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Photometric Redshift

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The purpose of this project is to understand the photometric data (images and magnitudes), the use we can do to select the kind of objects and the redshift estimation using underlying model. The program is to understand different concepts as:

- Expanding Universe and redshift
- Distances in cosmology
- What are a flux and a luminosity .... in an expanding Universe
- How to convert a flux in magnitude a for given system of reference
- How to find a best model using data (minimum χ^2 method and error evaluation using $\Delta\chi^2$)
- How to calculate the Fisher Matrix

and use it all of them to:

- Find the best photometric redshift for real red galaxies and verified your result with the spectroscopic information.
- Apply Fisher Matrix approximation for estimating the photometric redshift precision for specific surveys.

1 Equations for Cosmology

We define the scale factor of the Universe by "a" which is, by definition, equal to 1 today ($a(t=0)=1$). $a(t)$ defines the ratio you have to multiply with to obtain a distance today at the time t to compensate the expansion of the Universe. It does not concern the distances of collapsed objects, for which there is no expansion inside. The Hubble function, which express the rate of expansion of the Universe (expressed as a velocity by distance unit: km.s^{-1}/Mpc) at a given moment (which can be t, z or a) is given by:

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3c^4}\sum_i \rho_i - \frac{k}{a^2}, \tag{1}$$

where the ρ_i are the density of the different energetic component of the Universe (baryonic matter, dark matter, dark energy, radiation) and k the isotropic curvature of the Universe ($k=0$ if flat Universe which will be the case for the lecture). If we divide this equation by the value of $H(a)$ today ($H_0 \sim 70$km.s^{-1}/Mpc), and knowing that the critical density of the Universe ρ_{crit} today is defined as $\rho_{crit,0} = \frac{3H_0^2c^4}{8\pi G}$ we find that:

$$\frac{H^2(a)}{H_0^2} = \sum_i \frac{\rho_i(a)}{\rho_{crit,0}} - \frac{k}{a^2\rho_{crit,0}}. \tag{2}$$

We will see further (see appendix A) that the evolution with the scale factor can be wrote as:

$$\rho_i(a) = \rho_i(a=1)\times a^{-\alpha_i} \qquad \alpha_b = \alpha_{DM} = 3, \alpha_r = 4, \alpha_{DE} = 0. \tag{3}$$

This just mean that density of matter (baryons and Dark Matter) is diluted by the volume, the radiation density is diluted by the volume and a stretching effect more on the wavelength of the photons which reduce the energy by a scale factor too. The Dark Energy is not diluting with time. Considering a flat Universe (k=0), we can so write the Hubble function as:

$$H^2(a) = H_0^2\sum_i \frac{\rho_{i,0}}{\rho_{crit,0}}\times a^{-\alpha_i} = H_0^2\sum_i \frac{\rho_{i,0}}{\rho_{crit,0}}\times(1+z)^{\alpha_i}, \tag{4}$$

where the $\rho_{i,0}$ are the actual values for density energy of matter, radiation and dark energy we measure in the local Universe. We generally expressed the quantities in term of critical density that we not:

$$\Omega_i = \frac{\rho_{i,0}}{\rho_{crit,0}} \tag{5}$$

so the expression we generally use for the Hubble function is given by:

$$\boxed{H(z) = H_0\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}.} \tag{6}$$

One implication of the expansion of the Universe is the effect on the photons because the expansion stretch the wavelength λ which correspond to down the energy of them. The relation between the wavelength of an emitted photon at time t_{emit} far away and the one observed today is:

$$\frac{\lambda_{obs}}{\lambda_{emit}} = \frac{a(t_{obs})}{a(t_{emit})}. \tag{7}$$

If we interpret this "redshift" z (stretch of the wavelength) as a Doppler effect, then we can rely the "recessing velocity" with the redshift as:

$$\lambda_{obs} = \lambda_{emit}(1 + \frac{v}{c}) = \lambda_{emit}(1+z). \tag{8}$$

Considering that the scale factor today (t_{obs}) is equal to 1, it comes directly the relation between the redshift and the scale factor:

$$\lambda_{obs} = \lambda_{emit}\frac{a(t_{obs})}{a(t_{emit})}\,, \quad a(t_{obs}) = 1 \Rightarrow a(t) = \frac{1}{1+z(t)}. \tag{9}$$

Two important equations, known as FRW equations, allowing to calculate the evolution of the expansion of the Universe considering a dominating energetic component are

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G\rho}{3}, \tag{10}$$

and

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0, \tag{11}$$

where p is the pressure. Because we consider generally the different energetic component as perfect fluids, we can simplify the last equation considering $p = w\rho$ where w is the equation of state of the considered component. For example, $w = 0$ for the dark matter and $w = 1/3$ for the baryonic matter.

