0.1 ls.mixed: Mixed effects Linear Regression

Use multi-level linear regression if you have covariates that are grouped according to one or more classification factors and a continuous dependent variable.

While generally called multi-level models in the social sciences, this class of models is often referred to as mixed-effects models in the statistics literature and as hierarchical models in a Bayesian setting. This general class of models consists of linear models that are expressed as a function of both *fixed effects*, parameters corresponding to an entire population or certain repeatable levels of experimental factors, and *random effects*, parameters corresponding to individual experimental units drawn at random from a population.

Syntax

Inputs

zelig() takes the following arguments for multi:

• formula: a two-sided linear formula object describing the systematic component of the model, with the response on the left of a operator and the fixed effects terms, separated by + operators, on the right. Any random effects terms are included with the notation tag(z1 + ... + zn | g) with z1 + ... + zn specifying the model for the random effects and g the grouping structure. Random intercept terms are included with the notation tag(1 | g).

Alternatively, formula may be a list where the first entry, mu, is a two-sided linear formula object describing the systematic component of the model, with the repsonse on the left of a operator and the fixed effects terms, separated by + operators, on the right. Any random effects terms are included with the notation tag(z1, gamma | g) with z1 specifying the individual level model for the random effects, g the grouping structure and gamma references the second equation in the list. The gamma equation is one-sided linear formula object with the group level model for the random effects on the right side of a operator. The model is specified with the notation tag(w1 + ... + wn | g) with w1 + ... + wn specifying the group level model and g the grouping structure.

Additional Inputs

In addition, zelig() accepts the following additional arguments for model specification:

- data: An optional data frame containing the variables named in formula. By default, the variables are taken from the environment from which zelig() is called.
- family: A GLM family, see glm and family in the stats package. If family is missing then a linear mixed model is fit; otherwise a generalized linear mixed model is fit. In the later case only gaussian family with "log" link is supported at the moment.
- na.action: A function that indicates what should happen when the data contain NAs. The default action (na.fail) causes zelig() to print an error message and terminate if there are any incomplete observations.

Additionally, users may wish to refer to lmer in the package lme4 for more information, including control parameters for the estimation algorithm and their defaults.

Examples

1. Basic Example with First Differences

Attach sample data:

```
> data(voteincome)
```

Estimate model:

```
> z.out1 <- zelig(income ~ education + age + female + tag(1 | state),
+ data = voteincome, model = "ls.mixed")</pre>
```

Summarize regression coefficients and estimated variance of random effects:

```
> summary(z.out1)
```

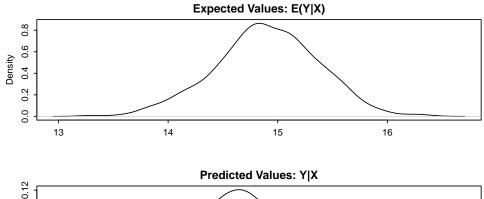
Set explanatory variables to their default values, with high (80th percentile) and low (20th percentile) values for education:

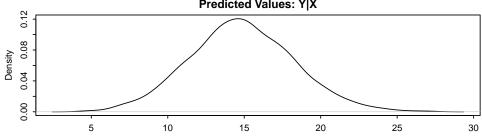
```
> x.high <- setx(z.out1, education = quantile(voteincome$education,
+ 0.8))
> x.low <- setx(z.out1, education = quantile(voteincome$education,
+ 0.2))</pre>
```

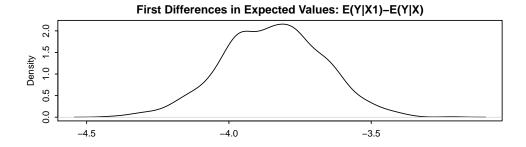
Generate first differences for the effect of high versus low education on income:

```
> s.out1 <- sim(z.out1, x = x.high, x1 = x.low)
> summary(s.out1)
```

> plot(s.out1)







Mixed effects linear regression model

Let Y_{ij} be the continuous dependent variable, realized for observation j in group i as y_{ij} , for i = 1, ..., M, $j = 1, ..., n_i$.

