Lecturer

Set up MATLAB

In [2]:

```
cd matlab
pwd
clear all
format compact
```

```
ans =

    '/Users/eechris/dev/eglm03-textbook/content/05/2/matlab'
```

Frequency Response Design of a Lag Compensator

This MATLAB Live Script examines the design of phase-lag cascade compensators using Bode diagrams.

Analysis

The plant is a type 1 servomechanism with transfer function:

$$G(s) = \frac{1}{s(s+1)}.$$

The system has unity gain feedback and the compensated closed-loop system is to have a static velocity error constant of 10 and a phase margin of 45°.

Defining the system in Matlab

In [5]:

```
nG = 1; dG = [1, 1, 0];
G = tf(nG,dG);
H = tf(1,1);
Go = G;
```

The first part of the analysis is the same as we went through for the Lead Compensation case (../1/freqlead) but we repeat the commands so that the code in the MATLAB workspace will be consistent. You should refer to the otherdocument for the detail.

As before, I want to show the uncompensated frequency response diagrams plotted with the asymptotic bode curves and again we use the function |asymp| to achieve this.

We predefine the frequency values that we want:

In [6]:

```
w = logspace(-2,1);
```

Now we calculate the magnitude and phase

In [7]:

```
[m0,p0] = bode(Go,w);
```

The result of | `[m0,p0] = bode(Go,w)` | produces a data structure. We need to convert the magnitude to decibels and extract the data into column vectors for plotting.

In [8]:

```
m0dB = 20*log10(m0);
m0dB = reshape(m0dB,length(w),1);
p0 = reshape(p0,length(w),1);
```

For the asymptotic magnitude. We need the state space matrices:

In [10]:

```
[Ao,Bo,Co,Do]=ssdata(Go);
```

and then the asymptotic response is computed using the function |asymp|:

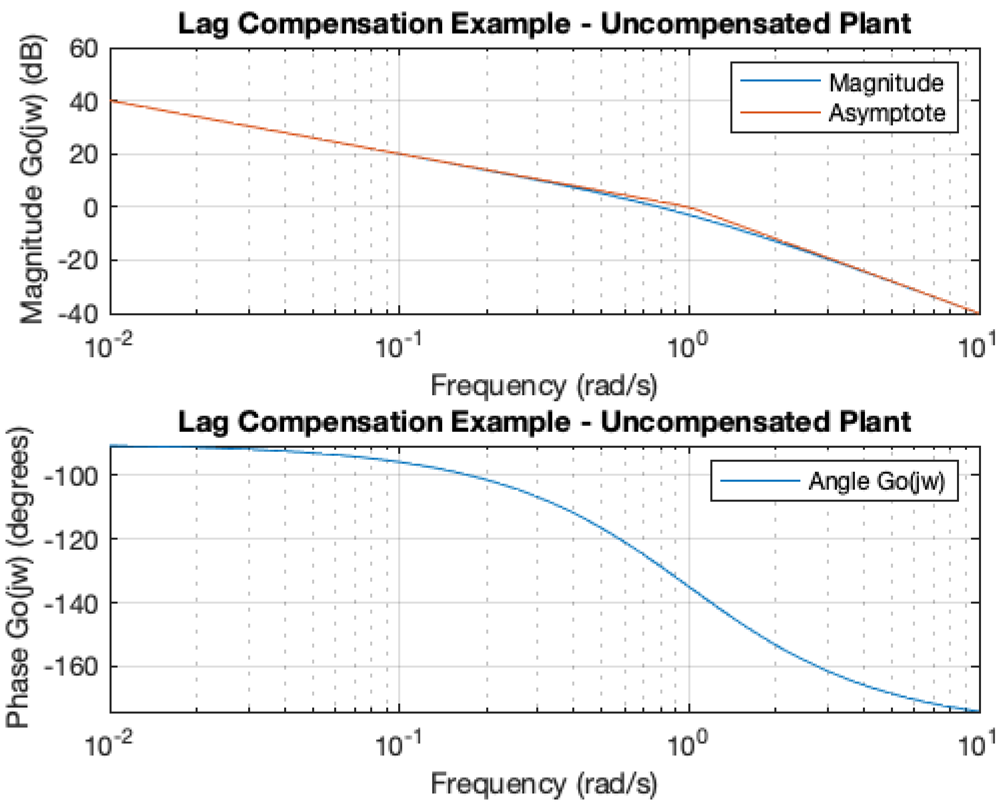
In [11]:

```
am0dB = asymp(Ao,Bo,Co,Do,w);
```

