Lecturer

Set up MATLAB

```
In [2]:
```

```
%cd matlab
pwd
clear all
format compact
```

```
/Users/eechris/dev/eglm03-textbook/content/04/2/matlab/matlab
ans =
    '/Users/eechris/dev/eglm03-textbook/content/04/2/matlab'
```

Tuning a PID Compensator

This Section presents an example that illustrates the effects of tuning the PID parameters on the step and error response of a simple system. The system model used is that of Figure 3.8 in Franklin, Powell and Emami-Naeni's "Feedback Control of Dynamic Systems" (2nd Ed.).

The PID Compensator

A PID compensator has the transfer function:

$$D(s) = \frac{U(s)}{E(s)} = K_{\mathrm{prop}} + \frac{K_I}{s} + sK_D(s)$$

$$D(s) = \frac{K_D s^2 + K_{\mathrm{prop}} s + K_I}{s}$$

This can be rearranged into a form that is often found in commercial implementations of the PID compensator:

$$\begin{aligned} D(s) &= \frac{K_{\mathrm{prop}}\left(\left(K_D/K_{\mathrm{prop}} s^2\right) + s + \left(K_I/K_{\mathrm{prop}}\right)\right)}{s} \\ &= \frac{K_{\mathrm{prop}}\left(T_D s^2 + s + 1/T_I\right)}{s} \\ D(s) &= K_{\mathrm{prop}}\left(1 + T_D s + 1/\left(T_I s\right)\right) \end{aligned}$$

where K_{prop} is called the "proportional gain" (with units of percentage, i.e. 100% is equivalent to $D(s) = 1$), $T_D = K_D/K_{\mathrm{prop}}$ is known as the "derivative time", $T_I = K_I/K_{\mathrm{prop}}$ is the "integral or *reset time*" and $1/T_I$ is called the "reset rate". The tuning parameters provided with commercial PID compensators are often calibrated in these units.

Tuning the PID

In this example a simple type 0 control system with open loop transfer function:

$$G(s) = \frac{1}{(s+1)(5s+1)}$$

is to be compensated with a PID compensator and the step and error responses are considered as each parameter is adjusted.

We first define the plant:

In [3]:

```
s = tf('s');
G = 1/((s + 1)*(5*s + 1))
```

```
G =

         1
  ---------------
  5 s^2 + 6 s + 1

Continuous-time transfer function.
```

We then obtain the step response and error responses of the plant with unity gain feedback and no controller.

The closed loop transfer function of the system is

$$G_c(s) = \frac{G(s)}{1 + G(s)}$$

In [4]:

```
Gc=feedback(G,1)
```

```
Gc =

         1
  ---------------
  5 s^2 + 6 s + 2

Continuous-time transfer function.
```

The error transfer function is given by

$$G_e(s) = \frac{1}{1 + G(s)}$$

In [5]:

