Lecture 10: Random Variables

COMS10014 Mathematics for Computer Science A

cs-uob.github.io/COMS10014/ and github.com/coms10011/2020_21

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A random variable

A **random variable** is a map from outcomes to real numbers.

A dice example

If a pair of dice are rolled the space of outcomes might be the set of values:

$X = \{(1,1), (1,2), (1,3), (1,4), (1,5), (1,6), (2,1), (2,2), (2,3), (2,4), (2,5), (2,6), (3,1), (3,2), (3,3), (3,4), (3,5), (3,6), (4,1), (4,2), (4,3), (4,4), (4,5), (4,6), (5,1), (5,2), (5,3), (5,4), (5,5), (5,6), (6,1), (6,2), (6,3), (6,4), (6,5), (6,6)\}$

An random variable

Let S be the random variable which maps a dice roll to the total value:

$$S : (n, m) \mapsto n + m$$

so $S[(3, 2)] = 5$.

An event

A value of the random variable $S = 5$ corresponds to an event:

$$\{x \in X | S(x) = 5\} = \{(1, 4), (2, 3), (3, 2), (4, 1)\}$$

Probabilities of random variables

We write $p_S(s)$ to mean the probability of the event where $S = s$:

$$p_S(s) = P(\{x \in X | S(x) = s\})$$

Probability example

$$p_S(5) = P(\{(1,4),(2,3),(3,2),(4,1)\}) = \frac{4}{36} = \frac{1}{9}$$

Probability distribution

S	2	3	4	5	6	7	8	9	10	11	12
p_S	$\frac{1}{36}$	$\frac{1}{18}$	$\frac{1}{12}$	$\frac{1}{9}$	$\frac{5}{36}$	$\frac{1}{6}$	$\frac{5}{36}$	$\frac{1}{9}$	$\frac{1}{12}$	$\frac{1}{18}$	$\frac{1}{36}$

A table like this is called a **probability distribution**.

Frequencies

Often we interpret the probability as the frequency:

$$\frac{\text{number of times we get the value } S = s}{\text{number of samples we take}} \to p_S(s)$$