# Typesetting Flow Graphs with tikz

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These notes attempt to explain how to use the LATEX package tikz to type-set flow graphs, used for indicating the flow of information in different kinds of proofs. The code can be downloaded from https://github.com/consequently/flowgraphs.

In your LATEX document, to set things up, load tikz in the preamble and define some colours.

```
\usepackage{tikz}
\definecolor{Red}{rgb}{0.66,0,0}
\definecolor{Green}{rgb}{0,0.5,0}
\definecolor{Blue}{rgb}{0,0,0.66}
```

In this document, I will typeset some proofs and derivations. For that, I'll use ebproof, so I need to load that package too:

#### \usepackage{ebproof}

Both tikz and ebproof are available on ctan, and they should be already on any reasonably up-to-date LATEX installation.

To typeset flow graphs with tikz, you use two definitions. The first, \tm (short for "atom") defines the nodes for the graph:

```
\newcommand{\tm}[2]{%
  \ensuremath{\mathord{%}
  \tikz[remember picture,baseline=(#2.base)]%
  \node[inner sep=.5pt,outer sep=.5pt](#2){\({#1}\)};%
}}%
}
```

 $\mathbf{p}_{p}^{p}$  typesets p in math mode, inside a tikz picture, as a node with label p1, which can be used as the source or a target for a link in a graph, typest later in the code. To typest the links, you use a tikzpicture with the flowgraph style:

Such a tikzpicture is typeset as an *overlay* (it is typeset over the other text), and the default links are set in the color Red, with the tikz style thick, as arrows with the "stealth" arrowhead, and shortened at the end by a little bit (for clarity). Of course, these defaults can all be overridden. (In the examples below you'll see Blue and Green links, too.)

#### The General Idea

You can mark an atom like this p, typesetting it in a small tikz picture, where you remember its node name and location, and do the same with another one like this—p—and then typeset a link from the one to the other.

```
You can mark an atom like this \tm{p}{p1}, typesetting it in a small tikz picture, where you remember its node name and location, and do the same with another one like this---\tm{p}{p2}---and then typeset a link from the one to the other.

\begin{tikzpicture}[flowgraph]
\draw (p1) -- (p2);
\end{tikzpicture}
```

Here are some concrete examples of using flow graphs in proofs.

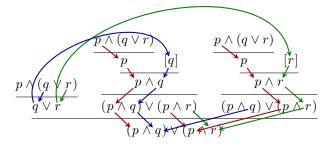
## Example 1: In an Array



```
\[
\begin{array}{c}
\tm{p}{pp}\land(\tm{q}{qp}\lor \tm{r}{rp}) \\[0.66cm]
(\tm{p}{p1c}\land \tm{q}{qc})\lor(\tm{p}{p2c}\land \tm{r}{rc})
\end{array}
\]
\begin{tikzpicture}[flowgraph]
\draw (pp) -- (p1c); \draw (pp) -- (p2c);
```

```
\draw (qp) -- (qc); \draw (rp) -- (rc);
\end{tikzpicture}
```