2 Distances in Cosmology

2.1 Comoving distance

In order to calculate distances in cosmology, we refer to the distance done by the photon to move from one event (time and position) to another space-time event. The distance we are mostly interested with is the one between us ($a = 1$, $z = 0$ or $t = 0$) and an object far away. The metric considering an isotropic Universe without curvature is given by:

$$ds^2 = -c^2dt^2 + a^2(t)d\chi^2\,, \qquad \gamma \Rightarrow ds^2 = 0, \tag{12}$$

where $d\chi^2$ is the infinitesimal radial distance element. The second equation is verified only by a massless particle and allows us to rely the infinitesimal space and time elements :

$$c^2dt^2 = a^2(t)d\chi^2 \quad \Rightarrow \quad \frac{c^2}{a^2(t)}dt^2 = d\chi^2. \tag{13}$$

Thanks to this relation, we can calculate the distance integrating the inverse of the scale factor with during the time travel of the photon:

$$d\chi = \frac{c}{a(t)}dt \quad \Rightarrow \quad \chi(t_{emit}) = c\int_{t_{emit}}^{t_0}\frac{dt}{a(t)}. \tag{14}$$

Doing the substitution from t to a using the Hubble function, we obtain:

$$H(t) = \frac{\dot{a}(t)}{a(t)}\,;\ \dot{a} = \frac{da}{dt} \quad \Rightarrow \quad dt = \frac{da}{a(t)H(t)} \chi(a_{emit}) = c \int_{a_{emit}}^{a(t_0)=1} \frac{da}{a^2 H(a)}. \tag{15}$$

Now, we do the substitution from the scale factor a to the redshift z:

$$a = \frac{1}{1+z} \quad \Rightarrow \quad da = \frac{-dz}{(1+z)^2}, \tag{16}$$

$$a = \frac{1}{1+z} \Rightarrow da = \frac{-dz}{(1+z)^2}, \tag{17}$$

and we obtain finally the following integral

$$\chi(z_{emit}) = -c \int_{z_{emit}}^{z(t_0)=0} \frac{dz(1+z)^2}{(1+z)^2 H(z)}, \tag{18}$$

and inverting the integration limits, to integrate from us to the object, we obtain finally:

$$\chi(z_{emit}) = c \int_0^{z_{emit}} \frac{dz}{H(z)} \tag{19}$$

$$H(z) = H_0 \sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda} \tag{20}$$

$$\boxed{\chi(z_{emit}) = \frac{c}{H_0} \int_0^{z_{emit}} \frac{dz}{\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}}} \tag{21}$$

2.2 Luminosity distance and Angular distance

We have to define two others distances which are the "Luminosity distance" ($D_L(z)$) and the "Angular distance" ($D_A(z)$) . The first one come from the relation between the luminosity and the flux of an object. They are expressed as following:

$$\boxed{D_l(z_{emit}) = (1+z_{emit}) \times \chi(z_{emit}) = \frac{c(1+z_{emit})}{H_0} \int_0^{z_{emit}} \frac{dz}{\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}},} \tag{22}$$

and

$$D_A(z_{emit}) = \frac{\chi(z_{emit})}{(1+z_{emit})} = \frac{c}{(1+z_{emit})H_0} \int_0^{z_{emit}} \frac{dz}{\sqrt{\Omega_{M,0}(1+z)^3 + \Omega_{R,0}(1+z)^4 + \Omega_\Lambda}}. \tag{23}$$

We will see in details on the next section why the Luminosity distance has this factor $(1+z)$.

3 Equations for the astronomical observations

In astronomy, the flux is defined as the energy from photons we receive during 1s on a surface of 1cm^2 at a given frequency (i.e wavelength). For this reason, the unit of a flux is generally expressed in erg.s^{-1}.cm^{-2}.Hz^{-1} where the erg is an astronomical unit of energy.

The luminosity of an object is the integral of the photons emitted during the same second. So we have to integrate over all the surface of the shell. If we consider an isotropic emission, we just have to multiply the flux we measure by the the surface of the shell with radius equal to the distance between us and the object. This surface have to be expressed in cm^2 because of the definition of the flux.

So, the relation between the flux and the luminosity, for a given wavelength λ, without expansion (for example a star in our galaxy) is

$$f(\lambda) = \frac{L(\lambda)}{4\pi d^2}, \tag{24}$$

where d is the distance between us and the star expressed in cm.

However, we know that the Universe expands implying 2 things:

1. photons are redshifted
2. the distance has to be calculated in a similar way than the comoving distance

So, the frequency or the wavelength of a photon emitted from a far galaxy is different when we measure it :

$$\lambda_{obs} = (1+z)\lambda_{emit} \qquad \text{or} \qquad \nu_{obs} = \frac{\nu_{emit}}{(1+z)}, \tag{25}$$

which mean that the energy change of size by a factor $(1+z)$. The energy for a photon is given by:

$$E = h\nu = \frac{hc}{\lambda} \tag{26}$$

so the difference between the emitted energy and the one observed is

$$\boxed{E_{obs} = h\nu_{obs} = \frac{h\nu_{emit}}{(1+z)} = \frac{E_{emit}}{(1+z)}.} \tag{27}$$

We also have a difference for the number of photons we observe per second. Because of the distance between the photons increase by a factor $(1+z)$, the photons hosted in box with size of 1 light second are in a box ox $(1+z)$ light second. So, in one second we receive only $1/(1+z)$ of the photons. Another way to think about it is to say that we need (1+z) seconds to receive the photons emitted in a 1 second range. So, we loose another factor $(1+z)$ on the flux. For this reason, we measure :

$$f = \frac{L}{4\pi\chi^2(z)(1+z)^2}, \tag{28}$$

where we did not specified the wavelength on purpose and we can see immediately that

$$D_L^2(z) = (1+z)^2\chi^2(z) \qquad \Rightarrow \qquad \boxed{D_L(z) = (1+z)\chi(z).} \tag{29}$$

Because the wavelength of the emitted photon is different from the one received, we have to explicitly write it:

$$\boxed{f(\lambda) = \frac{L(\lambda/(1+z))}{4\pi D_L^2(z)},} \tag{30}$$

which one of the main equations you need to know for this lecture.

