

Example Lecture Notes

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The following are my lecture notes from 2/12/24. I think they provide an adequate example of how to take notes using my framework. See homework.tex for an example of truth tables.

1 OR Elimination

Going for conclusion $(P \vee Q) \rightarrow R$:

$$\begin{array}{ccc} & P \vee Q & \\ & \begin{array}{cc} P & Q \\ P \rightarrow R & Q \rightarrow R \\ R & R \end{array} & \\ & (P \vee Q) \rightarrow R & \end{array}$$

Think about:

- Strategy of order
- OR elimination from two separate subproofs to get the desired conclusion

2 Truth tables

- All the answers we've gotten could have been gotten by truth tables

3 RAA

- Last rule
- Had to meditate before lecture – not a class about speculation

3.1 Philosophy of RAA

- Strange rule
- Says "if, from a set of premises, you can derive a contradiction, then something in that set of premises must be false"
- "Indirect proof" or "proof by contradiction"
- If you use logic to derive something that no sane person would believe, it means that, somewhere, there is a rotten apple that you need to get rid of

3.2 Example

Goal: To show DeMorgan's Law $\neg(P \vee Q) \vdash \neg P$

1	(1)	$\neg(P \vee Q)$	A
2	(2)	P	A (goal: contradiction)
2	(3)	$P \vee Q$	$\vee I$ 2
1, 2	(4)	$(P \vee Q) \wedge \neg(P \vee Q)$	$\wedge I$ 3,1
1	(5)	$\neg P$	RA 2,4

3.3 Famous Result – Ex Falso Quodlibet (From the False, Everything Follows)

$P, \neg P \vdash Q$

1	(1)	P	A
2	(2)	$\neg P$	A
3	(3)	$\neg Q$	A
1, 2	(4)	$P \wedge \neg P$	$\wedge I$ 1,2
1, 2	(5)	$\neg\neg Q$	RA 3,4
1, 2	(6)	Q	DN 5

- Don't worry—not bad logic
- As long as you believe consistent things, you can't do this

3.4 Instead of $\vee E$

$P \vee Q, \neg P \vdash Q$

1	(1)	$P \vee Q$	A
2	(2)	$\neg P$	A
3	(3)	$\neg Q$	A
2, 3	(4)	$\neg P \wedge \neg Q$	$\wedge I$ 2,3
1, 2, 3	(5)	$(P \vee Q) \wedge (\neg P \wedge \neg Q)$	$\wedge I$ 1,4
1, 2	(6)	Q	RA 3,5

3.5 Show $\neg(\neg P \vee Q) \vdash \neg(P \rightarrow Q)$

1	(1)	$\neg(\neg P \vee Q)$	A
2	(2)	$P \rightarrow Q$	A (goal: RA)
1	(3)	$\neg\neg P \wedge \neg Q$	DM 1
1	(4)	$\neg\neg P$	$\wedge E$ 3
1	(5)	P	DN 4
1	(6)	$\neg Q$	$\wedge E$ 3
1, 2	(7)	Q	MP 2,5
1, 2	(8)	$Q \wedge \neg Q$	$\wedge I$ 6,7
1	(9)	$\neg(P \rightarrow Q)$	RA 2,8

4 Laws

4.1 Law of Non-Contradiction

$\vdash \neg(P \wedge \neg P)$

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|---|-----|-------------------------|--------|
| 1 | (1) | $P \wedge \neg P$ | A |
| | (2) | $\neg(P \wedge \neg P)$ | RA 1,1 |

4.2 Excluded Middle

$\vdash P \vee \neg P$

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|------|-----|--|---|
| 1 | (1) | $\neg(P \vee \neg P)$ | A (goal: RA) |
| 2 | (2) | P | A |
| 2 | (3) | $P \vee \neg P$ | $\vee I$ 2 |
| 1, 2 | (4) | $(P \vee \neg P) \wedge \neg(P \vee \neg P)$ | $\wedge I$ 1,3 |
| 1 | (5) | $\neg P$ | RA 2,4 |
| 1 | (6) | $P \vee \neg P$ | $\vee I$ 5 |
| 1 | (7) | $(P \vee \neg P) \wedge \neg(P \vee \neg P)$ | $\wedge I$ 1,6 |
| | (8) | $\neg\neg(P \vee \neg P)$ | RA 1,7 |
| | (9) | $P \vee \neg P$ | DN 8 (for non-heretical mathematicians) |