Introduction to Root Locus Design

In this section we will engage in a short exploration of compensator design in the time domain with a look at root-locus design of a velocity-feedback compensator for a simple "double integrator" system. This serves as an introduction to the topic of phase lead compensation which is used to improve transient performance and relative stability.

Gain compensation

First design example (Satellite Attitude Control). The system may be represented in block diagram form as shown in Figure 1. (Simulink model: satellite.slx (satellite.slx))

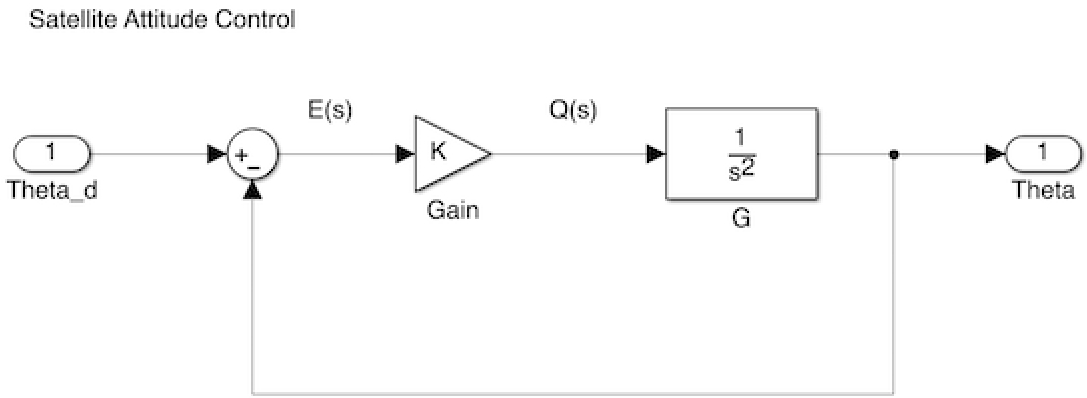


Figure 1 Satellite control with gain modulated torque

For this system the plant transfer function is

$$G(s) = \frac{1}{s^2}$$

Feedback:

$$H(s) = 1$$

Controller:

$$D(s) = K$$

The root locus equation is:

$$1 + KG(s)H(s) = 0$$

with root locus parameter = K.

Defining the problem in Matlab

In [1]:

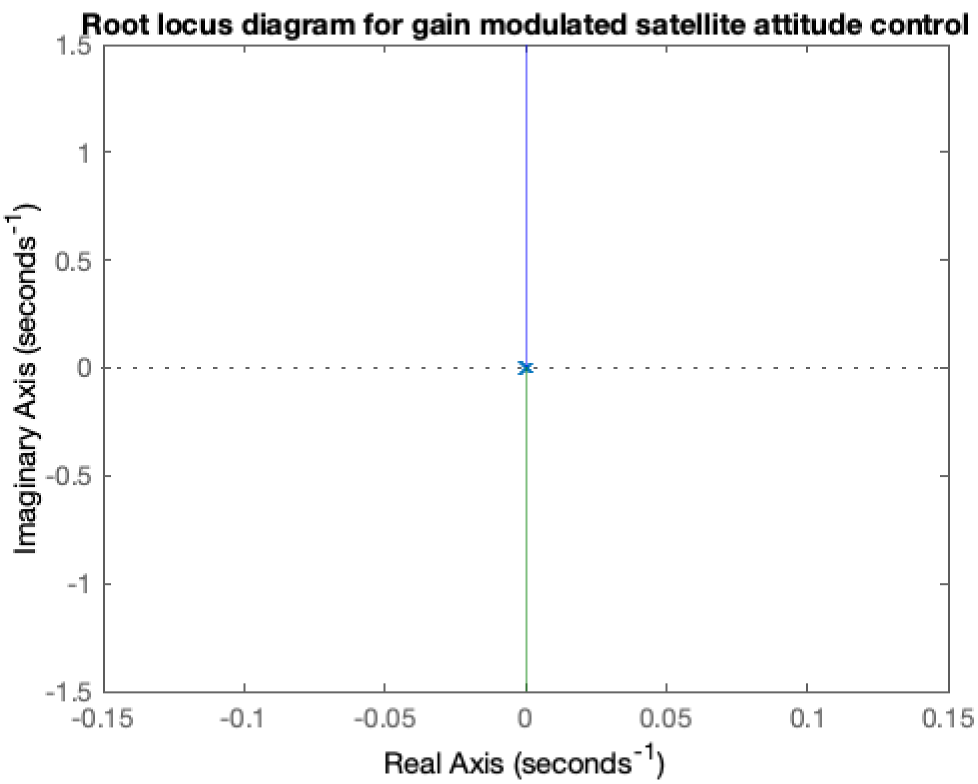
```
G = tf(1,conv([1,0],[1,0]));
H = tf(1,1);
Go = G*H
```

```
Go =

    1
   ---
   s^2

Continuous-time transfer function.
```