However, we generally do not use directly the flux to define the brightness of a star or a galaxy We generally use the magnitude in a given band of observation.

4 Astronomical Magnitude

In astronomy, magnitude is a logarithmic measurement of the brightness of an object in a defined band observation. Magnitudes should be quoted for a specific wavelength range since real detectors are not sensitive to the entire EM spectrum. Astronomers use two different definitions of magnitude: apparent magnitude and absolute magnitude. The apparent magnitude is the brightness of an object as it appears in the night sky from Earth, while the absolute magnitude describes the intrinsic brightness of an object as it would appear if it were placed at 10 parsecs distance from Earth.

4.1 Apparent Magnitud

This quantity is a logarithmic comparison between the measured flux and another of reference in the same band of observation. During many years, the spectrum from the star Vega (Vega, also designated Alpha Lyrae is the brightest star in the constellation of Lyra) was used as the reference, but we often prefer to use the AB system which use a constant flux independently of the frequency. The definition of a magnitude in a band b is given by:

$$mag_b = -2.5\log_{10}\left(\frac{f_b^{mes}}{f_b^{ref}}\right), \tag{31}$$

where f_b^{mes} and f_b^{ref} are the measured flux and the one of reference. For example, the AB system use a constant flux of 3.631×10^{-20}erg.s^{-1}cm^{-2}Hz^{-1}. We understand that we need to make corrections when we want to compare various magnitudes from different systems.
Because the flux is the sum of the stars, we can say that the flux is $f^{mes} = M_* \times f_b^{mes}(1M_\odot)$, where the M_* is the stellar mass and $f_b^{mes}(1M_\odot)$ is the flux for a galaxy of only 1 solar mass. For this reason, we can write the difference between to magnitudes as:

$$mag_{b1} - mag_{b2} = -2.5 \left[\log_{10}\left(\frac{f_{b1}^{mes}}{f_{b1}^{ref}}\right) - \log_{10}\left(\frac{f_{b2}^{mes}}{f_{b2}^{ref}}\right) \right] \tag{32}$$

$$mag_{b1} - mag_{b2} = -2.5 \left[\log_{10}\left(\frac{f_{b1}^{mes} f_{b2}^{ref}}{f_{b1}^{ref} f_{b2}^{mes}}\right) \right] \tag{33}$$

$$mag_{b1} - mag_{b2} = -2.5 \left[\log_{10}\left(\frac{M_* \times f_{b1}^{mes}(1M_\odot) f_{b2}^{ref}}{M_* \times f_{b2}^{mes}(1M_\odot f_{b1}^{ref})}\right) \right] \tag{34}$$

so we can simplify the M_* and we find that the difference between two magnitudes do not depend on the stellar mass:

$$mag_{b1} - mag_{b2} = -2.5 \left[\log_{10}\left(\frac{f_{b1}^{mes}(1M_\odot) f_{b2}^{ref}}{f_{b2}^{mes}(1M_\odot f_{b1}^{ref})}\right) \right] = mag_{b1}(1M_\odot) - mag_{b2}(1M_\odot) \tag{35}$$

The difference between 2 magnitudes is named a color and is independent of the stellar mass of the galaxy. So we can create son color selection for galaxies that works independently of the luminosity. (The reality is a bit more complex because two galaxies of same type but with different stellar mass will probably have a slightly different metalicity that can complicate a bit the work.)

We will see how we can generate synthetic spectra and magnitudes of galaxies taking into account all these effects on the next section.

5 SDSS-I-I-II -IV

SDSS has imaged about one-third of the night sky in five broad bands (ugriz). The resulting catalog includes photometry for almost half a billion unique objects. Understanding how to use SDSS imaging data requires some knowledge of how the data were collected. This page `http://www.sdss.org/dr14/imaging/imaging_basics/` explains what you need to know about SDSS imaging data. What are the bands in SDSS

To learn more about current dark energy experiment read (`https://arxiv.org/abs/1508.04473`)

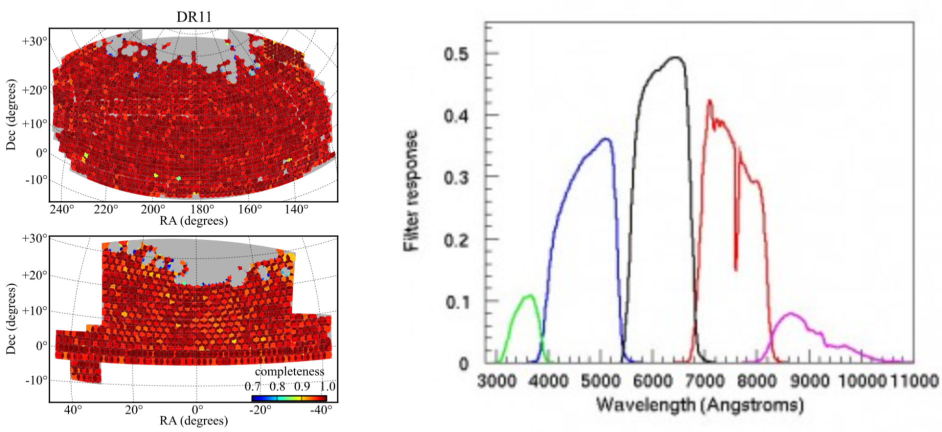

Figure 1: BOSS footprint.

