Adjusting for missed tags in salmon escapement surveys

Karim N. Rajwani and Carl J. Schwarz

Abstract: One of the assumptions in mark–recapture studies is that all tags recovered are correctly identified. In some cases, tags are overlooked, and a subsample of fish previously classified as untagged is examined to look for missed tags. In this paper, a method for estimating the salmon escapement using a mark–recapture methodology after correcting for missed tags is developed. The variance of the revised estimator is derived and the loss of efficiency compared with a Petersen estimate, where no tags are overlooked, is found. Optimal allocation of effort between the initial and second examination is considered. Finally, a numerical example using actual data from the 1994 Fraser River sockeye salmon (*Oncorhynchus nerka*) season is used to illustrate the results.

Résumé : Une des suppositions que l'on fait dans les études de marquage–recapture, c'est que toutes les marques récupérées sont correctement identifiées. Dans certains cas, les marques passent inaperçues, et un sous-échantillon de poissons antérieurement classés comme non marqués est examiné pour trouver des marques ayant passé inaperçues. Dans la présente communication, on décrit une méthode pour estimer l'échappement de saumons à l'aide d'une méthode de marquage–recapture après correction pour tenir compte des marques passées inaperçues. La variance de l'estimateur révisé est dérivée et la perte d'efficacité comparée à l'estimation de Petersen, où aucune marque ne passe inaperçue, est déterminée. L'allocation optimale de l'effort entre l'examen initial et le deuxième examen est prise en considération. Enfin, un exemple numérique utilisant des données réelles de la saison de pêche du saumon sockeye (*Oncorhynchus nerka*) du fleuve Fraser de 1994 vient illustrer les résultats.

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Introduction

Salmon escapement surveys often use a mark–recapture method to obtain an estimate of the returning population size. A common method used in British Columbia is to capture n_1 fish as they return to spawn, tag them with Petersen disk tags, and release them. After spawning, the fish die and some of them get washed onto the banks of the spawning area. Survey teams examine n_2 carcasses, and m_2^* carcasses are observed with tags present. The simple Petersen estimate (Seber 1982, p. 59) of the number of fish that return to spawn (N, the escapement) is $\hat{N} = n_1 n_2 / m_2^*$.

One of the assumptions of all mark–recapture experiments is that all fish captured are correctly identified as to tagging status, i.e., no tags are overlooked and no untagged fish are erroneously classified as being tagged. However, field conditions for the carcass recoveries are often severe and carcasses are not in pristine condition, and so mistakes can be made. The most common error is that a tagged fish can be classified as being untagged. This results in an overestimate of N. Mistakes of the other kind are unlikely, as the tag number must be recorded for each tagged fish observed. To account for these misclassification errors, a subsample from the $n_2 - m_2^*$ fish identified as being without tags is examined more carefully for missed tags. This second examination is assumed to be infallible and is used to estimate the total number of tags that were present in the initial sample.

Current practice is to use a simple moment estimator to estimate a revised m_2, which is then used in the usual Petersen formula. Precision is estimated as if this revised m_2 was known exactly and does not account for the fact that m_2 has been estimated. As well, little examination of the optimal allocation of effort between the initial carcass recovery and the subsequent reexamination of carcasses has been done.

The Fraser River Sockeye Public Review Board (1995) was asked to investigate issues dealing with the 1994 sockeye salmon (*Oncorhynchus nerka*) returns. Among its recommendations (p. 96) was that "the resampling program for missed tags should definitely be more structured. A different formula also should be developed and the uncertainty over tag loss be incorporated into a better variance estimate. In addition, the method for constructing confidence limits should be revised in light of recent developments in mark–recapture and general statistical theory."

Paulik (1961) discussed the detection of incomplete reporting of tags. He derived a preliminary guide to determine the number of fish that should be tagged, and the number that should be recaptured and examined for tags, to be reasonably sure of discovering nonreporting of a certain magnitude. His plan to estimate "incomplete reporting should not be confused with the plan of examining the catch for tags that the fisherman failed to remove" (p. 828). Hilborn (1988) derived a method for determining the percentage of recaptured tagged fish that are represented by returned tags for cases in which tags are examined sequentially, such as at the time of harvesting and then at the time of processing. Both Hilborn and Paulik discussed ways to adjust for the nonreporting of tags, but neither expanded their work to estimate the population size.

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Table 1. Notation for statistics and parameters.

Symbol	Definition
	Statistics
n_1	Number of tags applied
n_2	Number of carcasses initially examined
m_2^*	Number of tags recovered from the n_2 carcasses examined in the first survey
n_3	Number of carcasses subsampled from the $n_2 - m_2^*$ fish initially classified as being without tags
r	Subsampling rate, so $r = n_3/(n_2 - m_2^*)$
m_3	Number of additional tags recovered from the n_3 carcasses examined in the subsample
$\hat{m}_2$	Estimated number of marks in the original sample of n_2 carcasses
C_0	Total cost
c_2	Cost per examination of a carcass in the first carcass survey
c_3	Cost per examination of a carcass in the second survey (cost of subsampling)
	Parameters
N	Population size (escapement)
λ	Miss rate; P(failing to observe a tag on a tagged fish)
p_1	Tagging rate; P(carcass has a tag) = n_1/N
m_2	Actual number of marked fish in the first recovery sample

The problem of incorrectly classifying tagged fish as untagged is similar to the problem of misclassifying objects in a quality control setting. Here the object is to estimate the percentage of defective components, which is analogous to the percentage of fish originally tagged.

