COMS10011 sample paper

This is a sample paper, it has the same rubric and the same style of question as the real paper, the layout is slightly different to the official exam layout.

Rubric

This paper contains *two* parts.
The first section contains *20* short questions.
Each question is worth *two marks* and all should be attempted.
The second section contains *three* long questions.
Each long question is worth *25 marks*.
The best *two* long question answers will be used for assessment.
The maximum for this paper is *90 marks*.
Calculators must have the Faculty of Engineering Seal of Approval.

Section A: short questions - answer all questions

1. A coin is flipped four times. What is the sample space?

   Solution: $\{TTTT, TTTH, TTHT, TTHH, THTT, THTH, THHT, THHH, HTTT, HTTH, HTHT, HTHH, HHTT, HHTH, HHHT, HHHH\}$.

2. A probability is a map P from events to real numbers. What are the three defining properties of a probability?

   Solution:

   (a) $P(A) \geq 0$ for all events.

   (b) $P(X) = 1$

   (c) If $A \cap B = \emptyset$ for two events A and B then

   $$P(A \cup B) = P(A) + P(B) \tag{1}$$

   where $A \cap B$ and $A \cup B$ are the intersection and union of A and B and $\emptyset$ is the empty set.

3. In stud poker you are given five cards from a normal pack of poker cards, that is with 13 cards labelled A, 2 to 10 and J, Q and K. Each value comes in one of four suits, spades, diamonds, clubs and hearts. A straight has all the cards in order, for example A2345 or 56789. Even though A is the lowest card the sequence 10JQKA is allowed, however, A2345 and 10JQKA are the only two sequences allowed which include A. In a straight flush the cards are all the same suit. What is the probability of getting a straight flush? You may use

   $$\left(\begin{array}{c} 52 \\ 5 \end{array} \right) = \frac{52 \times 51 \times 50 \times 49 \times 48}{1 \times 2 \times 3 \times 4 \times 5} = 2598960$$

and the answer can be left as a fraction.

Solution:

$$P = 40/2598960 = 1/64974$$

or if you exclude the royal flushes

$$P = 36/2598960$$

either answer is acceptable.

4. $X = \{1, 2, 3, 4, 5\}$ and $B = \{1, 5\}$ what is $X \setminus B$?

   Solution:

   $$x \setminus B = \{2, 3, 4\}$$

5. In a library where all books have blue or yellow spines, four fifths of books with yellow spines are about mathematics but only a fifth of books with blue spines are about mathematics. There are the same number of yellow and blue spined books, you come upom a book open on a table; the book is about mathematics. What is the chance it has a yellow spine?

   Solution: In the obvious notation we need $p(M)$:

   $$p(M) = p(M|B)p(B) + p(M|Y)p(Y) = 0.1 + 0.4 = 0.5$$

   and using Bayes:

   $$p(Y|M) = p(M|Y)p(Y)/p(M) = 0.4/0.5 = 0.8$$

6. Two four-sided dice are rolled and the two face values added up. Write down the probability table for the result.

   Solution: $p(2) = 1/16$, $p(3) = 1/8$, $p(4) = 3/16$, $p(5) = 1/4$, $p(6) = 3/16$, $p(7) = 1/8$ and $p(8) = 1/16$.

7. A six-sided dice is rolled five times, what is the chance of getting one exactly three times? You can leave your result as a fraction with powers.

   Solution:

   $$p = \left(\begin{array}{c} 5 \\ 3 \end{array} \right) \frac{1}{6}^3 \frac{5}{6}^2 = \frac{20 \times 5^2}{2 \times 6^5}$$

8. On average three buses arrive at a bus stop every hour in the afternoon. What is the chance that no buses will arrive between 1pm and 2pm?

   Solution:

   $$p = e^{-3}$$

9. What is

   $$\lim_{n \to \infty} \left(1 - \frac{x}{n}\right)^n$$

   Solution:

   $$\lim_{n \to \infty} \left(1 - \frac{x}{n}\right)^n = e^{-x}$$

10. If $F(x)$ is the cumulative for a continuous probability distribution, what is $\lim_{x\to\infty} F(x)$?

    Solution:

$$\lim_{x\to\infty} F(x) = 1$$

11. A Friedman test is a statistic test for comparing parametric data, true or false? Explain yourself in one sentence.

    (Answer: False this is used for comparing non-parametric data, i.e. with a skewed distribution)

12. A researcher has created five new drugs (A B C D E) for weight loss and he wishes to verify which one is the best. He devises an experiment in which he tests the effect of each drugs on weight loss (number of grams lost). The researcher wants to use T-test in order to compare the effect of the drugs. Using Bonferroni corrections, what is the new significance level that the researcher should use when looking for significant results when comparing each pair?

    (Answer: 0.005/10 because there are 10 comparisons made)

13. A politician claims that the dropout rate for schools is less than 25%. Last year, 190 out of 603 students dropped out. A researcher is aiming to looking for an evidence to reject the politician's claim, should he use a one tail or a two-tail statistical test and why?

