

Longest alternating subsequence

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In combinatorial mathematics, probability, and computer science, in the **longest alternating subsequence** problem, one wants to find a subsequence of a given sequence in which the elements are in alternating order, and in which the sequence is as long as possible.

Formally, if $\mathbf{x} = \{x_1, x_2, \dots, x_n\}$ is a sequence of distinct real numbers, then the subsequence $\{x_{i_1}, x_{i_2}, \dots, x_{i_k}\}$ is *alternating*^[1] (or *zigzag* or *down-up*) if

$$x_{i_1} > x_{i_2} < x_{i_3} > \cdots x_{i_k} \quad \text{and} \quad 1 \leq i_1 < i_2 < \cdots < i_k \leq n.$$

Similarly, \mathbf{x} is *reverse alternating* (or *up-down*) if

$$x_{i_1} < x_{i_2} > x_{i_3} < \cdots x_{i_k} \quad \text{and} \quad 1 \leq i_1 < i_2 < \cdots < i_k \leq n.$$

Let $\mathbf{as}_n(\mathbf{x})$ denote the length (number of terms) of the longest alternating subsequence of \mathbf{x} . For example, if we consider some of the permutations of the integers 1,2,3,4,5, we have that

- $\mathbf{as}_5(1, 2, 3, 4, 5) = 2$; because any sequence of 2 distinct digits are (by definition) alternating. (for example 1,2 or 1,4 or 3,5)
- $\mathbf{as}_5(1, 5, 3, 2, 4) = 4$, because 1,5,3,4 and 1,5,2,4 and 1,3,2,4 are all alternating, and there is no alternating subsequence with more elements;
- $\mathbf{as}_5(5, 3, 4, 1, 2) = 5$, because 5,3,4,1,2 is itself alternating.

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Efficient algorithms

The longest alternating subsequence problem is solvable in time $O(n)$, where n is the length of the original sequence.

Distributional results

If \mathbf{x} is a random permutation of the integers $1, 2, \dots, n$ and $A_n \equiv \mathbf{as}_n(\mathbf{x})$, then it is possible to show^{[2][3][4]} that

$$E[A_n] = \frac{2n}{3} + \frac{1}{6} \quad \text{and} \quad \text{Var}[A_n] = \frac{8n}{45} - \frac{13}{180}.$$

Moreover, as $n \rightarrow \infty$, the random variable A_n , appropriately centered and scaled, converges to a standard normal distribution.

Online algorithms

The longest alternating subsequence problem has also been studied in the setting of online algorithms, in which the elements of \mathbf{x} are presented in an online fashion, and a decision maker needs to decide whether to include or exclude each element at the time it is first presented, without any knowledge of the elements that will be presented in the future, and without the possibility of recalling on preceding observations.

Given a sequence X_1, X_2, \dots, X_n of independent random variables with common continuous distribution F , it is possible to construct a selection procedure that maximizes the expected number of alternating selections. Such expected values can be tightly estimated, and it equals $(2 - \sqrt{2})n + O(1)$.^[5]

As $n \rightarrow \infty$, the optimal number of online alternating selections appropriately centered and scaled converges to a normal distribution.^[6]

See also

- Alternating permutation
- Permutation pattern and pattern avoidance
- Counting local maxima and/or local minima in a given sequence
- Turning point tests for testing statistical independence of n observations
- Number of alternating runs
- Longest increasing subsequence
- Longest common subsequence problem

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