Tenenbein (1970) considered a double sampling scheme using a true and fallible device to estimate the proportion of defective devices. A random sample of n_2 units is selected from a population of interest. Then, a subsample of n_3 units from the n_2 units is classified by both devices, while the remaining $n_2 - n_3$ units are classified by the fallible device only. Tenenbein developed an estimate for the proportion defective with an appropriate variance formula. However, in Tenenbein's approach, a proportion of fish correctly identified as having tags would be reexamined. This can be considered an unnecessary expense under the assumption that untagged carcasses cannot be incorrectly classified as having tags.

Haitovsky and Rapp (1992) expanded on Tenenbein's work. They developed a conditional resampling scheme to improve the estimators of multinomial classification probabilities in the presence of a fallible classifier. Different sampling rates could be used on each category, which would then make it possible to exclude a category from being resampled. Haitovsky and Rapp considered improving the estimate for the proportion in each class; we extend the results to estimating the escapement.

In this paper, a general statistical theory for this type of sampling experiment will be developed. The variance for the estimate of the escapement accounting for missed tags will be developed, and the loss of efficiency compared with the simple Petersen estimate will be found. The optimal allocation of effort between the two samples will also be examined. Lastly, an example is presented, using data provided by the Fraser River Sockeye Public Review Board. Note that other problems with the survey design, e.g., tagging and recovery occurring over a period of time rather than a single point in time, are not examined in this paper.

Methods

The notation used in the remainder of the paper is described in Table 1.

Survey protocol

The standard survey protocol used to estimate escapement was outlined by the Fraser River Sockeye Salmon Management Review Team 1994 Spawning Escapement Estimation Working Group (1995) and is as follows.

As fish return to their spawning sites, n_1 are captured using seine nets. A Petersen disk tag is attached and the fish is released. If a captured fish appears to be stressed, at an advanced stage of maturation, or physically damaged, then it is released without a tag. Tagging starts when a significant number of fish are first observed and continues through the period of arrival at the spawning ground. The number of fish caught and tagged on a given day is determined either by standardizing the daily application effort or by tagging in proportion to estimated daily abundance; abundance is estimated from the previous day's visual counts on or below the spawning grounds.

After spawning, the fish die and the carcasses are often washed onto the banks of the spawning area. Survey teams walk along the banks looking for carcasses. When a carcass is found, it is examined for a tag. After enumeration, all tags are cut from the carcasses, and only those carcasses are removed from the study area by cutting them into two with a machete and returning them to the river. Untagged carcasses are left where found. Recapture commences when the first dead fish is observed and continues until die-off is complete, and is conducted over the entire spawning area. More surveyors are deployed at peak of carcass abundance than at tails. A total of n_2 carcasses are examined and m_2^* marks are found.

Later in the season, a second team examines some of those carcasses identified as being without tags to check for tags missed in the initial survey. Some of the carcasses will have been washed away by freshets and not all spawning areas are reexamined (typically only those areas with large numbers of carcasses are reexamined). Presumably, the freshet removes carcasses at random, and it is also assumed that marks were uniformly spread over all spawning areas so that it does not matter which spawning areas are reexamined. A total of n_3 carcasses are reexamined (subsampled) and m_3 new tags are found.

Conditional model

Because the initial tagging effort, the recovery effort, and the subsampling effort are controlled by the experimenter, a conditional model for this experiment will be developed. In this conditional model, n_1, n_2, and r are treated as fixed quantities, dependent only on the allocation of resources. The remaining random variables are m_2^* and m_3. Rajwani (1995) also developed results for the unconditional model where n_1 and n_2 are random variables.

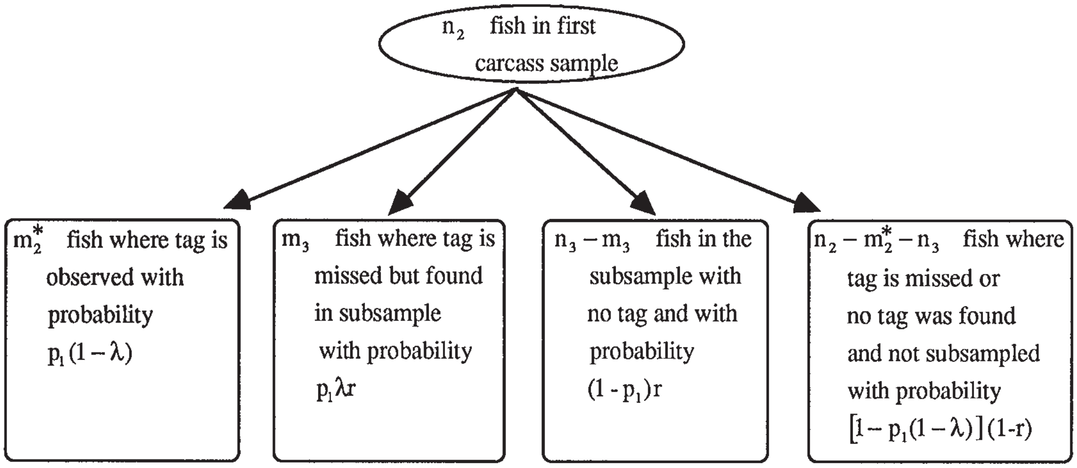
The usual assumptions of mark–recapture experiments are made: the population is closed, i.e., the number of individuals does not change during the study through immigration and (or) emigration; tag loss does not occur during the study; capture and tagging do not affect subsequent catchability or survival of a fish; each fish has a constant and equal probability of capture and recapture during the study; and the subsample is a random sample of those fish initially classified as untagged.

