## Laplace and *z* Transforms Transform Pairs

f(t)	F(s)	F(z)
$\delta(t)$	1	1
$\varepsilon(t)$	$\frac{1}{s}$	<u>z</u>
t		$\frac{\overline{z-1}}{(z-1)^2}$
$t^2$	$\frac{1}{s^2}$ $\frac{2}{s^3}$	
$e^{-at}$	s <sup>3</sup> 1	$\frac{T^2 z(z+1)}{(z-1)^3}$ $\frac{z}{z-e^{-aT}}$
te <sup>-at</sup>	$\frac{s+a}{1}$	$\frac{\overline{z - e^{-aT}}}{Tze^{-aT}}$
	$\frac{1}{(s+a)^2}$	$\frac{Tze^{-aT}}{(z-e^{-aT})^2}$
sin bt	$\frac{b}{s^2 + b^2}$	$\frac{z\sin bT}{z^2 - 2z\cos bT + 1}$
$\cos bt$	$\frac{s}{s^2 + b^2}$	$\frac{z(z-\cos bT)}{z^2 - 2z\cos bT + 1}$
$\sin(bt + \phi)$	$\frac{s\sin\phi + b\cos\phi}{s^2 + b^2}$	$\frac{z[z\sin\phi + \sin(bT - \phi)]}{z^2 - 2z\cos bT + 1}$
$\cos(bt + \phi)$	$s\cos\phi - b\sin\phi$	$\frac{z[z\cos\phi + \cos(bT - \phi)]}{z^2 - 2z\cos bT + 1}$
$e^{-at}\sin bt$	$\frac{s^2 + b^2}{b}$ $\frac{b}{(s+a)^2 + b^2}$	$\frac{ze^{-aT}\sin bT}{z^2 - 2ze^{-aT}\cos bT + e^{-2aT}}$
$e^{-at}\cos bt$	$\frac{s+a}{(s+a)^2+b^2}$	$\frac{z(z - e^{-aT}\sin bT)}{z^2 - 2ze^{-aT}\cos bT + e^{-2aT}}$
$1-e^{-at}$	$\frac{a}{s(s+a)}$	$\frac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$at - 1 + e^{-at}$	$a^2$	$z[aT(z-e^{-aT})-(z-1)(1-e^{-aT})]$
$\delta(t-mT)$	$s^{2}(s+a)$ $e^{-smT}$	$(z-1)^2(z-e^{-aT})$ $z^{-m}$
$\varepsilon(t-mT)$	$\frac{e^{-smT}}{c}$	$\frac{Z^{-(m-1)}}{Z}$
$\varepsilon(t) - \varepsilon(t - mT)$	$\frac{1 - e^{-sT}}{1 - e^{-sT}}$	$\frac{z-1}{z-1}$
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Properties Overleaf

## Laplace and z Transforms Properties

f(t)	F(s)	F(z)
f(t) = 0  for  t < 0	$\int_0^\infty f(t)e^{-st}dt$	$\sum_{0}^{\infty} f(nT)z^{-n}$
$f(\Omega t)$	$\frac{1}{\Omega}F\left(\frac{s}{\Omega}\right)$	
$k_1 f_1(t) + k_2 f_2(t)$	$k_1 F_1(s) + k_2 F_2(s)$	$k_1 F_1(z) + k_2 F_2(z)$
$\int_{o}^{t} f_{1}(\tau) f_{2}(t-\tau) d\tau$	$F_1(s)F_2(s)$	
$\frac{df(t)}{dt}$	sF(s) - f(0)	
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$	
tf(t)	$-\frac{dF(s)}{ds}$	$-Tz\frac{dF(z)}{dz}$
$\frac{f(t)}{t}$ $f(t - mT)$	$\int_{s}^{\infty} F(\Omega) d\Omega$	
f(t-mT)	$e^{-smT}F(s)$	$z^{-m}F(z)$
$e^{-at}f(t)$	F(s+a)	$F(e^{aT}z)$
$\sum_{m=0}^{\infty} f(t - mT)$	$\frac{1}{1 - e^{-aT}} F(s)$	$\frac{z}{z-1}F(z)$
$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$	$\lim_{z\to\infty} F(z)$
$\lim_{t\to 0} f(t)$	$\lim_{s\to 0} sF(s)$	$\lim_{z\to 1}(z-1)F(z)$

Transform Pairs Overleaf