

$$m\dot{\mathbf{r}}(t) \equiv \nabla S_c(\mathbf{r}, t) \equiv \nabla U(\mathbf{r}, t)$$