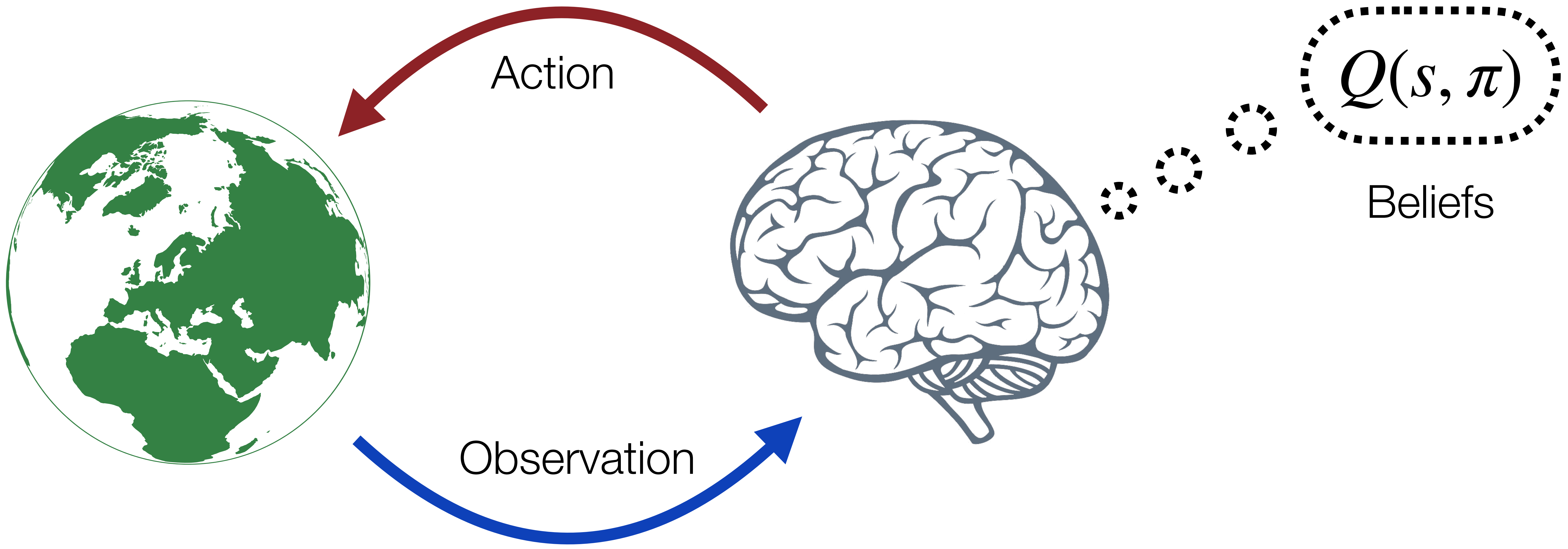
Tutorial B: Part II

Hands-on Active Inference with **pymdp**



Conor Heins and Daphne Demekas

Tutorial B: Part II

When specifying a POMDP model for performing active inference, it's often useful to *factorize* the state-space of observations and hidden states

$$\mathbf{o}_t = \{o_t^1, o_t^2, \ldots, o_t^M\} \qquad \mathbf{s}_t = \{s_t^1, s_t^2, \ldots, s_t^F\}$$

Modalities

Factors

When specifying a POMDP model for performing active inference, it's often useful to *factorize* the state-space of observations and hidden states

Several benefits to doing this:

- Computational efficiency (both memory-wise and CPU-wise)
- Generative model interpretability / transparency
- Neuronally-plausible? c.f. factorised (aka 'modular') representations

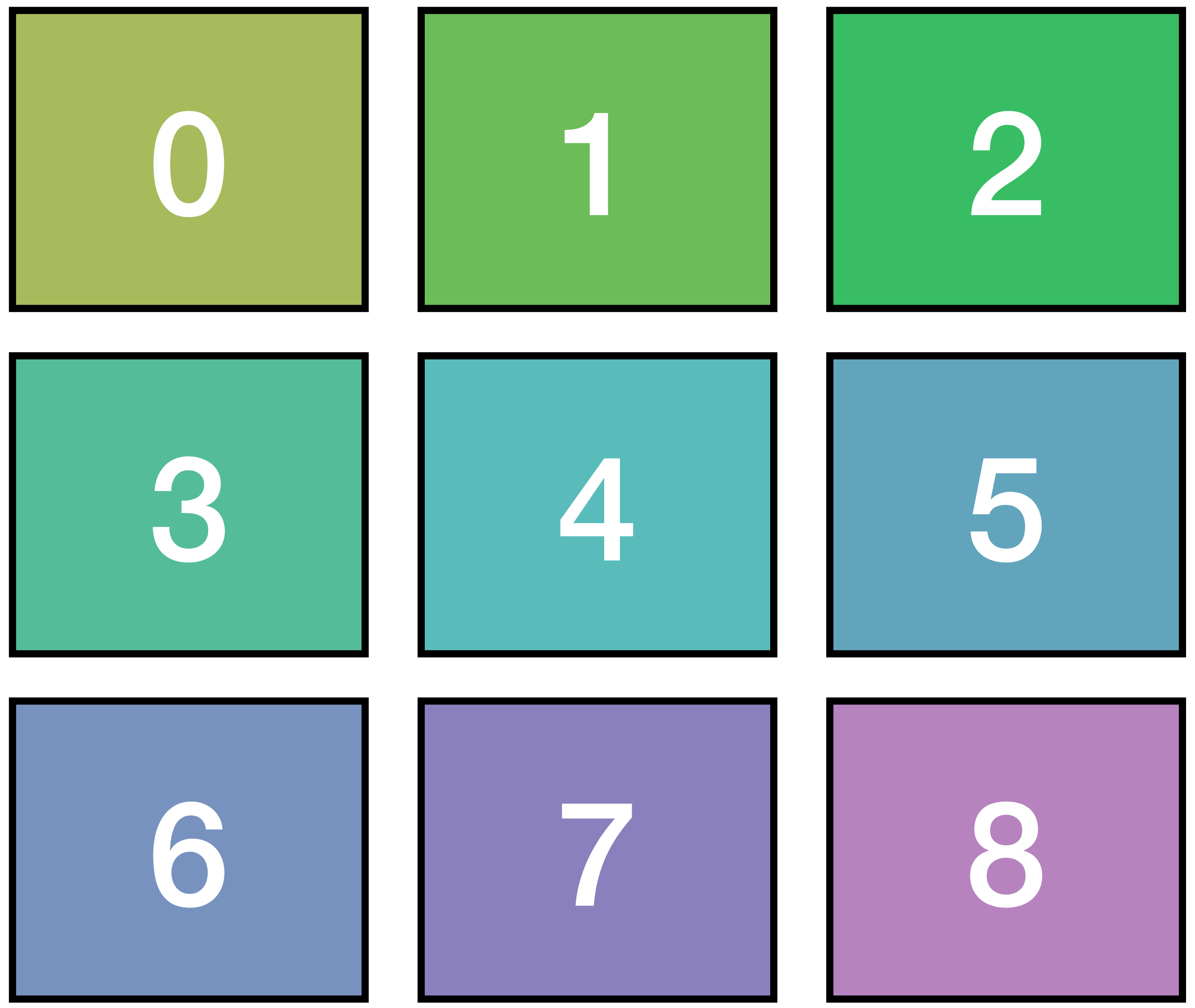
Recall Grid-World

One hidden state factor

$$\mathbf{s}_t = \{s_t^{Loc}\} = \boxed{0}\ \boxed{1}\ \boxed{2}\ \boxed{3}\ \boxed{4}\ \boxed{5}\ \boxed{6}\ \boxed{7}\ \boxed{8}$$

9 levels

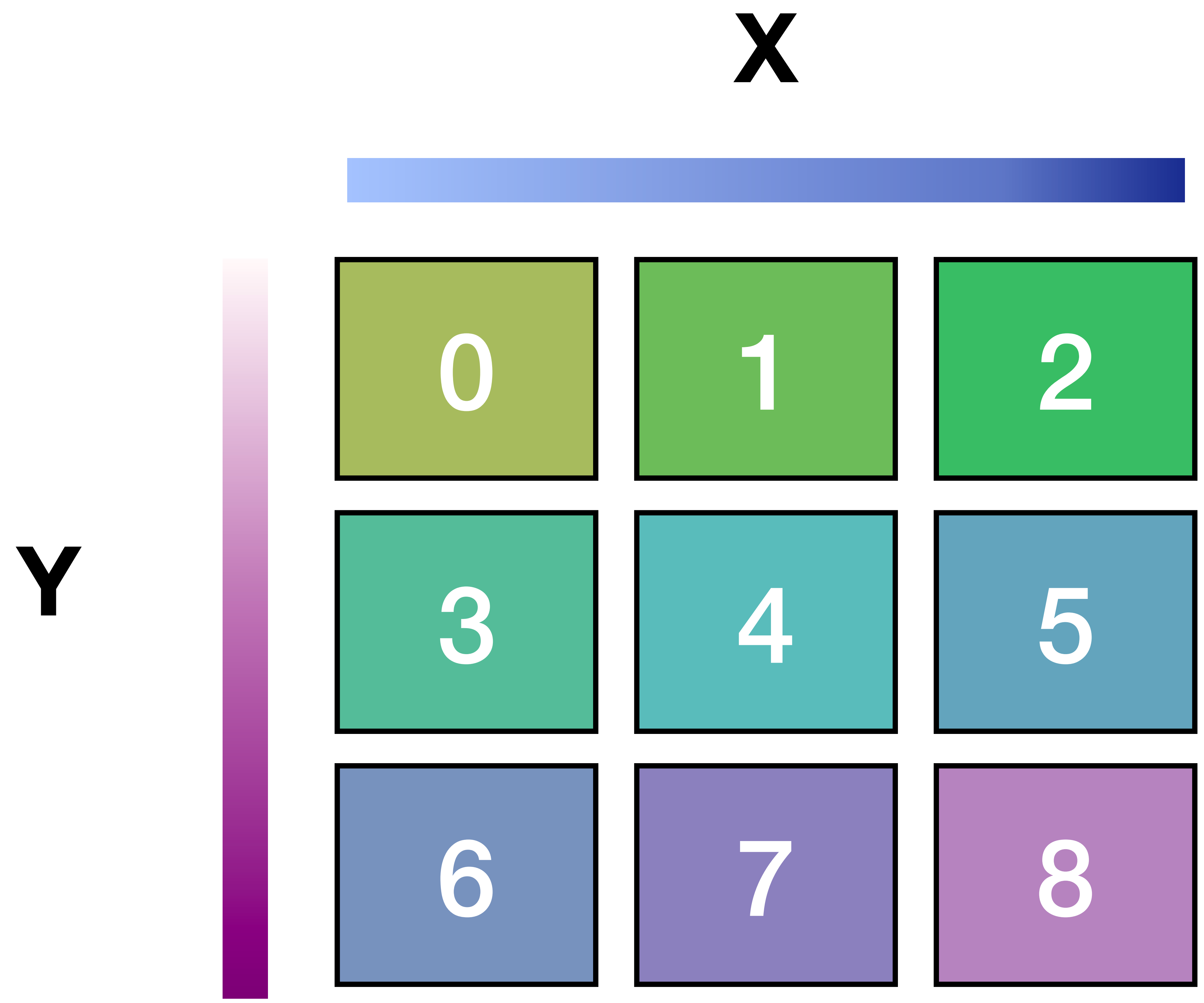
A “fully enumerated” state space



Recall Grid-World

$$\mathbf{s}_t = \{s_t^X, s_t^Y\} = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$$

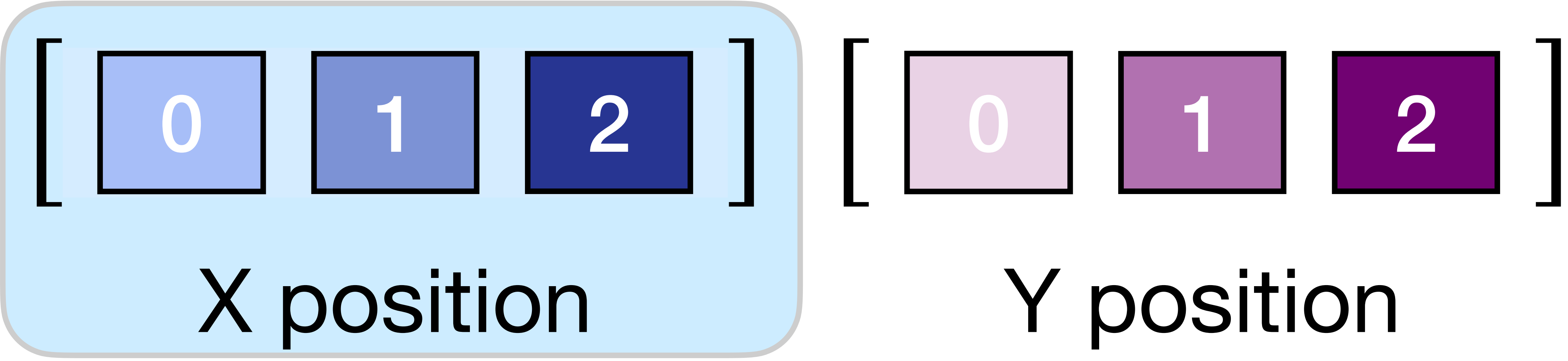
X position Y position



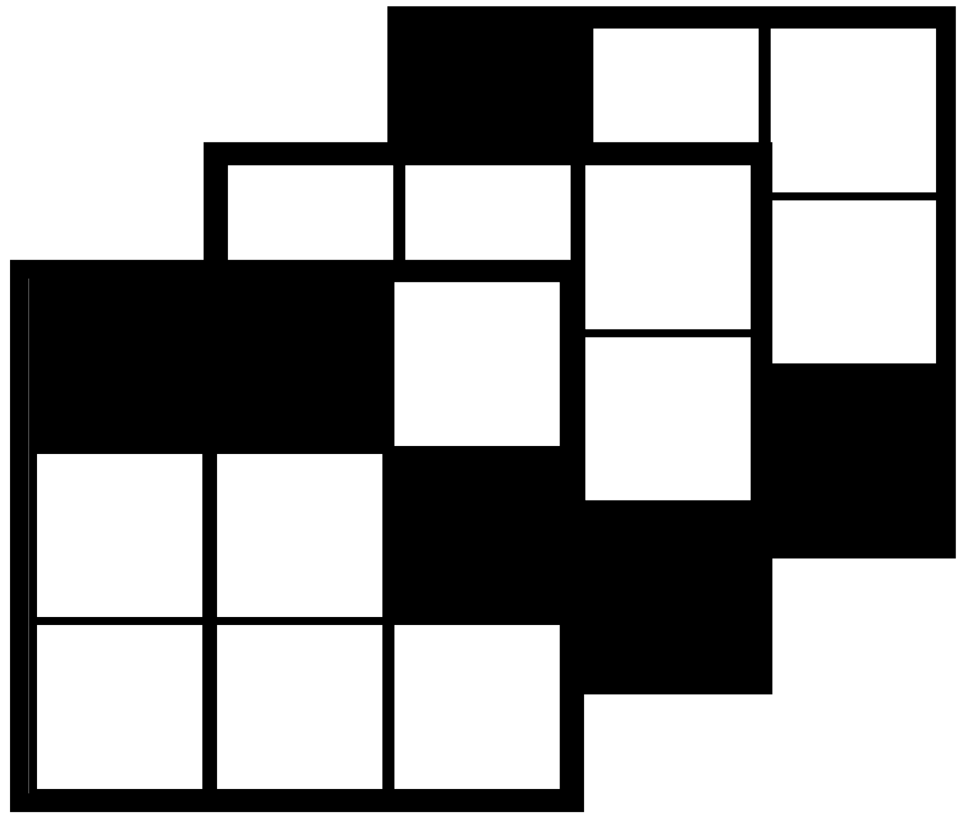
Each hidden state is now a pair of **two** hidden state factors

