Dynamic Programming

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June 11 – June 19, 2017

Dynamic Programming

1. 1D DP
2. 2D DP
3. DP on Graphs
4. The Knapsack Problem

$f^{(S(n))} = 1$

$f^{(S(n))} = 1$ (Problem 7.2)

$$f(n) = \begin{cases} n - 1 & \text{if } n \in \mathbb{Z}^+ \\ n/2 & \text{if } n\%2 = 0 \\ n/3 & \text{if } n\%3 = 0 \end{cases}$$

$S(n)$: minimum number of steps taking n to 1.

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$S(i)$: minimum number of steps taking i to 1

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$S(n)$: minimum number of steps taking n to 1.

$S(i)$: minimum number of steps taking i to 1

$$S(i) = 1 + \min\{N(i-1), N(i/2)(\text{if } n\%2 = 0), N(i/3)(\text{if } n\%3 = 0)\}$$

$$S(1) = 0$$

$f^{(S(n))} = 1$

Collatz $(3n+1)$ conjecture:

$$f(n) = \begin{cases} n/2 & \text{if } n\%2 = 0 \\ 3n+1 & \text{if } n\%2 = 1 \end{cases}$$

$$f^*(n) = 1?$$

$f^{(S(n))} = 1$

Collatz $(3n+1)$ conjecture:

$$f(n) = \begin{cases} n/2 & \text{if } n\%2 = 0 \\ 3n+1 & \text{if } n\%2 = 1 \end{cases}$$

$$f^*(n) = 1?$$

"Mathematics may not be ready for such problems."

— Paul Erdős

Longest increasing subsequence

Longest increasing subsequence (Problem 7.3)

- Given an integer array $A[1 \dots n]$
- To find (the length of) a longest increasing subseqence.

$$5, 2, 8, 6, 3, 6, 9, 7 \implies 2, 3, 6, 9$$

Longest increasing subsequence

Subproblem: $L(i)$: the length of the LIS of $A[1 \ldots i]$

Goal: $L(n)$

Longest increasing subsequence

Subproblem: $L(i)$: the length of the LIS of $A[1 \dots i]$

Goal: $L(n)$

Make choice: whether $A[i] \in LIS[1 \dots i]$?

Recurrence:

$$L(i) = \max\{L(i-1), 1 + \max_{j < i \wedge A[j] \leq A[i]} L(j)\}$$

Longest increasing subsequence

Subproblem: $L(i)$: the length of the LIS of $A[1 \dots i]$

Goal: $L(n)$

Make choice: whether $A[i] \in LIS[1 \dots i]$?

Recurrence:

$$L(i) = \max\{L(i-1), 1 + \max_{j<i \wedge A[j] \le A[i]} L(j)\}$$

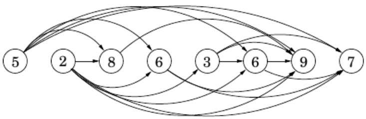
Init:

$$L(0) = 0$$

Time:

$$O(n^2) = \Theta(n) \cdot O(n)$$

Longest increasing subsequence



Longest path distance in the DAG!

Maximum-sum subarray

Maximum-sum subarray (Google Interview)

- Array $A[1 \cdots n], a_i >=< 0$
- To find (the sum of) a maximum-sum subarray of A
  - mss $= 0$ if all negative

$$A[-2, 1, -3, 4, -1, 2, 1, -5, 4] \implies [4, -1, 2, 1]$$

Maximum-sum subarray

Maximum-sum subarray (Google Interview)

- Array $A[1 \cdots n], a_i >=< 0$
- To find (the sum of) a maximum-sum subarray of A
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$$A[-2, 1, -3, 4, -1, 2, 1, -5, 4] \implies [4, -1, 2, 1]$$

Subproblem: MSS$[i]$: the sum of the MS$[i]$ of $A[1 \cdots i]$

Goal: mss $=$ MSS$[n]$

Maximum-sum subarray

Maximum-sum subarray (Google Interview)

- Array $A[1 \cdots n], a_i >=< 0$
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Subproblem: MSS$[i]$: the sum of the MS$[i]$ of $A[1 \cdots i]$

Goal: mss $=$ MSS$[n]$

Make choice: Is $a_i \in$ MS$[i]$?

