Vortex Rings
with rectangular initial conditions

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Abstract

The project was ambitious to try a 3D simulation, then it was just a proof of concept of a phenomena. It tested how a vortex ring behave when it goes through a rectangular hole. It worked and produced very interesting results; however, it is possible to better fine tune the simulation to see more oscillations. Further analysis could be done using a super computer to achieve a better spacial resolution.

1 Introduction

The project is about simulating vortexes using a square and a rectangle shape as initial conditions and confirm the oscillation characteristics of such geometries. To program used for the simulations was OpenFoam v1906.

1.1 Motivation

Dianna Cowern is a science communicator that has a YouTube channel called Physics Girl that is part of the PBS Digital Studios Network. The channel has almost 1.5 million subscribers and more than 100 million view. The channel cover a variety of topics in physics and mostly showing hands-on experiments.

On November 7, 2018, she released a video entitle "How to Make a SQUARE Vortex Ring! ft. 3blue1brown", the thumbnail is on Figure 1. The guest is another science communicator, whose name is Grant Sanderson and make video more math related.In the video, her colleagues and she tried different experiments with vortexes, including different geometries for the hole.

Figure 1: **How to Make a SQUARE Vortex Ring! ft. 3blue1brown.** Thumnail of video produced by the channel Physics Girl on YouTube.

The results using a rectangular shape hole are astonishing, it has some peculiar oscillating motion; therefore, the idea of this work is to simulated such system using OpenFOAM, which is a open source program to run hydrodynamics simulations with an extensive community.

1.2 Background

Many problems in fluid dynamics can be described by the Navier-Stokes equation, which describes the flow of incompressible fluids. It can be used to model complex vortexes, shock waves, and turbulence in 3 dimensional fluids/gases. The equation can be express as follows

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} = -\frac{\nabla P}{\rho} + v \nabla^2 \mathbf{u}, \tag{1}$$

where $\mathbf{u}$ is the fluid velocity vector, P is the fluid pressure, ρ is the fluid density, v is the kinematic viscosity [2].

A vortex is a when a fluid swirls around line; if the line is a closed loop, we have vortex ring, also know as toroidal vortex. The vortex can have any size, a the closed loop can have any geometry. For a circle, we can see an idealized representation in Figure 2, where the arrows represent the velocity field of the fluid around the vortex.

Looking at a cross section of a vortex ring like in Figure 3, we see that each vortex is spinning in opposite directions; as a result, a vortex ring propels itself forwards, it gets faster if it is a smaller ring because each vortexes are closer and interact more, in a symmetrical shape as a circle, all the parts are equidistant, so the whole shape move together.

However, if we break this symmetry, there will be parts of the loop that are closer together compared to other parts, then this parts are going to move faster, so they will go farther away. Therefore, the shape will not be constant, since different parts of the ring move with different speeds.

For example, in a square, the corners will move faster, sharp turns concentrate more the vector field that interacts with itself and increase the speed in that region. In the real world, the vortex

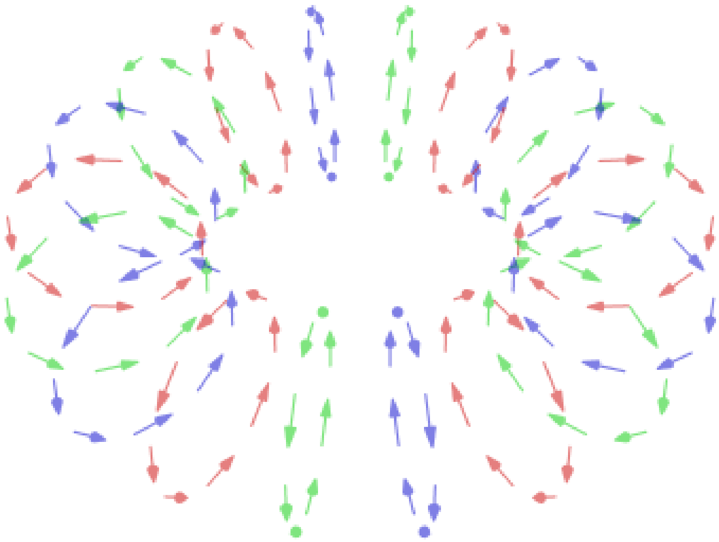

Figure 2: **Representation of a vortex.** Idealized representation of a vortex, velocity field of the fluid around the vortex is shown by vectors.

