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Note for Fisher Matrix calculation for photo-z

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This note provide the calculation of the Fisher Matrix elements for our project in photometric redshift estimation. As explained in the main note, the BC03 model we use depends of the formation redshift of the single starburst and of course of the redshift. In this note we will use the notation z_p to refer to the redshift (p for photometric) and z_f for the formation redshift. We can calculate the $\chi^2(z_p, z_f)$ for a value in the plan (z_p, z_f) for a given observed galaxy (for which we have the 5 magnitudes and associated errors) as follows:

$$\chi^2(z_p, z_f) = \sum_{col} \frac{(col_{th}(z_p, z_f) - col_{obs})^2}{\sigma^2_{col}}, \tag{1}$$

where $col_{th}(z_p, z_f)$ are the theoretical colors provided by the model and col_{obs} and σ_{col} are provided by the data. The idea of the Fisher Matrix analysis for future survey is to assume a fiducial model as the correct one and derive the errors over the estimation of quantity (here the photometric redshift). So in our case we can assume a formation redshift as the correct one and model the magnitudes in each bands for a galaxy with a given stellar mass. Knowing the specification of the survey we want to model, we can associate an error for each magnitudes. We remind you that the procedure is explained in the section "Error on magnitudes" in the main note. In this case the colors generated as data will perfectly correspond to the theoretical ones and the χ^2 will be equal to 0 by definition. However, we are interested on the propagation of the incertitude over the parameters (here z_p and z_f). All the subscript *obs* will refer to the generate data in the case of Fisher analysis (if used for future survey prediction). We start from the definition pof the Fisher matrix in case of gaussian likelihood (so with $\mathcal{L} = exp(-\chi^2/2)$)rovided on **Note_chi2.pdf** :

$$F_{ij} = \frac{1}{2}\left\langle \frac{\partial^2 \chi^2(\theta^b_1 \dots \theta^b_N)}{\partial\theta_i \partial\theta_j} \right\rangle, \tag{2}$$

that corresponds in our case to:

$$F = \begin{pmatrix} F_{z_p,z_p} & F_{z_p,z_f} \\ F_{z_f,z_p} & F_{z_f,z_f} \end{pmatrix} = \frac{1}{2}\begin{pmatrix} \left\langle \frac{\partial^2\chi^2(z_p,z_f)}{\partial z_p \partial z_p} \right\rangle & \left\langle \frac{\partial^2\chi^2(z_p,z_f)}{\partial z_p \partial z_f} \right\rangle \\ \left\langle \frac{\partial^2\chi^2(z_p,z_f)}{\partial z_f \partial z_p} \right\rangle & \left\langle \frac{\partial^2\chi^2(z_p,z_f)}{\partial z_f \partial z_f} \right\rangle \end{pmatrix}. \tag{3}$$

We can calculate the 3 elements of the matrix (the 2 off-diagonal terms are identical because our model is varying smoothly) in the context of our $\chi^2(z_p, z_f)$ estimator. First we calculate the term F_{z_p,z_p}:

$$F_{z_p z_p}(z'_p, z'_f) = -\left\langle \left. \frac{\partial^2 - 1/2\chi^2(z_p, z_f)}{\partial_{z_p}\partial_{z_p}} \right|_{z'_p, z'_f} \right\rangle \tag{4}$$

$$\chi^2(z_p, z_f) = \sum_{col} \frac{(col_{th}(z_p, z_f) - col_{obs})^2}{\sigma^2_{col}} \tag{5}$$

$$F_{z_p z_p}(z'_p, z'_f) = \left\langle \left. \frac{\partial^2 1/2 \sum_{col} \frac{(col_{th}(z_p,z_f)-col_{obs})^2}{\sigma^2_{col}}}{\partial_{z_p}\partial_{z_p}} \right|_{z'_f, z'_p} \right\rangle \tag{6}$$

$$F_{z_p z_p}(z'_p, z'_f) = \left\langle \left. \frac{\partial}{\partial_{z_p}} \right|_{z'_p} \sum_{col} \frac{(col_{th}(z_p, z_f) - col_{obs})}{\sigma^2_{col}} \left. \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_p}} \right|_{z'_p} \right\rangle \tag{7}$$

$$F_{z_p z_p}(z'_p, z'_f) = \left\langle \sum_{col} \frac{1}{\sigma^2_{col}} \left. \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_p}} \right|_{z'_p} \left. \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_p}} \right|_{z'_p} + \right. \tag{8}$$

$$\left. + \sum_{col} \frac{(col_{th}(z_p, z_f) - col_{obs})}{\sigma^2_{col}} \left. \frac{\partial^2 col_{th}(z_p, z_f)}{\partial_{z_p}\partial_{z_p}} \right|_{z'_f, z'_p} \right\rangle, \tag{9}$$

where the term:

$$\sum_{col} \frac{(col_{th}(z_p, z_f) - col_{obs})}{\sigma^2_{col}} \frac{\partial^2 col_{th}(z_p, z_f)}{\partial_{z_p} \partial_{z_p}}\bigg|_{z'_f, z'_p} \Bigg\rangle = 0 \tag{10}$$

because $col_{th}(z_p, z_f) \equiv col_{obs}$ here. So we finally get the simple results:

$$\boxed{F_{z_p z_p}(z'_p, z'_f) = \left\langle \sum_{col} \frac{1}{\sigma^2_{col}} \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_p}}\bigg|_{z'_p} \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_p}}\bigg|_{z'_p} \right\rangle} \tag{11}$$

In fact the caluation is strictly the same for the 2 other terms and we have that:

$$\boxed{F_{z_f z_f}(z'_p, z'_f) = \left\langle \sum_{col} \frac{1}{\sigma^2_{col}} \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_f}}\bigg|_{z'_f} \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_f}}\bigg|_{z'_f} \right\rangle,} \tag{12}$$

and

$$\boxed{F_{z_f z_p}(z'_p, z'_f) = F_{z_p z_f}(z'_p, z'_f) = \left\langle \sum_{col} \frac{1}{\sigma^2_{col}} \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_f}}\bigg|_{z'_f} \frac{\partial col_{th}(z_p, z_f)}{\partial_{z_p}}\bigg|_{z'_p} \right\rangle.} \tag{13}$$

Finally, the error on the photometric redshift estimation taking into account the effect of the formation redshift incertitude is given by:

$$\boxed{\sigma_{z_p}(z'_p, z'_f) = \sqrt{F^{-1}_{z_f z_p}(z'_p, z'_f)}}, \tag{14}$$

where $F^{-1}_{z_f z_p}(z'_p, z'_f)$ is the upper left element of the inverse matrix!