Given these assumptions, each of the n_2 recaptured fish can end up in one of the four categories, as shown in Fig. 1, and given that n_1 is typically small relative to N, the number of fish in these four categories has an approximate multinomial distribution with probability

$$L = \binom{n_2}{m_2^*,\ m_3,\ n_3 - m_3,\ n_2 - m_2^* - n_3} \times (p_1(1-\lambda))^{m_2^*}(p_1\lambda r)^{m_3}((1-p_1)r)^{(n_3 - m_3)} \times ((1 - p_1(1-\lambda))(1-r))^{(n_2 - m_2^* - n_3)}$$

Estimates of N and λ are found by maximizing the log-likelihood function, $\ell = \ln(L)$, and are (Rajwani 1995)

Fig. 1. Diagram of categories into which a recaptured fish can be placed on the basis of tag status, accuracy of identification, and probability of subsampling.



$$(1)\qquad \hat{N} = \frac{n_1 n_2}{(m_2^* n_3 + (n_2 - m_2^*)m_3)/n_3}$$

$$(2)\qquad \hat{\lambda} = \frac{(n_2 m_2^*)}{(m_2^* n_3 + (n_2 - m_2^*)m_3)/m_3}.$$

If we let

$$\hat{m}_2 = \frac{m_2^* n_3 + (n_2 - m_2^*)m_3}{n_3} = m_2^* + \frac{m_3}{r},$$

then eqs. 1 and 2 can be rewritten as

$$\hat{N} = \frac{n_1 n_2}{\hat{m}_2}$$

and

$$\hat{\lambda} = \frac{(n_2 - m_2^*)m_3}{\hat{m}_2 n_3} = \frac{m_3/r}{\hat{m}_2}.$$

Notice that the maximum likelihood estimate for N is similar in format to that of the Petersen estimate and is the same as the moment estimator currently in use.

The variance–covariance matrix derived by Rajwani (1995) is then found by inverting the information matrix to obtain

$$V(\hat{N}) = \frac{N^2(N - n_1)(N\lambda + (N - n_1)(1 - \lambda)r)}{n_1 n_2 r(N - n_1(1 - \lambda))}$$

$$(3)\qquad V(\hat{\lambda}) = \frac{N(1 - \lambda)(N\lambda r + (N - n_1)(1 - \lambda))}{n_1 n_2 r(N - n_1(1 - \lambda))}$$

$$C(\hat{N},\hat{\lambda}) = \frac{-N^2\lambda(N - n_1)(1 - \lambda)(1 - r)}{n_1 n_2 r(N - n_1(1 - \lambda))}.$$

Estimates of these variances and covariance are obtained by replacing parameters with their maximum likelihood estimates. Large sample confidence intervals can be formed in the usual fashion.

Variance inflation factor

If all carcasses were correctly classified when initially examined, the variance for the Petersen estimate (assuming that exactly m_2 tags were present) could be written as (Ricker 1975, p. 78)

$$V(\hat{N}_{\text{Petersen}}) = \frac{N^2(N - n_1)}{n_1 n_2}$$

after replacing random variables with their expected values. The increase in variance caused by misclassification and subsequent subsampling is

$$\text{VIF} = \frac{V(\hat{N})}{V(\hat{N}_{\text{Petersen}})} = \frac{N\lambda + (N - n_1)(1 - \lambda)r}{(N - n_1(1 - \lambda))r} = 1 + \frac{N\lambda(1 - r)}{(N - n_1(1 - \lambda))r}.$$

There is no inflation of the variance (VIF = 1) when no tags are missed in the recapture ($\lambda = 0$) or all carcasses identified as being without tags are reexamined ($r = 100\%$). Also, for fixed values of λ, the VIF monotonically approaches 1 and $V(\hat{N}_{\text{Petersen}})$ approaches $V(\hat{N})$ as the subsampling rate increases. Similarly, for a fixed value of r, the VIF monotonically approaches 1 and $V(\hat{N}_{\text{Petersen}})$ approaches $V(\hat{N})$ as the miss rate decreases.

For large N, $N \approx N - n_1(1 - \lambda)$, and $\text{VIF}_{\text{approx}} = 1 + (\lambda(1 - r)/r)$ is a function only of the miss rate (λ) and the subsample rate (r). Figure 2 is a plot of $\text{VIF}_{\text{approx}}$ for various combinations of the miss rate and subsample rate.

Small-sample bias corrections

It is well known that the usual Petersen estimator is severely positively biased when the number of tags recovered is small. (Technically, the expectation is infinite because of the nonzero probability of observing no tags.) Since our estimator has the same form as a Petersen estimator, it too is biased when the expected number of marks recovered ($E(m_2)$) is small. Following a procedure similar to that in Bailey (1951), it can be shown that

$$E(\hat{N}) \approx N(1 + \text{CV}(\hat{m}_2)) \approx N\left(1 + \frac{1 - p_1 + \lambda\left(\frac{1}{r} - 1\right)}{E(m_2)}\right),$$

where CV(·) is the coefficient of variation. Again following the methods in Bailey (1951), the revised estimator

$$\hat{N}_{\text{corrected}} = \frac{n_1(n_2 + 1)}{(\hat{m}_2 + 1)}$$

removes the first-order bias in the original estimator. However, as seen in simulations later, second-order bias terms are still important in cases where the miss rate is large, the subsampling rate is small, and the expected number of tags is small; the revised estimator still has a large positive bias.