• The stochastic component is described by a univariate normal model with a vector of means μ_{ij} and scalar variance σ^2 .

$$Y_{ij} \sim \text{Normal}(y_{ij}|\mu_{ij}, \sigma^2)$$

• The q-dimensional vector of random effects, b_i , is restricted to be mean zero, and therefore is completely characterized by the variance covarance matrix Ψ , a $(q \times q)$ symmetric positive semi-definite matrix.

$$b_i \sim Normal(0, \Psi)$$

• The *systematic component* is

$$\mu_{ij} \equiv X_{ij}\beta + Z_{ij}b_i$$

where X_{ij} is the $(n_i \times p \times M)$ array of known fixed effects explanatory variables, β is the p-dimensional vector of fixed effects coefficients, Z_{ij} is the $(n_i \times q \times M)$ array of known random effects explanatory variables and b_i is the q-dimensional vector of random effects.

Quantities of Interest

• The predicted values (qi\$pr) are draws from the normal distribution defined by mean μ_{ij} and variance σ^2 ,

$$\mu_{ij} = X_{ij}\beta + Z_{ij}b_i$$

given X_{ij} and Z_{ij} and simulations of β and b_i from their posterior distributions. The estimated variance covariance matrices are taken as correct and are themselves not simulated.

• The expected values (qi\$ev) are averaged over the stochastic components and are given by

$$E(Y_{ij}|X_{ij}) = X_{ij}\beta.$$

• The first difference (qi\$fd) is given by the difference in expected values, conditional on X_{ij} and X'_{ij} , representing different values of the explanatory variables.

$$FD(Y_{ij}|X_{ij}, X'_{ij}) = E(Y_{ij}|X_{ij}) - E(Y_{ij}|X'_{ij})$$

• In conditional prediction models, the average predicted treatment effect (qi\$att.pr) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^{M} \sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^{M} \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij}=1) - \widehat{Y_{ij}(t_{ij}=0)}\},$$

where t_{ij} is a binary explanatory variable defining the treatment $(t_{ij} = 1)$ and control $(t_{ij} = 0)$ groups. Variation in the simulations is due to uncertainty in simulating $Y_{ij}(t_{ij} = 0)$, the counterfactual predicted value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_{ij} = 0$.

• In conditional prediction models, the average expected treatment effect (qi\$att.ev) for the treatment group is given by

$$\frac{1}{\sum_{i=1}^{M} \sum_{j=1}^{n_i} t_{ij}} \sum_{i=1}^{M} \sum_{j:t_{ij}=1}^{n_i} \{Y_{ij}(t_{ij}=1) - E[Y_{ij}(t_{ij}=0)]\},$$

where t_{ij} is a binary explanatory variable defining the treatment $(t_{ij} = 1)$ and control $(t_{ij} = 0)$ groups. Variation in the simulations is due to uncertainty in simulating $E[Y_{ij}(t_{ij} = 0)]$, the counterfactual expected value of Y_{ij} for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_{ij} = 0$.

• If "log" link is used, expected values are computed as above and then exponentiated, while predicted values are draws from the log-normal distribution whose logarithm has mean and variance equal to μ_{ij} and σ^2 , respectively.

Output Values

The output of each Zelig command contains useful information which you may view. You may examine the available information in z.out by using slotNames(z.out), see the fixed effect coefficients by using summary(z.out)@coefs, and a default summary of information through summary(z.out). Other elements available through the operator are listed below.

- From the zelig() output stored in summary(z.out), you may extract:
 - fixef: numeric vector containing the conditional estimates of the fixed effects.
 - ranef: numeric vector containing the conditional modes of the random effects.
 - frame: the model frame for the model.
- From the sim() output stored in s.out, you may extract quantities of interest stored in a data frame:
 - qi\$pr: the simulated predicted values drawn from the distributions defined by the expected values.
 - qi\$ev: the simulated expected values for the specified values of x.
 - qi\$fd: the simulated first differences in the expected values for the values specified in x and x1.
 - qi\$ate.pr: the simulated average predicted treatment effect for the treated from conditional prediction models.
 - qi\$ate.ev: the simulated average expected treatment effect for the treated from conditional prediction models.

How to Cite

To cite the *ls.mixed* Zelig model:

Delia Bailey and Ferdinand Alimadhi. 2007. "Is.mixed: Mixed effects linear model" in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software, "http://gking.harvard.edu/zelig

To cite Zelig as a whole, please reference these two sources:

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See also

Mixed effects linear regression is part of lme4 package by Douglas M. Bates (Bates 2007). For a detailed discussion of mixed-effects models, please see?

Bibliography

Bates, D. (2007), lme4: Fit linear and generalized linear mixed-effects models.