    (Answer: one-tail is enough because the researcher is only interested in one direction. i.e. the dropout rate below 25%. If the question was 'equal to' we would use a two-tail test)

14. You are comparing two data sets and have done a two-tail unpaired t-test and the p-value returned is 0.08. How can you change the experimental design to increase your chances to have a p-values below the significant level of 0.05? Give two possible ways to do so.

    (Answers possible: increase the sample size, used paired data (within subject experiment), design the experiment as one-tail)

15. One research is performing a crowd-source study to understand the effect of web advertising layouts on the number of items bought by a website visitor. He has gathered data from 5000 participants and he is trying to verify if the data are following a normal distribution. He is using a Shapiro-Wilk test to do this. Is this a correct method to test the assumption of normality? Yes or No. Explain why.

    (Answer: False. Kolmogorov-Smirnov test is more appropriate for large sample size $N > 50$ and Shapiro-Wilk for small sample size $N <= 50$)

16. A scientist has found that the number of tornadoes since 2002 follow a very similar pattern than the number of shark attacks since 2002 (see graph). He claims that Tornadoes are causing the sharks to be stressed

and to attack more people. Is the scientist's reasoning correct? Yes or no. Explain why.

(Answer: No. The graphs show a correlation between the two data sets but not causality. A correlation does not mean that the change in one variable is the cause of the change in the values of the other variable.

17. Below is a list of variables that might be measured in a research study:

    (a) A person's length recorded in cm.

    (b) Whether a person is smoking, recorded as 'Yes' or 'No'.

    (c) How long a person worked for, recorded as the number of years.

    (d) The value of a house, recorded as 'under £100 000', '£100 000 - £500 000', '£500 000 - £1 000 000', 'over £1 000 000'.

    (e) The treatment group a person was in, recorded as 'Group 1', 'Group 2' and 'Group 3'.

    (f) The change in concentration of sugar in a person's blood, recorded as a percentage of the original. Write down whether each variable is categorical or numerical.

    (Answer: b, d, e are categorical; a, c, f are numerical.)

18. In a study, 20 participants were asked to write text using two different keyboard layouts: the classical QWERTY vs. a new one called DVORAK. DVORAK proponents claim the layout requires less finger motion and thus reduces errors compared to the standard QWERTY. Half of the participants started the task on the QWERTY layout and then the DVORAK and the other half of the participants started the task on the DVORAK layout and then the QWERTY. The number of words typed per minute was collected for each participant and each layout. Choose the most appropriate procedure to decide which layout allow participants to type the fastest.

    A Chi-squared test

    B Paired T-test

    C Unpaired T-test

    D Linear regression

    (Answer: B)

19. In a study, 40 participants were randomized to two groups. One group received a drug to decrease hair loss and the other group received a placebo (a pill of sugar). At the end of the program, the percentage hair loss for each patient was recorded. Choose the most appropriate procedure to decide if there is a relationship between the use of the drug and the percentage of hair loss:

A Chi-squared test

B Paired T-test

C Unpaired T-test

D Linear regression

(Answer: C)

20. A study attempted to find out if the age of an animal had any relationship to their athletic ability. The researchers took the data of 104 cheetahs, calculating their age and running a test to measure their speed. Choose the most appropriate procedure to decide if the age has any relationship with the run speed:

    A Chi-squared test

    B Paired T-test

    C Unpaired T-test

    D Linear regression

    (Answer: D)

Section B: long questions - answer two questions

1. This question is largely about the binomial distribution.

    (a) [5] A six-sided dice is rolled four times. What is the change of getting exactly two threes?

    (b) [5] A six-sided dice is rolled four times. What is the change of getting at least two threes?

    (c) [5] An n-sided dice is rolled s times with $n > 3$. What is the change of getting exactly r threes?

    (d) [5] If the chance of getting exactly r threes is $p(r)$ show

$$\sum_{r=0}^{s} p(r) = 1$$

    (e) [5] You have a three-sided dice and a six-sided dice. In a game you flip a coin, if the coin comes up heads, you roll the three-sided dice, if it comes up tails, you roll the six-sided dice. What is the chance of getting a three.