How does this change the generative model representation?

$$\mathbf{s}_t = \{s_t^X, s_t^Y\} =$$



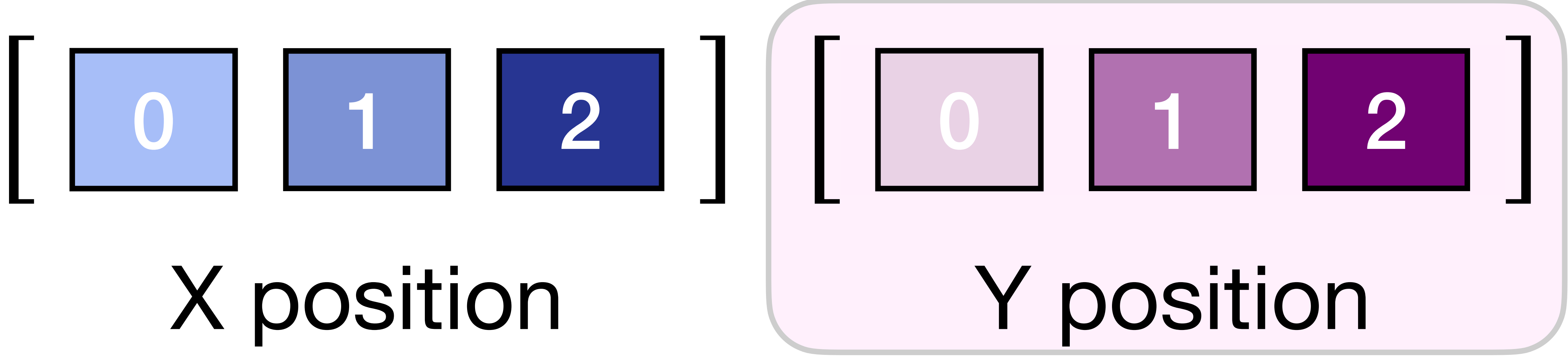
$$P(s_t^X \mid s_{t-1}^X, u_{t-1}^X)$$



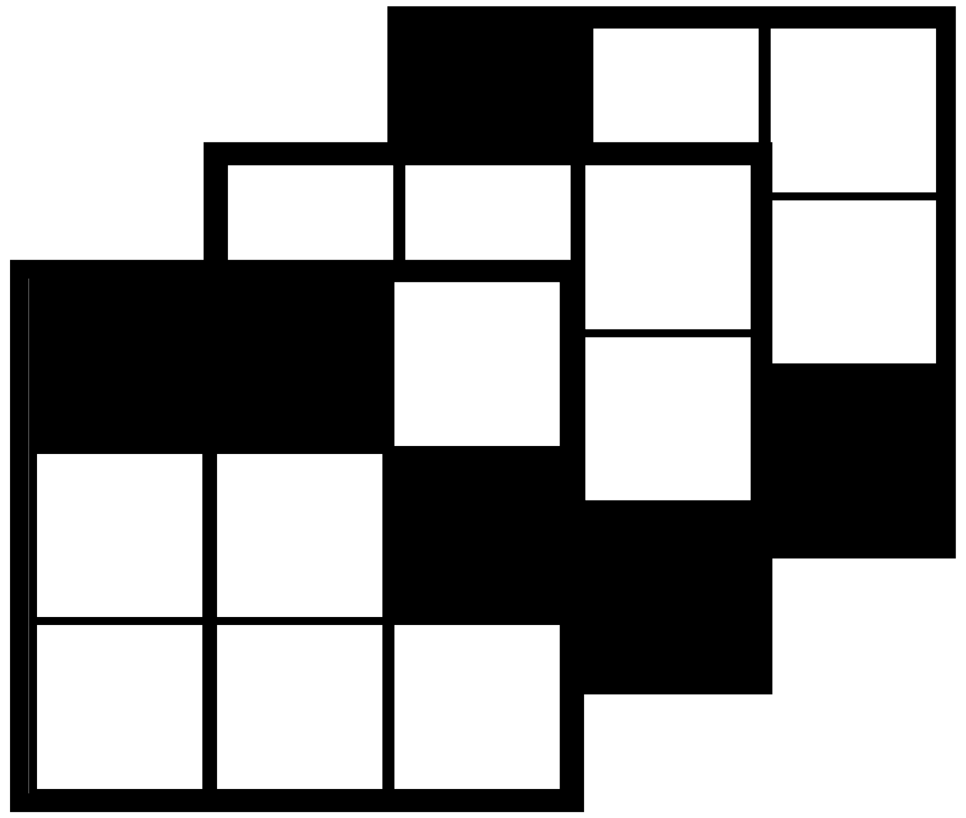
B = B[0]

How does this change the generative model representation?

$$\mathbf{s}_t = \{s_t^X, s_t^Y\} =$$



$$P(s_t^Y \mid s_{t-1}^Y, u_{t-1}^Y)$$



MOVE DOWN STAY

B = B[1]

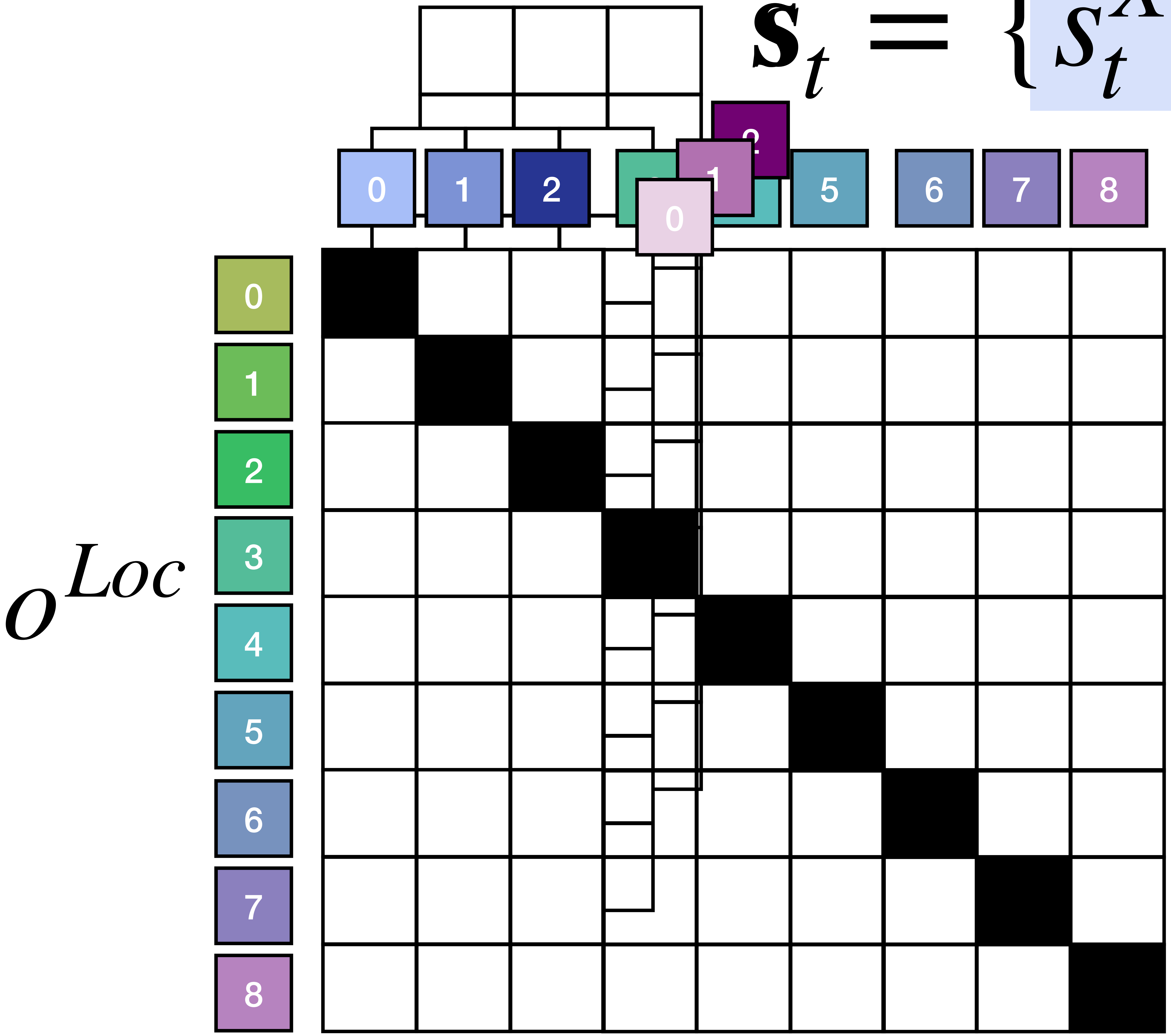
How do we represent this in `pymdp`?

(Go to Colab)

Let's start with a single observation modality

$$\mathbf{s}_t = \{s_t^X, s_t^Y\}$$

$$\mathbf{o}_t = \{o_t^{Loc}\}$$



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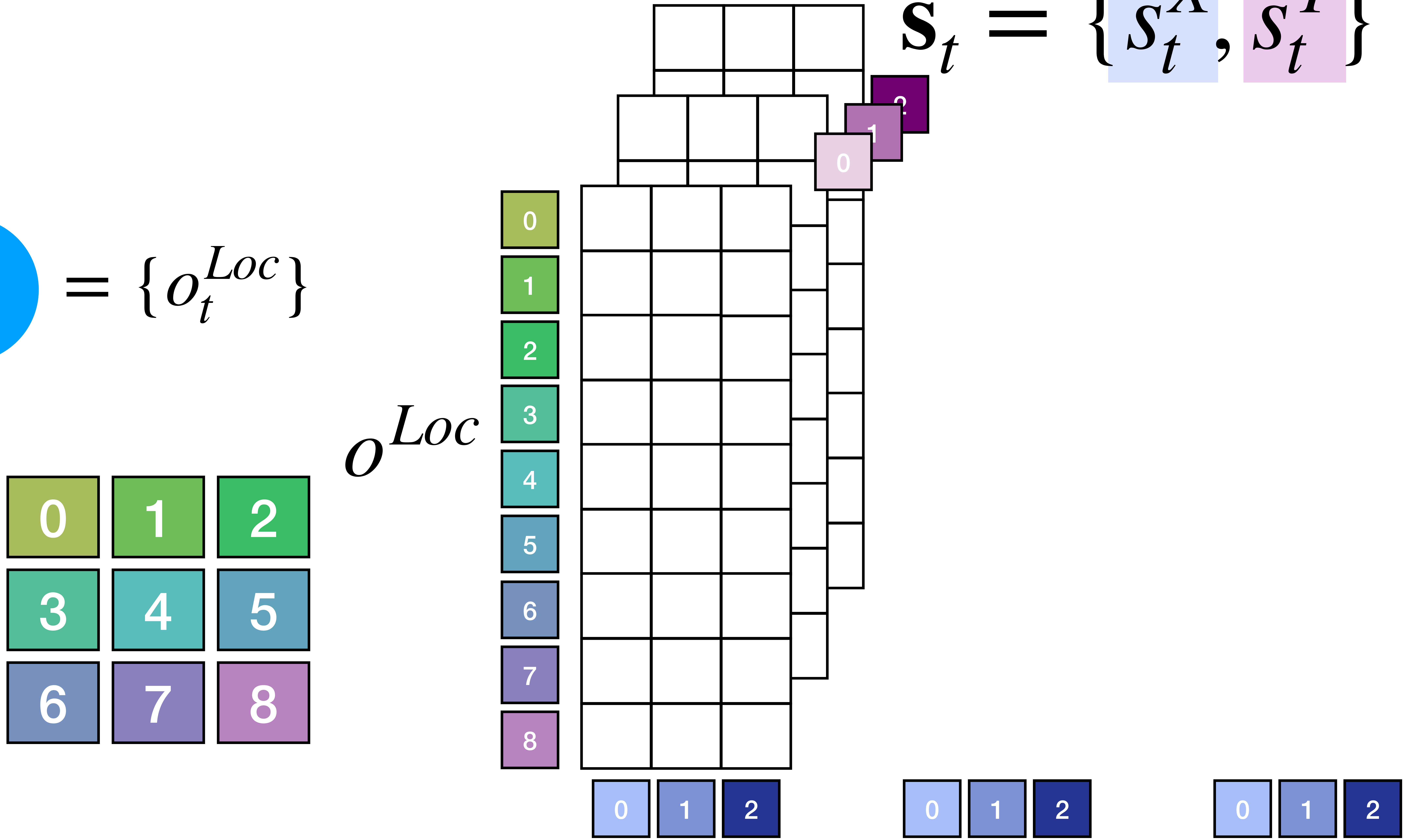
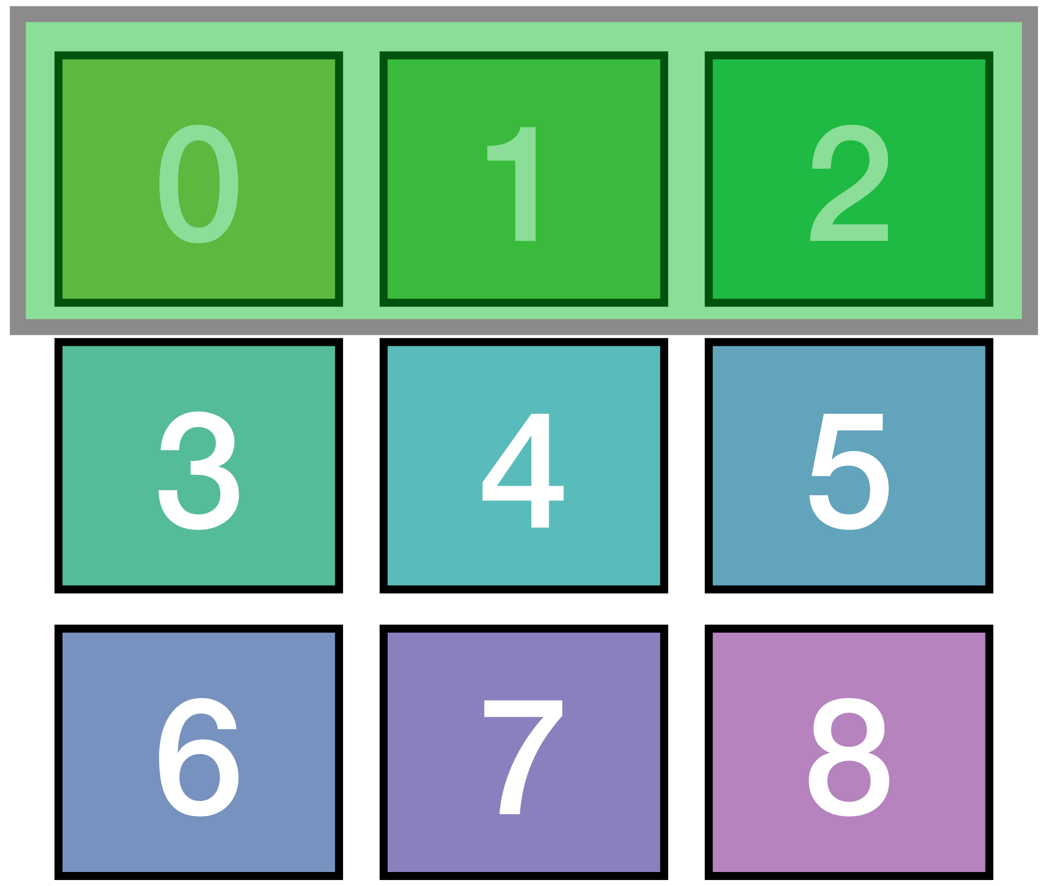


