Decompositions of Graphs

Hengfeng Wei

hfwei@nju.edu.cn

May 20, 2017 – May 24, 2017

Decompositions of Graphs

Turing Award

John Hopcroft

Robert Tarjan

"For fundamental achievements in the design and analysis of algorithms and data structures."

— *Turing Award, 1986*

Depth-first search

SIAM J. COMPUT.
Vol. 1, No. 2, June 1972

DEPTH-FIRST SEARCH AND LINEAR GRAPH ALGORITHMS*

ROBERT TARJAN†

Abstract. The value of depth-first search or "backtracking" as a technique for solving problems is illustrated by two examples. An improved version of an algorithm for finding the strongly connected components of a directed graph and an algorithm for finding the biconnected components of an un-direct graph are presented. The space and time requirements of both algorithms are bounded by $k_1V + k_2E + k_3$ for some constants k_1, k_2, and k_3, where V is the number of vertices and E is the number of edges of the graph being examined.

Key words. Algorithm, backtracking, biconnectivity, connectivity, depth-first, graph, search, spanning tree, strong-connectivity.

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"We have seen how the depth-first search method may be used in the construction of very efficient graph algorithms. . . .
Depth-first search is a powerful technique with many applications."

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Graph decomposition

Graph decomposition vs. Graph traversal

Structures!

Graph decomposition

Graph decomposition vs. Graph traversal

Structures!

1. states of vertices
2. types of edges
3. lifetime of vertices (DFS)
   - v : d$[v]$, f$[v]$
   - f$[v]$: DAG, SCC
   - d$[v]$: biconnectivity

Types of edges

Definition (Classifying edges)

Given a DFS/BFS traversal $\Rightarrow$ DFS/BFS tree:

Tree edge: $\rightarrow$ child

Back edge: $\rightarrow$ ancestor

Forward edge: $\rightarrow$ *nonchild* descendant

Cross edge: $\rightarrow$ neither ancestor nor descendant

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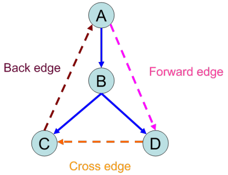
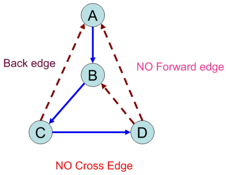
Remarks

- applicable to both DFS and BFS
- w.r.t. DFS/BFS trees

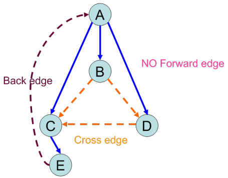
Types of edges (Problem 5.18)



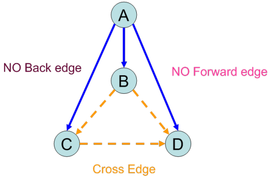
(a) DFS on directed graph.



(b) DFS on undirected graph.



(c) BFS on directed graph.



(d) BFS on undirected graph.

Types of edges

DFS tree and BFS tree coincide (Additional)

$G = (V, E), v \in V$.
DFS tree T = BFS tree T'.

- G is an undirected graph $\implies G = T$
- G is a digraph $\overset{?}{\implies} G = T$

Types of edges

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- T: tree + back *vs.* T': tree + cross

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- T: tree + back + forward + cross *vs.* T': tree + back + cross

Lifttime of vertices in DFS

Theorem (Disjoint or contained)

$$\forall u, v :$$
$$[_u \;]_u \cap [_v \;]_v = \emptyset$$
$$\bigvee$$
$$([_u \;]_u \subsetneqq [_v \;]_v \vee [_v \;]_v \subsetneqq [_u \;]_u)$$

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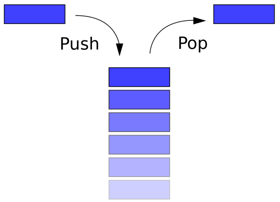
Proof.