    If a six sided dice if rolled four times then the probability of exactly two threes is

$$p(2) = \left(\begin{array}{c} 4 \\ 2 \end{array} \right) \left(\frac{1}{6}\right)^2 \left(\frac{5}{6}\right)^2 \tag{2}$$

The chance of at least two threes is the same as the chance of anything except zero threes or one three:

$$p(\geq 2) = 1 - \left(\frac{5}{6}\right)^4 - 4\left(\frac{1}{6}\right)\left(\frac{5}{6}\right) \tag{3}$$

The more general case is

$$p(r) = \left(\begin{array}{c} s \\ r \end{array} \right) \left(\frac{1}{6}\right)^r \left(\frac{5}{6}\right)^{s-r} \tag{4}$$

The sum of $p(r)$ is one follows from the binomial theorem

$$(p+q)^s = \sum_{r=0}^{s} \left(\begin{array}{c} s \\ r \end{array} \right) p^r q^{s-r} \tag{5}$$

but with $q = 1 - p$. Finally the probability of getting a three in the coin flipping game is

$$p = \frac{1}{2}\frac{1}{6} + \frac{1}{2}\frac{1}{3} \tag{6}$$

2. This question has two parts, one is about Bayes's theorem, the other about Pearson's chi-square test.

   (a) [10] You want to go for a walk. However, when you wake up the day is cloudy and half of all raining days start of cloudy. On the other hand, two days in five start off cloudy and it's been rather dry recently with only rain only on one day in ten. What is the chance it will rain?

   (b) [15] A genetics engineer was attempting to cross a tiger and a cheetah. She predicted a phenotypic outcome of the traits she was observing to be in the following ratio: 4 stripes only; 3 spots only; 9 both stripes and spots. When the cross was performed, and she counted the individuals she found 50 with stripes only, 41 with spots only and 85 with both. According to the Chi-square test, did she get the predicted outcome? Use the table below with a significance level of 0.05 to check.

Degrees of freedom (df)	χ^2 value										
1	0.004	0.02	0.06	0.15	0.46	1.07	1.64	2.71	3.84	6.63	10.83
2	0.10	0.21	0.45	0.71	1.39	2.41	3.22	4.61	5.99	9.21	13.82
3	0.35	0.58	1.01	1.42	2.37	3.66	4.64	6.25	7.81	11.34	16.27
4	0.71	1.06	1.65	2.20	3.36	4.88	5.99	7.78	9.49	13.28	18.47
5	1.14	1.61	2.34	3.00	4.35	6.06	7.29	9.24	11.07	15.09	20.52
6	1.63	2.20	3.07	3.83	5.35	7.23	8.56	10.64	12.59	16.81	22.46
7	2.17	2.83	3.82	4.67	6.35	8.38	9.80	12.02	14.07	18.48	24.32
8	2.73	3.49	4.59	5.53	7.34	9.52	11.03	13.36	15.51	20.09	26.12
9	3.32	4.17	5.38	6.39	8.34	10.66	12.24	14.68	16.92	21.67	27.88
10	3.94	4.87	6.18	7.27	9.34	11.78	13.44	15.99	18.31	23.21	29.59
P value (Probability)	0.95	0.90	0.80	0.70	0.50	0.30	0.20	0.10	0.05	0.01	0.001

For the Bayes question

$$p(\text{rain}|\text{cloudy}) = \frac{p(\text{cloudy}|\text{rain})p(\text{rain})}{p(\text{cloudy})} = \frac{0.5 * 0.1}{0.4} = 0.125 \qquad (7)$$

For the chi-square question

expected ratio	observed #	expected #	$O - E$	$(O - E)^2$	$(O - E)^2/E$
4 stripe	50	44	6	36	0.82
3 spots	41	33	8	64	1.94
9 stripes/spots	85	99	-14	196	1.98
total	176	176	0		4.74

where $4/16 * 176 = 44$, $3/16 * 176 = 33$, $9/16 * 176 = 99$ and the degrees of freedom are $3 - 1 = 2$ since there are three characteristics: spots, stripes and both. 4.74 is less than 5.991 so we can accept the null hypothesis.

3. You have designed a horror game in which a sound of heartbeat is played during the game. Your assumption is that the sound of the heartbeat make the game frightening. You wish to design an experiment to investigate if adding the sound of the heartbeat does increase the actual heartbeat of the player. You set out to make 20 participants play two versions of your game: one with the sound of heartbeat is played, one with no sound. Using a heartbeat sensor, you measure the heart rate of the participants before playing and after and note the difference.

   (a) [6] What are your independent and dependent variables
   Answer: dependant = different in heartbeat / independent = sound of heartbeat or not

(b) [8] Are you doing a within or between experiment and why?

Answer: it is possible to do a within-subjects experiment as long as the portion of the game tested are changed between the sound vs. no sound. You would have to make sure the portion of the game used are comparable (same level of fear). Doing it within would be better because the data are paired and thus you would need less participants. You can also do a between-subjects study in which one group of participants would test the same portion of the game with sound and another group of participants would test it without. In such case you will be surer that the games are comparable (in term of level of fear) but you will need more participants.

(c) [6] Do you need to use counterbalancing or not and why?

Answer: in the within case, you would need to make one half of the participants start with the 'no sound' condition and then the 'sound' one. The other half of participants would do the opposite.

(d) [5] Imagine what is the task that the participants are going to do. [4 marks]

Answer: the participants are asked to sit down in front of the TV screen and to take the Xbox controller. The experimenter first measures the participants rest heartbeat. The game starts, and the participants are asked to find a specific item in the game. Once finished the experimenter measures the participants current heartbeat.