Note: The root locus gain K is implied in Matlab (it does not need to be defined)

In [2]:

```
rlocus(Go),title('Root locus diagram for gain modulated satellite attitude contr
ol')
```



Pick off an arbitrary gain

In [3]:

```
[K]=rlocfind(Go,3/4j)
```

```
K =

    0.5625
```

Closed-loop transfer function

In [9]:

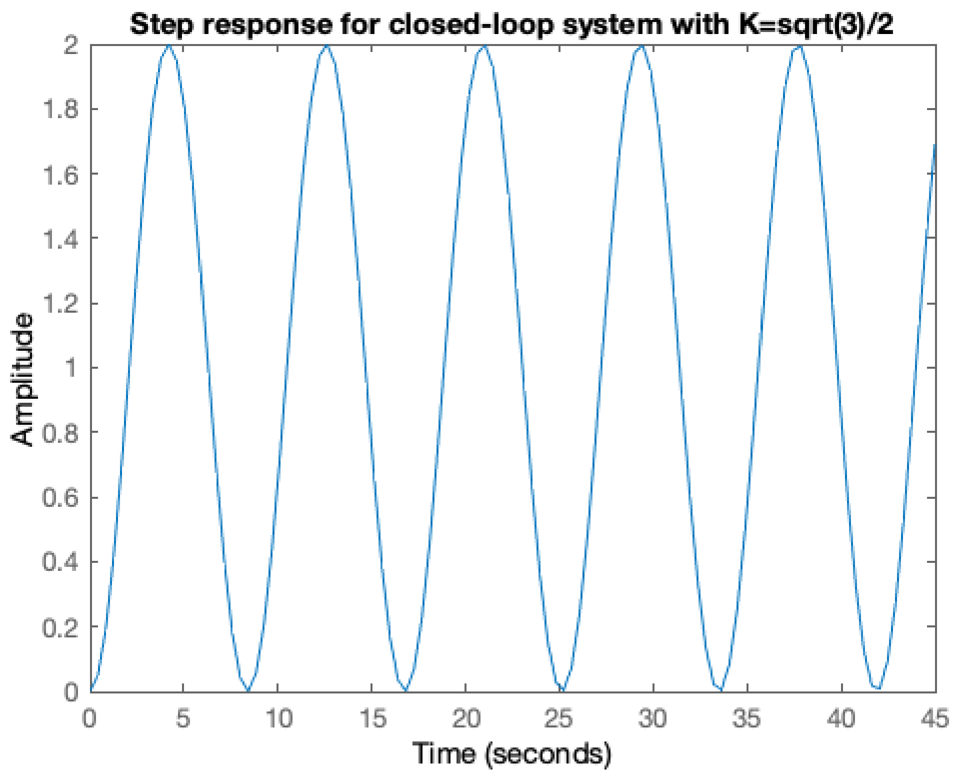
```
Gc = feedback(K*G,H)
```

```
Gc =

      0.5625
  ------------
  s^2 + 0.5625

Continuous-time transfer function.
```

$$G_c(s) = \frac{0.5625}{s^s + 0.5625}$$

In [5]:

```
step(Gc,45),title('Step response for closed-loop system with K=sqrt(3)/2')
```



With velocity feedback,

The block diagram becomes that shown in Figure 2 (Simulink model: velfb.slx (velfb.slx)).

The root locus equation is

$$1 + \frac{KK_T(s + 1/K_T)}{s^2} = 0$$

where KK_T is the root locus gain.

