Swansea University
School of Engineering

EGLM03: Modern Control Systems

Control System Design Methods, Compensation Strategies and Design Criteria[1]

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1 Aims of Control Systems Analysis and Design

1.1 Analysis

- Objectives: Determination of the following system characteristics:
  - degree and extent of system stability
  - steady-state (d-c) performance (accuracy)
  - transient performance
- Methods of analysis
  - determine differential equations or transfer functions for each component
  - choose suitable representation, e.g. block diagram
  - formulate system model by connecting components together
  - determine system response.

  For the latter there are several methods used:
  - Direct solution to find steady-state and transient solutions. (Difficult with high order systems — relative stability difficult to study in time domain)
  - Four graphical methods (described in Section 2)
    - Root locus
    - Bode diagram
    - Nyquist diagram
    - Nichols chart

    (The last three are all frequency response methods)

1.2 Design Objectives

To meet performance specifications. Performance specifications are constraints on the system characteristics which are a number of ways of specifying these constraints. These fall into two broad classes

- Time domain criteria
- Frequency domain criteria

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[1] Abstracted from DiSteffano, Stubberud and Williams, "Feedback and Control Systems", 2nd Ed., Schaum's Outline Series, McGraw & Hill, 1989.

Required response is prescribed in either or both forms. In general the constraints define:

- speed of response
- relative stability
- system accuracy or allowable error

These are described in Section 4.

2 Design Methods

2.1 Bode

Bode diagrams can be used for the quick design of simple systems. Provided that the open-loop contains no more than two integrators and no right half plane poles or zeros (almost certainly true if the data has been obtained experimentally). The stability is easily assessed and the open-loop gain can be adjusted to give the desired gain and phase margins.

Theoretical Bode diagrams can be plotted accurately by calculation (use Matlab command *bode*), or from summation of first and second order templates (Figures 16, 17 and 18), but often straight line segment approximations are adequate especially for the gain plot. See also Appendix 1.

Furthermore, for simple designs such as operational amplifier stabilization where generous stability margins can be allowed and where no time delays exist, the stability can often be estimated on the basis of the gain diagram alone. Observing that a steady slope of -20 dB/decade corresponds to a phase lag of 90°, -40 dB/decade to 180° lag, etc., it may readily be verified that the amplifier will be very stable if the gain falls at no more than -20 dB/decade for a decade on either side of the unity open-loop gain frequency ω_1.

For digital systems, the bilinear transform

$$z = \frac{1 + \omega T/2}{1 - \omega T/2}$$

(Matlab function *z2w*) is used to map the *z*-transfer function into a *w*-transfer function which obeys the same stability laws as the *s*-plane. Bode diagrams may be plotted for these *w* transfer functions.

2.2 Root Locus

This is a more advanced method that requires an accurate mathematical model of the system. Example loci for typical systems are given in Appendix 1. These can be deduced and their important features quantified by using simple construction rules. However, the full strength of the method is only revealed when computerized locus plotting is available (see Matlab command *rlocus*).

The main features of the method are the good correlation that may be obtained with either time or frequency domain criteria, the direct assessment that is made of the sensitivity, and the ease with which the right half plane poles or zeros are included.

The correlation with the time and frequency domain criteria is usually defined in terms of a 2^{nd} order model and can be extended to higher order systems because of the effect of "dominant poles". The following general conclusions can be made:

- Poles in the right-half plane are unstable.
- Zeros in the right half plane are stable but give an undesirable reverse transient.
- Poles and zeros further from the origin give a faster response.
- Poles subtending larger angles with the negative real axis give an under-damped response.

- Poles and zeros with the largest negative real parts dominate the response. Poles and zeros with real parts more than perhaps two to five times more negative can generally be neglected.

The response of a dominant first-order pole is easily assessed and examining standard responses of a pure second order can assess dominant second-order poles. Figure 15 shows the family of under-damped second-order step responses, Figure 9 examines the settling time, and Figures 6, 14 and 11 shows the relationship of damping ratio ζ to percentage overshoot M_p, phase margin ϕ_m and resonant peak $M_{\max}$ respectively. The relationship between these show that the damping ratio should generally be greater than 0.25 and will usually be better placed between 0.5 and 0.8.

The root-locus is equally applicable to digital control systems. The only differences are that in the *z*-plane, the stability boundary is the unit circle and poles and zeros in the negative half plane produce responses that alternate positive and negative, so dominant poles are kept in the right half of the unit circle.

Finally note that the root locus may also be employed the examine sensitivity to parameters other than loop gain.

2.3 Nyquist

The Nyquist diagram can be used with practical as well as theoretical data but is more powerful than the Bode diagram enabling peak magnification $M_{\max}$ and closed-loop bandwidth to be evaluated. Sample diagrams are given in Appendix 1, but note the error in item 5 and other examples with the factor $1/s(s\tau_1+1)$ of assuming that the real part tends to zero when $\omega \to \infty$.

By examination of the magnitude and phase of the closed-loop frequency response

$$G_c(j\omega) = \frac{G_o(j\omega)}{1+G_o(j\omega)}$$

it is fairly easy to derive equations for curves of constant closed-loop magnitude (magnification) and constant closed-loop phase, both or which turn out to be circles (called respectively the M and N circles) in the $G_o(j\omega)$ plane. By noting at what frequency the Nyquist curve crosses these circles it is possible, if tedious, to develop the closed-loop frequency response. Of particular importance is the M-circle which is tangential to the Nyquist diagram since this represents the peak magnification $M_{\max}$ — a useful design parameter. The formulae for the M and N circles are[2]:

M-circle (for $M = |G_c(j\omega)|$): Centre: $M^2/(1-M^2) + j0$, Radius: $\left|M/(1-M^2)\right|$

N-circle (for $\alpha = \angle G_c(j\omega)$): ($N = \tan\alpha$) Centre: $1/2 + j(1/2N)$, Radius: $\frac{1}{2}\sqrt{1+(1/N^2)}$

Examples are given in Figure 1. Collections of M and N circles are available as design charts (called Hall Charts).

Design in the Nyquist domain is somewhat tedious, due to the fact that any modification in gain or phase requires a reshaping of the frequency response. This is somewhat less of a problem in Matlab where the functions *nyquist*, *mcircle* and *ncircle* have been provided to do the donkeywork.

The Nyquist diagram can also cope with open-loop poles and zeros in the right-half plane and with more than two integrations, provided that these hazards are enumerated.

2.4 Nichols

The Nichols chart (complete with M and N curves) is a variation of the Nyquist diagram which is better suited to iterative design, particularly by hand. The choice of axes (phase in degrees versus

[2] For derivation and examples of the use of M and N circles see, e.g. Dorf section 8.5, DiSteffano *et al* Section 11.2.

gain in dB) makes the choice of open-loop gain trivial and dynamic compensation can be accomplished by the straightforward addition of standard templates (see for example Chapters 17 and 18 of DiSteffano *et al*). When computer-aided tools are available, the Nyquist diagram is as easy to work with!

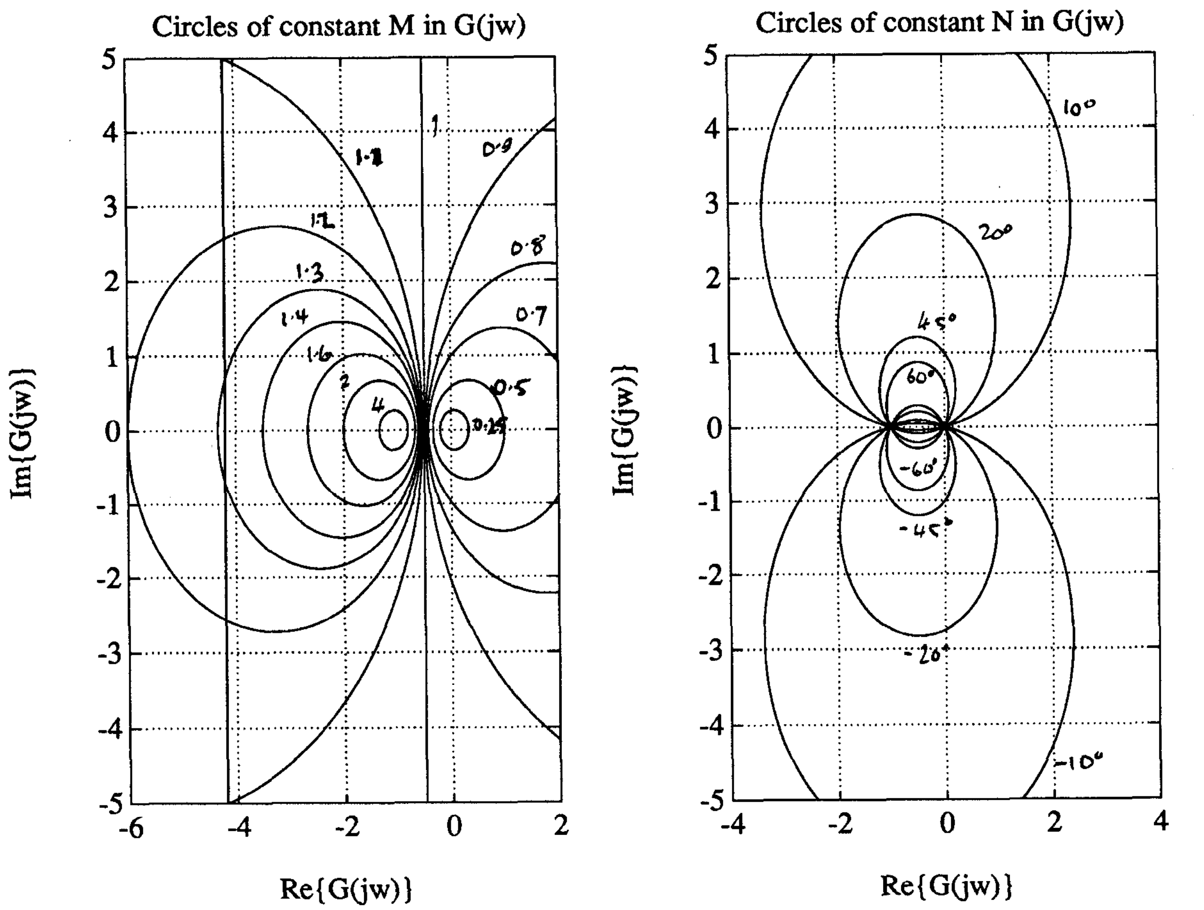

Figure 1 M and N Circles

2.5 Inverse Nyquist

This technique is more appropriate for parallel compensation such as velocity feedback. The adjustment of open-loop gain is accomplished by a change of scale and is straightforward because the contours of constant closed-loop gain (M-circles) are concentric circles, and parallel compensation is achieved by straightforward addition.

