General questions from "Parameter and state estimations with a time-dependent adjoint marine ice sheet model"

1. Table 2 and Table 3 appear as identical. Is this on purpose?

Constructing a Forward Model/Adjoint Method Workflow

Forward Model

Begin With:
  u_0: From observations
  β: Homogeneous, or calculated using Shallow Ice Approximation

These determine:
  $\overline{\nu}^0 = F_v(u^0, \beta^0)$
  $\beta^0_{Eff} = F_B(u^0, \overline{\nu}^0, \beta^0)$

Loop through Picard Iterations: 1,2,...,n
  Construct: $A_n = F_A(\beta^n_{Eff}, \overline{\nu}^n)$
  Solve: $u^{n+1} = A_n^{-1} * \tau$
  Update: $v^{n+1} = F_v(u^{n+1}, \overline{\nu}^n, \beta^n)$
  $B^{n+1}_{Eff} = F_{B_{Eff}}(\overline{\nu}^{n+1}, \beta^n)$
  Check Convergence
End

Adjoint Model

Adjoint of Cost Function $\delta^* u^{n+1} = (\frac{\partial F_J}{\partial u})^T$

Loop through Picard Iterations in Reverse: n,n-1,...,1
  Reverse_Update:
  [Pass on initial iteration since $\delta^* u^{n+1}$ comes from the adjoint of the cost function]
  $\delta^* u^{n+1} = (\frac{\partial F_{\overline{\nu}}}{\partial u})^T \delta^* \overline{\nu}^{n+1}$
  $\delta^* \beta^n = (\frac{\partial F_\beta}{\partial \beta^n})^T \delta^* \beta^{n+1}_{Eff}$

  Reverse_Solve: $A_n^T b = \delta^* u^{n+1}$ [For b]
  $\delta^* A = -b(u^{n+1})^T$

  Reverse_Construct: $\delta^* \beta_{Eff} = (\frac{\partial F_A}{\partial \beta_{Eff}})^T \delta^* A$
  $\delta^* \overline{\nu} = (\frac{\partial F_A}{\partial \overline{\nu}})^T \delta^* A$
End

Pass $\delta^* \beta_{Eff}$ to Optimization Algorithm

Important Functions

Cost Function:
$J = F_J(u) = \int\int \frac{1}{2}(u_{obs} - u_s)^2 dxdy$ where $u_s = \frac{u(1+\beta F_1)}{1+\beta F_2}$

Integrated Viscosity:
$\overline{\nu} = F_{\overline{\nu}}(u, \overline{\nu}\ [optionally], \beta) = \frac{1}{h} \int \frac{1}{2}\beta((\partial_x \overline{u})^2 + (\partial_y \overline{v})^2 + (\partial_x \overline{u})(\partial_y \overline{v}) + \frac{1}{4}(\partial_y \overline{u} + \partial_x \overline{v})^{\frac{1}{2}} + \frac{1}{4}(\partial_z u)^2 + \frac{1}{4}(\partial_z v)^2 + \epsilon^2)^{\frac{1-n}{2n}}$ dz

where: $\partial_z u = \frac{\sigma_{xz}}{\overline{\nu}} = \frac{\tau_{bx}(s-z)}{\overline{\nu}h} = \frac{\beta_{Eff}\overline{u}(s-z)}{\overline{\nu}h}$ and similarly for $\partial_z v$

Effective Basal Slipperiness:
$\beta_{Eff} = F_{B_{Eff}}(\beta, \overline{\nu}) = \frac{\beta}{1+\beta F_2}$ where $F_2 = \int_b^s \frac{1}{\overline{\nu}}(\frac{s-z}{h})^2 dz$

Basal Slipperiness:
$\beta = F_B(\beta_{Eff}, \overline{\nu}) = \frac{\beta_{Eff}}{1-\beta_{Eff}F_2}$

Automatic Differentiation

Required on the following: $F_J, F_{\overline{\nu}}, F_A, F_B, F_{B_{Eff}}$