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$$E^A = K_A' \frac{(A_{tot} - A_{tot}^0)^2}{A_{tot}^0} + K_A^2 \sum_i \frac{(A_i - A_i^0)^2}{A_i^0}$$

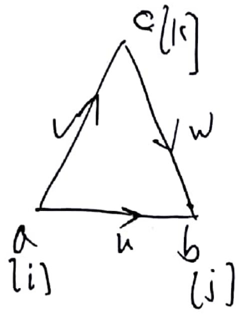
$$\frac{\partial E^A}{\partial X_l} = 2K_A' \frac{(A_{tot} - A_{tot}^0)}{A_{tot}^0}\left(\sum_i \frac{\partial A_i}{\partial X_l}\right) + 2K_A^2 \sum_i \frac{(A_i - A_i^0)}{A_i^0}\left(\frac{\partial A_i}{\partial X_l}\right)$$

Local area. conservation

$$N = \begin{pmatrix} N_x \\ N_y \\ N_z \end{pmatrix}$$

$A_{tot} = \sum_i A_i$

$A_{tot}^0 = \sum_i A_i^0$



$N = u \times v$.

$A_t = \frac{1}{2}|u \times v|$: t is index for triangle.

$$F = -\frac{\partial E^A}{\partial X_l}$$

$$= \frac{1}{2}\left[(u\times v)\cdot(u\times v)\right]^{\frac{1}{2}}$$

$$= \frac{1}{2}(N\cdot N)^{\frac{1}{2}} = \frac{1}{2}(N^2)^{\frac{1}{2}}$$

$u = X_b - X_a$

$v = X_c - X_a$

$$\frac{\partial A_t}{\partial X_l} = \frac{1}{2}\cdot\frac{1}{2}\frac{2\cdot N\cdot\frac{\partial N}{\partial X_l}}{(N\cdot N)^{\frac{1}{2}}}$$

: l is index for position, with which the derivative is taken

$$= \frac{1}{4A_t}\left(N\cdot\frac{\partial N}{\partial X_l}\right)$$

$$= \frac{1}{4A_t}\left[N\cdot\left(\frac{\partial u}{\partial X_l}\times v\right) + N\cdot\left(u\times\frac{\partial v}{\partial X_l}\right)\right]$$

$$= \frac{1}{4A_t}\left[\frac{\partial u}{\partial X_l}\cdot(v\times N) - \frac{\partial v}{\partial X_l}(u\times N)\right]$$

1) [i]: X_l is X_a or X_i

So. $\frac{\partial A_t}{\partial X_a} = \frac{1}{4A_t}\left[-(v\times N) + (u\times N)\right] = \frac{1}{4A_t}(u-v)\times N = \frac{W}{4A_t}\times N$

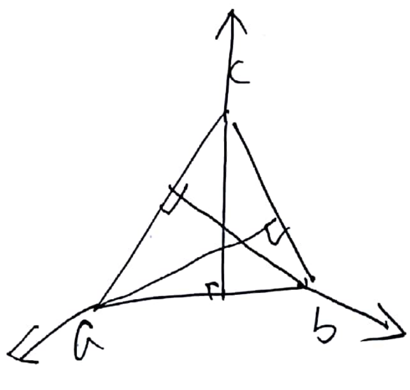
2) [j]: X_l is X_b or X_j.

So. $\frac{\partial A_t}{\partial X_b} = \frac{1}{4A_t}\left[v\times N\right]$

3) [k]: X_l is X_c or X_k.

So $\frac{\partial A}{\partial X_c} = \frac{1}{4A_t}\left[-u\times N\right]$

Note the derivative is always in the plane of triangle and outwards ~~inwards~~ to the triangle to increase the area, which makes sense for the positive derivatives.



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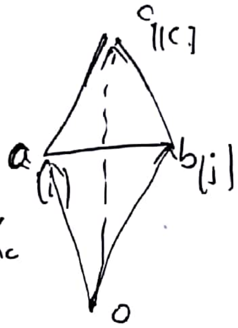
global. volume conservation



$$V_{tot} = \sum_t V_t = \sum_i \frac{1}{6}(X_a \times X_b)\cdot X_c$$

$$E^V = K_V \frac{(V_{tot}-V_{tot}^0)^2}{2V_{tot}^0}. \qquad \frac{\partial E^V}{\partial X_l} = \frac{2K_V}{V_{tot}^0}(V_{tot}-V_{tot}^0)\sum_t \frac{\partial V_t}{\partial X_l}$$

$$\frac{\partial V_t}{\partial X_l} = \frac{1}{6}\frac{\partial\left[(X_a \times X_b)\cdot X_c\right]}{\partial X_l} = \frac{1}{6}\left[\left(\frac{\partial X_a}{\partial X_l}\times X_b\right)\cdot X_c + \left(X_a \times \frac{\partial X_b}{X_l}\right)\cdot X_c + (X_a \times X_b)\cdot \frac{\partial X_c}{\partial X_l}\right]$$

$$= \frac{1}{6}\left\{[X_b \times X_c]\cdot\frac{\partial X_a}{\partial X_l} + (X_c \times X_a)\cdot\frac{\partial X_b}{X_l} + (X_a \times X_b)\cdot\frac{\partial X_c}{\partial X_l}\right)$$

when

1) l : a or [i]. $\frac{\partial V_t}{\partial X_l} = \frac{1}{6}[X_b \times X_c]\cdot\frac{\partial X_a}{\partial X_a} = \frac{1}{6}[X_b \times X_c]$

2) l : b or [j] $\frac{\partial V_t}{\partial X_l} = \frac{1}{6}[X_c \times X_a]\cdot\frac{\partial X_b}{\partial X_b} = \frac{1}{6}[X_c \times X_a)$

3) l : c or [k] $\frac{\partial V_t}{\partial X_l} = \frac{1}{6}[X_a \times X_b]\frac{\partial X_c}{\partial X_c} = \frac{1}{6}(X_a \times X_b)$

$$F = -\frac{\partial E^V}{\partial X_l}.$$