2.6 Simulation

For complex control systems an analogue or digital computer simulation enables the above methods to be tested by examining the exact design criteria of interest. An analogue or hybrid simulation also enables the electronic controller to be validated with a simulated plant before it is used on the real plant.

2.7 Experiment

In the end there is no alternative but to experiment with the real system. With the results of one or more of the above methods this should be a fairly painless process though some final adjustments are usually required. In some cases (particularly in process control) the above methods may be difficult and use of the Ziegler-Nichols design rules may be an adequate starting point (ref. Franklin, Powell and Emami-Naeini, "*Feedback Control of Dynamic Systems*", pp 103-106, Addison-Wesley, 1986).

3 Design Strategies

3.1 Proportional Control

For simple systems, a loop gain can often be chosen which is high enough to give acceptably small errors, low sensitivity and compliance (see Section 4.1.4) an adequate speed of response and yet good stability. When these specifications cannot be satisfied simultaneously a more sophisticated strategy is required.

3.2 Phase Lag Compensation

The lag compensator

$$\frac{K\left(1+T_G s\right)}{1+\alpha T_G s}, \alpha > 1$$

and proportional-plus-integral (PPI) compensator

$$K\left(1+1/(T_I s)\right)$$

are used either to increase the loop gain at low frequencies and so reduce the errors and the sensitivity, or to reduce the loop gain at higher frequencies (specifically around the region of 180º phase lag (phase cross-over frequency ω_π) to reduce the band-width and improve stability. PPI action may be preferred for the former (it increases system type by 1 and so eliminates steady state-errors) and a lag compensator for the latter. In any case the phase lag is incidental and since it is a destabilizing influence it should be used well below the unity loop gain frequency (gain cross-over frequency ω_1). When there is nothing else to choose between the two, the smaller phase lag of the lag controller (PPI is always 90º at zero frequency) and its freedom from 'warm-up' problems will lead to its use.

3.3 Phase Lead Compensation

The lead compensator

$$\frac{K(1+T_L s)}{1+\alpha T_L s}, \alpha < 1$$

is used to reduce the phase lag around the gain cross-over frequency (unity loop gain frequency) ω_1. It is therefore used to increase stability without reducing bandwidth. Naturally it may also permit the use of a higher proportional gain than would otherwise be possible. The gain of the lead compensator at high frequencies is a destabilizing influence so that it should be used at the highest effective frequency and a should not be reduced below 0.1. The high gain will also amplify any noise in the measurement sensor, which may be a problem.

3.4 The Three-Term Controller

This combines the effect of both phase lag and phase lead compensation. A special case is the proportional + integral + derivative (PID) compensator

$$K\left(1+T_D+1/(T_I s)\right)$$

3.5 Parallel Compensation

Parallel compensation (seen in earlier courses as velocity feedback) can be used to produce the same stabilizing effect as phase lead compensation without amplifying the measurement noise. For this reason it may be possible to introduce a little more phase advance. Note that velocity feedback will reduce the bandwidth a little.

3.6 Feed-forward Control

Feed-forward control is used when the compliance is too high but when the disturbances can be measured. If the load disturbances can be measured, feeding the measurement 'forward' to the error comparator can reduce compliance errors. It does not affect the stability.

4 Design Criteria

4.1 Steady-State Criteria

These criteria are measures of the 'tracking accuracy' and robustness of control systems.

4.1.1 Type 0 systems: position constant (K_p)

For the simple, unity-gain feedback system shown in Figure 2 the error is given by[3]

$$E(s) = \frac{R(s)}{1 + G_o(s)}$$

For a type 0 system, the position constant, K_p is the d-c open loop gain:

$$K_p = \lim_{s \to 0} G_o(s)$$

and implies a steady state error of

$$\frac{1}{1 + K_p}$$

following a unit step input. Thus a large position constant implies a small steady state error. For a type 1 control system $G_o(s)$ contains an integration so that $K_p \to \infty$ and the error tends to zero.

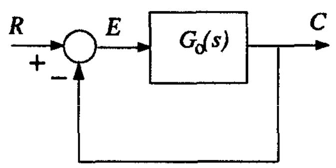

Figure 2 Unity gain feedback control system

4.1.2 Type 1 systems: velocity constant (K_v)

For the same system,

$$K_v = \lim_{s \to 0} sG_o(s).$$

The steady state position error for a constant velocity input is $1/K_v$, so a large velocity constant implies a small steady-state error. For a type 2 system $G_0(s)$ contains two integrations so that $K_v \to \infty$ and the error to a constant velocity input tends to zero.

4.1.3 Type 2 systems: acceleration constant (K_a)

Similarly

$$K_a = \lim_{s \to 0} s^2 G_o(s).$$

but this is not often specified.

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[3] for more general systems with desired output $C(s)$ and demanded output $R(s)$ the formula $E(s) = R(s) - C(s)$ must be used to derive a relationship between $E(s)$ and $R(s)$.

4.1.4 Compliance

For the system illustrated in Figure 3, with a disturbance $W(s)$, the error is given by

$$E(s) = \frac{R(s) + G(s)W(s)}{1 + D(s)G(s)}$$

The compliance is the steady-state error following a unit step change in $W(s)$. A small compliance requires $D(s) >> 1$ and zero compliance is obtained when $D(s)$ contains an integrator.

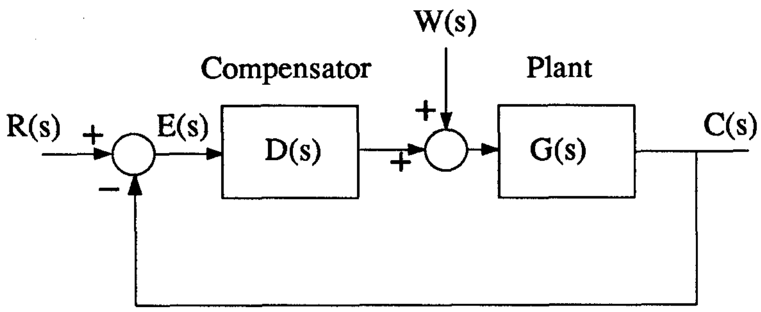


Figure 3 Compliance

4.1.5 Sensitivity

The sensitivity is the proportional variation in the closed-loop system caused by a variation in one of the open-loop components. A system will always be sensitive to variations of input, feedback and feed-forward paths, but the sensitivity to variations in the forward path is reduced by a factor of 1/(1 + loop gain).

4.2 Time Domain Criteria

The time domain criteria defined below are derived from the model second order under-damped output response to a step input shown in Figure 4. Similar criteria may be defined from the impulse or ramp response. The performance measures are related to the s-plane via the pole-zero diagram shown in Figure 5 where the poles are solutions of the equation

$$s^2 + 2\zeta\omega_n s + \omega_n^2 = 0$$

and α, σ and ω_d are defined as

$$\alpha = \cos^{-1}\zeta$$
$$\sigma = \zeta\omega_n$$
$$\omega_d = \omega_n\sqrt{1-\xi^2}$$

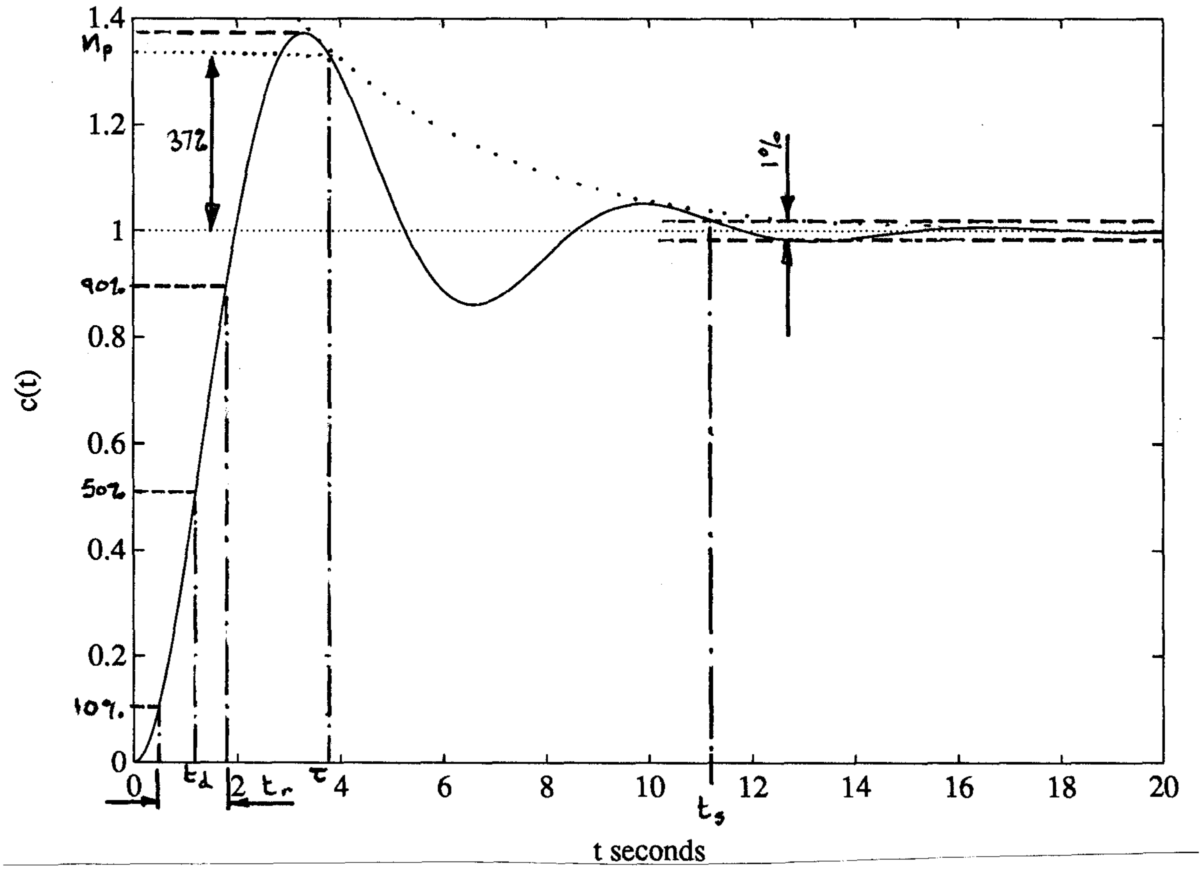


Figure 4 Model second-order under-damped step response

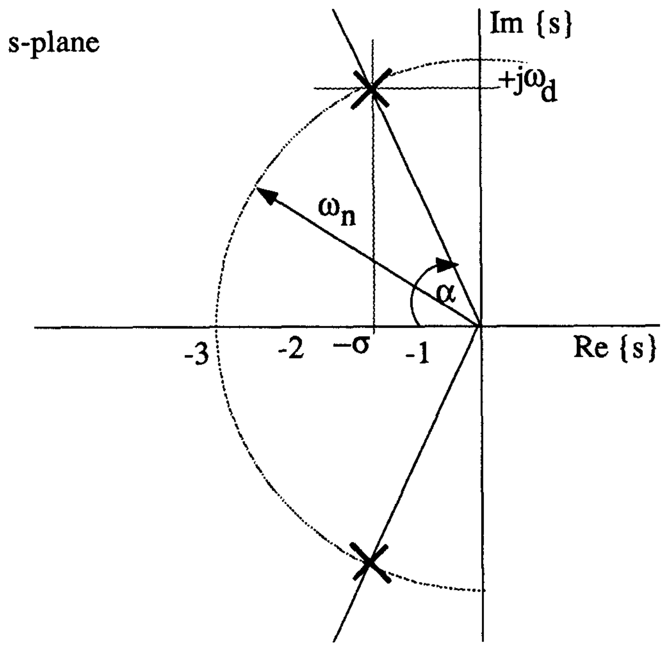


Figure 5 Location of second-order complex poles

4.2.1 Percentage overshoot (M_p)

Maximum difference between transient and steady state response to a unit step input. A measure of relative stability. Often quoted as a percentage of the final value of the response.

