Related Topics

Fourier transform, lenses, Fraunhofer diffraction, index of refraction, Huygens' principle.

Principle

The hallmark of Fourier optics is the use of the spatial frequency domain as the conjugate of the spatial domain, and the use of terms and concepts from signal processing, such as: transform theory, spectrum, bandwidth, window functions, sampling, etc. In this part observations with and without optical filtration (low-pass filtration) in the Fourier plane should be investigated. Such kind of optical filtration can be used as a spatial frequency filter to eliminate the disturbances of the wave front, which result from soiling of the lenses. Another direct application of low-pass filtration is the elimination of raster lines in composite images. For example, in astronomy, composite satellite images are freed from their raster.

Equipment

1	Optical base plate w. rubber ft.	08700.00
1	He/Ne Laser, 5 mW with holder	08701.00
1	Power supply f. laser head 5 mW	08702.93
2	Adjusting support 35x35 mm	08711.00
2	Surface mirror 30x30 mm	08711.01
9	Magnetic foot f. opt. base plt.	08710.00
2	Holder f. diaphr./beam splitter	08719.00
3	Lens, mounted, f = +100 mm	08021.01
3	Lensholder f. optical base plate	08723.00
1	Screen, white, 150x150 mm	09826.00
1	Slide -Emperor Maximilian-	82140.00
1	Screen, with arrow slit	08133.01
1	Diffraction grating, 4 lines/mm	08532.00
1	Diffraction grating, 50 lines/mm	08543.00
1	Diaphragms, d = 1, 2, 3, 5 mm	09815.00
1	Screen, with diffracting elements	08577.02
1	Achromatic objective 20x N.A. 0.45	62174.20
1	Sliding device, horizontal	08713.00
2	xy shifting device	08714.00
1	Adapter ring device	08714.01
1	Pin hole 30 μm	08743.00
1	Rule, plastic, l = 200 mm	09937.01

Fig. 1: Set-up of experiment P2261200 with He/Ne Laser, 5 mW

Tasks

1. Optical Filtration of diffraction objects in 4f set-up.
2. Reconstruction of a filtered image.

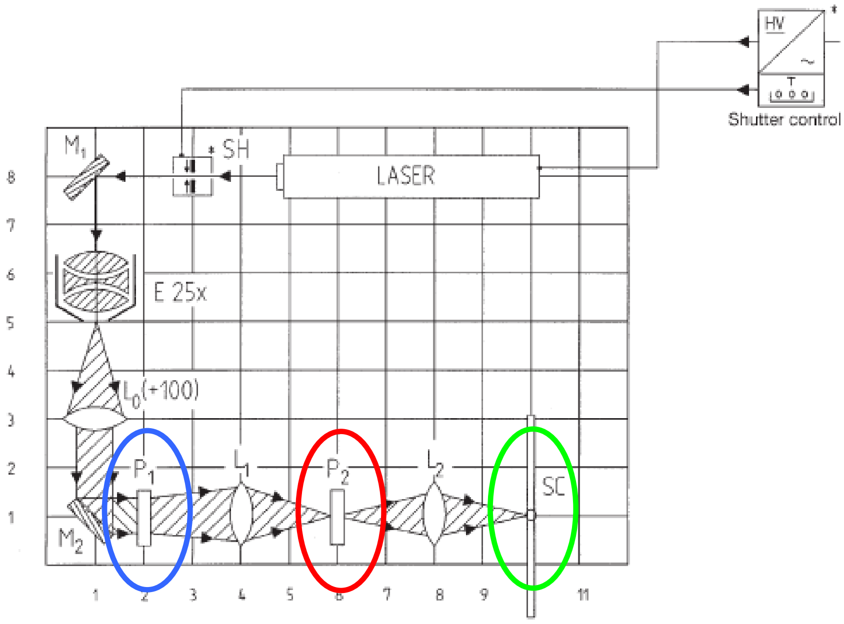


Fig. 1: Sketch of the experimental set-up (object plane: blue, Fourier plane: red, observation plane: green)

Set-up and Procedure

Set up the equipment as seen in Fig. 1 and Fig. 2.

