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## Control and protection of the HVDC/AC electrical grids

### Assignment 2: MTDC control

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The material for this assignment is adapted from the paper:

M. Aghahadi, L. Piegari, **A. Lekić** and A. Shetgaonkar, "Sliding Mode Control of the MMC-based Power System," *IECON 2022 – 48th Annual Conference of the IEEE Industrial Electronics Society*, Brussels, Belgium, 2022, pp. 1-6, doi: 10.1109/IECON49645.2022.9968871.

# 1. System description

## 1.1 overview

The analyzed system is considered by four MMCs connected by the MTDC scheme in the system under study (Fig. 1).

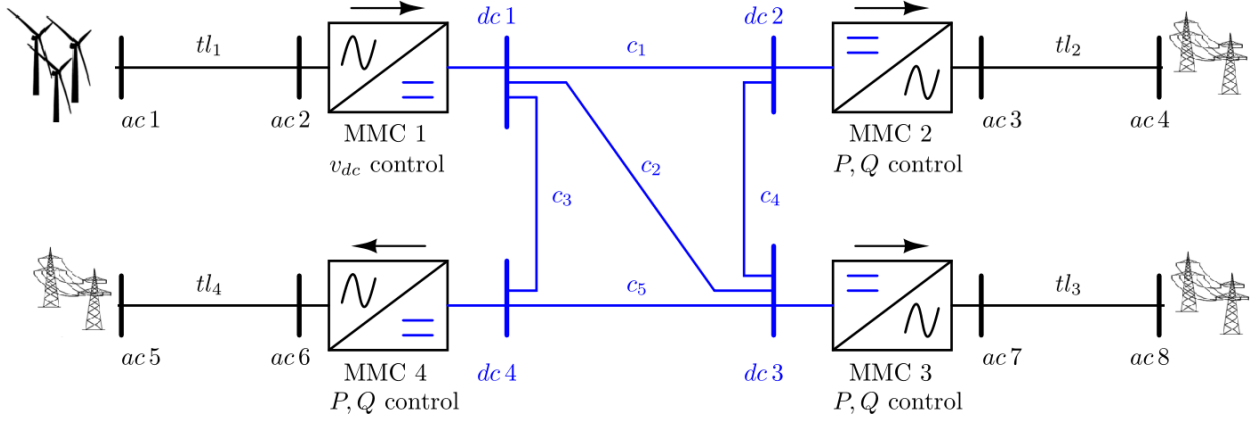


Fig. 1. The MTDC system under study.

In the master-slave control approach, the master MMC regulates the voltage across the system, while other MMCs are responsible for controlling active power flows, and reactive power can be controlled by all MMCs independently. For convenience, it is assumed that all MMCs and transmission lines have the same characteristics, such as modeling and parameters, although they can be different in practical cases.

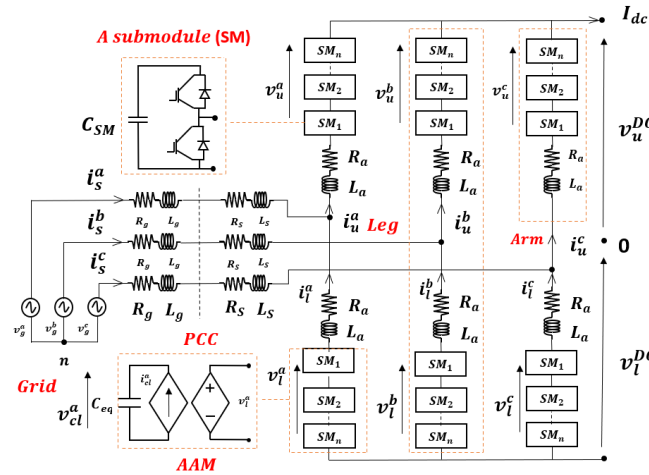


Fig. 2. Circuit diagram of the MMC and the grid Thevenin equivalent.

A typical MMC (Fig.2) equivalently has three legs and three phases. Each leg has two arms with  $N_{arm}$  half-bridge submodules connected to a capacitor ( $C_{sm}$ ). The key point of the control scheme is to calculate the voltages of six arms ( $v_{ul}^{abc}$ ) for balancing the energy to avoid extra energy losses in the MMC and to exchange the power from the AC side to the DC side or vice versa smoothly.

A typical control scheme for the MMC is shown in Fig. 3. At the top, the output current control is presented, consisting of a voltage controller for the master MMC, or an active power controller for the slave MMC and reactive power control for both.