Design values: M_p should be kept below 40% and < 25% is usually required. For a second order system (or a system with a pair of dominant complex poles) M_p may be related to the damping ratio ζ by the equation

$$M_p = e^{-\pi\zeta/\sqrt{1-\zeta^2}} \quad 0 \le \zeta < 1$$

A plot of this curve is shown in Figure 6.

A useful approximation to M_p is given by the straight-line equation:

$$M_p \approx 1 - \zeta/0.6 \quad 0 \le \zeta \le 0.6$$

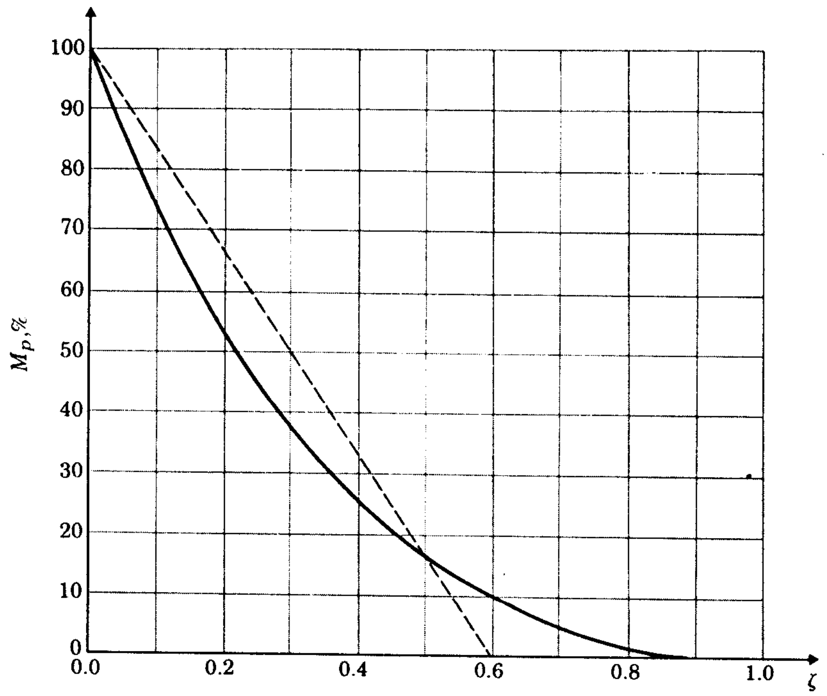


Figure 6 Plot of the peak overshoot M_P versus the damping ratio ζ for the second-order system

4.2.2 Rise time (t_r)

Usually defined as the time taken for the response to rise from 10% to 90% of its final value. A measure of the speed of response of the system. Figure 7 shows the relationship between rise time

Design values: For a second order system (or a system with a pair of complex dominant poles) rise time may be related to the natural frequency ω_n by

$$t_r \approx \frac{1.8}{\omega_n}$$

For process control, the step response is often over-damped and an alternative definition of rise time for such a response is illustrated in Figure 8.

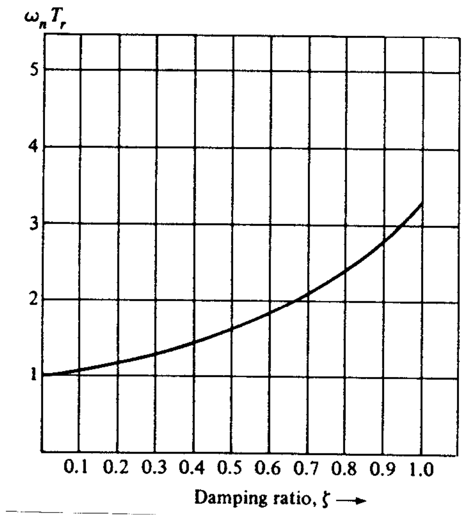


Figure 7 Normalized rise time vs damping ratio for a. second order system

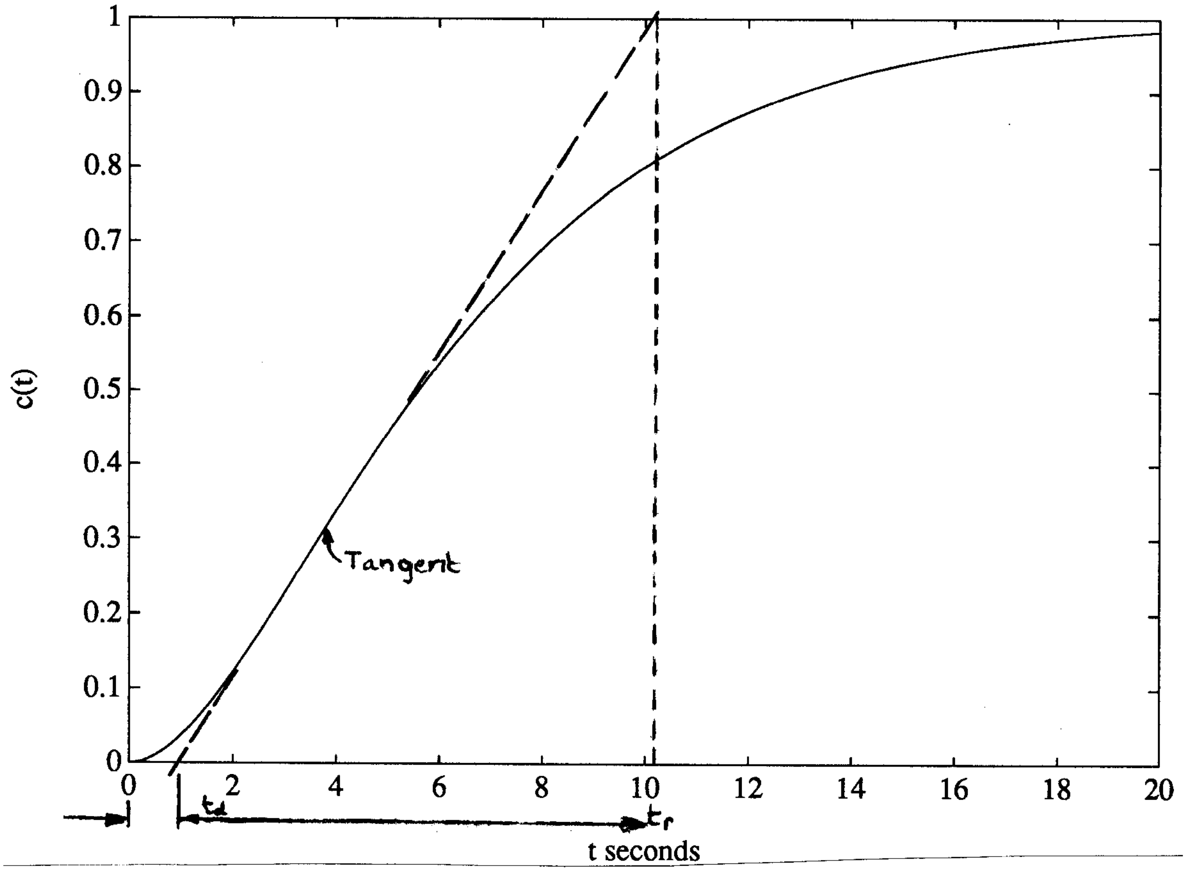


Figure 8 Typical process response curve

4.2.3 Settling time (t_s)

Usually defined as the time taken for the response to reach and remain within some percentage of its final (steady-state) value (often 1, 2, or 5%). This parameter is related to the real part of the dominant poles $\sigma = \zeta\omega_n$. Figure 9 shows the relationship between settling time and damping ratio for 5% settling time.

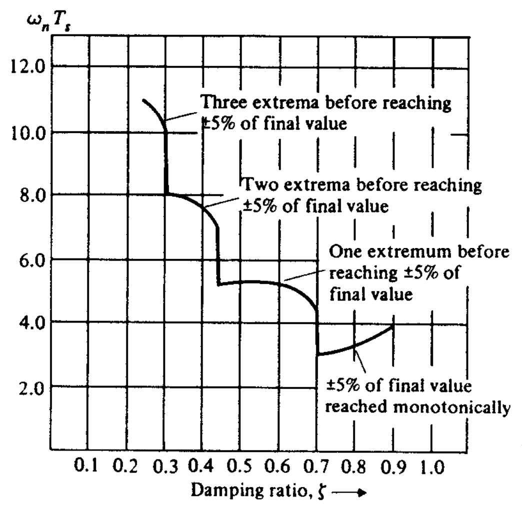


Figure 9 Normalized 5% settling time vs damping ratio for a second order system

Design values: For a second order system (or a system with a pair of dominant complex poles) rise time may be related to σ by

$$t_s = \frac{-\ln\left(\frac{\%\text{of final value}}{100}\right)}{\sigma}$$

Table 1 Typical settling time values

Percentage of Final Value	1%	2%	5%	10%
Settling Time t_s seconds	$\frac{4.6}{\sigma}$	$\frac{3.9}{\sigma}$	$\frac{3}{\sigma}$	$\frac{2.3}{\sigma}$

Other parameters sometimes given are

4.2.4 Delay time (t_d)

Usually defined as the time taken for the response to reach 50% of its final value. An alternative definition favoured for process control (under-damped responses) is illustrated in Figure 8.

4.2.5 Predominant time constant (τ)

Sometimes given as an alternative to t_s. The exponential envelope of the second order response decays to 37% of its final value in τ seconds.

Design values: for a second order system (or a system with a pair of dominant second order poles)

$$\tau = \frac{1}{\zeta\omega_n}.$$

The second order parameters ζ and ω_n may also be given as figures of merit. They can be useful in the design of higher order systems.

