

# 1 Problem Definitions and Statement

**Definition 1.** A **point cloud**  $A$  over  $\mathbb{R}^n$  is a sequence of vectors in  $\mathbb{R}^n$

$$A = (a_1, \dots, a_k) \quad \forall i \in [k], a_i \in \mathbb{R}^n$$

and we let  $|A| = k$ .

**Definition 2.** The **centroid** of a point cloud  $A$  is defined to be

$$\bar{a} = \frac{1}{|A|} \sum_{a \in A} a$$

**Definition 3.** A **rigid-body transformation** on a point cloud  $A$  is an affine map  $T : \mathbb{R}^n \rightarrow \mathbb{R}^n$  determined by an rotation matrix  $R \in \mathbb{R}^{n \times n}$  and translation  $t \in \mathbb{R}^n$  and given by

$$T : a \mapsto R(a - \bar{a}) + \bar{a} + t$$

that is, a rotation around the centroid of followed by a translation.

The **POINT-CLOUD REGISTRATION PROBLEM** is, given two point clouds  $A, B$  and a pairing function  $\phi(T, \cdot)$  based on a rigid-body transformation  $T$  such that  $\forall a \in A, \phi(T, a) \in B$ , to determine the rigid-body transformation  $T$  that optimizes

$$\min_T \sum_{a \in A} \|T(a) - \phi(T, a)\|^2 \quad (1)$$

**Definition 4.** The **point-to-point metric** for two point clouds  $A, B$  is given by

$$\phi(T, a) = \operatorname{argmin}_{b \in B} \|T(a) - b\|^2 \quad (2)$$

## 2 Solution

In general, there is no closed-form solution that minimizes (??). When there is no translation, Horn (1987) derives a closed-form solution; therefore, if an initial odometry estimate is precise enough, this translation can be applied to all the points in the point cloud, producing a problem instance with zero translation.