The plots

In [12]:

```
clf
subplot(211)
semilogx(w,m0dB,w,am0dB),...
   axis([0.01, 10, -40, 60]),...
   title('Lag Compensation Example - Uncompensated Plant'),...
   legend('Magnitude','Asymptote'),ylabel('Magnitude Go(jw) (dB)'),xlabel('Frequ
ency (rad/s)'),...
   grid
subplot(212)
semilogx(w,p0),...
   title('Lag Compensation Example - Uncompensated Plant'),...
   legend('Angle Go(jw)'),ylabel('Phase Go(jw) (degrees)'),xlabel('Frequency (ra
d/s)'),...
   grid
```



The gain cut-off frequency $\omega_m \approx 1$ rad/s and the phase margin $\phi_m \approx 45^{\circ}$. Thus the transient performance requirements are already satisfied.

Lag Compensation for Steady-State Performance

For this system:

$$K_v = sG_o(s)|_{s=0} = \left.\frac{1}{s(s+1)}\right|_{s=0} = 1$$

We want $K_v = 10$ so, lag compensation is needed to raise the low frequency gain to 10. The lag compensator has transfer function

$D(s) = K_c\left(\frac{1+Ts}{1+\alpha Ts}\right),\ \alpha > 1.$

Examining the frequency response of this compensator we see that

$D(j\omega) = K_c\left(\frac{1+Tj\omega}{1+\alpha Tj\omega}\right).$

The low frequency response is

$$D(j\omega)|_{\omega=0} = K_c,$$

and the high frequency response is

$$D(j\omega)|_{\omega=\infty} = \frac{K_c}{\alpha}.$$

Since $\alpha > 1$, the low frequency gain is higher than the high frequency gain. The problem with a lag compensator is that the price we pay for this increase in the low frequency gain is a phase-lag. To illustrate this, let us take an example system:

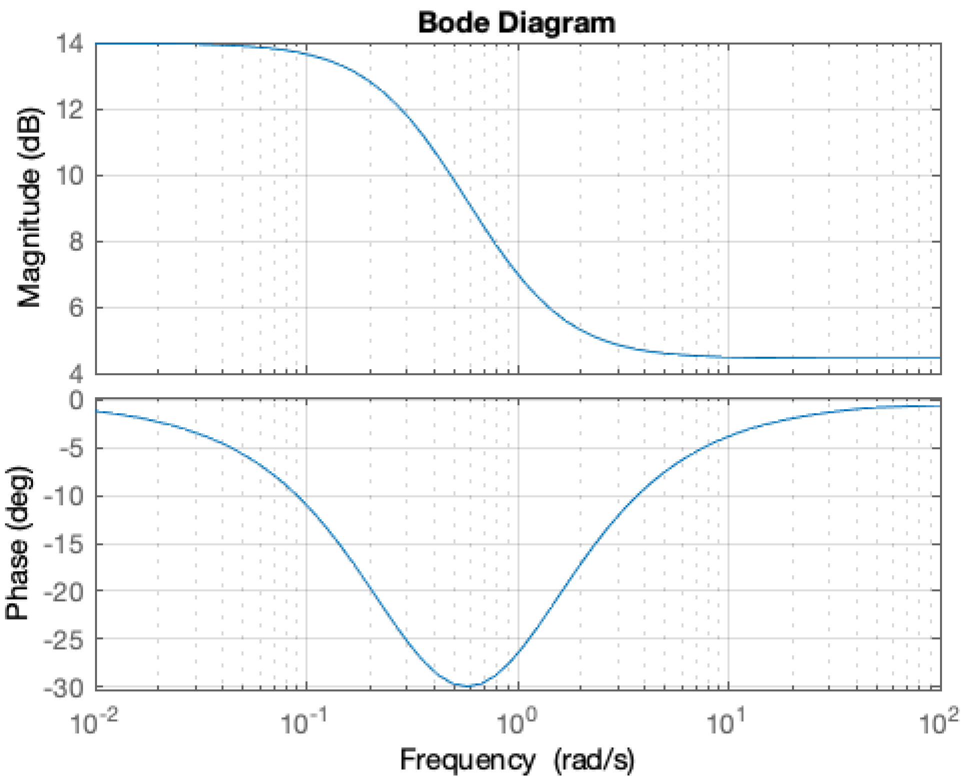
In [13]:

```
Kc = 5; alpha = 3; T = 1;
Deg = tf(Kc*[T, 1],[alpha*T, 1])
clf
bode(Deg)
grid
```

```
Deg =

  5 s + 5
  -------
  3 s + 1

Continuous-time transfer function.
```



There is a significant phase lag at the centre frequency. We must avoid adding any affects of this lag to the plant transfer[1] function at the gain cut-off frequency as this would reduce the phase margin and hence stability. So, we arrange the lag compensator as follows.