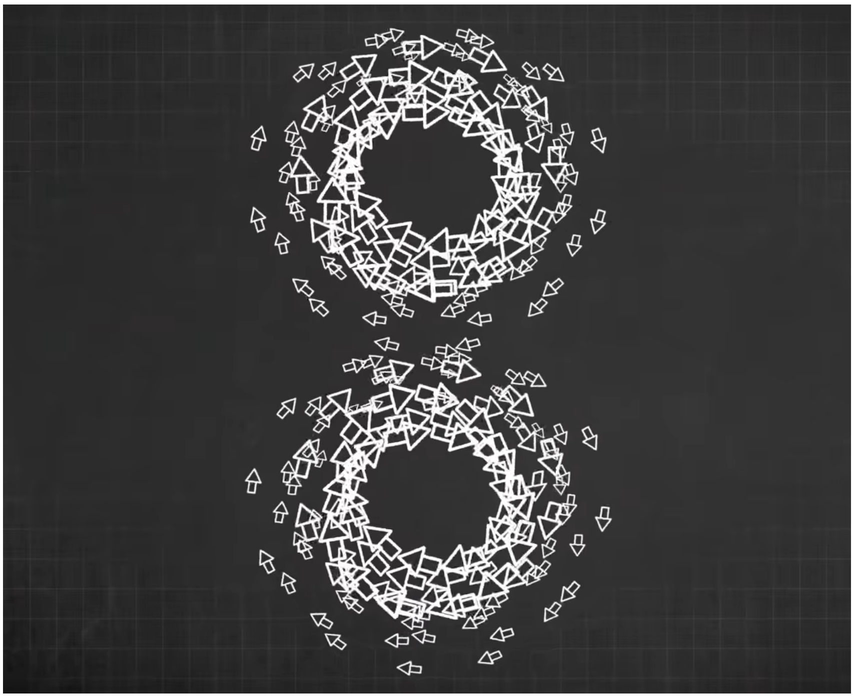

Figure 3: **Cross-section of a vortex ring.** The cross section allows us to see how the vortex ring interacts with itself.

will tend to be circular, but we are going to see it wobble, it will oscillate between the initial square of the hole and another one rotate 45 degrees. This effect is represented in Figure 4.

In a rectangle, this effect is even more strong, it also creates other oscillations. The principle is the same, but since there is no symmetry in the sizes of the sides, the regions that gets farther

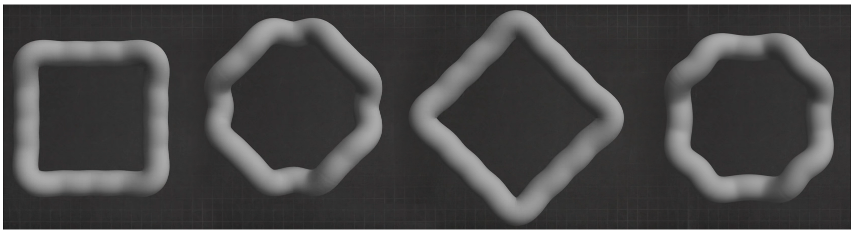

Figure 4: **Representation of the square vortex.** Idealized representation of the square vortex, it oscillates because of the sharp corners.

ahead gets even farther than the square and creates a vortex in a C shape when we look side ways, and after that closer corners flip, and results in an inverted C shape. Therefore, in this case the vortex will oscillate between an C and an inverted C, and between a rectangle and the same rectangle rotated 90 degrees (in reality, it looks more like an oval shape not a rectangle). There is also an representation in Figure 5 [1].

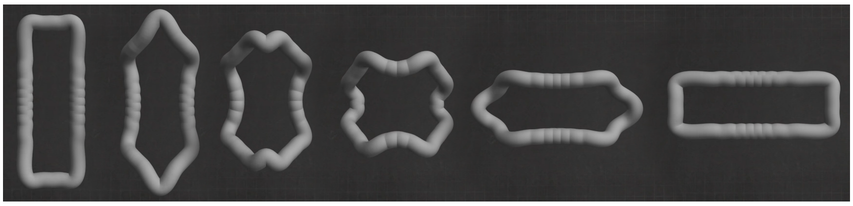

Figure 5: **Representation of the rectangular vortex.** Idealized representation of the rectangular vortex, it oscillates in two different ways, due to the shape and its corners and sizes.

2 Methods

As seen before, the program used for the simulation was OpenFoam v1906, it also includes a visualization tool called paraView, where it was possible to measure the position of the vortex and create a 3D representation of the vortex. In addition, some other analysis was made using Python, and some specific libraries such as numpy, and matplotlib.