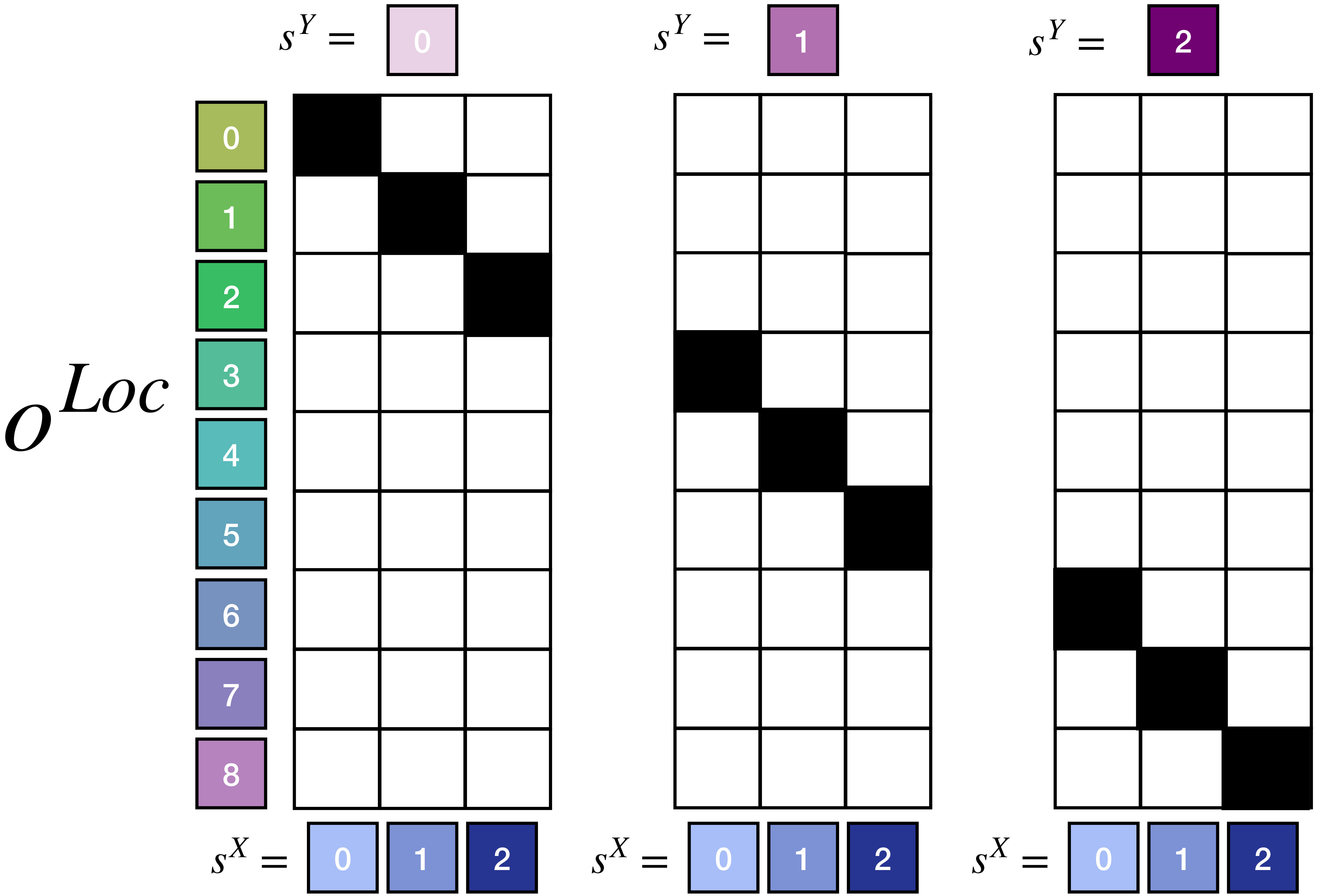
Let’s start with a single observation modality

$$\mathbf{s}_t = \{s_t^X, s_t^Y\}$$

```
A[0][6:9,:,2] = np.eye(3)
```

$$\mathbf{o}_t = \{o_t^{Loc}\}$$





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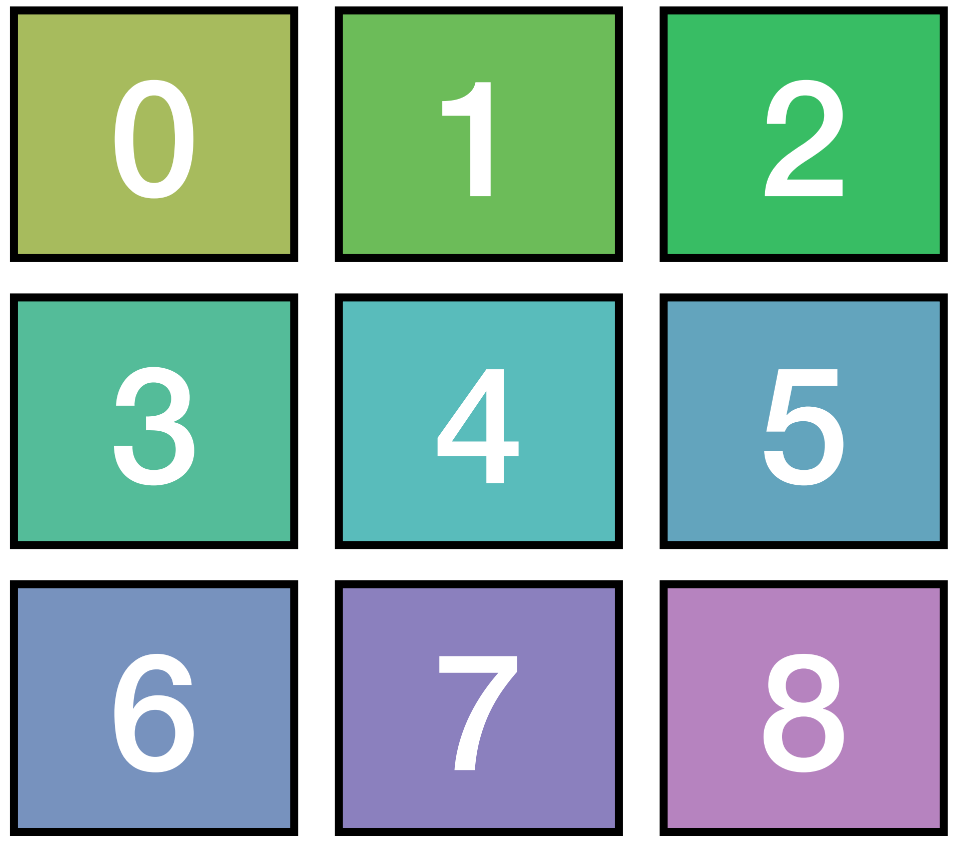
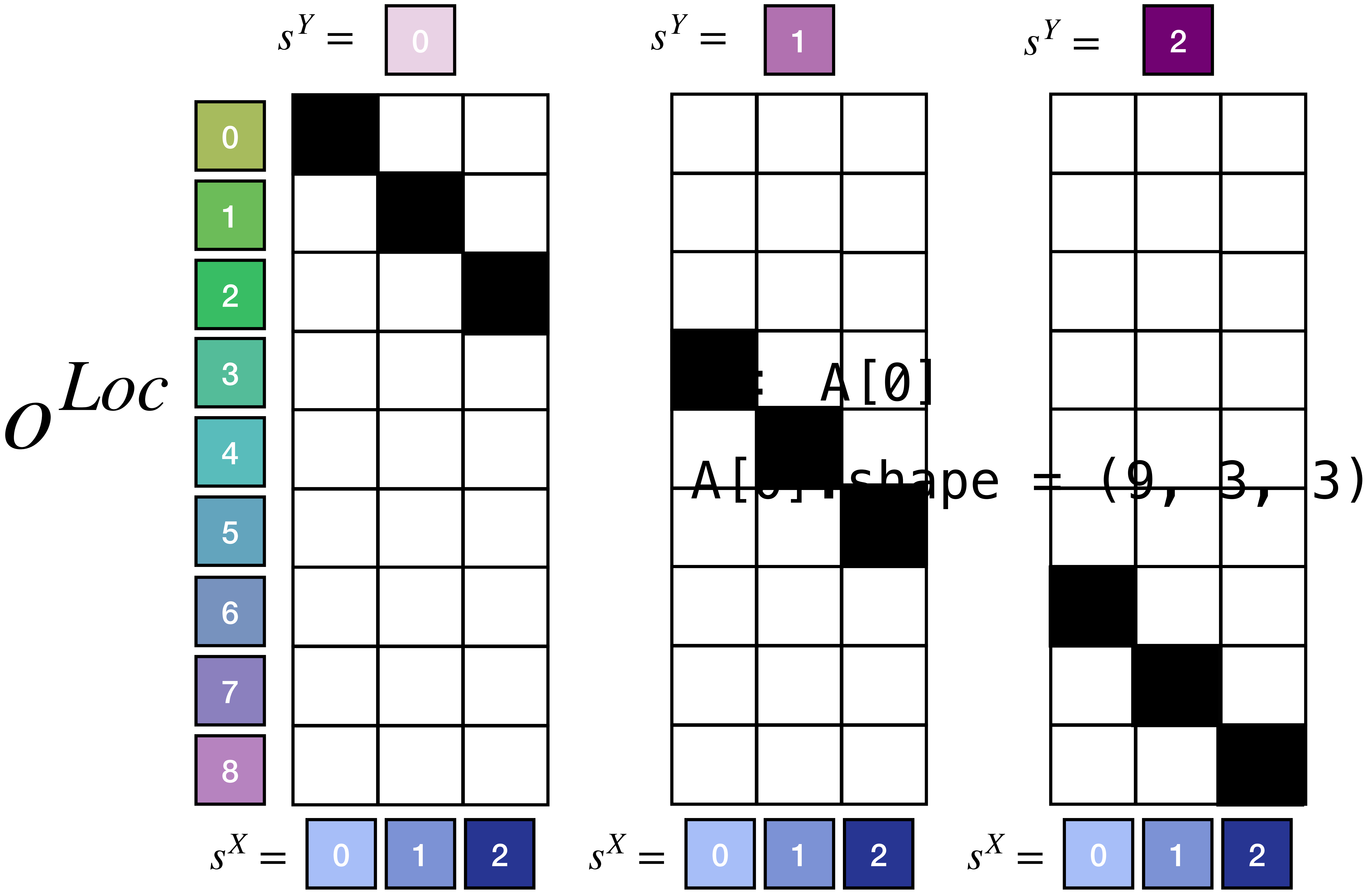
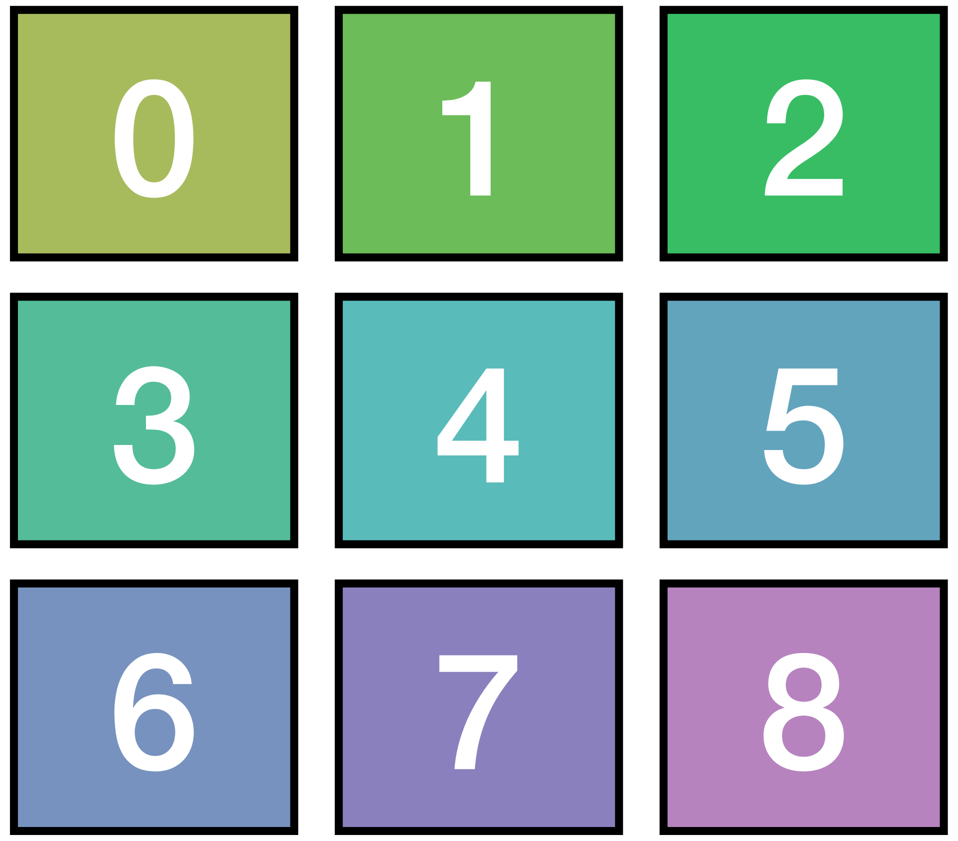