In the following, the pairs of numbers in brackets refer to the coordinates on the optical base plate in accordance with Fig. 2. These coordinates are intended to help with coarse adjustment. The recommended set-up height (beam path height) is about 130 mm.

- The **E25x** beam expansion system (magnetic foot at [1,5.5]) and the lens $\mathbf{L_0}$ [1,3] are not to be used for the first beam adjustment.
- When adjusting the beam path with the adjustable mirrors $\mathbf{M_1}$ [1,8] and $\mathbf{M_2}$ [1,1], the beam is set along the 1,x and 1,y coordinates of the base plate.
- Now place the **E25x** [1,5.5] beam expansion system without its objective and pinhole, but equipped instead only with the adjustment diaphragm, in the beam path. Orient it such that the beam passes through the circular stops without obstruction.
- Now replace these diaphragms with the objective and the pinhole diaphragm. Move the pinhole diaphragm toward the focus of the objective. In the process, first ensure that a maximum of diffuse light strikes the pinhole diaphragm and later the expanded beam. Successively adjust the lateral positions of the objective and the pinhole diaphragm while approaching the focus in order to ultimately provide an expanded beam without diffraction phenomena.
- The $\mathbf{L_0}$ [1,3] (f= +100 mm) is now positioned at a distance exactly equal to the focal length behind the pinhole diaphragm such that parallel light now emerges from the lens. No divergence of the light spot should occur with increasing separation. (testing for parallelism via the light spot diameter with a ruler

at various distances behind the lens L_0 in a range of approximately 1 m).
- Place a plate holder P_1 [2,1] in the object plane.
- Position the lens L_1 [5,1] at the focus (f = 150 mm) and the screen **SC** [8,1] at the same distance behind the lens.
- Place a plate holder P_1 [2,1] in the object plane. Position the lens L_1 [4,1] at the focus (f = +100 mm) and the second plate holder P_2 [6,1] at the same distance behind the lens.
- Additionally, place another lens L_2 [8,1] at a distance equal to the focus f = +100 mm and at the same distance of 10 cm set up the screen **SC** [10,1]. The parallel light beam that strikes the lens L_1 must appear on the screen **SC** at the same height and with the same extension (Check with the ruler!).

Note the terms of the "object plane" at P_1 (blue), the "Fourier plane" at P_1 (red) and the "observation plane" at the screen **SC** (green).

Procedure

Different diffraction structures are placed into the plate holder P_1 (object plane) and filter in the plate holder P_2 (Fourier plane). Observe the patterns in the Fourier plane and the observation plane **SC** using the screen and compare them to the theoretical predictions (see experimental manual).

Observations without optical filtration

a) Diaphragm with the arrow

Clamp the diaphragm with the arrow (arrow pointing upwards) as the first diffracting structure in plate holder P_1 [2,1] in the object plane (Fig. 3) and shift it laterally in such a manner that the light from the mirror M_2 strikes the arrow head. An arrow also appears on the observation screen (compare with theory).

The arrow is now turned 90° so that it points in a horizontal direction (Fig. 4, shift the diaphragm laterally until the arrow head is will illuminated). In this case also, compare the image on the screen with the theory.

b) Photographic slide

The photographic slide of Emperor Maximilian serves as the next diffracting structure (Fig. 5). It is placed in plate holder P_1 and laterally shifted until the light beam illuminates significant contours of the face (e.g. the nose) (in a recognizable form!). Observe, describe and explain the image in the observation plane (what is different to the original?).

c) Grid

Place the grid (4 lines/mm) in the object plane P_1 as a further diffracting structure (Fig. 6). In the process, observe the Fourier-transformed image in the Fourier plane P_2 [6,1] with the screen and subsequently examine it in the observation plane, **SC** at [10,1]. One ascertains that the lines of the grid cannot be resolved in the image (and they cannot be resolved in the real grid either).

