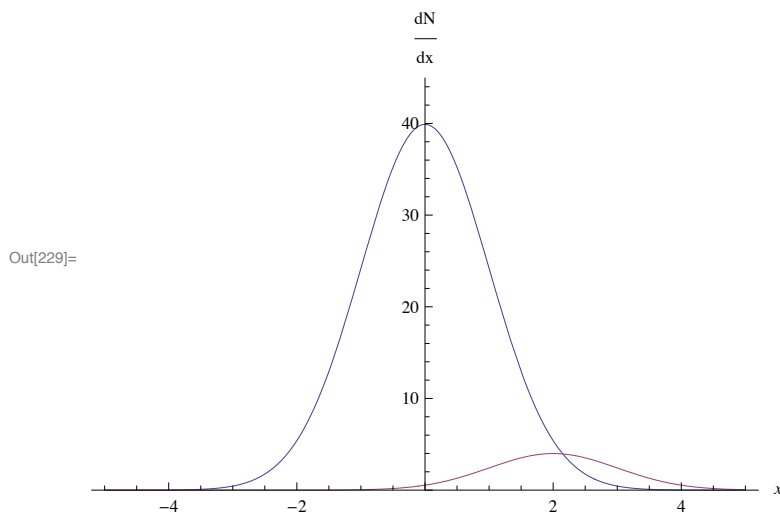# An example demonstraiting different optimal points using simple and improved significance estimate

Kyle Cranmer, July 8, 2014

Here's a plot of an example background (blue) and signal (red) distribution. The goal is to optimize the cut on x.

In[229]:= `Plot[{100 * PDF[NormalDistribution[0, 1], x]`
`, 10 * PDF[NormalDistribution[2, 1], x]},`
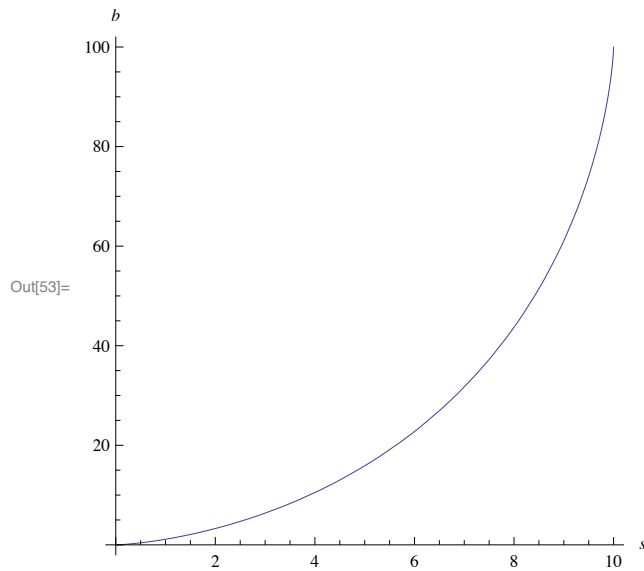`{x, -5, 5}, PlotRange → {0, 45}, AxesLabel → {x, dN / dx}]`

Out[229]=

Define number of signal and background satisfying cut.

In[49]:= `s[cut_] := 10 * (1 - CDF[NormalDistribution[1, 1], cut])`

In[50]:= `b[cut_] := 100 * (1 - CDF[NormalDistribution[0, 1], cut])`

In[53]:= `ParametricPlot[{s[cut], b[cut]}, {cut, -5, 5}, AxesLabel → {s, b}, AspectRatio → 1]`

Out[53]=

Define simple and improved significance estimates, find cut value that maximizes them.

In[54]:= `Simple[S_, B_, sigma_] := S / Sqrt[B + sigma^2]`

In[55]:= `ZA[S_, B_, sigma_] :=`
`  Sqrt[2 ((S + B) Log[((S + B) (B + sigma^2)) / (B^2 + (S + B) sigma^2)] -`
`      (B^2 / sigma^2) Log[1 + (S sigma^2) / (B (B + sigma^2))])]`

In[122]:= `Maximize[N[Simple[s[x], b[x], 1]], x]`

Out[122]= $\{1.23101, \{x \to 0.710368\}\}$

In[123]:= `Maximize[N[ZA[s[x], b[x], 1]], x]`

Out[123]= $\{1.18309, \{x \to 0.597082\}\}$

In[165]:= `SimplePoint = {N[s[0.71]], N[b[0.71]]}`

Out[165]= $\{6.14092, 23.8852\}$
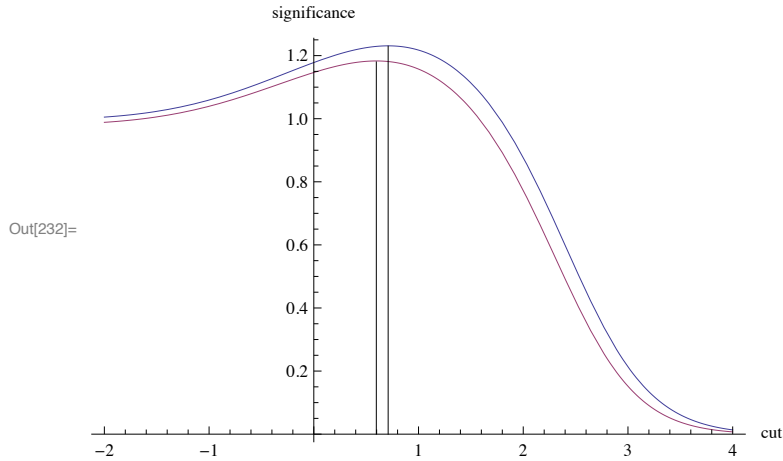
In[166]:= `ZAPoint = {N[s[0.597]], N[b[0.597]]}`

