12. Mar. 2019.

derivative of reduced volume

$$A = 4\pi R^2 \Rightarrow R = \sqrt{\frac{A}{4\pi}}.$$

$$V_s = \frac{4}{3}\pi R^3 = \frac{4}{3}\pi\left(\frac{A}{4\pi}\right)^{\frac{3}{2}} \Rightarrow \frac{dV_s}{dx} = \frac{4}{3}\pi\cdot\frac{3}{2}\cdot\frac{1}{4\pi}\cdot\left(\frac{A}{4\pi}\right)^{\frac{1}{2}}\underbrace{\frac{dA}{dx}}_{\text{primitive}} = \frac{R}{2}\frac{dA}{dx}$$

reduce volume $v = \frac{V}{V_s}.$

Constrain $C = v - \underbrace{v_0}_{\text{target reduced volume}}$

Energy for penalty

$$E = \frac{\mu}{2}C^2 = \frac{\mu}{2}(v-v_0)^2$$

$$\Downarrow$$

$$\frac{dE}{dx} = \mu(v-v_0)\frac{dv}{dx}$$

$$\frac{dv}{dx} = \frac{d\left(\frac{V}{V_s}\right)}{dx} = \frac{1}{V_s}\underbrace{\frac{dV}{dx}}_{\text{primitive}} - \frac{V}{V_s^2}\frac{dV_s}{dx} = \frac{1}{V_s}\frac{dV}{dx} - \frac{VR}{2V_s^2}\frac{dA}{dx}.$$