Prove that if the weights on the edges of a connected, undirected graph are distinct, then there is a unique minimum spanning tree.

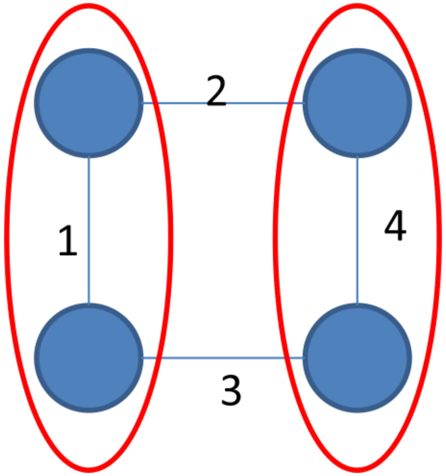
Ans:

1. Say we have an algorithm that finds an MST (which we will call ***A***) based on the structure of the graph and the order of the edges when ordered by weight.
2. Assume MST ***A*** is not unique.
3. There is another spanning tree with equal weight, say MST **B**.
4. Let **e1** be an edge that is in ***A*** but not in **B**.
5. Then **B** should include at least one edge **e2** that is not in ***A***.
6. Assume the weight of **e1** is less than that of **e2**.
7. As **B** is a MST, {**e1**} ∪**B** must contain a cycle.
8. Replace **e2** with **e1** in **B** yields the spanning tree {**e1**} ∪**B** - {**e2**} which has a smaller weight compared to **B**.
9. Contradiction. As we assumed **B** is a MST but it is not.

23.2-8

Professor Borden proposes a new divide-and-conquer algorithm for computing minimum spanning trees, which goes as follows. Given a graph G=(V,E), partition the set V of vertices into two sets V1 and V2 such that |V1| and |V2| differ by at most 1. Let E1 be the set if edges that are incident only on the vertices in V1, and let E2 be the set of edges that are incident only on vertices in V2. Recursively solve a minimum-spanning-tree problem on each of the two subgraphs G1=(V1,E1) and G2=(E2,V2).
Finally, select the minimum-weight edge in E that crosses the cut (V1,V2), and use this edge to unite the resulting two minimum spanning trees into a single spanning tree.
Either argue that the algorithm correctly computes a minimum spanning tree of G, or provide an example for which the algorithms fails.

ANS:
FAILS



如圖 若以教授的找法切成兩個橢圓的話是1+4+2
但實際上最小為1+2+3