2.1 OpenFOAM

OpenFOAM is a free and open source software developed primarily by OpenCFD Ltd. The first version was released in 2004. It is very versatile and used for many physical scenarios, since it can solve anything from complex fluid flows involving chemical reactions, turbulence and heat transfer, to acoustics, solid mechanics and electromagnetics and some other phenomena. Because of that many scientist and engineers use it to simulate their systems.

It is frequently updated with a new version every six months (in June and December), and it includes features created from the developers and the community which is really extensive. There are many tutorial and an active forum for solving problems and reporting bugs.

The programs is also user friendly, the user guide includes a tutorial with basic simulations to understand the basics in how to use OpenFOAM. The tutorials can be easily modified to create new interesting systems, and other more complicated features can be learn reading the documentation.

There is also a very useful visualization tool called paraView that have many filters that can be applied in the simulation for the best output image. It can also produce animations to see the phenomenon over time.

3 Results

An 3D simulation is very computational intensive, so it was not possible to have a very accurate description of the system studied. The initial idea was to create the obstacle like in the Figure 6. However, it just over complicates the simulation, the corners increases the necessary computer power, it also increases the necessary time to finish and require a better fine tuning of the parameters to properly see the effect.

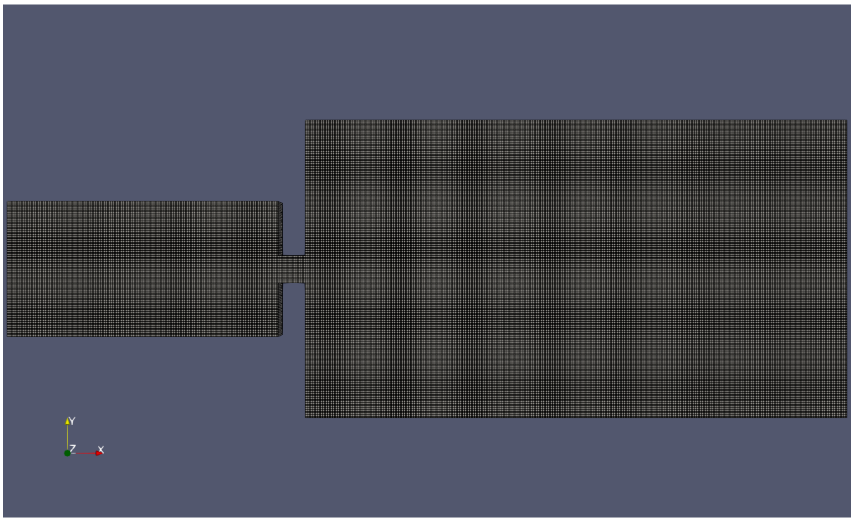

Figure 6: **Initial proposed mesh.** The environment of the simulation considering the fluid passing through an obstacle, but it turn out to be just an over complication.

The only important part is the actual initial condition not the obstacle itself, so to simplify the simulation, the mesh is just a simple box and the effect is achieve by applying initial conditions after the obstacle. Therefore, it is created a higher pressure and with higher velocity region than the environment surrounding it, the region is a thin square or rectangle.

The box is a closed box, because letting it as an open boundary couldn't create the effect, the higher pressure/speed region would quickly spread and fade out without creating any vortex.

The square was tested first, but since there is not a good spacial resolution, it was not possible to see the oscillations, the simulation just created a circular vortex, some snapshots of it is shown in Figure 7.

Even though, it was not possible to see vortex wobbling, it is enough as proof of concept for the simplifications. In other words, the phenomena can be simulated in simple modern laptop, it would be better to have access to supercomputer to increase the spacial resolution, but to test, understand, and learn how OpenFOAM works in a interesting system is enough.

As mentioned in the background section, the effect is more pronounced in a rectangular hole, so it was also simulated, and in this case, it was possible to see it oscillating between two different ovals,but the C and inverted C is hard to see, again it is because of the low spacial resolution.

