In [18]:

```
clear all
format compact
```

Proportional Plus Derivative Compensation

Introduction

First design example (Satellite Attitude Control).

Plant:

$$G(s) = \frac{1}{s^2}$$

Feedback:

$$H(s) = 1$$

With velocity feedback the system is as shown in Figure 1.

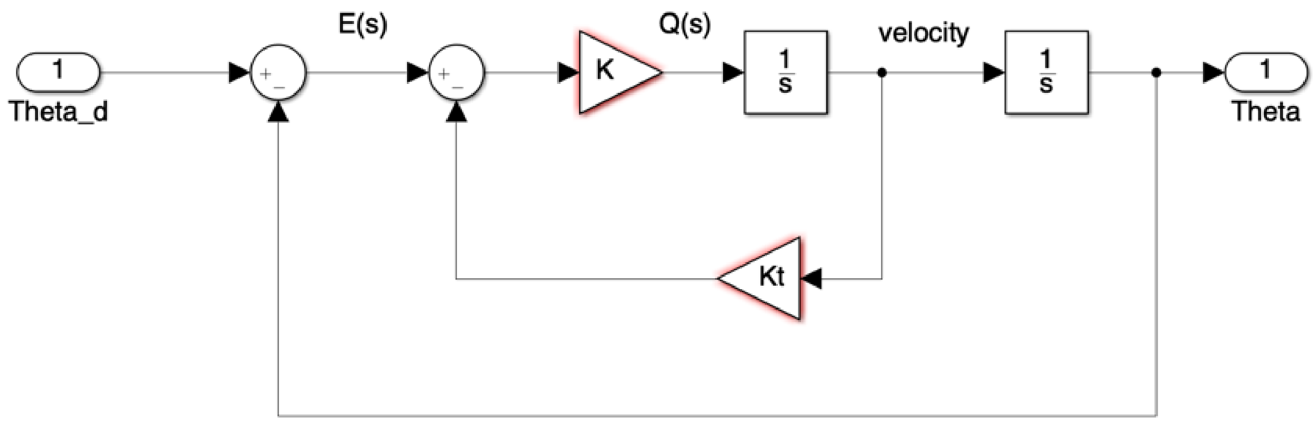


Figure 1 Satellite Attitude Control with Velocity Feedback

For this system, the root locus equation is

$$1 + \frac{KK_T\left(s + \frac{1}{K_T}\right)}{s^2}$$

and the design parameters where calculated to be

In [19]:

```
Kt = 0.5; K = 8;
```

The closed-loop characteristic equation is

In [20]:

```
clce1 = [1, K*Kt, K];
```

The closed-loop transfer function is then:

In [21]:

```
Gc1 = tf(K,clce1)
```

```
Gc1 =

        8
  -------------
  s^2 + 4 s + 8

Continuous-time transfer function.
```

In this document we illustrate how we may implement a silimilar control law using cascade compensation.

Cascade compensator

An alternative compensation architecture is the cascade compensator illustrated in Figure 2.