Ancestor/descendant relation

Preprocessing for ancestor/descendant relation (Problem 5.23)

- binary tree $T = (V, E)$ (tree)
- $r \in V$

$$v : \mathsf{d}[v], \mathsf{f}[v]$$

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$\forall v$: how many descendants?

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Question

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$$(\mathsf{f}[v] - \mathsf{d}[v] - 1)/2$$

Edge types and lifetime of vertices in DFS

Edge types and lifetime of vertices in DFS (Problem 5.2)

$\forall u \to v$:

- tree/forward edge: $[_u \; [_v \;]_v \;]_u$
- back edge: $[_v \; [_u \;]_u \;]_v$
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Remark

- f$[v] <$ d$[u]$: cross edge
- f$[u] <$ f$[v]$: back edge

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Remark

- $\mathsf{f}[v] < \mathsf{d}[u]$: cross edge
- $\mathsf{f}[u] < \mathsf{f}[v]$: back edge

$$u \to v \iff \mathsf{f}[v] < \mathsf{f}[u]$$

Height and diameter of tree

Height and diameter of tree (Problem 5.21)

Binary tree $T = (V, E)$ with $|V| = n$:

- height ($O(n)$)
- diameter ($O(n)$)

Question

Diameter of a tree *without* designated root?

Perfect subtree

Perfect subtree (Problem 5.22)

- binary tree $T = (V, E)$
- root $r \in V$
- goal: find all perfect subtrees

Counting shortest paths

Counting shortest paths (Problem 5.26)

Decompositions of Graphs

Cycle detection

Cycle detection (Problem 5.24)

	Digraph	Undirected graph
DFS		
BFS		

Cycle detection

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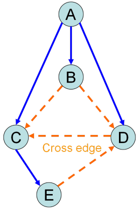

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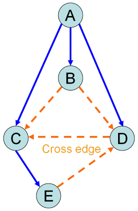

Remark

How to identify back edges?

Edge deletion

Edge deletion (Problem 5.20)

- connected, undirected graph G
- $\exists ? e \in E : G \setminus e$ is connected?
- $O(|V|)$

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- connected, undirected graph G
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- $O(|V|)$

$$\exists \text{ cycle} \iff \exists \text{ such } e$$

$$\text{tree: } |E| = |V| - 1 \implies \text{ check } |E| \geq |V|$$

Orientation of undirected graph

Orientation of undirected graph (Problem 5.9)

- undirected (connected) graph G
- edges oriented *s.t.*

$$\forall v, \mathsf{in}[v] \geq 1$$

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- undirected (connected) graph G
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$$\forall v, \text{in}[v] \geq 1$$

$$\text{orientation} \iff \exists \text{ cycle } C$$

$$\text{BFS/DFS from } v \in C$$

Shortest cycle of undirected graph

Shortest cycle of undirected graph (Problem 5.8)

Shortest cycle of G:

- DFS on G
- $\forall v$: level$[v]$
- back edge $u \to v$: level$[u]$ $-$ level$[v] + 1$

Shortest cycle of undirected graph

Shortest cycle of undirected graph (Problem 5.8)

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- $\forall v$: level$[v]$
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Question

What about digraphs?

Decompositions of Graphs

1. DFS and BFS
2. Cycles
3. DAG
4. SCC
5. Biconnectivity

DAG

no back edge $\iff$ DAG

DAG

no back edge $\iff$ DAG $\iff$ $\exists$ topo. ordering

Toposort algorithm by Tarjan (probably), 1976

DFS on digraph, $u \to v$:

- ~~back edge:~~ $\mathsf{f}[u] < \mathsf{f}[v]$
- others: $\mathsf{f}[u] > \mathsf{f}[v]$

DAG

$$\text{no back edge} \iff \text{DAG} \iff \exists \text{ topo. ordering}$$

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Toposort: sort vertices in *decreasing* order of their *finish* times.