It was tested a few different initial values for the pressure and velocity to try to fine tune the problem and get the best visualization of the system. The initial values for all the simulations

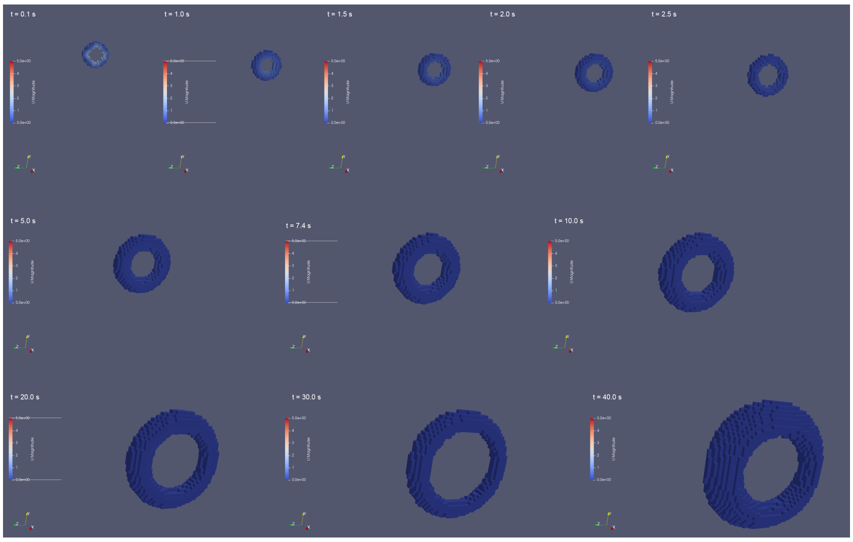

Figure 7: **Snapshots of the square vortex.** The propagation of the square vortex though the fluid, it is not possible to it oscillating, due to low spacial resolution.

can be seen in the Table 1, including the values for the square case.

Simulation	Position (x y z) (m)	Pressure (kg/m^2)	Velocity (x)(m/s)
Square	(0.0 0.4 0.4) - (0.1 0.6 0.6)	0.2	5.0
Rectangle 1	(0.0 0.3 0.4) - (0.1 0.7 0.6)	0.2	5.0
Rectangle 2	(0.0 0.3 0.4) - (0.1 0.7 0.6)	0.5	8.0
Rectangle 3	(0.0 0.3 0.4) - (0.1 0.7 0.6)	1.0	10.0
Rectangle 4	(0.0 0.3 0.4) - (0.1 0.7 0.6)	2.0	20.0
High Resolution	(0.0 0.3 0.45) - (0.05 0.7 0.55)	5.0	10.0

Table 1: Initial Conditions for the simulations

The transport properties were all the same for all simulation, with $\nu = 1 \cdot 10^{-5}$. The constant properties were copied from the High Reynolds number flow tutorial, but it could also be fine tuned for a better result.

The Figure 8, 9, 10 and 11 show snapshot of the rectangular simulations, respectively. There is also attached GIFs for each simulation. From those simulations, it was measured the position of the vertex over time, and the summary can be seen in Figure 12.

As we can see in the plot (Figure 12), the fluid slows down the vortex in logarithmic form, higher pressure and speed can go farther, but they all slow down fast.

For a final test, it was also tried a simulation with a better resolution than the previous ones. The initial conditions are also in Table 1. For this one the snapshots have a clearer oscillation from a C to an inverted C, and it can been seen in Figure 13.

4 Discussion

This project was more a proof of concept than trying to have a better description of the phenomenon. It was ambitions for being in 3 dimension, it requires a lot of computational power for the simulation and visualization. It was also difficult to find the best way to show it, since

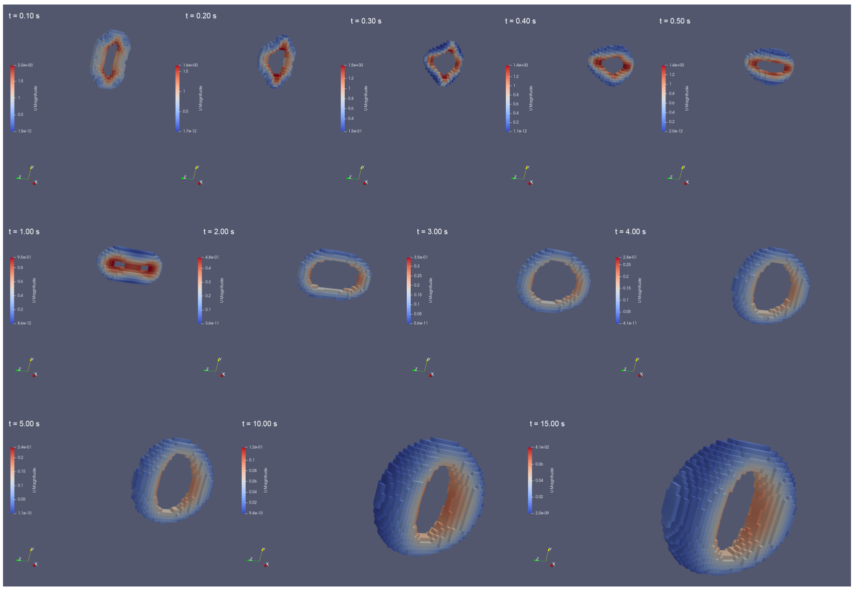

Figure 8: **Snapshots of the Rectangle 1 simulation.** The propagation of the rectangular vortex though the fluid. The initial conditions are $p = 0.2\,kg/m^2$ and $v_x = 5.0\,m/s$