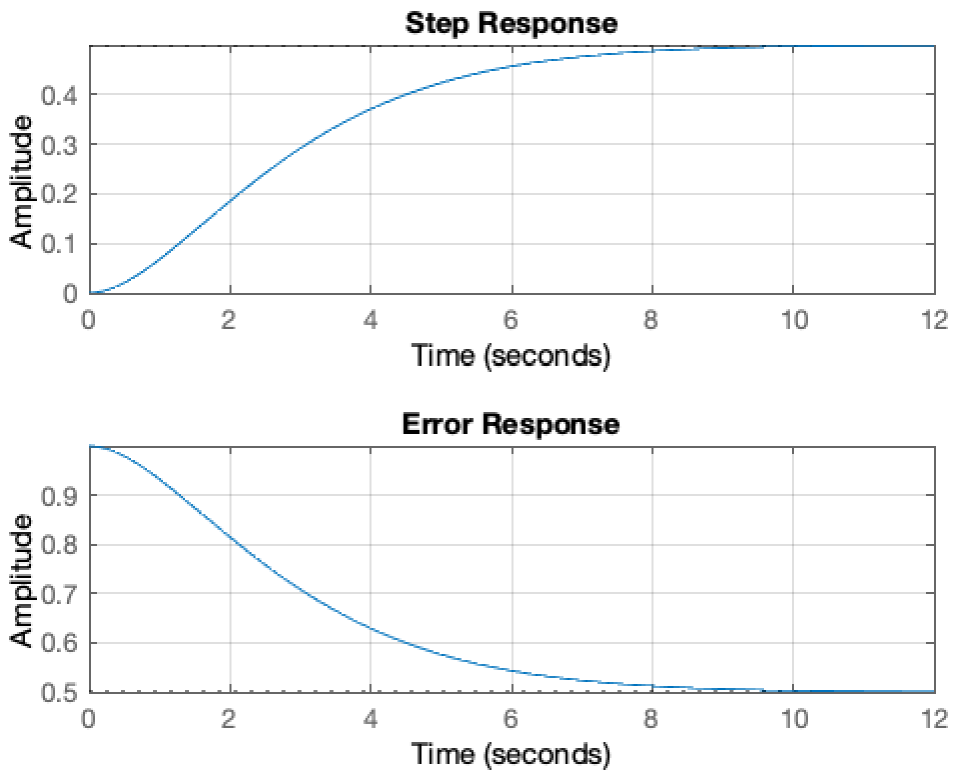
```
Ge=1/(1+G)
```

```
Ge =

   5 s^2 + 6 s + 1
   ---------------
   5 s^2 + 6 s + 2

Continuous-time transfer function.
```

To simulate these responses:

In [6]:

```
subplot(211),step(Gc),title('Step Response'),grid
subplot(212),step(Ge),title('Error Response'),grid
```



The incompensated system has a large steady-state step error of 0.5 (or 50%). It is also overdamped and therefore slow.

First Tuning Stage: Add Proportional Gain

Let us add some proportional control to increase the rise time and to reduce the error to 5%. Increasing the proportional gain of the system can satisfy the error requirement

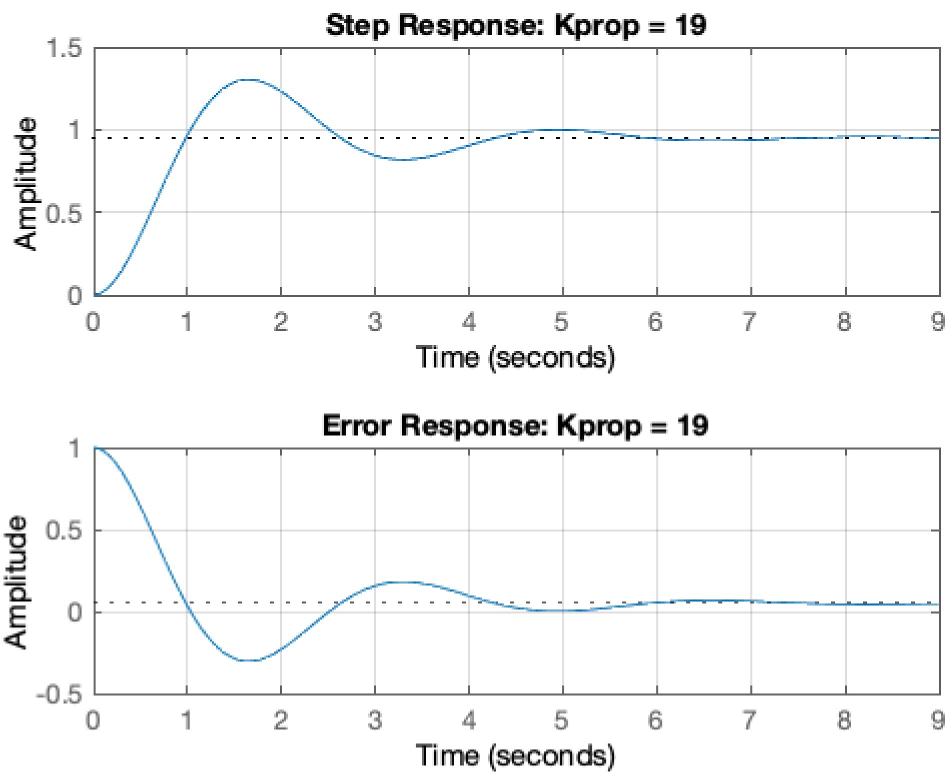
In [7]:

```
Kprop = 19;
D = Kprop;
Go = D*G
```

```
Go =
 
        19
  ---------------
  5 s^2 + 6 s + 1
 
Continuous-time transfer function.
```

