Swansea University

College of Engineering

EGLM03 Modern Control Systems

Homework 6: Digital Systems Revision

Problems

1. Given that $z = e^{sT} = e^{\sigma T} \angle \pm \omega T$, where T is the sampling period of a digital system, show that

   a. points with frequencies of $\pm\omega_s/2$ (where ω_s is the sampling rate $2\pi/T$) in the s-domain map onto the negative real axis in the z-domain.

   b. the imaginary axis in the s-domain maps onto the unit circle
   $$|z| = 1$$
   in the z-domain

   c. lines of constant σ in the s-domain map onto concentric circles centred at $z = 0$ in the z-domain

   d. lines of constant frequency in the s-domain map onto radial lines centred at $z = 0$ in the z-domain.

   Derive equations for curves of constant second-order damping ratio ζ and natural frequency ω_n in the z-domain. Comment on the effect of high sampling rates on the poles of a discrete system.
2. Determine the z-transform of the function $f(t) = e^{-at}$, where a is real > 0.
3. Given that
   $$G(s) = \frac{s+4}{(s+1)(s+3)},$$
   determine $G(z)$. [*Hint*: determine $g(t)$ first.]
4. A discrete transfer function $G(z)$ is given by
   $$G(z) = \frac{0.387z^2}{(z-1)(z^2 - 2.37z + 025)}.$$
   Determine the value of $g(nT)$ as $nT \to \infty$.
5. Use long division to find the inverse z-transform of
   $$F(z) = \frac{\left(1 - e^{-aT}\right) z}{z^2 - \left(1 - e^{-aT}\right) z + e^{-aT}}$$
   .
6. The system type of a unity gain feedback digital control system
   $$\frac{C(z)}{R(z)} = \frac{G_o(z)}{1 + G_o(z)}$$
   is given by the number of poles at $z = 1$. Thus a Type 0 system has no poles at $z = 1$, a Type 1 system has one, and so on. Use the discrete version of the final value theorem
   $$\lim_{n\to\infty} x(nT) = \lim_{z\to 1} \left(1 - z^{-1}\right) X(z)$$
   to derive formulae for the steady-state error of a digital control system to step, ramp and parabolic inputs.
7. (not examinable) Apply Jury's test (MATLAB function `jury`) to determine if the following characteristic equations have any roots outside the unit circle:

a. $z^2 + 0.25 = 0$

b. $z^3 - 1.1z^2 + 0.01z + 0.405 = 0$

c. $z^3 - 3.6z^2 + 4z - 1.6 = 0.$