Kahn's toposort algorithm

Kahn's toposort algorithm (1962; Problem 5.11)

- queue for source vertices ($\text{in}[v] = 0$)
- repeat: dequeue v, delete it, output it

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Lemma

Every DAG has at least one source (and at least one sink vertex).

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Lemma

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Question

What if G is not a DAG?

Taking courses

Taking courses in few semesters (Problem 5.14)

- n courses
- $c_1 \to c_2$
- goal: taking courses in few semesters

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critical path *OR* longest path

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- n courses
- $c_1 \to c_2$
- goal: taking courses in few semesters

critical path *OR* longest path

Remark

For general digraph, LONGEST-PATH is NP-hard.

Line up

Line up (Problem 5.16)

1. i hates j: $i \prec j$
2. i hates j: $\#i < \#j$

Hamiltonian path in DAG

Hamiltonian path in DAG (Problem 5.10)

- DAG G
- HP: path visiting each vertex once

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For general (di)graph, HP is NP-hard.

$$\text{DAG: } \exists \text{ HP} \iff \exists! \text{ topo. ordering}$$

Proof.

$\Longleftarrow$: By contradiction. $\square$

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- DAG G
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For general (di)graph, HP is NP-hard.

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Algorithms:

1. toposort, check edges
2. the Kahn toposort algorithm

Decompositions of Graphs

1. DFS and BFS
2. Cycles
3. DAG
4. SCC
5. Biconnectivity

Digraph as DAG

Digraph as DAG (Problem 5.3)

Every digraph is a dag of its SCCs.

Remark

Two tiered structure of digraphs:

- digraph $\equiv$ a dag of SCCs
- SCC: equivalence class over reachability

SCC

Kosaraju SCC algorithm, 1978

> *"SCCs can be topo-sorted in decreasing order of their highest finish time."*

SCC

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The vertice with the highest finish time is in a source SCC.

SCC

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> *"SCCs can be topo-sorted in decreasing order of their highest finish time."*

The vertice with the highest finish time is in a source SCC.

Remark

- DFS on G; DFS/BFS on G^T
- DFS on G^T; DFS/BFS on G

SCC

Kosaraju SCC algorithm, 1978 (Problem 5.4)

- 1st DFS $\overset{?}{\Longrightarrow}$ BFS
- 2nd DFS $\overset{?}{\Longrightarrow}$ BFS

One-to-all reachability

One-to-all reachability (Problem 5.12)

Digraph $G = (V, E)$:

- given $v : v \leadsto^{?} \forall u$
- $\exists ?\ v : v \leadsto \forall u$

One-to-all reachability

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One-to-all reachability

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Digraph $G = (V, E)$:

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Proof.

- $\Longleftarrow$: (1) source (2) $\exists!$

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Proof.

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- $\Longrightarrow$: By contradiction.

$$\exists u : v \not\leadsto u \land \text{in}[u] > 0 \implies \exists u' \to u \land v \not\leadsto u' \implies \exists \text{ cycle}$$

$\square$

Impacts of vertices

Impacts of vertices (Problem 5.13)

Digraph G:

$$\mathsf{impact}(v) = |\{w : v \leadsto w\}|$$

- $\arg\min_v \mathsf{impact}(v)$
- $\arg\max_v \mathsf{impact}(v)$

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Question

$\forall v$: computing $\text{impact}(v)$.

One-way streets

One-way streets (Problem 5.15)

Digraph G for city:

1. $\forall u, v : u \leftrightsquigarrow v$
2. $s : s \rightsquigarrow v \rightsquigarrow s$

One-way streets

One-way streets (Problem 5.15)

Digraph G for city:

1. $\forall u, v : u \leftrightsquigarrow v$
2. $s : s \rightsquigarrow v \rightsquigarrow s$

$(2)\ \{v \mid s \rightsquigarrow v\}$ is an SCC

Connectivity

Connectivity (Problem 5.7)