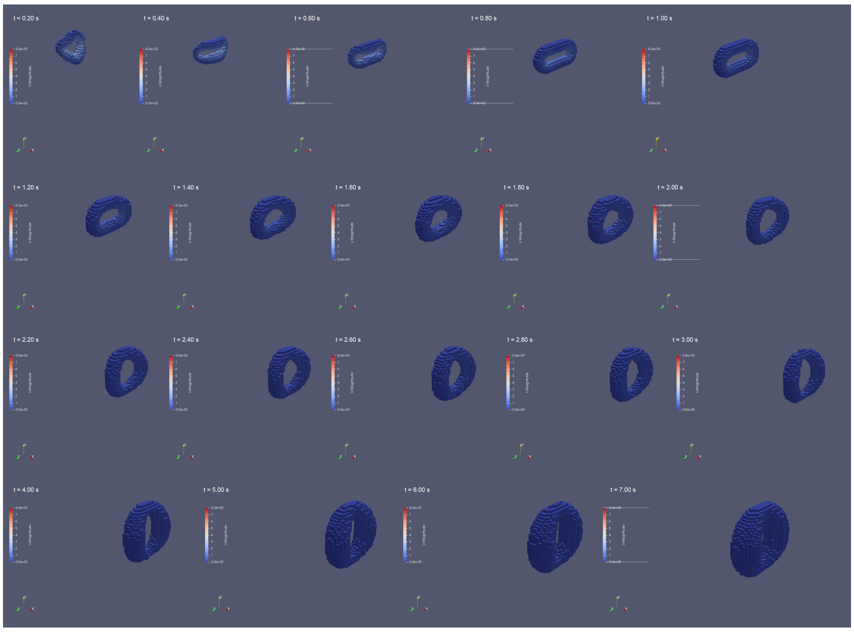

Figure 9: **Snapshots of the Rectangle 1 simulation.** The propagation of the rectangular vortex though the fluid. The initial conditions are $p = 0.5\,kg/m^2$ and $v_x = 8.0\,m/s$

the variables change a lot, so the color scale for the image had to be redone for every single frame.

Further analysis studies are possible, such as fine tuning the parameters to create a good vortex that oscillates more. It is also important to try even higher resolutions for more details and for that use multiprocessor at once. In this project it was just used one core for each simulation.

Overall, it was very useful to learn how to use another scientific program that is much more user friendly than Gadget-2. OpenFOAM also is more collaborative, it has a good community and

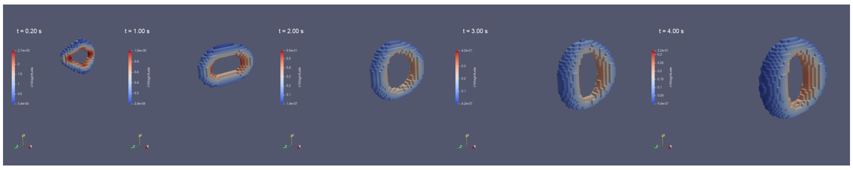

Figure 10: **Snapshots of the Rectangle 1 simulation.** The propagation of the rectangular vortex though the fluid. The initial conditions are $p = 1.0\,kg/m^2$ and $v_x = 10.0\,m/s$

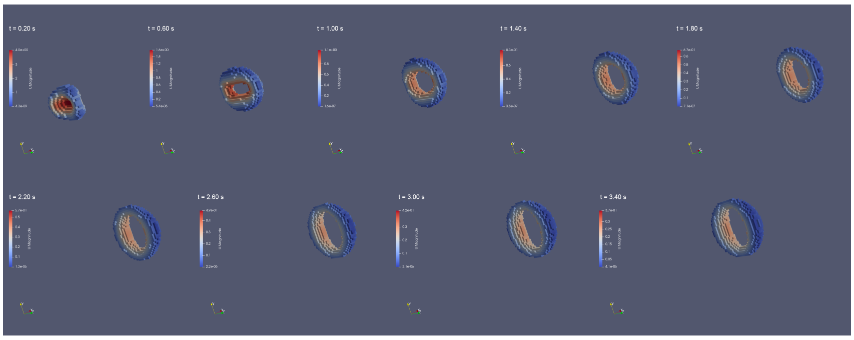

Figure 11: **Snapshots of the Rectangle 1 simulation.** The propagation of the rectangular vortex though the fluid. The initial conditions are $p = 2.0\,kg/m^2$ and $v_x = 20.0\,m/s$