At the bottom, the circulating current (CC) and energy balancing control are presented. At the top-left, a phase-locked loop (PLL) is presented that can estimate an angle between grid voltage and the point of common connection (PCC) for control loops.

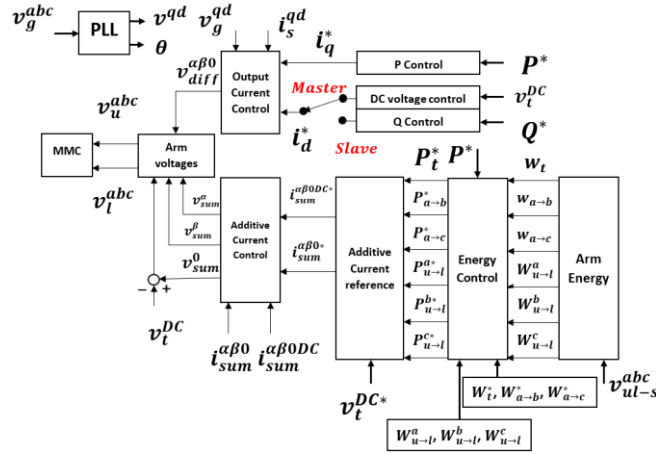


Fig. 3. A classical control scheme of MMC.

## 1.2 Dynamic modeling of the system:

### 1.2.1 MMC and Grid

Applying the KVL to each phase ( $k = a, b, c$ ), it is possible to write:

$$V_u^{DC} - v_u^k - v_g^k - v_n = R_a i_u^k + L_a \frac{di_u^k}{dt} + (R_s + R_g) i_s^k + (L_s + L_g) \frac{di_s^k}{dt} \quad (1)$$

$$-V_l^{DC} - v_l^k - v_g^k - v_n = -R_a i_l^k - L_a \frac{di_l^k}{dt} + (R_s + R_g) i_s^k + (L_s + L_g) \frac{di_s^k}{dt} \quad (2)$$

Where the lower and upper arms, and grid currents for the generic  $k$  phase are indicated by  $i_l^k$ ,  $i_u^k$ , and  $i_s^k$  respectively. The resistances and inductances of the grid and arm are denoted by  $R_g$ ,  $R_a$ ,  $L_g$ , and  $L_a$ . Furthermore, the voltages related to the DC side, grid, and arm are denoted by  $V_u^{DC}$ ,  $V_l^{DC}$ ,  $V_g^k$ , and  $V_u^k$ ,  $V_l^k$  respectively.

To obtain the expressions that describe the differential and additive quantities, it is necessary to sum and subtract equations (1) and (2). These equations provide useful insights into the way MMC works. Additionally, these diagonalize the dynamic operation and can decouple voltage and current variables. New variables are defined as follows:  $v_{diff}^k \triangleq 0.5 (-v_u^k + v_l^k)$ , differential voltage.  $v_{sum}^k \triangleq v_u^k + v_l^k$ , additive voltage.  $i_{sum}^k \triangleq 0.5 (i_u^k + i_l^k)$ , additive current  $R_{eq} \triangleq R_s + R_g + \frac{R_a}{2}$ .  $L_{eq} \triangleq L_s + L_g + \frac{L_a}{2}$  ( $R_s$ ,  $L_s$  are neglected as just slight voltage drop),  $v_t^{DC} \triangleq v_u^{DC} + v_l^{DC}$ , ( $v_u^{DC} \approx v_l^{DC}$ ).

Therefore, MMC can be defined by two equations as follows:

$$V_{diff}^k - v_g^k - v_{n0} = R_{eq} i_s^k + L_{eq} \frac{di_s^k}{dt} \quad (3)$$

Differential equation interprets the relation of the MMC and AC side.

$$V_{sum}^k - v_t^{DC} = -2R_{eq}i_{sum}^k - 2L_{eq}\frac{di_{sum}^k}{dt} \quad (4)$$

The additive equation interprets the relation of the MMC and circulating currents. Equations (3), (4) can identify the key degrees of freedom of MMC and help to design the controller more conveniently. Generally, the currents  $i_{sum}^k$  and  $i_s^k$  have AC and DC components. Indeed, each component has a special responsibility in controlling MMC and power exchange.

For control purposes, analyzing these currents in positive, negative, and zero sequences is suggested.

According to the average arm model (AAM), the relation between charging current and real capacitor voltage for six arms when power is exchanged can be expressed as [1]:

$$i_{Cul}^{abc} = C_{eq} \mathbf{I}_6 \frac{dV_{Cul}^{abc}}{dt} \quad (5)$$

Where,  $\mathbf{I}_6 \in \mathcal{R}^{6 \times 6}$ , is an identity matrix.

