Paths in Graphs

Hengfeng Wei

hfwei@nju.edu.cn

June 03, 2017 – June 05, 2017

Paths in Graphs

Dijkstra's algorithm for SSSP

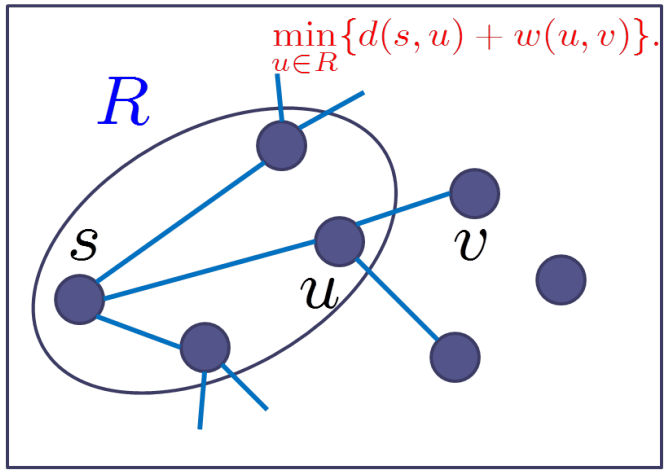
$$R \triangleq \{v \mid s \rightsquigarrow v \text{ is known}\}$$

Finding shortest paths from s to other nodes t in increasing order of $\mathrm{d}(s,t)$.

Theorem (Invariant)

$$\exists d : \begin{cases} d(s,v) \leq l, & \forall v \in R, \\ d(s,v) > l, & \forall v \notin R \end{cases}$$

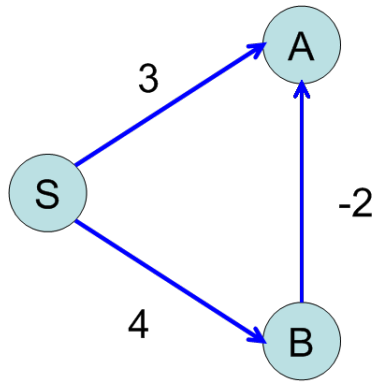
Dijkstra's algorithm for SSSP



Negative edges

Negative edges (Problem 6.16)

Dijkstra's algorithm may fail if $w(e) < 0$.



Negative edges

Negative edges from s (Problem 6.21)

All negative edges are from s.

$$\arg\min_{(s,v)} w(s,v)$$

Generalized shortest path problem

Generalized shortest path problem (Problem 6.20)

Shortest paths among nodes

Shortest paths among nodes (Problem 6.27)

Shortest path through v_0

Shortest paths through v_0 (Problem 6.28)

Shortest path in maze

Shortest paths in maze (Problem 6.24)

Bellman-Ford algorithm

Bellman-Ford algorithm (Problem 6.30)

Paths in Graphs

Dijkstra's algorithm

$$\mathsf{d}(v) = \min_{u \in N(v)} \{\mathsf{d}(u) + l(u, v)\}$$

for all $v \in V$ **do**
    $\mathsf{d}[v] \leftarrow \infty$
$\mathsf{d}[s] \leftarrow 0$

$Q \leftarrow \mathsf{MinPQ}(V)$

while $Q \neq \emptyset$ **do**
    $u \leftarrow \mathsf{deleteMin}(Q)$
    for all $(u, v) \in E \wedge v \in Q$ **do**
        if $\mathsf{d}[v] > \mathsf{d}[u] + l(u, v)$ **then**
            $\mathsf{d}[v] \leftarrow \mathsf{d}[u] + l(u, v)$
            decreaseKey(Q, v)

$$O(n + (n + m) \log n) \implies O((n + m) \log n) \implies O(m \log n)$$

Dijkstra's algorithm

Unique shortest paths

Unique shortest paths (Problem 6.18)

Number of shortest paths

Number of shortest paths (Problem 6.31, 5.26)

Min-max path problem

Min-max path problem (Problem 6.23)

Max-min path problem (Problem 6.26)

Paths in Graphs

Floyd-Warshall algorithm

Routing table

Routing table (Problem 6.25)

Shortest cycle in digraph

Shortest cycle in digraph

Paths in Graphs

Hamiltonian path in Tournament graph

Hamiltonian path in Tournament graph (Problem 6.22)

$$\forall u, v : (u \to v \vee v \to u) \tag{1}$$

$$\wedge \not(u \to v \wedge v \to u) \tag{2}$$


THANK YOU