# Example 2: In a Natural Deduction Proof

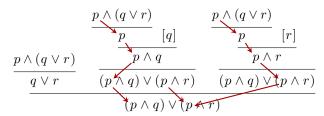


Using different colours can help with distinguishing different paths.

```
% Vertical space to give room for looping links
\vspace{0.5cm}
\begin{prooftree}[rule margin=.9ex, separation=0.2em]
\mbox{Hypo}{p\and(\tm{q}{q1}\lor \tm{r}{r1})}
\label{limit} $$ \prod_{q}{q2}\leq \lim_{r}{r2}$
\t \{p}{p1}\\ r)
Infer1{tm{p}{p3}}
\Hypo{[\tm{q}{q3}]}
\Infer2{\tm{p}{p5}\land \tm{q}{q4}}
\t {p}{p2}\t r)}
\Infer1{	m{p}{p4}}
\Hypo{[\tm{r}{r}]}
\Infer2{\tm{p}{p6}\land \tm{r}{r4}}
\label{limits} $$ \prod_{p}{p1}\quad \int_{q}{q7} \color(\tm{p}{p12}\) \
\end{prooftree}
\]
%% p links
\begin{tikzpicture}[flowgraph,bend angle=90]
\draw (p1) -- (p3); \draw (p3) -- (p5); \draw (p5) -- (p7); \draw (p7) -- (p11); \draw (p2) -- (p4); \draw (p4) -- (p6);
\draw (p6) -- (p10); \draw (p8) -- (p12);
\draw (p10) -- (p12);
%% q links
\begin{scope}[Blue]
\draw (q1) -- (q2);
\path (q2) edge[bend left] (q3);
```

```
\draw (q3) -- (q4); \draw (q4) -- (q5);
\draw (q5) -- (q7); \draw (q6) -- (q7);
\end{scope}
%% r links
\begin{scope}[Green]
\draw (r1) -- (r2);
\path (r2) edge[bend left] (r3);
\draw (r3) -- (r4); \draw (r4) -- (r6);
\draw (r5) -- (r7); \draw (r6) -- (r7);
\end{scope}
\end{tikzpicture}
```

However, there it is still difficult to follow such a complex flow graph with many different links at play. You can reuse the same derivation code and link up only *some* of the atoms. The new node labels take precedence over the old ones.

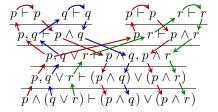


Here's the code:

```
١[
\begin{prooftree}[rule margin=.9ex, separation=0.2em]
\mbox{Hypo}{p\and(\tm{q}{q1}\lor \tm{r}{r1})}
\left\{ \frac{q}{q2} \right\} 
\t \{p}{p1}\
\Infer1{	m{p}{p3}}
\Hypo{[\tm{q}{q3}]}
\Infer2{\tm{p}{p5}\land \tm{q}{q4}}
\label{limin_to_the_p} $$ \prod_{p}{p7} \land \\ tm{q}{q5} \to (tm{p}{p8} \land tm{r}{r5}) $$
\t \{p}{p2}\t r)
\Infer1{\{tm\{p\}\{p4\}\}}
\Hypo{[\tm{r}{r3}]}
\label{limits} $$ \prod_{p}{p11}\quad \lim_{q}{q7} \or(\lim_{p}{p12}\and \lim_{r}{r7}) $$
\end{prooftree}
\]
\begin{tikzpicture}[flowgraph,bend angle=90]
\draw (p1) -- (p3); \draw (p3) -- (p5);
\draw (p5) -- (p7); \draw (p7) -- (p11);
\draw (p2) -- (p4); \draw (p4) -- (p6);
\draw (p6) -- (p10); \draw (p8) -- (p12);
```

```
\draw (p10) -- (p12);
\end{tikzpicture}
```

