$$\dot{x} = A x + Bu$$

$$y = C x + du$$

$$A = \begin{bmatrix} -6 & -5 \\ 1 & 0 \end{bmatrix} \quad B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$y = \begin{bmatrix} 3, & 1 \end{bmatrix}$$

Controllability

$$B = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \quad A = \begin{bmatrix} -6 & -5 \\ 1 & 0 \end{bmatrix}$$

$$AB = \begin{bmatrix} -6 & -5 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} -6 \\ 1 \end{bmatrix}$$

$$CM = \begin{bmatrix} 1 & -6 \\ 0 & 1 \end{bmatrix} \quad \begin{bmatrix} B & AB \end{bmatrix}$$

$$\det \begin{bmatrix} 1 & -6 \\ 0 & 1 \end{bmatrix} \neq 0$$

$$1 - (-6 \times 0) = 1$$

Observability

$$C^T = \begin{bmatrix} 3 \\ 1 \end{bmatrix} \qquad A^T C^T = \begin{bmatrix} -6 & 1 \\ -5 & 0 \end{bmatrix} \begin{bmatrix} 3 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -17 \\ -15 \end{bmatrix}$$

$$OM = \begin{bmatrix} 3 & -17 \\ 1 & -15 \end{bmatrix}$$

$$\det(OM) = -28 \neq 0$$

Use diagonalization to confirm result.