University College of Swansea
DEPARTMENT OF ELECTRICAL AND ELECTRONIC ENGINEERING

EE208: Part II Control Systems

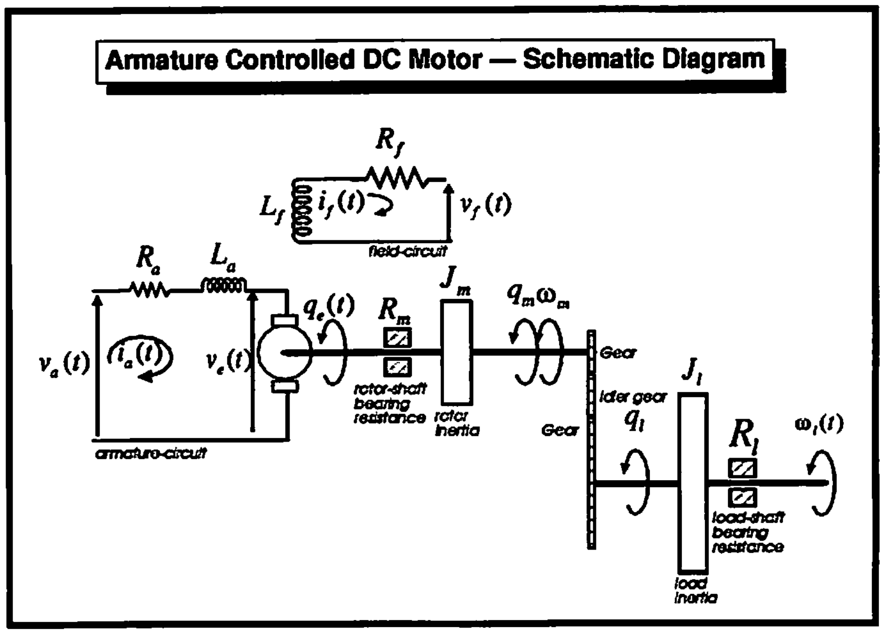
Mathematical Modelling — The Position Control System

Lecture 1

1 Modelling the Azimuth Position Control System

Consider the azimuth control system we have already seen. The plant for this is illustrated in Slide 1. It is an *armature-voltage controlled dc motor* which drives the load through a gearbox.

lide 1



There are some features of this model that will be unfamiliar to you, for example the gearbox. We shall build up to a full block diagram in stages starting from the mechanical side, introducing the gearbox and finally adding in the electrical side.

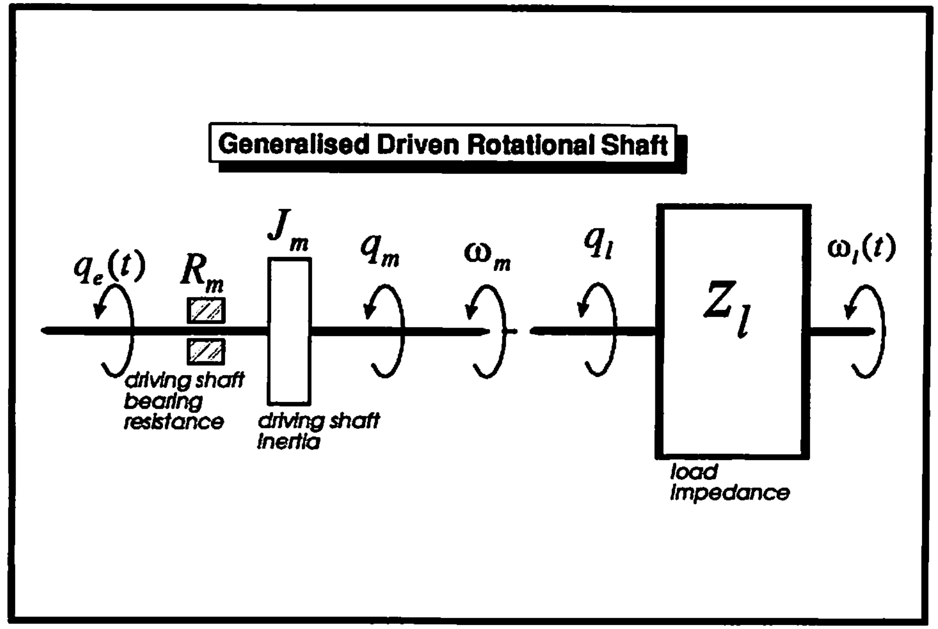
1.1 Mechanical Side

The rotor of the electric motor has inertia J_m kg m^2 and bearing resistance R_m N m/(rad/s). Assume that we can lump the load effects into a single rotational mechanical impedance

$$\frac{Q_l(s)}{\Omega_l(s)} = Z_l(s)$$

then, ignoring the gearbox for the moment, a schematic diagram of a driven shaft is that shown in Slide 2.

Slide 2



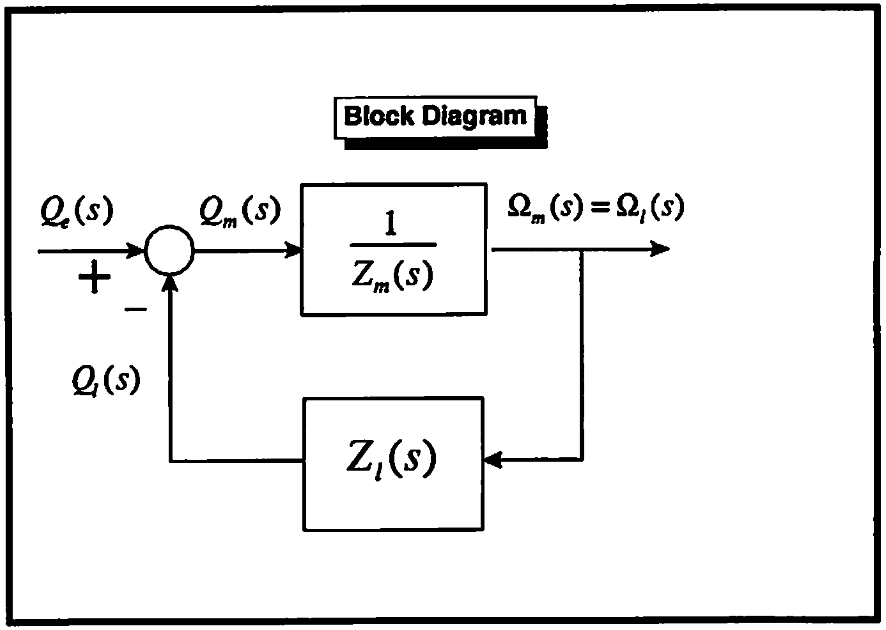
The impedence of the driving shaft is $Z_m(s) = sJ_m + R_m$ and $Q_e(s) = Q_m(s) + Q_l(s)$, $\Omega_m(s) = \Omega_l(s)$ hence a block diagram for the driven shaft is that shown in Slide 3, which has $Q_e(s)$ as input and $\Omega_l(s)$ as output. Reducing the block diagram gives