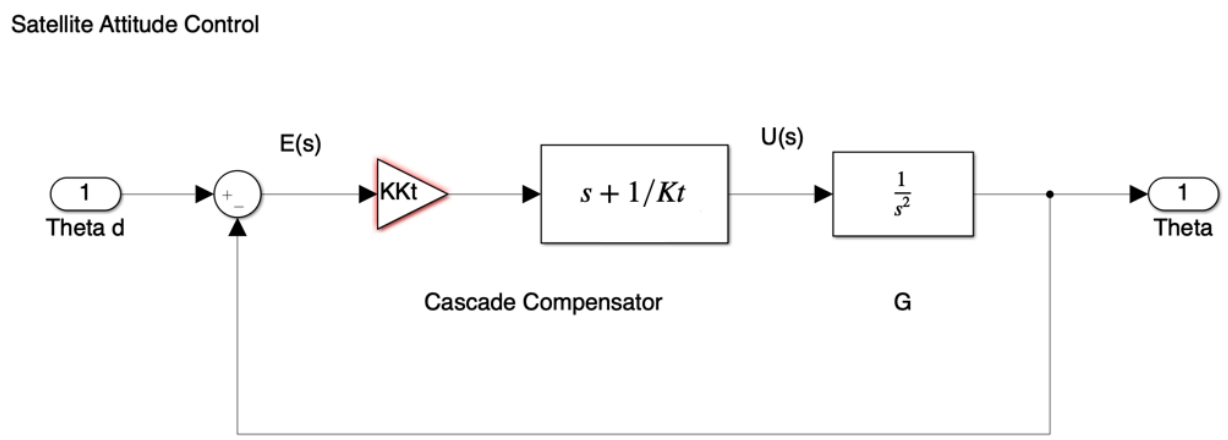


Figure 2 The cascade compensator

The compensator is in series with the plant so that, in general, if the compensator transfer function is

$$D(s) = \frac{K_c(s+z_1)\dots(s+z_r)}{(s+p_1)\dots(s+p_q)}$$

and the compensator poles and zeros are simply added to the poles and zeros of the plant.

If we wish to achieve the same root-locus equation as the previous design (1) then the compensator must have transfer function

$$D(s) = K_c(s + z_1)$$

where

$$K_c = KK_t = 4$$

$$z_1 = 1/K_t = 2$$

Let us verify that this gives the same results as the previous example:

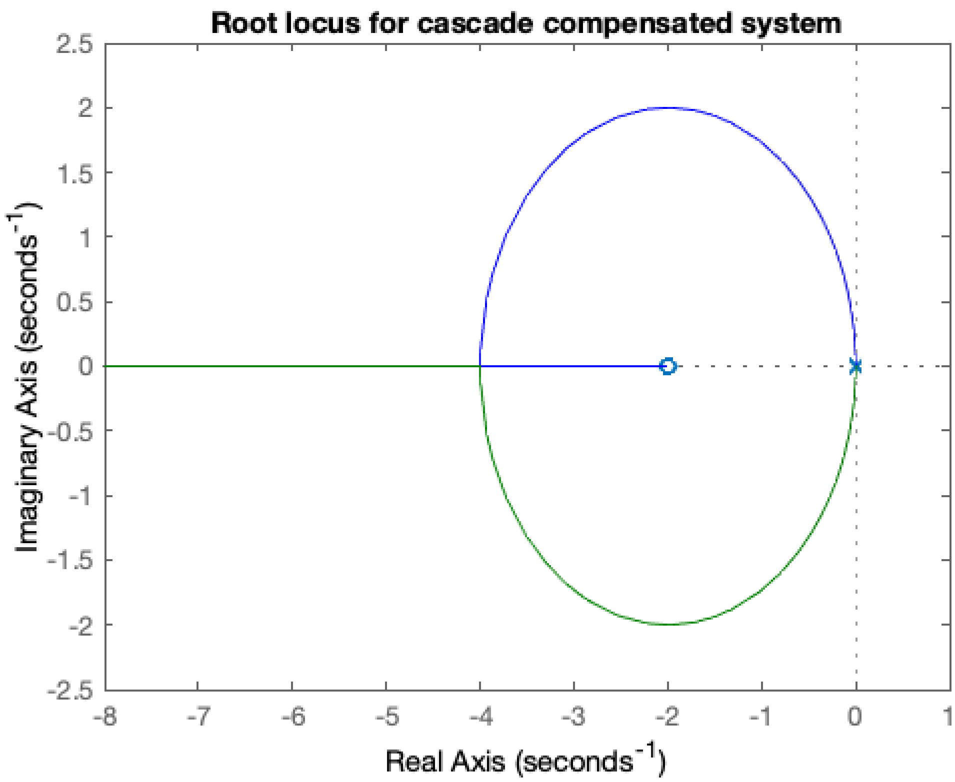
In [22]:

```
z1 = -1/Kt;
Go = zpk(z1,[0, 0],1) % root locus gain initially set to unity
```

```
Go =

  (s+2)
  -----
   s^2

Continuous-time zero/pole/gain model.
```

In [23]:

```
rlocus(Go),title('Root locus for cascade compensated system')
```



Find the root locus gain at the point on the root locus where the poles are located at $s = -2 + j2$.