6 Spectral Energy Density (SED)

In order to model the galaxy spectra (what is a galaxy?), it is necessary to take into account the sum of the stars spectra and dust effects. I propose to use the synthetic model from Bruzual & Charlot 2003 ([1])

The SED of a galaxy is the sum of all the star spectra considering their mass and ages, adding if necessary the effect of the dust (which will not be for our case). The changes between different galaxies at a same redshift will depend exclusively on the composition in mass and ages of the stars inside the galaxy. Thus for modeling the SED we need at least 2 ingredients: the distribution initial of stars per bin of mass called Initial Mass Function (IMF) and the distribution of ages.

6.1 Initial Mass Function (IMF)

It allows to choose the distribution on mass for the stars formed during a burst. For our project, we propose to use the standard Chabrier IMF. (see [2], for new kind of IMF read [4] and [3])This prescription is given by:

$$\phi(\log(m)) \propto \begin{cases} \exp\left[\frac{-(\log(m)-\log(m_c))^2}{2\sigma^2}\right] & \text{for } m \leq 1M_\odot \\ m^{-1.3} & \text{for } m > 1M_\odot \end{cases} \tag{36}$$

where $m_c = 0.08M_\odot$ and $\sigma = 0.69$. The results provided by the files with name like "couleurs_yjhbk_zf_70.txt" are following this prescription.

6.2 Star Formation History (SFH)

One time we choose the distribution of star masses are formed during a burst, we have to decide how much stellar masses are formed and when. This is exactly the definition of the star formation history (SFH). In our project, we propose to use simple single-burst. This suppose that all the stars were formed at the same time (just one burst) at a given redshift of formation z_f.

The files "couleurs_yjhbk_zf_#.txt" provides the SDSS magnitudes (u,g,r,i,z) in the AB system for **stellar mass $\mathbf{M_* = 1M_\odot}$** at all redshift between 0 and 3 taking into account the distance, the redshift evolution and the star evolutions for $z_f = \#/10$. You will play with these files to find the best photometric redshift of red galaxies from the SDSS survey.

The SED is given in the rest-frame of the galaxy so, we have to take into account the evolution of the spectrum with the redshift (Distance and stretch) and consider that the age of the galaxy is defined by the time between the observed redshift and the formation redshift. Using a single burst model, all the stars formed at a formation redshift z_f. The age of these stars in a galaxy we observe at z_{obs} is the time passing between these 2 redshifts. This time is generally named look-back time and corresponds to the time travelling of the photon between the 2 redshift (so the comoving distance divided by c). So we have the age defined as:

$$age(z_{obs}) = t_{lb}(z_{obs}, z_f) = \int_{z_{obs}}^{z_f} \frac{dz}{H(z)} \tag{37}$$

6.3 Photometric redshift (Photo-z)

Using this simple model, you will see that you have a nice tool to determine the photometric redshift. But what is a photo-z? As you can see on the figure 2, the spectrum of a red galaxy present a break at 4000Å in the galaxy restframe. This break creates a difference in the magnitude of the bands which strongly depends on the redshift of observation. Using the colors, so the difference between magnitudes, we can determine which redshift allows to model the best the observed color.

You will play with the various files to create an algorithm which allows to determine this photometric redshift for SDSS galaxies.

7 Spectra and bolometric flux from BC03

In figure 2, we can see the evolution (without the dilution, just the strecth) of a typical spectrum of red galaxy formed a redshift=7 and observed at $z = 0.$, $z = 0.2$ and $z = 0.4$. Are represented the 5 bands of the Sloan Digital Sky Survey instrument (SDSS). We can see immediately that the difference of magnitudes in the bands

G, R, I change with the redshift. So, we see that the colors $G - R$ and $R - I$ will be important to determine the photometric redshift of these galaxies.

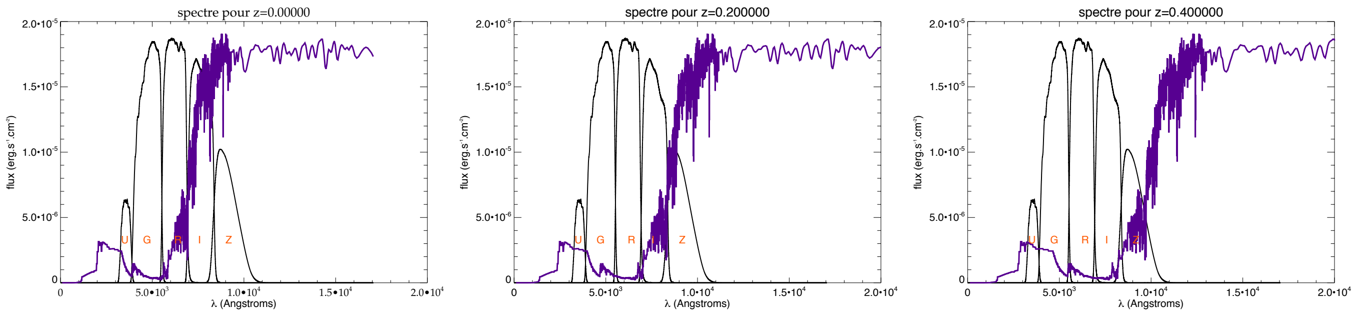

Figure 2: Typical spectrum of red galaxy formed a redshift=7 and observed at $z = 0.$, $z = 0.2$ and $z = 0.4$ with the SDSS bands. The spectrum here is just stretched by the redshift but non effect of dilution and evolution are take into account. Is just a sketch to understand the magnitude and color variations.