$$\Omega_l(s) = \frac{1}{Z_m(s) + Z_l(s)} Q_e(s)$$

or

$$Q_l(s) = \frac{Z_l(s)}{Z_m(s) + Z_l(s)} Q_e(s)$$

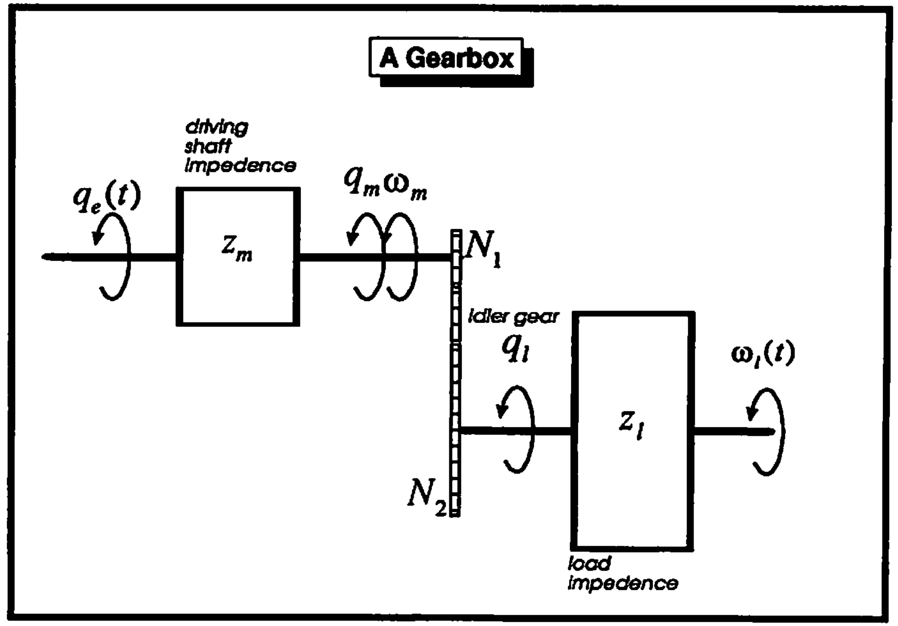
lide 3



1.2 Effect of the Gearbox

A gearbox is an example of a *transformer*: a passive element which couples two systems of the same kind by transmitting energy without loss from one system to the other. The element is a sink in one system and a source in the other. The roles of source and sink are interchangeable and depend on the direction of the net energy flow.

Slide 4



A side view of a typical gearbox is shown in Slide 4. In a simple gearbox. the driving shaft is connected to a gear-wheel that has N_1 teeth. This drives a second gear-wheel that is attached to the driven shaft. The second gear has N_2 teeth. One input rotation of the driving gear produces N_1/N_2 rotations of the driven shaft in the opposite direction. If the direction of rotation is to be in the same direction as the driving shaft, an *idler gear* is placed between the driving and driven gear-wheels (as in the illustration).

The ratio N_2/N_1 is called the gear-ratio r and is always > 1. Thus:

$$\frac{\theta_l}{\theta_m} = \frac{\omega_l}{\omega_m} = \frac{N_1}{N_2} = \frac{1}{r}. \qquad (1)$$

We assume that there are no losses in the gearbox[1] so that the energy into the gearbox equals the energy out (2):

$$\omega_m q_m = \omega_l q_l \tag{2}$$

and

$$\omega_m = r\omega_l \tag{3}$$

hence

$$\frac{q_l}{q_m} = \frac{\omega_m}{\omega_l} = r. \tag{4}$$

The purpose of a gearbox, a pure *rotational transformer*, is:

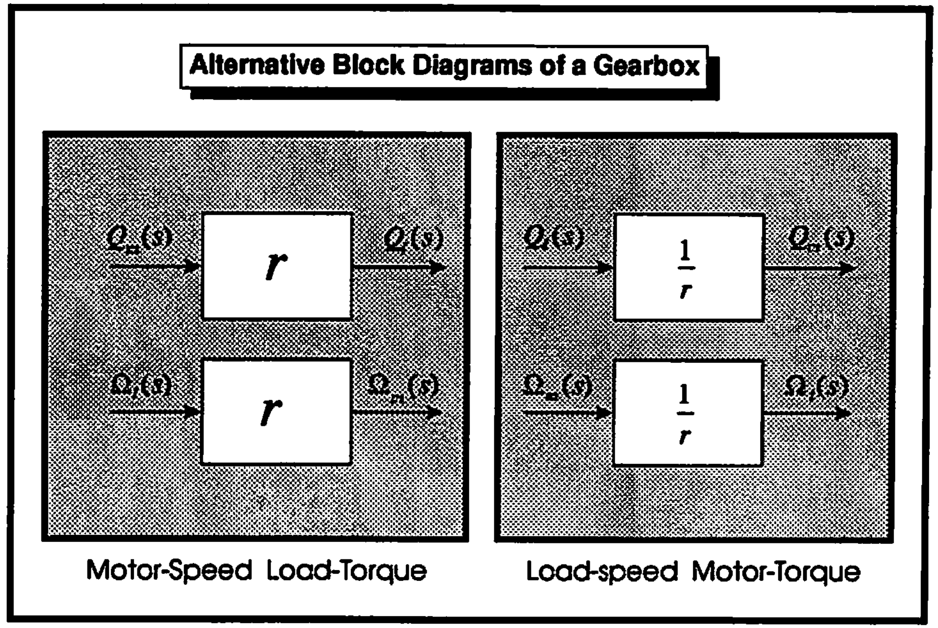
1. to change the speed of a power source, i.e. a motor, to meet the need for a different output speed, e.g. car wheels.
2. to change the torque of a power source to meet the need for a different output torque.

In most cases, the power source is a high-speed low-torque device and the load is a low-speed high-torque device. So in general $r > 1$ and often $r \gg 1$.

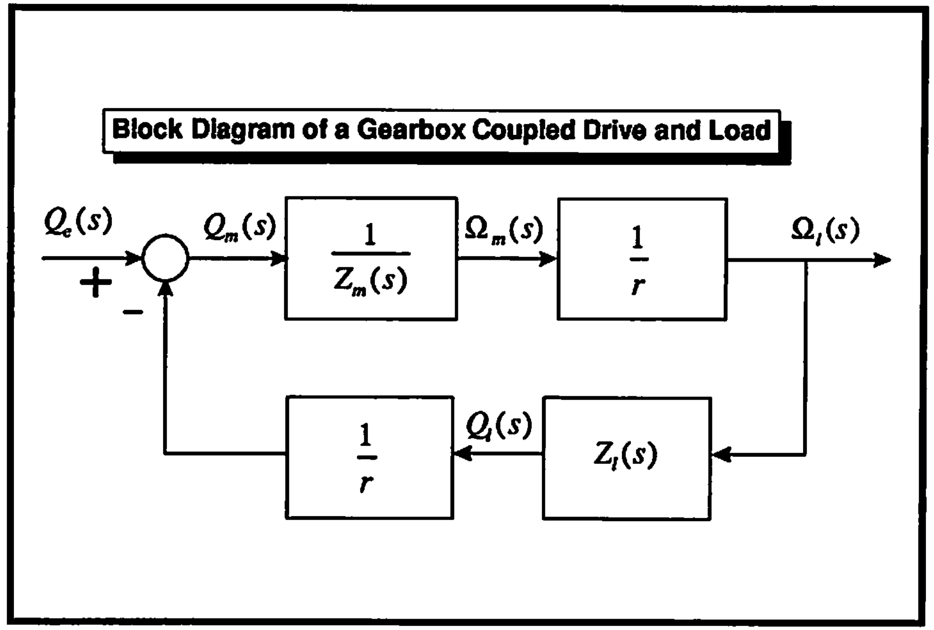
When developing a mathematical model for a gearbox, it is important to recognise that the energy equations (2) and (4) are fundamental. As a result of this equation there are a pair of constraints on the driven and driving speeds and torques that must be satisfied. In block diagram terms this means that the gearbox is either represented by the two blocks shown on the left of Slide 5 or, alternatively as the two blocks shown at the right of Slide 5 (the equations represented by these block diagrams are easily derived from (4)).