The compensator is designed to have unity high frequency gain.

$$D(j\omega)|_{\omega=\infty} = \frac{K_c}{\alpha} = 1.$$

$$K_c = \alpha.$$

This will avoid any change in the gain cut-off frequency ω_1. The low frequency gain is thus given by

$$D(j\omega)|_{\omega=0} = K_c = \alpha.$$

We also ensure that the lag effect is restricted to the low frequency region: so we make sure that the break-frequency of the zero ($\omega_z = 1/T$) is located at least a decade lower than the gain cut-off frquency ω_1[2].

Let us set up a lag compensator with these considerations in place.

In [14]:

```
Kv = 10; w1 = 1; % rad/s
alpha = Kv; % low frequency gain required
Kc = alpha; % compensator gain
wz = w1/10; % 1 decade below gain cut-off
T = 1/wz;   % zero value
D = tf(Kc*[T, 1],[alpha*T, 1])
```

```
D =

  100 s + 10
  ----------
  100 s + 1

Continuous-time transfer function.
```

We produce a new Bode diagram for the gain compensated system. The same commands are issued as before.

In [16]:

```
DGo = series(D,Go);
[m1,p1] = bode(DGo,w);
m1dB = 20*log10(m1);
m1dB = reshape(m1dB,length(w),1);
p1 = reshape(p1,length(w),1);
```

Asymptotic magnitude

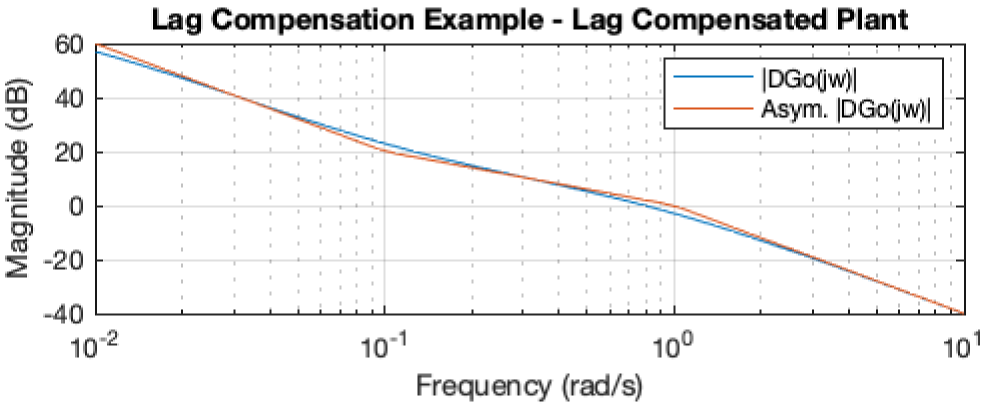
In [17]:

```
[A1,B1,C1,D1]=ssdata(DGo);
am1dB = asymp(A1,B1,C1,D1,w);
```

Plots

In [32]:

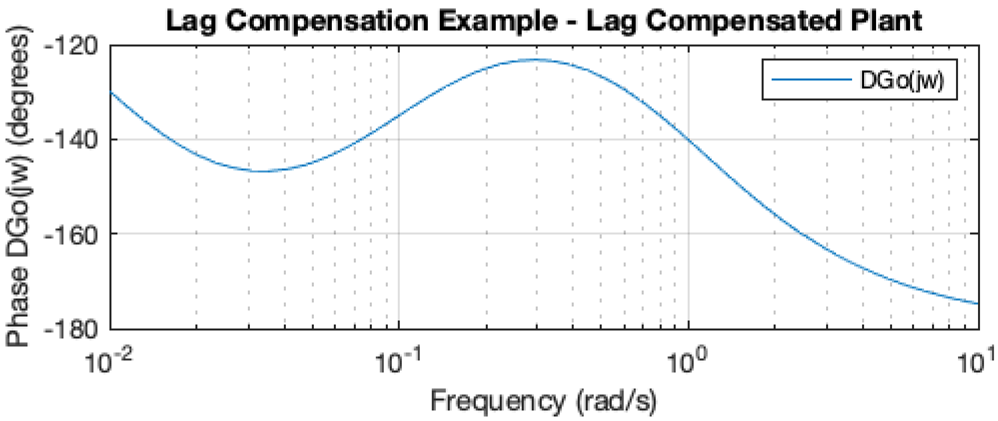
```
clf
subplot(211)
semilogx(w,m1dB,w,am1dB),...
   axis([0.01, 10, -40, 60]),...
   title('Lag Compensation Example - Lag Compensated Plant'),...
   legend('|DGo(jw)|','Asym. |DGo(jw)|'),...
   ylabel('Magnitude (dB)'),xlabel('Frequency (rad/s)'),...
   grid
```