The asymptotic theoretical variance (eq. 3) will be an underestimate of the true variation when few tags are recovered because of the extreme skewness in the distribution of $\hat{N}$. On the other hand, because

Fig. 2. Plot of approximate variance inflation factor ($\mathrm{VIF}_{\mathrm{approx}}$) versus miss rate ($\lambda$) for various subsampling rates r.

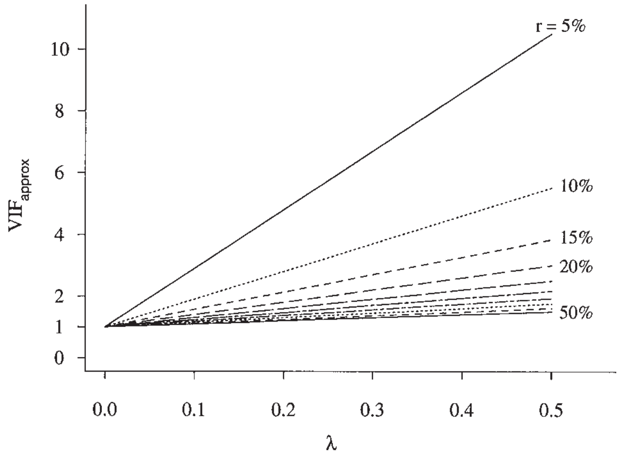
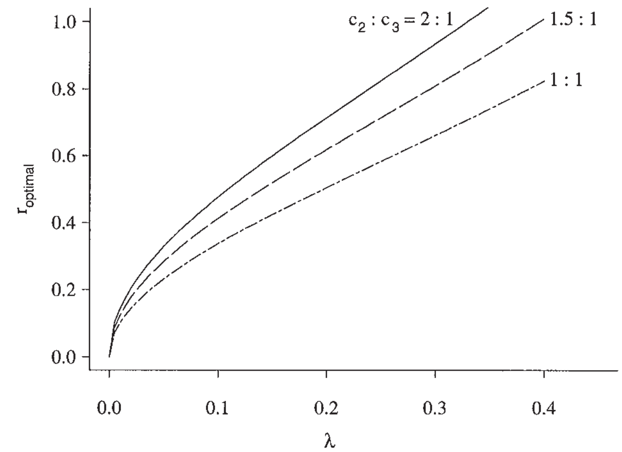


Fig. 3. Plot of optimal subsampling (r_{optimal}) versus miss rate (λ) at various cost ratios (c_2: c_3) with a tagging rate (p_1) of 0.01.



the estimated variance uses $\hat{N}$ (which is positively biased), it is difficult to examine theoretically the performance of the variance estimators.

Unfortunately, because the distribution of $\hat{m}_2$ is not straightforward, there does not appear to be a method analogous to that used by Bailey (1951) to obtain less biased estimators of the variance of $\hat{N}$ in the case of small estimated number of marks. Two alternative estimates of the variance of $\hat{N}$ that may be useful in small samples are to either replace $\hat{N}$ by $\hat{N}_{\mathrm{corrected}}$ in (eq. 3) or use the Bailey (1951) Petersen variance estimator multiplied by the estimated VIF.

Regardless of the form of the variance estimator, they are all a function of $\hat{N}$ and are correlated with $\hat{N}$. Sprott (1981) showed that $\hat{N}^{-1/3}_{\mathrm{Petersen}}$ is much less skewed, and confidence intervals formed on the transformed scale and then back-transformed to the original scale have actual confidence interval coverages close to the nominal level. Consequently, we recommend (on the basis of a large number of simulations as noted later) the following four-step procedure (see also Arnason et al. 1991) to estimate a confidence interval for N in the case of small estimated marks.

1. Compute $\hat{N}_{\mathrm{corrected}}$ and $\hat{V}(\hat{N}_{\mathrm{corrected}})$.

2. Transform to $\hat{N}^{\mathrm{transformed}}_{\mathrm{corrected}} = \hat{N}^{-1/3}_{\mathrm{corrected}}$ and $\hat{V}(\hat{N}^{\mathrm{transformed}}_{\mathrm{corrected}}) = V(\hat{N}_{\mathrm{corrected}}) \times (\hat{N}^{-8/3}_{\mathrm{corrected}})/9$.

3. Compute a confidence interval as $(L_{\mathrm{transformed}}, U_{\mathrm{transformed}}) = \hat{N}^{\mathrm{transformed}}_{\mathrm{corrected}} \pm z_{\alpha/2}(\hat{V}(\hat{N}^{\mathrm{transformed}}_{\mathrm{corrected}})^{1/2}$.

4. Back-transform the lower and upper limits by $(L,U) = (L^{-3}_{\mathrm{transformed}}, U^{-3}_{\mathrm{transformed}})$.

Alternatively, profile likelihood methods could be used to construct a confidence interval directly.

Optimal allocation

The total cost of the survey can be approximated by a linear cost function because carcasses are spread throughout the watershed and the number of carcasses examined is roughly proportional to the total effort expended. Assuming that the number of tags applied (n_1) is fixed, the total cost of the experiment can be written as $C = c_2n_2 + c_3(n_2 - m_2^*)r \le C_0$ and can be solved for r to give $r = (C_0 - c_2n_2)/(c_3(n_2 - m_2^*))$. If this is then substituted for r, m_2^* replaced by its expected value $\mathrm{E}(m_2^*) = n_2p_1(1-\lambda)$, and n_1 replaced by Np_1, then

$$V(\hat{N}) = \frac{N^2c_3(1-p_1)\left(\lambda + \dfrac{(1-p_1)(1-\lambda)(C_0 - c_2n_2)}{c_3n_2(1-p_1(1-\lambda))}\right)}{p_1(C_0 - c_2n_2)},$$