4.3 Frequency Domain Criteria

Frequency domain specifications are usually given in terms of the steady-state response of a system to a sinusoidal input.

4.3.1 Gain margin (GM)

A measure of relative stability. It is defined as the reciprocal of the magnitude of the open-loop transfer function evaluated at the frequency ω_π, at which the phase angle is -180º (-π radians). That is

$$GM \equiv \frac{1}{|GH(j\omega_\pi)|}$$

where $\angle GH(j\omega_\pi) = -180^\circ = -\pi$ radians. ω_π is called the 'phase-cross-over frequency'. See Figure 10 for an illustration of gain margin determination from frequency response data.

Design values: for absolute stability we require $|GH(j\omega_\pi)| < 1$ or GM > 1 (0 dB). Typical values are 3dB < GM < 8dB.

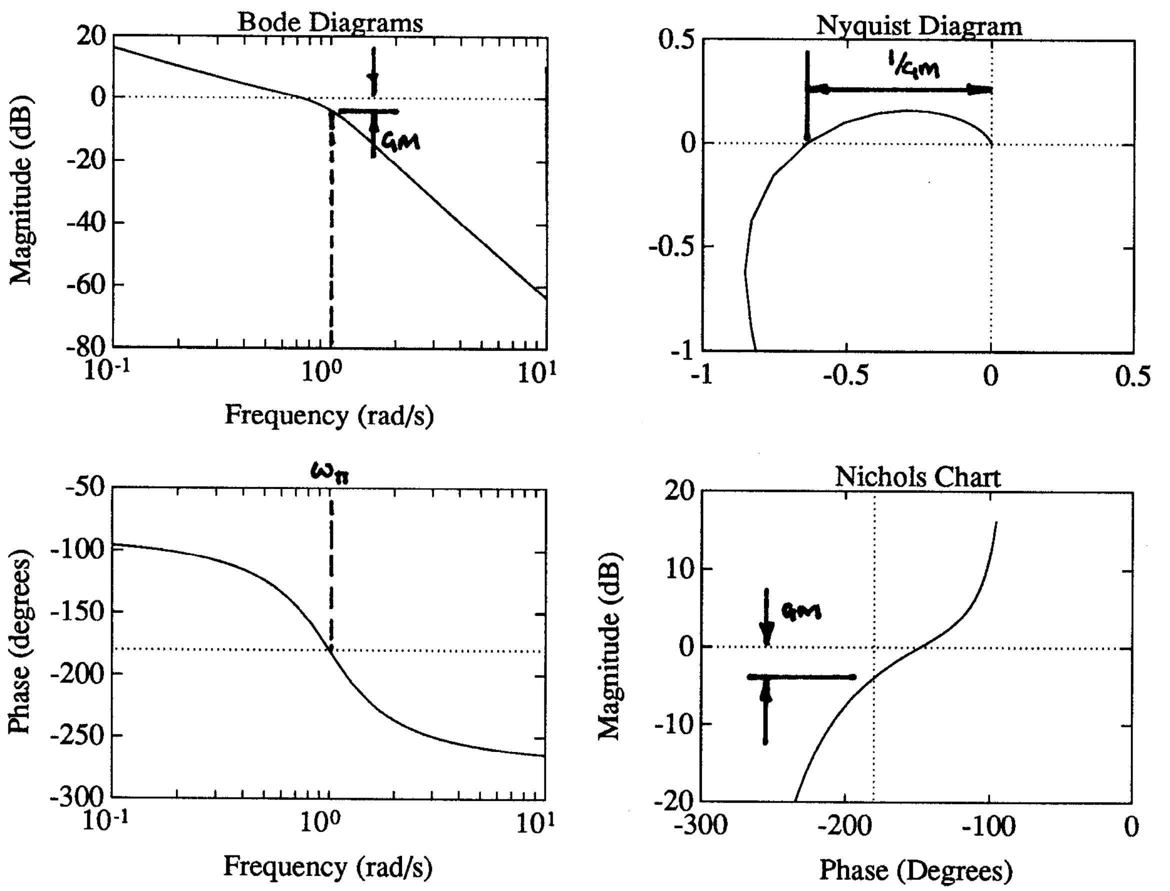


Figure 10 Determination of gain margin (GM) from frequency response diagrams

4.3.3 Phase margin (PM or ϕ_m)

Another measure of relative stability. It is defined as 180° plus the phase angle ϕ of the loop-transfer function at unity gain. That is

$\phi_m \equiv 180^\circ + \angle GH(j\omega_1)$&z = 180 + LGH(jwl)

where $|GH(j\omega_1)| = 1$ (0 dB). Frequency ω_1 is called the gain cross-over frequency. See Figure 11 for illustration of phase margin determination from frequency response data.

Design values: for absolute stability we require $\angle GH(j\omega_1) > -180^\circ$ or $\phi_m > 0º$. Typical values used are $\phi_m > 30º$, preferably $45º < \phi_m < 60º$. Note that if the Nyquist diagram is of the form illustrated in

Figure 12, then ϕ_m is related to the damping ratio ζ by the curve illustrated in Figure 12, for which $\zeta \approx \phi_m/100$.

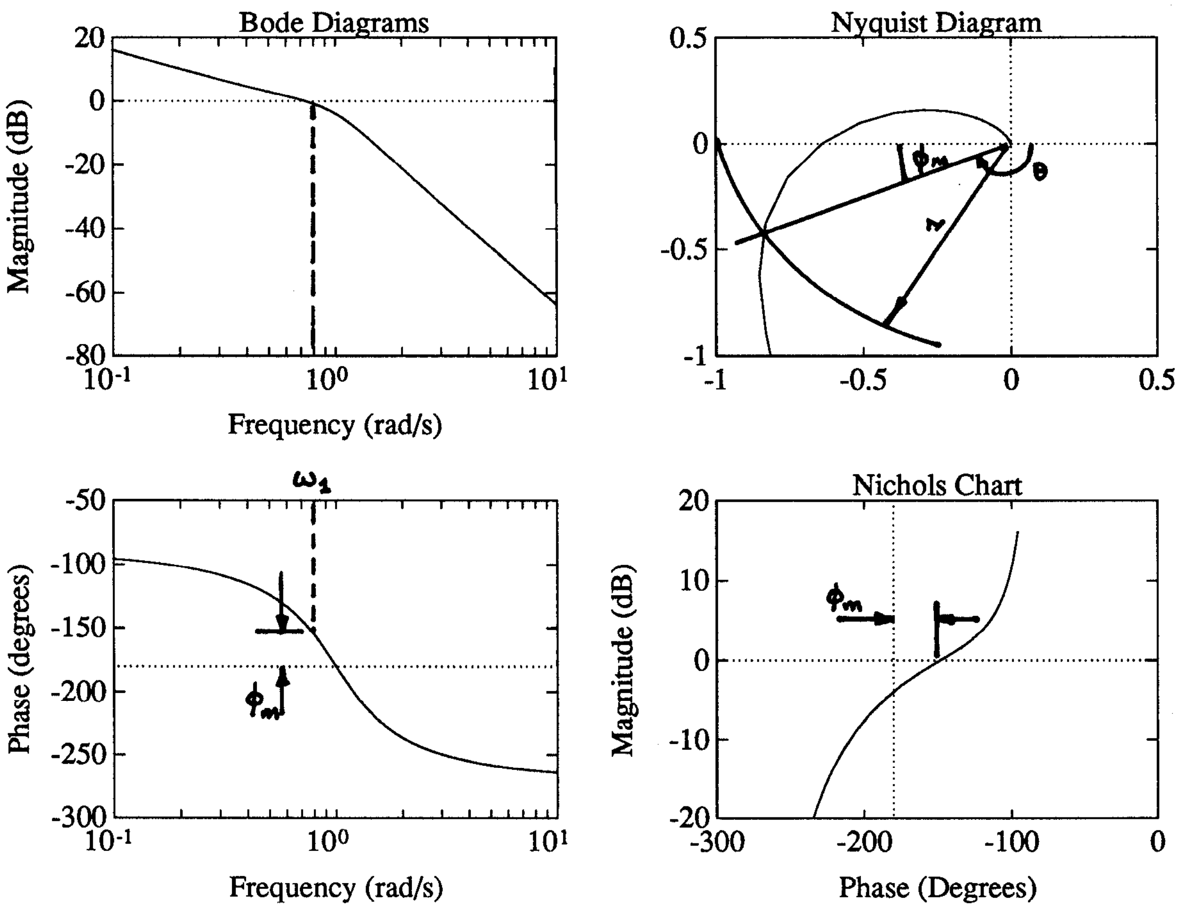


Figure 11 Determination of phase margin (ϕ_m) from frequency response diagrams

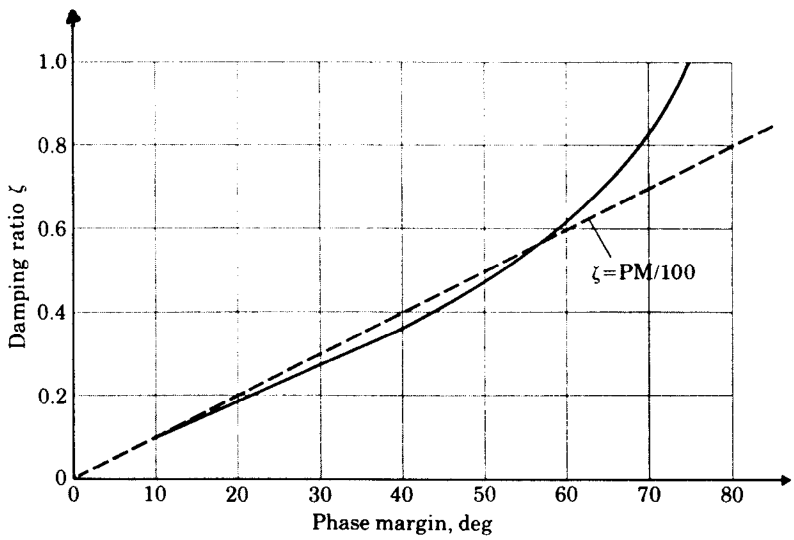


Figure 12 Damping ratio ζ versus phase margin ϕ_m

4.3.4 Band-width (ω_{BN})

A measure of the speed of response of a system. In control systems, this is defined as the frequency at which the magnitude ratio $|C(j\omega)/R(j\omega)|$ is 3 dB down from the magnitude at $\omega = 0$. ω_{BW} is then equal to the cut-off frequency ω_c. See Figure 13. ω_{BW} is approximately equal to $2\omega_1$ where ω_1 is the gain cross-over frequency.

