1 Problem Definitions and Statement

Definition 1. A **point cloud** A over $\mathbb{R}^n$ is a sequence of vectors in R^n

$$A = (a_1, \dots, a_k) \quad \forall i \in [k],\, a_i \in \mathbb{R}^n$$

and we let $|A| = k$.

Definition 2. The **centroid** of a point cloud A is defined to be

$$\bar{a} = \frac{1}{|A|} \sum_{a \in A} a$$

Definition 3. A **rigid-body transformation** on a point cloud A is an affine map $T : \mathbb{R}^n \to \mathbb{R}^n$ determined by an rotation matrix $R \in \mathbb{R}^{n \times n}$ and translation $t \in \mathbb{R}^n$ and given by

$$T : a \mapsto R(a - \bar{a}) + \bar{a} + t$$

that is, a rotation around the centroid of followed by a translation.

The Point-Cloud Registration Problem is, given two point clouds A, B and a pairing function $\phi(T, \cdot)$ based on a rigid-body transformation T such that $\forall a \in A,\, \phi(T, a) \in B$, to determine the rigid-body transformation T that optimizes

$$\min_T \sum_{a \in A} \|T(a) - \phi(T, a)\|^2 \tag{1}$$

Definition 4. The **point-to-point metric** for two point clouds A, B is given by

$$\phi(T, a) = \operatorname*{argmin}_{b \in B} \|T(a) - b\|^2 \tag{2}$$

2 Solution

In general, there is no closed-form solution that minimizes (**??**). When there is no translation, Horn (1987) derives a closed-form solution; therefore, if an initial odometry estimate is precise enough, this translation can be applied to all the points in the point cloud, producing a problem instance with zero translation.