The figure 3 shows the evolution of standard mix of stars spectra, given a Initial Mass Function (IMF), with the age which is indicated in billion years. The galaxy spectrum is the sum of the stars spectra, with different ages depending on the Star Formation History (SFH) and the presence or not of the dust.

The BC03 code provide to us the luminosity (so in the rest-frame of the galaxy) for a stellar mass $M_* = 1M_\odot$ allowing us to generate the luminosity for the stellar mass we want as:

$$L(\lambda, M_*, age) = M_* \times L(\lambda, 1M_\odot, age), \tag{38}$$

where the age is the time between the starburst (z_f) and the redshift of observation of the galaxy z_{obs} as define in Eq. 37. This is the quantity we get from the BC03 code. We can finally derive the bolometric flux that should observe a perfect spectrograph that takes into account the redshift (i.e. $\lambda_{obs} = (1 + z)\lambda_{emitted}$) and the damping due to the sphere propagation as:

$$F(\lambda, M_*, age) = M_* \times \frac{L(\lambda/(1+z), 1M_\odot, age)}{4\pi D_L^2(z)}, \tag{39}$$

8 Magnitude estimation from spectra

One time we have the possibility to generate spectra for a given stellar population at any time we can derive the flux we should observe for a given experiment. In order to do that, we need to know the bands of observation, that means the efficiency of photon detection at each wavelength. This efficiency is the convolution of :

- mirror efficiency
- filter efficiency
- CCD efficiency
- Optics efficiency

and we define the efficiency of the band b as the function $R_b(\lambda)$ with values in [0,1] for all wavelength λ as it is shown on the figure 3 for the 5 bands of SDSS. We can derive the theoretical magnitude of a galaxy with a stellar mass M_* as follow:

$$mag_b(z, M_*) = M_* \times \frac{\int_0^\infty d\lambda L(\lambda/(1+z))R_b(\lambda)}{4\pi D_L^2(z)\int_0^\infty d\lambda R_b(\lambda)}, \tag{40}$$

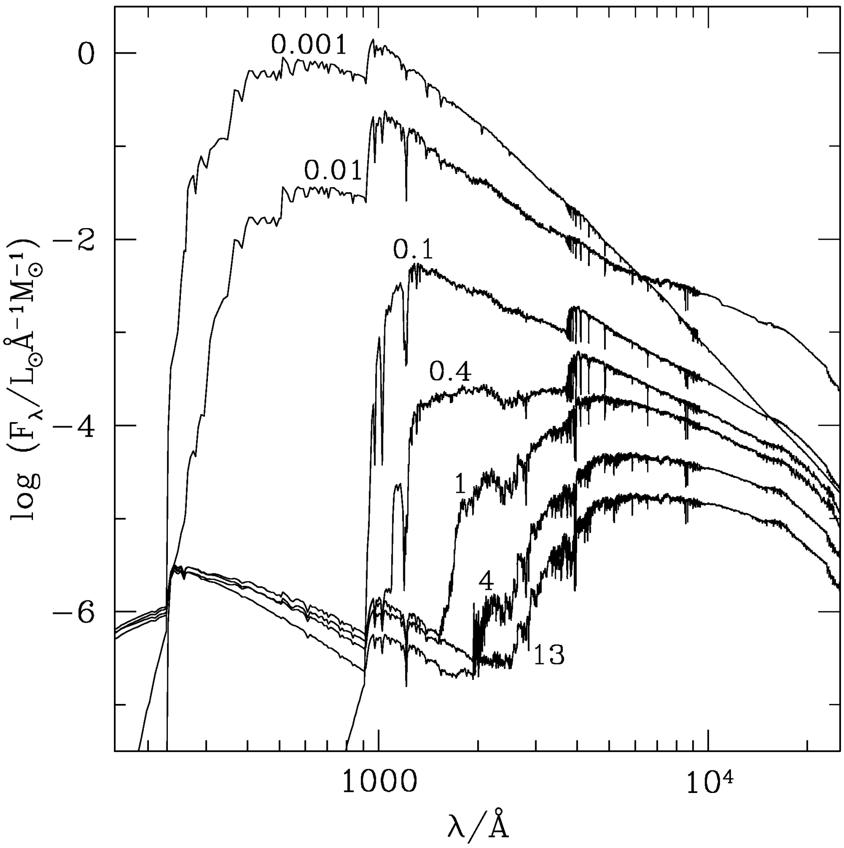

Figure 3: Age evolution of a single burst galaxy spectrum. The ages are given in billion years

where $L(\lambda) = L(\lambda, 1M_\odot, age)$ in order to simplify the notation in the equation. The files we provide to you are these magnitudes for $M_* = 1M_\odot$ for different models of single burst, so different formation redshifts z_f.

