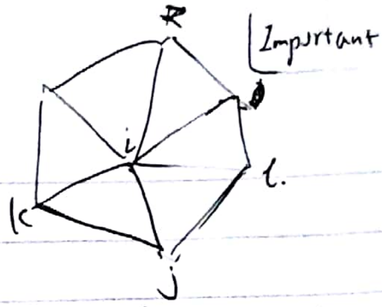
Force calculation

14.Sep.2018 and revised on 17.Sep.2018 Jülicher 1996



✓ $E = K_b \sum_i^{N_v} E_i = K_b \sum 2(H_i - H_0)^2 A_i$ $\quad H_i = \frac{\hat{H}_i}{A_i}$ $> E_i = 2\left(\frac{\hat{H}_i}{A_i} - H_0\right)^2 A_i$ $\quad \hat{H}_i = \frac{1}{4}\sum_e l_e\theta_e$ $> E_i = 2\left(\frac{1}{4A_i}\sum_e l_e\theta_e - H_0\right)^2 A_i$ ✓

$$E_i = 2\left(\frac{1}{16A_i^2}\left(\sum_e l_e\theta_e\right)^2 - \frac{H_0}{2A_i}\sum_e l_e\theta_e + H_0^2\right)A_i \;✓$$

$$= \frac{1}{8A_i}\left(\sum_e l_e\theta_e\right)^2 - H_0\sum_e l_e\theta_e + 2H_0^2A_i \;✓ \qquad A_i = \frac{1}{3}\sum_{t:\langle ijk\rangle} A_t \;✓$$

$$-\frac{\partial E_i}{\partial s} = -\frac{2\sum_e l_e\theta_e}{8A_i}\sum_e\left(\frac{\partial l_e}{\partial s}\theta_e + l_e\frac{\partial\theta_e}{\partial s}\right) + \left(\sum_e l_e\theta_e\right)^2\frac{1}{8A_i^2}\frac{\partial A_i}{\partial s} + H_0\sum_e\left(\frac{\partial l_e}{\partial s}\theta_e + l_e\frac{\partial\theta_e}{\partial s}\right) - 2H_0^2\frac{\partial A_i}{\partial s}$$

$$= -\frac{\sum_e l_e\theta_e}{4A_i}\sum_e\left(\underbrace{\frac{\partial l_e}{\partial s}\theta_e}_{1a} + \underbrace{l_e\frac{\partial\theta_e}{\partial s}}_{1b}\right) + \frac{\left(\sum_e l_e\theta_e\right)^2}{8A_i^2}\underbrace{\frac{\partial A_i}{\partial s}}_{1c} + H_0\sum_e\left(\underbrace{\frac{\partial l_e}{\partial s}\theta_e}_{2a} + \underbrace{l_e\frac{\partial\theta_e}{\partial s}}_{2b}\right) - 2H_0^2\underbrace{\frac{\partial A_i}{\partial s}}_{2c}$$

Implementation: pseudo code.

loop 1: loop over each edge $e:\langle i,j\rangle$ and calculate l_e and θ_e. so that at ~~each~~ each vertices i and j, we ~~have~~ lentheta[i] ~~=~~ lentheta[j] ~~+~~
$\quad$ += $l_e \cdot \theta_e$ $\quad$ += $l_e \cdot \theta_e$.

loop 2: loop over each triangle t and calculate each A_t. and split into $t:\langle i,j,k\rangle$ with
~~$A_i = A$~~: A_i += 1/3 A_t. A_j += 1/3 A_t. A_k += 1/3 A_t.

loop 3: loop over ~~each~~ each edge again $\begin{cases}\text{calculate } \frac{\partial l_e}{\partial s}. \\ \text{calculate } \frac{\partial\theta_e}{\partial s}: \text{ refer to Kantor Nelson force calculation}\end{cases}$

$$l_e = l_{ij} = \left[(X_i - X_j)\cdot(X_i - X_j)\right]^{1/2}$$

$$\frac{\partial l_e}{\partial X_k} = \frac{1}{2}\frac{1}{l_e}\frac{\partial(X_i - X_j)^2}{\partial X} = \frac{(X_i - X_j)}{l_e}\frac{\partial(X_i - X_j)}{\partial X_k} = \frac{(X_i - X_j)}{l_e}(\delta_{ik} - \delta_{jk})$$

loop 4. loop over each triangle t. and calculate $\frac{\partial A_t}{\partial s}$ and split contribution to. i. j. k. vertices

$$\frac{\partial A_i}{\partial s} = \frac{1}{3}\sum\frac{\partial A_t}{\partial s}$$