Probability and Combinatorics Worksheet 7

Useful facts

- **Integrating a polnomial**

$$\int x^n dx = \frac{x^{n+1}}{n+1} \tag{1}$$

so the definite integral is

$$\int_a^b x^n dx = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1} \tag{2}$$

- **Integrating an exponential**

$$\int_{x_1}^{x_2} e^{ax} dx = \frac{1}{a}\left(e^{ax_2} - e^{ax_1}\right) \tag{3}$$

- **Integrating by parts**

$$\int_a^b udv = uv]_a^b - \int_a^b vdu \tag{4}$$

Questions

These are the questions you should make sure you work on in the workshop.

1. A distribution x has the form

$$p(x) = \begin{cases} x & 0 \leq x < 1 \\ 2 - x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

What is the probability $x < 1$; what is the probability $x < 1.5$? What is the probability $0.5 < x < 1.5$?

2. The distribution of tree heights in a christmas tree forest is

$$p(h) = \begin{cases} 0.3 & 0 \leq h < 2 \\ 0.2 & 2 \leq h < 4 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

What is the mean height of trees in the forest?

3. Work out the mean and variance for the distribution

$$p(x) = \begin{cases} 2/a & x \in [-a, a] \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

4. Another useful distribution is the exponential distribution:

$$p(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability $\text{Prob}(x_1 < x < x_2)$ where x_1 and x_2 are both positive.

Extra questions

Do these in the workshop if you have time.

1. By integrating, what is the mean of the exponential distribution?
2. Work out the mean of the exponential distribution by integrating

$$1 = Z = \int_0^\infty p(x)dx$$

3. What is the variance of the exponential distribution?