In [24]:

```
Kc = rlocfind(Go,-2+2j)
```

```
Kc =
     4
```

Now add this to the compensator

In [25]:

```
D = tf(Kc*[1 -z1],1)
```

```
D =

  4 s + 8

Continuous-time transfer function.
```

$$D(s) = 4s + 8$$

Analysis of this compensator reveals that it is of a type known as "*proportional plus derivative*" (P+D). The output of the compensator is of the form

$$U(s) = K_D sE(s) + K_{\text{prop}}E(s)$$

$$u(t) = K_d \frac{de(t)}{dt} + K_{\text{prop}}e(t)$$

and is made up of a "proportion" of the error plus a proportion of the rate-of-change (or derivative) of the error. It is the derivative term that gives the dampening effect required to allow the frictionless system to come to rest.

Closed-loop response

The closed-loop tranfer function is given by

$$G_c(s) = \frac{D(s)G(s)}{1 + D(s)G(s)}$$

In [26]:

```
G=tf(1,[1,0,0])
```

```
G =

   1
  ---
  s^2

Continuous-time transfer function.
```

In [27]:

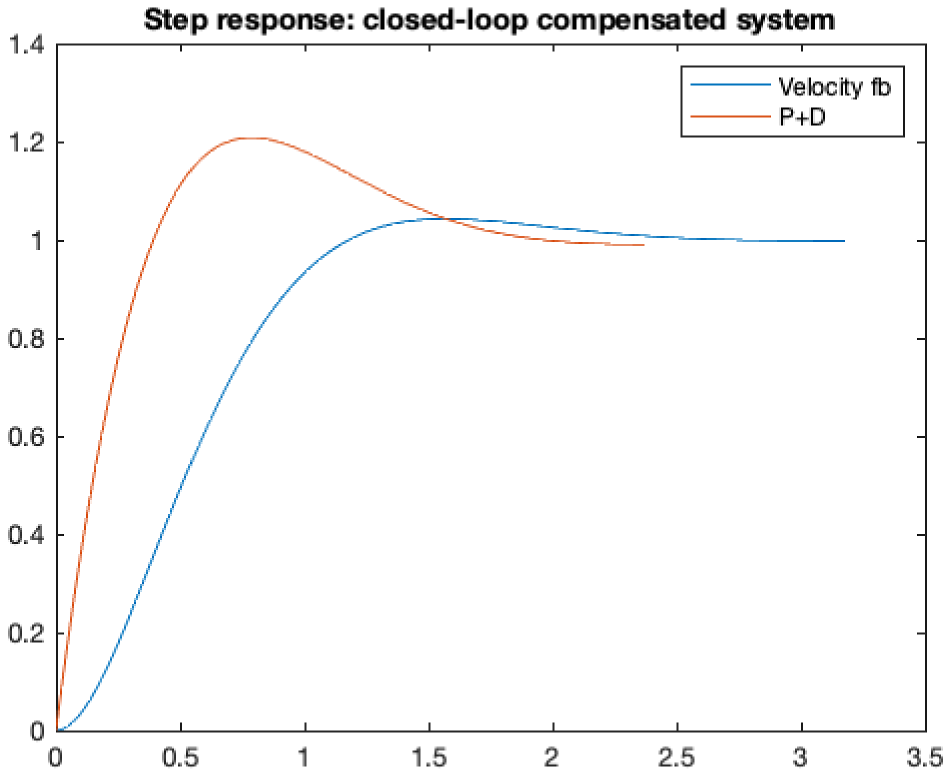
```
Gc2 = feedback(D*G,1)
```

```
Gc2 =

      4 s + 8
  -------------
  s^2 + 4 s + 8

Continuous-time transfer function.
```

Let us plot and compare the step responses of the P+D and velocity feedback results.

In [28]:

```
[y1,t1]=step(Gc1);
[y2,t2]=step(Gc2);
plot(t1,y1,t2,y2),...
 legend('Velocity fb','P+D'),...
 title('Step response: closed-loop compensated system')
```



Notes

Notice that, although the settling time is about the same in both designs, the overshoot is considerably larger in the P+D compensated system. This is because the zero added by the P+D compensator appears in the numerator of the closed-loop transfer function. (refer back to Contact Hour 2 for an explanation).