates**) and like Hebbian learning, respectively



Rao & Ballard 1999 ***Nat Neurosci***

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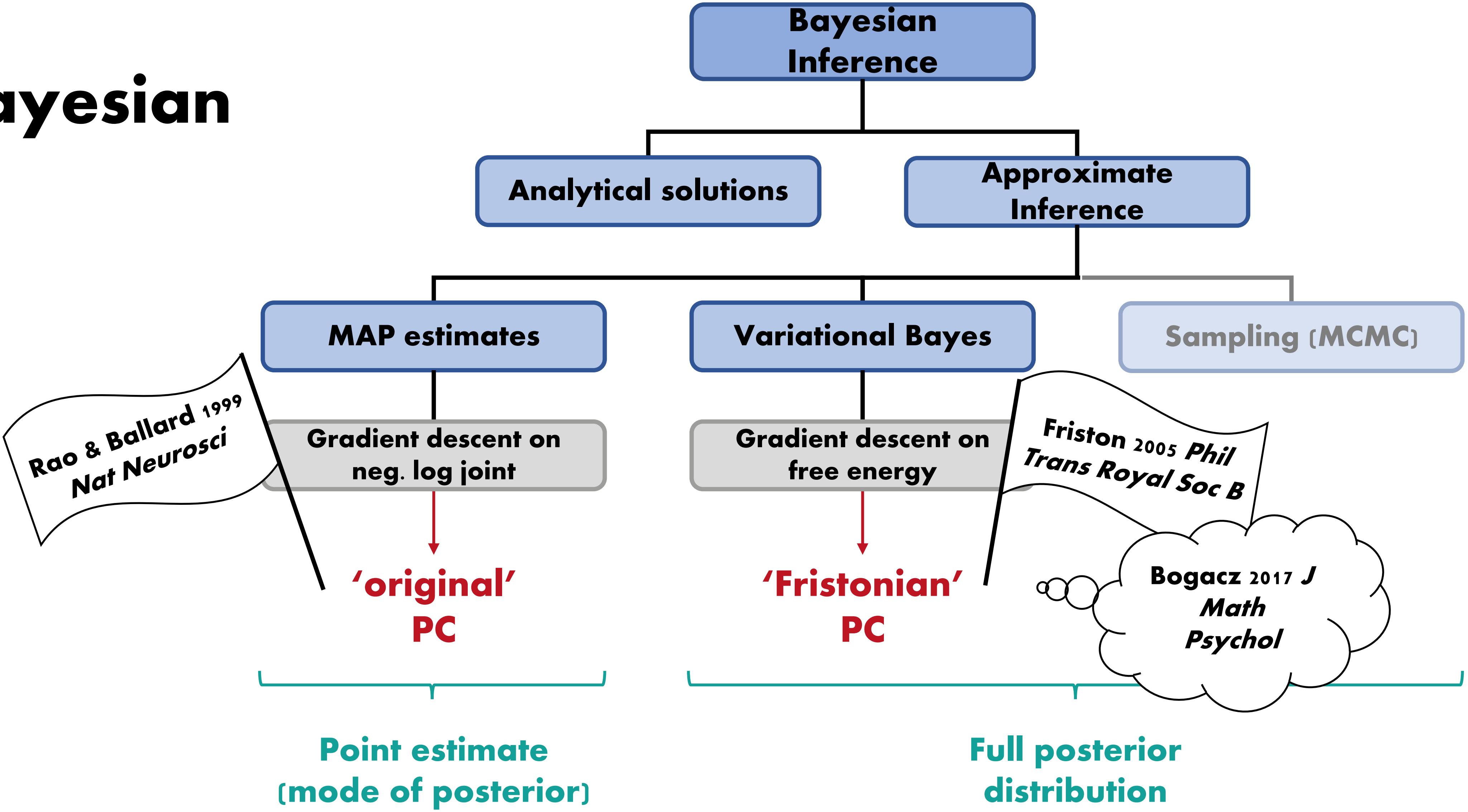
The Bayesian Brain



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$$p(x|y) = \frac{p(x)p(y|x)}{p(y)} \qquad p(y) = \int p(x)p(y|x)\,\mathrm{d}x$$

Representing uncertainty

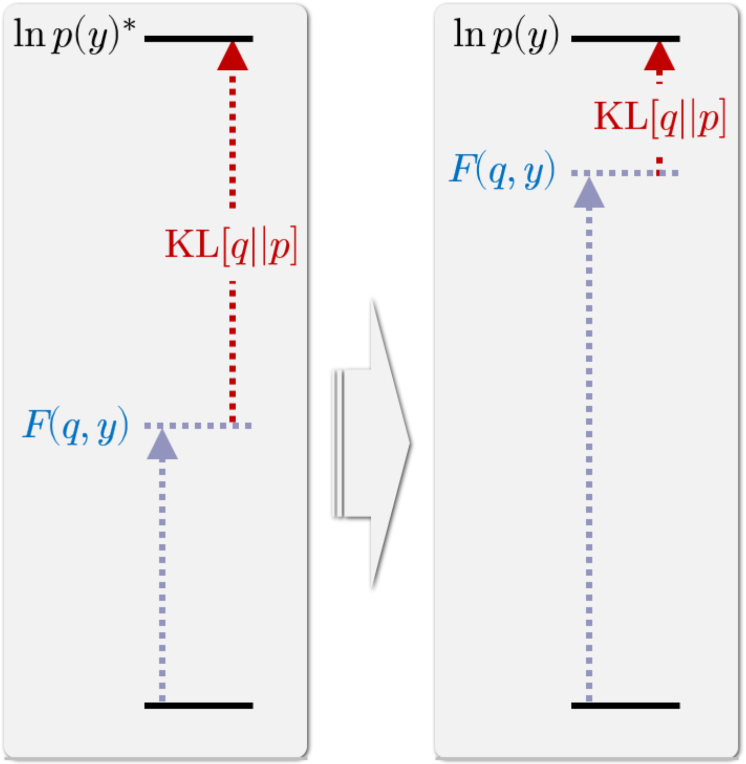
So far, we've only computed the MAP.
To be fully Bayesian: care about your uncertainty!

$q(x)$: our best guess about $p(x|y)$

$$\mathrm{KL}(q(x), p(x|y)) = \ln p(y) - \int q(x) \ln \frac{p(y,x)}{q(x)}\,\mathrm{d}x$$
$$= \ln p(y) - F$$

$$\ln p(y|m) = \mathrm{KL}(q(x), p(x|y,m)) + F(q(x), p(x,y|m))$$

$$F = \int q(x) \ln \frac{p(y,x)}{q(x)}\,\mathrm{d}x$$



Bishop, 2008; **Friston et al.** 2007 ***NeuroImage***

Predictive Coding can be viewed as the implementation of a particular scheme of approximate Bayesian inference in the brain.