[1] there are actually several losses in practical gearboxes, including friction, backlash, gear-wheel inertia, etc.

Slide 5



Slide 6



To derive equations for the gearbox coupled mechanical system illustrated in Slide 4, we note that

$$
\begin{aligned}
Q_l(s) &= Z_l(s)\Omega_l(s) && (5)\\
\Omega_m(s) &= \frac{1}{Z_m(s)}Q_m(s) && (6)\\
\Omega_l(s) &= \frac{1}{r}\Omega_m(s) && (7)\\
Q_e(s) &= Q_m(s) + \frac{1}{r}Q_l(s) && (8)
\end{aligned}
$$

A block diagram representing these equations is shown in Slide 6.

From (6) and (7) we have

$$\Omega_l(s) = \frac{1}{rZ_m(s)}Q_m(s) \qquad (9)$$

and from (5) and 8):

$$Q_m(s) = Q_e(s) - \frac{Z_l(s)}{r}\Omega_l(s) \qquad (10)$$

Hence:

$$
\begin{aligned}
Q_e(s) &= rZ_m(s) + \frac{Z_l(s)}{r}\Omega_l(s) && (11)\\
\frac{Q_e(s)}{\Omega_m(s)} &= Z_m(s) + \frac{Z_l(s)}{r^2} && (12)
\end{aligned}
$$

Equation (12) is of particular interest since it tells us that the impedance of the load shaft as seen at the input shaft of the gearbox is reduced by r^2. Thus for a motor driving a load through a gearbox we have:

$$\frac{\Omega_m(s)}{Q_e(s)} = \frac{1}{Z_m(s) + Z_l(s)/r^2} \qquad (13)$$

which should be compared with the results derived in Section 1.1. The interpretation of this result is that a gearbox allows quite large transfers of energy with modest torques. For example, a gearbox with a gear ratio of 20 : 1 is able to move an inertia of 400 times larger than the directly connected driven-shaft intertia with the *same* amount of input effort.

1.2.1 An Ideally Matched Gearbox

Transformers are often used to match parameters in one system to those of another, normally to optimize some aspect of performance. A gearbox could be used to maximise the rotational acceleration $\frac{d\omega(t)}{dt}$ in one system achievable by a given torque in the other. Taking the model of driving shaft and driven load that is represented by the schematic in Slide 1, then, assuming that $Z_m(s) = sJ_m + R_m$ and $Z_l(s) = sJ_l + R_l$, from equation (13) we have:

$$
\begin{aligned}
\Omega_m(s) &= \frac{1}{(sJ_m + R_m) + (sJ_l + R_l)/r^2}Q_e(s) && \\
Q_e(s) &= (r(sJ_m + R_m) + (sJ_l + R_l)/r)\Omega_l(s) && (14)\\
&= (rJ_m + J_l/r)s\Omega_l(s) + (rR_m + R_l/r)\Omega(s) && (15)
\end{aligned}
$$

Thus to minimize the torque due to acceleration ($s\Omega_l(s)$), we must minimize $(rJ_m + J_l/r)$ with respect to r. That is:

$$
\begin{aligned}
\frac{d}{dr}(rJ_m + J_l/r) &= 0 && (16)\\
-\frac{1}{r^2}J_l + J_m &= 0 && (17)\\
r &= \sqrt{\frac{J_l}{J_m}} && (18)
\end{aligned}
$$

Such an ***ideally matched gearbox*** ensures that the inertia in one side matches the inertia in the other side because each contributes $\sqrt{J_lJ_m}$.

Lecture 2

1.3 Electrical Side

The equations for the electrical side have been introduced in the Dynamic Systems II Course.

Essentially we have the basic laws for a dc motor which are:

$$
\begin{aligned}
q_e(t) &= K_m i_a(t) i_f(t) && (19)\\
v_e(t) &= K_m i_f(t)\omega_m(t) && (20)
\end{aligned}
$$

where $i_f(t)$ is the field circuit current; $i_a(t)$ is the armature circuit current; $\omega_m(t)$ is the rotational speed of the rotor of the motor; $q_e(t)$ is the electrically generated torque applied to the rotor shaft by the interactions of the electrical fields produced by the field and armature coils; $v_e(t)$ is the back-emf generated across the brushes of the motor when the rotor rotates and which opposes the armature circuit voltage; and K_m is an electromagnetic coupling constant.

These equations are nonlinear. To make them linear, either the field current or the armature current is kept constant and the motor speed is then controlled by the current flowing in the other circuit. We thus have four basic configurations for the dc motor. If the armature circuit current is kept constant then the motor is said to be *field-controlled*. The basic equation of motion becomes:

$$q_e(t) = K_{mf} i_f(t) \tag{21}$$

where $K_{mf} = K_m i_a =$ constant is the field-circuit controlled electromagnetic ccoupling constant which has units N m/A. If the field current is used to control the motor the motor is said to be *field-current controlled* and (21) suffices. If the field voltage is used to control the motor we need an extra equation to take into account the field circuit impedance which is taken to be the field coil's inductance and resistance in series. The motor is then said to be *field-voltage controlled.*

If the field circuit current is kept constant then the motor is said to be *armature-controlled*. The basic equations of motion become:

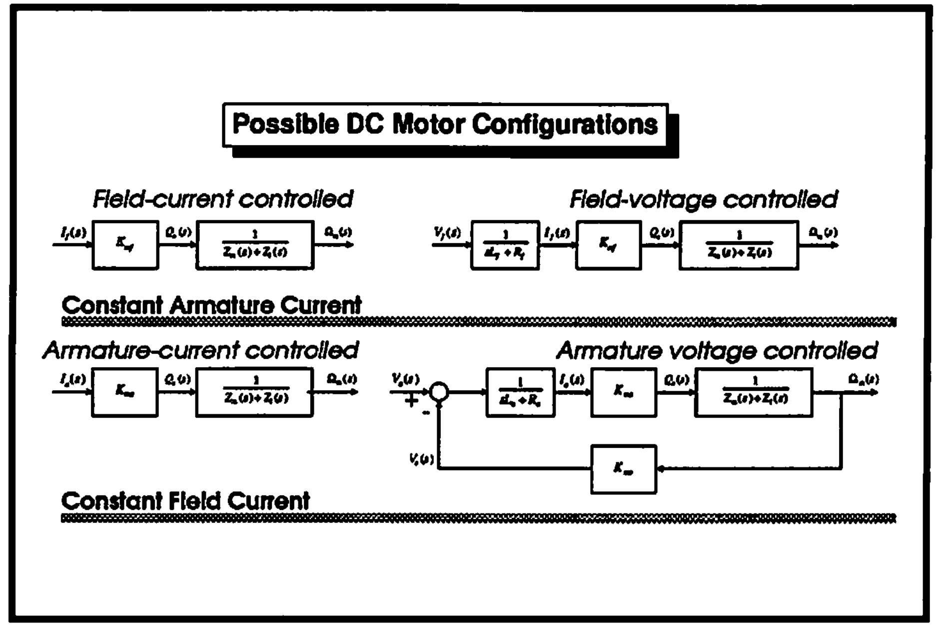
$$q_e(t) = K_{ma} i_a(t) \tag{22}$$
$$v_e(t) = K_{ma} \omega_m(t) \tag{23}$$