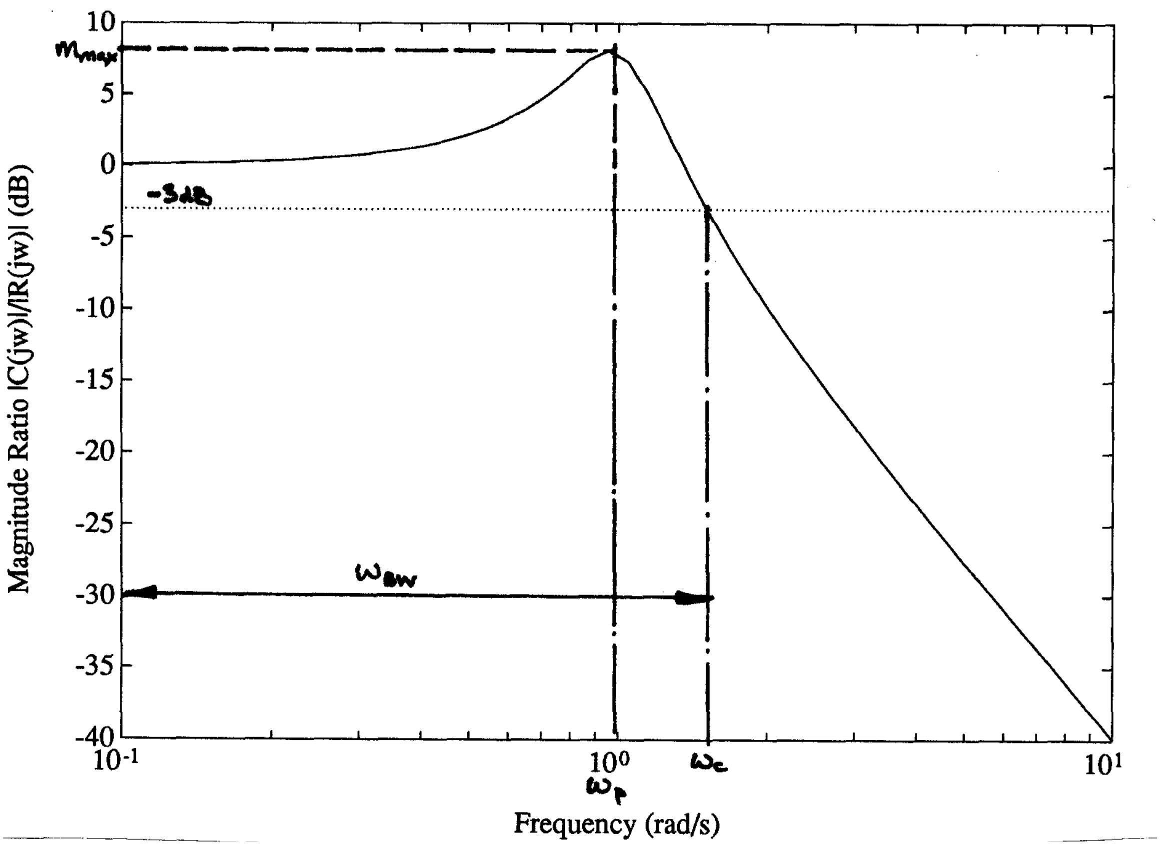


Figure 13 Typical second-order closed-loop frequency response

4.3.5 Cut-off rate
The rate at which the magnitude ratio decreases beyond cut-off. E.g. 6 dB/octave = 20 dB/decade.

4.3.6 Resonant peak (M_{max})
A good measure of stability, maximum value of the closed-loop frequency response (see Figures 13 and 14).

$$M_{\max} = \max_{\omega} \frac{|C(j\omega)|}{|R(j\omega)|}$$

Design values: a useful rule-of thumb is $M_{\max} = |C(j\omega_1)|/|R(j\omega_1)|$. That is the closed-loop resonant peak occurs at about the open-loop gain cross-over frequency. A more accurate value can be obtained by finding the M-circle that is tangential to the Nyquist curve (see Section 2.3). We usually require $M_{\max} < 2$ (6 dB), preferably $M_{\max} < 1.3$ (2.3 dB).

4.3.7 Resonant frequency (ω_p)
The frequency ω_p at which M_{max} occurs.

A final frequency response criterion sometimes encountered is

4.3.8 Delay time (T_d)
A measure of the speed of response is given by

$$T_d(\omega) = \frac{d\gamma}{d\omega}$$

where $\gamma = \angle C(j\omega)/R(j\omega)$ = phase of closed-loop transfer function. The average value of $T_d(\omega)$ of the frequency range of interest is usually specified.

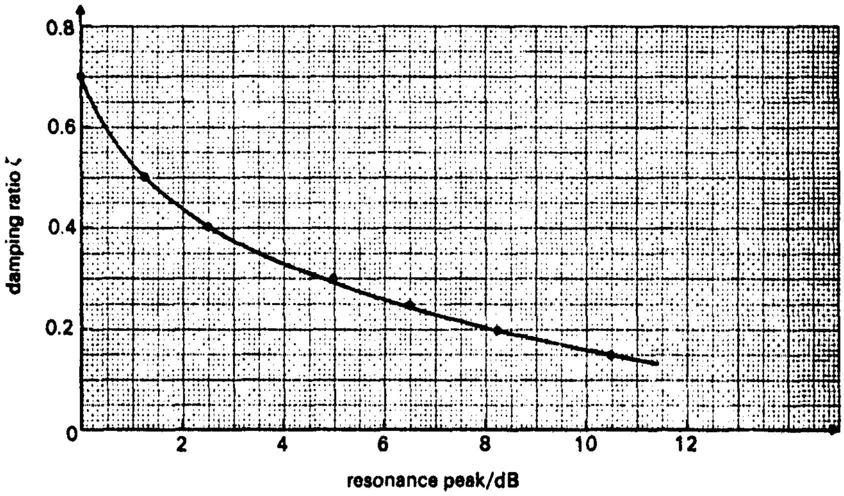


Figure 14 Relationship between resonant peak M_{max} and damping ratio ζ

5 Some useful design curves

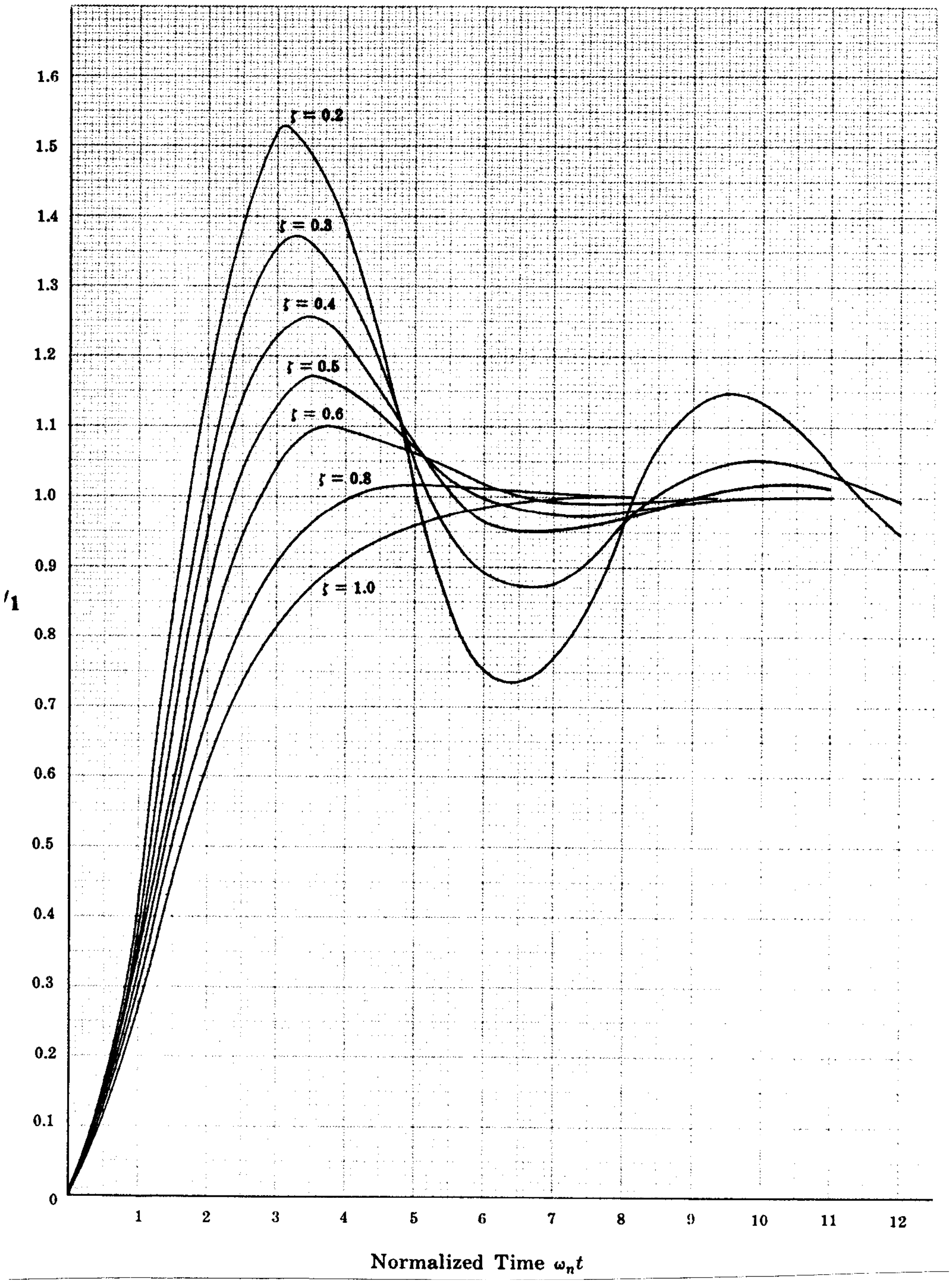


Figure 15 Unit step response vs normalized time $\omega_n t$ for various values of ζ (2nd order)

