Laplace and z Transforms – Transform Pairs

Note, in the following table, $T = 1/f_s$ s is the sampling period, $f_s = 1/T$ Hz is the sampling frequency. In angular measure, the sampling frequency is $\omega_s = 2\pi f_s$ rad/s.

$f(t)$	$F(s)$	$F(z)$
$\delta(t)$	1	1
$\varepsilon(t)$	$\dfrac{1}{s}$	$\dfrac{z}{z-1}$
t	$\dfrac{1}{s^2}$	$\dfrac{Tz}{(z-1)^2}$
t^2	$\dfrac{2}{s^3}$	$\dfrac{T^2 z(z+1)}{(z-1)^3}$
e^{-at}	$\dfrac{1}{s+a}$	$\dfrac{z}{z-e^{-aT}}$
te^{-at}	$\dfrac{1}{(s+a)^2}$	$\dfrac{Tze^{-aT}}{(z-e^{-aT})^2}$
$\sin bt$	$\dfrac{b}{s^2+b^2}$	$\dfrac{z\sin bT}{z^2-2z\cos bT+1}$
$\cos bt$	$\dfrac{s}{s^2+b^2}$	$\dfrac{z(z-\cos bT)}{z^2-2z\cos bT+1}$
$\sin(bt+\phi)$	$\dfrac{s\sin\phi+b\cos\phi}{s^2+b^2}$	$\dfrac{z\left[z\sin\phi+\sin(bT-\phi)\right]}{z^2-2z\cos bT+1}$
$\cos(bt+\phi)$	$\dfrac{s\cos\phi-b\sin\phi}{s^2+b^2}$	$\dfrac{z\left[z\cos\phi+\cos(bT-\phi)\right]}{z^2-2z\cos bT+1}$
$e^{-at}\sin bt$	$\dfrac{b}{(s+a)^2+b^2}$	$\dfrac{ze^{-aT}\sin bT}{z^2-2ze^{-aT}\cos bT+e^{-2aT}}$
$e^{-at}\cos bt$	$\dfrac{s+a}{(s+a)^2+b^2}$	$\dfrac{z(z-e^{-aT}\sin bT)}{z^2-2ze^{-aT}\cos bT+e^{-2aT}}$
$1-e^{-at}$	$\dfrac{a}{s(s+a)}$	$\dfrac{z(1-e^{-aT})}{(z-1)(z-e^{-aT})}$
$at-1+e^{-at}$	$\dfrac{a^2}{s^2(s+a)}$	$\dfrac{z\left[aT(z-e^{-aT})-(z-1)(1-e^{-aT})\right]}{(z-1)^2(z-e^{-aT})}$
$\delta(t-mT)$	e^{-smT}	z^{-m}
$\varepsilon(t-mT)$	$\dfrac{e^{-smT}}{s}$	$\dfrac{z^{-(m-1)}}{z-1}$
$\varepsilon(t)-\varepsilon(t-mT)$	$\dfrac{1-e^{-sT}}{s}$	1

Laplace and z Transforms – Properties

Note, in the following table, $T = 1/f_s$ s is the sampling period, $f_s = 1/T$ Hz is the sampling frequency. In angular measure, the sampling frequency is $\omega_s = 2\pi f_s$ rad/s.

$f(t)$	$F(s)$	$F(z)$
$f(t) = 0$ for $t < 0$	$\int_0^\infty f(t)e^{-st}dt$	$\sum_0^\infty f(nT)z^{-n}$
$f(\Omega t)$	$\frac{1}{\Omega}F\left(\frac{s}{\Omega}\right)$	
$k_1 f_1(t) + k_2 f_2(t)$	$k_1 F_1(s) + k_2 F_2(s)$	$k_1 F_1(z) + k_2 F_2(z)$
$\int_o^t f_1(\tau)f_2(t-\tau)d\tau$	$F_1(s)F_2(s)$	
$\frac{df(t)}{dt}$	$sF(s) - f(0)$	
$\int_0^t f(\tau)d\tau$	$\frac{F(s)}{s}$	
$tf(t)$	$\frac{dF(s)}{ds}$	$-Tz\frac{dF(z)}{dz}$
$\frac{f(t)}{t}$	$\int_s^\infty F(\Omega)d\Omega$	
$f(t - mT)$	$e^{-smT}F(s)$	$z^{-m}F(z)$
$e^{-at}f(t)$	$F(s+a)$	$F(e^{aT}z)$
$\sum_{m=0}^\infty f(t - mT)$	$\frac{1}{1-e^{-aT}}F(s)$	$\frac{z}{z-1}F(z)$
$\lim_{t\to 0} f(t)$	$\lim_{s\to\infty} sF(s)$	$\lim_{z\to\infty} F(z)$
$\lim_{t\to\infty} f(t)$	$\lim_{s\to 0} sF(s)$	$\lim_{z\to 1}(z-1)F(z)$

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