Recurrence:

$$\text{MSS}[i] = \max\{\text{MSS}[i-1], ???\}$$

Maximum-sum subarray

Subproblem: MSS$[i]$: the sum of the MS$[i]$ *ending with* a_i or 0

Goal: mss $= \max_{1 \le i \le n}$ MSS$[i]$

Maximum-sum subarray

Subproblem: MSS$[i]$: the sum of the MS$[i]$ *ending with* a_i or 0

Goal: mss $= \max_{1 \le i \le n}$ MSS$[i]$

Make choice: where does the MS$[i]$ start?

Recurrence:

$$\text{MSS}[i] = \max\{\text{MSS}[i-1] + a_i, 0\}$$ (prove it!)

Maximum-sum subarray

Subproblem: MSS$[i]$: the sum of the MS$[i]$ *ending with* a_i or 0

Goal: mss $= \max_{1 \le i \le n}$ MSS$[i]$

Make choice: where does the MS$[i]$ start?

Recurrence:

$$\text{MSS}[i] = \max\{\text{MSS}[i-1] + a_i, 0\}$$ (prove it!)

Init:

$$\text{MSS}[0] = 0$$

Maximum-sum subarray

Subproblem: MSS$[i]$: the sum of the MS$[i]$ *ending with* a_i or 0

Goal: mss $= \max_{1 \le i \le n}$ MSS$[i]$

Make choice: where does the MS$[i]$ start?

Recurrence:

$$\text{MSS}[i] = \max\{\text{MSS}[i-1] + a_i, 0\}$$ (prove it!)

Init:

$$\text{MSS}[0] = 0$$

Time: $\Theta(n)$

Maximum-sum subarray

Maximum-product subarray

Maximum-product subarray (Problem 7.4)

Reconstructing string

Reconstructing string (Problem 7.9)

- String $S[1 \cdots n]$
- Dict for *lookup*:

$$\text{dict}(w) = \begin{cases} \text{true} & \text{if } w \text{ is a valid word} \\ \text{false} & \text{o.w.} \end{cases}$$

- Is $S[1 \cdots n]$ valid (reconstructed as a sequence of valid words)?

Reconstructing string

Subproblem: $V[i]$: is $S[1 \cdots i]$ valid?

Goal: $V[n]$

Reconstructing string

Subproblem: $V[i]$: is $S[1 \cdots i]$ valid?

Goal: $V[n]$

Make choice: where does the last word start?

Recurrence:

$$V[i] = \bigvee_{j=1\ldots i} (V[j-1] \land \mathsf{dict}(S[j \cdots i]))$$

Reconstructing string

Subproblem: $V[i]$: is $S[1 \cdots i]$ valid?

Goal: $V[n]$

Make choice: where does the last word start?

Recurrence:

$$V[i] = \bigvee_{j=1\ldots i} (V[j-1] \wedge \mathsf{dict}(S[j \cdots i]))$$

Init:

$$V[0] = \mathsf{true}$$

Time: $O(n^2)$

Hotel along a trip

Hotel along a trip (Problem 7.15)

- Hotel sequence (distance): $a_0 = 0, a_1, \cdots, a_n$
- $a_0 \rightsquigarrow a_n$
- Stop at only hotels
- Cost: $(200 - x)^2$
- To minimize overall cost

Hotel along a trip

Subproblem: $C[i]$: minimum cost when the destination is a_i

Goal: $C[n]$

Hotel along a trip

Subproblem: $C[i]$: minimum cost when the destination is a_i

Goal: $C[n]$

Make choice: what is the last but one hotel a_j to stop?