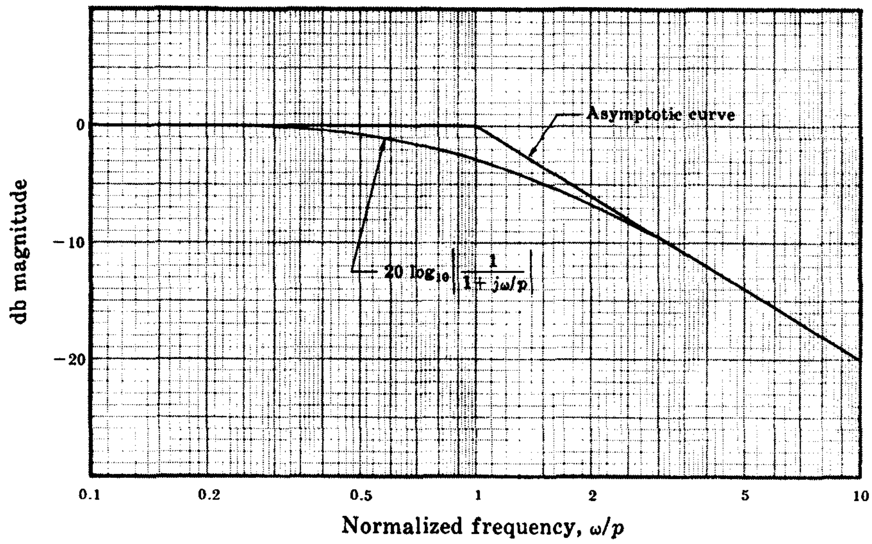


Fig. 15-7

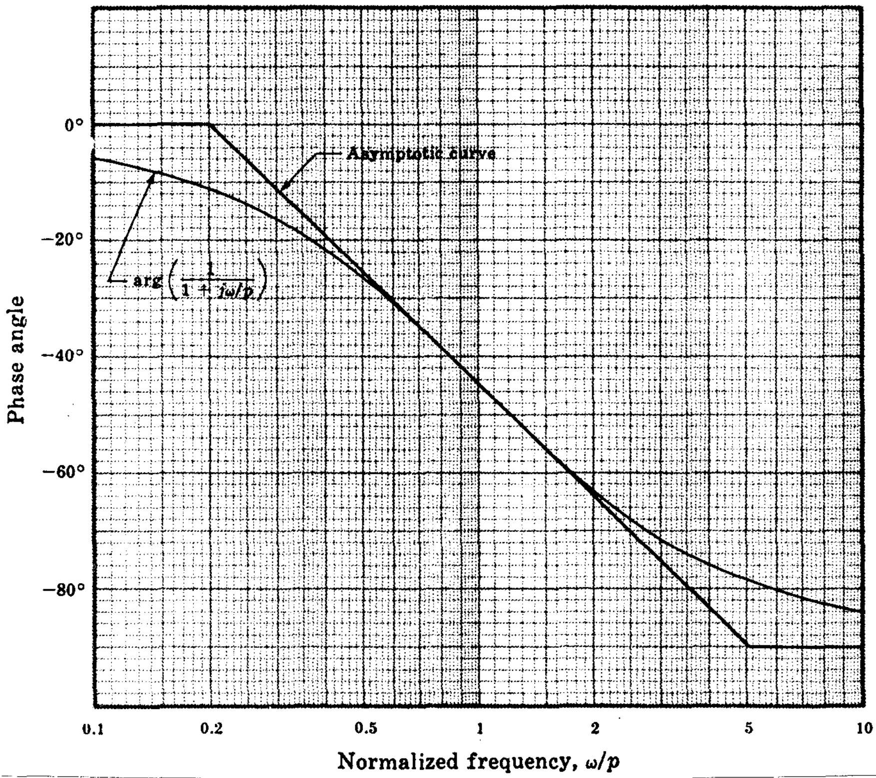


Figure 16 Bode diagram of a first order pole plotted against normalized frequency ω/p

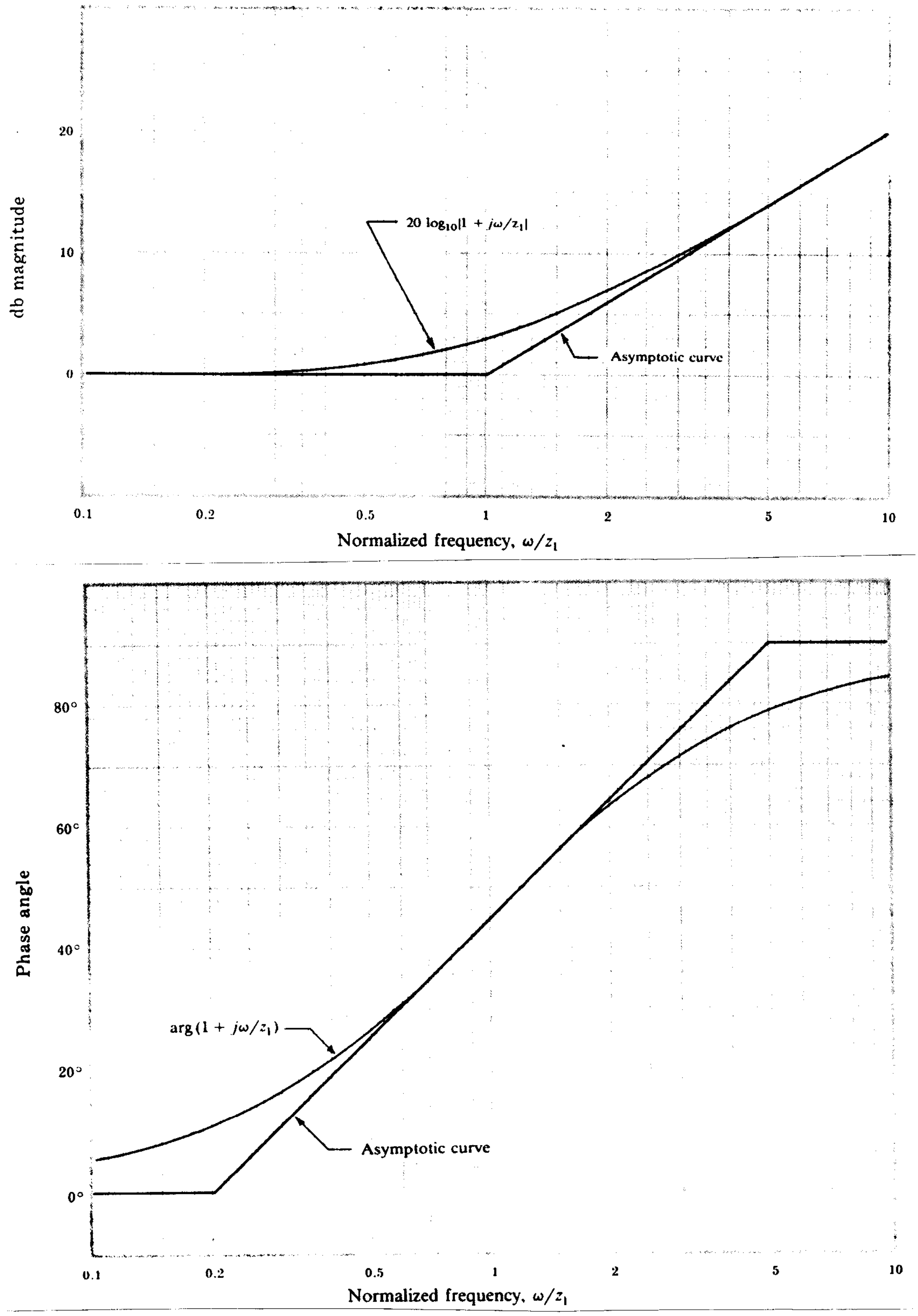


Figure 17 Bode diagram of a first order zero plotted against normalized frequency ω/z

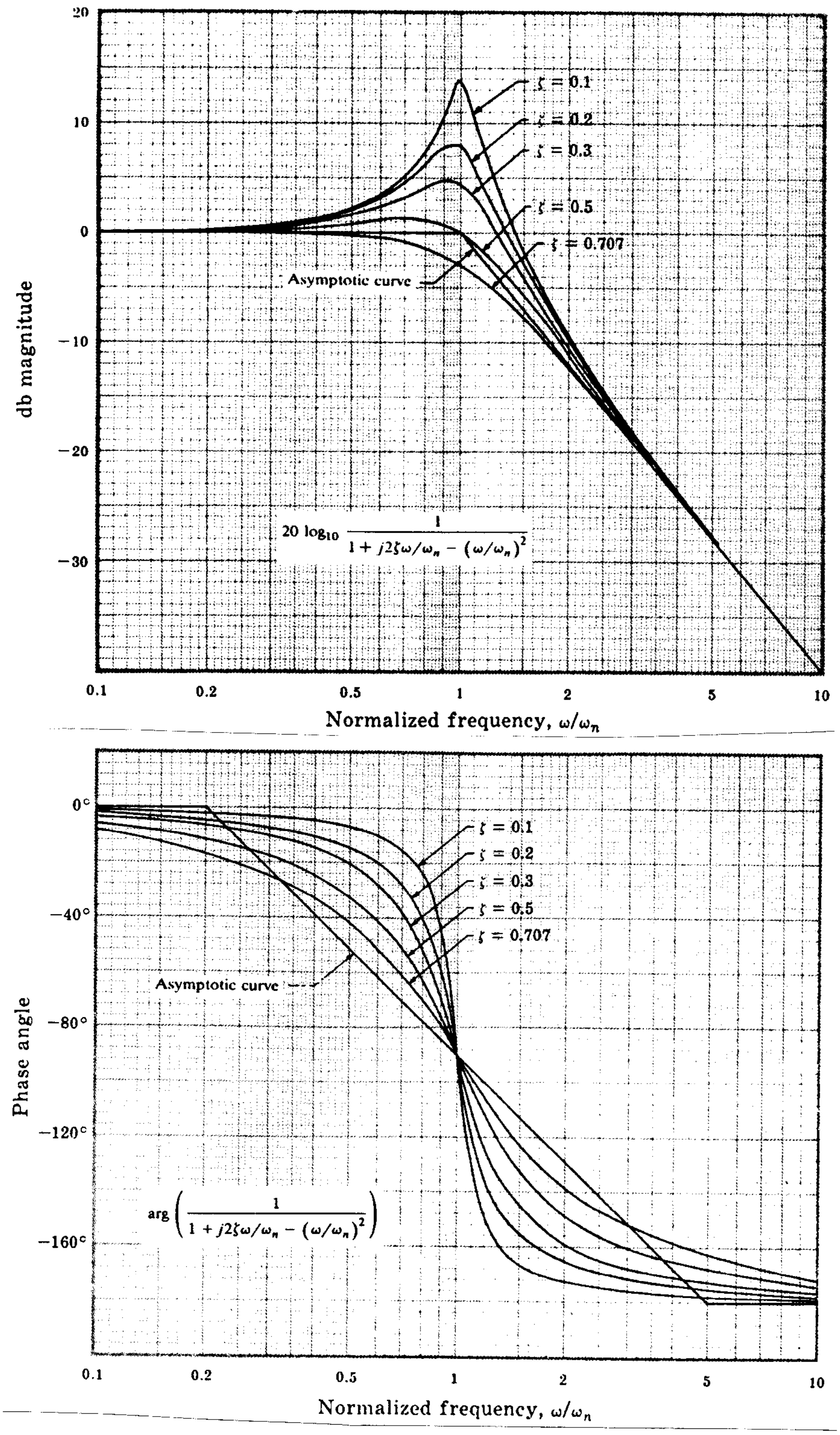


Figure 18 Bode diagram of a second order complex pair of poles plotted against normalized frequency ω/ω_n for various values of ζ

