Searching and Selection

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Reference

"The Art of Computer Programming, Vol 3: Sorting and Searching" by Donald E. Knuth.

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References

"Selecting the Top Three Elements" by Aigner, 1982.

The largest k elements (Problem 3.5)

Close to median (Problem 3.6)

Medians of sorted arrays (Problem 3.7)

Weighted median (Problem 3.9)

Searching and Selection

$\max$ / $\min$ differences (Problem 4.5)

(a) unsorted; $\max |x - y|$; $O(n)$

(b) sorted; $\max |x - y|$; $O(1)$

(c) unsorted; $\min |x - y|$; $O(n \log n)$

(d) sorted; $\min |x - y|$; $O(n)$

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- M: matrix $m \times n$
- row: increasing from left to right
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$$i + j \leq n - 1 \implies x > M_{ij}$$
$$i + j > n - 1 \implies x < M_{ij}$$

Local