

Lecture 1: Introducing probability

COMS10014 Mathematics for Computer Science A

`cs-uob.github.io/COMS10014/` and `github.com/coms10011/2021_22`

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What is probability?

Probability is a set of mathematical definitions and theorems that are useful for reasoning about randomness and uncertainty

Working out probabilities



A fair coin has a one in two chance of turning up heads, if each spin is independent it has a probability of

$$P(\text{six heads in a row}) = \left(\frac{1}{2}\right)^6 = 0.015625$$

Frequentist statistics



A fair coin has a 1.5% chance of coming up heads six times in a row, so if you spun a coin six times and it came up heads each time you can be 98.5% certain the coin is not fair, maybe.

Frequentist statistics



A fair coin has a 1.5% chance of coming up heads six times in a row, so if you spun a coin six times and it came up harps each time you can be 98.5% certain the coin is not fair, maybe. If you include all heads and heads-harps-heads-harps-heads-harps and harps-heads-harps-heads-harps-heads in your list of 'weird' outcomes then there is a 6% chance of getting a 'weird' outcome.

Bayesian statistics

Aoife tells Brendan she has spun a coin six times and taken down the sequence. As far as Brendan is concerned all 64 outcomes are equally likely. Now, Aoife tells Brendan the coin turned out the same each time, so Brendan updates his belief about the sequence; he now believes there are two possibilities, all heads and all tails, each equally likely.

What is probability?

Probability is a set of mathematical definitions and theorems that are useful for reasoning about randomness and uncertainty

Some definitions - sample space

A **sample space** is a set of possible outcomes.

Sample space

To model the experiment where a coin is tossed once we'd use

$$S = \{H, T\}$$

where H and T correspond to heads and harps.

Sample space

To model the experiment where a coin is tossed twice in a row we'd use

$$S = \{HH, HT, TH, TT\}$$

and where two coins are tossed and we don't distinguish one from the other

$$S = \{HH, HT, TT\}$$

where *HT* stands for heads and harps in any order.

Infinite sample spaces

If you wanted to model the number of coin tosses before you get a head you'd use

$$S = \mathbb{N}$$

and to model the height of trees

$$S = [0, \infty)$$

Some definitions - event

An **event** in a sample space X is a subset $E \subset X$.

Event



If we have sample space

$$S = \{1, 2, 3, 4, 5, 6\}$$

corresponding to the face values of a dice¹, then the set of even sides

$$E = \{2, 4, 6\}$$

is an example event. This might be useful for working out the probability of an even roll.

¹or die - originally *dice* was the plural of *die*

Some definitions - probability

Formally, a **probability** is a map P from events to real numbers such that:

1. $P(A) \geq 0$ for all events.
2. $P(X) = 1$
3. If $A \cap B = \emptyset$ for two events A and B then

$$P(A \cup B) = P(A) + P(B)$$

Some definitions - probability mass function

The **probability mass function** is a map p from outcomes to real numbers such that:

1. $p(x) \geq 0$ for all $x \in X$
2. $\sum_{x \in X} p(x) = 1$

The probability and the probability mass function

$$P(A) = \sum_{x \in A} p(x)$$