Proof of Horner's Rule Correctness

- **Loop Invariant**

$$y = \sum_{k=0}^{n-(i+1)} a_{k+i+1} x^k$$

- **Initialization** Before the loop $y = 0$. At initialization, $i = n$.

$$y = \sum_{k=0}^{n-(n+1)} a_{k+i+1} x^k = \sum_{k=0}^{-1} a_{k+i+1} x^k = 0$$

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- **Loop Invariant**

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- **Maintenance** At iteration j of the loop, $i' = n - j + 1$ and $y' = \sum_{k=0}^{n-(i'+1)} a_{k+i'+1} x^k$. At iteration $j+1$, $i = n - j$ and $y = a_i + xy'$. Need to show

$$y = \sum_{k=0}^{n-i} a_{k+i} x^k$$

Proof of Horner's Rule Correctness

- **Maintenance** Need to show

$$y = \sum_{k=0}^{n-i} a_{k+i} x^k$$

$$\begin{aligned}
y &= a_i + x \left(\sum_{k=0}^{n-(i+1)} a_{k+i+1} x^k \right) && (1) \\
&= a_i + x \left(a_{0+i+1} x^0 + \ldots + a_{(n-i)+i} x^{n-(i+1)} \right) && (2) \\
&= a_i + \left(a_{0+i+1} x^1 + \ldots + a_{(n-i)+i} x^{n-i} \right) && (3) \\
&= a_i + \sum_{k=1}^{n-i} a_{k+i} x^k && (4) \\
&= a_{i+0} x^0 + \sum_{k=1}^{n-i} a_{k+i} x^k && (5) \\
&= \sum_{k=0}^{n-i} a_{k+i} x^k && (6)
\end{aligned}$$

Proof of Horner's Rule Correctness

- **Loop Invariant**

$$y = \sum_{k=0}^{n-(i+1)} a_{k+i+1} x^k$$

- **Termination** At the end of the loop, $i = -1$. Therefore

$$y = \sum_{k=0}^{n-(i+1)} a_{k+i+1} x^k = \sum_{k=0}^{n} a_k x^k$$