Problem Sheet 3

There are two questions from Conor and two from Anne; Anne's question come after the 'additional problem' from Conor's part of the problem sheet.

Useful facts

- **The Gaußian distribution**:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

- Working out **probabilities for the Gaußian**:

$$\text{Prob}(x_1 < x < x_2) = \frac{1}{2}[\text{erf}\,(z_2) - \text{erf}\,(z_1)]$$

  where

$$z = \frac{x-\mu}{\sqrt{2}\sigma}$$

  The error function

$$\text{erf}\,(z) = \frac{1}{\sqrt{\pi}} \int_{-z}^{z} e^{-y^2} dy = \frac{2}{\sqrt{\pi}} \int_0^z e^{-y^2} dy$$

  goes from -1 to one, so $\text{erf}\,(-\infty) = -1$ and $\text{erf}\,(\infty) = 1$.

- The **sum of two random variables** $Z = X + Y$ has a probability given by the convolution:

$$p_Z(z) = \int_{-\infty}^{\infty} p_X(x) p_Y(z-x) dx$$

Questions

1. The size of a standard croquet ball is 3 5/8 inches[1]. The height of a croquet hoop is 3 3/4 inches. If a not very good croquet-ball making machine makes croquet balls whose mean matches the standard and with standard deviation 1/8 inch, what is the chance it will make a ball too large to fit through the hoop? You can write the solution in terms of the error function.

2. This will look like a long question but it is almost all background and the question is not too bad when you actually read through it. In particle physics when a collider is being used to find a new particle like the Higgs boson or the top squark scientists don't detect the sought after particle directly since it usually decays almost straight away, instead they detect the more common particles that particle will decay into, for example, a Higgs boson can decay in to two photons and these can be detected. Roughly speaking scientists count these events. However, the whole situation is very messy and there will always be some events even if the particle doesn't exist at the energy being examined. The amount of these background events will fluctuate from experiment to experiment, typically like a Gaußian. The scientific team is allowed to claim they have discovered the particle if the number of events they measure is more than five standard deviations above what would be expected if the particle didn't exist. What is the probability of this 'discovery' happening by chance?

[1] Everything in croquet is measured in old timey units

Extra questions

- The Dirac delta function $\delta(x)$ is a special function that is zero everywhere but when $x=0$, but
$$\int_{-\infty}^{\infty} \delta(x)dx = 1$$
You can think of it as the $a \to 0$ limit of the uniform distribution with support $[-a, a]$. If the random variable X has $p(x) = \delta(x-1)$ it means $X = 1$. If Y is uniformely distributed from zero to two, what is the distribution for $Z = X + Y$?

- Australorp hens weigh on average 4kg with a standard deviation 0.25kg; australorps who weight less than 3.5kg are fed *patent chicken spicer*, a mixture of chalk, corn and pepper. What fraction of these hens are fed patent chicken spicer?