Why do these recurrences determine the number of ways of tiling a 3xN rectangle with 2x1 dominoes?

http://www.algorithmist.com/index.php/UVa_10918

The above link is a solution to UVa 10918 Problem. The problem is based on Dynamic Programming. I am not able to understand this approach to the problem. I have coded the solution but the approach is completely different. I want to understand the given approach. The problem is:

Determine in how many ways can a 3xN rectangle be completely tiled with 2x1 dominoes.

I only want to know how these recurrence relations came:

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f(n)=f(n-2)+2*g(n-1)
g(n)=f(n-1)+g(n-2)
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where f(n)=number of tilings of a 3xN rectangle g(n)= number of tilings of a 3xN rectangle with one of its corner squares removed

recurrence-relation dynamic-programming correctness-proof

edited May 5 '15 at 15:18
Raphael ♦
48.4k 17 116 255

asked May 5 '15 at 13:25
Temp Id
6 3

1 It is explained on the site. So can you be more explicit as to what you do not understand? – babou May 5 '15 at 14:12

1 What have you tried and where did you get stuck? Hint: correctness of recurrences is typically shown by induction (here on n). – Raphael ♦ May 5 '15 at 15:19

2 Answers

Suppose that the rectangle to be tiled has 3 rows and n columns. Consider a tiling of this rectangle using 2×1 dominos. There are two basic options:

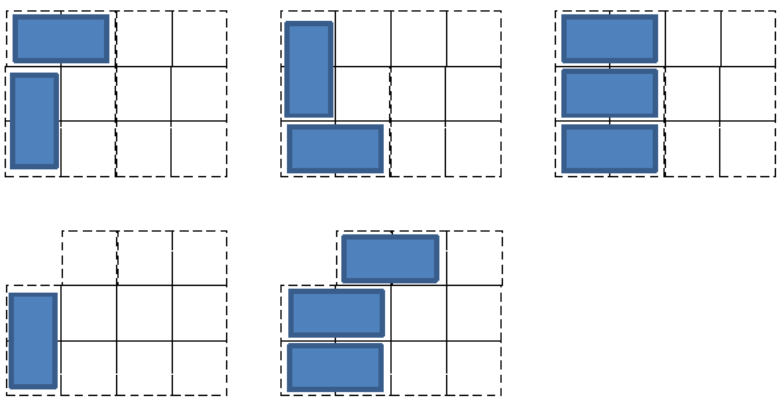
1. All tiles touching the nth column are horizontal. There must be 3 of them, and if you remove them, you get a tiling of a rectangle of size $3 \times (n-2)$.
2. Exactly one tile touching the nth column is vertical. It can either touch the top or the bottom. For each of these two options, if you remove it then you get a tiling of a rectangle of size $3 \times (n-1)$ with one corner square added. That square must be tiled by a horizontal tile, after whose removal you are left with a tiling of a $3 \times (n-1)$ rectangle with one corner square removed.

This explains the formula $f(n) = f(n-2) + 2g(n-1)$.

The same sort of analysis yields the formula for $g(n)$, but I leave that one for you.

answered May 5 '15 at 13:47
Yuval Filmus
141k 6 128 267

The picture should say more than words.



edited May 5 '15 at 14:36

answered May 5 '15 at 13:48
Hendrik Jan
16.7k 16 56

But if you need the words, see the answer by @Yuval-Filmus. – Hendrik Jan May 5 '15 at 13:51

Sorry. As @babou observes above the first link in the question has some explanation *and* these pictures (albeit as ascii art). So how can they help? – Hendrik Jan May 6 '15 at 23:34