Recurrence:

$$C[i] = \min_{0 \le j < i} \{C[j] + (200 - (a_i - a_j))^2\}$$

Hotel along a trip

Subproblem: $C[i]$: minimum cost when the destination is a_i

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Recurrence:

$$C[i] = \min_{0 \le j < i} \{C[j] + (200 - (a_i - a_j))^2\}$$

Init:

$$C[0] = 0$$

Time:

$$O(n^2) = \Theta(n) \cdot O(n)$$

Highway restaurants

Highway restaurants (Problem 7.16)

- Locations: $L[1 \ldots n]$
- Profits: $P[1 \ldots n]$
- Any two hotels should be $\geq k$ miles apart
- To maximize the total profit

Subproblem:

Goal:

Make choice:

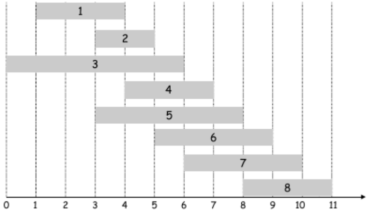
Recurrence:

Time:

Weighted interval/class scheduling

Weighted interval/class scheduling (Problem 7.14)

- Classes: $\mathcal{C} = \{c_1, c_2, \cdots, c_n\} \quad c_i \triangleq \langle g_i, s_i, f_i \rangle$
- Choosing non-conflicting classes to maximize your grades



sort $\mathcal{C}$ by finishing time.

Weighted interval/class scheduling

Greedy algorithms by finishing time or weights fail.

Weighted interval/class scheduling

Subproblem: $G[i]$: the maximal grades obtained from $\{c_1, c_2, \cdots, c_i\}$

Goal: $G[n]$

Weighted interval/class scheduling

Subproblem: $G[i]$: the maximal grades obtained from $\{c_1, c_2, \cdots, c_i\}$

Goal: $G[n]$

Make choice: choose c_i or not in $G[i]$?

Recurrence:

$$G[i] = \max\{G[i-1], G[p(i)] + g_i\}$$

$p(i)$: the largest index $j < i$ *s.t.* c_i and c_j are disjoint

Weighted interval/class scheduling

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Init:

$$G[0] = 0$$

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$p(i)$: the largest index $j < i$ *s.t.* c_i and c_j are disjoint

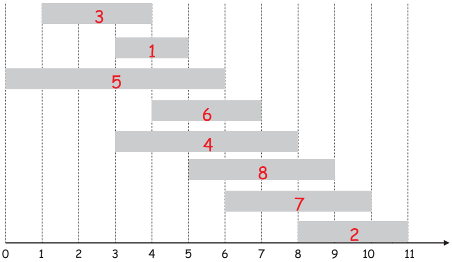
Init:

$$G[0] = 0$$

Time: $O(n \log n) + T(p(i)) + O(n) \cdot O(1)$

Weighted interval/class scheduling

Why is ordering necessary?

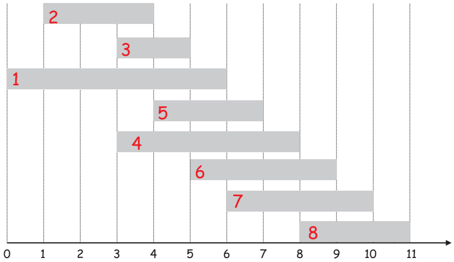


$$G[7] = \max\{G[6], G[\{1,3,5\}] + g_7\}$$

subproblems changed: all $O(2^n)$ subsets

Weighted interval/class scheduling

What about sorting by starting time?



$$G[6] = \max\{G[5], G[\{2,3\}] + g_6\}$$

subproblems changed: all $O(2^n)$ subsets

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Longest common subsequence

LCS: longest common subsequence (Problem 7.5)

$$X = X_1 \cdots X_m \quad Y = Y_1 \cdots Y_n$$

(1) Find (the length of) an LCS of X and Y

$$X = \langle A, B, C, B, D, A, B \rangle$$
$$Y = \langle B, D, C, A, B, A \rangle$$
$$Z = \langle B, C, B, A \rangle$$

Longest common subsequence

Subproblem: $L[i, j]$: the length of an LCS of $X[1 \cdots i]$ and $Y[1 \cdots j]$

Goal: $L[m, n]$

Longest common subsequence

Subproblem: $L[i, j]$: the length of an LCS of $X[1 \cdots i]$ and $Y[1 \cdots j]$

Goal: $L[m, n]$

Make choice: Is $X_i = Y_j$?