9 Error on magnitudes

A photometric survey depth is characterized by its limit magnitudes at N σ (one for each band). For example, if you take a look to SDSS characteristics you will find that the 10 σ magnitudes are (`https://www.sdss.org/dr16/algorithms/magnitudes/`):

- $mag^u_{10\sigma} = 22.12$
- $mag^g_{10\sigma} = 22.6$
- $mag^r_{10\sigma} = 22.29$
- $mag^i_{10\sigma} = 21.85$
- $mag^z_{10\sigma} = 20.32$

that we have to use for error on magnitude measurement. To do that, we can write that the observed flux in band U corresponding to $mag_u = 22.12$ is 10 times the noise flux in this band:

$$22.12 = -2.5log_{10}\left(\frac{10 * f_{u,noise}}{f_{u,0}}\right) \quad (41)$$

$$mag_u = -2.5log_{10}\left(\frac{f_u}{f_{u,0}}\right) \quad (42)$$

$$22.12 - mag_u = -2.5log_{10}\left(\frac{10 * f_{u,noise}}{f_u}\right) \quad (43)$$

$$\frac{S}{N} = 10 \times 10^{\frac{22.12 - mag_u}{2.5}} \quad (44)$$

where the signal to noise ratio is by definition $S/N = f_u/f_{u,noise}$. Now we can derive the theoretical error on magnitude in U band as follow:

$$\delta mag_u = -2.5\delta(log_{10}(f_u/f_{u,0})) = \frac{-2.5}{ln(10)} \times \delta(ln(f_u/f_{u,0})) \Rightarrow \quad (45)$$

$$\delta mag_u = \frac{-2.5}{ln(10)} \times \left(\frac{\delta f_u}{f_u}\right) \Rightarrow \delta mag_u = \frac{-2.5}{S/N \times ln(10)}. \quad (46)$$

The same work can be done for each band and we present the comparison between real errors on SDSS G band measurement and this formula in figure 4. So, you can use this formalism in order to predict the error in magnitude measurement for survey characteristics, in particular for future surveys that provide the expected goal on limit magnitud they pretend to reach.

10 Practice

We have 3 sessions for the practice. First session we will discuss the cosmology material in particular the distances. And if any time we will review briefly python.

10.1 Play with the data

1. If you do not have already install python and jupyter do it using the helping note.
2. Install numpy, astropy, astroquery packages if needed.
3. Using the notebook **Notebook_read_sdss_data.ipynb** (using astro_query.sdss method), get the magnitudes in u,g,r,i and z band of SDSS objects (you can find more information in the doc for using tables astropy: `http://docs.astropy.org/en/stable/table/`). We want to get the spectroscopic redshift for these objects too, reason why we use the cross information with spectroscopic observation in the example. We will use a radius of 60 arcminutes in order to have enough galaxies. Use one (ra,dec) covered by the SDSS mask (See Figure 1 of the footprint in equatorial Coordinate System).

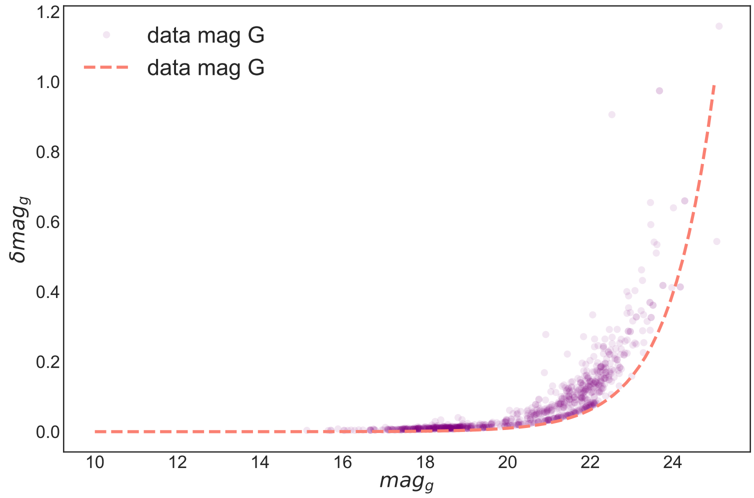

Figure 4: Comparison between real errors on SDSS G band measurement. We can see that it models pretty well the real errors. We can distinguish various generation of data observation that produce different trends. The errors for older observations are bit higher.

4. Read the model magnitudes for $z_f = 0.7$ and $z_f = 0.5$ and plot the colors $g - u$, $g - r$, $g - i$ and $g - z$ for redshift between 0 and 1.
5. Overplot the colors for the objects you download from the server.
6. Plot the color G-R Vs I magnitude for all the objects
7. Do the same plot but using a different marker for the galaxies (type 3) and stars (type 6). You can see that there is a segregation between the two populations around the color value G-R=0.5. That's how one create color magnitude selection cuts in order to select specific kind of objects. This is a very simplified version, search for the criteria used in the target selection of SDSS for Luminous Red Galaxies. (`https://arxiv.org/pdf/astro-ph/0108153.pdf`) for separating stars and galaxies.
8. select the red galaxies using a simple cut $g - r > 0.5$.

