Paths of Graphs

Hengfeng Wei

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Edsger W. Dijkstra ($1930 \sim 2002$)

for all $v \in V$ **do**
    $\text{dist}[v] \leftarrow \infty$
$\text{dist}[s] \leftarrow 0$

$Q \leftarrow \textsc{MinPQ}(V)$
while $Q \neq \emptyset$ **do**
    $u \leftarrow \textsc{DeleteMin}(Q)$
    for all $(u, v) \in E \land v \in Q$ **do**
        if $\text{dist}[v] > \text{dist}[u] + l(u, v)$ **then**
            $\text{dist}[v] \leftarrow \text{dist}[u] + l(u, v)$
            $\textsc{DecreaseKey}(Q, v)$

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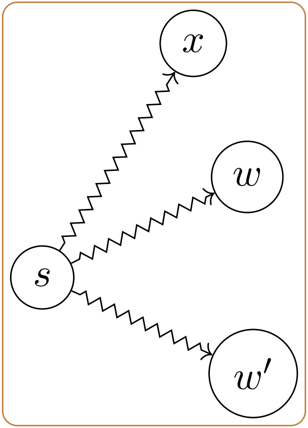
$$O(n + (n+m)\log n) \implies O((n+m)\log n) \implies O(m \log n)$$

$R \triangleq \{v \mid s \rightsquigarrow v \text{ is known}\}$



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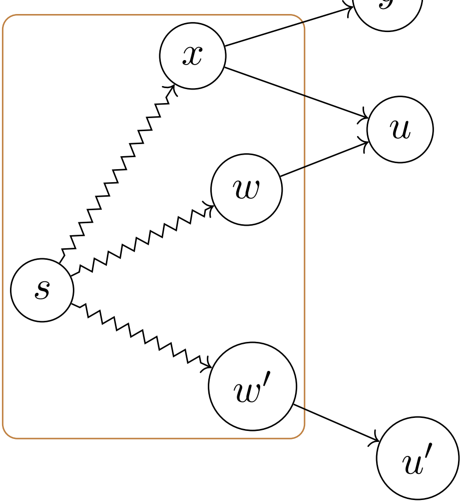

$$u : \min_{v \in R} \Big(\text{dist}(v) + l(v, u) \Big)$$

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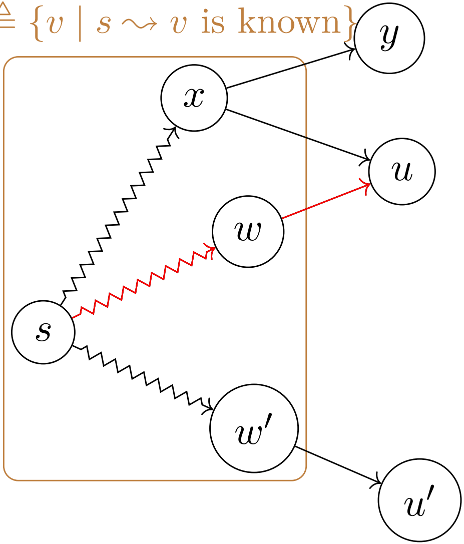

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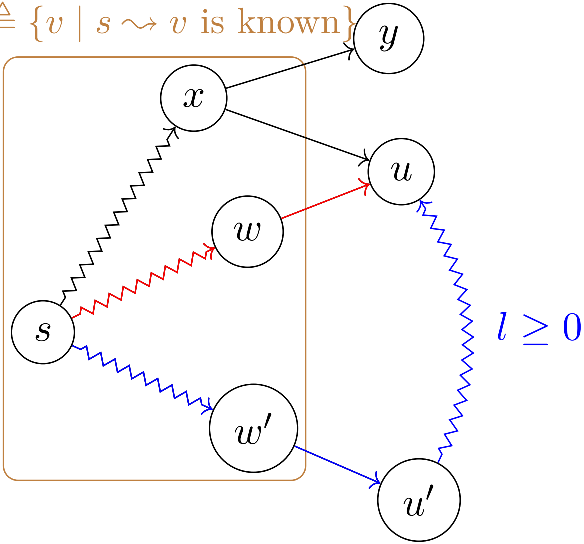

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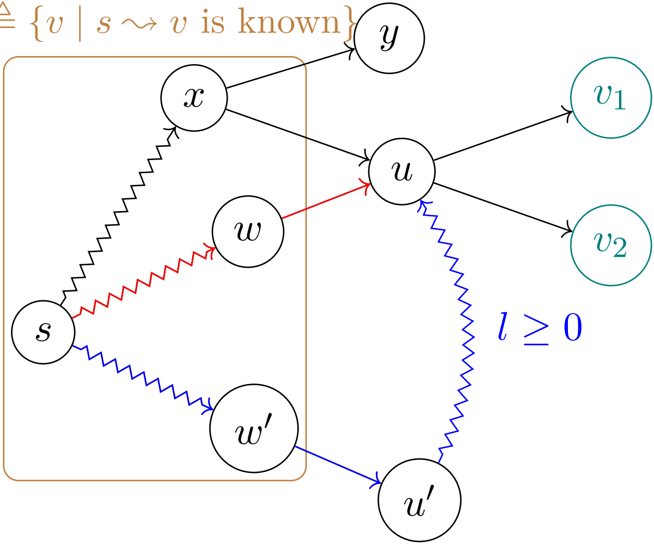

Negative Edges from s (Problem 11.9)

All negative edges are from s.