Recurrence: (Proof!)

$$L[i,j] = \begin{cases} L[i-1, j-1] + 1 & \text{if } X_i = Y_j \\ \max\{L[i-1, j], L[i, j-1]\} & \text{if } X_i \neq Y_j \end{cases}$$

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Init:

$$L[0, j] = 0, \; 0 \leq j \leq n$$
$$L[i, 0] = 0, \; 0 \leq i \leq m$$

Time: $\Theta(mn)$

Longest common subsequence

Longest common subsequence (Problem 7.5)

$$X = X_1 \cdots X_m \quad Y = Y_1 \cdots Y_n$$

(2) Allowing repetition of X

(3) Allowing repetition $\leq k$ of X

Longest common subsequence

Longest common subsequence (Problem 7.5)

$$X = X_1 \cdots X_m \quad Y = Y_1 \cdots Y_n$$

(2) Allowing repetition of X

(3) Allowing repetition $\leq k$ of X

$$L[i,j] = \begin{cases} L[i, j-1] + 1 & \text{if } X_i = Y_j \\ \max\{L[i-1,j], L[i,j-1]\} & \text{if } X_i \neq Y_j \end{cases}$$

Longest common subsequence

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(2) Allowing repetition of X

(3) Allowing repetition $\leq k$ of X

$$L[i,j] = \begin{cases} L[i, j-1] + 1 & \text{if } X_i = Y_j \\ \max\{L[i-1,j], L[i,j-1]\} & \text{if } X_i \neq Y_j \end{cases}$$

$$X \implies X^{(k)} \triangleq X_1^{(k)} \cdots X_m^{(k)}$$

Longest common substring

What about longest common *substring*?

Shortest common supersequence

Shortest common supersequence (Problem 7.6)

$$X = X_1 \cdots X_m \quad Y = Y_1 \cdots Y_n$$

- Find (the length of) a shortest common subsequence of X and Y

Shortest common supersequence

Subproblem: $L[i,j]$: the length of an SCS of $X[1 \cdots i]$ and $Y[1 \cdots j]$

Goal: $L[m,n]$

Shortest common supersequence

Subproblem: $L[i,j]$: the length of an SCS of $X[1 \cdots i]$ and $Y[1 \cdots j]$

Goal: $L[m,n]$

Make choice: Is $X_i = Y_j$?

Recurrence:

$$L[i,j] = \begin{cases} L[i-1,j-1]+1 & \text{if } X_i = Y_j \\ \max\{L[i-1,j]+1, L[i,j-1]+1\} & \text{if } X_i \neq Y_j \end{cases}$$

Shortest common supersequence

Subproblem: $L[i,j]$: the length of an SCS of $X[1 \cdots i]$ and $Y[1 \cdots j]$

Goal: $L[m,n]$

Make choice: Is $X_i = Y_j$?

Recurrence:

$$L[i,j] = \begin{cases} L[i-1,j-1]+1 & \text{if } X_i = Y_j \\ \max\{L[i-1,j]+1, L[i,j-1]+1\} & \text{if } X_i \neq Y_j \end{cases}$$

Init:

$$L[0,j] = j, \ 0 \le j \le n$$
$$L[i,0] = i, \ 0 \le i \le m$$

Shortest common supersequence

Subproblem: $L[i, j]$: the length of an SCS of $X[1 \cdots i]$ and $Y[1 \cdots j]$

Goal: $L[m, n]$

Make choice: Is $X_i = Y_j$?