### 1.3 Control System

#### 1.3.1 Overview

Generally, the designing control system is done in a synchronous reference  $dq$ -frame to control active and reactive power separately. The overall scheme is presented in Fig. 3. If PLL is locked properly ( $v_g^d = 0$ ), the active and reactive power references can be calculated as  $i_q^* = \frac{2}{3} \frac{P^*}{v_g^q}$ ,  $i_d^* = \frac{2}{3} \frac{Q^*}{v_g^q}$ . Generally, both equations are used for slave MMCs, but  $i_q^*$  must be calculated by a voltage regulator in master MMC to keep reference voltage constant. The equation (3) rewrites in  $dq$ -frame by Park transformation as follows:

$$\begin{aligned} V_{diff}^q &= -R_{eq}i_s^q - L_{eq}\frac{di_s^q}{dt} + v_g^q - L_{eq}\omega i_s^d \\ V_{diff}^d &= -R_{eq}i_s^d - L_{eq}\frac{di_s^d}{dt} + v_g^d + L_{eq}\omega i_s^q \end{aligned} \quad (6)$$

Where  $\omega$  is the grid angular frequency.

As mentioned before, an energy controller should balance the six MMC arms. To do that, total energy and leg horizontal difference energies calculated based on equation (5) as follows:

$$W_t = W_u^a + W_l^a + W_u^b + W_l^b + W_u^c + W_l^c \quad (7)$$

$$W_{a \rightarrow b} = (W_u^a + W_l^a) - (W_u^b + W_l^b)$$

$$W_{a \rightarrow c} = (W_u^a + W_l^a) - (W_u^c + W_l^c)$$

$$W_{u \rightarrow l}^k = W_u^k - W_l^k$$

Then, the equivalent reference powers  $P_t^*$ ,  $P_{a \rightarrow b}^*$ , and  $P_{a \rightarrow c}^*$  can be calculated by the values of equations (8) as well as their reference values. Generally, it can be done by a simple PI controller regarding the equation  $\frac{dW}{dt} = P$ .

$$W_t^* = 6 * 0.5 \frac{C_{SM}}{N_{cm}} (v_t^{DC*})^2 \quad (8)$$

The calculated reference powers in the  $abc$ -frame can be used for calculating the DC grids current ( $i_{sum}^{0DC*}$ ) and DC circulating current ( $i_{sum}^{\alpha\beta DC*}$ ) in the  $\alpha\beta 0$ -frame (Clarke transformation). This concept is shown as follows:

$$\begin{bmatrix} i_{sum}^{\alpha DC*} \\ i_{sum}^{\beta DC*} \\ i_{sum}^{0 DC*} \end{bmatrix} = \frac{1}{3v_t^{DC*}} \begin{bmatrix} 0 & 1 & 1 \\ 0 & \sqrt{3} & -\sqrt{3} \\ 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_t^* \\ P_{a \rightarrow b}^* \\ P_{a \rightarrow c}^* \end{bmatrix} \quad (9)$$

Where  $v_t^{DC*}$  is the reference value of the DC side

Finally, the additive voltages are calculated by the equivalent circuit in  $\alpha\beta 0$ -frame as follows:

$$\begin{bmatrix} v_{sum}^{\alpha} \\ v_{sum}^{\beta} \\ v_{sum}^0 \end{bmatrix} - v_t^{DC} \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = -2R_a \mathbf{I}_3 \begin{bmatrix} i_{sum}^{\alpha} \\ i_{sum}^{\beta} \\ i_{sum}^0 \end{bmatrix} - 2L_a \mathbf{I}_3 \frac{d}{dt} \begin{bmatrix} i_{sum}^{\alpha} \\ i_{sum}^{\beta} \\ i_{sum}^0 \end{bmatrix} \quad (60)$$

Where,  $\mathbf{I}_3 \in \mathcal{R}^{3 \times 3}$  is the identity matrix

Finally, modulation indices can be calculated using reference voltages  $v_{ul}^{abc}$  as follows:

$$m_{ul}^{abc} = \frac{v_{ul}^{abc}}{v_{cul}^{abc}} \quad (71)$$

## 1.4 Sliding mode control

It is proven that if there is a surface in state space that is equal to zero, states are inclined to move to this surface and if the system dynamic is stable, states will move to the desired point. In other words, by defining a surface so-called sliding surface, it is possible to work with a simple equation and scalar variables instead of working with the set of differential equations and vector variables.

