Finding the shortest path in the presence of negative cycles

Given a directed cyclic graph where the weight of each edge may be negative the concept of a "shortest path" only makes sense if there are no negative cycles, and in that case you can apply the Bellman-Ford algorithm.

However, I'm interested in finding the shortest-path between two vertices that doesn't involve cycling (ie. under the constraint that you may not visit the same vertex twice). Is this problem well studied? Can a variant of the Bellman-Ford algorithm be employed, and if not is there another solution?

I'm also interested in the equivalent all-pairs problem, for which I might otherwise apply Floyd–Warshall.

ds.algorithms graph-algorithms shortest-path

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1 Answer

Paths with no repeated vertices are called simple-paths, so you are looking for the shortest simple-path in a graph with negative-cycles.

This can be reduced from the longest-path problem. If there were a fast solver for your problem, then given a graph with only positive edge-weights, negating all the edge-weights and running your solver would give the longest path in the original graph.

Thus your problem is NP-Hard.

answered May 1 '13 at 20:40

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1 This is a beautiful answer. I've asked several people this IRL without any solutions and when I explained this to them their reaction was the same as mine - "of course, I feel so stupid now". – jleahy May 2 '13 at 9:24