By turning the screen around its vertical axis (Fig. 7), the image can be expanded in a distorted manner such that the grid lines can be discerned.

Observations with optical filtration

a) Filtration of a Grid

Clamp the grid (50 lines/mm) in the object plane P_1 in a plate holder (Fig. 8). Now perform a low-pass filtration in the Fourier plane P_2, by positioning a pinhole diaphragm (diameter: 1 or 2 mm) in such a manner that only a single, arbitrary diffraction maximum passes through. Observe the image on the screen **SC**.

b) Slide with grid

Clamp the slide (Emperor Maximilian) and the grid (4 lines/mm in vertical direction) together in the optical plane in the plate holder **P_1** (Fig. 9). To begin with, observe the Fourier spectrum in **P_2** [6,1] on the screen **SC** and subsequently the image on the screen **SC** at position [10,1].

In the Fourier plane, one sees primarily the grid spectrum with discrete diffraction maxima. On the observation screen **SC** [10,1], the slide (Emperor Maximilian) can be seen. To still be able to resolve the grid, it is necessary to turn the screen around its vertical axis to such an extent that it is nearly parallel to the direction of light propagation. In this manner, the fine structures will be enlarged and readily visible.

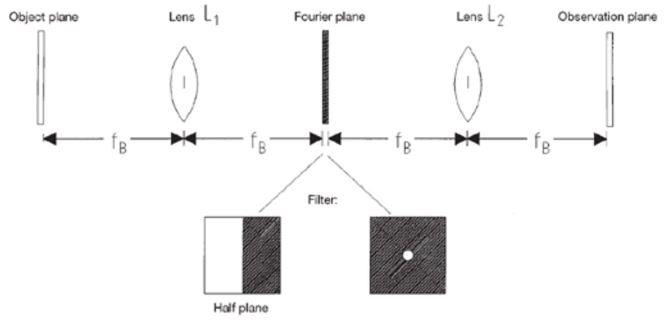


Fig. 3: Principle of the set-up for coherent optical filtration.

c) Slide with grid – filtration

Now perform a low-pass filtration with a pinhole diaphragm (diaphragm with diffraction objects, Fig. 10) in the Fourier plane **P_2** [6,1] by selectively filtering out all but a single arbitrary diffraction maximum. It is advisable to use the central zero diffraction maximum, as the greatest light intensity is located there. And accordingly, the image on the screen is the brightest

Use a pinhole diaphragm with a diameter of 0.25 mm and pinhole diaphragm has a diameter equal to 0.5 mm. Observe what happens to the grid structure in both cases. For a better resolution the screen should be turned nearly parallel to the direction of light propagation.

Theory and Evaluation

For information on the fundamentals of Fourier optics and the Fourier transformation by a lens, see the “Fourier optics – 2f Arrangement” experiment.

Coherent optical filtration By intervening in the Fourier spectrum, optical filtration can be performed which can result in image improvement, etc. The appropriate operation for making the original image visible again, in the inverse Fourier transform, which however cannot be used due to diffraction. The Fourier transform is again used; this leads to the 4f set-up (see Fig. 3).

Using the 1st lens (L1), the spectrum with the appropriate spatial frequencies is generated in the Fourier plane from the original diffracting structure t (x,y) (see the “Fourier optics – 2f Arrangement” experiment). In this plane, the spectrum can be altered by fading out specific spatial frequency fractions. A modified spectrum is created, which is again Fourier transformed by the 2nd lens (L2). If the spectrum is not altered, one obtains the original image in the inverse direction in the image plane (right focal plane of the 2nd lens) (partial experiment (a) with the arrow diaphragm). This follows from the calculation of the two-fold Fourier transformation:

$$\tilde{F}\left[\tilde{F}[f(x,y)]\right] = f(-x,-y) \qquad (1)$$

The simplest applications for optical filtration are the high- and low-pass filtration. Low-pass raster elimination In the experiment, the photographic slide was provided with a raster by superimposing grid lines on it in one direction. The scanning theory states that an non-raster image (in this case: Emperor Maximilian) can be exactly reconstructed if the image is band-limited in its spectrum, i. e. if it only contains spatial frequencies in the Fourier plane up to an upper limiting frequencies. The raster image can be described mathematically as follows:

(2)

$$b(x, y) = \text{comb}\,\frac{y}{b} \cdot g(x, y)$$

with the comb function

$$\text{comb}\,\frac{y}{b} = \sum_{-\infty}^{+\infty} \delta(y - nb) \qquad (3)$$

This describes the grid lines (the grid) and g (x,y) the non-raster image (in this case: the slide). The slit separation of the grid is b. The Fourier spectrum B(ν_x, ν_y) of the entire image becomes the following with the convolution law:

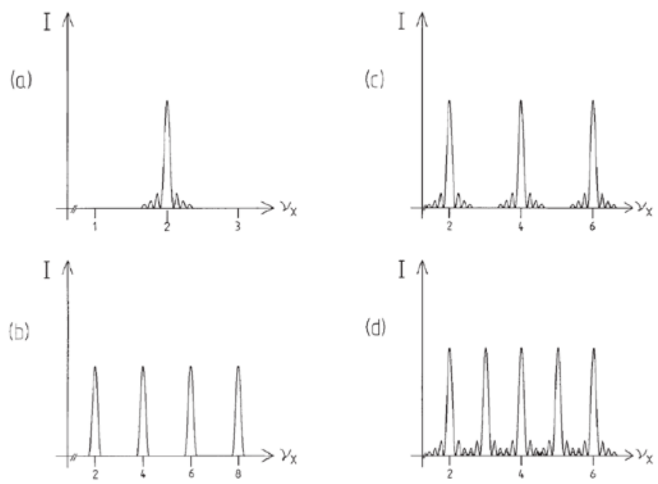


Fig. 4: Composite spectrum, e.g. raster image.

$$\tilde{F}[b(x, y)] = B(\nu_x, \nu_y) = b \cdot \text{comb}\,(b \cdot \nu_y) \cdot G(\nu_x, \nu_y) \qquad (4)$$

where G(ν_x, ν_y) is the Fourier transformation of the non-raster image. In addition, the fact that the Fourier transformation of an comb function is also a comb function. This means that the Fourier spectrum once again a grid which is formed by the reiteration of the spectrum of the non-raster image (see Fig. 4).

Each grid point with its immediate surroundings contains the total information of the non-raster image g(x,y). It is important that the grid points in the Fourier plane are sufficiently far apart that the spectra of the unlined image do not overlap. Only in this case is it possible to filter out a single image point with a pinhole diaphragm. This spatial frequency filtration can be considered as multiplication of the spectrum by an aperture function A(ν_x, ν_y) (pinhole diaphragm in the Fourier spectrum). In this case, an appropriate measuring dimension would be a diameter of _1/b. Therefore:

$$B_{filtered}(\nu_x, \nu_y) = B(\nu_x, \nu_y) \cdot A(\nu_x, \nu_y) = b(-x, -y) \cdot \tilde{F}[A(\nu_x, \nu_y)] \qquad (5)$$

Fog procedure

The fog procedure makes it possible to see phase objects (e.g. phase grids through a standing ultrasonic wave). In this case, the Fourier spectrum is filtered with a half-plane filter.

This amplitude filter should fade out exactly one half-plane of the spatial frequency spectrum including half of the zero order! The intensity of the image in the observation plane (in this case at Sc [10,1]) is then proportional to the gradient of the phase, where the direction of the observed gradient is a function of the position of the half-plane:

$$I(-x, -y) \sim \left| \frac{\partial \varphi(x, y)}{\partial x} \right| \qquad (6)$$

with X: phase of light.