### Example 3: In a Sequent Derivation



```
\begin{prooftree}
\footnote{The monotonian of the points of 
\footnote{Mypo{\tm{q}}{q1}\vdash \tm{q}{q2}}
\label{lem:linear} $$ \prod_{p}{p5}, \lim_{q}{q3}\varepsilon  \tm{p}{p6}\label{linear} $$ \operatorname{linear}(q4)$
<caption> \t \{p}{p3} \t \{p3}{p4}
\ \frac{1}\operatorname{tm}{r}{r}\
\label{lem:linear} $$ \prod_{p}{p},\int_{r}{r3}\operatorname{tm}{p}{p8}\label{linear} $$ \int_{r}{r4}} $$
\label{liminary} $$ \prod_{p}{p}, \lim_{q}{q5} \ln \tm{r}{r5} $$
\label{liminary} $$ \prod_{p}{p12}, \lim_{q}{q7} \ln \int_{r}{r7} \operatorname{d} \theta $$
\label{lambda} $$ (\int_{p}{p13}\ \int_{q}{q8}) \int_{p14}\ \int_{r}{r8})$
\end{prooftree}
\]
\begin{tikzpicture}[flowgraph,bend angle=60]
%%%
\path (p1) edge[bend left] (p2);
\path (p3) edge[bend left] (p4);
\draw (p15) -- (p12); \draw (p12) -- (p9);
\draw (p9) -- (p5); \draw (p9) -- (p7);
\draw (p7) -- (p3); \draw (p5) -- (p1);
\draw (p2) -- (p6); \draw (p6) -- (p10);
\draw (p10) -- (p13); \draw (p13) -- (p16);
\draw (p4) -- (p8); \draw (p8) -- (p11);
\draw (p11) -- (p14); \draw (p14) -- (p17);
%%%
\begin{scope}[Blue]
\path (q1) edge[bend left] (q2);
\draw (q3) -- (q1); \draw (q5) -- (q3);
\draw (q7) -- (q5); \draw (q9) -- (q7);
\draw (q2) -- (q4); \draw (q4) -- (q6);
```

```
\draw (q6) -- (q8); \draw (q8) -- (q10);
\end{scope}
%%%
\begin{scope}[Green]
\path (r1) edge[bend left] (r2);
\draw (r3) -- (r1); \draw (r5) -- (r3);
\draw (r7) -- (r5); \draw (r9) -- (r7);
\draw (r2) -- (r4); \draw (r4) -- (r6);
\draw (r6) -- (r8); \draw (r8) -- (r10);
\end{scope}
\end{tikzpicture}
```

Again, the thicket of arrows in over an entire sequent derivation makes a flow graph a little hard to scan. You can reuse the code for the derivation and draw a flow graph on its *endsequent*.

$$\frac{p \vdash p \quad q \vdash q}{\underbrace{p, q \vdash p \land q}} \quad \frac{p \vdash p \quad r \vdash r}{p, r \vdash p \land r}$$
$$\frac{p, q \lor r \vdash p \land q, p \land r}{\underbrace{p, q \lor r \vdash (p \land q) \lor (p \land r)}}$$
$$\underbrace{p \land (q \lor r) \vdash (p \land q) \lor (p \land r)}$$

```
١[
\begin{prooftree}
\Hypo{	tm{p}{p1}\vdash \tm{p}{p2}}
\Hypo{\tm{q}{q1}\vdash \tm{q}{q2}}
\label{lem:linear} $$ \prod_{p}{p5}, \lim_{q}{q3}\varepsilon  \tm{p}{p6}\label{linear} $$ \operatorname{linear}(q4)$
<caption> \t m{p}{p3}\v dash \t m{p}{p4}}
\Hypo{\tm{r}{r1}\vdash \tm{r}{r2}}
\label{lem:linear2} $$ \prod_{p}{p7}, \lim\{r}{r3}\operatorname{tm}{p}{p8}\label{linear2} $$ \operatorname{tm}{r}{r4}$$
\label{liminary} $$ \prod_{p}{p9}, \lim_{q}{q5} \ln \tm{r}{r5} $$
\tm{p}{p10}\land \\tm{q}{q6},\\tm{p}{p11}\land \\tm{r}{r6}\}
\label{limin_p} $$ \prod_{p}{p12}, \lim_{q}{q7} \ln \operatorname{tm}{r}{r7} \
\label{land $$ \int_{q}{q8}} \operatorname{tm}{p}{p13}\land $$ \int_{q}{q8}} \operatorname{tm}{p}{p14}\land $$ \int_{q}{q8}} 
\left( \frac{q}{q9} \right) 
\end{prooftree}
/]
\begin{tikzpicture}[flowgraph,bend angle=45]
\path (p15) edge[bend right] (p16);
\path (p15) edge[bend right] (p17);
\path (q9) edge[bend right, Blue] (q10);
\path (r9) edge[bend right, Green] (r10);
\end{tikzpicture}
```