# Probability and Combinatorics Worksheet 7

## Useful facts

- **Integrating a polnomial**

$$\int x^n dx = \frac{x^{n+1}}{n+1} \tag{1}$$

so the definite integral is

$$\int_a^b x^n dx = \frac{b^{n+1}}{n+1} - \frac{a^{n+1}}{n+1} \tag{2}$$

- **Integrating an exponential**

$$\int_{x_1}^{x_2} e^{ax} dx = \frac{1}{a}\left(e^{ax_2} - e^{ax_1}\right) \tag{3}$$

- **Integrating by parts**

$$\int_a^b udv = uv]_a^b - \int_a^b vdu \tag{4}$$

## Questions

These are the questions you should make sure you work on in the workshop.

1. A distribution $x$ has the form

$$p(x) = \begin{cases} x & 0 \leq x < 1 \\ 2 - x & 1 \leq x < 2 \\ 0 & \text{otherwise} \end{cases} \tag{5}$$

What is the probability $x < 1$; what is the probability $x < 1.5$? What is the probability $0.5 < x < 1.5$?

2. The distribution of tree heights in a pine tree forest is

$$p(h) = \begin{cases} 0.3 & 0 \leq h < 2 \\ 0.2 & 2 \leq h < 4 \\ 0 & \text{otherwise} \end{cases} \tag{6}$$

What is the mean height of trees in the forest?

3. Work out the mean and variance for the distribution

$$p(x) = \begin{cases} 1/2a & x \in [-a, a] \\ 0 & \text{otherwise} \end{cases} \tag{7}$$

4. Another useful distribution is the exponential distribution:

$$p(x) = \begin{cases} \lambda e^{-\lambda x} & x \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

What is the probability $\text{Prob}(x_1 < x < x_2)$ where $x_1$ and $x_2$ are both positive.

## Extra questions

Do these in the workshop if you have time.

1. By integrating the formula for $\langle X \rangle$, what is the mean of the exponential distribution?
2. Work out the mean of the exponential distribution by integrating

$$1 = Z = \int_0^\infty p(x)dx$$

3. What is the variance of the exponential distribution?