‘Fristonian’ PC

The free energy formulation of predictive coding

$$F = \int q(x) \ln \frac{p(y,x)}{q(x)} \, \mathrm{d}x$$

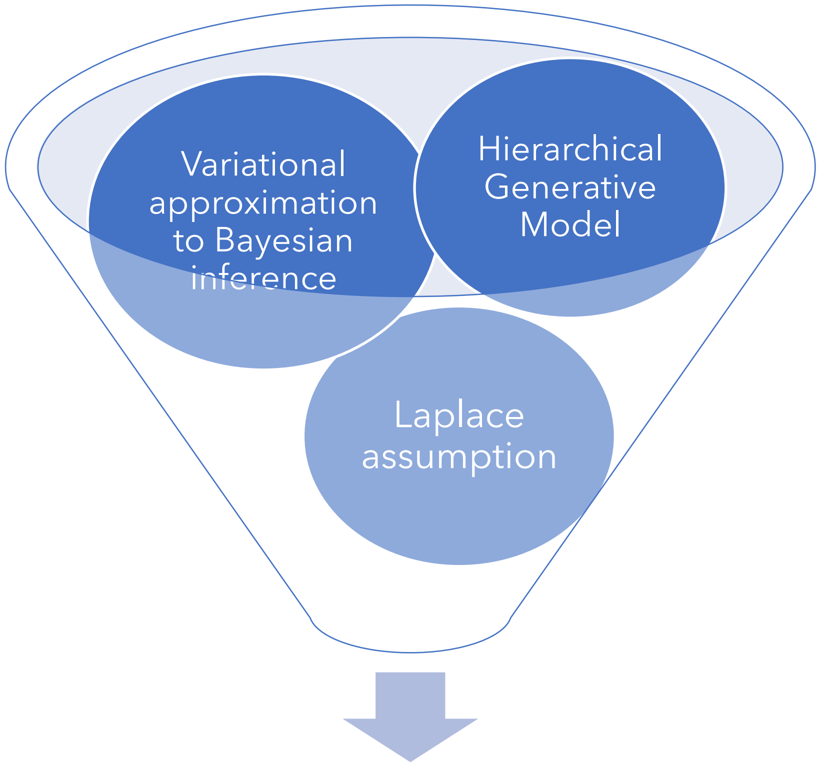
Inference: maximizing F to find the approximate posterior

Learning: maximizing F to improve the model and thereby reduce average surprise over time.

Additional assumption: *q(x)* is a Gaussian.
We only need to represent the mean and the variance.

→ *precision-weighted prediction errors*

→ *learning about the **precision** of beliefs*



Predictive Coding

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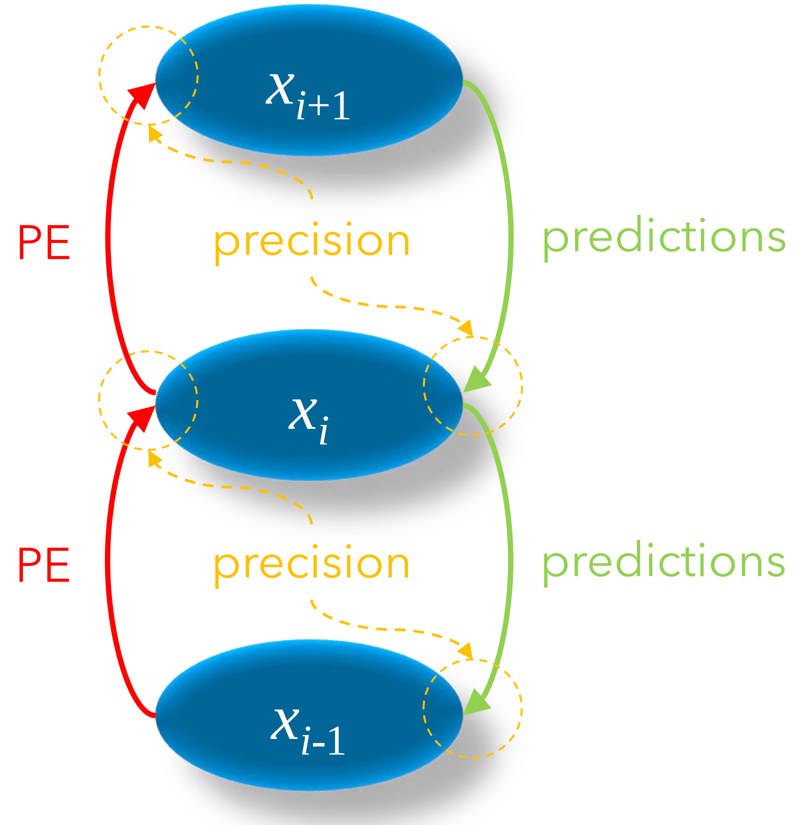
The main ingredients of predictive coding (from a computational point of view)

- A hierarchical generative model of sensory inputs
- The estimates of the causes generate predictions of sensory input
- The estimates of causes are updated in response to prediction errors (mismatches)
- The relative influence of PEs and predictions is determined by their relative precision (certainty)
- These computations underlie both perception (inference) and learning (model update)

Δ**belief** ~ **precision** × **PE**



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Adapted from Stephan et al. 2016 ***Brain***

Mapping predictive coding onto the brain

- **A hierarchical generative model of sensory inputs**
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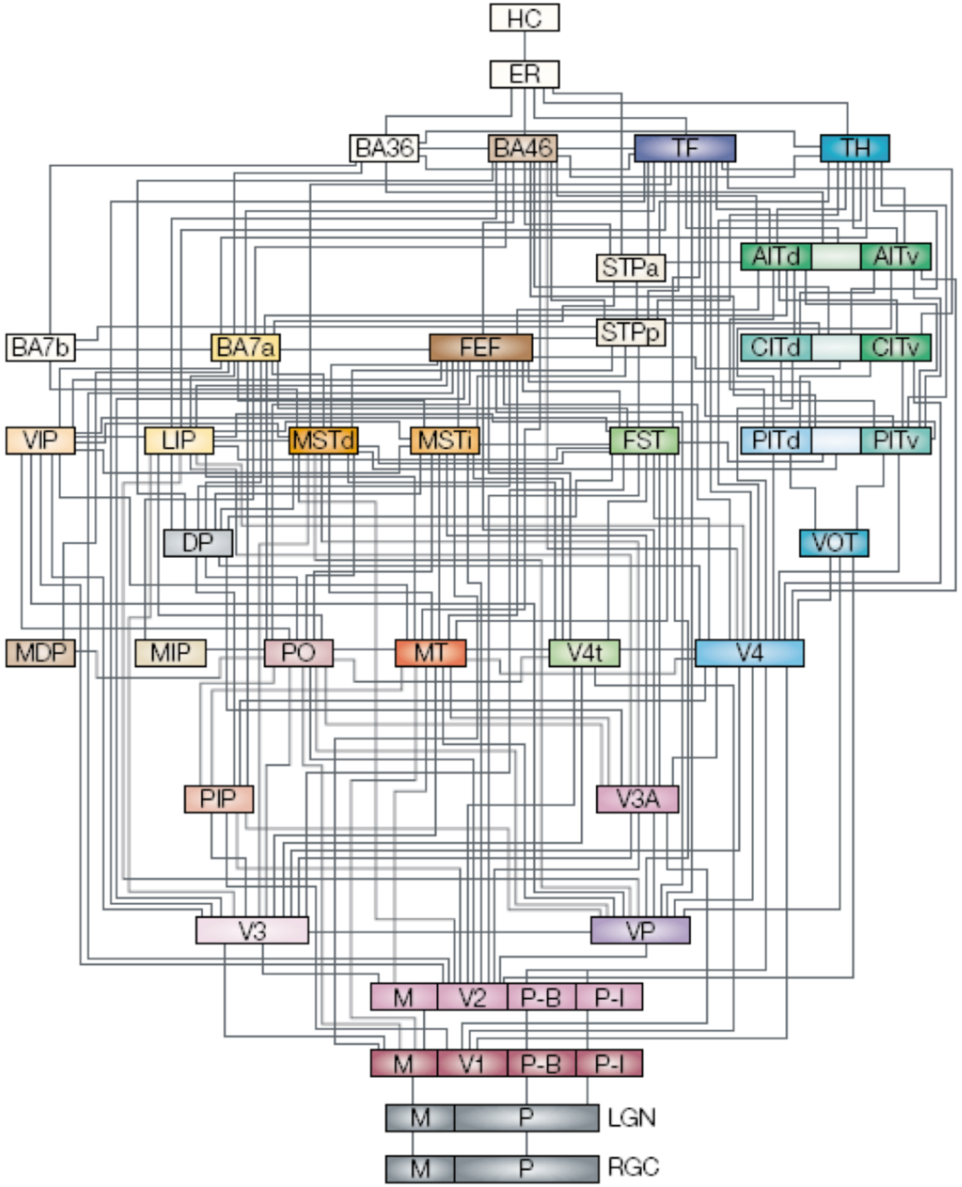