Phase is unchanged

In [19]:

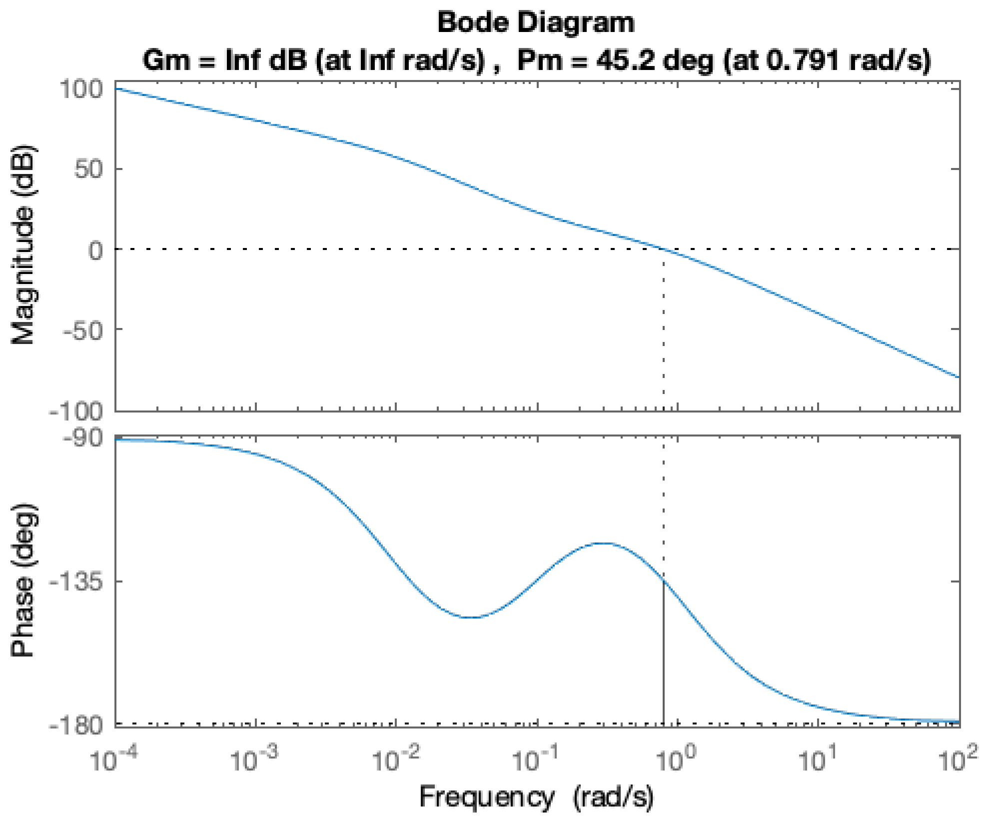
```
subplot(212)
semilogx(w,p1),...
   title('Lag Compensation Example - Lag Compensated Plant'),...
   legend('DGo(jw)'),ylabel('Phase DGo(jw) (degrees)'),xlabel('Frequency (rad/
s)'),...
   grid
```



How have we done? The low frequency gain has certainly increased by 20 dB (10). What about the phase margin?

In [20]:

```
clf
margin(DGo)
```



The phase margin is ϕ_m and $\omega_1 = 0.79$ rad/s.