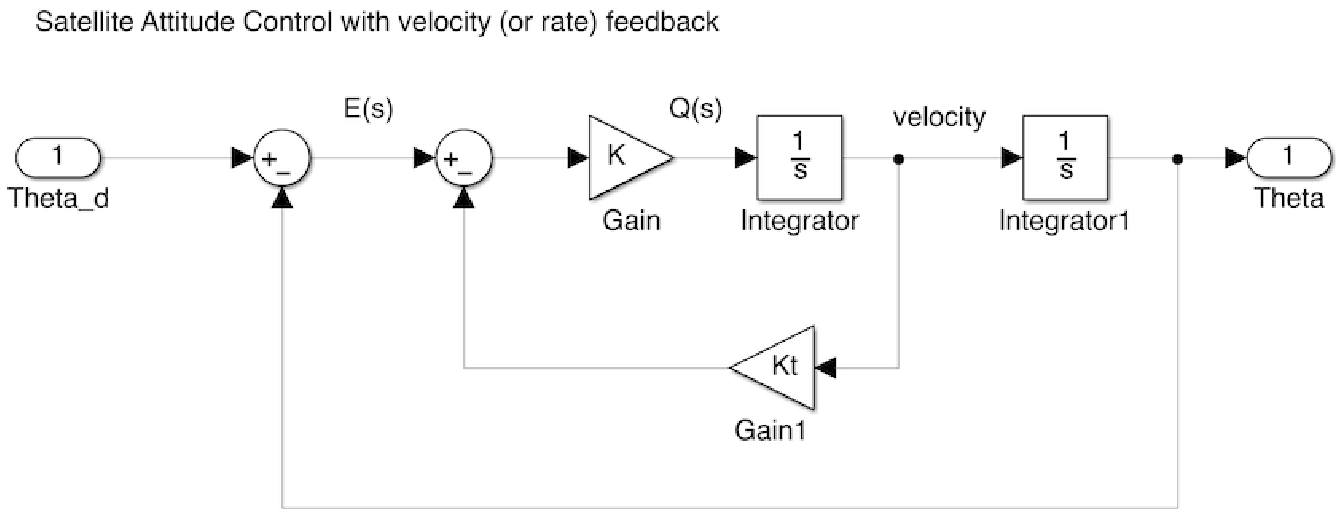
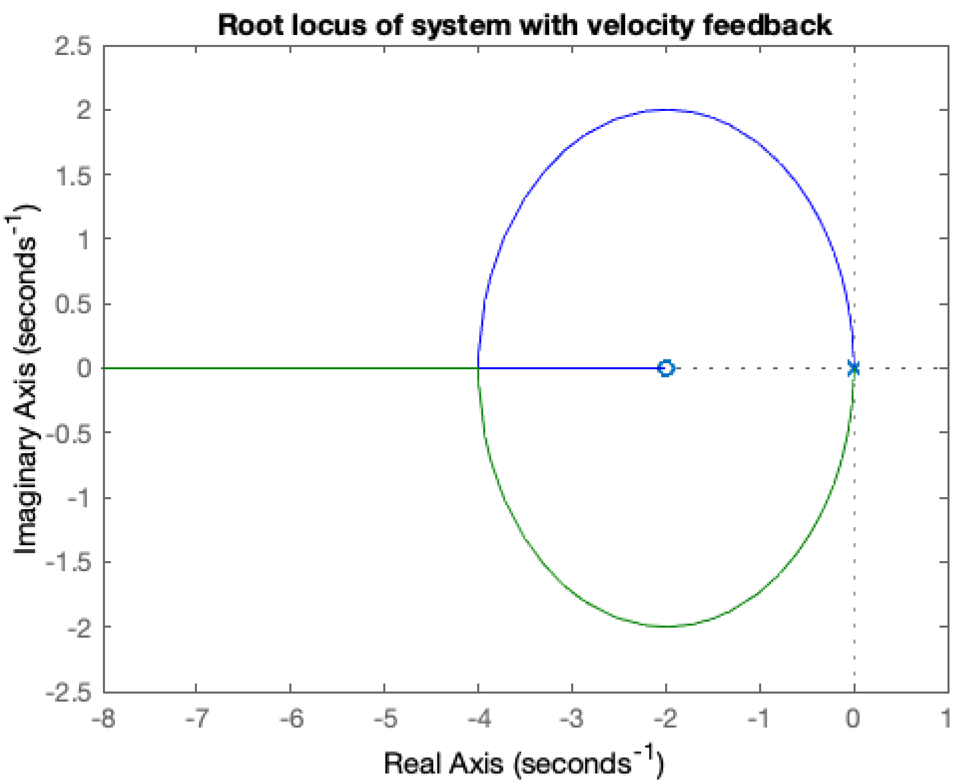


Figure 2 System with velocity feedback

In [6]:

```
Kt = 0.5;
Go2=tf(Kt*[1, 1/Kt],[1,0,0]);
rlocus(Go2),title('Root locus of system with velocity feedback')
```



Closed-loop step response

$$G_o(s) = \frac{1}{s} \times \frac{K/s}{1 + (KK_T)/s}$$

$$G_o(s) = \frac{K}{s(s + KK_T)}$$

$$G_c(s) = \frac{K}{s^2 + KK_T s + K}$$

In [7]:

```
Integrator=tf(1,[1,0]);
G1=feedback(K*Integrator,Kt)*Integrator;
Gc2=feedback(G1,1)
```

```
Gc2 =

          0.5625
  ----------------------
  s^2 + 0.2812 s + 0.5625

Continuous-time transfer function.
```

In [8]:

```
step(Gc2),title('Step response for velocity feedback (rate) compensated system')
```