

### Example 3

Laplace

$$s\underline{X}(s) - \underline{x}_0 = \underline{A}\underline{X}(s) + \underline{B}U(s)$$

$$(sI - A)\underline{X}(s) = \underline{x}_0 + \underline{B}U(s)$$

$$\underline{X}(s) = (sI - A)^{-1}\underline{x}_0 + (sI - A)^{-1}\underline{B}U(s)$$

$$(sI - A)^{-1} = \left( \begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} - \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix} \right)^{-1}$$

$$= \begin{bmatrix} s+3 & 2 \\ -1 & s \end{bmatrix}^{-1}$$

$$= \frac{\begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}}{s(s+3) + 2} = \frac{\begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}}{s^2 + 3s + 2}$$

$$= \frac{\begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}}{(s+1)(s+2)}$$

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$$= \frac{\begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix}}{(s+1)(s+2)}$$

$$\underline{X(s)} = \frac{1}{(s+1)(s+3)} \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$+ \frac{1}{(s+1)(s+3)} \begin{bmatrix} s & -2 \\ 1 & s+3 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \cdot \frac{1}{s}$$

$$X(s) = \begin{bmatrix} \frac{s-1}{(s+1)(s+3)} \\ \frac{s^2+4s+1}{s(s+1)(s+3)} \end{bmatrix}$$

$$x(t) = \int \begin{bmatrix} \cdot \\ \cdot \end{bmatrix} = \int \begin{bmatrix} \frac{-2}{s+1} + \frac{3}{s+2} \\ \frac{0.5}{s} + \frac{2}{s+1} - \frac{1.5}{s+2} \end{bmatrix}$$

$$= \begin{bmatrix} -2e^{-t} + 3e^{-2t} \\ 0.5 + 2e^{-t} - 1.5e^{-2t} \end{bmatrix} \leftarrow$$

$$\underline{Y(s)} = \underline{C} X(s) + D U(s)$$

$$\underline{y(t)} = \underline{C} x(t) + D u(t)$$