Term Project - Rollercasters

Group 14

Problem 1

The track of the Flip Flap Railway is a perfect circle. The radius of it is R.
The net force is $\frac{mv^2}{r}$, where r is the radius of curvature, which is the reciprocal of curvature.
For $\varphi(t) = \begin{pmatrix} R\cos t \\ R\sin t \end{pmatrix}$, we have

$$\varphi'(t) = \begin{pmatrix} -R\sin t \\ R\cos t \end{pmatrix},\ T \circ \varphi(t) = \begin{pmatrix} -\sin t \\ \cos t \end{pmatrix},\ (T \circ \varphi)(t) = \begin{pmatrix} -\cos t \\ -\sin t \end{pmatrix}$$

By substituting the quantities, we obtain

$$\kappa = \frac{||(T \circ \varphi)'(t)||}{||\varphi'(t)||} = \frac{1}{R},$$

which indicates that the radius of the curvature is a constant R.
Thus, when the Flip Flap Railway reaches the lowest position, the normal force reaches max, which is:

$$N = \frac{mv_{\text{bottom}}^2}{R} + mg. \tag{1}$$

Since it's weightless at the top, we have

$$\frac{mv_{\text{top}}^2}{R} = mg. \tag{2}$$

Thus,

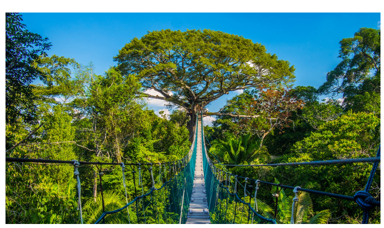
$$v_{\text{top}} = \sqrt{gR}.$$

Supposing that there is no frction, by the conservation of mechanical energy, we have

$$\frac{1}{2}mv_{\text{bottom}}^2 = mgh + \frac{1}{2}mv_{\text{top}}^2 \tag{3}$$

Combining Eq.(1)(2)(3), we obtain

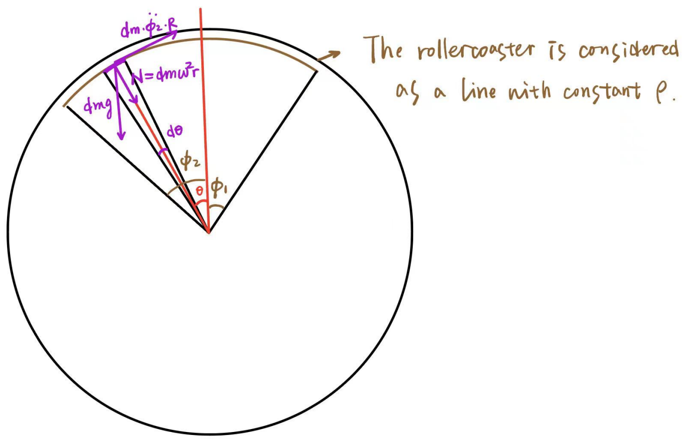
$$N = 6mg = 6G_s.$$



When entering the loop, the G-force suddenly acts on the passengers whose maximum value is $6G_s$.
(In [1], the author states that the maximum G-force can reach $12G_s$. It's not the condition of a perfect circle. Instead, it is possible for a curve with the curvature becomes larger in the process of from top to bottom.)

Problem 2

Problem 3



$$\phi_1+\phi_2=\phi=\text{const.}\qquad I=mR^2=\rho R^3\phi.$$

$$E=\rho R\phi R(1-\cos\varphi)+\frac{1}{2}\rho R^3\phi\dot{\theta}^2$$

$$E=\rho R\phi R(1-\cos(\phi_2-\frac{\phi}{2}))+\frac{1}{2}\rho R^3\phi\dot{\phi_2}^2$$

$$-\rho\phi Rg^2\sin(\phi_2-\frac{\phi}{2})=\rho R^3\phi\ddot{\phi}_2$$

$$\ddot{\phi}_2=-\frac{g}{R}\sin(\phi_2-\frac{\phi}{2})$$

$$\mathrm{d}m=\rho R\mathrm{d}\theta$$

$$T\mathrm{d}\theta+N+\mathrm{d}mg\cos\theta=\mathrm{d}m\omega^2 r$$

$$\mathrm{d}T=\mathrm{d}mg\cdot\sin\theta$$

$$\Rightarrow \mathrm{d}T=\rho Rg\sin\theta\mathrm{d}\theta+\underbrace{\rho gR\sin(\phi_2-\frac{\phi}{2})}_{\text{constant}}\mathrm{d}\theta$$

$$T|_{\phi_2}=T|_{\phi_1}=0$$

$$T(\theta)=\rho gR(\cos\phi_2-\cos\theta)+\rho gR\sin(\phi_2-\frac{\phi}{2})(\phi_2-\theta) \tag{4}$$

$$a_{*\&*\&}=\frac{N}{\mathrm{d}m}=\omega^2 r-g\cos\theta-T\frac{\mathrm{d}\theta}{\mathrm{d}m}=\omega^2 r-g\cos\theta-\frac{T}{\rho R} \tag{5}$$

Case 1: $\theta=0,\phi_1=0,\phi_2=\phi,T\approx 0.$

Case 2: $\theta=0,\phi_1=\phi,\phi_2=0,T\approx 0.$

Problem 3 continued on next page...

Case 3: $\theta = 0, \phi_1 = \frac{\phi}{2}, \phi_2 = \frac{\phi}{2}, T \approx \rho g R(\cos\frac{\phi}{2} - 1)$.

Substituting Eq.(2) into the equation above, we have

$$a_\lambda = \omega_{\text{top}}^2 r - g\cos\frac{\phi}{2}$$

$$\Delta\omega^2 = \frac{2g}{R}(1 - \cos\frac{\phi}{2})$$

$$a_\lambda = \omega^2 r - 2g(1 - \cos\frac{\phi}{2}) - g\cos\frac{\phi}{2} = \omega^2 r - 2g + g\cos\frac{\phi}{2} < \omega^2 r - g$$

$$\mathrm{d}m = \rho R \mathrm{d}\theta$$

Total length is ϕ.

So the accelerate of the two sides is larger than that of the middle.

Problem 4

Problem 5