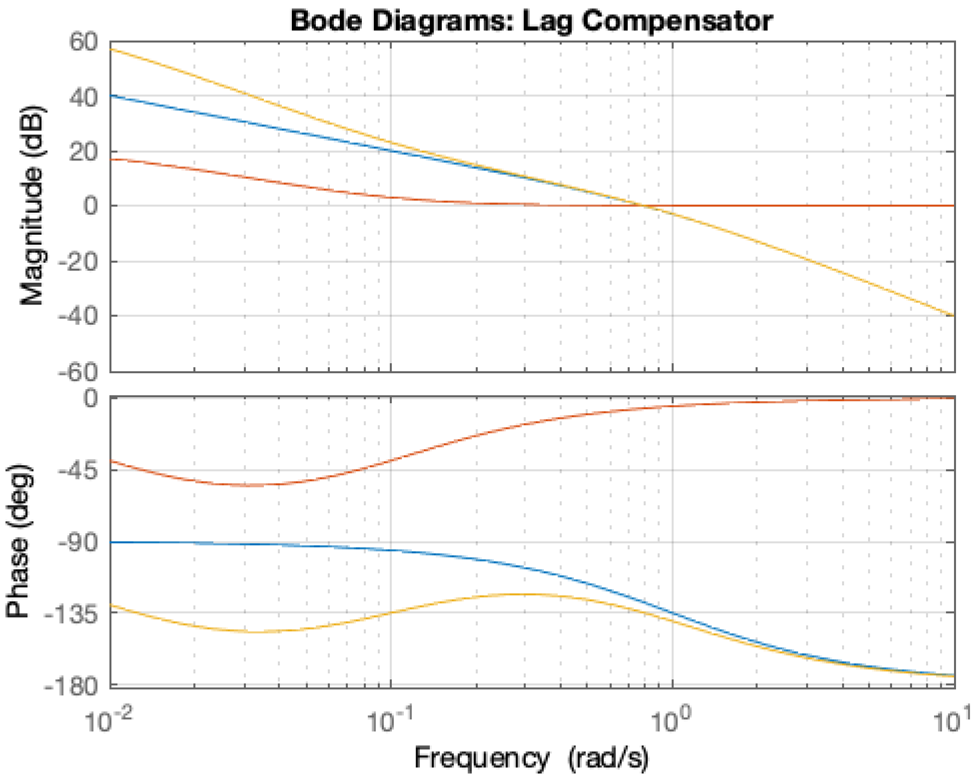
Evaluation of the Design

We now put everything together to evaluate the design.

Bode Plots

In [24]:

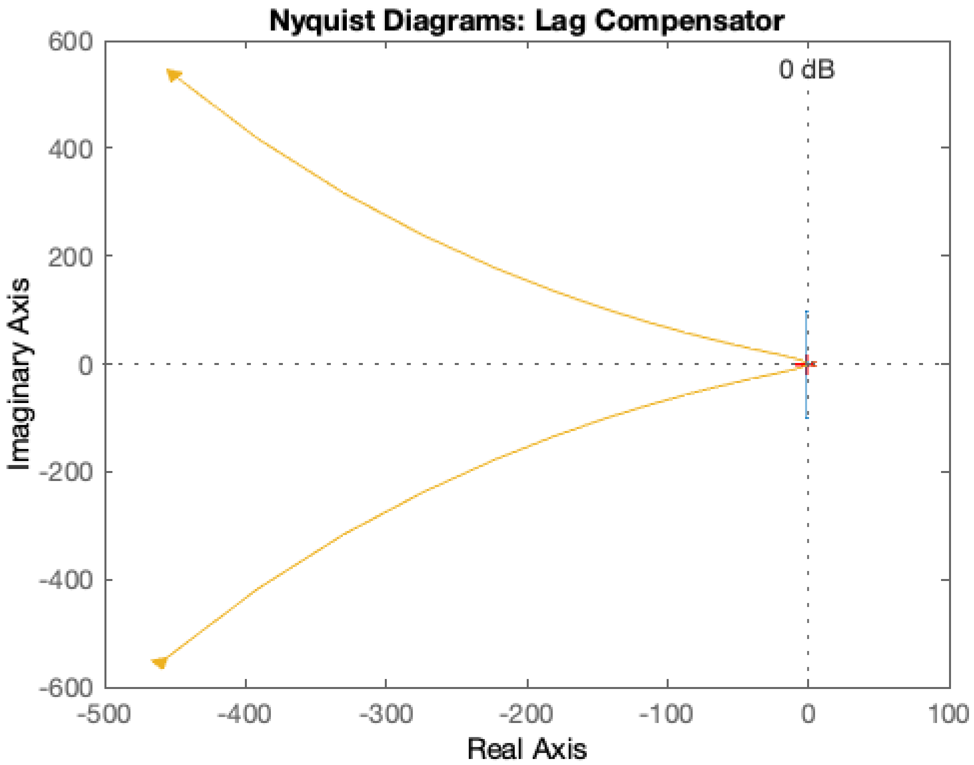
```
% Bode
%  blue - Uncompensated Go(jw)
%  green - Lag Compensator D(jw)
%  red - Lag compensated DGo(jw)
bode(Go,D,DGo,w),...
   title('Bode Diagrams: Lag Compensator'),...
   grid
```