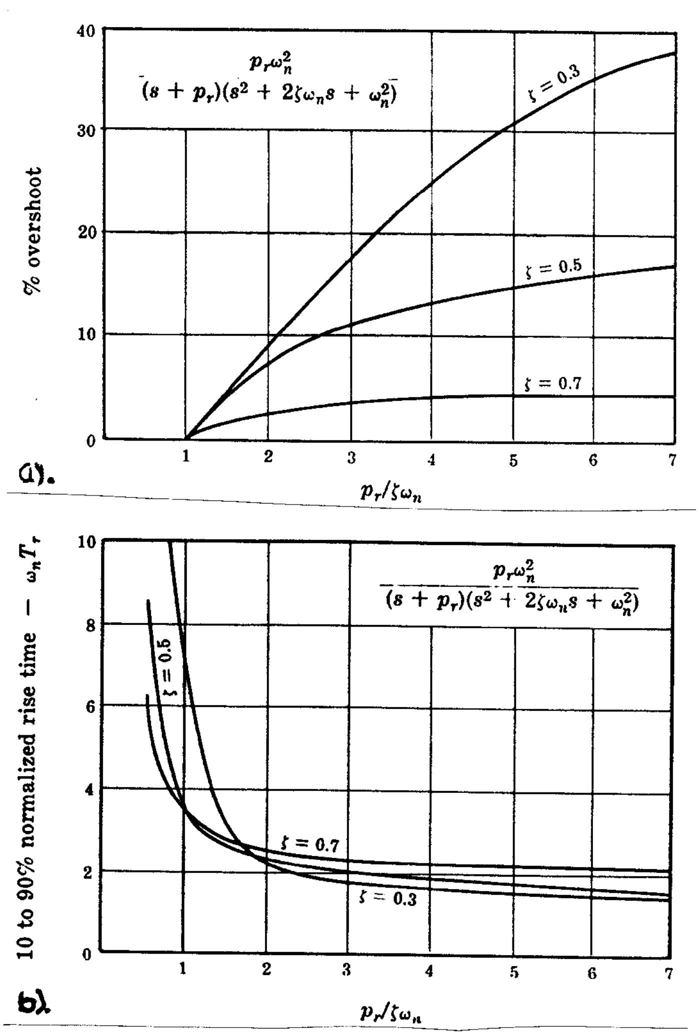


Figure 19 Effect of an extra pole at s = -p_r on a second order system
a) % overshoot M_P vs $p_r/\zeta\omega_n$
b) normalized rise time $\omega_n t_r$ vs $p_r/\zeta\omega_n$

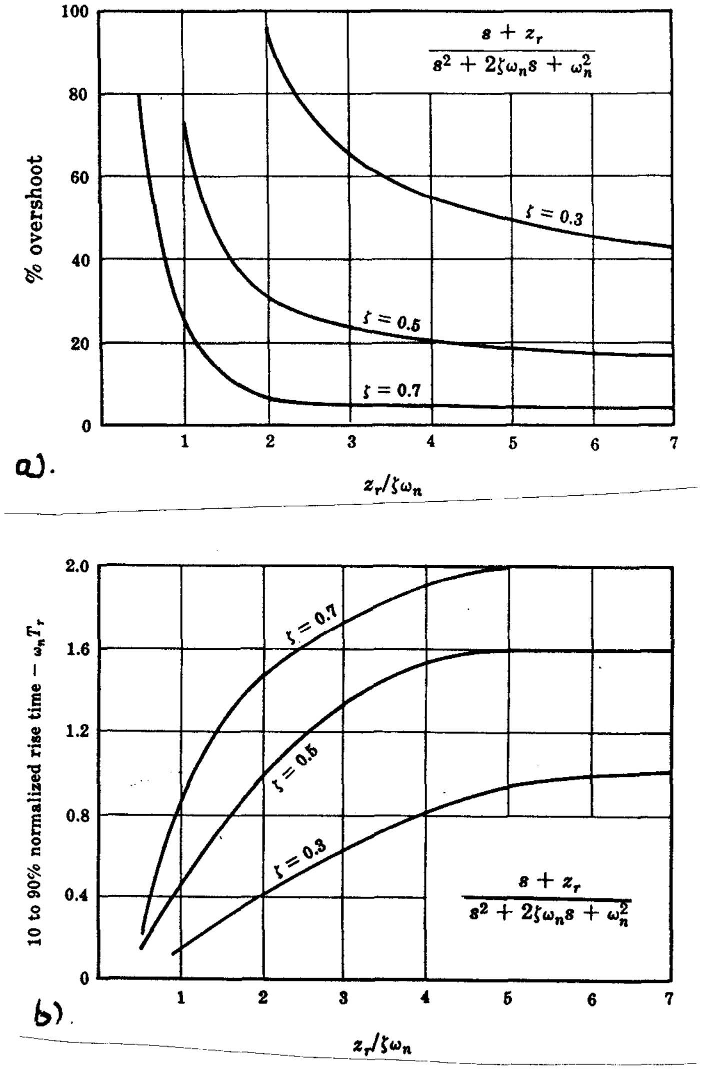
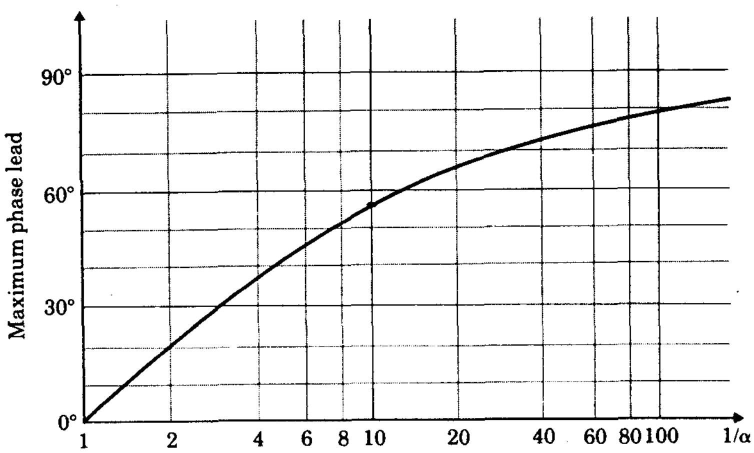


Figure 20 Effect of an extra zero at $s = -z_r$ on a second order system
a) % overshoot M_P vs $z_r/\zeta\omega_n$
b) normalized rise time $\omega_n t_r$ vs $z_r/\zeta\omega_n$



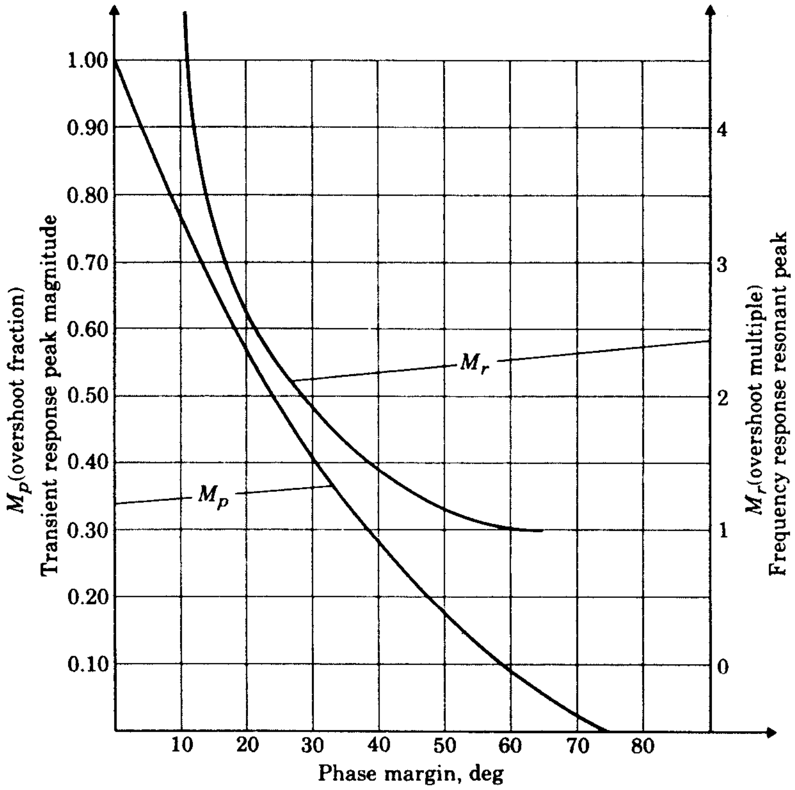


Figure 22 Transient response overshoot M_P and frequency response resonant peak vs phase margin for a second order system (note $M_r = M_{max}$)

6 Acknowledgements

Figures 6, 11,21 and 22 are reproduced from Franklin, Powell and Emami-Naeini, "*Feedback Control of Dynamic Systems*", Addison-Wesley, 1986.

Figures 15, 16, 17, 18, 19 and 20 are reproduced from DiStefano, Stubberud and Williams, "*Feedback and Control Systems*", 2nd Ed., Schaum's Outline Series, McGraw & Hill, 1989.

Figures 7 and 9 are reproduced from Hostetter, Savant and Stefani, "*Design of Feedback Control Systems*", 2nd Ed., Holt, Rinehart and Winston, 1989.

Figures in Appendix 1 are reproduced from Dorf, "*Modern Control Systems*", 5th Ed., Addison Wesley, 1989.

Figure 14 is reproduced from Open University, "Control Engineering", T391 Course Notes.

This handout is based, in part, on lecture notes written by Dr W.R. Moore for a second year course in Automatic Control given in the Department of Electronic Engineering, University of Hull, circa 1979.

Appendix 1 –Transfer Function Plots for Typical Transfer Functions

Note error in Nyquist diagrams for terms with denominator factor $s(s\tau_1+1)$. Real part of $G(j\omega)$ does not (necessarily) tend to zero as $\omega \to \infty$.