which is now a function of n_2. This can be minimized for a fixed cost, and optimal values for n_2 and r are then found to be

$$n_{2,\mathrm{optimal}} = \left(\frac{C_0}{c_2}\right)\left(\frac{1}{1 + \left(\left(\dfrac{c_3}{c_2}\right)\left(\dfrac{\lambda}{1-\lambda}\right)\left(\dfrac{1-p_1(1-\lambda)}{1-p_1}\right)\right)^{1/2}}\right)$$

$$r_{\mathrm{optimal}} = \left(\left(\frac{c_2}{c_3}\right)\left(\frac{\lambda}{1-\lambda}\right)\left(\frac{1}{(1-p_1)(1-p_1(1-\lambda))}\right)\right)^{1/2}.$$

The optimal quantities depend on the cost ratio (c_2/c_3), the miss rate (λ), and the initial tagging rate (p_1). If p_1 is small, then r_{optimal} can be approximated by

$$r_{\mathrm{optimal,approx}} = \left(\left(\frac{c_2}{c_3}\right)\left(\frac{\lambda}{1-\lambda}\right)\right)^{1/2}.$$

If no tags are missed ($\lambda = 0$), then the optimal strategy is to recapture the maximum possible ($n_{2,\mathrm{optimal}} = C_0/c_2$) and subsample nothing ($r_{\mathrm{optimal}} = 0$). For a fixed cost ratio and tagging rate, the number of fish recaptured declines and the proportion subsampled increases as the miss rate increases. For a fixed total cost, miss rate, and tagging rate the subsampling rate can be increased (because it is cheaper to subsample), and surprisingly, the recapture number can also be increased (because resources saved in the subsample can be diverted to the recapture) as the cost ratio increases. Lastly, for fixed values of cost ratio and miss rate, optimal values of recapture number and subsampling rate appear to be insensitive to changes in the initial tagging rate when p_1 is small. Figure 3 is a plot of the effect of various miss rates and cost ratios on the optimal subsampling rates for a typical tagging rate (p_1) of 0.01.

Results

As described by Fraser River Sockeye Salmon Management Review Team 1994 Spawning Escapement Estimation Working Group (1995, p. 2), the Chilko River is part of the Chilkotin River system, which drains a large portion of the west-central Fraser River watershed. Spawning occurs immediately downstream from the lake in a spawning channel on the upper Chilko River and on the shores along the north and south ends of Chilko Lake. Sockeye first arrive in August, with peak spawning in late September; die-off is complete by late October. The tagging site is located near Lingfield Creek, 5 km below the spawning grounds. Recovery surveys were conducted every 2–3 days in the river and north lake and every week in the south lake. Boat access to the south lake was restricted by weather and the fact that fish in this area spawn in a remote area that is logistically difficult to sample.

Neil Schubert, Department of Fisheries and Oceans, provided the information by sex on the number of tags applied, carcasses recaptured, tags observed in the recapture, carcasses subsampled, and tags observed in the subsample in the 1994 survey. From this information, the estimated miss rate, the estimated escapement size, and the standard errors for the Petersen and Conditional models were calculated (Table 2). As the number of marks observed was large, there is no need to use the bias-corrected estimator or to use the transformation method to find a confidence interval.

Given the subsample and miss rates, the correct standard error is about 20% larger than that for the Petersen estimate. Hence, previous estimates of precision found by treating $\hat{m}_2$ as a known quantity were too small, and a nominal 95% confidence interval for the total escapement treating $\hat{m}_2$ as a known number had serious undercoverage.

It is not possible at this time to go back and obtain exact cost ratios between the recovery and subsample phases of the survey, but the ratio is estimated to be about 2:1. The initial survey was more expensive because crews spent more time extracting partially buried carcasses and searching areas with few carcasses. If the cost ratio is 2:1, then a total of 49 826 (= 45 595 + (0.5)8462) units of effort was required for the male sockeye. Approximately 19% of those carcasses identified as being without tags were subsampled whereas the optimal would have been to subsample 48%. At the optimal subsampling rate, a total of 40 181 carcasses should have been recaptured, compared with 45 595 that were, and the standard error of the estimate of the escapement size could have been reduced by 7%.

A small simulation study was done to illustrate the magnitude of the small sample biases that could be present when the expected number of tags recovered is small. A series of simulated populations of 200 000 fish was generated, with "tags applied" at a rate of $p_1 = 0.01$, values similar to those in the Chilko survey. Values of p_2 were selected to give $E(m_2)$ ranging from 5 to 500. Each set was also simulated with values of λ ranging from 0 to 30% and with values of r ranging from 5 to 40%. For each replicate of each combination of p_1, p_2, $E(m_2)$, λ, and r, the maximum likelihood estimate (MLE) of N, its bias-corrected version, and the three variance estimators (using eq. 3, using eq. 3 with $\hat{N}_{\text{corrected}}$, and $\hat{V}_{\text{Bailey}}(\hat{N}_{\text{Petersen}}) \times \widehat{\text{VIF}}$) were computed. A 95% confidence interval using the suggested four-step method was computed using each variance estimator and the bias-corrected estimate of N and indicator variables were set if the confidence intervals contained the true population value.

