Swansea University

College of Engineering

EGLM03 Modern Control Systems

Homework 7: Modelling Systems in Statespace

Problems

1. Prove that the impulse response of a state-space model is

$$\begin{aligned}\mathbf{g}(t) &= \mathcal{L}^{-1}\left\{\mathbf{C}\Phi(s)\mathbf{B} + \mathbf{D}\right\}\\ &= \mathcal{L}^{-1}\left\{\mathbf{G}(s)\right\}\end{aligned}$$

2. Consider the circuit example introduced in Section 7.1 (../../07/2/tf4ss) and converted into transfer function matrix form in Section 7.2 (../../07/2/intro2ss).

   If $L = 100$ mH, $C = 1000\ \mu$F and $R = 20\ \Omega$, and the output is taken to be the voltage $v_{21}(t)$ across the resistor, determine the following:
   A. The state transition matrix.
   B. The zero input response given an initial voltage across the capacitor $v_{32}(0) = 10$ V and assuming the current in the inductor is initially 0 A.
   C. The zero state output and state variable responses given an input current of $i = 0.5e^{-t}$.
   D. The total output response,
3. Use state-space methods to find the total solution to the differential equation

$$\frac{d^2(t)}{dt^2} + 20\frac{dy(t)}{dt} + 10y(t) = 16u(t)$$

   given the initial conditions $\frac{dy(0)}{dt} = 1$ and $y(0) = 0$ and $u(t) = \epsilon(t)$.
4. Show that the state space models

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} 0 & 1 \\ -100 & -200 \end{bmatrix} + \begin{bmatrix} 0 \\ 200000 \end{bmatrix} u\\ y &= [1, 0]\mathbf{x}\end{aligned}$$

   and

$$\begin{aligned}\dot{\mathbf{x}} &= \begin{bmatrix} -0.50126 & 0 \\ 0 & -199.5 \end{bmatrix} + \begin{bmatrix} 1124.2 \\ 2.0051 \times 10^5 \end{bmatrix} u\\ y &= [0.89398 - 0.0050125]\mathbf{x}\end{aligned}$$

   have the same input-output relationship (transfer function).

   Construct the state transition matrix $\phi(t)$ and the zero state and zero input output response equations for both systems and observe how the modes of the system are represented in the system output equations.

   Compare the result with that of question 1 which has the same transfer function!

   What does this tell you about the transfer function as a representation of a system?