Felleman & Van Essen 1991 *Cereb Cortex*

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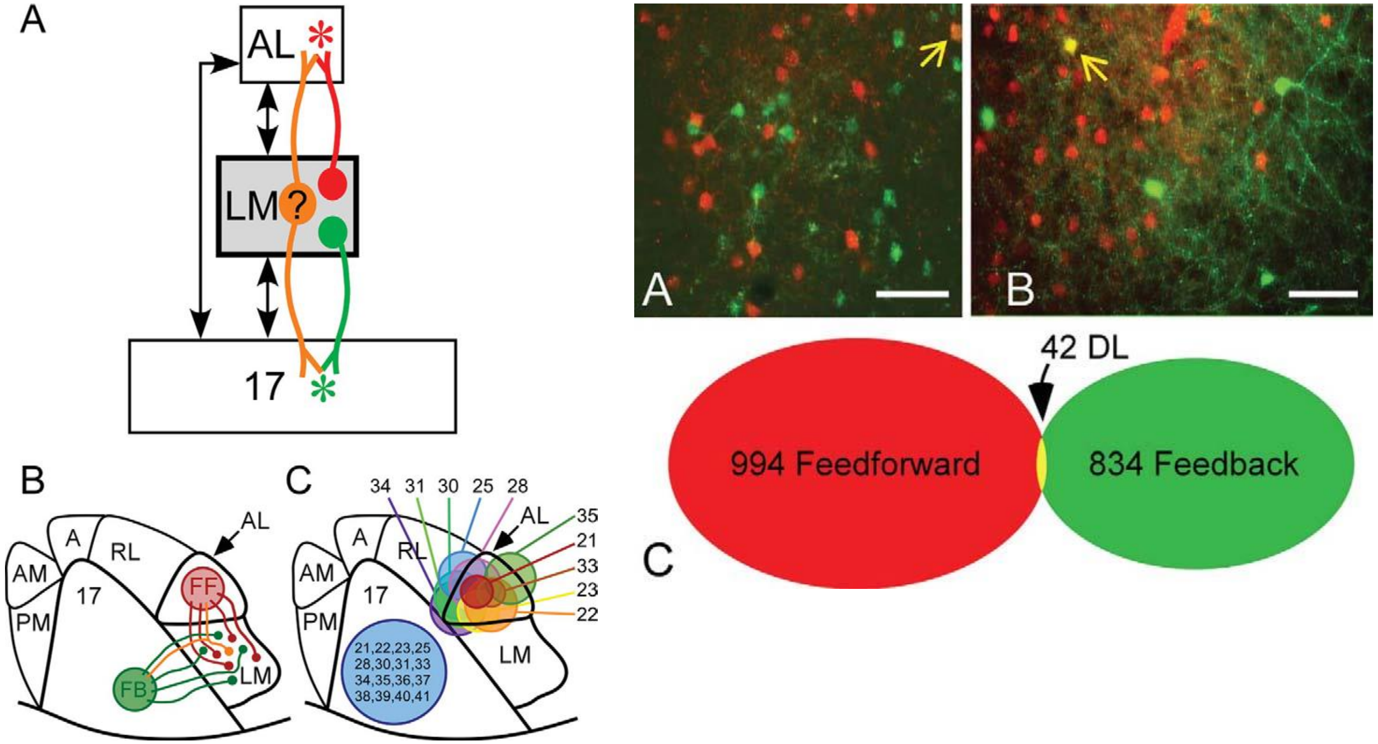
1. The source populations of forward and backward pathways should be completely separate, given their functional distinction.



Berezovskii et al. 2011 ***J Comp Neurol***

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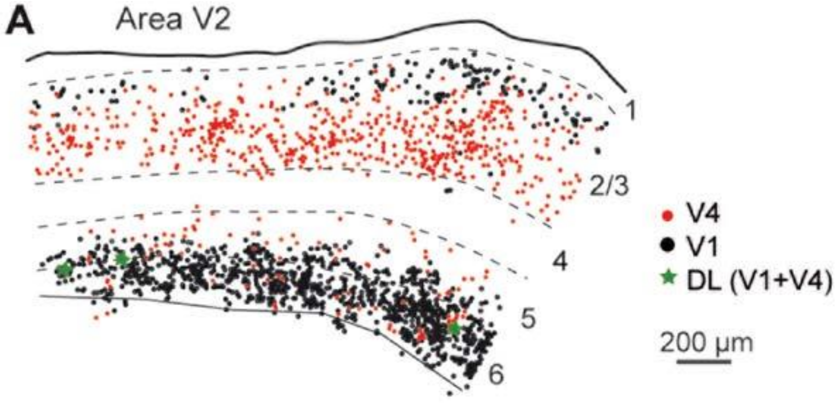