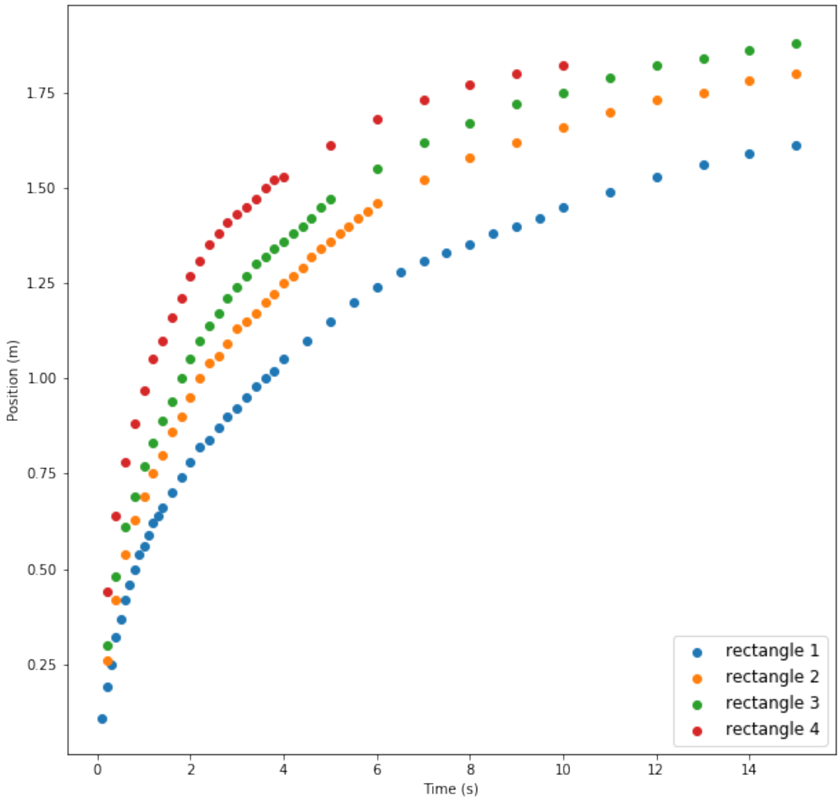

Figure 12: **Position of the vortex for the different rectangular simulations** The position of the vortexs follow a logarithmic curve, it slows down fast.

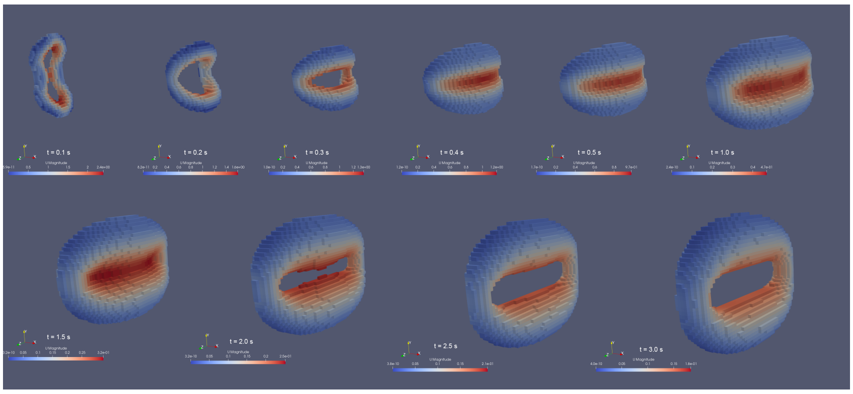

Figure 13: **Snapshots of the High Resolution simulation.** The propagation of the rectangular vortex though the fluid. The initial conditions are $p = 5.0\,kg/m^2$ and $v_x = 10.0\,m/s$

it is coded in such a form that it easier for the interactions.

References

[1] D. Cowern. How to make a square vortex ring! ft. 3blue1brown. *YouTube*, Nov 2018.

[2] W. L. Hosch. Navier-stokes equation. *Encyclopædia Britannica*, Aug 2009.