0.1 `logit`: Logistic Regression for Dichotomous Dependent Variables

Logistic regression specifies a dichotomous dependent variable as a function of a set of explanatory variables. For a Bayesian implementation, see Section **??**.

Syntax

```
> z.out <- zelig(Y ~ X1 + X2, model = "logit", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out, x1 = NULL)
```

Additional Inputs

In addition to the standard inputs, `zelig()` takes the following additional options for logistic regression:

- `robust`: defaults to `FALSE`. If `TRUE` is selected, `zelig()` computes robust standard errors via the `sandwich` package (see Zeileis (2004)). The default type of robust standard error is heteroskedastic and autocorrelation consistent (HAC), and assumes that observations are ordered by time index.

  In addition, `robust` may be a list with the following options:

  - `method`: Choose from
    * `"vcovHAC"`: (default if `robust = TRUE`) HAC standard errors.
    * `"kernHAC"`: HAC standard errors using the weights given in Andrews (1991).
    * `"weave"`: HAC standard errors using the weights given in Lumley and Heagerty (1999).
  - `order.by`: defaults to `NULL` (the observations are chronologically ordered as in the original data). Optionally, you may specify a vector of weights (either as `order.by = z`, where `z` exists outside the data frame; or as `order.by = ~z`, where `z` is a variable in the data frame) The observations are chronologically ordered by the size of `z`.
  - `...`: additional options passed to the functions specified in `method`. See the `sandwich` library and Zeileis (2004) for more options.

Examples

1. Basic Example

   Attaching the sample turnout dataset:

   ```
   > data(turnout)
   ```

Estimating parameter values for the logistic regression:

```
> z.out1 <- zelig(vote ~ age + race, model = "logit", data = turnout)
```

Setting values for the explanatory variables:
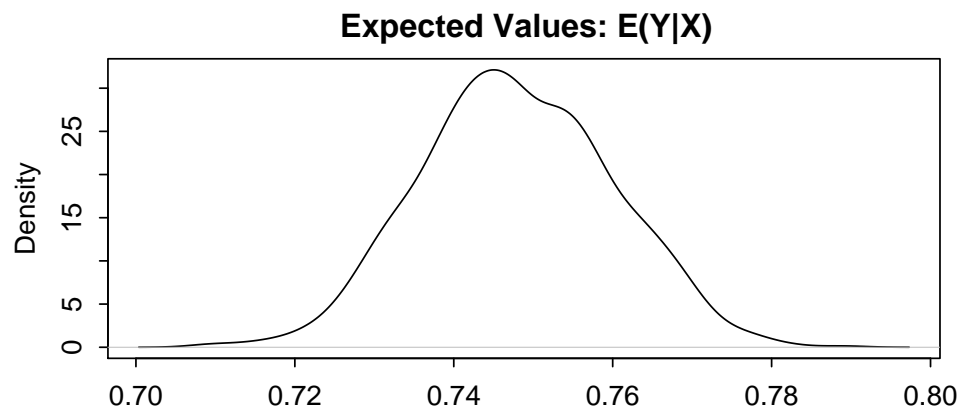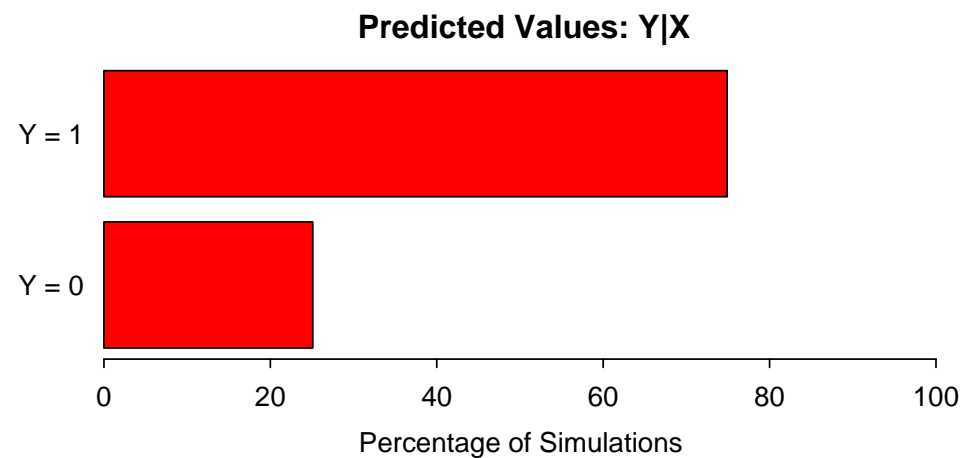
```
> x.out1 <- setx(z.out1, age = 36, race = "white")
```