Nyquist Diagrams

In [26]:

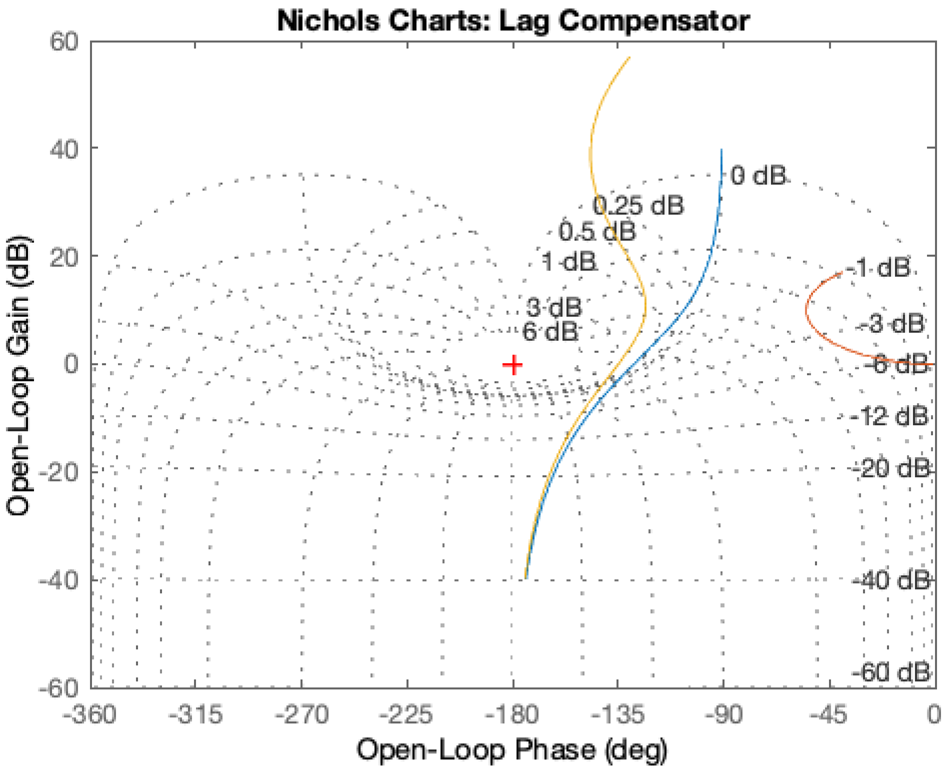
```
% Nyquist
% blue - Uncompensated Go(jw)
% green - Lag Compensator D(jw)
% red - Lag compensated DGo(jw)
nyquist(Go,D,DGo,w)
   title('Nyquist Diagrams: Lag Compensator'),...
   grid
```



Nichols Charts

In [27]:

```
% Nichols
% blue - Uncompensated Go(jw)
% green - Lag Compensator D(jw)
% red - Lag compensated DGo(jw)
nichols(Go,D,DGo,w)
   title('Nichols Charts: Lag Compensator'),...
   grid
```