In [8]:

```
Gc = feedback(Go,1);
Ge = 1/(1 + Go);
subplot(211),step(Gc),title('Step Response: Kprop = 19'),grid
subplot(212),step(Ge),title('Error Response: Kprop = 19'),grid
```



Note that the steady-state error is now 5% as required and that the proportional controller has speeded up the response time, but at the cost of reduced damping and excessive overshoot and settling time.

Second Tuning Stage: Add Derivative Action

To reduce the overshoot and settling time we add some derivative action to dampen the response without affecting the steady state error.

In [10]:

```
Td = 4/19;
Deriv = Td*s;
D = Kprop *(1 + Deriv);
Go = D*G;
```

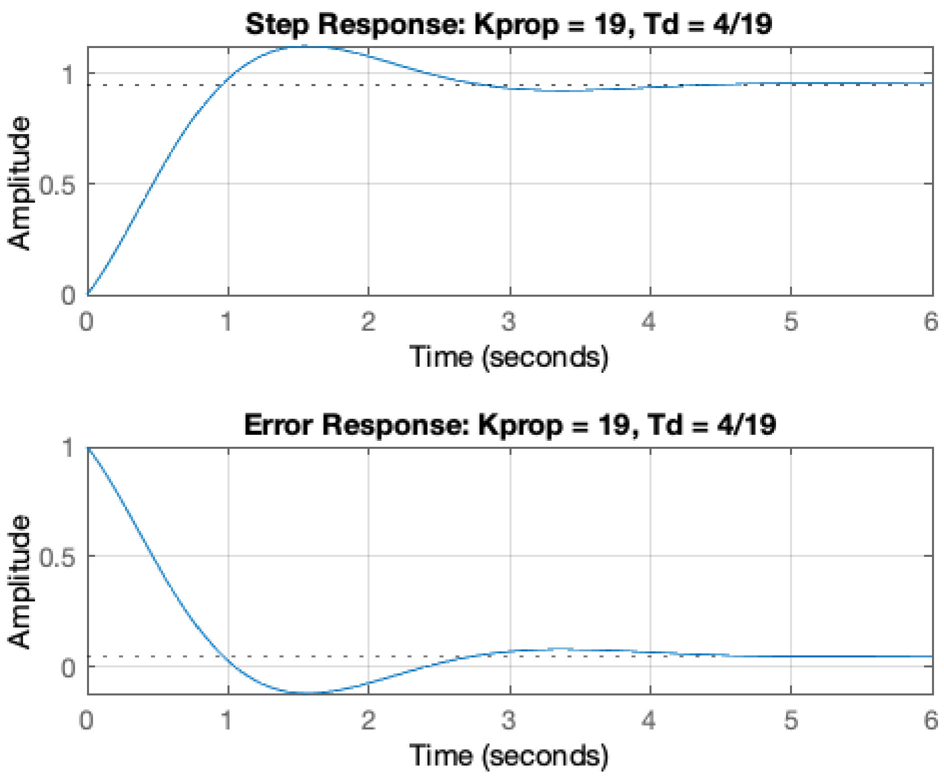
In [11]:

```
Gc = feedback(Go,1)
Ge=1/(1 + Go)
```

```
Gc =

        4 s + 19
  ------------------
  5 s^2 + 10 s + 20

Continuous-time transfer function.


Ge =

    5 s^2 + 6 s + 1
  ------------------
  5 s^2 + 10 s + 20

Continuous-time transfer function.
```

In [12]:

```
subplot(211),step(Gc),title('Step Response: Kprop = 19, Td = 4/19'),grid
subplot(212),step(Ge),title('Error Response: Kprop = 19, Td = 4/19'),grid
```



Third Tuning Stage: Add Integral Action

Finally adding some integral action will eliminate the steady-state step error altogether.