Simulating quantities of interest from the posterior distribution.

```
> s.out1 <- sim(z.out1, x = x.out1)
```

```
> summary(s.out1)
```

```
> plot(s.out1)
```

Predicted Values: Y|X

Expected Values: E(Y|X)
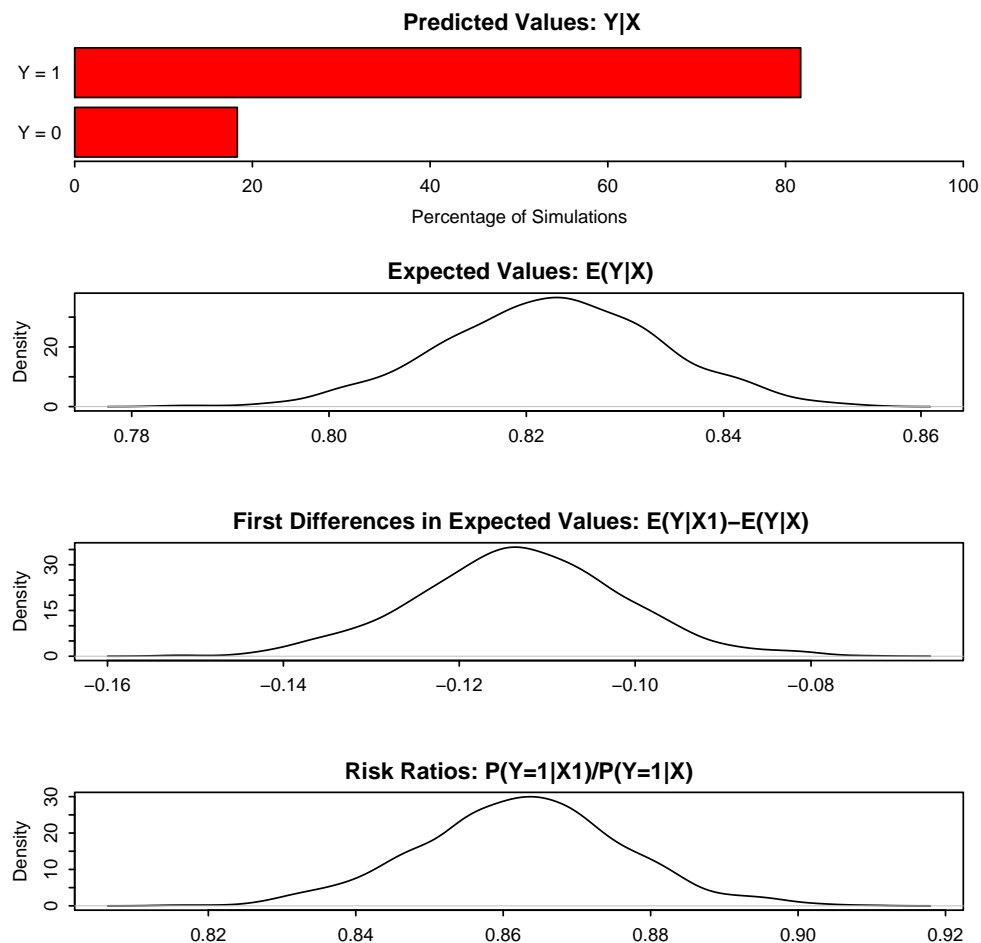
2. Simulating First Differences