The principle can be described as follows:

The total image information is contained in a half-plane of the spatial frequency spectrum. In order to obtain the original intensity distribution of the diffracting structure by repeated Fourier transform, the superimposition of the two spatial frequency half-planes is necessary. If this superimposition is prevented, the phase information of the diffracting structure is made visible.

Information on the ultrasonic wave:

The ultrasonic wave forms a standing sound wave between the ultrasonic transducer and the cell's bottom. This is a periodically oscillating pressure (and density) variation of the water with spatially fixed pressure nodes. Since the optical refraction index is proportional to the density of the medium, the propagation velocities of the light which is perpendicularly incident to the direction of sonic propagation are different in the various pressure regions. This results in a phase modulation of the light behind the ultrasonic wave (see Fig. 5)

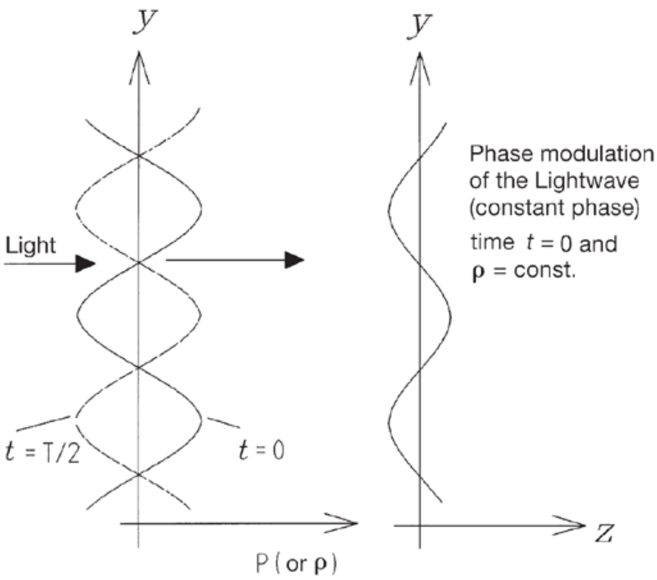


Fig 5: Schematic diagram of a standing ultrasonic wave based on a pressure change (or density change).

Since the ultrasonic wave is a standing wave, the location of the phase gradients does not change (it disappears only in the pressure maxima and minima of the sonic field), but the intensity changes periodically. Due to the sluggishness of perception, a temporal average is taken here as the sound frequency is approximate 800 kHz.

Application areas of optical filtration:

Low-pass filtration can be used as a spatial frequency filter to eliminate the disturbances of the wave front, which result from soiling of the lenses, in a beam expansion by a microscope's objective (as E25x in this experiment). Another direct application of low-pass filtration is the elimination of raster lines in composite images. For example, in astronomy, composite satellite images are freed from their raster.

In Fourier optics, numerous procedures exist for making phase objects or modulations visible:

1. The **phase-contrast procedure** effects the transformation of a phase modulation on a diffraction object into an amplitude modulation by inserting a M/4 wafer into the zero diffraction order. This modulation can then be observed in the observation plane. The phase-contrast microscope (Frits Zernicke, Nobel Prize in 1953) is a wide-spread application of this procedure.
2. The **fog technique** in which a half-plane filter is used in the Fourier plane makes the phase gradients visible (as described in this experiment).
3. The **dark-field technique** is a *high-pass filtration* in the Fourier plane, i.e. the zero diffraction order is filtered out by a small disk of the appropriate size. Using this filtration method, thin sections, organic preparation, currents of air and pressure waves in fluid dynamics research are made visible.

Additional fields of application result from the use of holographic filters. Images of Fourier spectra of certain diffracting structures on holographic photo material can be used as filters in the Fourier plane of the 4f set-up. These then are used for pattern recognition, i. e. for the repeated recognition of fundamental diffracting structure.

Sample results

In the following we will show typical patterns.

