Probability and Combinatorics Worksheet 2

Useful facts

- **Combinations**: The number of ways of splitting n items into sets of size r_1, r_2 through to r_k with

$$r_1 + r_2 + \ldots + r_k = n \tag{1}$$

is

$$\left(\begin{array}{c} n \\ r_1, r_2, \ldots, r_k \end{array} \right) = \frac{n!}{r_1! r_2! \ldots r_k!} \tag{2}$$

- The **conditional probability** of event R given C:

$$P(R|C) = \frac{P(R \cap C)}{P(C)} \tag{3}$$

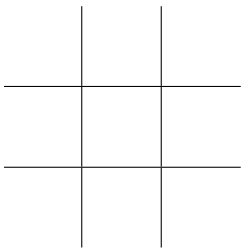
This is the probability of getting an outcome in event R if we know the outcome is in event C.

Questions

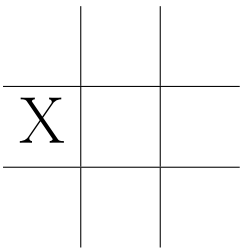
These are the questions you should make sure you work on in the workshop.

1. How many distinct anagrams has the word 'BOOKKEEPER'?
2. Two events A and B have probabilities $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cup B) = 0.4$. Find
   a) Find $P(A \cap B)$.
   b) Find $P(\bar{A} \cap \bar{B})$.
   c) Find $P(A|B)$.
3. In tic-tac-toe players take turns to write a X or O in a empty location in a 3×3 grid,

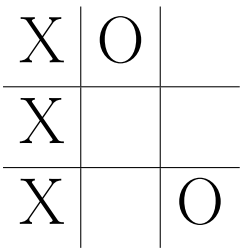
   the grid is traditionally drawn like this



   X goes first so after one move the board might look like



and if the O player is very poor, or suffering from despair, the game might end after five moves with



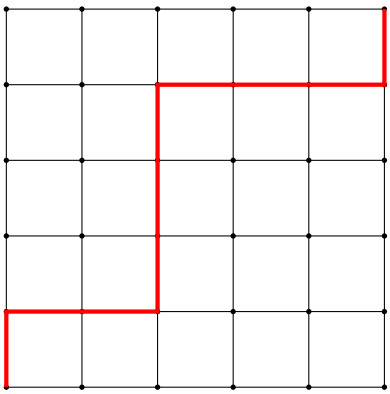
A player wins then they have a complete row, column or diagonal, in this case X has a column. How many five move games of tic-tac-toe are there?

4. You roll a dice twice, what is the probability the second roll has a lower value than the first? You take the ace, two, three, four, five and six of hearts to make a mini-pack of cards. You draw two cards, what is the probability the second will be lower than the first, with the ace counting as a one?

Extra questions

These are questions you can work on in the workshop.

1. In a lattice path problem you have a two-dimensional lattice whose points are located at points with coordinates (n, m) where n and m are integers. A lattice path can only have steps that increase either the x- or y-coordinate. This is a 5×5 with an example path from (0,0) to (5,5) in red.



How many paths are there from (0,0) to (4,5)? If you pick a random path from (0,0) to (4,5) what is the probability it goes through $(2, 2)$?

2. How many six move games of tic-tac-toe are there? Remember that a six move game can't have been won after five moves.

3. If p is a probability mass function and we define a function on events:

$$P(E) = \sum_{x \in E} p(x) \tag{4}$$

show P is a probability.

4. Ten different objects are put into three boxes. How many different configuations are there? They are sorted so that there are five in the first box, three in the second and two in the third. How many ways configurations are there?

Tic-tac-tic-tac-toe

Tic-tac-toe is a terrible game but tic-tac-tic-tac-toe is excellent. In tic-tac-tic-tac-toe nine tic-tac-toe boards are arranged in 3×3 grid so it looks like a big tic-tac-toe board with a little tic-tac-toe board in each of its nine squares. X can place their first move in any of the 81 small squares, but from then the move a player makes determines which small board the other player can play in; so if X plays the top left square of the small board in the middle then O must play in the small board in the top left, if they play the middle square of that small board, then X must play in the middle small board. When a small board is full, a player directed there can play in any available square. The winner is the person who wins the most small games. There is a better description on wikipedia which calls tic-tac-tic-tac-toe 'ultimate tic-tac-toe'.