After 1000 replications were simulated, the averages of the two estimators of abundance and the three estimators of the variance were computed. The relative bias in the estimates of abundance was computed as $(\bar{\hat{N}} - N)/N$ and $(\bar{\hat{N}}_{\text{corrected}} - N)/N$ and the relative bias in the three estimated variances was found as $(\bar{\hat{V}}_i -$ actual $\text{var}(\hat{N}_{\text{corrected}}))$/actual $\text{var}(\hat{N}_{\text{corrected}})$ where $i = 1$, 2, or 3 corresponds to the three proposed variances and actual $\text{var}(\hat{N}_{\text{corrected}})$ is the actual observed variance of $\hat{N}_{\text{corrected}}$ over the replicates. The coverage of the nominal 95% confidence intervals was estimated by the proportion of intervals that actually included the true value of N.

The relative bias in the estimates of abundance is shown in Fig. 4. The relative bias, even in the corrected estimates, can be severe for a small expected number of tags returned, particularly when the miss rate is large and the subsample rate is small. The bias is generally small when the expected number of tags is over 10 when λ is small and over 40 when λ is large.

Table 2. Summary statistics, estimates, standard errors, and VIF values of mark–recaptures conducted on the Chilko River (the optimal allocation of subsampling and recapture numbers was generated using a 2:1 cost ratio).

	Males	Females
n_1	1 510	2 074
n_2	45 595	63 752
m_2^*	279	467
n_3	8 462	12 708
m_3	6	9
r (%)	19	20
$\hat{\lambda}$	0.10	0.09
$\hat{m}_2$	311	512
Petersen model		
$\hat{N}$	221 284	258 337
$\widehat{\text{SE}}(\hat{N}_{\text{Petersen}})$	12 507	11 377
Conditional model		
$\hat{N}$	221 284	258 337
$\widehat{\text{SE}}(\hat{N}_{\text{Conditional}})$	15 068	13 220
$\widehat{\text{SE}}(\hat{N}_{\text{Conditional}})/\widehat{\text{SE}}(\hat{N}_{\text{Petersen}})$	1.21	1.16
$(\text{VIF}_{\text{approx}})^{1/2}$	1.20	1.16
p_1	0.006	0.008
c_2: c_3	2:1	2:1
Cost (C)	49 826	70 106
r_{optimal} (%)	48	44
$n_{2,\text{optimal}}$	40 181	57 505
$\text{SE}(\hat{N})_{\text{optimal}}$	14 039	12 625
Improvement in SE	1.07	1.05

The relative bias in the estimates of variance is shown in Fig. 5. Substantial bias exists even when large expected numbers of tags are recovered. This is not unexpected, as the variance of the estimates is essentially proportional to N^3, and so even a small positive bias in $\hat{N}$ is magnified when the variance is computed. The last two proposed estimators for the variance (using eq. 3 with $\hat{N}_{\text{corrected}}$, and $\hat{V}_{\text{Bailey}}(\hat{N}_{\text{Petersen}}) \times \widehat{\text{VIF}}$) seem to perform comparably unless the number of marks is small when no estimate seems to work well.

The estimated coverages using Sprott's (1981) procedure are shown in Fig. 6. Surprisingly, the actual coverage was fairly uniform regardless of the size of $E(m_2)$ and which estimator of abundance and variance was used. Coverage was slightly larger than the nominal 95% level.

Discussion

One of the assumptions of a simple Petersen experiment is that all tags are reported. One way of modifying the experiment to account for partial reporting of tags is to carefully examine a subsample of the fish previously classified as without tags and then to use this subsample to estimate the number of tags overlooked.

Fig. 4. Relative bias of the abundance estimators (solid line, bias-corrected; dashed line, MLE).