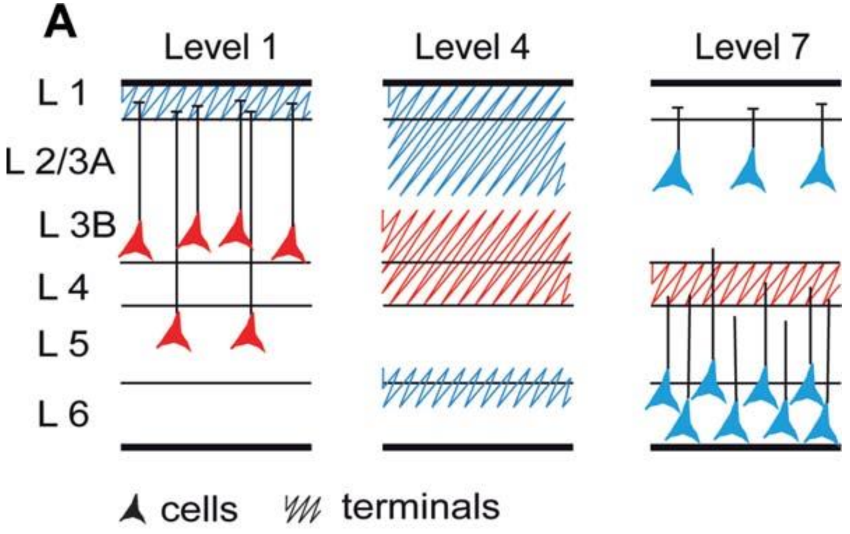

Markov et al. 2014 *J Comp Neurol*

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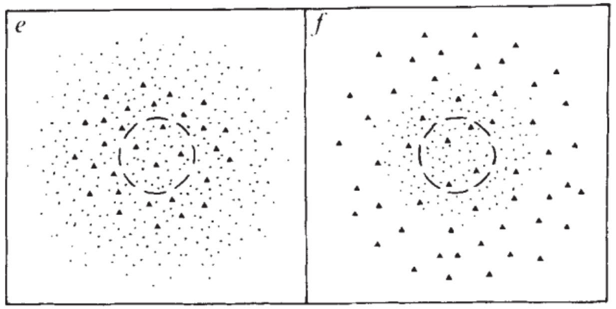
2. Backward connections encode predictions and expected precision, so they should be more divergent.



Zeki & Shipp 1988 ***Nature***

3. Causes interact non-linearly to generate data, so backward connections should be more modulatory.

Self et al. 2012 ***PNAS***
Olsen et al. 2012 ***Nature***
Zilles et al. 2004 ***J Anat***

4. Predictions are generated more slowly than prediction errors, suggesting a spectral asymmetry.

Roopun 2006 ***PNAS***
Roopun et al. 2008 ***Front. Cell. Neurosci***

Buffalo et al. 2011 ***PNAS***
Bosman et al. 2012 ***Neuron***
Bastos et al. 2015 ***NeuroImage***

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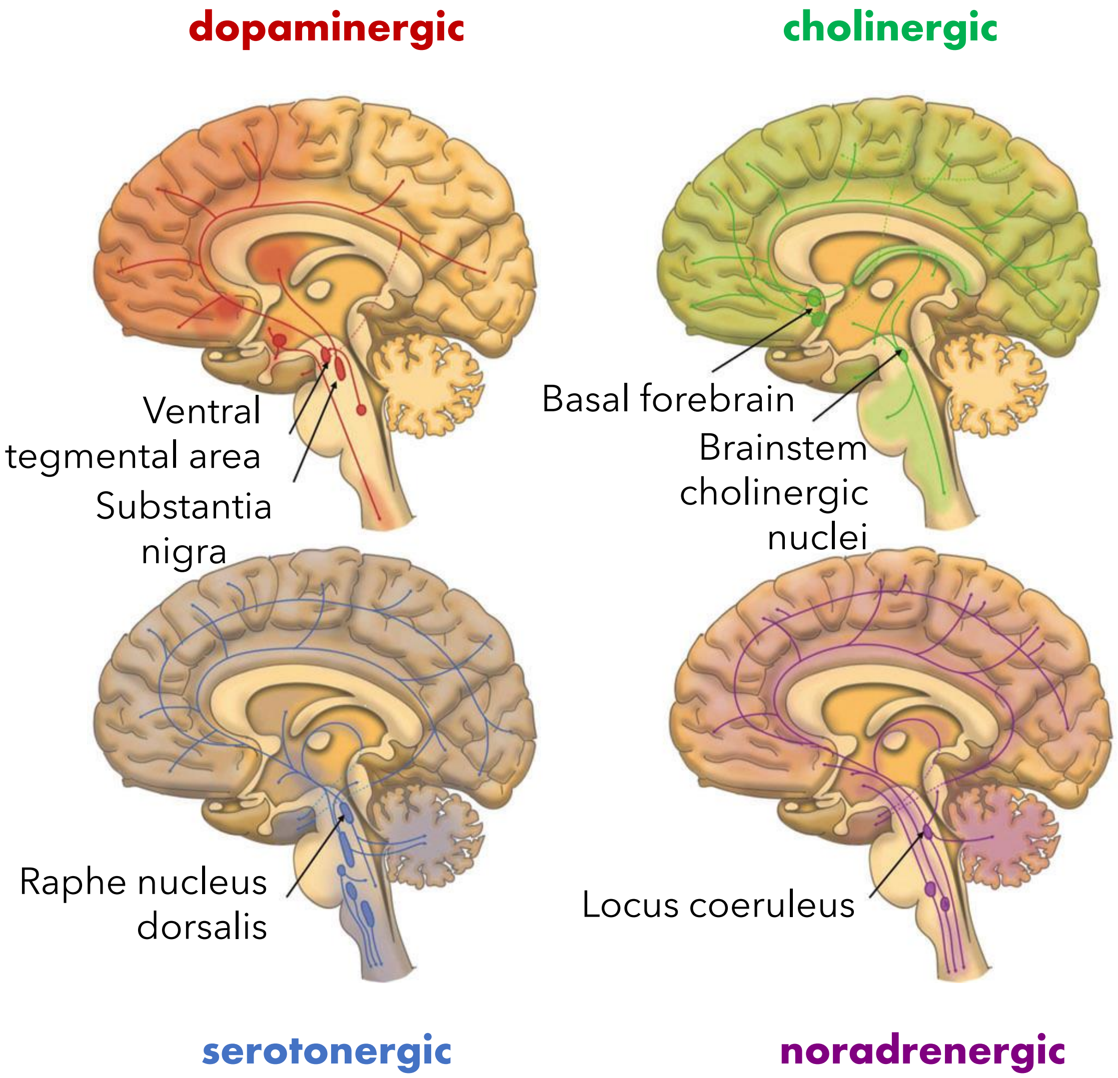

Adapted from **Iglesias et al.** 2017 ***WIRE Cogn Sci***

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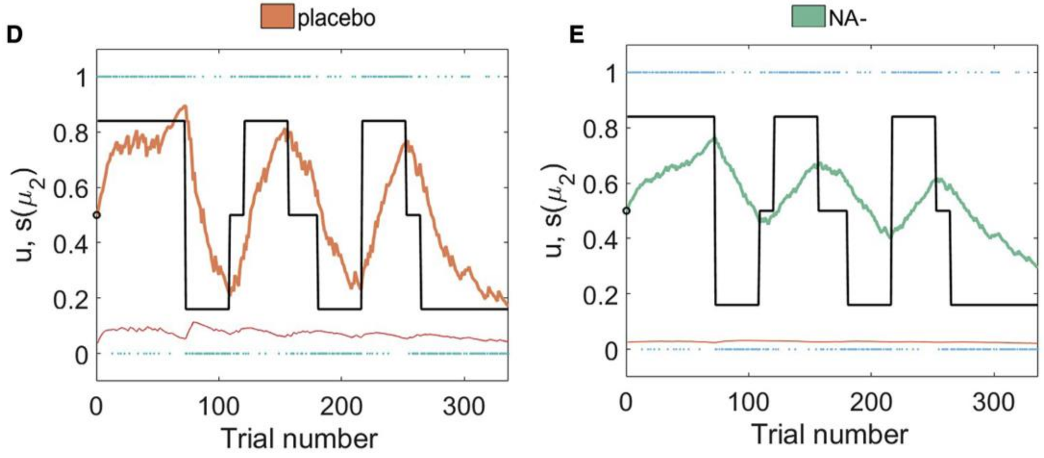
Dopamine
Fiorillo et al. 2003 *Science*
Tobler et al. 2005 *Science*

see also:
Fiorillo et al. 2005 *Behav Brain Func*
Bunzeck et al. 2010 *Hum. Brain. Map.*