Estimating the risk difference (and risk ratio) between low education (25th percentile) and high education (75th percentile) while all the other variables held at their default values.

```
> z.out2 <- zelig(vote ~ race + educate, model = "logit", data = turnout)
> x.high <- setx(z.out2, educate = quantile(turnout$educate, prob = 0.75))
> x.low <- setx(z.out2, educate = quantile(turnout$educate, prob = 0.25))

> s.out2 <- sim(z.out2, x = x.high, x1 = x.low)

> summary(s.out2)

> plot(s.out2)
```
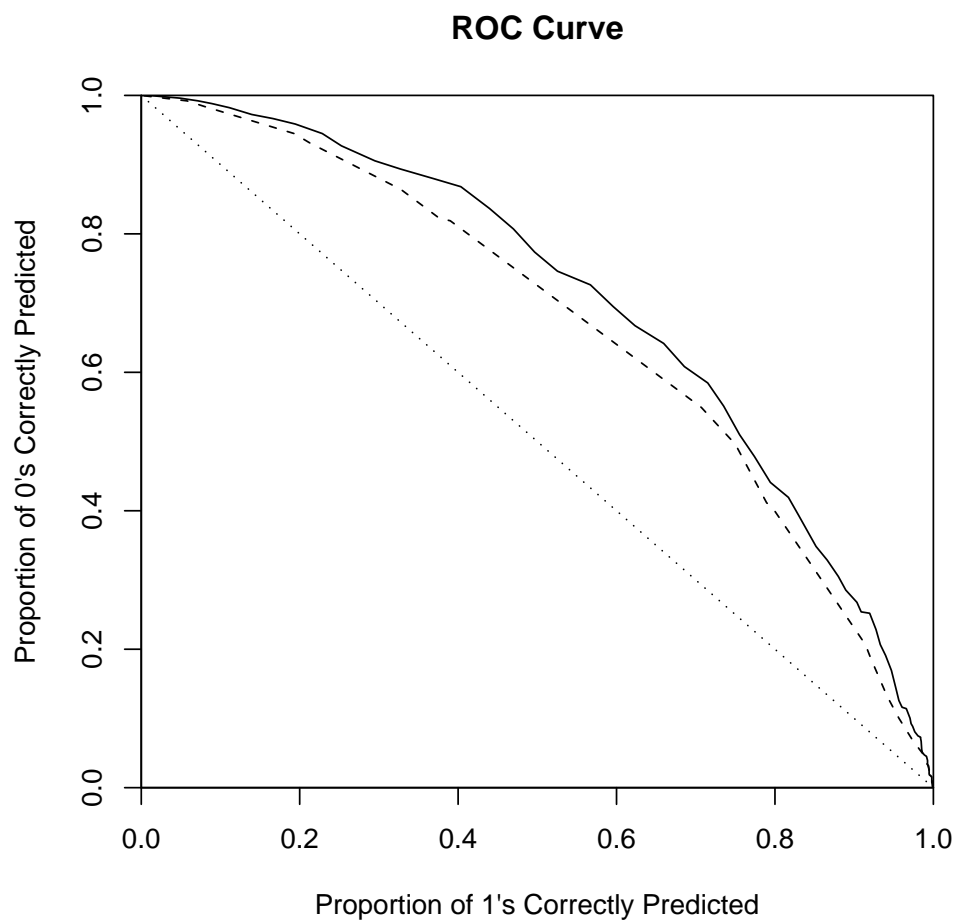
Predicted Values: Y|X

Expected Values: E(Y|X)

First Differences in Expected Values: E(Y|X1)–E(Y|X)

Risk Ratios: P(Y=1|X1)/P(Y=1|X)

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3. Presenting Results: An ROC Plot

One can use an ROC plot to evaluate the fit of alternative model specifications. (Use `demo(roc)` to view this example, or see King and Zeng (2002).)

```
> z.out1 <- zelig(vote ~ race + educate + age, model = "logit",
+      data = turnout)
> z.out2 <- zelig(vote ~ race + educate, model = "logit", data = turnout)


> rocplot(z.out1$y, z.out2$y, fitted(z.out1), fitted(z.out2))
```

ROC Curve

Proportion of 0's Correctly Predicted

Proportion of 1's Correctly Predicted

Model

Let Y_i be the binary dependent variable for observation i which takes the value of either 0 or 1.

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- The *stochastic component* is given by

$$
\begin{aligned}
Y_i &\sim \ \mathrm{Bernoulli}(y_i \mid \pi_i) \\
&= \ \pi_i^{y_i}(1 - \pi_i)^{1-y_i}
\end{aligned}
$$

  where $\pi_i = \Pr(Y_i = 1)$.

- The *systematic component* is given by:

$$
\pi_i \ = \ \frac{1}{1 + \exp(-x_i\beta)}.
$$

  where x_i is the vector of k explanatory variables for observation i and β is the vector of coefficients.

