0.1 net1s: Network Least Squares Regression for Continuous Proximity Matrix Dependent Variables

Use network least squares regression analysis to estimate the best linear predictor when the dependent variable is a continuously-valued proximity matrix (a.k.a. sociomatrices, adjacency matrices, or matrix representations of directed graphs).

Syntax

```
> z.out <- zelig(y ~ x1 + x2, model = "netls", data = mydata)
> x.out <- setx(z.out)
> s.out <- sim(z.out, x = x.out)</pre>
```

Examples

1. Basic Example with First Differences

Load sample data and format it for social networkx analysis:

```
> data(sna.ex)
```

Estimate model:

```
> z.out <- zelig(Var1 ~ Var2 + Var3 + Var4, model = "netls", data = sna.ex)
```

Summarize regression results:

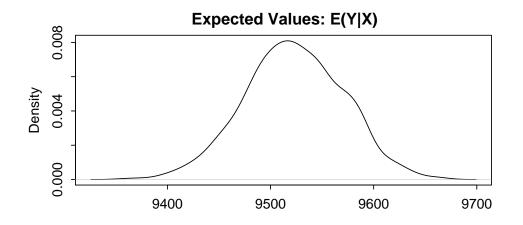
```
> summary(z.out)
```

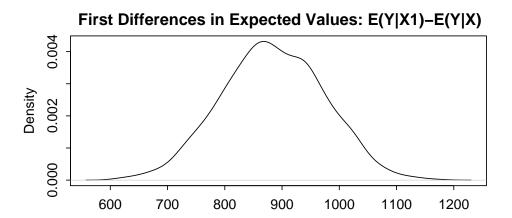
Set explanatory variables to their default (mean/mode) values, with high (80th percentile) and low (20th percentile) for the second explanatory variable (Var3).

```
> x.high <- setx(z.out, Var3 = quantile(sna.ex$Var3, 0.8))
> x.low <- setx(z.out, Var3 = quantile(sna.ex$Var3, 0.2))</pre>
```

Generate first differences for the effect of high versus low values of Var3 on the outcome variable.

```
> try(s.out <- sim(z.out, x = x.high, x1 = x.low))
> try(summary(s.out))
> plot(s.out)
```





Model

The netls model performs a least squares regression of the sociomatrix \mathbf{Y} , a $m \times m$ matrix representing network ties, on a set of sociomatrices \mathbf{X} . This network regression model is a directly analogue to standard least squares regression element-wise on the appropriately vectorized matrices. Sociomatrices are vectorized by creating Y, an $m^2 \times 1$ vector to represent the sociomatrix. The vectorization which produces the Y vector from the \mathbf{Y} matrix is preformed by simple row-concatenation of \mathbf{Y} . For example if \mathbf{Y} is a 15 × 15 matrix, the $\mathbf{Y}_{1,1}$ element is the first element of Y, and the \mathbf{Y}_{21} element is the second element of Y and so on. Once the input matrices are vectorized, standard least squares regression is performed. As such:

• The stochastic component is described by a density with mean μ_i and the common variance σ^2

$$Y_i \sim f(y_i|\mu_i,\sigma^2).$$

• The systematic component models the conditional mean as

$$\mu_i = x_i \beta$$

where x_i is the vector of covariates, and β is the vector of coefficients.

The least squares estimator is the best linear predictor of a dependent variable given x_i , and minimizes the sum of squared errors $\sum_{i=1}^{n} (Y_i - x_i \beta)^2$.

Quantities of Interest

The quantities of interest for the network least squares regression are the same as those for the standard least squares regression.

• The expected value (qi\$ev) is the mean of simulations from the stochastic component,

$$E(Y) = x_i \beta,$$

given a draw of β from its sampling distribution.

• The first difference (qi\$fd) is:

$$FD = E(Y|x_1) - E(Y|x)$$

Output Values

The output of each Zelig command contains useful information which you may view. For example, you run z.out <- zelig(y x, model="netls", data), then you may examine the available information in z.out by using names(z.out), see the coefficients by using z.out\$coefficients, and a default summary of information through summary(z.out). Other elements available through the \$ operator are listed below.

- From the zelig() output stored in z.out, you may extract:
 - coefficients: parameter estimates for the explanatory variables.
 - fitted.values: the vector of fitted values for the explanatory variables.
 - residuals: the working residuals in the final iteration of the IWLS fit.
 - df.residual: the residual degrees of freedom.
 - zelig.data: the input data frame if save.data = TRUE
- From summary(z.out), you may extract:
 - mod.coefficients: the parameter estimates with their associated standard errors, p-values, and t statistics.

$$\hat{\beta} = \left(\sum_{i=1}^{n} x_i' x_i\right)^{-1} \sum x_i y_i$$

- sigma: the square root of the estimate variance of the random error ε :

$$\hat{\sigma} = \frac{\sum (Y_i - x_i \hat{\beta})^2}{n - k}$$

- r.squared: the fraction of the variance explained by the model.

$$R^{2} = 1 - \frac{\sum (Y_{i} - x_{i}\hat{\beta})^{2}}{\sum (y_{i} - \bar{y})^{2}}$$

- adj.r.squared: the above \mathbb{R}^2 statistic, penalizing for an increased number of explanatory variables.
- cov.unscaled: a $k \times k$ matrix of unscaled covariances.
- From the sim() output stored in s.out, you may extract:
 - qi\$ev: the simulated expected values for the specified values of x.
 - qi\$fd: the simulated first differences (or differences in expected values) for the specified values of x and x1.

How to Cite

To cite the *netls* Zelig model:

Skyler J. Cranmer. 2007. "netls: Network Least Squares Regression for Continuous Proximity Matrix Dependent Variables," in Kosuke Imai, Gary King, and Olivia Lau, "Zelig: Everyone's Statistical Software," http://gking.harvard.edu/zelig.

To cite Zelig as a whole, please reference these two sources:

Kosuke Imai, Gary King, and Olivia Lau. 2007. "Zelig: Everyone's Statistical Software," http://GKing.harvard.edu/zelig.

Kosuke Imai, Gary King, and Olivia Lau. 2007. "Toward A Common Framework for Statistical Analysis and Development," http://gking.harvard.edu/files/abs/z-abs.shtml.

See also

The network least squares regression is part of the sna package by Carter T. Butts (Butts and Carley 2001). In addition, advanced users may wish to refer to help(netlm).

Bibliography

Butts, C. and Carley, K. (2001), "Multivariate Methods for Interstructural Analysis," Tech. rep., CASOS working paper, Carnegie Mellon University.