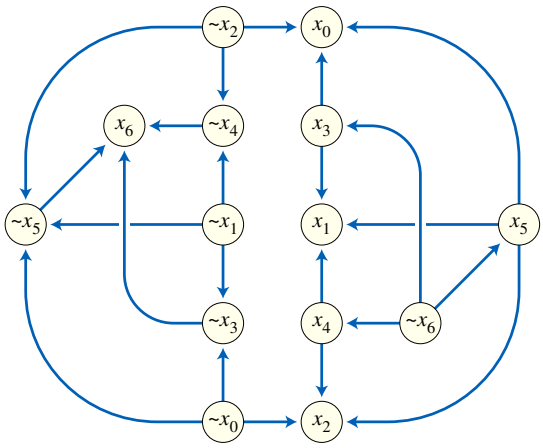
# Implication graph




*An implication graph representing the 2-satisfiability instance* $(x_0 \vee x_2) \wedge (x_0 \vee \neg x_3) \wedge (x_1 \vee \neg x_3) \wedge (x_1 \vee \neg x_4) \wedge (x_2 \vee \neg x_4) \wedge (x_0 \vee \neg x_5) \wedge (x_1 \vee \neg x_5) \wedge (x_2 \vee \neg x_5) \wedge (x_3 \vee x_6) \wedge (x_4 \vee x_6) \wedge (x_5 \vee x_6)$.

In mathematical logic, an **implication graph** is a skew-symmetric directed graph $G(V, E)$ composed of vertex set $V$ and directed edge set $E$. Each vertex in $V$ represents the truth status of a Boolean literal, and each directed edge from vertex $u$ to vertex $v$ represents the material implication "If the literal $u$ is true then the literal $v$ is also true". Implication graphs were originally used for analyzing complex Boolean expressions.

## 1 Applications

A 2-satisfiability instance in conjunctive normal form can be transformed into an implication graph by replacing each of its disjunctions by a pair of implications. For example, the statement $(x_0 \vee x_1)$ can be rewritten as the pair $(\neg x_0 \rightarrow x_1), (\neg x_1 \rightarrow x_0)$ . An instance is satisfiable if and only if no literal and its negation belong to the same strongly connected component of its implication graph; this characterization can be used to solve 2-satisfiability instances in linear time.[1]

In CDCL SAT-solvers, unit propagation can be naturally associated with an implication graph that captures all possible ways of deriving all implied literals from decision literals,[2] which is then used for clause learning.

## 2 References


[1] Aspvall, Bengt; Plass, Michael F.; Tarjan, Robert E. (1979). "A linear-time algorithm for testing the truth of certain quantified boolean formulas". *Information Processing Letters*. **8** (3): 121–123. doi:10.1016/0020-0190(79)90002-4.

[2] Paul Beame; Henry Kautz; Ashish Sabharwal (2003). *Understanding the Power of Clause Learning* (PDF). IJCAI. pp. 1194–1201.

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## 3.1 Text

- **Implication graph** *Source:* https://en.wikipedia.org/wiki/Implication_graph?oldid=745821013 *Contributors:* Altenmann, Vadmium, PWilkinson, GregorB, CBM, Thisisraja, David Eppstein, DavidCBryant, R0uge, DOI bot, Twri, Dcirovic, ClueBot NG, BG19bot, 0a.io and Anonymous: 3

## 3.2 Images

- **File:Implication_graph.svg** *Source:* https://upload.wikimedia.org/wikipedia/commons/2/2f/Implication_graph.svg *License:* Public domain *Contributors:* Own work *Original artist:* David Eppstein

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