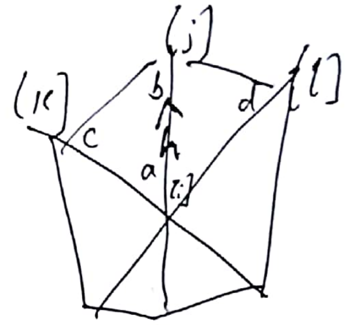
8.6.2018

Total energy



$$E = K_B \sum_i E_i^H + \underbrace{\frac{K_B\tau}{2AD^2}}_{\alpha} (\Delta A - \Delta A_0)^2$$
$$= E^H + E^{ADE}$$

local Helfrich energy.

$$E_i^H = \frac{1}{2} k_B (2H_i - 2H_0)^2 A_i \quad \text{with } H_0 = \frac{C_0}{2}$$

non-local ADE energy.

$$\Delta A = 2D \sum_i H_i A_i \qquad \Delta A_0 = 2D H_0 A$$

$$E^{AD} = \alpha \left[2D \left(\sum_i H_i A_i - H_0 A \right) \right]^2$$

$$H_i = \frac{1}{2} \Delta_s X \cdot n$$
$$\Downarrow$$
$$H_i^2 = \frac{1}{4} (\Delta_s X)^2$$

$$\frac{\partial H_i}{\partial X_k} = \frac{1}{2H_i} \frac{\partial (H_i^2)}{\partial X_k} = \frac{1}{8H_i} \frac{\partial (\Delta_s X^2)}{\partial X_k}$$

$$F = -\frac{\partial E_i^H}{\partial X_k}$$

$$\frac{\partial E_i^H}{\partial X_k} = k_B (2H_i - 2H_0) A_i \cdot 2 \frac{\partial H_i}{\partial X_k} + \underbrace{\frac{1}{2}(2H_i - 2H_0)^2}_{2(H_i - H_0)^2} \frac{\partial A_i}{\partial X_k}$$

$$= 2(H_i - H_0) k_B \left[2A_i \frac{\partial H_i}{\partial X_k} + (H_i - H_0) \frac{\partial A_i}{\partial X_k} \right]$$

$$= \underbrace{2(H_i - H_0) k_B \left[\frac{A_i}{4H_i} \frac{\partial (\Delta_s X^2)}{\partial X_k}\right.}_{\mathrm{I.\ II}} + \underbrace{\left.(H_i - H_0) \frac{\partial A_i}{\partial X_k} \right]}_{\mathrm{III}} \checkmark$$

$$F = \frac{\partial E^{AD}}{\partial X_k}$$

$$\frac{\partial E^{AD}}{\partial X_k} = 2\alpha \left[2D \left(\sum_i H_i A_i - H_0 A \right) \right] \left[2D \sum_i \left(\frac{\partial H_i}{\partial X_k} A_i + H_i \frac{\partial A_i}{\partial X_k} \right) \right]$$

$$= 8\alpha D^2 \left(\sum_i H_i A_i - H_0 A \right) \sum_i \left(\frac{\partial H_i}{\partial X_k} A_i + H_i \frac{\partial A_i}{\partial X_k} \right)$$

$$= \underbrace{\frac{4K_B\tau}{A} \left(\sum_i H_i A_i - H_0 A \right) \sum_i \left[\frac{A_i}{8H_i} \frac{\partial (\Delta_s X^2)}{\partial X_k}\right.}_{\mathrm{I.\ II.}} + \underbrace{\left.H_i \frac{\partial A_i}{\partial X_k} \right]}_{\mathrm{III}} \checkmark$$

$$\Delta_s X_i = \frac{\sum_{ij}(\cot t_1^{ij}+\cot t_2^{ij})(X_i-X_j)}{2A_i = \frac{1}{4}\sum_{ij}(\cot t_1^{ij}+\cot t_2^{ij})\, X_{ij}^2} = 4\frac{\sum_{ij} T_{ij} X_{ij}}{\sum_{ij} T_{ij} X_{ij}^2}$$

$$A_i = \frac{1}{8}\sum_{ij}(\cot t_1^{ij}+\cot t_2^{ij})\,|X_i - X_j|^2 = \frac{1}{8}\sum_{ij} T_{ij} X_{ij}^2$$

$$(\Delta_s X_i)^2 = 16\left[\frac{\sum_{ij}(T_{ij})(X_{ij})}{\sum_{ij} T_{ij} X_{ij}^2}\right]^2 \qquad \frac{1}{8}\sum_{ij} T_{ij} X_{ij}^2 = A_i$$

$$\begin{cases} T_{ij} = (\cot t_1^{ij}+\cot t_2^{ij}) \\ X_{ij} = X_i - X_j \end{cases}$$

i. j. k are indices for position. m is direction such as. x. y. z.

$$\frac{\partial(\Delta_s X_i)^2}{\partial X_k^m} = 32\left(\frac{\sum_{ij} T_{ij} X_{ij}}{\sum_{ij} T_{ij} X_{ij}^2}\right)\cdot\left[\frac{\sum_{ij}\left(\frac{\partial X_{ij}}{\partial X_k^m}T_{ij} + \frac{\partial T_{ij}}{\partial X_k^m}X_{ij}\right)}{\sum_{ij} T_{ij} X_{ij}^2} - \frac{\left(\sum_{ij} T_{ij} X_{ij}\right)\left[\sum_{ij} 2T_{ij}X_{ij}\frac{\partial X_{ij}}{X_k^m} + X_{ij}^2\frac{\partial T_{ij}}{\partial X_k^m}\right]}{\left(\sum_{ij} T_{ij} X_{ij}^2\right)^2}\right]$$

$$= 8\Delta_s X_i \cdot \left[\frac{\sum_{ij}\frac{\partial X_{ij}}{\partial X_k}T_{ij} + \frac{\partial T_{ij}}{\partial X_k}X_{ij}}{8A_i}\right] - 8\Delta_s X_i \frac{1}{4}\cdot\Delta_s X_i \frac{1}{8A_i}\left[\sum_{ij} 2T_{ij}X_{ij}\frac{\partial X_{ij}}{\partial X_k^k} + X_{ij}^2\frac{\partial T_{ij}}{\partial X_k^k}\right]$$

$$= \Delta_s X_i \cdot \frac{\sum_{ij}\frac{\partial X_{ij}}{\partial X_k}T_{ij}}{A_i} - \frac{(\Delta_s X_i)^2}{4A_i}\sum_{ij} 2T_{ij}X_{ij}\cdot\frac{\partial X_{ij}}{X_k^k}$$

$$+ \Delta_s X_i \cdot \frac{\sum_{ij}\frac{\partial T_{ij}}{\partial X_k}X_{ij}}{A_i} - \frac{(\Delta_s X_i)^2}{4A_i}\sum_{ij} X_{ij}^2\frac{\partial T_{ij}}{\partial X_k^m}$$

$$\frac{\partial A_i}{\partial X_k} = \frac{1}{8}\sum_{ij}\frac{\partial T_{ij}}{\partial X_k^m}\cdot X_{ij}^2 + T_{ij}\frac{\partial X_{ij}^2}{\partial X_k^m} = \frac{1}{8}\sum_{ij} 2T_{ij}X_{ij}\cdot\frac{\partial X_{ij}}{\partial X_k}$$

$$+ \frac{1}{8}\sum_{ij} X_{ij}^2\frac{\partial T_{ij}}{\partial X_k}$$