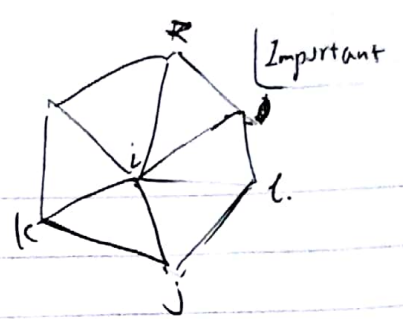Force calculation

14. Sep. 2018 and revised on 17. Sep. 2018   Jülicher 1996

$$\checkmark \quad E = K_b \sum_i^{N_v} E_i = K_b \sum 2(H_i - H_0)^2 A_i \quad > E_i = 2\left(\frac{\hat{H}_i}{A_i} - H_0\right)^2 A_i \quad > E_i = 2\left(\frac{1}{4A_i}\sum_e l_e \theta_e - H_0\right)^2 A_i \checkmark$$

$$H_i = \frac{\hat{H}_i}{A_i}$$

$$\hat{H}_i = \frac{1}{4}\sum_e l_e \theta_e$$

$$E_i = 2\left(\frac{1}{16A_i^2}\left(\sum_e l_e\theta_e\right)^2 - \frac{H_0}{2A_i}\sum_e l_e\theta_e + H_0^2\right)A_i \quad \checkmark$$

$$= \frac{1}{8A_i}\left(\sum_e l_e\theta_e\right)^2 - H_0\sum_e l_e\theta_e + 2H_0^2 A_i \quad \checkmark \qquad A_i = \frac{1}{3}\sum_{t:\langle i,j,k\rangle} A_t \quad \checkmark$$

$$-\frac{\partial E_i}{\partial s}\bigg)\bigg)$$

$$= -\frac{2\sum_e l_e\theta_e}{8A_i}\sum_e\left(\frac{\partial l_e}{\partial s}\theta_e + l_e\frac{\partial\theta_e}{\partial s}\right) + \left(\sum_e l_e\theta_e\right)^2\frac{1}{8A_i^2}\frac{\partial A_i}{\partial s} + H_0\sum_e\left(\frac{\partial l_e}{\partial s}\theta_e + l_e\frac{\partial\theta_e}{\partial s}\right)\overline{\ominus} 2H_0^2\frac{\partial A_i}{\partial s}$$

$$= -\frac{\sum_e l_e\theta_e}{4A_i}\sum_e\left(\underbrace{\frac{\partial l_e}{\partial s}\theta_e}_{1a} + \underbrace{l_e\frac{\partial\theta_e}{\partial s}}_{1b}\right) + \underbrace{\frac{\left(\sum_e l_e\theta_e\right)^2}{8A_i^2}\frac{\partial A_i}{\partial s}}_{1c} + H_0\sum_e\left(\underbrace{\frac{\partial l_e}{\partial s}\theta_e}_{2a} + \underbrace{l_e\frac{\partial\theta_e}{\partial s}}_{2b}\right)\overline{\ominus}\underbrace{2H_0^2\frac{\partial A_i}{\partial s}}_{2c}$$

Implementation: Pseudo code.

loop 1: loop over each edge $l_e^{\langle i,j\rangle}$ and calculate l_e and θ_e so that at each
vertices i and j, we have   len theta [i] $\doteq$ len theta [j] $\neq$
$+= l_e \cdot \theta_e \qquad += l_e \cdot \theta_e$.

loop 2: loop over each triangle t and calculate each A_t and split into $t:\langle i,j,k\rangle$ with
$A_i \doteq A: \quad A_i += \frac{1}{3}A_t$. $\quad A_j += \frac{1}{3}A_t$. $\quad A_k += \frac{1}{3}A_t$.

loop 3: loop over each edge again $\begin{cases}\text{calculate } \frac{\partial l_e}{\partial s}. \\ \text{calculate } \frac{\partial\theta_e}{\partial s}: \text{refer to Kantor & Nelson force calculation}\end{cases}$

$$l_e = l_{ij} = \left[(X_i - V_j)\cdot(X_i - V_j)\right]^{1/2}$$

$$\frac{\partial l_e}{\partial X_k} = \frac{1}{2}\frac{1}{l_e}\frac{\partial(X_i-X_j)^2}{\partial X} = \frac{(X-X_j)}{l_e}\frac{\partial(X_i-X_j)}{\partial X_k} = \frac{(X_i-V_j)(\delta_{ik}-\delta_{jk})}{l_e}$$

loop 4: loop over each triangle t and calculate $\frac{\partial A_t}{\partial s}$ and split contribution to i, j, k vertices.

$$\frac{\partial A_i}{\partial s} = \frac{1}{3}\sum\frac{\partial A_t}{\partial s}$$

Scanned by CamScanner