where $K_{ma} = K_m i_f =$ constant is the armature-circuit controlled electromagnetic coupling constant which has units N m/A. If the armature current is used to control the motor the motor is said to be *armature-current controlled* and (22) suffices. If the armature voltage is used to control the motor we need extra equations to take into account the armature circuit impedance and the back e.m.f. (23). The motor is then said to be *armature-voltage controlled.*

Block diagrams for the possible dc motor configurations are easy to derive and they are all illustrated in Slide 7. Note that we have used the driven-load equations derived in Section 1.1 to model the mechanical side of the motor. You should be comfortable with deriving models for all these configurations of motor, and to that end, Exercises 1–1 to 1–4 are provided to give you some practice.

As an aside, the same basic equations are used to derive models for electrical generators. In that case, the input is the rotor speed $\omega_m(t)$ and the output is the back e.m.f $v_e(t)$. The model is linearised by either keeping the rotor speed or the field current constant.

Slide 7



Exercises

- **1–1** A field voltage controlled motor drives a load with resistance R N m/(rad/s) and inertia J kg m^2. Determine the transfer function relating the load speed to the field voltage.

- **1–2** An armature current controlled motor drives a load with negligible resistance and inertia J kg m^2 through a long shaft with compliance C rad/N m. Determine the transfer function relating the load speed to the armature current.

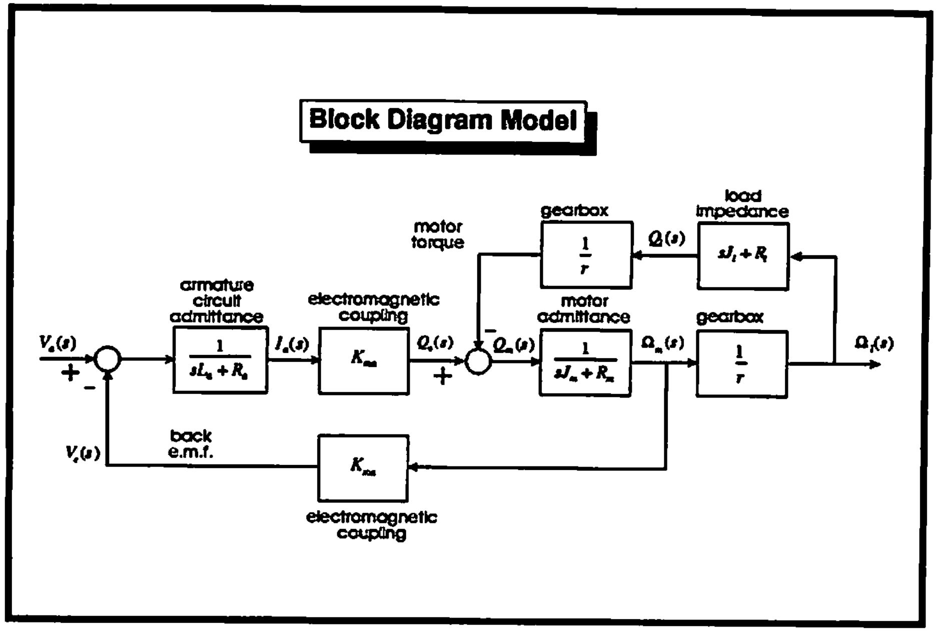
- **1–3** An armature voltage controlled motor drives a load with resistance R N m/(rad/s) and inertia J kg m^2. Obtain an electrical network for which the input impedance is the same as the input impedance of the armature circuit.

- **1–4** In a field voltage controlled motor with field resistance 1 Ω, field inductance 5 H, rotor resistance 0.5 N m/(rad/s) and rotor inertia 2 kg m^2, the electromechanical coupling constant relating torque to field current is 10 N m/A. If the motor drives the load with resistance 0.5 N m/(rad/s) and inertia 8 kg m^2, determine the output speed following a step input of 20 V applied to the field circuit when the motor is at rest.

We are now ready to put together these results to construct a model for the dc motor used to control the azimuth position of the antenna.

Slide 8

1.3.1 Armature-Voltage Controlled DC Motor



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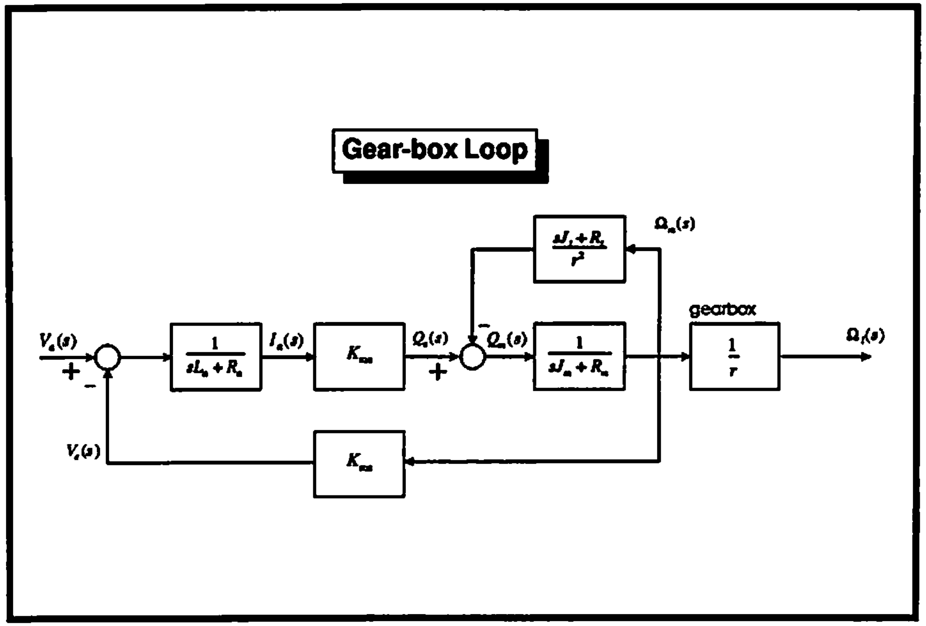


For the azimuth position control system we shall use the armature-voltage controlled dc motor shown in schematic form in Slide 1.

By putting together all we know so far, the block diagram for this kind of motor is shown in Slide 8.