The sliding surface or switching surface in a first-order system can be defined as  $S \triangleq$  error of the system. Generally, the sliding surface is defined with tracking error, therefore,  $S$  is inclined to zero, and all states reach desired values that are reference values. When the states reach the surface, the controller for keeping the condition must use the sign function ( $\text{sign}(S)$ ). It leads to chattering phenomena. The sign function is a discontinuous function, and the control law will be discontinuous around  $S = 0$ . Indeed, chattering is a big problem for the SMC; it leads to high control activity and can enable the high order modes which are eliminated during modelling. Chattering can be decreased significantly by using high-order SMC [12].

The sliding condition is based on this concept if a signal is positive and the derivative of that is negative, that signal will move to zero and it is defined as follows:

$$S\dot{S} \leq -\eta|S| \quad (12)$$

$\eta$  is a constant and is one of the parameters of design and  $\dot{S}$  is the first derivative of the sliding surface.

The control input in SMC is defined as follows:

$$v = v_{eq} + v_{sw} \quad (14)$$

$v_{eq}$  can be calculated according to Filippov's Construction of the Equivalent Dynamics by solving the equation  $\dot{S}=0$ . Moreover, there are many options to find the switching input ( $v_{sw}$ ) concerning equation (12). But one of the best choices is defined as follows:

$$v_{sw} = -k_1 \text{sign}(S) - k_2 S \quad (15)$$

The variables  $k_1$  and  $k_2$  can be tuned to achieve the best response. Additionally, this equation is modified in this paper, while for the standard first-order SMC,  $k_2 = 0$ . In fact, the equivalent input holds the system trajectory on the sliding surface, whereas switching input keeps the response near the sliding surface.

### 1.4.1 Designing output current control using SMC

Equation (7) shows the dynamic of the output current of MMC. The current error in the  $dq$ -frame is defined as:

$$e_q = i_s^{q*} - i_s^q \quad (16)$$

$$e_d = i_s^{d*} - i_s^d \quad (178)$$

The sliding surfaces are defined as equal to errors and the derivative of sliding surfaces are represented as:

$$\dot{S}_q = \dot{e}_q = -\frac{di_s^q}{dt} - \frac{1}{L_{eq}}(v_{diff}^q + R_{eq}i_s^q - v_g^q + L_{eq}\omega i_s^d) \quad (18)$$

$$\dot{S}_d = \dot{e}_d = -\frac{di_s^d}{dt} - \frac{1}{L_{eq}}(v_{diff}^d + R_{eq}i_s^d - v_g^d - L_{eq}\omega i_s^q) \quad (19)$$

Equivalent inputs are calculated from equations (18)-(19) and  $\dot{S}=0$  given as:

$$v_{eq}^q = -R_{eq}i_s^{q*} + v_g^q - L_{eq}\omega i_s^{d*} \quad (20)$$

$$v_{eq}^d = -R_{eq}i_s^{d*} + v_g^d + L_{eq}\omega i_s^{q*} \quad (21)$$

In addition, switching inputs can be as follows:

$$v_{sw}^q = -L_{eq}(k_{1q} \text{sign}(S_q) + k_{2q}S_q) \quad (22)$$

$$v_{sw}^d = -L_{eq}(k_{1d} \text{sign}(S_d) + k_{2d}S_d) \quad (22)$$

And finally, control outputs are represented:

$$v_{diff}^q = v_{eq}^q + v_{sw}^q \quad (23)$$

$$v_{diff}^d = v_{eq}^d + v_{sw}^d \quad (24)$$

The second terms switching inputs lead to states moving to sliding surfaces; they have an inverse relationship with rise time. In addition, oscillation can be decreased by reducing the ,  $k_{1q}, k_{1d}$ . The control scheme is shown in Fig. 4.

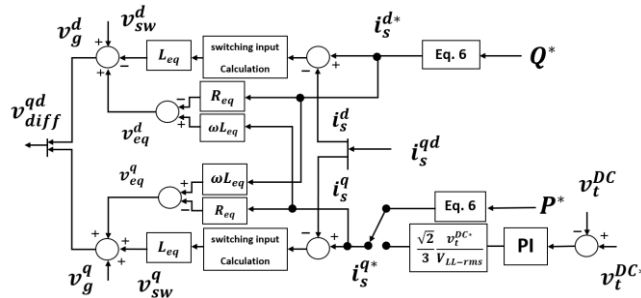


Fig. 4. Control scheme of output current control using SMC.