Out[166]= $\{6.56526, 27.5254\}$

Make a plot showing significance estimates peaking in different places

```
In[230]:= line1 = Line[{{0.71, 0}, {0.71, 1.23}}];
          line2 = Line[{{0.597, 0}, {0.597, 1.18}}];
          Plot[{Simple[s[cut], b[cut], 1],
            ZA[s[cut], b[cut], 1]},
           {cut, -2, 4}, Epilog → {line1, line2}, AxesLabel → {cut, significance}]
```
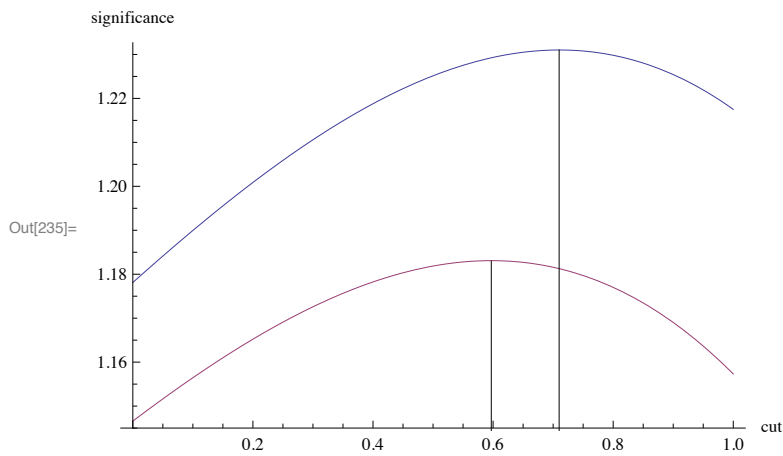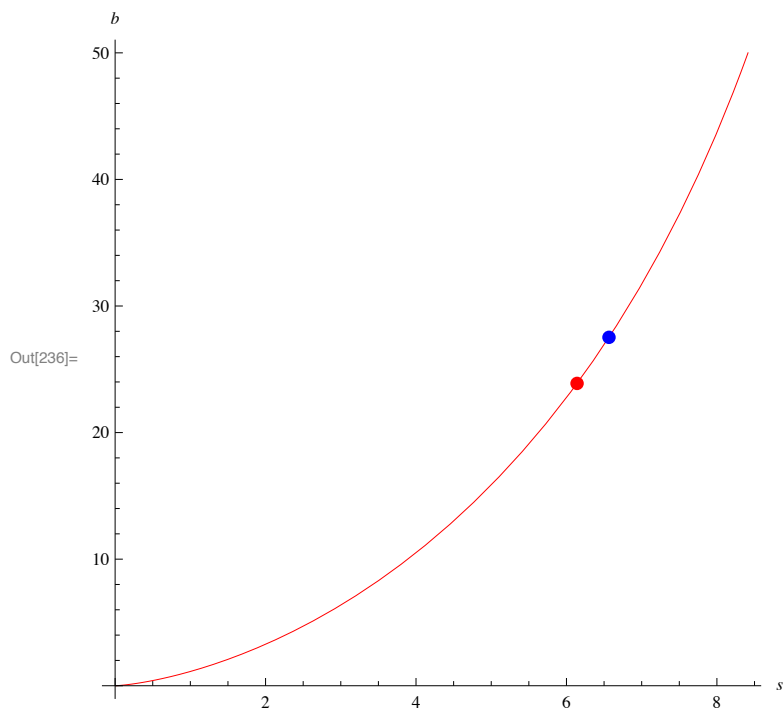
Out[232]=

```
In[233]:= line1 = Line[{{0.71, 0}, {0.71, 1.231}}];
          line2 = Line[{{0.597, 0}, {0.597, 1.183}}];
          Plot[{Simple[s[cut], b[cut], 1],
            ZA[s[cut], b[cut], 1]},
           {cut, 0, 1}, Epilog → {line1, line2}, AxesLabel → {cut, significance}]
```

Out[235]=

In[236]:=
```
plot = ParametricPlot[{s[cut], b[cut]}, {cut, 0, 4},
    PlotStyle → {Red}, AspectRatio → 1, AxesLabel → {s, b},
    Epilog → {PointSize[Large], Red, Point[{SimplePoint}], Blue, Point[ZAPoint]}]
```

Out[236]=

Make contour plots of two significance estimates

In[239]:= **Show[plot, ContourPlot[{Simple[S, B, 1]}, {S, 0, 10}, {B, 0, 50}], plot]**
Show[plot, ContourPlot[{ZA[S, B, 1]}, {S, 0, 10},
  {B, 0, 50}, Epilog → {PointSize[Large], Red, Point[ZAPoint]}], plot]

Out[239]=

Out[240]=