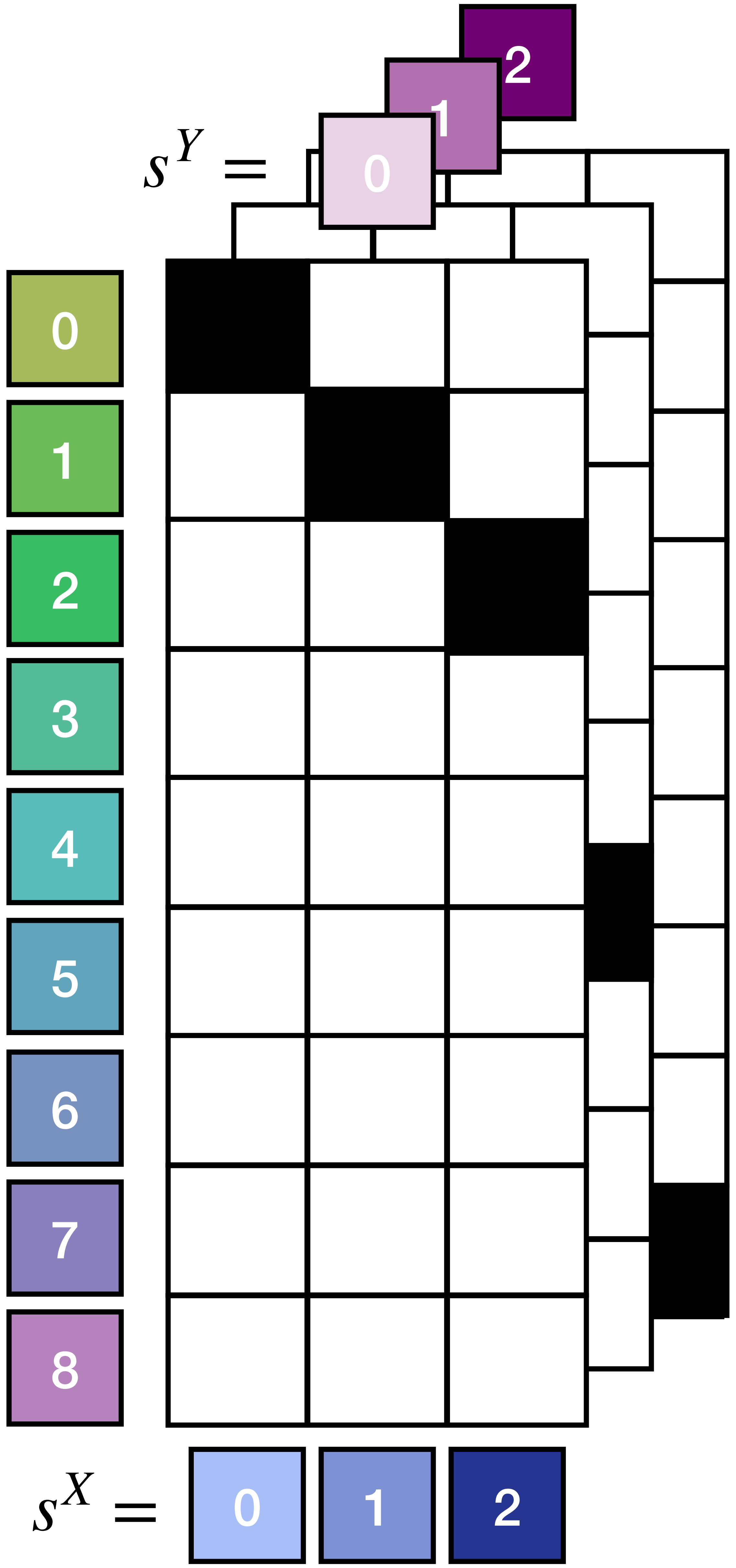
Let's start with a single observation modality

$$\mathbf{s}_t = \{s_t^X, s_t^Y\}$$

$$\mathbf{o}_t = \{o_t^{Loc}\}$$

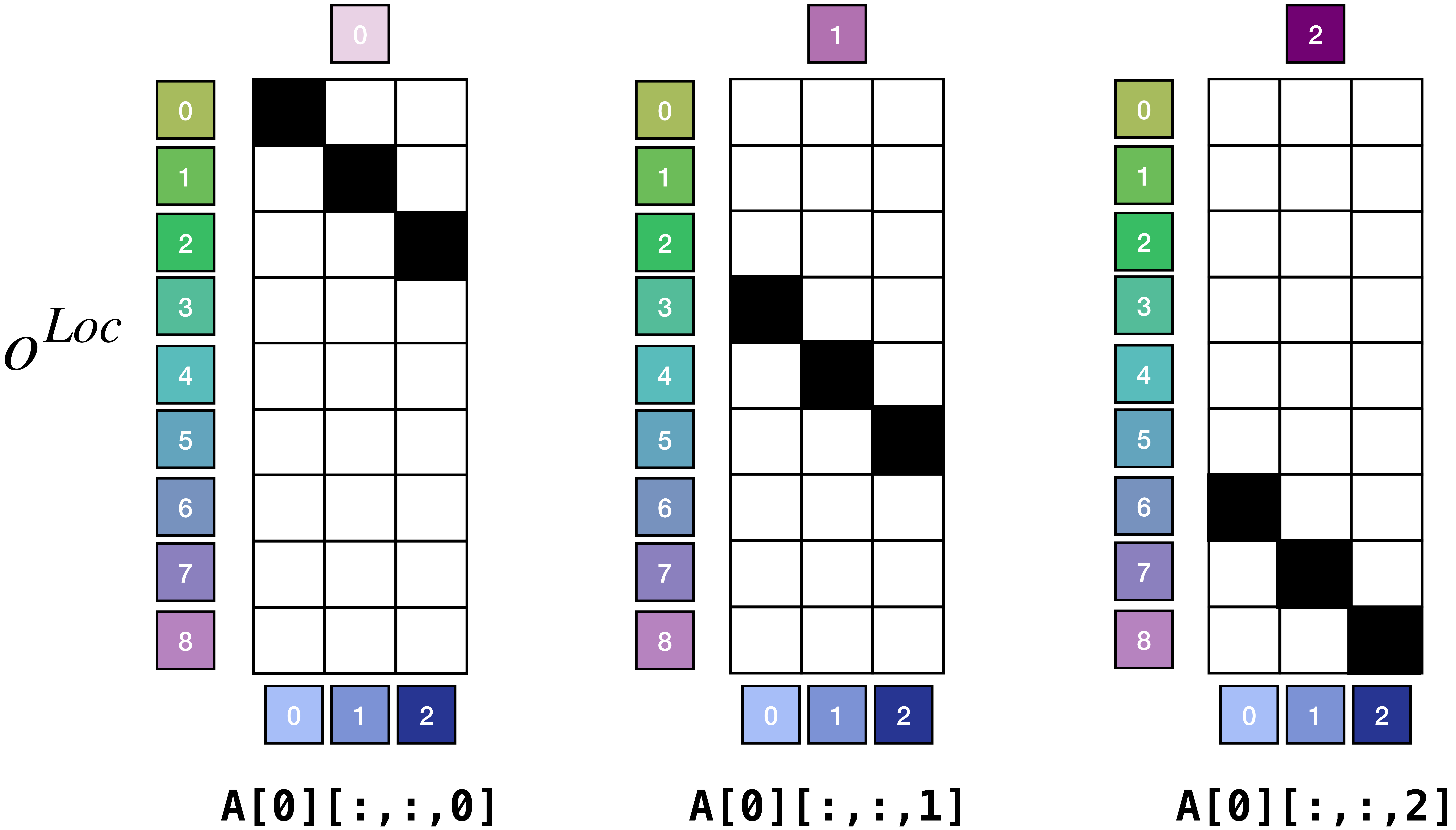


o^{Loc}



`A[0][:,i,j] =`

$$P(o^{Loc} \mid s^X = i, s^Y = j)$$



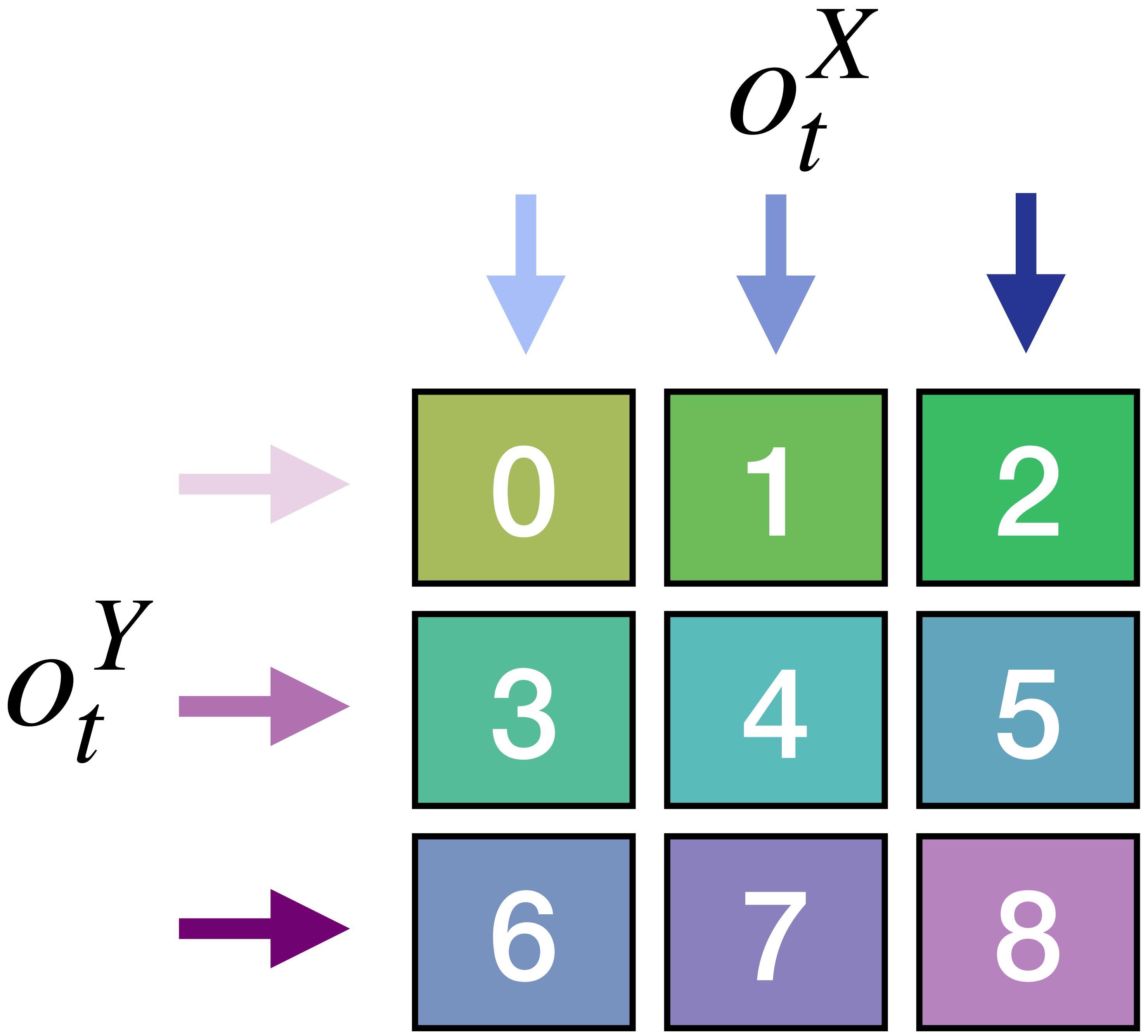
o^{Loc}
0
1
2
A[0][:,:,0]
A[0][:,:,1]
A[0][:,:,2]