Diederen et al. 2016 *Neuron*
Diederen et al. 2017 *J Neurosci*

Noradrenalin



Lawson et al. 2021 *Curr Biol*

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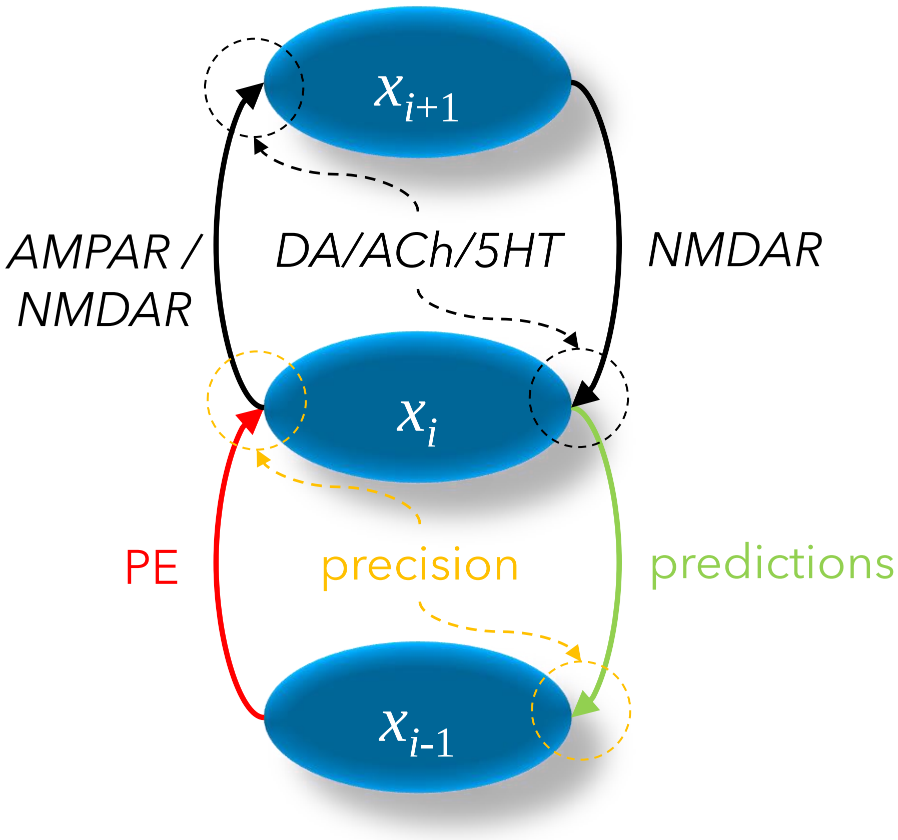

Stephan et al. 2016 *Brain*

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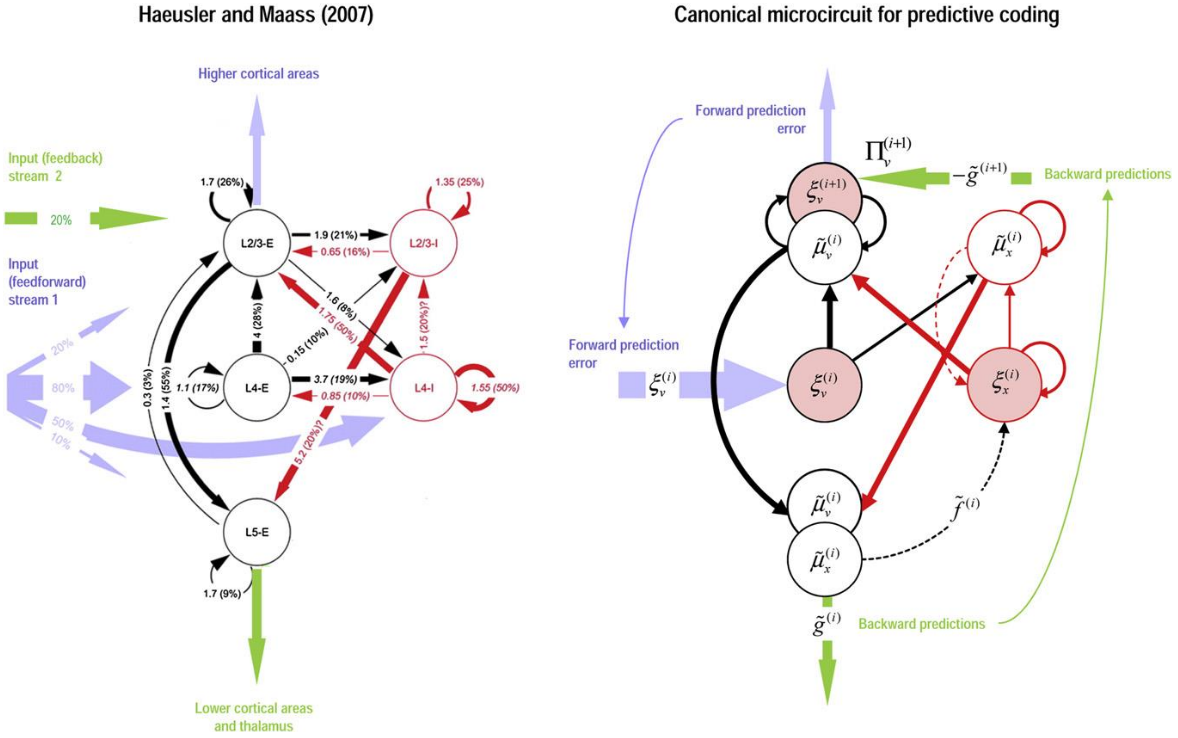
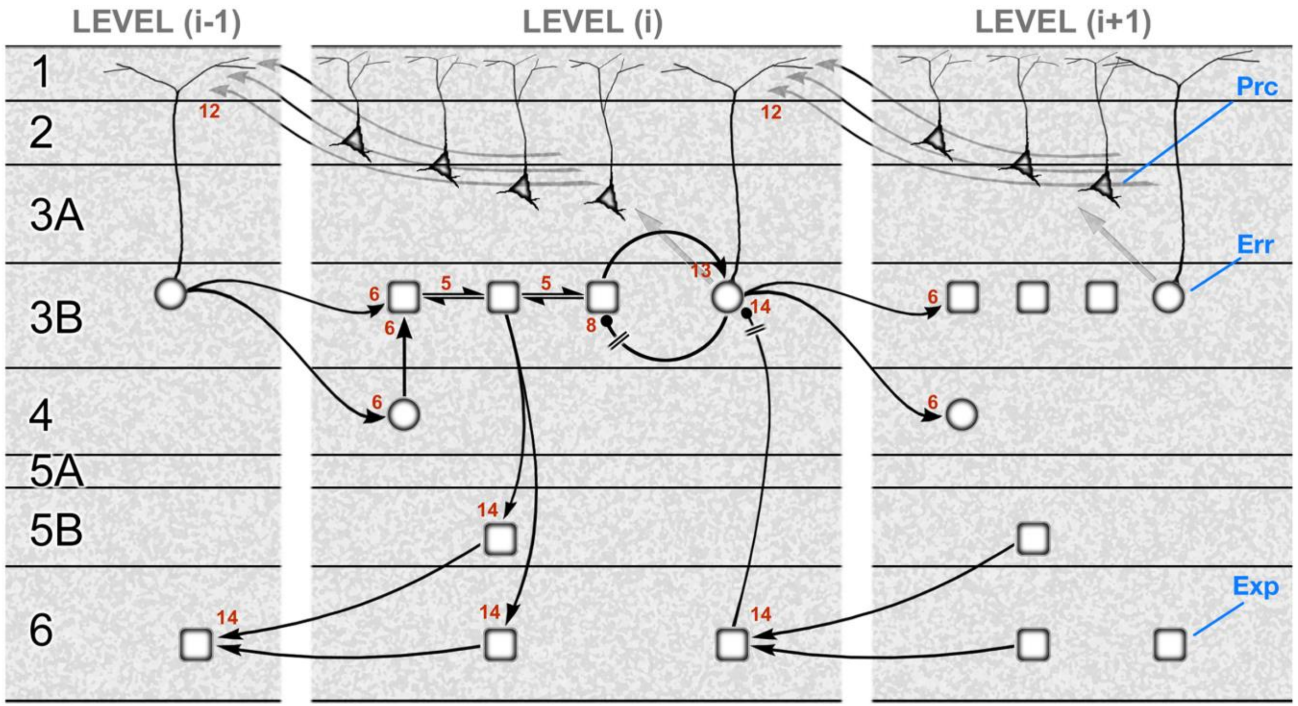
Mapping predictive coding onto the brain … *advanced*



