Paths of Graphs

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$$\textbf{for all } v \in V \textbf{ do}$$
$$\quad \text{dist}[v] \leftarrow \infty$$
$$\text{dist}[s] \leftarrow 0$$

$$Q \leftarrow \textsc{MinPQ}(V)$$
$$\textbf{while } Q \neq \emptyset \textbf{ do}$$
$$\quad u \leftarrow \textsc{DeleteMin}(Q)$$

$$\quad \textbf{for all } (u, v) \in E \wedge v \notin Q \textbf{ do}$$
$$\quad\quad \textbf{if } \text{dist}[v] > \text{dist}[u] + l(u, v) \textbf{ then}$$
$$\quad\quad\quad \text{dist}[v] \leftarrow \text{dist}[u] + l(u, v)$$
$$\quad\quad\quad \textsc{DecreaseKey}(Q, v)$$

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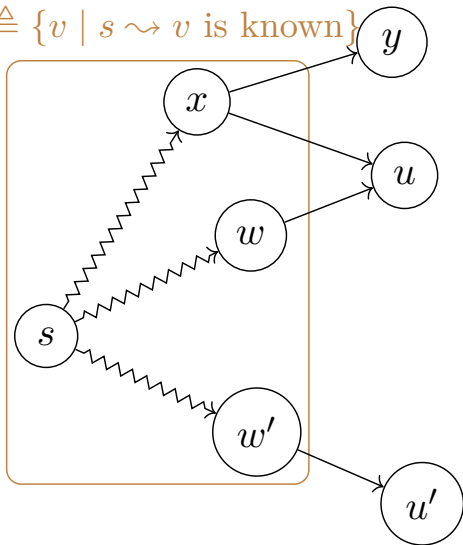$$\quad\quad\quad\quad \text{DecreaseKey}(Q, v)$$

$$O(n + (n + m)\log n) \implies O((n + m)\log n) \implies O(m \log n)$$

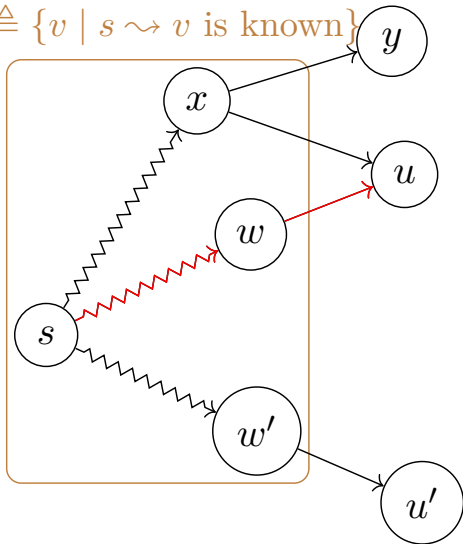$$R \triangleq \{v \mid s \rightsquigarrow v \text{ is known}\}$$
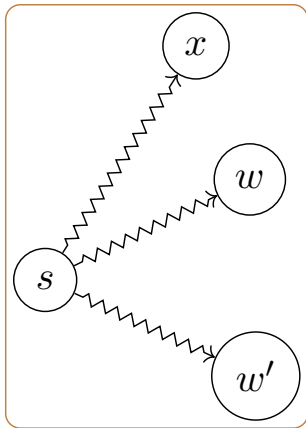
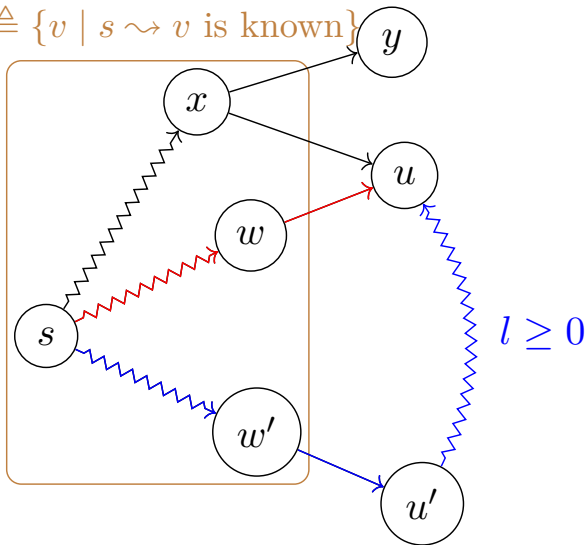$R \triangleq \{v \mid s \rightsquigarrow v \text{ is known}\}$

