Evasiveness of acyclicity of undirected graph

The lecture note by Jeff Erickson discusses "Evasive Graph Properties":

> We call a graph property *evasive* if we have to look at all $\binom{n}{2}$ entries in the adjacent matrix to decide whether an (undirected) graph has that property.

As an example, it shows that connectivity is evasive.

I want to know: Is acyclicity evasive? How to prove/disprove it?

My attempt: A rather straightforward adaptation I made from the proof for connectivity does not work for acyclicity.

algorithms graph-theory lower-bounds

edited just now

asked 14 hours ago
hengxin
5,884 10 37

1 This appears to be exercise 4 in the lecture notes you linked. That exercise gives a hint. I haven't verified whether the hint is helpful, but it seems worth a try. Also, Conjecture 1 in those lecture notes also suggests that it ought to be true, which says to me it's worth focusing on finding a proof (rather than a disproof). – D.W. ♦ 13 hours ago

question eligible for bounty tomorrow

1 Answer

Consider any algorithm for acyclicity. We will use the following adversary:

> Whenever the algorithm asks for an edge, say that the edge exists if it connects two different connected components. (That is, if it doesn't close a cycle.)

You can show that the graph constructed by this dialog is always a forest, and moreover if we the constructed graph has the same connected components as the query graph (which contains all queried edges). In particular, when the last edge is asked, the graph is a tree (assuming $n \geq 3$; otherwise every graph is acyclic), and the graph is acyclic if and only if this edge does not belong to the graph.

answered 7 hours ago
Yuval Filmus
138k 6 125 261

Is there a word missing in "moreover if we the constructed graph"? – D.W. ♦ 3 hours ago

Answer Your Question