

Swansea University

College of Engineering

EGLM03 Modern Control Systems

Homework 8: State Space Modelling

Problems

- For each of the systems which have the differential equations shown below. determine the state space model in companion, controller canonical and observer canonical forms.

a.

$$2\frac{dy}{dt} + 8y = 10u$$

b.

$$2\frac{d^2y}{dt^2} + 6\frac{dy}{dt} + 8y = 10\frac{d^2u}{dt^2} + 40\frac{du}{dt} + 30u$$

c.

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} + 4y = 10\frac{du}{dt} + 200u$$

d.

$$\frac{d^2y}{dt^2} + \frac{dy}{dt} + 4y = 10u$$

e;

$$\frac{d^3y}{dt^3} + 7\frac{d^2y}{dt^2} + 14\frac{dy}{dt} + 8y = 10\frac{du}{dt} + 30u$$

- Calculate the eigenvalues and eigenvectors of the state matrices of each of the systems derived in the answers to Question 1 and hence or otherwise construct the normal canonical forms of each system.
- Find the similarity transform that converts the solution for Question 1(e) from observer canonical form to the normal form derived in Question 2.
- Find the similarity transform that converts the solution for Question 1(e) from controller canonical form to the normal form derived in Question 2.

- For each of the systems with transfer functions shown below, construct the state space model in block diagonal (Jordan) form.

a.
$$\frac{d^3 y}{dt^3} + 9\frac{d^2 y}{dt^2} + 24\frac{dy}{dt} + 16y = 5\frac{du}{dt} + 10u$$

b.
$$\frac{d^3 y}{dt^3} + 10\frac{d^2 y}{dt^2} + 90\frac{dy}{dt} + 81y = 10\frac{du}{dt} + 20u$$

- Determine the system transform matrices that will convert a system in observer canonical form to one in controller canonical form. Demonstrate your result by applying the transform to the solutions to Question 1.
- For the controller canonical forms of the system shown in Question 1(c) calculate the system response for $u = 1$ $t \geq 0$, with initial conditions $X_1(0) = x_2(0) = 1$.

Use

- the state-transition matrix
 - transformation to normal form and
 - inverse Laplace transform.
- For each of the systems of Question 1, determine the controllability and observability using the controllability canonical form or observer canonical form as appropriate.
 - Use Ackermann's equations to confirm the results of Question 8.