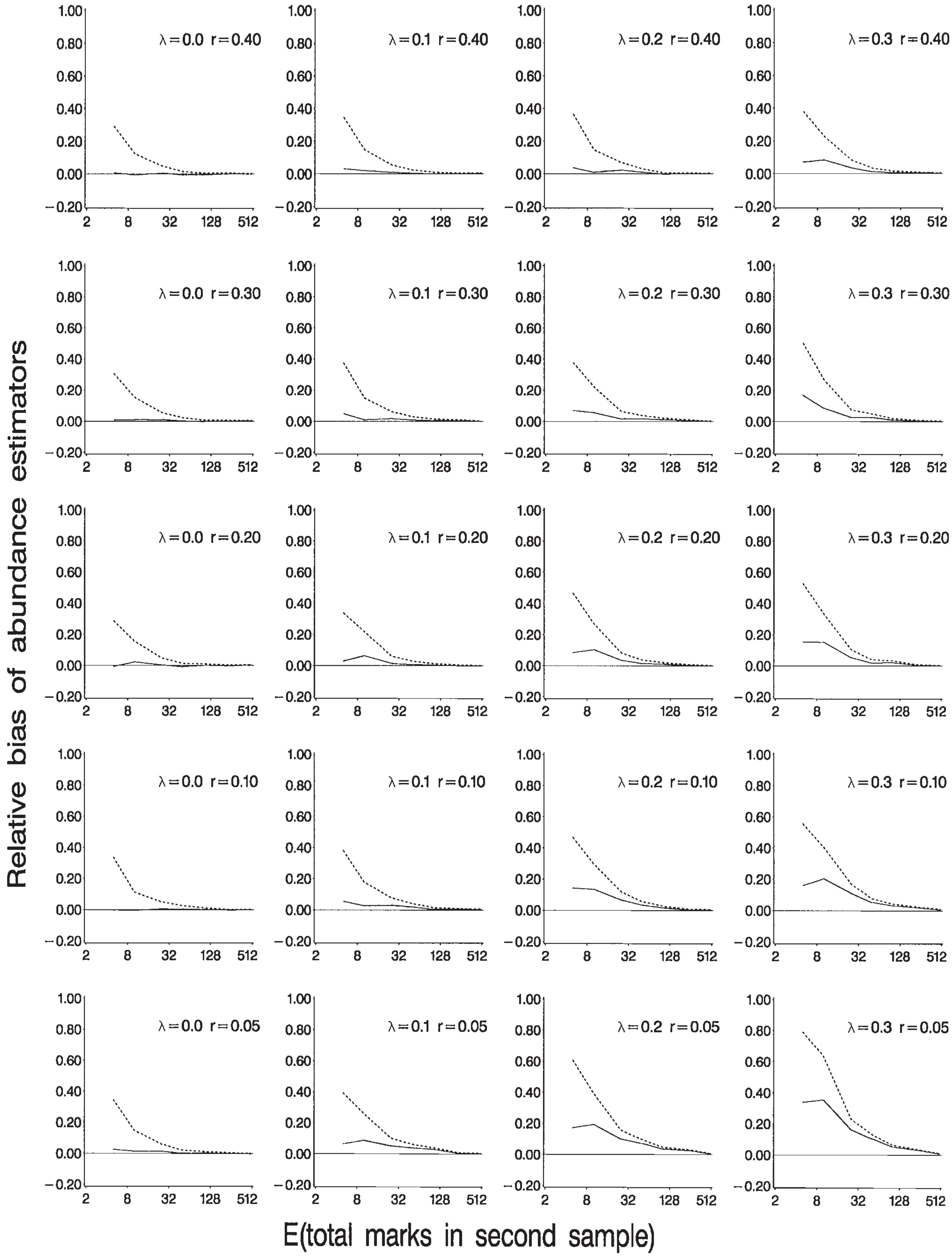


Fig. 5. Relative bias of the variance estimators (solid line, bias-corrected; dashed line, MLE; dash-dotted line, Petersen × VIF).