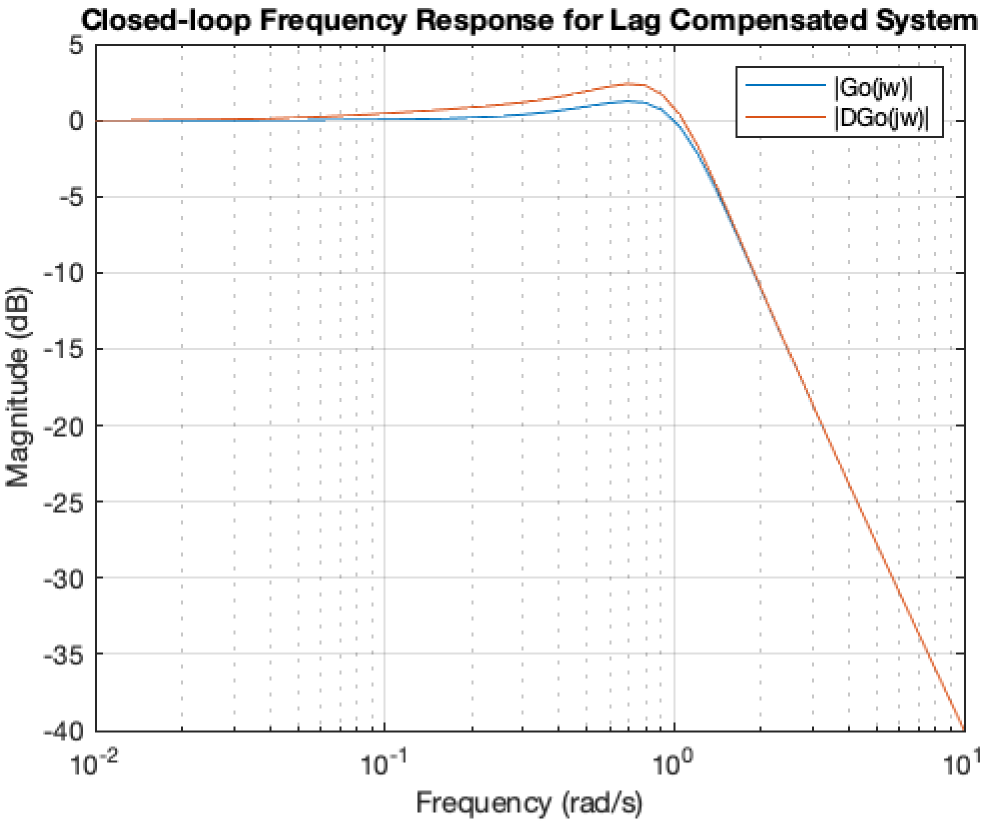


Closed-Loop Frequency Response

Now we examine the closed-loop frequency responses. Notice the slight increase in peak magnification $M_{\max}$.

In [28]:

```
Gc0 = feedback(Go,1);
Gc1 = feedback(DGo,1);
[mc0,pc0]=bode(Gc0,w);
[mc1,pc1]=bode(Gc1,w);
mc0 = 20.*log10(reshape(mc0,length(w),1));
mc1 = 20.*log10(reshape(mc1,length(w),1));
semilogx(w,mc0,w,mc1),...
   grid,...
   title('Closed-loop Frequency Response for Lag Compensated System'),...
   legend('|Go(jw)|','|DGo(jw)|'),...
   xlabel('Frequency (rad/s)'),...
   ylabel('Magnitude (dB)')
```