The "gearbox" loop can be reduced to that shown in Slide 9 from which it is clear that the motor speed is related to the electrically generated torque by the transfer function

$$\begin{aligned}\frac{\Omega_m(s)}{Q_e(s)} &= \frac{\frac{1}{sJ_m+R_m}}{1+\frac{sJ_l+R_l}{r^2}\frac{1}{sJ_m+R_m}} \\ &= \frac{1}{s(J_m+J_l/r^2)+(R_m+R_l/r^2)} \qquad (24)\end{aligned}$$

A shorthand for (24) is

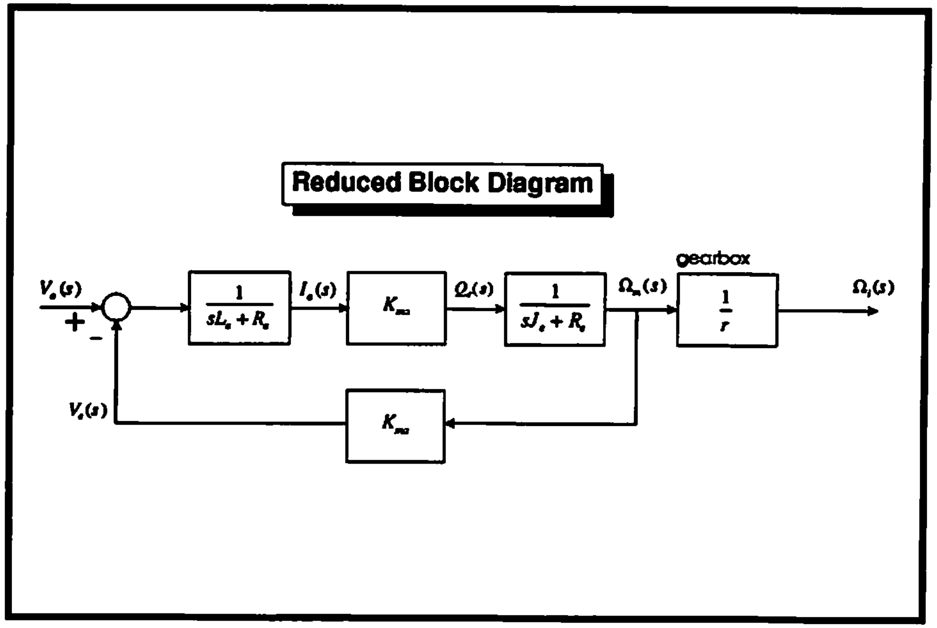
$$\frac{\Omega_m(s)}{Q_e(s)} = \frac{1}{sJ_e+R_e} \qquad (25)$$

where $R_e = R_m + R_l/r^2$ and $J_e = J_m + J_l/r^2$ are the effective resistance and inertia as seen at the motor shaft. $(sJ_l + R_l)/r^2$ is called the *reflected* load impedance.

The block diagram has now been reduced to that shown in Slide 10, from which it is easy to show that

$$\frac{\Omega_m(s)}{V_a(s)} = \frac{K_{ma}}{(R_a+sL_a)(sJ_e+R_e)+K_{ma}^2} \qquad (26)$$

Slide 10



Now $\Omega_m(s) = s\Theta_m(s)$ where $\Theta_m(s)$ is the transformed motor shaft position $\theta_m(t)$. If we neglect the armature inductance L_a then:

$$\begin{aligned}V_a(s) &= \left(\frac{R_a(sJ_e+R_e)+K_{ma}^2}{K_{ma}}\right)s\Theta_m \\ &= \left[\frac{R_a}{K_{ma}}(sJ_e+R_e)+K_{ma}\right]s\Theta_m \qquad (27)\end{aligned}$$

Hence

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{\frac{K_{ma}}{R_aJ_e}}{s\left(s+\frac{1}{J_e}\left(R_e+\frac{K_{ma}^2}{R_a}\right)\right)}. \qquad (28)$$

Equation (28) is a relatively simple transfer function of the form:

$$T(s) = \frac{K}{s(s+\alpha)} \tag{29}$$

of which, more later.

Lecture 3

How do we determine the coupling constants and hence find a suitable motor for a given load? Recall that for the motor

$$\begin{aligned} V_e(s) &= K_{ma}\Omega_m(s) && (30)\\ &= K_{ma}s\Theta(s) && (31)\end{aligned}$$

and

$$Q_e(s) = K_{ma}I_a(s). \tag{32}$$

Hence, for the armature circuit

$$\begin{aligned} I_a &= \frac{V_a - V_e}{sL_a + R_a}\\ (R_a + sL_a)I_a &= V_a - V_e\\ &= V_a - K_{ma}s\Theta_m \end{aligned}$$

$$V_a = (R_a + sL_a)\frac{Q_e}{K_{ma}} + K_{ma}s\Theta_m. \tag{33}$$

Assuming that the armature winding's inductance, L_a, is negligible then

$$V_a = \frac{R_a}{K_{ma}}Q_e + K_{ma}s\Theta_m. \tag{34}$$

In the time domain:

$$v_a(t) = \frac{R_a}{K_{ma}}q_e(t) + K_{ma}\omega_m(t). \tag{35}$$

If a constant dc voltage v_a is applied to a given motor, the motor will run at a constant speed ω_m with a constant torque q_e, hence in the steady-state

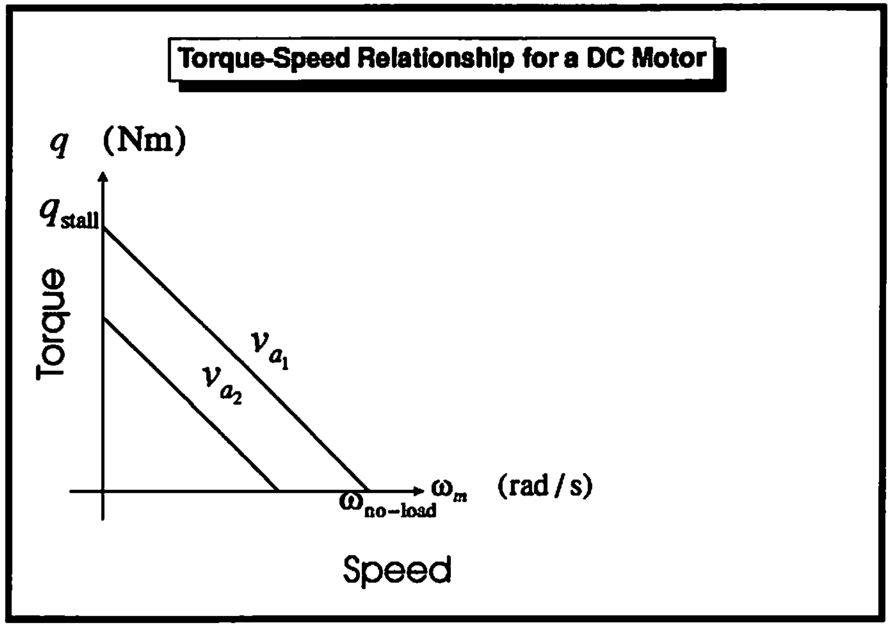
$$v_a = \frac{R_a}{K_{ma}}q_e + K_{ma}\omega_m. \tag{36}$$

Solving for q_e:

$$q_e = -\frac{K_{ma}^2}{R_a}\omega_m + \frac{K_{ma}}{R_a}v_a. \tag{37}$$

This is a straight-line relationship q_e versus ω_m as shown in Slide 11, and we use a *dynameter* to measure this torque-speed characteristic for a given v_a using a set-up as shown in Slide 12.

