Shortest Paths

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Shortest Paths

Dijkstra's algorithm

Invariant: maintain $R \subseteq V$: $\forall u \in R : s \rightsquigarrow u$ is known

1. choose the next v and (u, v):

$$\min_{u \in R} \mathsf{dist}(s, u) + w(u, v)$$

2. key points for the correctness proof

    2.1 $u_1 \to v$

    2.2 $u_1 \to x \rightsquigarrow v$

Dijkstra' algorithm

Negative edges [Problem: 3.7.9]

Dijkstra's algorithm on graphs with negative edges

Dijkstra' algorithm

Negative edges leaving s [Problem: 3.7.17]

- digraph $G = (V, E, w)$
- all negative edges are from s

Solution.

Dijkstra's algorithm works.

Dijkstra's algorithm

Uniqueness of shortest path [Problem: 3.7.7]

Dijkstra's algorithm

Uniqueness of shortest path [Problem: 3.7.16]

- digraph $G = (V, E, w)$
- $w(e) \geq 0$
- $S \cap T = \emptyset$
- to compute $\forall s \in S, \forall t \in T, s \leadsto t$ shortest paths

Solution.

- adding s_0
- $s_0 \to s \in S$
- $w(s_0 \to s) = 0$

Shortest Paths

Cycles

4-Cycle in undirected graph [Problem: 3.7.1]

Cycles

Shortest cycle in digraph [Problem: 3.7.4]

Solution.

Floyd-Warshall: $W^{(0)}[i][i] = \infty$

Cycles

Shortest cycle in undirected graph [Problem: 3.7.14]

Solution.

Cycles

Shortest cycle containing a specific edge [Problem: 3.7.5]

- undirected edge $G = (V, E)$

Solution.

$P_{u,v} + (u, v)$

Cycles

Hamiltonian path in tournament graph [Problem: 3.7.18]

- digraph $G = (V, E)$
- $\forall u, v : (u \to v \lor v \to u) \land \neg(u \to v \land v \to u)$
- hamiltonial path

Solution.

- existence
- algorithm $O(n^2)$


THANK YOU