Recurrence:

$$L[i, j] = \begin{cases} L[i-1, j-1] + 1 & \text{if } X_i = Y_j \\ \max\{L[i-1, j] + 1, L[i, j-1] + 1\} & \text{if } X_i \neq Y_j \end{cases}$$

Init:

$$L[0, j] = j, \ 0 \leq j \leq n$$
$$L[i, 0] = i, \ 0 \leq i \leq m$$

Remark

$$\max(m, n) \leq L(m, n) \leq m + n$$

Variants of LCS

Variants of LCS (Problem 7.7)

Longest contiguous substring both forward and backward

Longest contiguous substring both forward and backward (Problem 7.8)

- String $T[1 \cdots n]$
- Find a longest contiguous substring (LCS) both forward and backward

dynamicprogrammingmanytimes

- try subproblem $L[i]$: the length of an LCS in $T[1 \cdots i]$
- try subproblem $L[i, j]$: the length of an LCS in $T[i \cdots j]$

Longest contiguous substring both forward and backward

Subproblem: $L[i, j]$: the length of an LCS starting with T_i and ending with T_j

Goal: $\max_{1 \leq i \leq j \leq n} L[i, j]$

Longest contiguous substring both forward and backward

Subproblem: $L[i, j]$: the length of an LCS starting with T_i and ending with T_j

Goal: $\max_{1 \le i \le j \le n} L[i, j]$

Make choice: Is $T_i = T_j$?

Recurrence:

$$L[i, j] = \begin{cases} 0 & \text{if } T_i \neq T_j \\ L[i+1, j-1] + 1 & \text{if } T_i = T_j \end{cases}$$

Longest contiguous substring both forward and backward

Subproblem: $L[i, j]$: the length of an LCS starting with T_i and ending with T_j

Goal: $\max_{1 \leq i \leq j \leq n} L[i, j]$

Make choice: Is $T_i = T_j$?

Recurrence:

$$L[i, j] = \begin{cases} 0 & \text{if } T_i \neq T_j \\ L[i+1, j-1] + 1 & \text{if } T_i = T_j \end{cases}$$

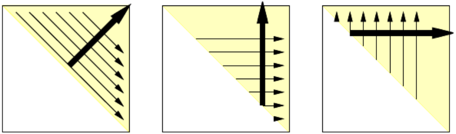
Init:

$$L[i, i] = 0, \ 0 \leq i \leq n$$

$$L[i, i+1] = \begin{cases} 1 & \text{if } T_i = T_{i+1} \\ 0 & \text{if } T_i \neq T_{i+1} \end{cases}$$

Longest contiguous substring both forward and backward

Code: three ways of filling the table



```
for d = 2 to n-1
  for i = 1 to n-d
    j = i + d
    ...
return max_{1 <= i <= j <= n} L[i,j]
```

Longest palindrome subsequence

Longest palindrome subsequence (Problem 7.10)

(1) Find (the length of) a longest palindrome subsequence of $S[1 \cdots n]$

Subproblem: $L[i, j]$: the length of an LSP of $S[i \cdots j]$

Goal: $L[1, n]$

Longest palindrome subsequence

Longest palindrome subsequence (Problem 7.10)

(1) Find (the length of) a longest palindrome subsequence of $S[1 \cdots n]$

Subproblem: $L[i,j]$: the length of an LSP of $S[i \cdots j]$

Goal: $L[1,n]$

Make choice: Is $S[i] = S[j]$?

Recurrence:

$$L[i,j] = \begin{cases} L[i+1, j-1] + 2 & \text{if } S[i] = S[j] \\ \max L[i+1, j], L[i, j-1] & \text{if } S[i] \neq S[j] \end{cases}$$

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Make choice: Is $S[i] = S[j]$?

Recurrence:

$$L[i,j] = \begin{cases} L[i+1, j-1] + 2 & \text{if } S[i] = S[j] \\ \max L[i+1,j], L[i,j-1] & \text{if } S[i] \neq S[j] \end{cases}$$

Init:

$$L[i,i] = 1, \; \forall 1 \leq i \leq n$$

Palindrome splitting

Palindrome splitting (Problem 7.10)

(2) Split a string $S[1 \dots n]$ into minimum number of palindromes ($\#$ cuts)

Subproblem: $C[i, j]$: minimum number of cuts for string $S[i \dots j]$

Goal: $C[1, n] + 1$

Palindrome splitting

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Subproblem: $C[i, j]$: minimum number of cuts for string $S[i \ldots j]$

Goal: $C[1, n] + 1$

Make choice: Where is the first cut?