Bastos et al. 2012 ***Neuron***



Shipp 2016 ***Frontiers in Psychology***

… and testing it: **Walsh et al.** 2020 ***Ann N Y Acad Sci***

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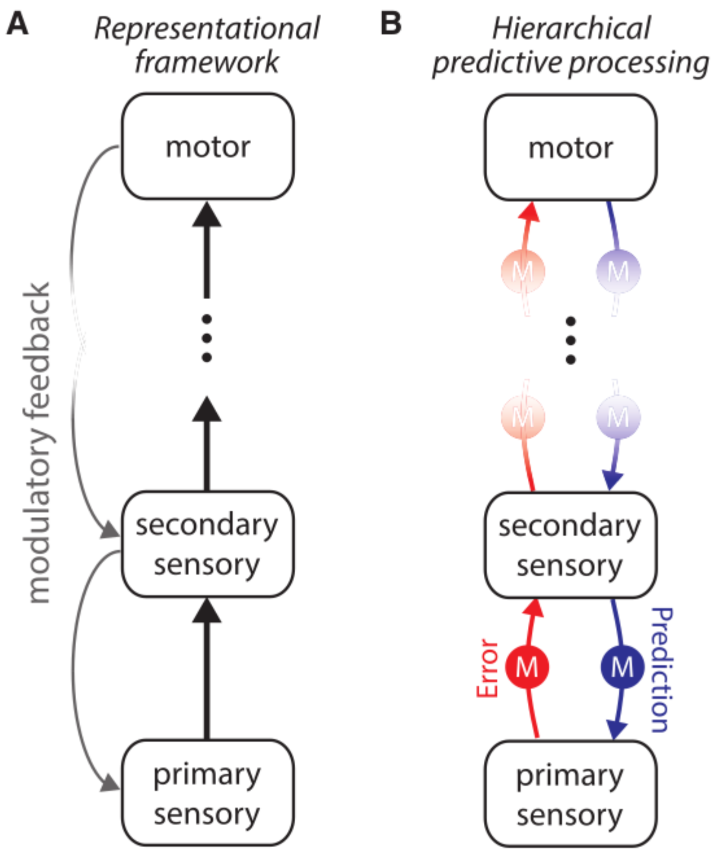
Does the brain use PC?

- locomotion is sufficient to drive activity in mouse V1, even in the complete absence of visual input (**Keller et al.**, 2012, **Saleem et al.**, 2013)
- layer 2/3 of sensory cortices signals a mismatch between predicted and actual sensory feedback (**Eliades and Wang**, 2008, **Keller et al.**, 2012)
- this results from a comparison of an excitatory motor-related input and an inhibitory visual input (**Attinger et al.**, 2017, **Zmarz and Keller**, 2016)
- A24b/M2 provides a strong and dense projection to V1, which conveys motor-related signals that depend on the mouse's visuomotor experience. This projection fulfills all the criteria necessary to be interpreted as a prediction of visual flow given a motor output (**Leinweber et al.**, 2017)



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Keller & Mrsic-Flogel 2018 ***Neuron***

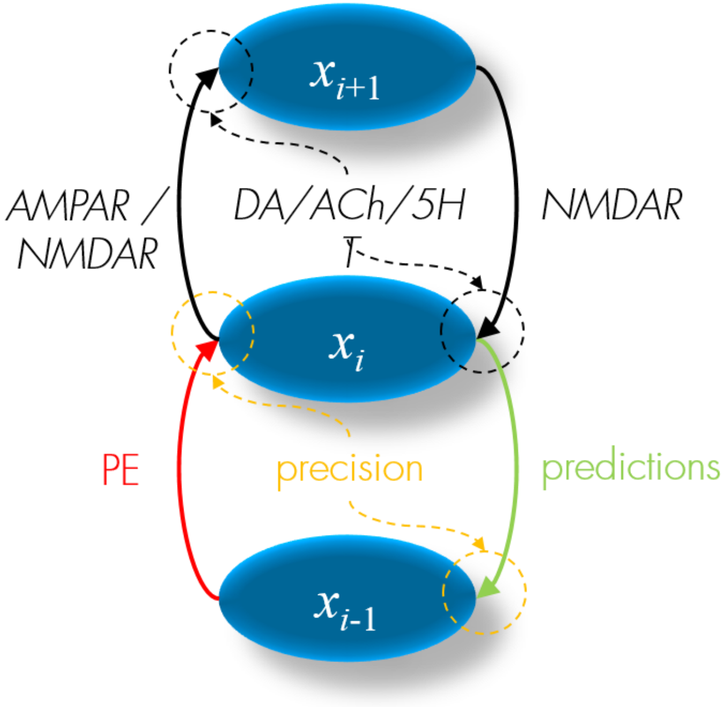
Hierarchical Bayesian Inference in Computational Psychiatry

Possible primary disruption at:

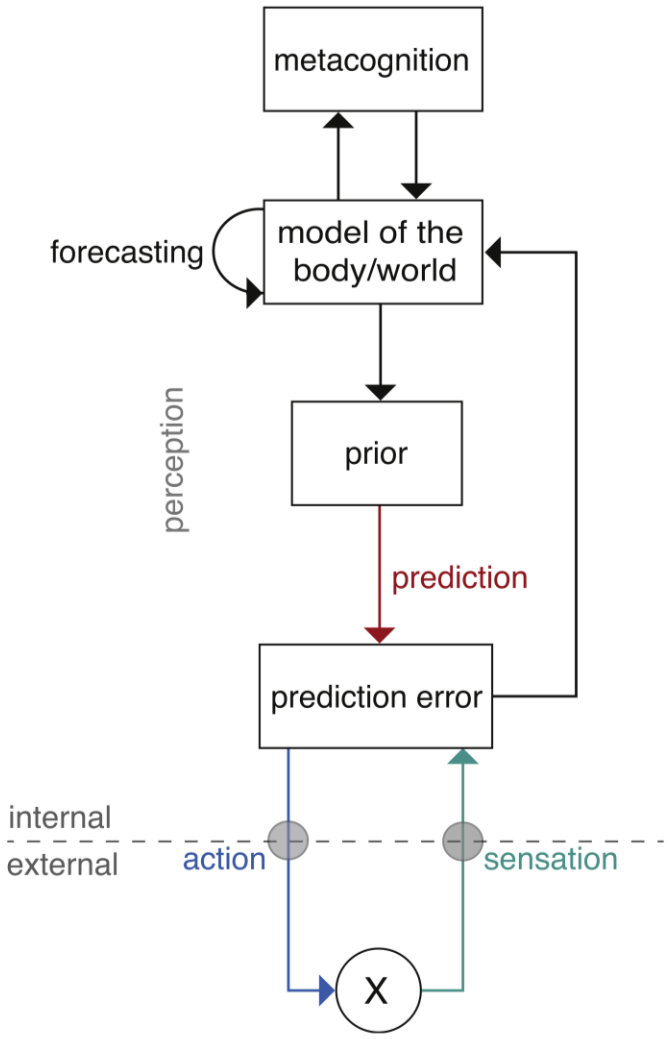
1. Sensory inputs (sensations)
2. Inference (perception)
3. Forecasting
4. Control (action)
5. Metacognition

At any of these, possible disturbance of:

- Prediction error (PE) computation
- Predictions/Expectations
- Estimation of their precision.



Stephan et al. 2016 ***Brain***



Petzschner et al. 2017 ***Biol Psychiatry***

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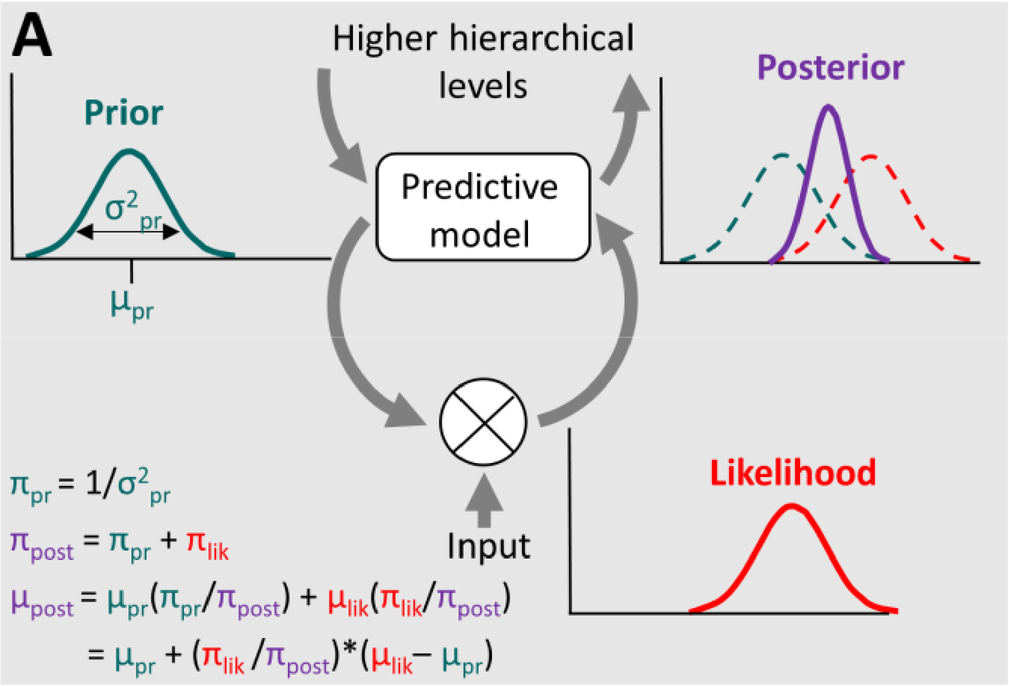
Precision: The right balance

Schizophrenia/Psychosis

Sterzer et al. 2018 *Biol Psychiatry*; Adams et al. 2013 *Front Psychiatry*; Corlett et al. 2011 *NPP*; Stephan et al. 2006 *Biol Psychiatry*; 2009; Powers et al. 2017 *Science*

Autism spectrum disorder

Pellicano & Burr 2012 *TiCS*; Van de Cruys et al. 2014 *Psychol Rev*; Lawson et al. 2014 *Front Hum Neurosci*; Lawson et al. 2017 *Nat Neurosci*



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Sterzer et al. 2018 *Biol Psychiatry*