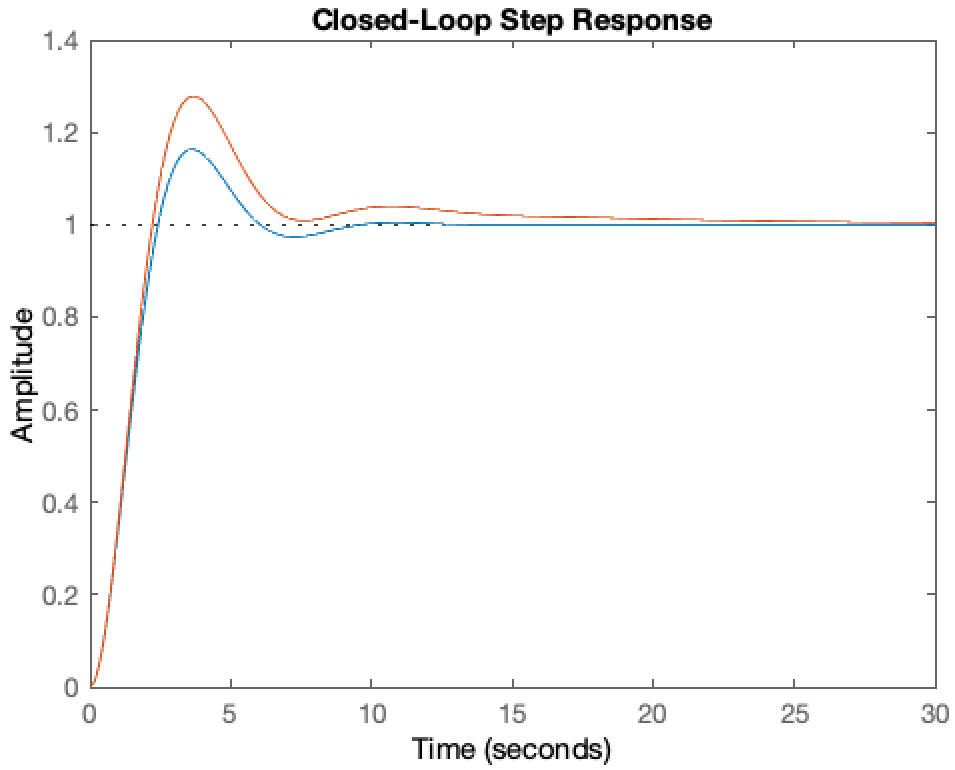


Closed-Loop Step Responses

First we examine the closed-loop step responses. Notice the increase in peak overshoot %OS and the slighly longer settling time.

In [30]:

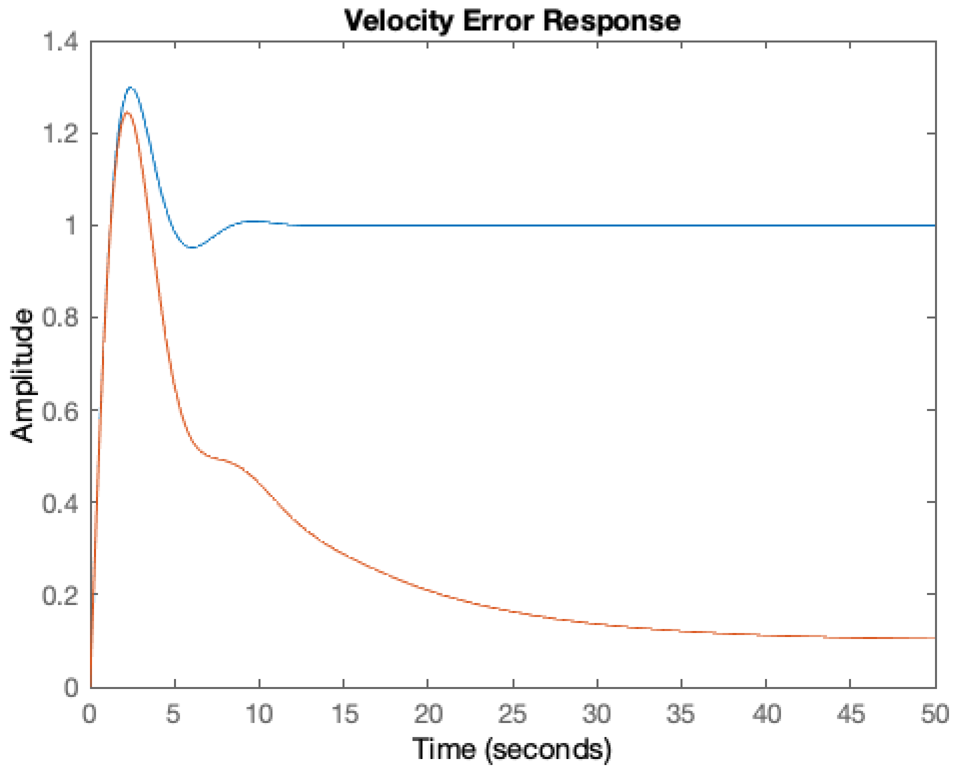
```
% Blue - uncompensated
% Green - Lag compensated
clf
step(Gc0,Gc1),title('Closed-Loop Step Response')
```



To compute the ramp error response we need to add an integrator to the error reponse:

In [31]:

```
integ=tf(1,[1 0]);
re0 = integ*(1/(1 + Go));
re1 = integ*(1/(1 +DGo));
clf
step(re0,re1), title('Velocity Error Response')
```



The velocity error is reduced to 10% from 100% showing that K_v is indeed 10. However, note the considerable time taken to reach the final value. This is quite typical of a ramp response response for a lag compensator.

Footnotes

[1] Or lead compensated plant in a lag-lead compensated system.

[2] The phase lag will then be close to zero at the gain cut-off frequency.

Resources

An executable version of this document is available to download as a MATLAB Live Script file freqlag.mlx (matlab/freqlag.mlx) [asymp.m (matlab/asymp.m)].