Table 2 Transfer function plots for typical transfer functions

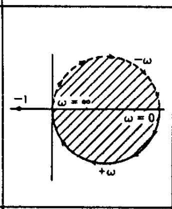
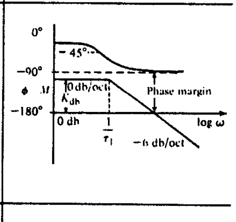
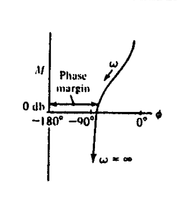
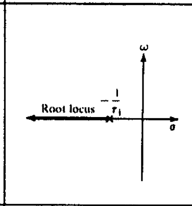
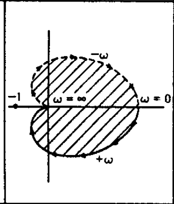
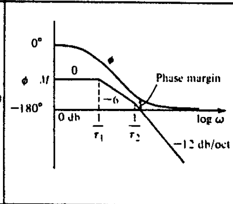
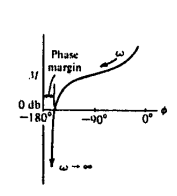
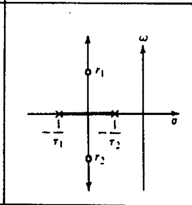
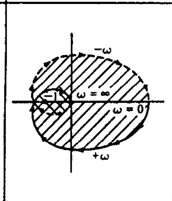
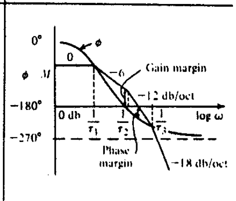
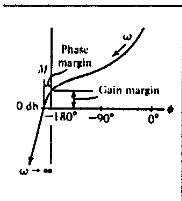
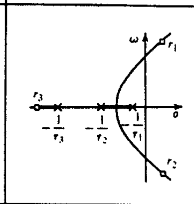
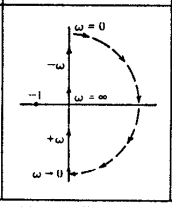
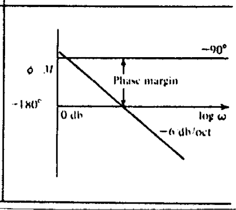
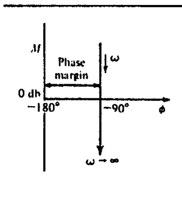
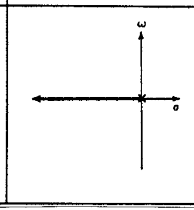
	$G(s)$	Polar Plot	Bode Diagram	Nichols Diagram	Root Locus	Comments
1.	$\dfrac{K}{s\tau_1+1}$					Stable; gain margin = ∞
2.	$\dfrac{K}{(s\tau_1+1)(s\tau_2+1)}$					Elementary regulator; stable; gain margin = ∞
3.	$\dfrac{K}{(s\tau_1+1)(s\tau_2+1)(s\tau_3+1)}$					Regulator with additional energy-storage component; unstable, but can be made stable by reducing gain
4.	$\dfrac{K}{s}$					Ideal integrator; stable

Table 3 Transfer function plots for typical transfer functions (continued)

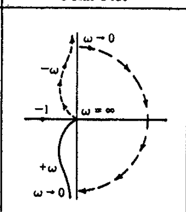
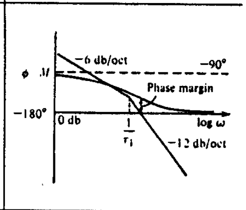
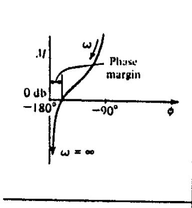
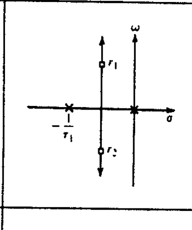
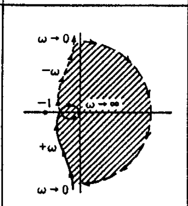
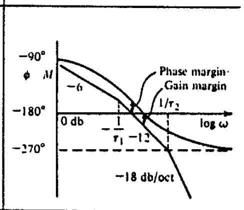
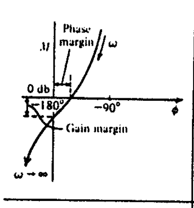
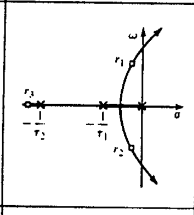
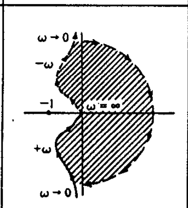
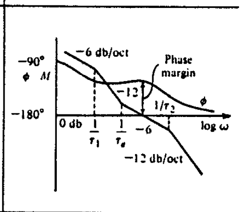
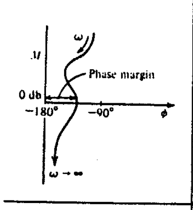
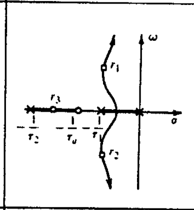
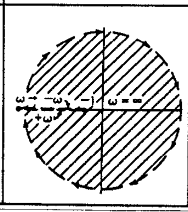
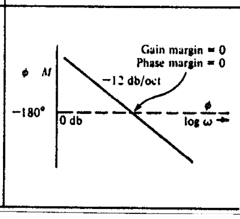
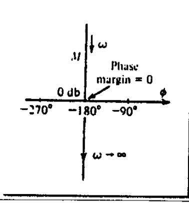
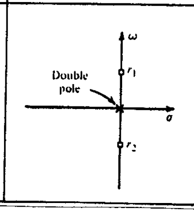
G(s)	Polar Plot	Bode Diagram	Nichols Diagram	Root Locus	Comments
5. $\frac{K}{s(s\tau_1+1)}$					Elementary instrument servo; inherently stable; gain margin = ∞
6. $\frac{K}{s(s\tau_1+1)(s\tau_2+1)}$					Instrument servo with field-control motor or power servo with elementary Ward–Leonard drive; stable as shown, but may become unstable with increased gain
7. $\frac{K(s\tau_a+1)}{s(s\tau_1+1)(s\tau_2+1)}$					Elementary instrument servo with phase-lead (derivative) compensator; stable
8. $\frac{K}{s^2}$					Inherently unstable; must be compensated

Table 4 Transfer function plots for typical transfer functions (continued)

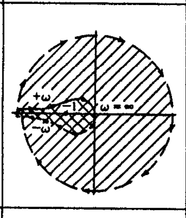
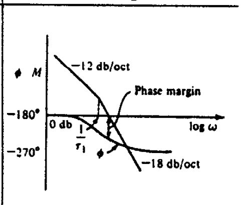
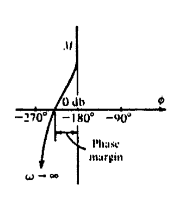
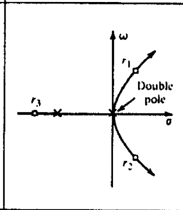
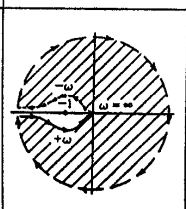
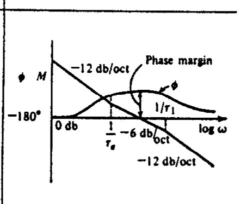
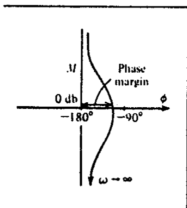
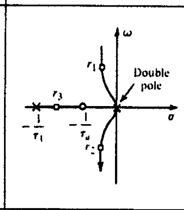
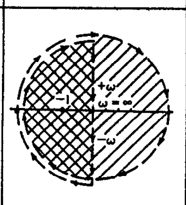
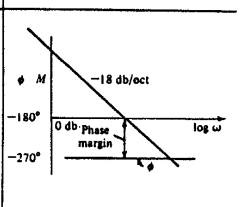
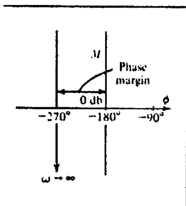
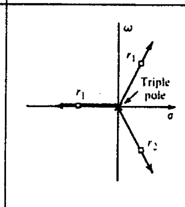
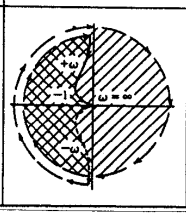
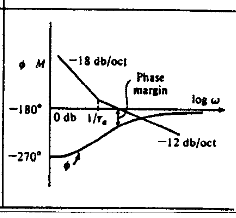
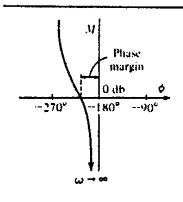
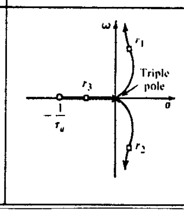
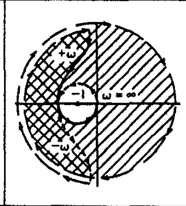
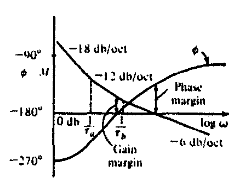
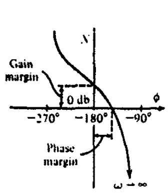
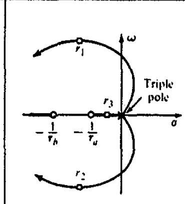
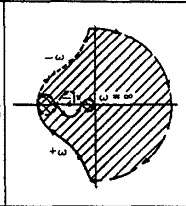
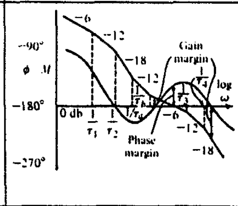
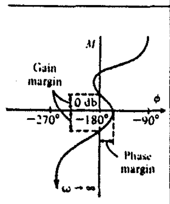
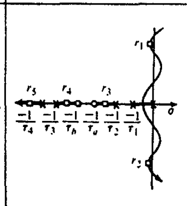
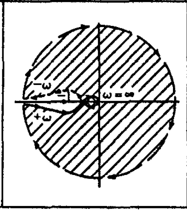
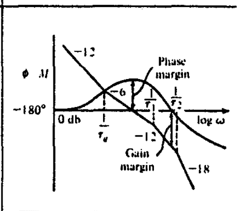
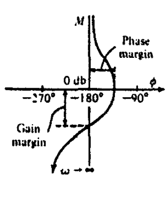
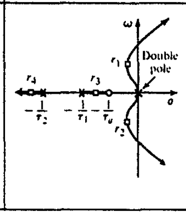
G(s)	Polar Plot	Bode Diagram	Nichols Diagram	Root Locus	Comments
9. $\frac{K}{s^2(s\tau_1+1)}$					Inherently unstable; must be compensated
10. $\frac{K(s\tau_a+1)}{s^2(s\tau_1+1)}$, $\tau_a > \tau_1$					Stable for all gains
11. $\frac{K}{s^3}$					Inherently unstable
12. $\frac{K(s\tau_a+1)}{s^3}$					Inherently unstable

Table 5 Transfer function plots for typical transfer functions (continued)

G(s)	Polar Plot	Bode Diagram	Nichols Diagram	Root Locus	Comments
13. $\dfrac{K(s\tau_a + 1)(s\tau_b + 1)}{s^3}$					Conditionally stable; becomes unstable if gain is too low
14. $\dfrac{K(s\tau_a + 1)(s\tau_b + 1)}{(s\tau_1 + 1)(s\tau_2 + 1)(s\tau_3 + 1)(s\tau_4 + 1)}$					Conditionally stable; stable at low gain, becomes unstable as gain is raised, again becomes stable as gain is further increased, and becomes unstable for very high gains
15. $\dfrac{K(s\tau_a + 1)}{s^2(s\tau_1 + 1)(s\tau_2 + 1)}$					Conditionally stable; becomes unstable at high gain