Slide 11



To find $\Theta_l(s)/V_a(s)$ we note that $\omega_l = \omega_m/r$ hence $\theta_l = \theta_m/r$ so

$$\frac{\Theta_l(s)}{V_a(s)} = \frac{0.0417}{s(s+1.1667)}.$$

A More Difficult Problem

Exercises

This problem is similar to the example except that the gearbox has non-negligible inertia.

- **1–5** A field voltage controlled dc. motor whose torque-speed characteristics are shown in Figure 2 drives a load with inertia 16 kg m^2 and bearing resistance 32 N m/(rad/s) through the gearbox, illustrated in Figure 3, in which some of the gears have non-negligible inertia. Find the transfer function relating the load speed to the armature voltage. (Adapted from Nise [?], Problem 2.39, page 106).

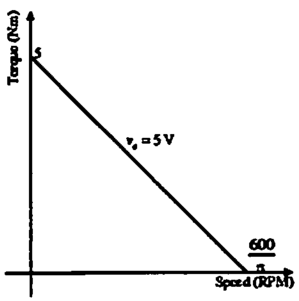


Figure 2: Torque-Speed Characteristics

Hint: reflect all the inertias and resistances of the gear-wheels and the load shaft to the drive shaft using the rule "equivalent impedance = impedance/r^2".

Summary

We have now derived a mathematical model, in block diagram form, of the dc motor and load which forms the *actuator* and *plant* of the azimuth position control system. We are still some way from a mathematical model of the complete closed-loop control system. To create this we need to add sensors for actual and demanded position (and perhaps velocity), signal and power amplifiers. This will be the topic of the next lecture at the end of which we shall be able to describe, in block diagram form, suitable control systems for both position and speed control.

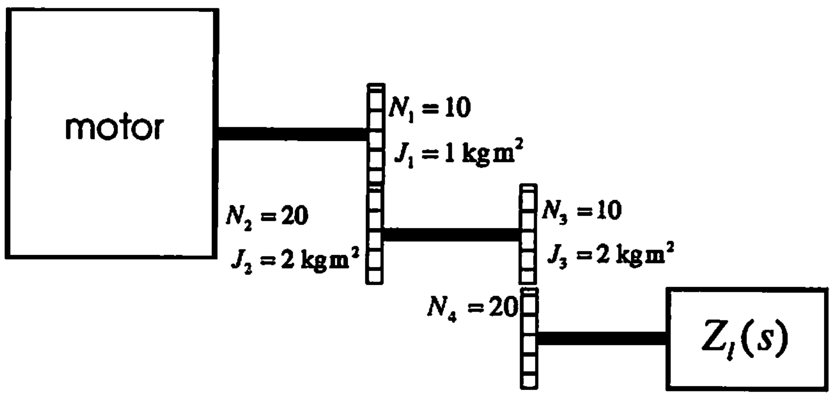


Figure 3: Gear-box

1.4 The Rest of the Control System

The motor fits into the azimuth position control system as shown in Fig. 4. Note that the potentiometers, amplifiers, gearbox, inertias and bearing resistances have been given physical values.

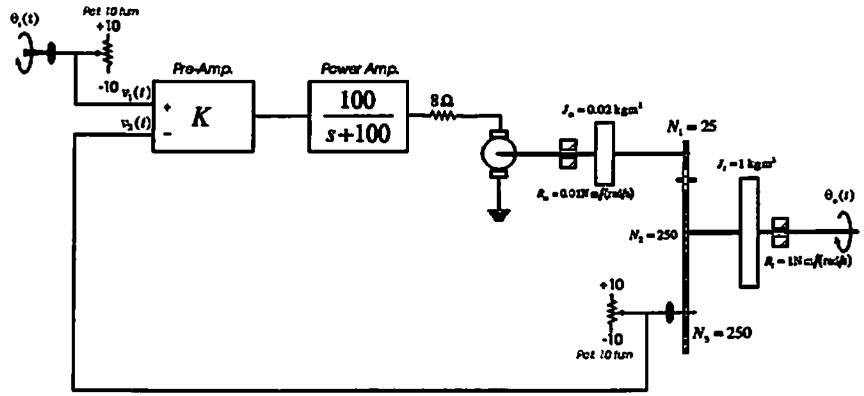


Figure 4: Position Control System Schematic

The various new components are described below and then a complete mathematical model of the system, in block diagram form, will be developed.

1.4.1 Position sensors

A potentiometer (Fig. 5) produces $v_{out} \propto \theta_{in}$. There are no dynamics.

$$\frac{v_{out}}{\theta_{in}} = \frac{v_{max} - v_{min}}{2\pi n}.$$

In this case 10 turns produces 20 V hence

$$\frac{v_{out}}{\theta_{in}} = \frac{20}{20\pi} = \frac{1}{\pi}\ V/rad$$

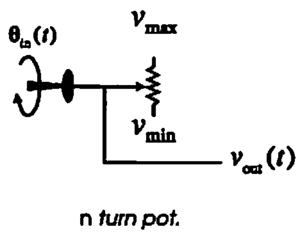


Figure 5: Potentiometer

1.4.2 Velocity sensors

A tacho-generator (Fig. 6) is used to sense the speed of a motor. $v_{out} \propto \omega_{in}$. Provided that the load-circuit impedance is high and the tacho-generator is physically small with respect to the driven shaft then the device can be assumed to have no dynamics.

$$\frac{v_{out}}{\omega_{in}} = K_T \; V/(rad/s).$$

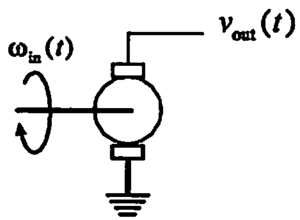


Figure 6: Tacho-generator

1.4.3 Pre-amplifier

The pre-amplifier is assumed to be a small current differencing op-amp circuit as shown in Fig. 7.

$$\begin{aligned} v_p(t) &= \frac{R_f}{R_i}\{v_i(t) - v_o(t)\} \\ &= K\{v_i(t) - v_o(t)\} \end{aligned}$$

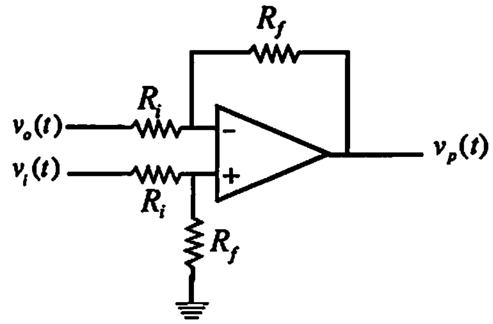


Figure 7: Differencing pre-amplifier

1.4.4 Power Amplifier

This produces a high-current voltage output from a low-current voltage input. Such an amplifier will often have dynamics which cannot be neglected. In this case we assume

$$\frac{V_a(s)}{V_p(s)} = \frac{100}{s+100}$$

indicating that the amplifier has unity DC gain and a time constant of 1/100 seconds. This means that the amplifier would reach 63% of its final output voltage in 0.01 seconds following a step change in the input voltage (see Fig. 8).