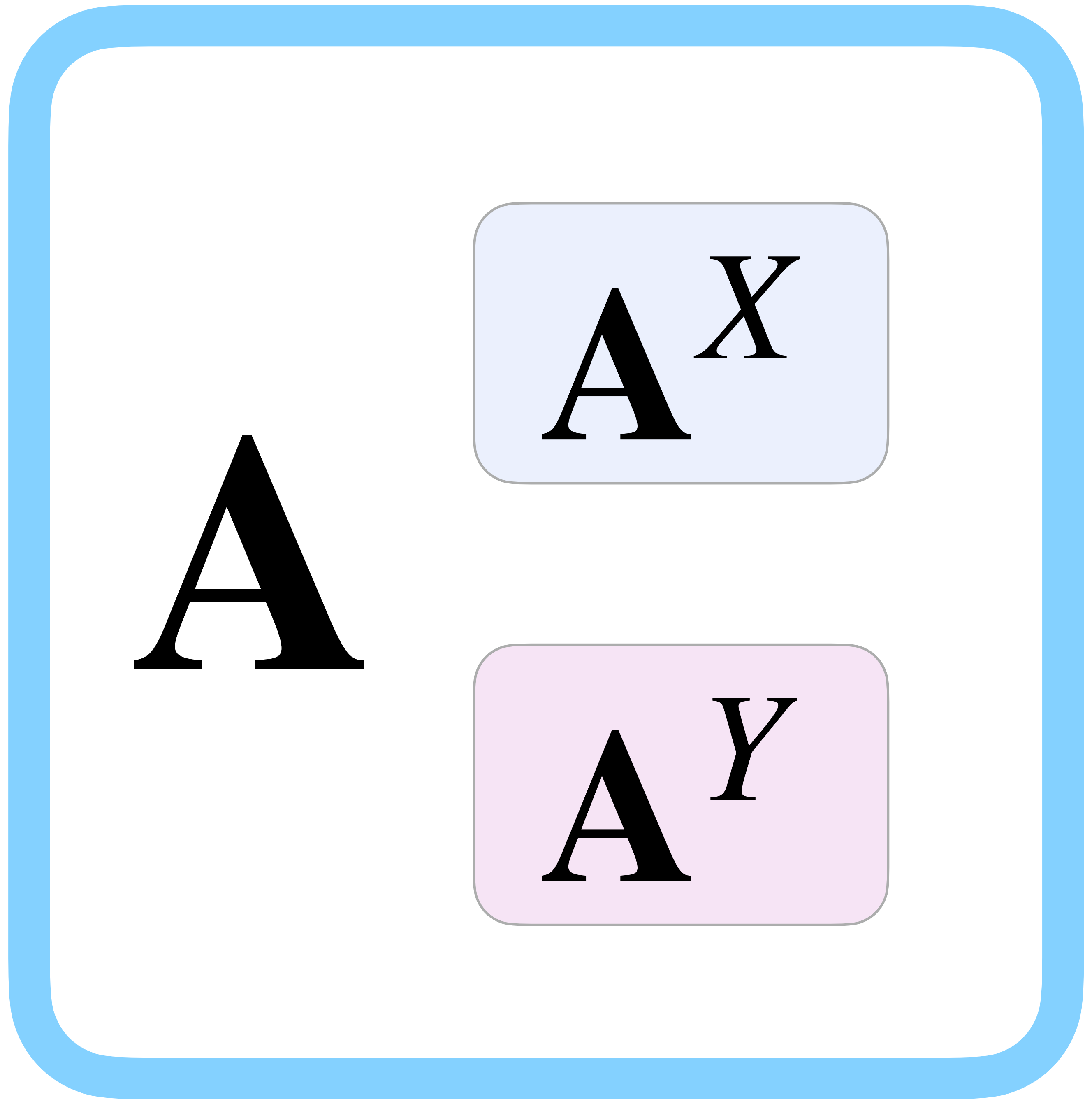
Back to Colab…

Multiple observation modalities

$$\mathbf{o}_t = \{o_t^X, o_t^Y\}$$

```
num_observations = [3, 3]
num_modalities = len(num_observations)
A = obj_array(num_modalities)
A[0] = …
A[1] = …
```





Multiple observation modalities

$$\mathbf{o}_t = \{o_t^X, o_t^Y\}$$

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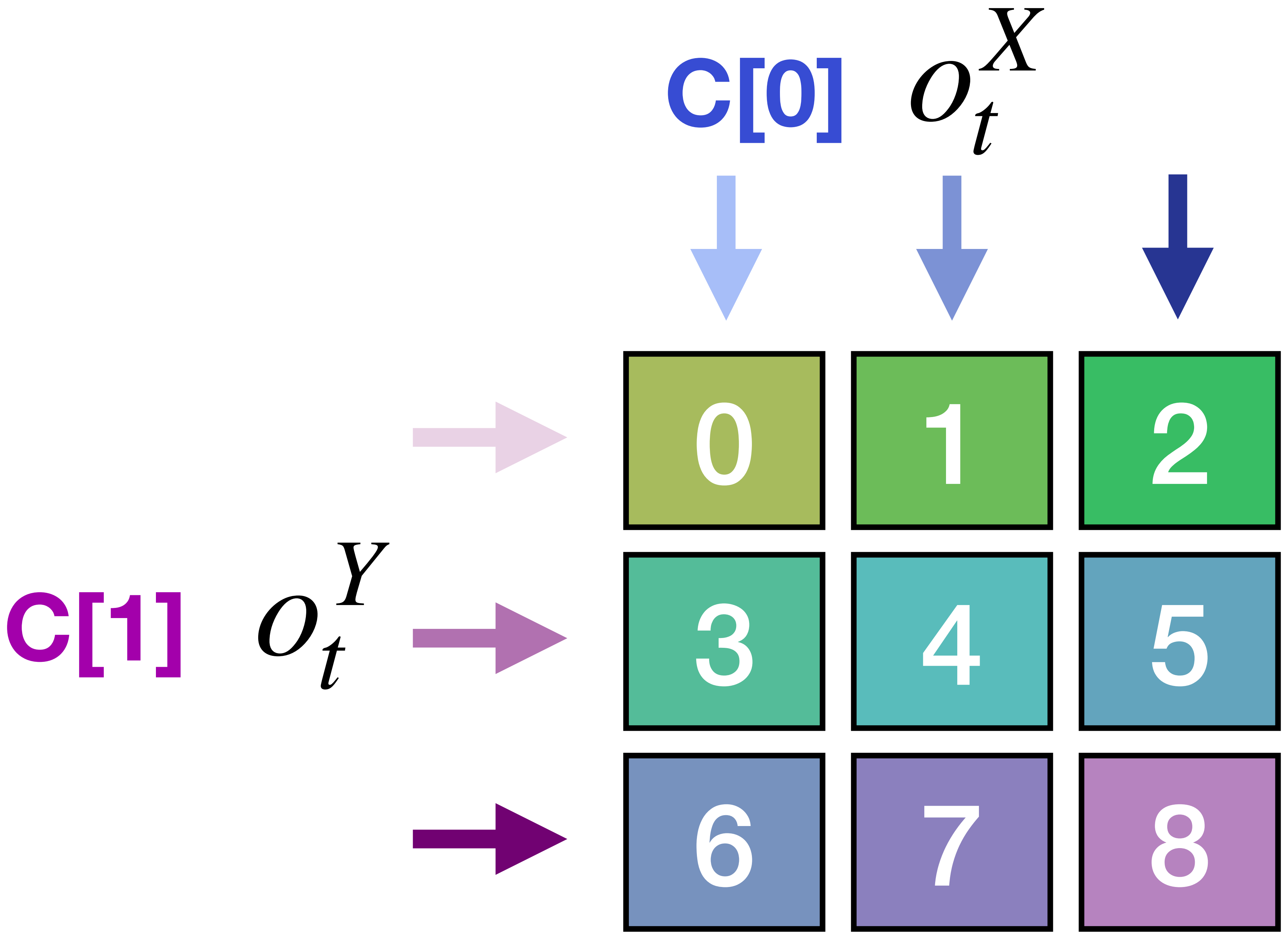
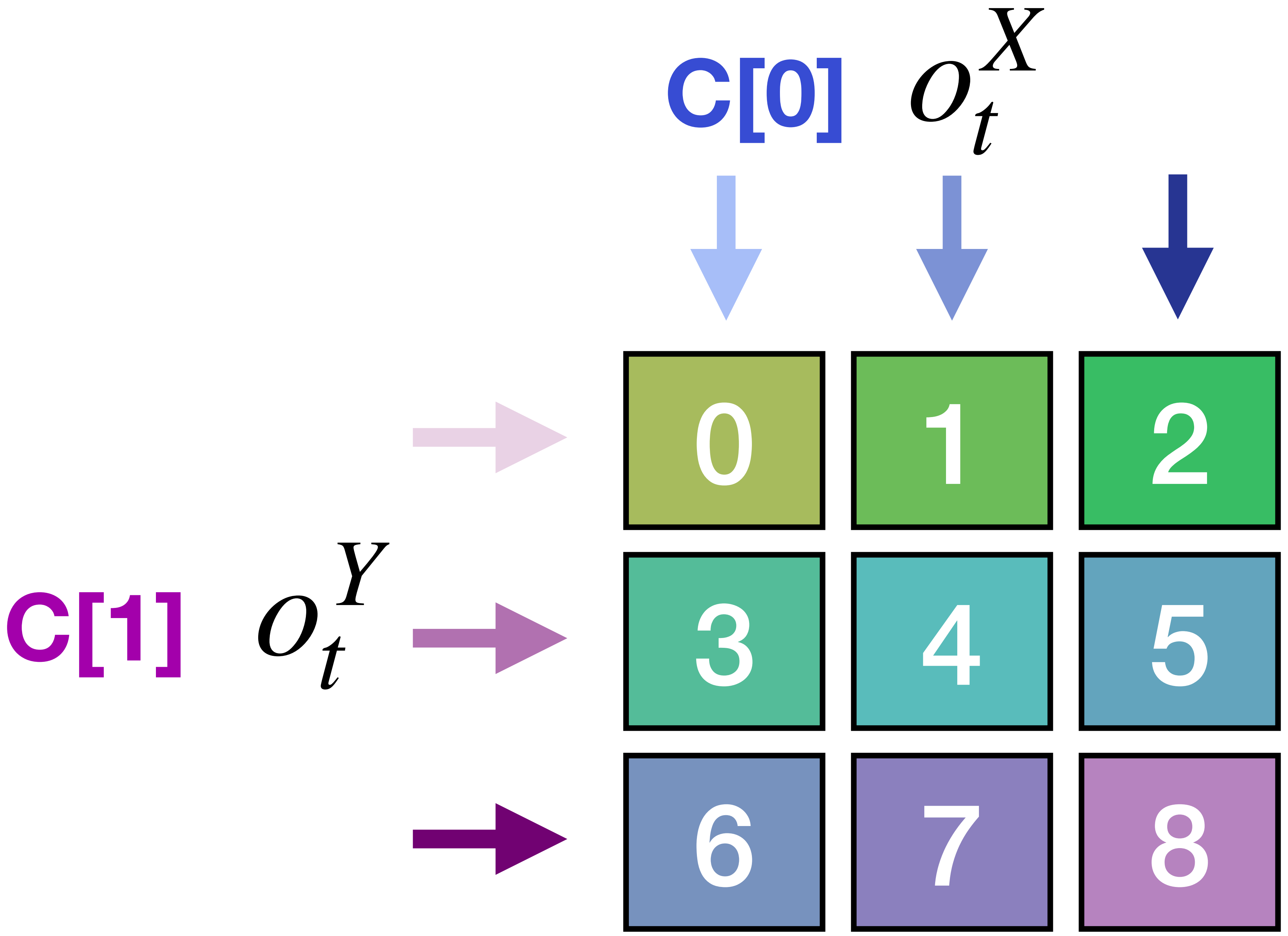
```

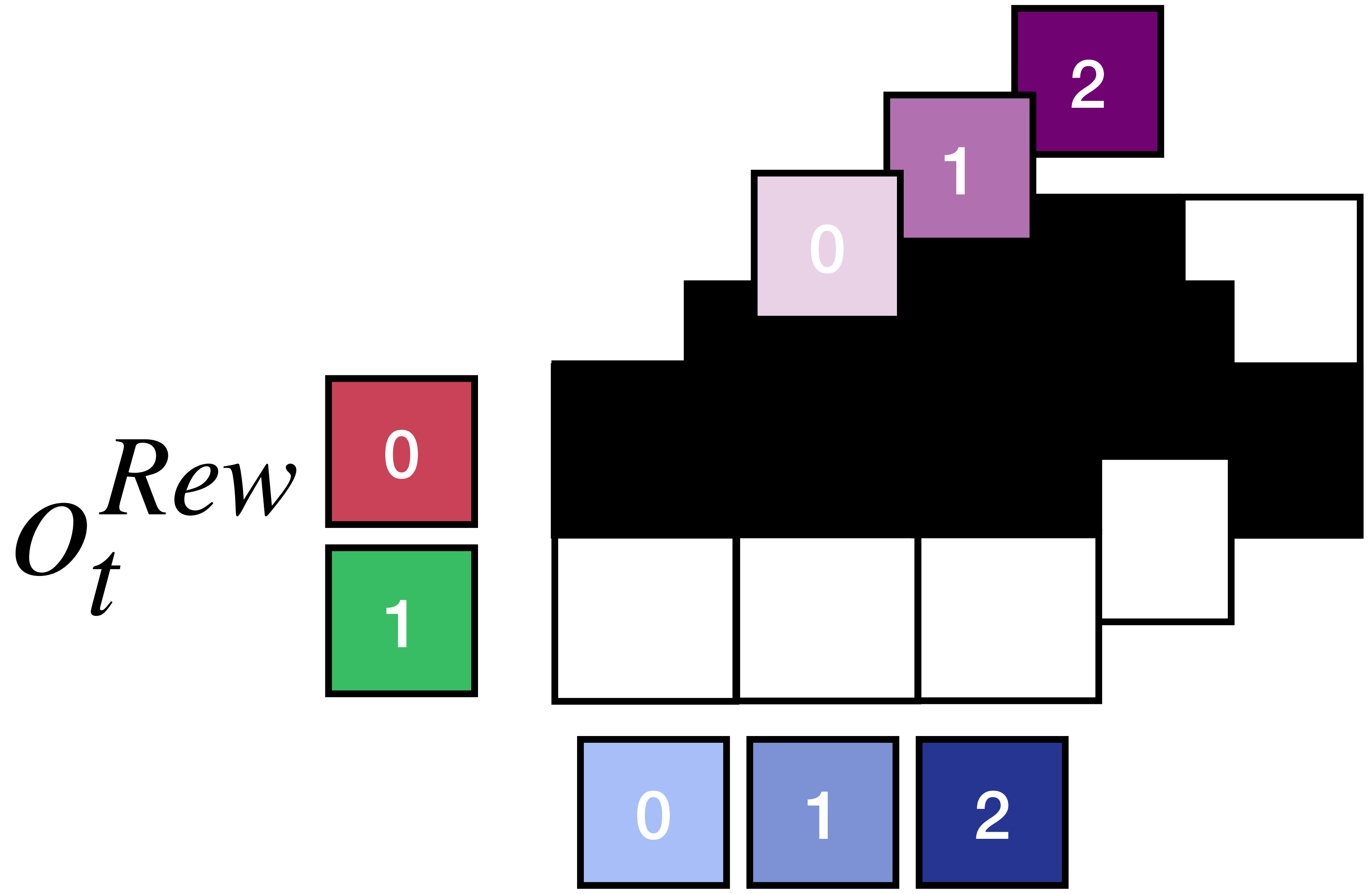


Multiple observation modalities

$$\mathbf{o}_t = \{o_t^X, o_t^Y, o_t^{Rew}\}$$