Quantities of Interest

- The expected values (`qi$ev`) for the logit model are simulations of the predicted probability of a success:

$$
E(Y) = \pi_i = \frac{1}{1 + \exp(-x_i\beta)},
$$

  given draws of β from its sampling distribution.

- The predicted values (`qi$pr`) are draws from the Binomial distribution with mean equal to the simulated expected value π_i.

- The first difference (`qi$fd`) for the logit model is defined as

$$
\mathrm{FD} = \Pr(Y = 1 \mid x_1) - \Pr(Y = 1 \mid x).
$$

- The risk ratio (`qi$rr`) is defined as

$$
\mathrm{RR} = \Pr(Y = 1 \mid x_1) \ / \ \Pr(Y = 1 \mid x).
$$

- In conditional prediction models, the average expected treatment effect (`att.ev`) for the treatment group is

$$
\frac{1}{\sum_{i=1}^{n} t_i} \sum_{i:t_i=1}^{n} \left\{ Y_i(t_i = 1) - E[Y_i(t_i = 0)] \right\},
$$

  where t_i is a binary explanatory variable defining the treatment $(t_i = 1)$ and control $(t_i = 0)$ groups. Variation in the simulations are due to uncertainty in simulating $E[Y_i(t_i = 0)]$, the counterfactual expected value of Y_i for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

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- In conditional prediction models, the average predicted treatment effect (`att.pr`) for the treatment group is

$$\frac{1}{\sum_{i=1}^{n} t_i} \sum_{i:t_i=1}^{n} \left\{ Y_i(t_i = 1) - \widehat{Y_i(t_i = 0)} \right\},$$

where t_i is a binary explanatory variable defining the treatment ($t_i = 1$) and control ($t_i = 0$) groups. Variation in the simulations are due to uncertainty in simulating $\widehat{Y_i(t_i = 0)}$, the counterfactual predicted value of Y_i for observations in the treatment group, under the assumption that everything stays the same except that the treatment indicator is switched to $t_i = 0$.

Output Values

The output of each Zelig command contains useful information which you may view. For example, if you run `z.out <- zelig(y ~ x, model = "logit", data)`, then you may examine the available information in `z.out` by using `names(z.out)`, see the `coefficients` by using `z.out$coefficients`, and a default summary of information through `summary(z.out)`. Other elements available through the `$` operator are listed below.

- From the `zelig()` output object `z.out`, you may extract:

    - `coefficients`: parameter estimates for the explanatory variables.
    - `residuals`: the working residuals in the final iteration of the IWLS fit.
    - `fitted.values`: the vector of fitted values for the systemic component, π_i.
    - `linear.predictors`: the vector of $x_i\beta$
    - `aic`: Akaike's Information Criterion (minus twice the maximized log-likelihood plus twice the number of coefficients).
    - `df.residual`: the residual degrees of freedom.
    - `df.null`: the residual degrees of freedom for the null model.
    - `data`: the name of the input data frame.

- From `summary(z.out)`, you may extract:

    - `coefficients`: the parameter estimates with their associated standard errors, p-values, and t-statistics.
    - `cov.scaled`: a $k \times k$ matrix of scaled covariances.
    - `cov.unscaled`: a $k \times k$ matrix of unscaled covariances.

- From the `sim()` output object `s.out`, you may extract quantities of interest arranged as matrices indexed by simulation × `x`-observation (for more than one `x`-observation). Available quantities are:

- qi$ev: the simulated expected probabilities for the specified values of x.

- qi$pr: the simulated predicted values for the specified values of x.

- qi$fd: the simulated first difference in the expected probabilities for the values specified in x and x1.

- qi$rr: the simulated risk ratio for the expected probabilities simulated from x and x1.

- qi$att.ev: the simulated average expected treatment effect for the treated from conditional prediction models.

- qi$att.pr: the simulated average predicted treatment effect for the treated from conditional prediction models.

How to Cite

To cite the *logit* Zelig model:

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> Kosuke Imai, Gary King, and Olivia Lau. 2007. "Toward A Common Framework for Statistical Analysis and Development," `http://gking.harvard.edu/files/abs/z-abs.shtml`.

See also

The logit model is part of the stats package by Venables and Ripley (2002). Advanced users may wish to refer to `help(glm)` and `help(family)`, as well as McCullagh and Nelder (1989). Robust standard errors are implemented via the sandwich package by Zeileis (2004). Sample data are from King et al. (2000).

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