Observations without optical filtration

a) Diaphragm with the arrow

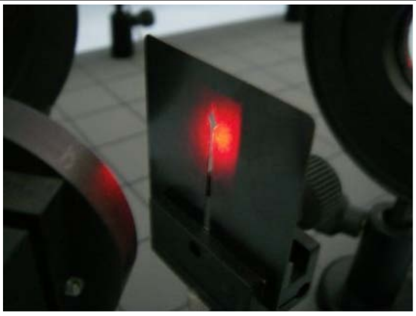
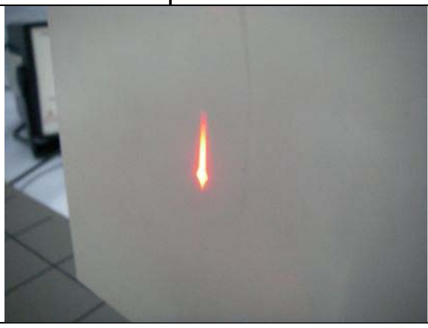
Object plane	Observation plane	Description/Explanation
		The arrow is pointing down, i.e. it has a reversed direction. You can explain this easily just by basic geometric optics or (in this case recommended) by the Fourier transformation of a plane wave (see theory in LEP 2.6.11-00).

Fig. 2: Diaphragm with the arrow pointing upwards

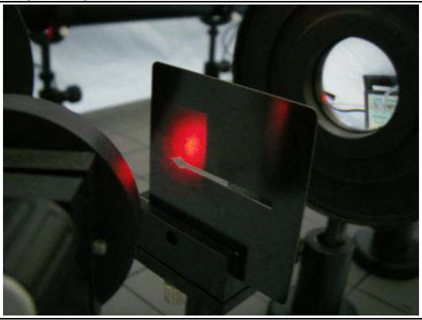
Object plane	Observation plane	Description/Explanation
		The arrow is pointing to the right, i.e. it has a reversed direction. You can explain this easily just by basic geometric optics or (in this case recommended) by the Fourier transformation of a plane wave (see theory in LEP 2.6.11-00).

Fig. 3: Diaphragm with the arrow pointing sideward

b) Photographic slide

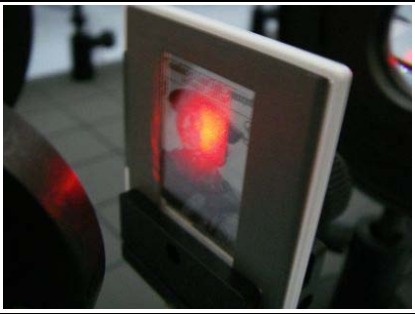
Object plane	Observation plane	Description/Explanation
		The picture is completely reversed. You can explain this easily just by basic geometric optics or (in this case recommended) by the Fourier transformation of a plane wave (see theory in LEP 2.6.11-00).

Fig. 4: Photographic slide

c) Grid

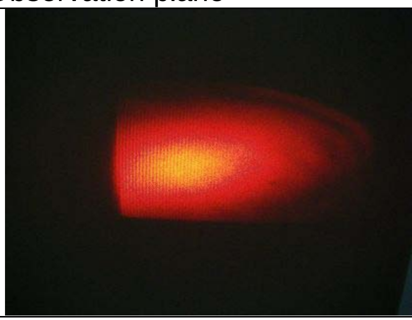
Object plane	Fourier plane	Observation plane
		