```
num_observations = [3, 3, 2]
num_modalities = len(num_observations)
A = obj_array(num_modalities)
A[0] = …
A[1] = …
A[2] = …
```





`A[2][1,2,2] = 1.0`

`A[2].shape = (2, 3, 3)`

Questions ?

A new generative model: the epistemic two armed bandit

A new generative model: the epistemic two armed bandit

Left vs. Right vs. Hint

$$\mathbf{o} = \{o^{Hint}, o^{Reward}, o^{Choice}\}$$

$$\mathbf{s} = \{s^{Context}, s^{Choice}\}$$

$$\mathbf{u} = \{u^{Context}, u^{Choice}\}$$

A new generative model: the epistemic two armed bandit

$$\mathbf{s} = \{s^{Context}, s^{Choice}\}$$

$$s^{Context} \in \{\text{Left-Better, Right-Better}\}$$

$$s^{Choice} \in \{\text{Start, Hint, Left, Right}\}$$

$$\mathbf{u} = \{u^{Context}, u^{Choice}\}$$

$$u^{Context} \in \{\text{Do-nothing}\}$$

$$u^{Choice} \in \{\text{Move-Start, Get-Hint, Play-Left, Play-Right}\}$$

A new generative model: the epistemic two armed bandit

$$o^{Hint} \in \{\text{Null, Hint-left, Hint-right}\}$$

$$o^{Reward} \in \{\text{Null, Loss, Reward}\}$$

$$o^{Choice} \in \{\text{Start, Hint, Left, Right}\}$$

$$\mathbf{o} = \{o^{Hint}, o^{Reward}, o^{Choice}\}$$

$$\mathbf{s} = \{s^{Context}, s^{Choice}\}$$

$$\mathbf{u} = \{u^{Context}, u^{Choice}\}$$

The epistemic two armed bandit

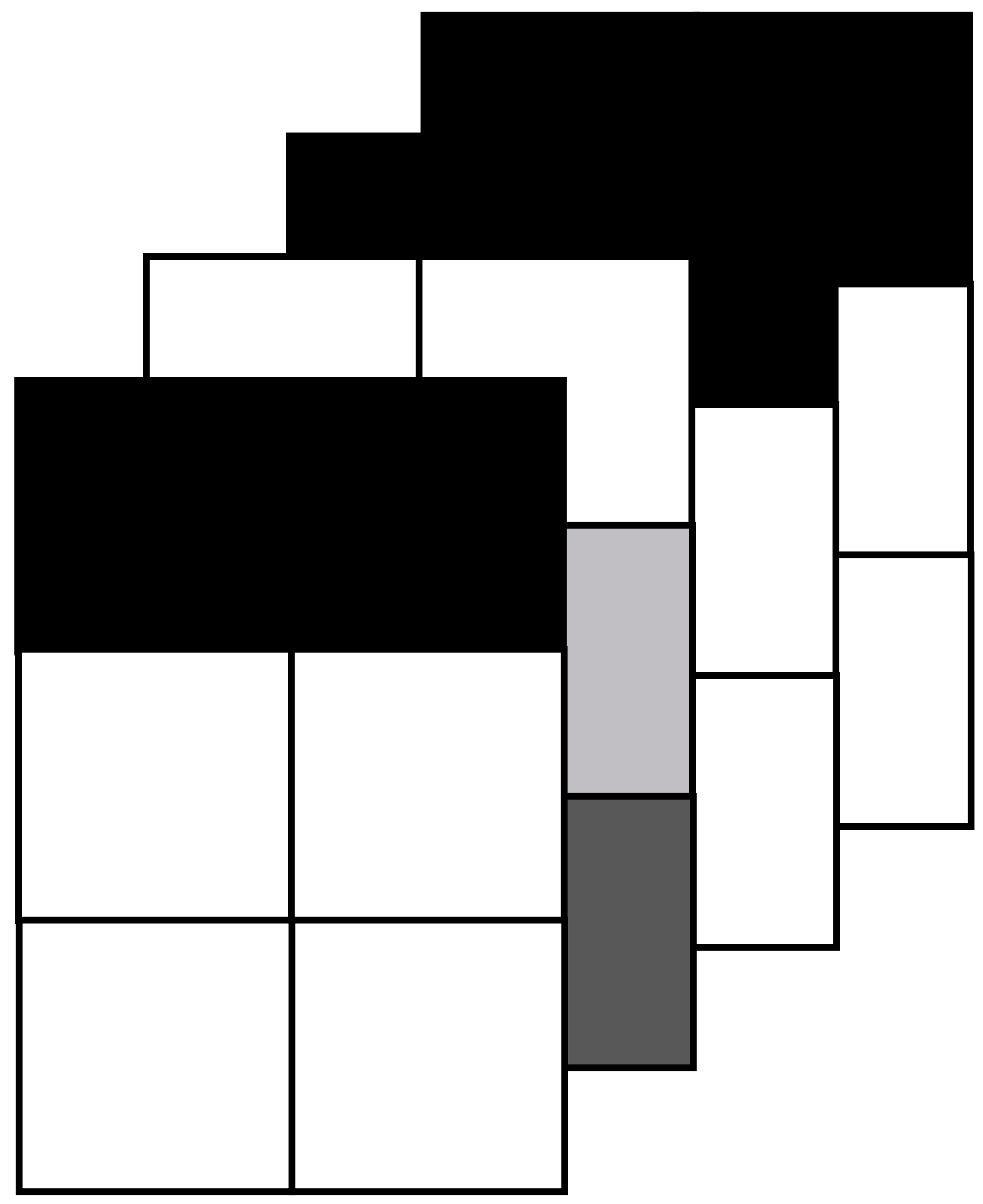
The **A** Matrix