1.4.5 Block Diagram of Plant

For the system shown in Fig 4 we may take $K_{ma} = 0.5$ N m/A. The motor-load gearbox ratio is

$$r = \frac{N_2}{N_1} = \frac{250}{25} = 10.$$

The equivalent motor-load inertia as reflected back to the motor shaft is

$$J_e = J_m + \frac{J_l}{r^2} = 0.02 + 1 \times \frac{1}{100} = 0.03.$$

The equivalent bearing resistance is:

$$R_e = R_m + \frac{R_l}{r^2} = 0.01 + 1 \times \frac{1}{100} = 0.02.$$

Hence from the transfer function derived earlier (Equation 28)

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{K_{ma}/(R_a J_e)}{s\left(s + \frac{1}{J_e}\left(R_e + \frac{K_{ma}^2}{R_a}\right)\right)} \qquad (42)$$

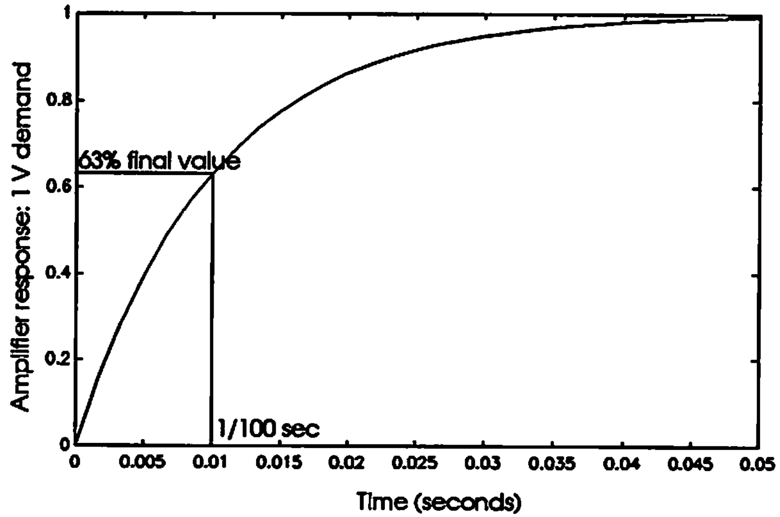
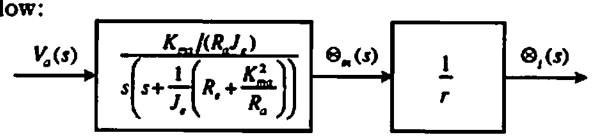


Figure 8: Response of the power amplifier

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{0.5/(8\times 0.03)}{s\left(s+\frac{1}{0.03}\left(0.02+\frac{0.5^2}{8}\right)\right)}$$
$$= \frac{2.083}{s(s+1.71)}$$

Thus the block diagram relating armature voltage to the position of the load is that shown below:



The transfer function is

$$\frac{\Theta_o(s)}{V_a(s)} = \frac{1}{r}\frac{K_{ma}/(R_aJ_e)}{s\left(s+\frac{1}{J_e}\left(R_e+\frac{K_{ma}^2}{R_a}\right)\right)} \quad (43)$$
$$= \frac{0.2083}{s(s+1.71)}.$$

The load-pot gearbox has unity gear-ratio so that the pot moves at the same speed as the load. So, putting everything together we end up with the block diagram shown in Fig. 9.

It is convenient to reduce the block diagram to the ***unity-gain feedback canonical form*** shown in Fig. 10.

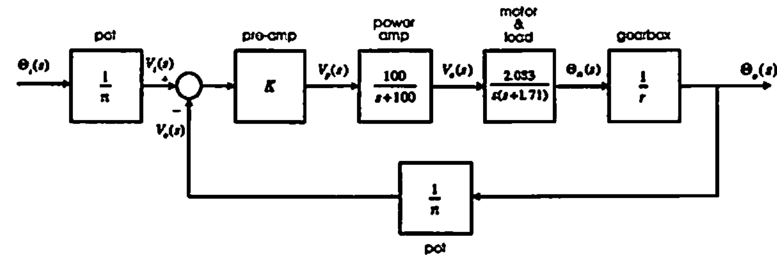


Figure 9: Complete position control system

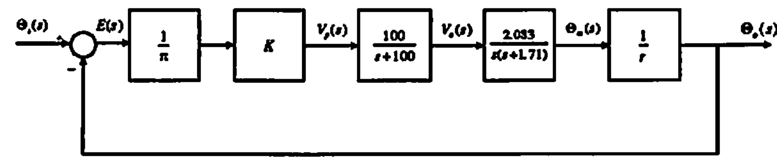


Figure 10: Reduced block diagram

The *open-loop* transfer function is then:

$$G_o(s) = \frac{\Theta_o(s)}{E(s)} = \frac{66.3K}{s(s+100)(s+1.71)},$$

and the *closed-loop* transfer function is

$$G_c(s) = \frac{\Theta_o(s)}{\Theta_i(s)} = \frac{66.3K}{s^3+101.7s^2+171s+66.3K}.$$

Note that in the steady-state, i.e. when all terms in s are removed

$$\theta_i = \theta_o.$$

Another name for this type of control system is a "*servomechanism*".

A "velodyne" is a load-speed control system. A velodyne can be made from the components seen so far if a tacho-generator with gain K_T v/(rad/s) is used in the feedback loop and a potentiometer with gain K_T is used as a demanded speed sensor. Such a set up is shown in Fig. 11.

On manipulating this block diagram we get the unity-gain feedback control system shown in Fig. 12.

The closed-loop transfer function for the velodyne is

$$G_c(s) = \frac{\Omega_o(s)}{\Omega_i(s)} = \frac{66.3KK_T}{s^2+101.71s+(171+66.3KK_T)}$$

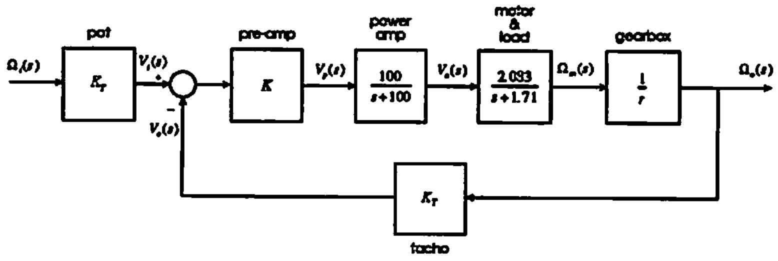


Figure 11: A velocity control system (Velodyne)

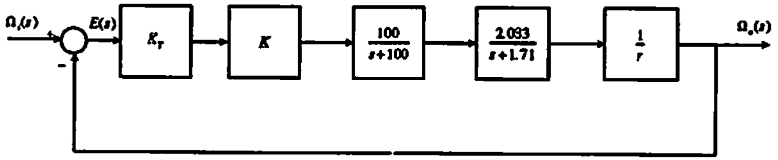


Figure 12: Reduced block diagram

In the steady-state

$$\frac{\omega_o}{\omega_i} = \frac{66.3KK_T}{171 + 66.3KK_T}$$

that is $\omega_o \neq \omega_i$ but $\omega_o \approx \omega_i$ if $66.3KK_T \gg 171$.