10.2 Determine photo-z and associate error

1. Define one set of colors (you have different combinations that use the 5 bands in 4 pairs.)
2. Find the best redshift for all the galaxies using the set of 4 colors you choose using the model of single starburst at $z_f = 6$. Plot the χ^2 and mark the minimun with a vertical line and the $\Delta\chi^2$ as an horizontal line, and derive the confidence interval at 1σ using the $\Delta\chi^2$ law (Take into account the d.o.f).
3. Compare them with the spectroscopic redshift get from the SDSS data in a scatter plot adding the error bars you just derived from $\Delta\chi^2$. Make a distribution of $z_{photo} - z_{spec}$ and derive the mean photometric error per redshift bins.
4. You can use the value of best-fit to eliminate some outliers. To do that, you can reject the best-fit with a χ^2 greater than a cut corresponding to 95 %. In other words, you have to find the value χ^2_{max} defined as:

$$\int_0^{\chi^2_{max}} d\chi^2 p(\chi^2, N_{d.o.f}) = 0.95, \tag{47}$$

   for the correct number of degree of freedom $N_{d.o.f}$.
5. Compare the distribution owith and without outliers, should improve the results.
6. Up to now we used colors which are independent of the total stellar mass. Reason why the magnitudes from the model files are given for 1 solar mass. The stellar mass can be derived as $M_* = 10^{\left(\frac{mag(1M_\odot) - mag_{obs}}{2.5}\right)}$. Another way to discard outliers is to conserving only the stellar masses corresponding to a galaxy (i.e.

$M_* \sim 10^9 - 10^{12} M_\odot$). A value outside this range should indicate that we are in presence of a star or that we associate a wrong redshift to a galaxy leading to a strange associate stellar mass. Calculate the corresponding stellar mass for each galaxy using the band r and check if it is coherent with the typical stellar masses we find in the literature for this kind of galaxies.

7. If you want to do more, do the same work with the other models we provide to you in order to improve the results. In order to do that, you can expand the parameter space with the formation redshift and so evaluate the 2-dimension $\chi^2(z, z_f)$. Do not forget to adapt the numero of degree of freedom.

10.3 Fisher Matrix analysis

The idea of the last part is to work on the Fisher Matrix estimation of the errors for the photometric redshift estimation using one model. You will first read the note we provide to you to calculate the Fisher matrix in the context of this project. Then you will implement:

1. Use the model BC03 for $z_f = 0.5$ as the fiducial one. So we assume that this model is the correct one and that the magnitude of the galaxies we will generate follow this model. Generate the theoretical magnitudes (u,g,r,i,z) for a galaxy of stellar mass $M_* \sim 10^{10} M_\odot$ for all the following redshift values $z \in [0.1, 0.3, 0.5, 0.7, 0.9]$

2. Generate the associated errors we expect to have observing this galaxy for the following limit magnitudes :

   - $mag^u_{10\sigma} = 25.12$
   - $mag^g_{10\sigma} = 25.6$
   - $mag^r_{10\sigma} = 25.29$
   - $mag^i_{10\sigma} = 24.85$
   - $mag^z_{10\sigma} = 23.32$

3. Evaluate the Fisher matrix $F_{ij} = -\left\langle \frac{\partial^2 lnL}{\partial_i \partial_j} \right\rangle$ with $i = z_p, z_f$ for $z \in [0.1, 0.3, 0.5, 0.7, 0.9]$ (So it is a 2 by 2 matrix). You will need to use the model for $z_f = 4.5$ and $z_f = 5.5$ in order to evaluate the derivative over z_f.

4. Deduce the expected photometric redshift estimation error defined as :

$$\sigma_{z_p} = \sqrt{F^{-1}_{z_p z_p}(z_p, z_f)} \tag{48}$$

5. Do the same work for the following survey characteristics:

   - $mag^u_{10\sigma} = 26.12$
   - $mag^g_{10\sigma} = 26.6$
   - $mag^r_{10\sigma} = 26.29$
   - $mag^i_{10\sigma} = 25.85$
   - $mag^z_{10\sigma} = 24.32$

   and compare the expted errors on photometric redshift estimation between the 2 surveys.

A Energy density evolution

Using the equations of Hubble and Friedmann-Robertson-Walker, we can develop the evolution of the scale factor of the Universe considering a dominant component of energy. We will first postulate that each component dominates at one time and we will develop the results. Then, using these results and using the actual values of the component, we will derive the time each of the component was the dominant one during the Universe history.

Let start with the FRW equations relying the pressure $p$and the energy density ρ with the scale factor for a dominating component:

$$\dot{\rho} + 3\frac{\dot{a}}{a}(\rho + p) = 0 \quad \Rightarrow \quad \dot{\rho} + 3\frac{\dot{a}}{a}\rho(1+\omega) = 0 \tag{49}$$

where we substitute the pressure with its relation to the density assuming a perfect fluid with equation of state $p = w\rho$. We can rewrite this equation as follow

$$\frac{\dot{\rho}}{\rho} + 3\frac{\dot{a}}{a}(1+\omega) = 0 \quad \Rightarrow \quad \frac{d}{dt}ln(\rho) + 3(1+\omega)\frac{d}{dt}ln(a) = 0, \tag{50}$$

where we can recognize the time derivative of the log of a and ρ. Using the log property between the multiplication and the power law of the argument we directly find:

$$\frac{d}{dt}ln(\rho) = -3(1+\omega)\frac{d}{dt}ln(a) \quad \Rightarrow \quad \frac{d}{dt}ln(\rho) = \frac{d}{dt}ln(a^{-3(1+\omega)}), \tag{51}$$

which give th trivial result:

$$\rho \propto a^{-3(1+\omega)}, \qquad \rho = \rho_0 a^{-3(1+\omega)}. \tag{52}$$

