class test - solutions

short questions

1. A bag contains one red ball, 1000 blue balls and 1000 yellow balls; two balls are taken from the bag. What is the sample space?
   solution:
   $$\{YY, BB, YB, BY, YR, BR, RY, RB\}$$

2. What is an event?
   solution: It is a subset of the sample space.

3. Ireland is divided into four provinces: Munster, Leinster, Ulster and Connaught. Two provinces are selected at random; what is the event that both end in 'ster' and what is the probability of this event?
   solution:
   $$\{ML, LM, MU, UM, UL, LU\}$$
   and
   $$1/2$$

4. A sample space consists of the words {stately, plump, buck, mulligan, came, from, the, stairhead}. The event A consists of all words with four or fewer letters. The event B consists of all words ending in a vowel. What is $p(B)$ and $p(B|A)$?
   solution:
   $$p(B) = 1/4$$
   and
   $$p(B|A) = 1/2$$
   also allow answer that included "y" as a vowel.

5. What is the 'naïve' aspect of the naïve Bayes estimator?
   solution: It assumes conditional independence of the evidence
   $$p(\mathbf{w}|s) = p(w_1|s)p(w_2|s)\dots p(w_n|s)$$

6. If X is a random variable define the expected value of X?
   solution:
   $$\langle X \rangle = \sum_i x_i p(x_i)$$

7. A shopkeeper says she has two new baby beagles to show you, but she doesn't know whether they're male, female, or one of each. You tell her that you want only a male, and she telephones the fellow who's giving them a bath. "Is at least one a male?" she asks him. "Yes!" she informs you with a smile. What is the probability that the other one is a male? (This question is taken from Marilyn von Savant's Ask Marilyn column).
   solution:
   $$1/3$$

8. A six sided dice is rolled ten times, what is the probability of getting exact three sixes?
   solution:
   $$p = \left(\begin{array}{c} 10 \\ 3 \end{array} \right) \frac{1}{6^3} \frac{5^7}{6^7}$$

long question

This question is about calculating probabilities in the context of the card game baccarat. In the game of baccarat the player and dealer are dealt cards; two initial and possibily a third. Each hand is given a score and, ultimately the hand with the highest score wins. The Ten, Jack, Queen and King are worth zero points, the Ace is worth one point and all the other cards have a value which corresponds to their face value; a seven is worth seven points for example. The value of a hand is worth the sum of the points in it, modulo ten; or put another way, it is equal to the last digit, so if a player has a King and a nine, the hand is worth nine points; if they have two nines, the hand is worth eight points since $9 + 9 = 18$ and eight is the final digit of 18.

1. If a player is dealt two cards from a 52 card deck what is the chance of getting zero?
2. If the player is dealt two cards from a 52 card deck what is the chance of a getting eight or nine points?
3. If the player is dealt an eight or nine, what is the probability they have in fact a nine?

solution: 1) So zero comes from picking two out of the 16 cards that have the form 10, J, Q, K which is 120 possibilities, or from getting two fives, six possibilities, or 1+9 or 2+8 or 3+7 or 4+6 which is $64=4 \times 16$ possibilities hence 120+6+64=190. (52 2)=1360 so the probability is $190/1326 \approx 0.14$.
2) So the possible combinations that give eight are 1+7 through to 3+5 and 4+4 which is $16 \times 3 + 6$ or eight and a 10, J, Q, K which is $4 \times 4 \times 4 = 64$. Finally there is 9+9 with 4 choose 2 suits making

$$3 + 6 + 64 + 6 = 124 \tag{1}$$

as for nines, there is 1+8 to 4+5 along with 9 + 10,J,Q,K giving

$$4 \times 16 + 4 \times 16 = 128 \tag{2}$$

so the overall odds are $252/(522) \approx 0.19$.
3) So this is 128/(124+128)=0.507.