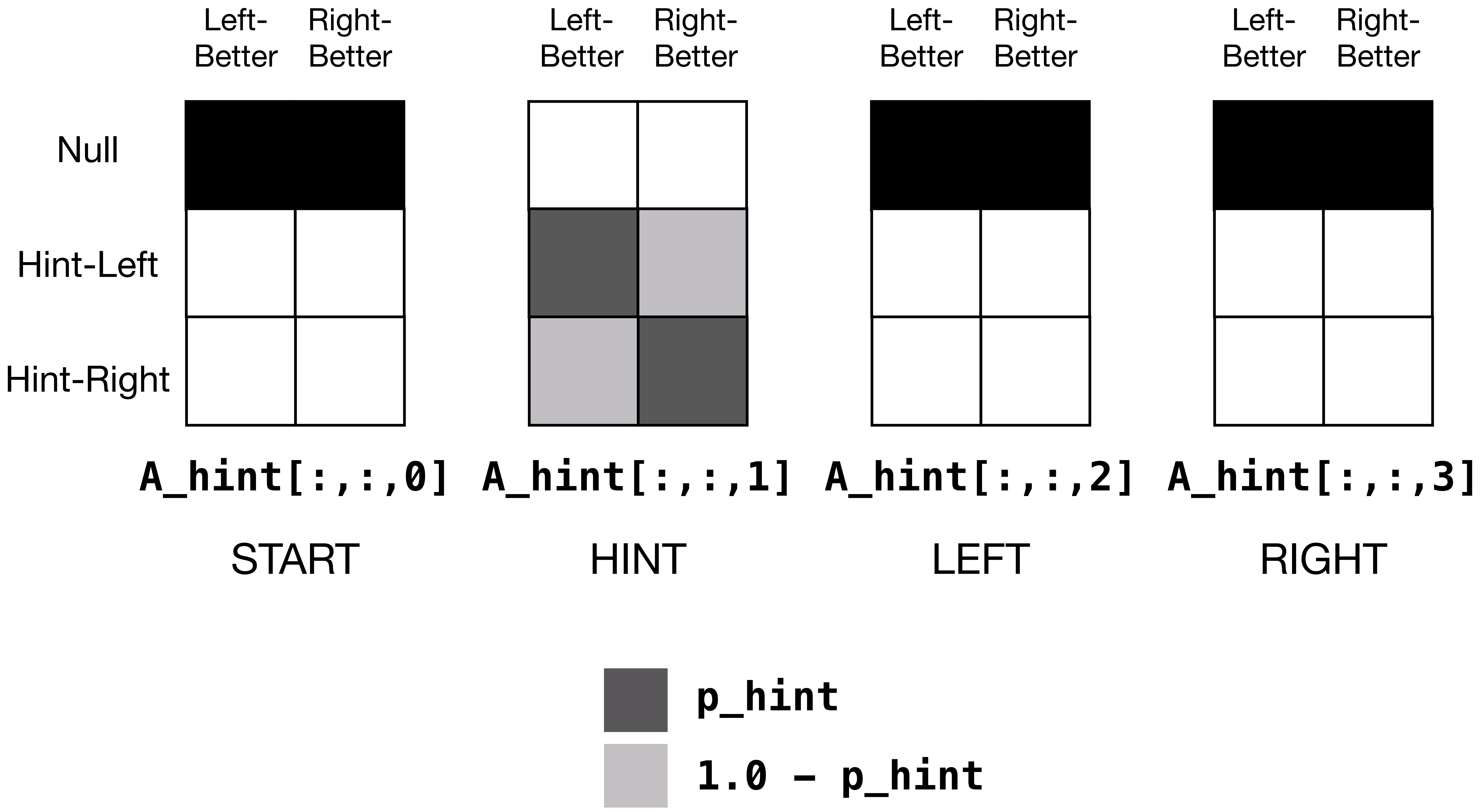
AKA $P(o \mid s)$

$$o^{Hint} \in \{\text{ Null, Hint-Left, Hint-Right }\}$$

	Left-Better	Right-Better
Null		
Hint-Left		
Hint-Right		

o^{Hint}




Left-Better
Right-Better
Null
Hint-Left
Hint-Right
A_hint[:,:,0]
A_hint[:,:,1]
A_hint[:,:,2]
A_hint[:,:,3]
START
HINT
LEFT
RIGHT
p_hint
1.0 - p_hint

The epistemic two armed bandit

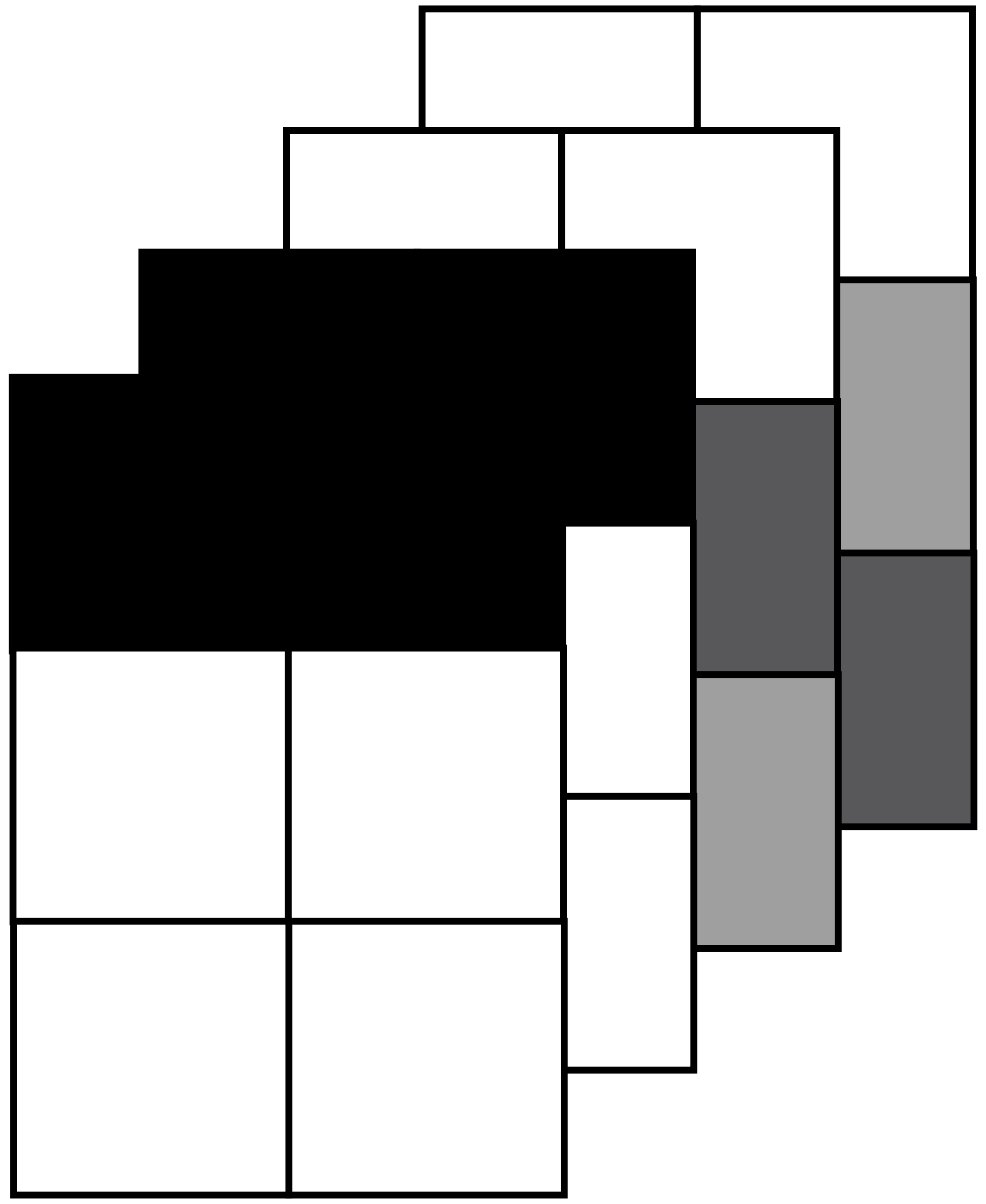
The **A** Matrix

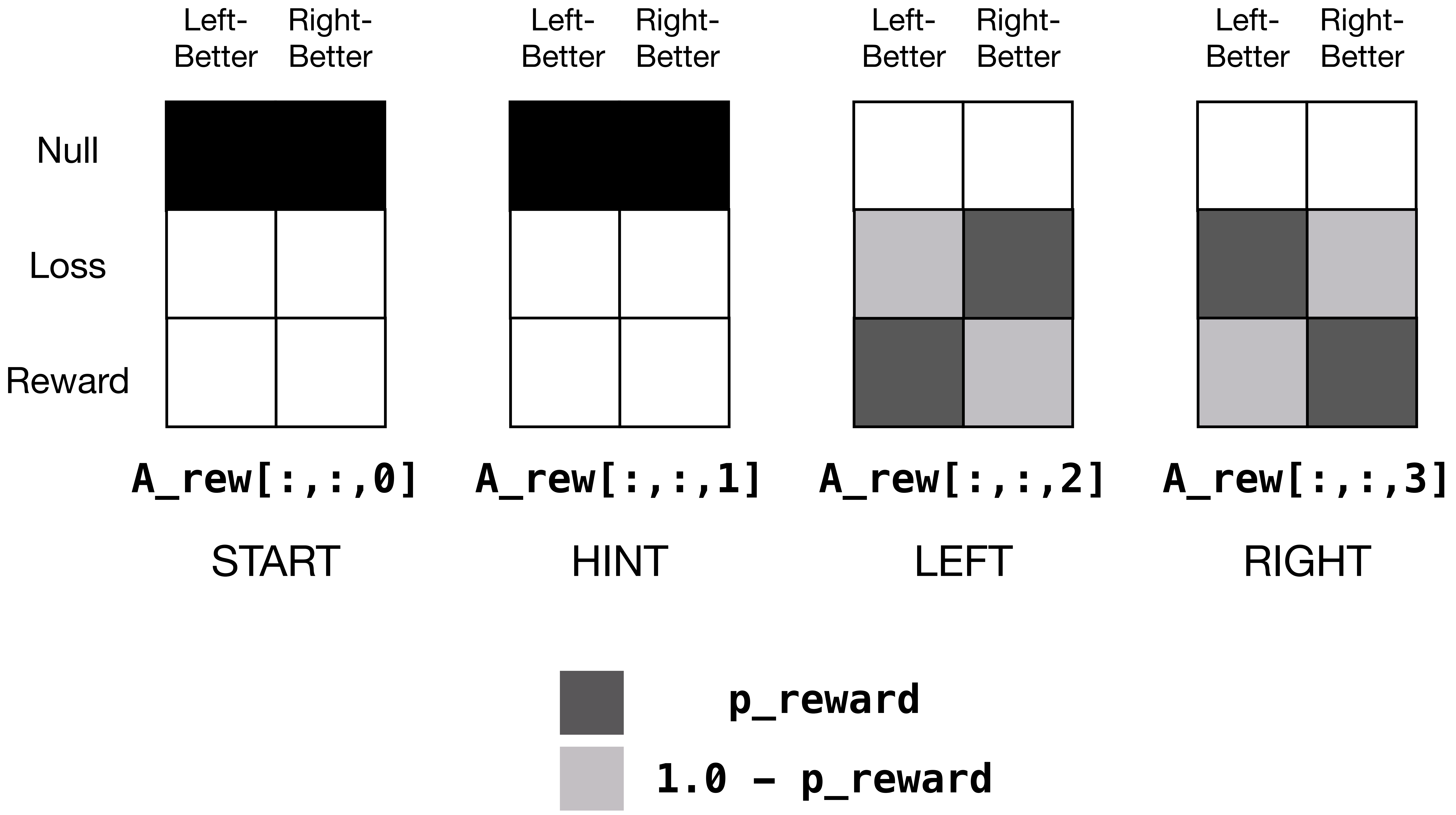
AKA $P(o \mid s)$

$$o^{Reward} \in \{\text{ Null, Loss, Reward }\}$$

	Left-Better	Right-Better
Null		
Loss		
Reward		

o^{Reward}





Left-Better
Right-Better
Null
Loss
Reward
A_rew[:,:,0]
A_rew[:,:,1]
A_rew[:,:,2]
A_rew[:,:,3]
START
HINT
LEFT
RIGHT
p_reward
1.0 - p_reward

The epistemic two armed bandit

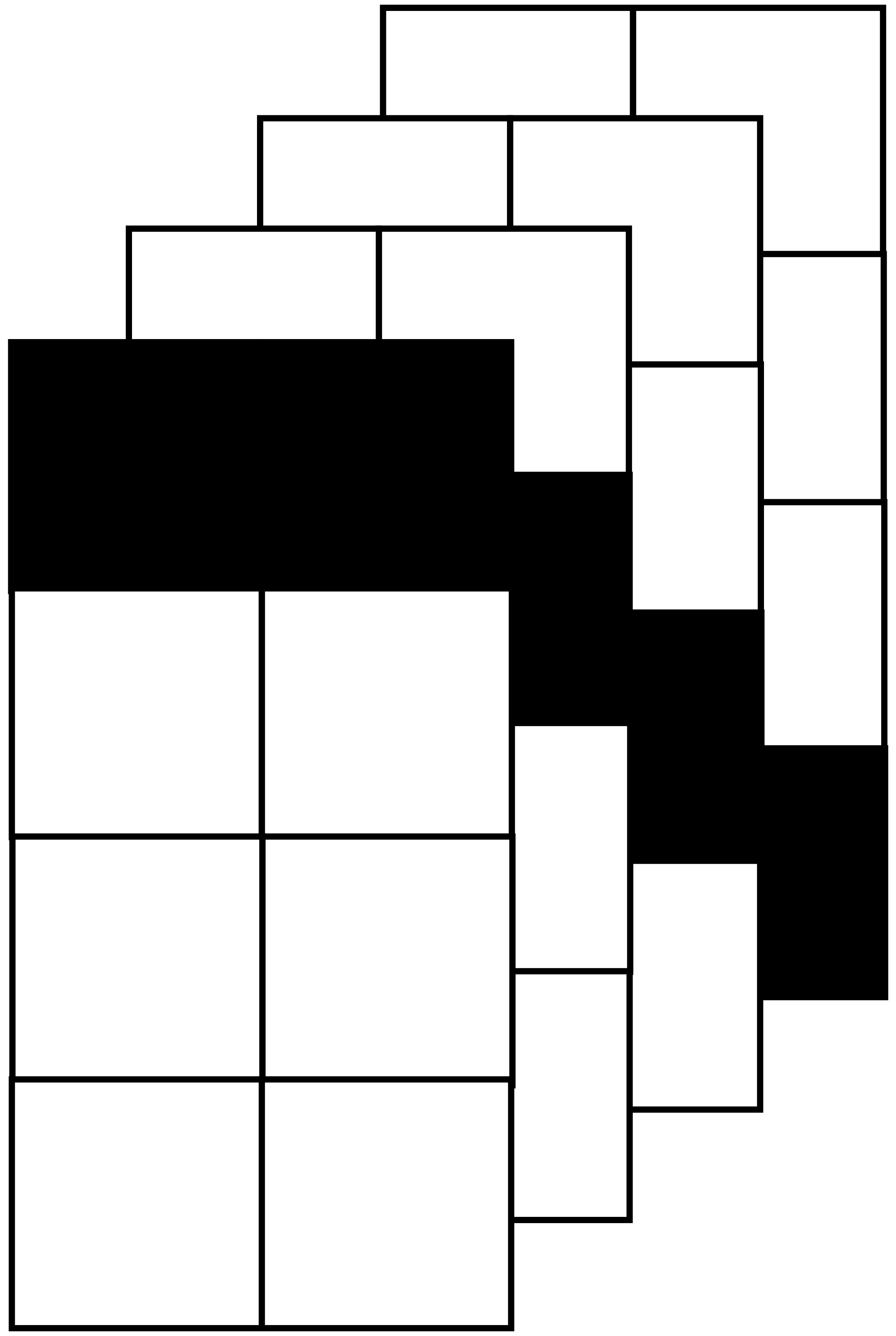
The **A** Matrix

AKA $P(o \mid s)$

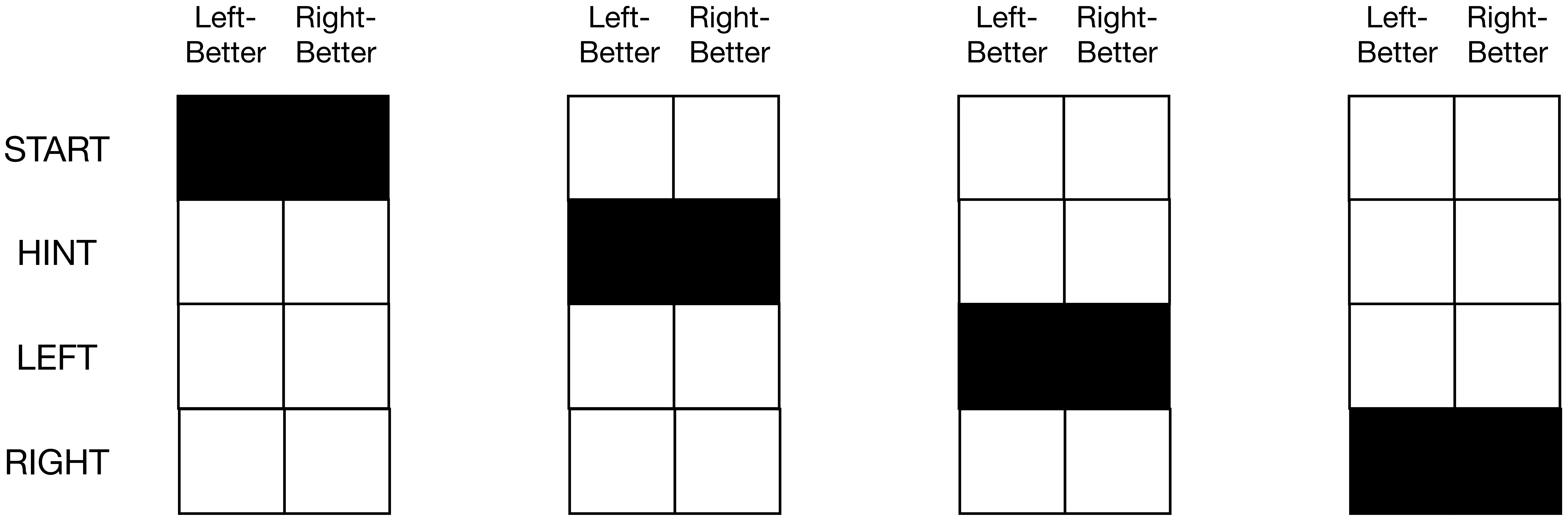
$$o^{Choice} \in \{\text{Start, Hint, Left, Right}\}$$

	Left-Better	Right-Better
Start		
Hint		
Left		
Right		

o^{Choice}



START HINT LEFT RIGHT



`A_choice[:,:,0]` `A_choice[:,:,1]` `A_choice[:,:,2]` `A_choice[:,:,3]`

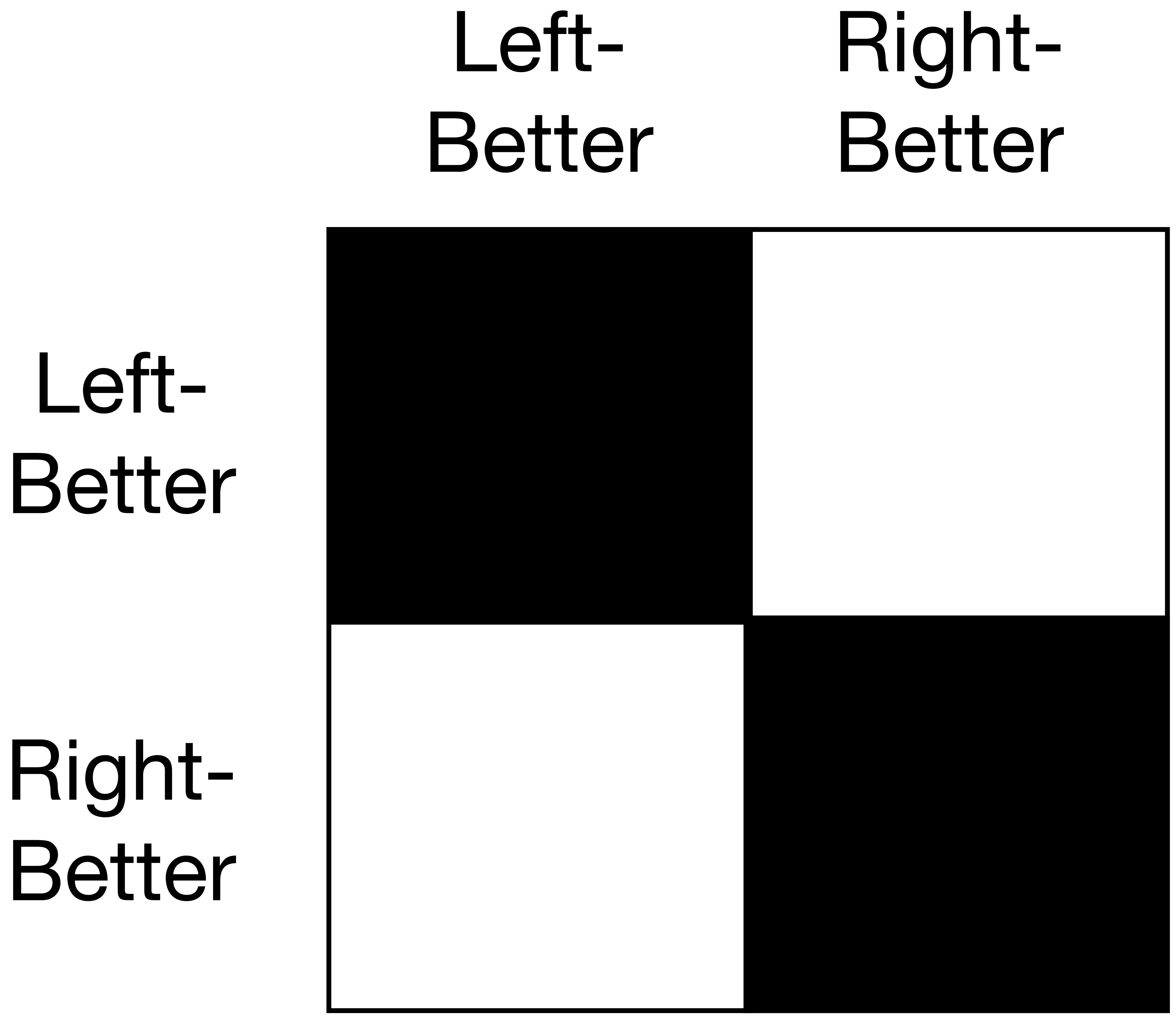
START HINT LEFT RIGHT

Back to Colab…

The epistemic two armed bandit

The **B** Matrix

AKA $P(s_t \mid s_{t-1}, u_{t-1})$

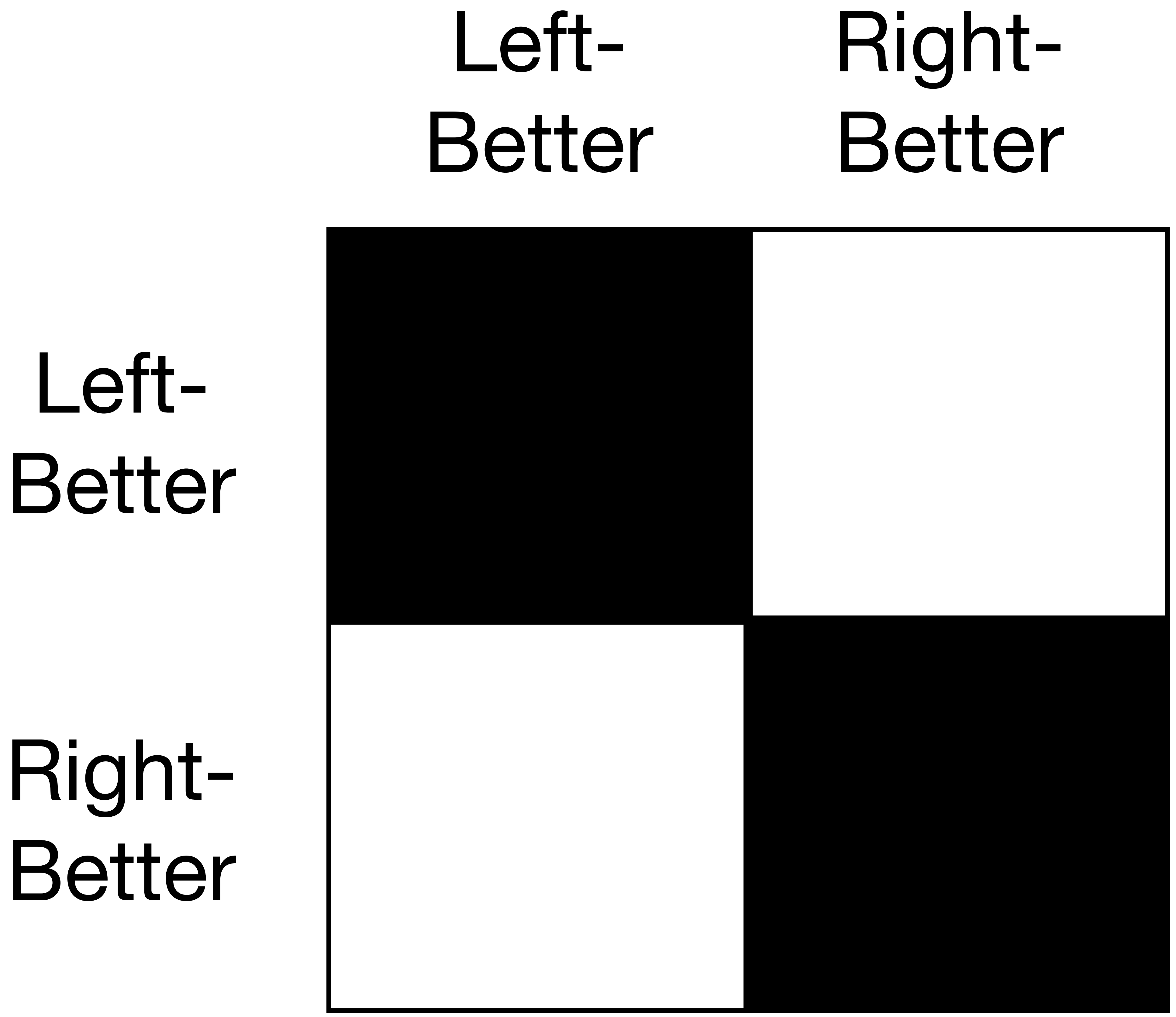