Considering the different values of the equations of state for the matter, radiation and Dark Energy (specific case of it is $\omega = -1$ which correspond to the cosmological constant):

$$\omega_m = 0 \quad \Longleftrightarrow \quad \rho_m(a) = \rho_{m,0} \times a^{-3} \tag{53}$$
$$\omega_R = 1/3 \quad \Longleftrightarrow \quad \rho_R(a) = \rho_{R,0} \times a^{-4} \tag{54}$$
$$\omega_\Lambda = -1 \quad \Longleftrightarrow \quad \rho_\Lambda(a) = Cte \tag{55}$$
$$\omega_{DE} < -1/3 \tag{56}$$

We will see in the next appendix why the Dark Energy condition is $\omega_{DE} < -1/3$.

B Scale factor evolution with the time

The goal of this appendix is to derive the time evolution of the scale factor of the Universe with the time over the different epoch of domination. Starting from the Hubble equation, in the case of a dominating component of energy we obtain (By definition, the sum disappear and we just keep one):

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3}. \tag{57}$$

in which we assume that the scale factor evolution follows a power law of time: Then we assume a

$$a \propto t^n \tag{58}$$
$$\dot{a} \propto t^{n-1} \tag{59}$$
$$\frac{\dot{a}}{a} \propto t^{-1} \tag{60}$$
$$\left(\frac{\dot{a}}{a}\right)^2 \propto t^{-2} \tag{61}$$

Using the results from the last appendix and reintroducing on this result we finally obtain that:

$$\rho \propto t^{-3(1+\omega)\times n} \tag{62}$$
$$-2 = -3(1+\omega)\times n \tag{63}$$
$$n = \frac{2}{3(1+\omega)} \tag{64}$$
$$a(t) \propto t^{\frac{2}{3(1+\omega)}}. \tag{65}$$

The specific cases of evolution are:

$$\omega_R = 1/3 \quad \Rightarrow \quad a(t) \propto t^{1/2} \tag{66}$$
$$\omega_m = 0 \quad \Rightarrow \quad a(t) \propto t^{2/3} \tag{67}$$

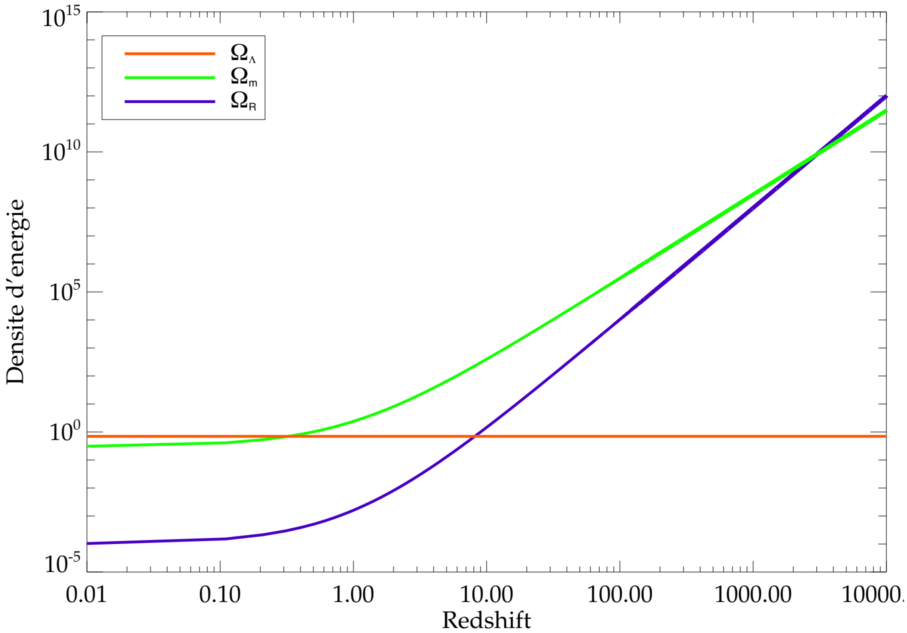

Figure 5: Evolution of the energy density starting with the local values. We can see that today the Universe is dominated by the Dark Energy/Cosmological constant. It was dominated by the matter before; and first by the radiation far away in the past.

Exercise: Show the condition over the value of ω to have an acceleration of the expansion of the Universe. Tip: it's equivalent to show the condition to have $\ddot{a} > 0$

However, in the case of $\omega = -1$ then we can not use this method because it is equivalent to devide by 0. But the solution is easier because we now since the previous appendix that it correspond to constant value of the energy density. Using this, we simply find that:

$$\rho_\Lambda = cte \Leftrightarrow \dot{\rho_\Lambda} = 0 \tag{68}$$

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho_\Lambda}{3} = cte \tag{69}$$

$$\boxed{\omega_\Lambda = -1 \qquad \Rightarrow \qquad \frac{\dot{a}}{a} = \sqrt{\frac{8\pi G\rho_\Lambda}{3}} \qquad \Rightarrow \qquad a(t) \propto \exp^{\sqrt{\frac{8\pi G\rho_\Lambda}{3}}}} \tag{70}$$

which is an exponential evolution with the time!

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