$$u : \min_{v \in R} \big(\mathrm{dist}(v) + l(v, u)\big)$$

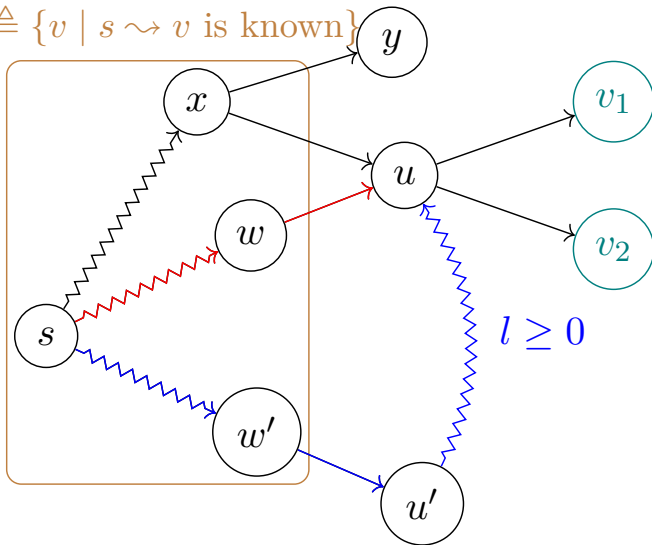$$R \triangleq \{v \mid s \rightsquigarrow v \text{ is known}\}$$

y

x

v_1

u

w

v_2

$l \geq 0$

s

w'

u'

Negative Edges from s (Problem 11.9)

All negative edges are from s.
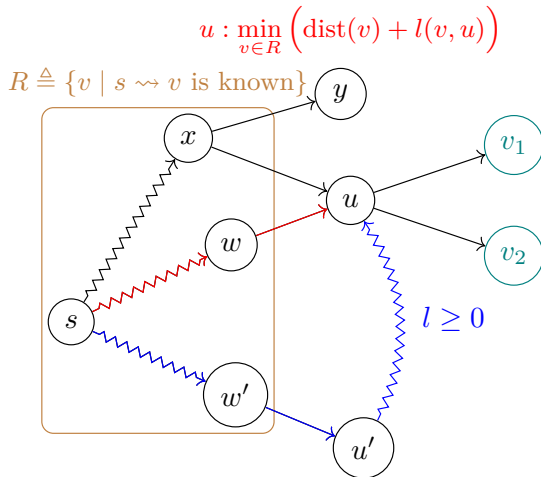
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$l \geq 0$

Shortest Paths Through v_0 (Problem 13.7)

Strongly connected digraph $G = (V, E)$, $\quad w(e) > 0$

$$v_0 \in V$$

Find shortest paths $s \rightsquigarrow^{\text{SP}} t$ through v_0.

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Strongly connected digraph $G = (V, E), \quad w(e) > 0$

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Find shortest paths $s \leadsto^{\text{SP}} t$ through v_0.

$$s \leadsto^{\text{SP}} v_0 \leadsto^{\text{SP}} t$$

Shortest Paths Through v_0 (Problem 13.7)

Strongly connected digraph $G = (V, E), \quad w(e) > 0$

$$v_0 \in V$$

Find shortest paths $s \rightsquigarrow^{\text{SP}} t$ through v_0.