`B[0][:,:,0]`

The epistemic two armed bandit

The **B** Matrix

AKA $P(s_t \mid s_{t-1}, u_{t-1})$

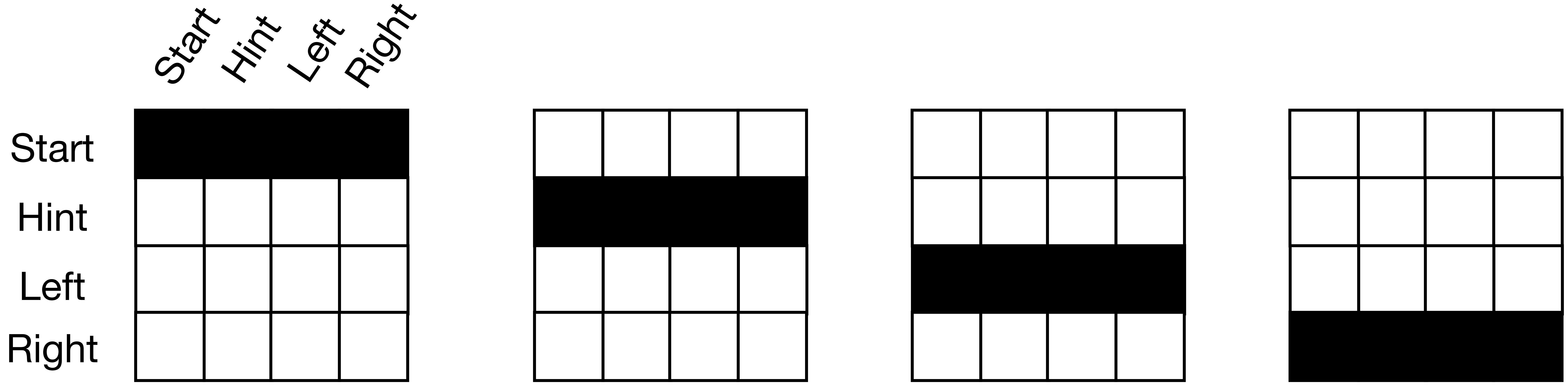


`B_context[:,:,0]`

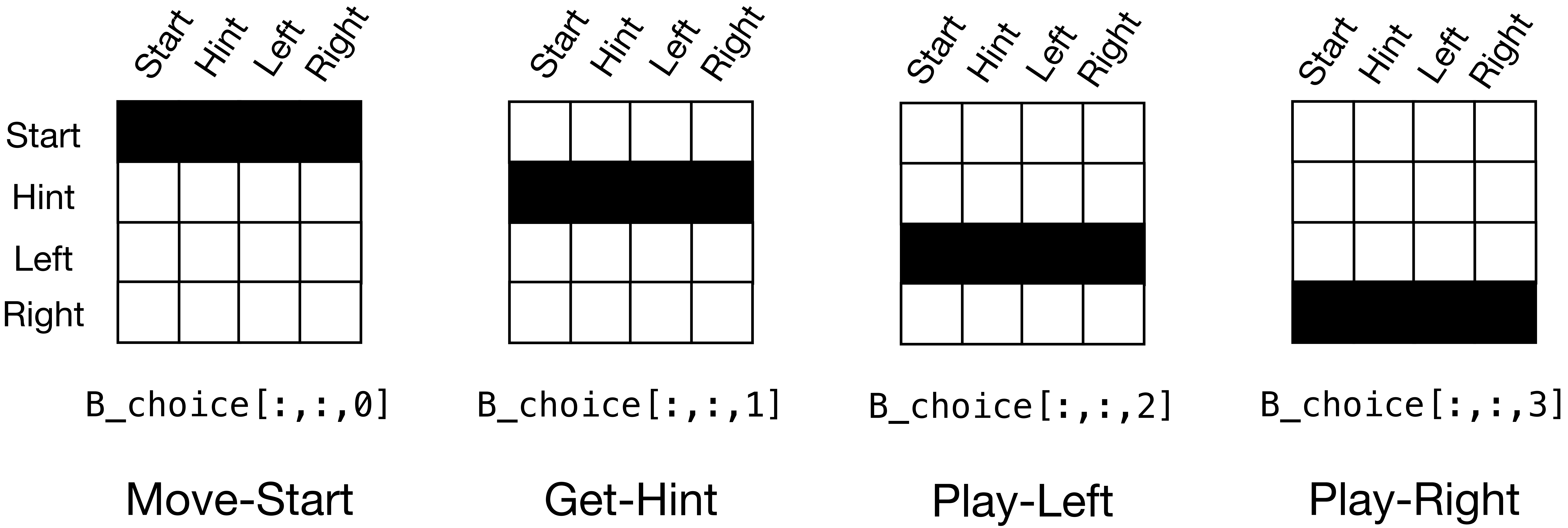
The epistemic two armed bandit

The **B** Matrix

AKA $P(s_t \mid s_{t-1}, u_{t-1})$



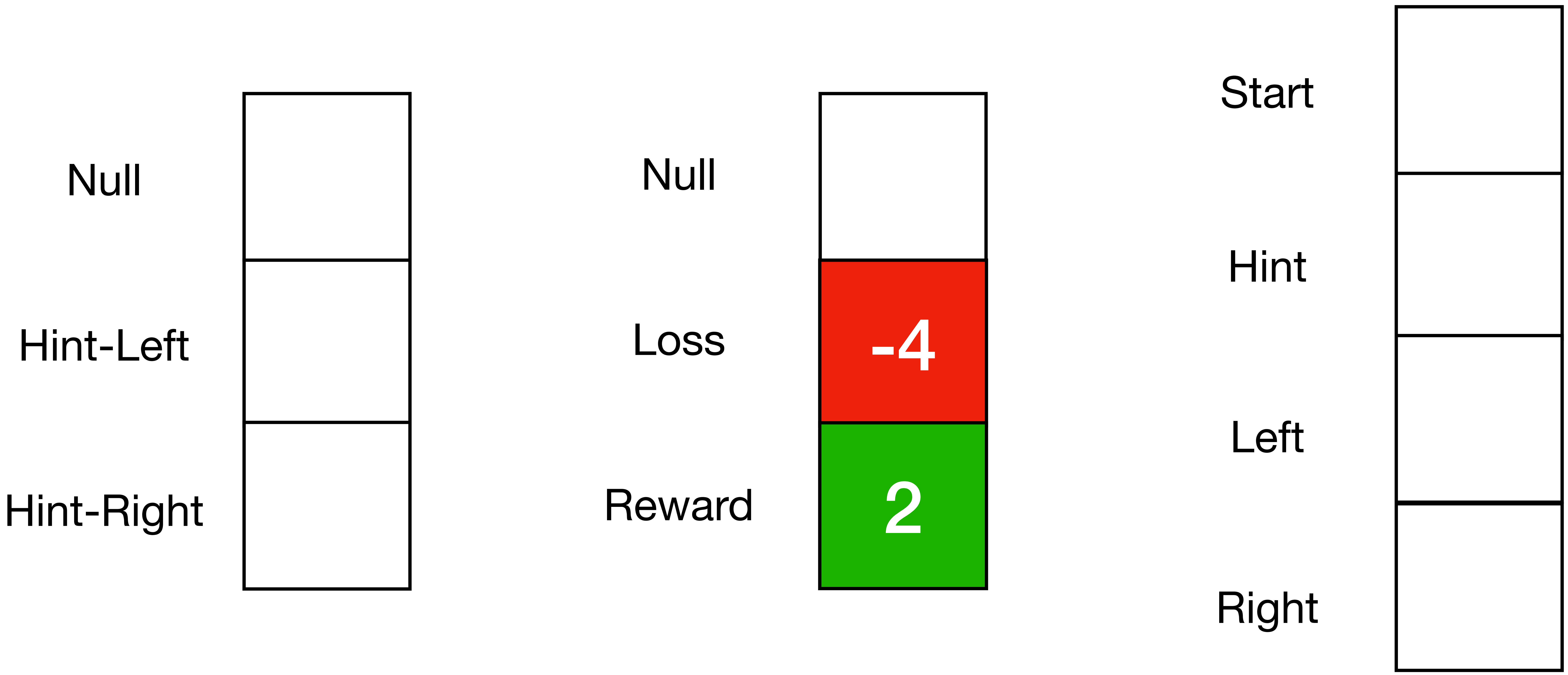
`B[1][:,:,0]`	`B[1][:,:,1]`	`B[1][:,:,2]`	`B[1][:,:,3]`
Move-Start	Get-Hint	Play-Left	Play-Right


Start
Hint
Left
Right
Start
Hint
Left
Right
B_choice[:,:,0]
Move-Start
B_choice[:,:,1]
Get-Hint
B_choice[:,:,2]
Play-Left
B_choice[:,:,3]
Play-Right

Back to Colab…

The epistemic two armed bandit

The C Vector



The epistemic two armed bandit

The C Vector

$$P(o \mid \mathbf{C}) = \sigma(\mathbf{C}) \qquad \sigma(\mathbf{C}_i) = \frac{\exp(C_i)}{\sum_i \exp(C_i)}$$

$$\begin{array}{r} \text{Null} \\ \text{Loss} \\ \text{Reward} \end{array} \begin{bmatrix} 0.119 \\ 0.002 \\ 0.88 \end{bmatrix} = \sigma\left(\begin{bmatrix} \; \\ -4 \\ 2 \end{bmatrix}\right)$$

The **D** Vector

AKA $P(s_0)$

Back to Colab…

A **B** **C** **D**

- `my_agent.infer_states(observation)`
- `my_agent.infer_policies()`
- `chosen_action = my_agent.sample_action()`

```
from pymdp.agent import Agent
my_agent = Agent(A = A, B = B, C = C, D = D)
```

Back to Colab…

Thank you for your attention!