Prove: connected undirected graph G:

$$\exists v : G \setminus v \text{ is still connected}$$

Example: strongly connected digraph G:

$$\exists v : G \setminus v \text{ is not strongly connected}$$

Example: digraph G with 2 SCCs:

$$(G + e) \text{ is not strongly connected}$$

2SAT

2SAT (Problem 5.17)

$$I : (x_1 \vee \overline{x_2}) \wedge (\overline{x_1} \vee \overline{x_3}) \wedge (x_1 \vee x_2) \wedge (\overline{x_3} \vee x_4) \wedge (\overline{x_1} \vee x_4)$$

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$$\alpha \vee \beta \equiv \overline{\alpha} \to \beta \equiv \overline{\beta} \to \alpha$$

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Implication graph G_I.

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Implication graph G_I.

Theorem

$$\exists \textit{ SCC } \exists x : v_x \in \textit{SCC} \wedge v_{\overline{x}} \in \textit{SCC} \iff I \textit{ is not satisfiable.}$$

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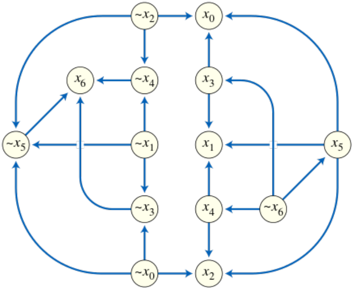
Reference

- "A Linear-time Algorithm for Testing the Truth of Certain Quantified Boolean Formulas" by Bengt Aspvall, Michael Plass, and Robert Tarjan, 1979.

2SAT

$$(x_0 \vee x_2) \wedge (x_0 \vee \neg x_3) \wedge (x_1 \vee \neg x_3) \wedge (x_1 \vee \neg x_4) \wedge$$
$$(x_2 \vee \neg x_4) \wedge (x_0 \vee \neg x_5) \wedge (x_1 \vee \neg x_5) \wedge (x_2 \vee \neg x_5) \wedge$$
$$(x_3 \vee x_6) \wedge (x_4 \vee x_6) \wedge (x_5 \vee x_6)$$



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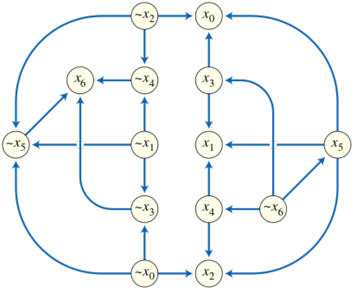
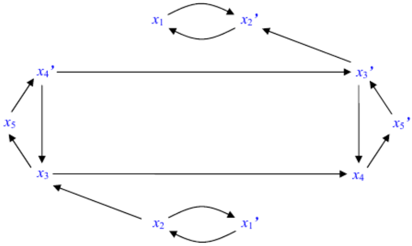
$$(x_0 \vee x_2) \wedge (x_0 \vee \neg x_3) \wedge (x_1 \vee \neg x_3) \wedge (x_1 \vee \neg x_4) \wedge$$
$$(x_2 \vee \neg x_4) \wedge (x_0 \vee \neg x_5) \wedge (x_1 \vee \neg x_5) \wedge (x_2 \vee \neg x_5) \wedge$$
$$(x_3 \vee x_6) \wedge (x_4 \vee x_6) \wedge (x_5 \vee x_6)$$





Odd cycle in digraph

Odd cycle in digraph (Additional)

Find an odd cycle in a digraph G.

Odd cycle in digraph

Odd cycle in digraph (Additional)

Find an odd cycle in a digraph G.

Lemma

A digraph G has an odd directed cycle $\iff$ $\exists$scc : scc is non-bipartite (when treated undirected).

Odd cycle in digraph

Odd cycle in digraph (Additional)

Find an odd cycle in a digraph G.

Lemma

A digraph G has an odd directed cycle $\iff$ $\exists$scc : scc is non-bipartite (when treated undirected).

Question

To prove the lemma and design an algorithm.

Decompositions of Graphs


THANK YOU