Servomechanisms

Exercises

• **1–6** In a control system for rotating a radar aerial assembly, an electric motor with inertia 0.05 g m^2 and resistance 0.02 N m/(rad/s) is required to drive the inertia with inertia 500 kg m^2 and resistance 50 N m/(rad/s) through a gearbox. Determine a suitable gear ratio, and the transfer function relating aerial speed to motor torque if such a gearbox is used. What is the motor power required to rotate the aerial at 10 rev/min?

• **1–7** In a servomechanism using a field voltage controlled motor, the ratio of the motor torque to the error between the demanded and actual load position is 100 N m/rad. If the inertia and resistances refererred to the motor shaft are 20 kg m^2 and 3 N m/(rad/s), and the motor drives the load through a gearbox with ratio 50:1, determine the overall transfer function of the system.

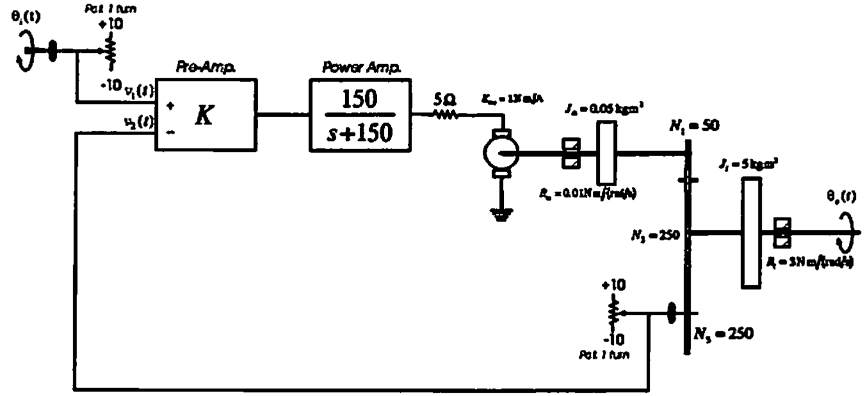
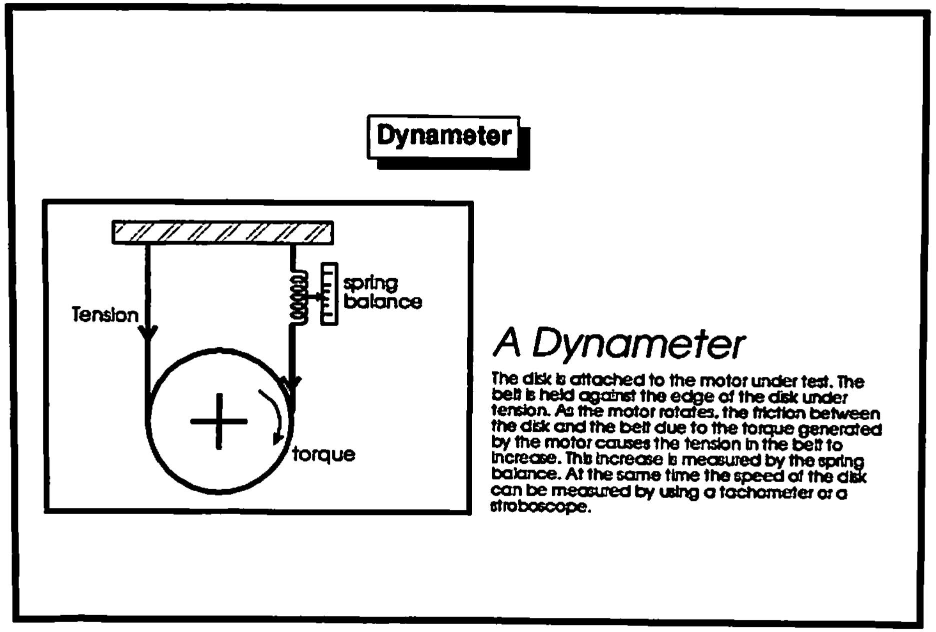


Figure 13:

• **1–8** A position control system is illustrated in Figure 13. Evaluate the transfer function of each subsystem and determine the closed-loop transfer function $\Theta_o(s)/\Theta_i(s)$.

(Adapted from *Chapter 2 Objective Problem*, Nise [?], Exercise 48, page 109).



When the speed ω_m is zero, the curve intercepts the torque axis at a value that is called the *stall-torque* q_{stall}.

$$q_{stall} = \frac{K_{ma}}{R_a} v_a. \tag{38}$$

When the torque is zero we have a speed called $\omega_{no-load}$

$$\omega_{no-load} = \frac{v_a}{K_{ma}} \tag{39}$$

Hence the electrical constants are:

$$\frac{K_{ma}}{R_a} = \frac{q_{stall}}{v_a} \tag{40}$$

$$K_{ma} = \frac{v_a}{\omega_{no-load}} \tag{41}$$

Example 1.1 *For the motor with the torque-speed characteristic shown in Figure 1 find the transfer function $\Theta_l(s)/V_a(s)$ for an armature-voltage controlled dc motor which drives a load with inertia 700 kg m^2 and bearing resistance 800 N m/(rad/s) through a gearbox with gear ratio $r = 10$. The rotor inertia of the motor is 5 kg m^2 and bearing resistance is 2 N m/(rad/s).*

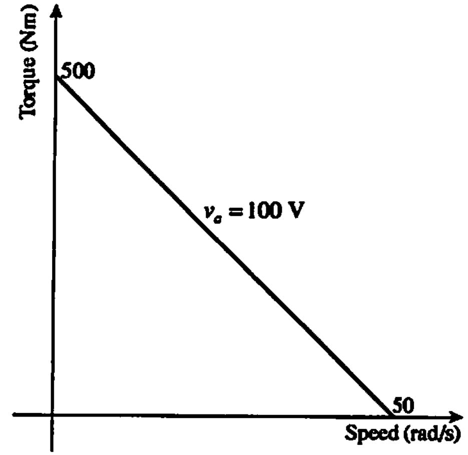


Figure 1:

Solution:

$$J_e = J_m + J_l/r^2 = 5 + \frac{700}{10^2} = 12$$

$$R_e = R_m + R_l/r^2 = 2 + \frac{800}{10^2} = 10$$

$q_{stall} = 500$ N m, $\omega_{no-load} = 50$ rad/s, $v_a = 100$ V. Hence

$$\frac{K_{ma}}{R_a} = \frac{q_{stall}}{v_a} = \frac{500}{100} = 5.$$

$$K_{ma} = \frac{v_a}{\omega_{no-load}} = \frac{100}{50} = 2.$$

Given that

$$\frac{\Theta_m(s)}{V_a(s)} = \frac{\frac{K_{ma}}{R_a} \cdot \frac{1}{J_e}}{s\left(s + \frac{1}{J_e}\left(R_e + K_{ma}\frac{K_{ma}}{R_a}\right)\right)}$$

then

$$\begin{aligned} \frac{\Theta_m(s)}{V_a(s)} &= \frac{5.\frac{1}{2}}{s\left(s + \frac{1}{12}(10 + 2 \times 5)\right)} \\ &= \frac{0.417}{s(s + 1.1667)}. \end{aligned}$$