In [13]:

```
Ti = 2;
Integ = (1/Ti)/s;
D = Kprop*(1 + Deriv + Integ);
Go = D*G
```

```
Go =

  4 s^2 + 19 s + 9.5
  ------------------
  5 s^3 + 6 s^2 + s

Continuous-time transfer function.
```

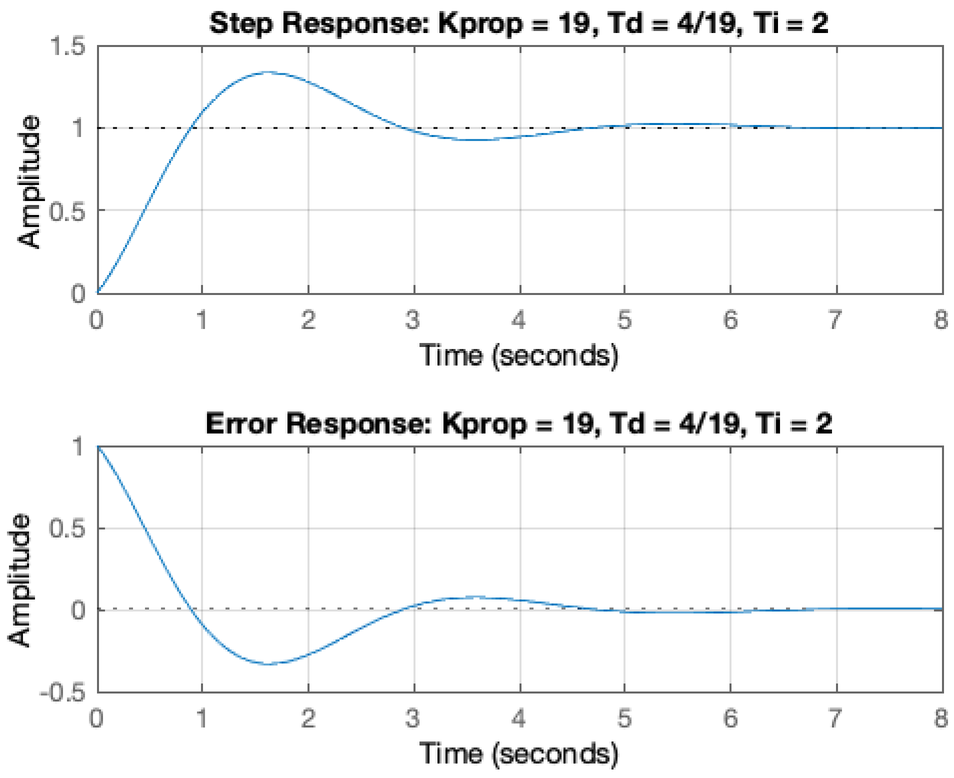
In [14]:

```
Gc=feedback(Go,1)
Ge=1/(1 + Go)
```

```
Gc =

      4 s^2 + 19 s + 9.5
  ---------------------------
  5 s^3 + 10 s^2 + 20 s + 9.5

Continuous-time transfer function.


Ge =

       5 s^3 + 6 s^2 + s
  ---------------------------
  5 s^3 + 10 s^2 + 20 s + 9.5

Continuous-time transfer function.
```

In [15]:

```
subplot(211),step(Gc),title('Step Response: Kprop = 19, Td = 4/19, Ti = 2'),grid
subplot(212),step(Ge),title('Error Response: Kprop = 19, Td = 4/19, Ti = 2'),gri
d
```



Comments

One would probably try further adjustments to reduce the overshoot that the addition of integral action produces

Exercises

You might like to try further adjustments of the parameters K_{prop}, T_d and T_I.

See also the companion documents on the Analytical design of a PID compensator (../3/analpid) and Zeigler-Nichols tuning (../2/zeigler) and Autotuning a PID Compensator with MATLAB (../4/autotune).

Resources

An executable version of this document is available to download as a MATLAB Live Script file pid_tuning.mlx (matlab/pid_tuning.mlx). You can simulate this design using the file pid_cl.slx (matlab/pid_cl.slx).