Interoceptive predictive coding

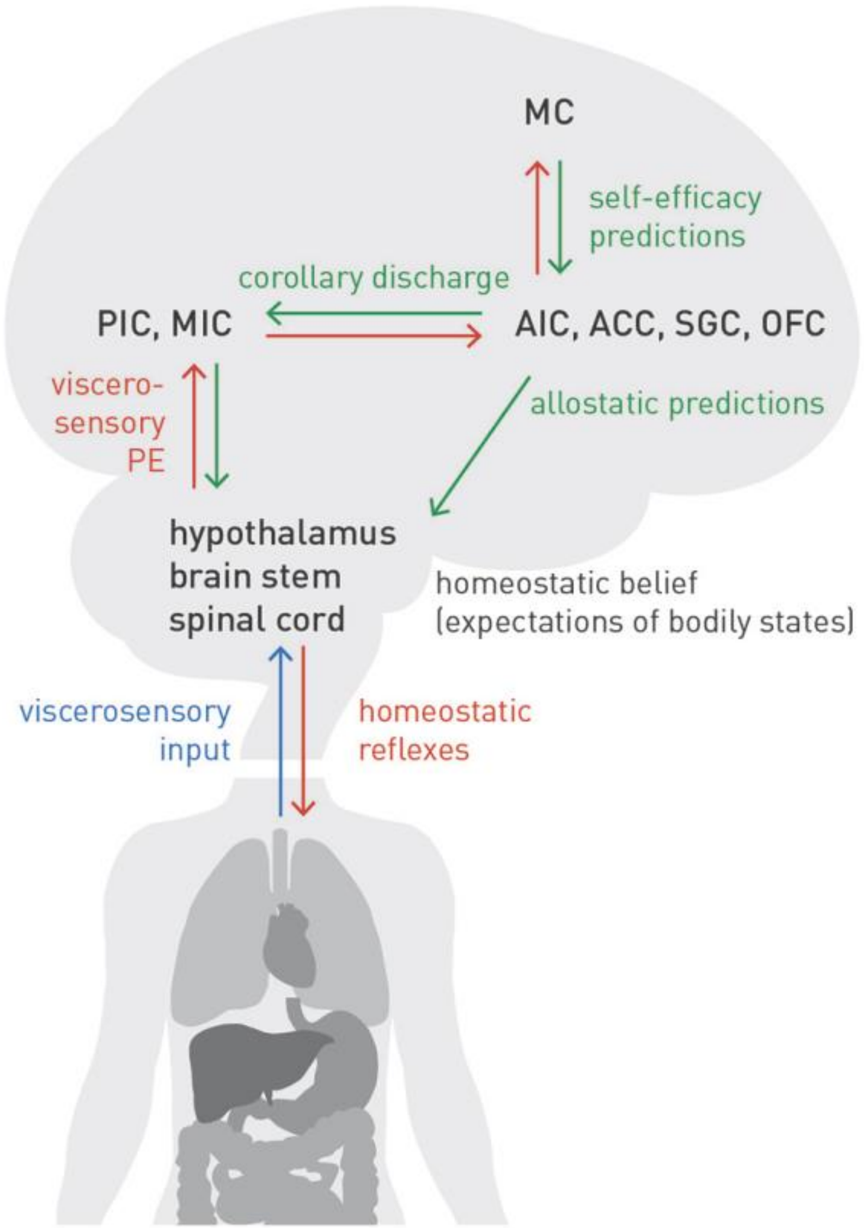
Seth 2013 *TiCS*

Barrett & Simmons 2015 *Nature Rev Neurosci*

Seth & Friston 2016 *Phil Trans Royal Soc B*

Stephan et al. 2016 *Front Hum Neurosci*

Petzschner et al. 2017 *Biol Psychiatry*



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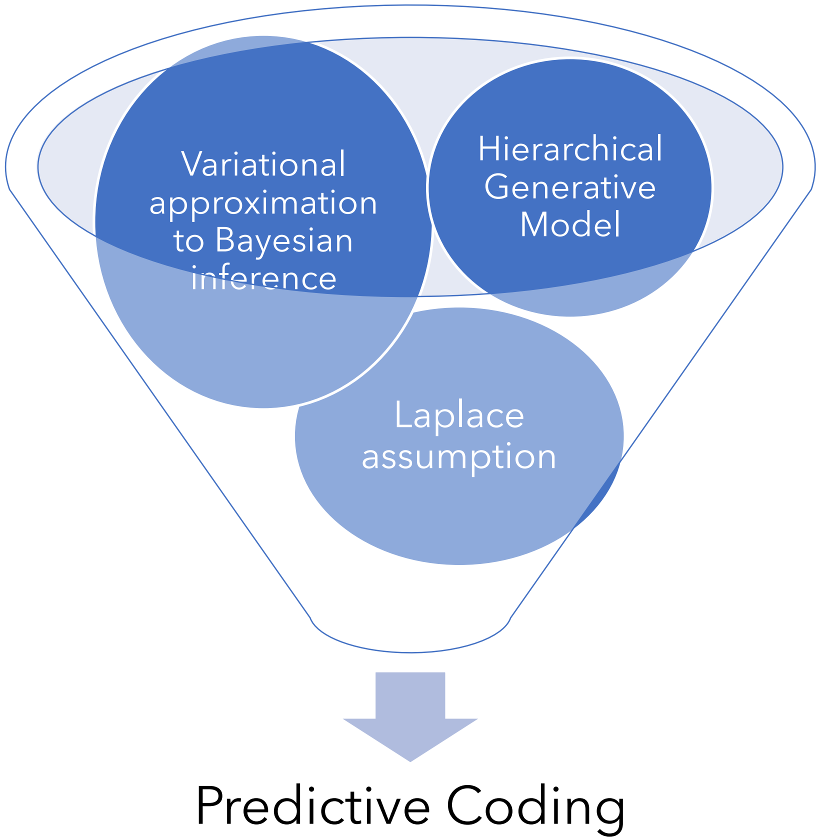
Building a model with PC

1. **Predict the input**
   - **generative model of the sensory inputs**
2. **Invert the model**
   - **gradient descent on the neg. log joint**
   - **or: be fully Bayesian and use F**
3. **Simulate!**
4. **Fitting: Be the scientist**
   1. **Specify a mapping to observable data**
   2. **Write down the likelihood, specify your priors**
   3. **Acquire data & invert your model**



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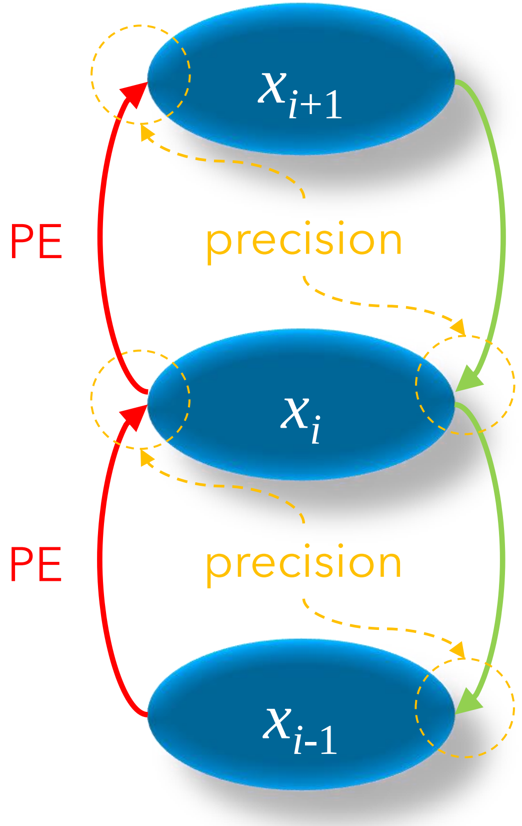
Thank you

Translational Neuromodeling Unit

Rick Adams

Rafal Bogacz



Δ**belief** ~
precision × **PE**

Resources

- *all things PC (very comprehensive):*
  - Millidge, Seth, Buckley 2021 *arXiv:2107.12979*
- *evidence for PC in the brain:*
  - Walsh et al. 2020 *Ann N Y Acad Sci*
  - Keller & Mrsic-Flogel 2018 *Neuron*
  - Heilbron & Chait 2018 *Neurosci*
- *tutorials on the free energy formulation of PC*
  - Bogacz 2017 *J Math Psychol*
  - Buckley et al. 2017 *J Math Psychol*
- *further thoughts on precision*
  - Yon & Frith 2021 *Curr Biol*
- *Recent PC perspectives on delusions and hallucinations*
  - Corlett & Fletcher 2021 *Cogn Neuropsych* (delusions)
  - Corlett et al. 2019 *TiCS* (hallucinations)