$$s \rightsquigarrow^{\text{SP}} v_0 \rightsquigarrow^{\text{SP}} t$$

$$\forall v : v_0 \rightsquigarrow^{\text{SP}} v$$

for all $v \in V$ **do**
  $\text{dist}[v] \leftarrow \infty$
$\text{dist}[s] \leftarrow 0$

$Q \leftarrow \textsc{MinPQ}(V)$
while $Q \neq \emptyset$ **do**
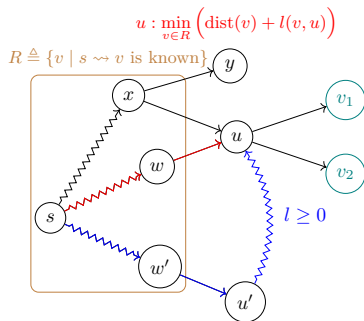 $u \leftarrow \textsc{DeleteMin}(Q)$

  for all $(u, v) \in E \wedge v \notin Q$ **do**
    if $\text{dist}[v] > \text{dist}[u] + l(u, v)$ **then**
      $\text{dist}[v] \leftarrow \text{dist}[u] + l(u, v)$
      $\textsc{DecreaseKey}(Q, v)$

$u : \min\limits_{v \in R} \Big(\text{dist}(v) + l(v, u) \Big)$

$R \triangleq \{v \mid s \rightsquigarrow v \text{ is known}\}$

$l \geq 0$

Min-Max Path (Problem 11.12)

$$G = (V, E) : \text{ network of highways}$$

$$l_e : \text{ road length} \quad L : \text{ tank capacity}$$

Given G, to compute $\min L$ in $O(m \log n)$ from s to t.

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Given G, to compute $\min L$ in $O(m \log n)$ from s to t.

$$Q \leftarrow \text{MinPQ}(V)$$

for all $v \in V$ **do**
  $L[v] \leftarrow \infty$
$L[s] \leftarrow 0$

if $L[v] > \max\Big(L[u], l(u, v)\Big)$ **then**
  $L[v] \leftarrow \max\Big(L[u], l(u, v)\Big)$

Max-Min Path (Problem 13.2 (1))

$$G = (V, E) : \text{ network of oil pipelines}$$

$$c(u, v) : \text{ capacity of } (u, v)$$

$$\text{cap}(s, t) : \max \min s \rightsquigarrow t$$

Given s, to compute $\text{cap}(s, v)$.

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$$Q \leftarrow \text{MaxPQ}(V)$$

for all $v \in V$ **do**
$\quad \text{cap}[v] \leftarrow -\infty$
$\text{cap}[s] \leftarrow 0$

if $\text{cap}[v] < \min\Big(\text{cap}[u], c(u, v)\Big)$ **then**
$\quad \text{cap}[v] \leftarrow \min\Big(\text{cap}[u], c(u, v)\Big)$

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Compute all-pair $\mathrm{cap}(i, j)$.

Max-Min Path (Problem 13.2 (2))

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$$c(u, v) : \text{ capacity of } (u, v)$$

$$\text{cap}(s, t) : \max \min s \rightsquigarrow t$$

Compute all-pair $\text{cap}(i, j)$.

$$\text{cap}(i, j, k) = \max\Big(\text{cap}(i, j, k - 1), \min\big(\text{cap}(i, k, k - 1), \text{cap}(k, j, k - 1)\big)\Big)$$

Max-Min Path (Problem 13.2 (2))
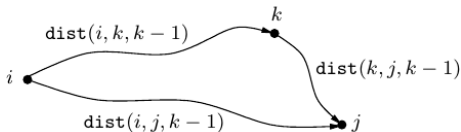
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Most Critical Edge (Problem 11.3)

$$s, t \in V$$

$$e : E \setminus \{e\} \implies \text{dist}(s, t) \text{ increases most}$$

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"Most Vital Links and Nodes in Weighted Networks", 1992

$$O(m \log n)$$

Bitonic Shortest Path (Problem 11.7)

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