Probability and Combinatorics Worksheet 1

Useful facts

- The **power set**, $\mathcal{P}(A)$, is the set of subsets of A:

$$\#(\mathcal{P}(A)) = 2^{\#(A)} \tag{1}$$

- **Factorial**: the number of ways to order n elements is

$$n! = n \times (n-1) \times \ldots \times 2 \times 1 \tag{2}$$

- **Combinations**: The number of ways of choosing r items out of n is

$$\binom{n}{r} = \frac{n(n-1)(n-2)\ldots(n-r+1)}{r(r-1)(r-2)\ldots 1} = \frac{n!}{r!(n-r)!} \tag{3}$$

- **Combinations**: The number of ways of splitting n items into sets of size r_1, r_2 through to r_k with

$$r_1 + r_2 + \ldots + r_k = n \tag{4}$$

  is

$$\binom{n}{r_1, r_2, \ldots, r_k} = \frac{n!}{r_1! r_2! \ldots r_k!} \tag{5}$$

- **Cards**: 52 cards made up of four suits; in each suit there are 13 values, ace, two through to ten and the jack, queen, king.

- **Poker hands**: the number of poker hands is

$$\binom{52}{5} = 2598960 \tag{6}$$

Questions

These are the questions you should make sure you work on in the workshop.

1. In the poker hand *two pair* there are two pairs of cards with each card in the pair matched by value; the fifth card has a different value to either pair. What is the probability of two pairs when five cards are drawn randomly.

2. In a *full house* there is one pair and one triple, what is the probability of getting a full house?

3. How many anagrams are there of the word 'COVID'?

4. How many distinct anagrams are there of the word 'CUMMINGS'?

Extra questions

These are extra questions you might attempt in the workshop or at a later time.

1. When it started in 1987 the Irish lottery has 36 numbers; participants paid 50 Irish pence to buy a combination of six different numbers; they would win if these numbers matched the six drawn. In the last week in May in 1992 a syndicate tried to buy all combinations of numbers. If they had succeeded how many numbers would they have bought? In the event the lottery shut down lots of the lottery machines so they only bought most of the numbers, they nonetheless had the winning number but shared the prize three ways. However, because of the roll-over prize and the match-5 and match-4 prizes, they are thought to have made a substantial profit. The lottery was redesigned after this to have more numbers.

2. From a group of three undergraduates and five graduate students, four students are randomly selected to act as TAs. What is the chance there will be exactly two undergraduate TAs?

3. Prove
$$\left(\begin{array}{c} n \\ r \end{array} \right) = \left(\begin{array}{c} n \\ n-r \end{array} \right) \tag{7}$$

4. How many distinct anagrams has the word 'OROONOKO'?