Recurrence:

$$C[i, j] = \begin{cases} 0 & \text{if } S[i \ldots j] \text{ is a palindrome} \\ \min\limits_{i+1 \le k \le j-1} C[i, k] + 1 + C[k+1, j] & \text{o.w.} \end{cases}$$

Palindrome splitting

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$$C[i,j] = \begin{cases} 0 & \text{if } S[i \dots j] \text{ is a palindrome} \\ \min\limits_{i+1 \le k \le j-1} C[i,k] + 1 + C[k+1,j] & \text{o.w.} \end{cases}$$

Init: $C[i,i] = 0$

Time: $O(n^3)$

Palindrome splitting

Palindrome splitting (Problem 7.10)

(2) Split a string $S[1 \dots n]$ into minimum number of palindromes

Subproblem: $P[i]$: minimum number of palindromes for $S[1 \cdots i]$

Goal: $P[n]$

Palindrome splitting

Palindrome splitting (Problem 7.10)

(2) Split a string $S[1 \dots n]$ into minimum number of palindromes

Subproblem: $P[i]$: minimum number of palindromes for $S[1 \cdots i]$

Goal: $P[n]$

Make choice: Where does the last palindrome start from?

Recurrence:

$$P[i] = \min_{\substack{1 \le k \le i \\ S[k \dots i] \text{ is a palindrome}}} P[k-1] + 1$$

Palindrome splitting

Palindrome splitting (Problem 7.10)

(2) Split a string $S[1 \dots n]$ into minimum number of palindromes

Subproblem: $P[i]$: minimum number of palindromes for $S[1 \cdots i]$

Goal: $P[n]$

Make choice: Where does the last palindrome start from?

Recurrence:

$$P[i] = \min_{\substack{1 \le k \le i \\ S[k \dots i] \text{ is a palindrome}}} P[k-1] + 1$$

Init: $P[0] = 1$

Time: $O(n^2)$

String splitting

String splitting (Problem 7.11)

- Split a string S into many pieces
- Cost $|S| = n \implies n$
- Given locations of m cuts: $C_0, C_1, \cdots, C_m, C_{m+1}$
- Find the minimum cost of splitting S into $m+1$ pieces $S_0 \cdots S_m$

String splitting

Subproblem: $C[i, j]$: the minimum cost of splitting substring $S_i \cdots S_{j-1}$ using cuts $C_{i+1} \cdots C_{j-1}$

Goal: $C[0, m+1]$

String splitting

Subproblem: $C[i,j]$: the minimum cost of splitting substring $S_i \cdots S_{j-1}$ using cuts $C_{i+1} \cdots C_{j-1}$

Goal: $C[0, m+1]$

Make choice: What is the first cut in $C_{i+1} \cdots C_{j-1}$?

Recurrence:

$$C[i,j] = \min_{i<k<j} \left(C[i,k] + C[k,j] + l(S_i \cdots S_{j-1}) \right)$$

String splitting

Subproblem: $C[i,j]$: the minimum cost of splitting substring $S_i \cdots S_{j-1}$ using cuts $C_{i+1} \cdots C_{j-1}$

Goal: $C[0, m+1]$

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Recurrence:

$$C[i,j] = \min_{i<k<j} \left(C[i,k] + C[k,j] + l(S_i \cdots S_{j-1}) \right)$$

Init: $C[i, i+1] = 0$

Subproblem:

Subproblem:

Goal:

Make choice:

Subproblem:

Goal:

Make choice:

Recurrence:

Init:

Time:

Dynamic Programming

1. 1D DP
2. 2D DP
3. DP on Graphs
4. The Knapsack Problem

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THANK YOU