The lumped model can be in the simple and  $\pi$  models which is dependent on the geometry of the transmission line, and it is not suitable for state-space form. The distributed model is created by modelling the cable into  $N$  parallel branches and  $M$  series circuits. When numbers of  $n$ , and  $m$  are considered infinite, a wideband, or universal line model (ULM) [15] is proposed which is frequency dependent. Additionally, there are many models that can be found in the best path project [16]. In this paper, an equivalent circuit with  $N = 3$ , and  $M = 1$  is considered for the sake of simplicity (Fig. 6).

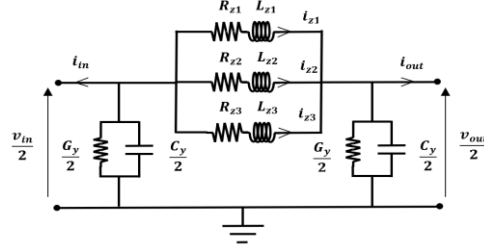


Fig. 6. Equivalent circuit for an underground cable.

All the lines in this paper have the same characteristics but with different lengths. The cable parameters are presented in Table 3.

## 2. Simulation results

During the case study lasting 3 s, two different scenarios are analyzed to depict the performance of the controller. The test is based on loading and disconnecting the load by maximum changes from zero to nominal load or vice versa (the worst cases). MMC1 and MMC3 behave as the power senders, while MMC2 and MMC4 as the power receivers.

**Case A:** The active and reactive power demands are connected at  $t = 0.5$  s, and  $t = 1$  s, respectively from 0 % to 100 % nominal load in the system.

**Case B:** A sudden disconnection of the load from 100% to 0% nominal load at  $t = 2$  s.

For sake of simplicity, it is assumed that all MMCs and transmission lines have the same characteristics such as modelling and parameters, although they can be different in practical cases. Given this assumption, the power changes in MMC1 and MMC2 are similar. While this behavior is exactly the opposite of MMC2 and MMC4. So, the plots related to power changes can be drawn for only one pair of converters such as receivers' converters.

All simulation values are given in Tables 1-3.

Table 1: Grid and MMC parameters

Symbol	Value	Units
$P^*$	1000	MW
$Q^*$	5	MVAR
$v_t^{DC}$	640	kV
$R_g + jL_g$	$0.512 + j 0.0587$	$\Omega$
$R_a + jL_a$	$1.024 + 0.0489$	$\Omega$
$N_{arm}$	400	-
C	9.5	mF
$\omega$	314.1593	$rad/s$



Table 1: SMC coefficients

Algorithm	controller	$k_{1qd}$	$k_{2qd}$	$\alpha_1$	$\alpha_2$	$V$	$\lambda$	$k_3$
First-order SMC	OC	0.05	600	-	-	-	-	-
Twisting	OC	-	-	1000	7500	-	-	-
PLCS	OC	-	-	-	-	20000	3900	
Super-twisting SMC	OC	-	-	-	-	-	-	4000
	CC	-	-	-	-	-	-	0.8

Table 3: Cable parameters

Symbol	Value	Units	Symbol	Value	Units
$r_{z1}$	0.1265	$\Omega/km$	$l_{z1}$	0.2644	$mH/km$
$r_{z2}$	0.1504	$\Omega/km$	$l_{z2}$	7.2865	$mH/km$
$r_{z3}$	0.0178	$\Omega/km$	$l_{z3}$	3.6198	$mH/km$
$c_y$	0.1616	$\mu F/km$	$g_y$	0.1015	$\mu S/km$

### 3. Assignment

For running the model, MATLAB is required.

Unzip the MATLAB models archive. In the archive you will find three files:

- MMC\_PARAMETERS.m – file with the parameters of each MMC converter;
- CONTROL\_PARAMETERS.m - file with the parameters of control loops;
- MTDC\_MODEL.slx – Simulink file containing the grid from Fig. 1.

First two files are used for the initialization of the system and should be run before running the Simulink file. By running the Simulink model, all data necessary for this assignment will be solved.

1. Identify the voltages of DC terminals and plot them. Attach diagrams in your report.
2. Identify which control methodology is currently used for each MMC. Determine how can you change the applied control. Try running simulations for all possible cases: PI, first-order SMC, ST-SMC, and PLCS.
3. Identify measured active power on DC terminals for MMC1 and MMC3 for PI and SMC. Attach them in your report.  
What do you observe?
4. Identify measured active power on DC terminals for MMC2 and MMC4 for PI and SMC. Attach them in your report.  
What do you observe?
5. Repeat steps 2 and 3 for the reactive power measurements.
6. Try changing control parameters. What effect(s) do you observe?
7. Try changing the rated power. What do you observe?