Graph Decomposition (2)

— BFS and its Applications

Hengfeng Wei

hengxin0912@gmail.com

November 18, 2014

Outline

Outline

Problem

Problem: Shortest Path

Given an undirected graph $G = (V, E)$ and a source vertex s, to compute distance $\delta(s, u)$ for each vertex u.

$\delta(s, u) \equiv \#$ of edges of the shortest path between s and u.

BFS on Undirected Graph

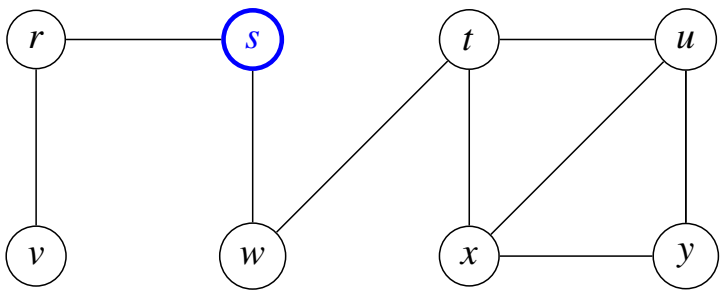
BFS with source vertex s:

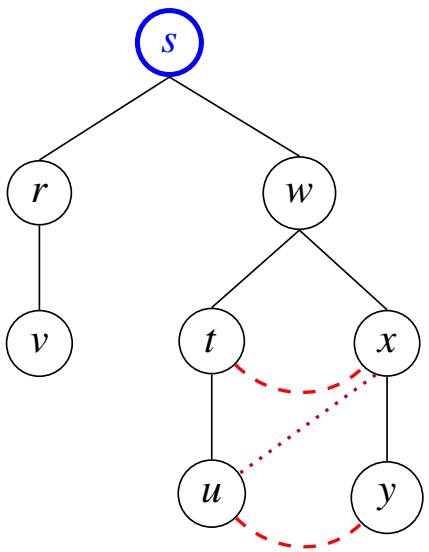
- exploring every vertex u reachable from s
- computing $\delta(s, u)$, for all reachable u

BFS as a framework:

- Prim's MST algorithm
- Dijkstra's SSSP algorithm

A Physical Algorithm of BFS (Phys-BFS)





Edge Properties of Phys-BFS

$$(u, v) \in E \Rightarrow d(u) \leq d(v) \leq d(u) + 1$$

A Parallel Algorithm of BFS (`Para-BFS`)

graph: network of computers

vertex: computer

edge: network connections

To disseminate a computer virus from computer s.

Color Properties in `Para-BFS`

$$\text{states of computers} = \begin{cases} \texttt{WHITE} & \text{if healthy} \\ \texttt{GRAY} & \text{if infected} \end{cases}$$

A Parallel Algorithm of BFS (Para-BFS)

Algorithm 1 A Parallel Algorithm of BFS (Para-BFS).

procedure PARA-BFS(G, s)
  for all $u \in V$ **do**
    color$[u] \leftarrow$ WHITE
    d$[u] \leftarrow \infty$

  color$[s] \leftarrow$ GRAY
  d$[s] \leftarrow \infty$

  Q $\leftarrow \{s\}$

while Q $\neq \emptyset$ **do**
  $u \leftarrow$ Deq(Q)
  for all $(u, v) \in E$ **do**
    if color$[v]$ = WHITE **then**
      color$[v]$ = GRAY
      d$[v]$ = d$[u]$ + 1
      Enq(Q, v)
  color$[u] \leftarrow$ BLACK

Color Properties of Para-BFS

States of vertices:

- WHILE: undiscovered
- GRAY: discorvered but
- BLACK: discorvered and all its neighbors has been discorvered

$$\texttt{WHITE} \Rightarrow \texttt{GRAY} \Rightarrow \texttt{BLACK}$$

1. $(u, v) \in E$, u is BLACK $\Rightarrow$ v is either BLACK or GRAY
2. GRAY vertex may have adjacent WHITE vertices
   $\Rightarrow$ "frontier" between discovered and undiscovered vertices
3. Invariant: at any time, all GRAY vertices are in the Queue

Correctness Proof `Para-BFS`

BFS Algorithm

Outline

Color Properties

Queue Properties

Correctness Proof

Edges Properties

Outline

Testing Bipartiteness

figure/thankyou.jpg