Fig. 5: Grid

Observations with optical filtration

a) Filtration of a Grid

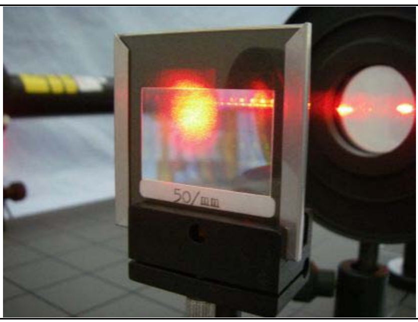
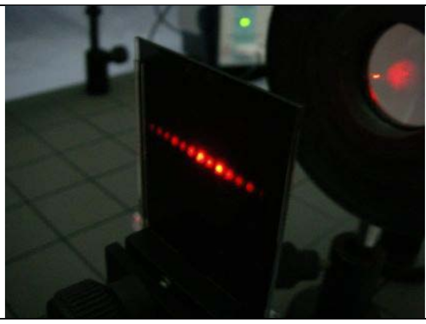
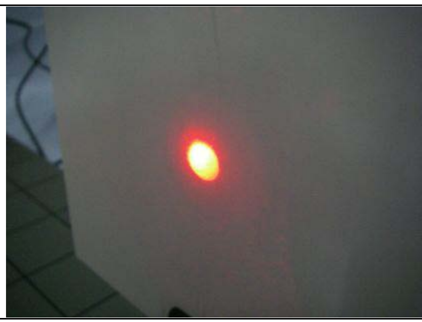
Object plane	Fourier plane	Observation plane
		

Fig. 6: Low-pass filtration using a pinhole diaphragm

b) Slide with grid

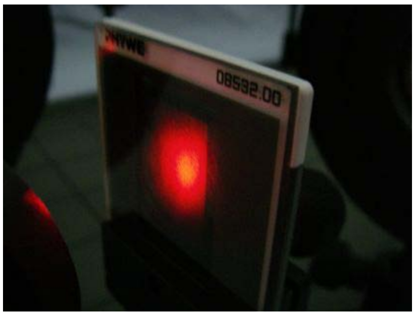
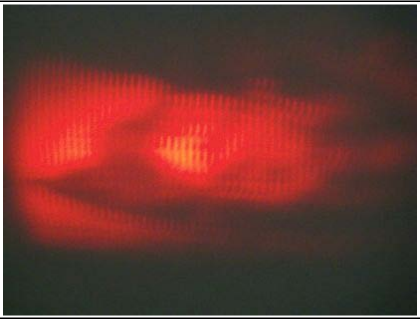
Object plane	Fourier plane	Observation plane
		

Fig. 7: Slide with grid

c) Slide with grid – filtration

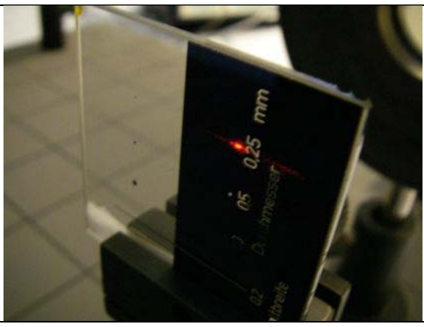
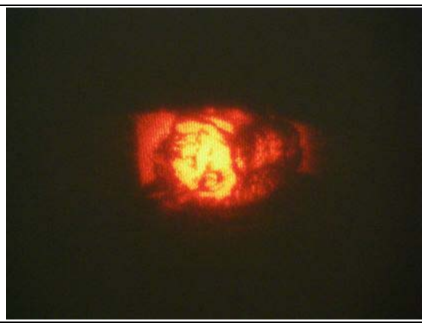
Fourier plane	Observation plane (pinhole diameter: 0.25 mm)	Observation plane (pinhole diameter: 0.5 mm)
		

Fig. 8: Filtering using pinhole diaphragm

Accordingly, the image on the screen is the brighter with a pinhole diameter of 0.5 mm then with 0.25 mm. On using the pinhole diaphragm with a diameter of 0.25 mm, the grid structure disappears. The image of the slide (Emperor Maximilian) is not affected when the screen is in a perpendicular position (no distortion). If the screen is turned nearly parallel to the direction of light propagation, the grid structure cannot be seen.

However, as soon as the pinhole diaphragm has a diameter equal to 0.5 mm, it is impossible to filter out the grid structure. When the screen is turned to the horizontal position, the image of the slide always has superimposed grid lines.

