MA2002 Midterm Cheatsheets

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1 Definition of Limits

Let f be defined on an **open** interval about c, **possibly excluding** c. The limit of f as x approaches c is L, denoted as

$$\lim_{x \to c} f(x) = L$$

if $\forall \epsilon > 0$, $\exists \delta > 0$ such that

$$0 < |x - c| < \delta \implies |f(x) - L| < \epsilon$$

2 Limit Rules

If **both limits exist**, then any well-defined algebraic operation can be performed on the limits (limits can be added, multiplied, taken to rational power, etc).

3 Limit Inequality Theorem

Suppose $f(x) \geq g(x)$ $\forall x$ in open interval containing c, except possibly at $x = c$ itself.

$$\textbf{If, } \lim_{x \to c} f(x) = L \text{ and } \lim_{x \to c} g(x) = M$$

$$\textbf{Then, } L \geq M$$

4 Squeeze Theorem

Suppose $g(x) \leq f(x) \leq h(x)$ $\forall x$ in open interval containing c, except possibly at $x = c$ itself.

$$\textbf{If, } \lim_{x \to c} g(x) = \lim_{x \to c} h(x) = L$$

$$\textbf{Then, } \lim_{x \to c} f(x) = L$$

Suppose $\lim_{x \to a} \frac{f(x)}{g(x)} = L$ and $\lim_{x \to a} g(x) = 0$, then $\lim_{x \to a} f(x) = 0$. Otherwise, we can use product limit laws to create a contradiction.

4.1 Definition of Limits as $x \to \pm\infty$