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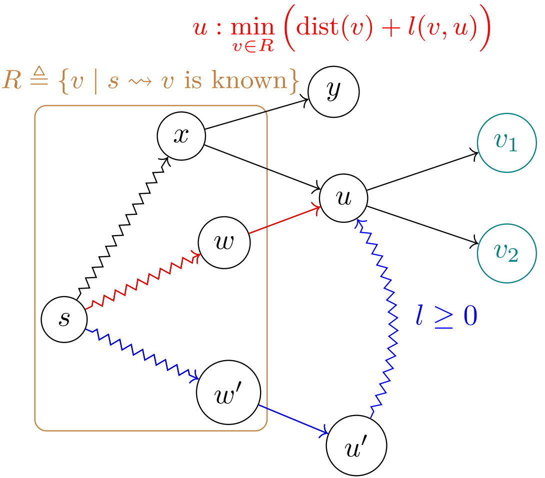

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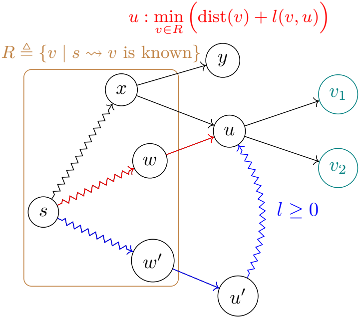
      $\textsc{DecreaseKey}(Q, v)$



Min-Max Path (Problem 11.12)

$$G = (V, E): \text{ network of highways}$$

$$l_e: \text{ road length} \quad L: \text{ tank capacity}$$

Given G, to compute $\min L$ in $O(m \log n)$ from s to t.

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for all $v \in V$ **do**
$L[v] \leftarrow \infty$
$L[s] \leftarrow 0$

if $L[v] > \max\Big(L[u], l(u,v)\Big)$ **then**
$L[v] \leftarrow \max\Big(L[u], l(u,v)\Big)$

Max-Min Path (Problem 13.2 (1))

$$G = (V, E) : \text{ network of oil pipelines}$$

$$c(u, v) : \text{ capacity of } (u, v)$$

$$\text{cap}(s, t) : \max \min s \rightsquigarrow t$$

Given s, to compute $\text{cap}(s, v)$.

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$$Q \leftarrow \text{MaxPQ}(V)$$

for all $v \in V$ **do**
  $\text{cap}[v] \leftarrow -\infty$
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if $\text{cap}[v] < \min\Big(\text{cap}[u], c(u, v)\Big)$ **then**
  $\text{cap}[v] \leftarrow \min\Big(\text{cap}[u], c(u, v)\Big)$

Max-Min Path (Problem 13.2 (2))

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$$\text{cap}(i, j, k) = \max\Big(\text{cap}(i, j, k-1), \min(\text{cap}(i, k, k-1), \text{cap}(k, j, k-1))\Big)$$

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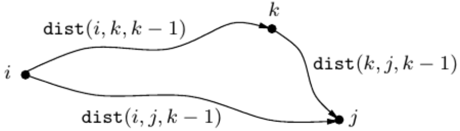

Shortest Paths Through v_0 (Problem 13.7)

$$\text{Strongly connected digraph } G = (V, E), \quad w(e) > 0$$

$$v_0 \in V$$

$$\text{Find shortest paths } s \overset{\text{SP}}{\rightsquigarrow} t \text{ through } v_0.$$

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Find shortest paths $s \rightsquigarrow^{\text{SP}} t$ through v_0.

$$s \rightsquigarrow^{\text{SP}} v_0 \rightsquigarrow^{\text{SP}} t$$

$$\forall v : v_0 \rightsquigarrow^{\text{SP}} v$$

Most Critical Edge (Problem 11.3)

$$s, t \in V$$

$$e : E \setminus \{e\} \implies \text{dist}(s, t) \text{ increases most}$$

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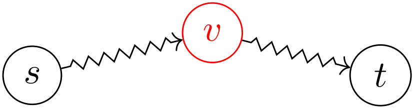
"Most Vital Links and Nodes in Weighted Networks", 1992

$$O(m \log n)$$

Bitonic Shortest Path (Problem 11.7)



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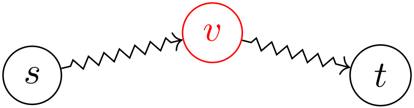
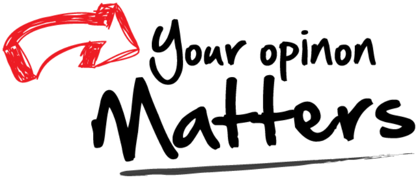



THANK YOU



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