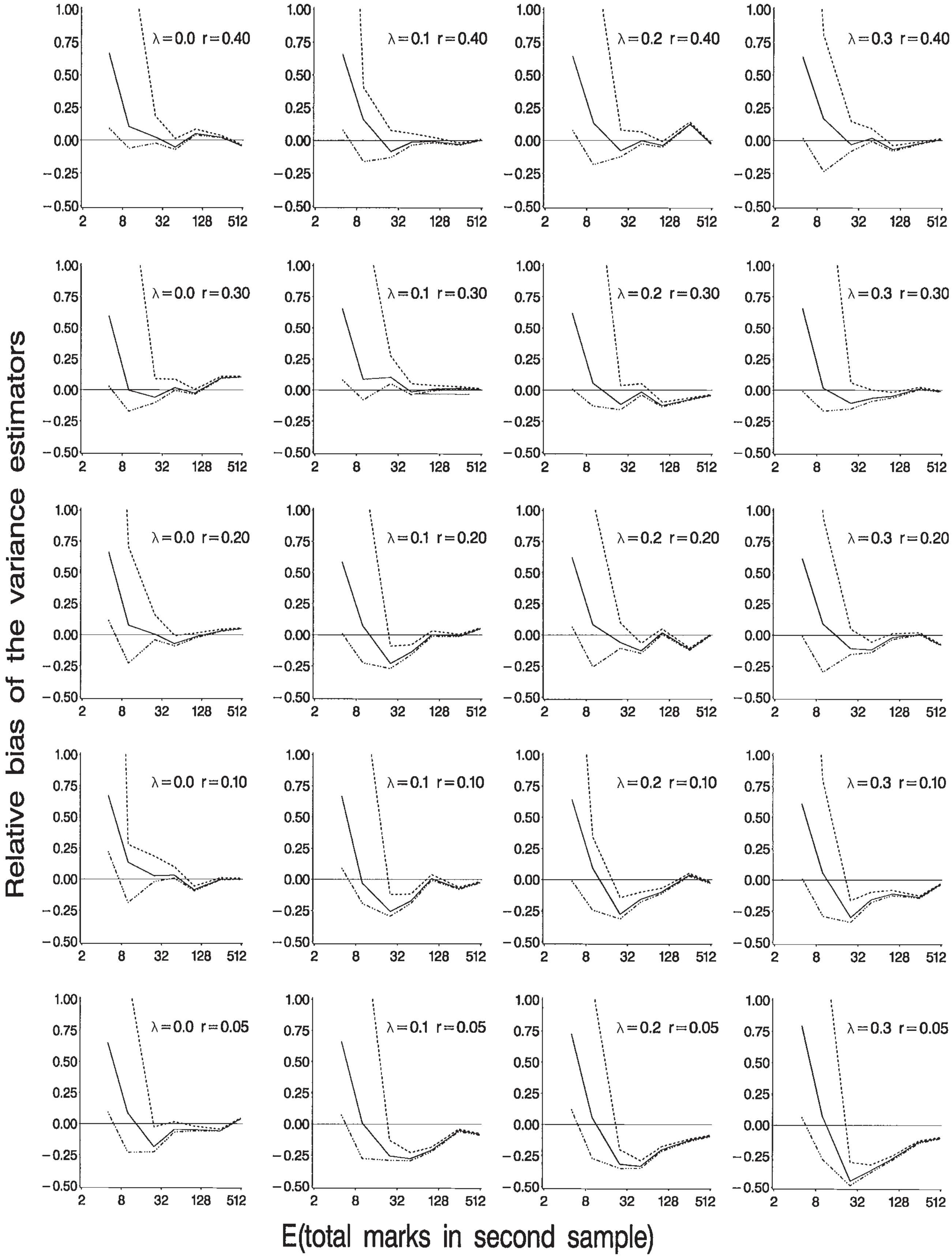
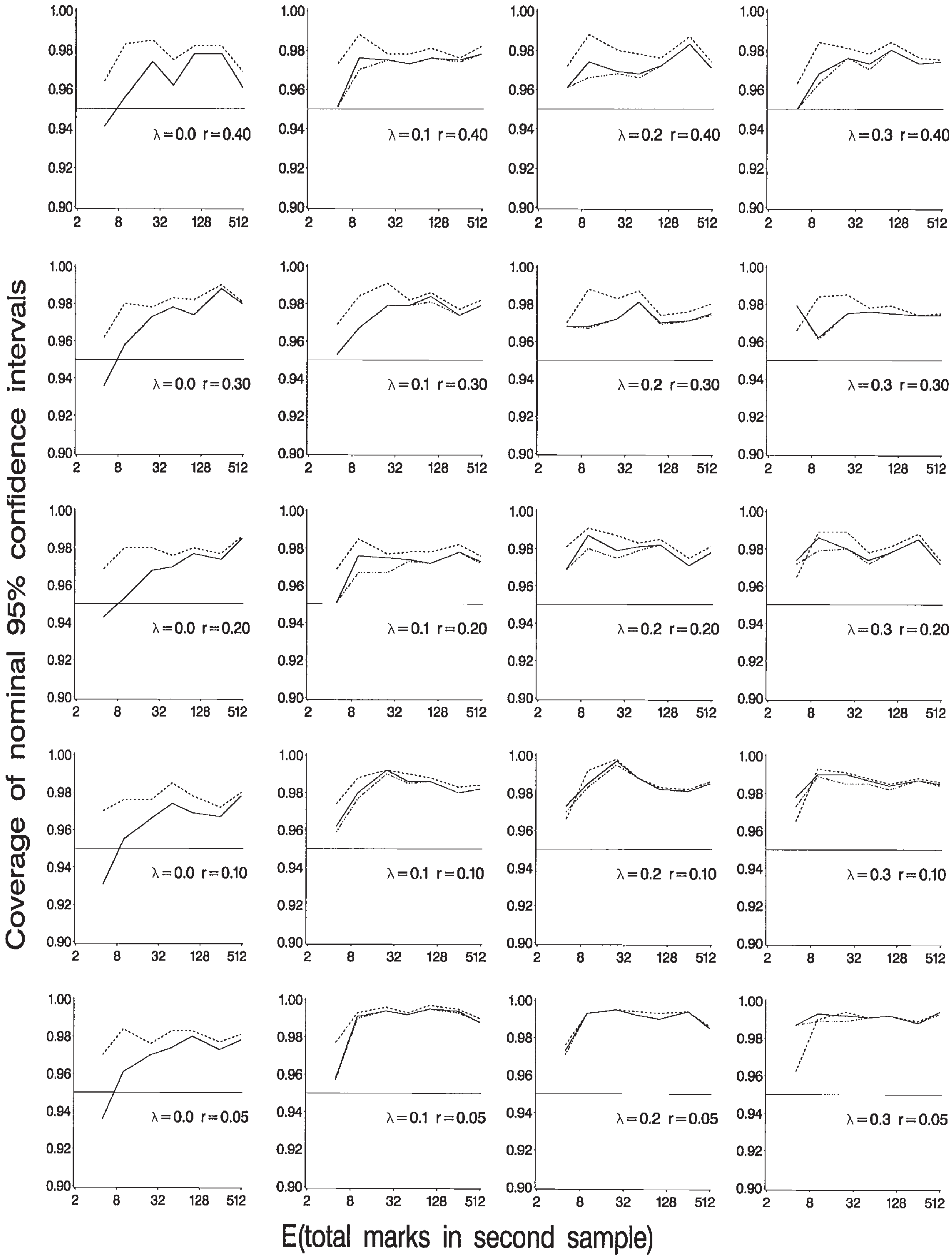


Fig. 6. Coverage of nominal 95% confidence intervals (solid line, bias-corrected; dashed line, MLE; dash-dotted line, Petersen × VIF).



Using this estimate in a simple Petersen estimate results in a better estimate of the escapement but a biased estimate of its standard error. This estimate of the standard error is approximately $(1 + \lambda(1 - r)/r)^{1/2} = (\mathrm{VIF}_{\mathrm{approx}})^{1/2}$ times smaller than the correct estimate of standard error that treats the number of tags found as an estimated value of the total number of tags in the recapture. As with the simple Petersen estimator, this revised estimator can be severely biased when the expected number of tags present is small. We recommend that the expected number of tags in the recapture sample be at least 10 and preferably at least 25.

In some surveys, the whole issue of two-stage sampling can be avoided by a more careful examination of the original sample so that no tags are overlooked. In the example of the sockeye spawning ground surveys, the emphasis could be changed from examining as many carcasses as possible to a more careful initial examination of the carcasses but examining fewer of them so that no tags are overlooked.

However, there are cases where this is not feasible. For example, tags may be voluntarily returned by fishers, and it is impossible to carefully examine the entire catch. A subsample of the catch can be reexamined and the same methods applied to estimate the number of tags overlooked; or, a known number of tags can be "planted" in certain subsamples to estimate the efficiency of the tag-extraction method, e.g., tags found by a factory processing the catch.

Lastly, another method to estimate the number of tags overlooked is the use of reward tags, where a certain percentage of the tags have a reward associated with their return and it is assumed that all recovered reward tags will be returned.

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