ASSIGNMENT-1.

1. Ans.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
0.1	1.4			
		0.16		
0.2	1.56		0.04	
		0.2		
0.3	1.76		0.04	0
		0.24		
0.4	2.00		0.04	
		0.28		
0.5	2.28			

$$x = x_0 + ph$$
$$= 0.1 + p \times 0.1$$
$$\Rightarrow \quad p = \left(\frac{x - 0.1}{0.1}\right) = 10(x - 0.1) = (10x-1)$$

$$f(x) = 1.4 + (10x-1).(0.16) + \frac{(10x-1)(10x-2)}{2!} \times (0.04)$$
$$= 1.4 + (10x-1)(0.16) + (10x-1)(10x-2)(0.02)$$

(Foward diff polynomial)

Backward diff. polynomial

$$x = 0.5 + p \times 0.1$$
$$\Rightarrow \quad p = \left(\frac{x - 0.5}{0.1}\right) = 10\left(x - \frac{5}{10}\right) = (10x-5).$$

$$f(x) = 2.28 + (10x-5).(0.28) + \frac{(10x-5)(10x-4)}{2!} \times (0.04)$$
$$= 2.28 + (10x-5)(0.28) + (10x-5)(10x-4) \times (0.02)$$
$$= 2.28 + (10x-5)\left[0.28 + 0.2x - 0.08\right].$$
$$= 2.28 + (10x-5) \quad (0.2 + 0.2x)$$
$$= 2.28 + (10x-5)(0.2).(1+x)$$

Taking Gauss's forward intp formula at point $x = 0.$

$0.35 = 0.3 + p \times 0.1$

$\Rightarrow 0.05 = p \times 0.1 \Rightarrow p = \frac{0.05}{0.1} = 0.5.$

$f(0.35) = 1.76 + 0.5 \times 0.24 + \frac{0.5 \times (0.5-1)}{2!} \times (0.04)$

$= 1.875$

Taking Gauss' backward intp. formula at point $x =$

$0.25 = 0.3 + p \times (0.1)$

$\Rightarrow -0.05 = p \times (0.1) \Rightarrow p = -0.5$

~~ffa·~~ $f(0.25) = 1.76 + (-0.5) \times (0.2) + \frac{(-0.5+1)\cdot(-0.5)}{2!} \times 0.04$

$= 1.655.$

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2. ① P.T $\nabla - \Delta = -\Delta\nabla$

Let test funcⁿ be $u(x)$

$\Delta u(x) = u(x+h) - u(x)$

$\nabla u(x) = u(x) - u(x-h)$

$-\Delta u(x) + \nabla u(x).$

$= u(x) - u(x-h) - u(x+h) + u(x)$

RHS $-\Delta\nabla u(x)$

$= -\Delta\{u(x) - u(x-h)\}$

$= -\{u(x+h) - u(x-h+h) - [u(x) - u(x-h)]\}$

$= -\{u(x+h) - u(x) - u(x) + u(x-h)\}$

$= -u(x+h) + u(x) + u(x) - u(x-h)$

$= u(x) - u(x-h) - [u(x+h) - u(x)]$

$= \nabla u(x) - \Delta u(x)$

$\Rightarrow -\Delta\nabla = \nabla - \Delta.$

(ii) $\Delta = E - 1$

$\nabla = 1 - E^{-1}$

$\Delta/\nabla - \nabla/\Delta = \frac{E-1}{1-E^{-1}} - \frac{1-E^{-1}}{E-1}$

$= \frac{E(E-1)}{E-1} - \frac{1-E^{-1}}{E-1}$

$= \frac{E(E-1)}{E-1} - \frac{(E-1)}{E(E-1)} = E - \frac{1}{E}$

$= \frac{\cancel{E^2(E-1) - (E-1)}}{\cancel{E(E-1)}} = E - E^{-1}$

$= E - 1 - E^{-1} + 1$

$= (E-1) + (1-E^{-1}) = \Delta + \nabla$

3.

x	y	Δy	$\Delta^2 y$
0	2.00		
		0.11	
0.1	2.11		0.06
		0.17	
0.2	2.28		−0.06
		0.11	
0.3	2.39		0.06
		0.17	
0.4	2.56		

For Max

~~$0.06 - 2\varepsilon = 0$~~

$\therefore -2\varepsilon = -0.06$

$\Rightarrow \varepsilon = 0.03$

Corrected ~~[illegible]~~ value of y

$= 2.28 - 0.03$

$= 2.25$

④ $u_{-1} = 10,\ u_1 = 8,\ u_2 = 10,\ u_4 = 50$, $\underset{(a)}{u_0}$ and $\underset{(b)}{u_3}$?

4th order diff is zero

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
-1	10				
		a-10			
0	a		8-2a+10 = 18-2a		
		8-a		-24-3a	
1	8		-6+a		b+18+2a
		2		b-6-a	
2	10		b-12		78-4b+a
		b-10		72-3b	
3	b		60-2b		
		50-b			
4	50				

Now, $78 - 4b + a = 0 \Rightarrow$ ~~$a = 4b - 78$~~

and $b + 18 + 2a = 0$

$$b + 18 + 2a = 0$$
$$-8b\ 156 + 2a = 0$$

$$-9b + 138 = 0$$

$$78 \times 2 = 156$$
$$156 - 18 = 138$$

$$\Rightarrow b = \frac{\cancel{138}\ 46}{\cancel{9}\ 3} = \frac{46}{3}.$$

$$a = 4b - 78$$
$$= 4 \times \frac{46}{3} - 78$$
$$= \frac{138 - 234}{3} = \frac{-96}{3} = -32.$$

6.

x	50	52	54	56
y	3·684	3·732	3·779	3·825

Lagrange's Method,

~~$x = \frac{(y-52)(y-54)(y-56)}{(50-52)\ 50-}$~~

7.5.2 ·7, 5. 3 ·(−2) ·3 ·(a−3)(a−5) ·(−6) 92 + 35 + 27 3/35 + (a−6)3

6.

x	50	52	54	56
y	3·684	3·732	3·779	3·825

$$x = \frac{(y-3.732)(y-3.779)(y-3.825)}{(3.684-3.732)(3.684-3.779)(3.684-3.825)} \cdot 50$$
$$+ \frac{(y-3.684)(y-3.779)(y-3.825)}{(3.732-3.684)(3.732-3.779)(3.732-3.825)} \cdot 52$$
$$+ \frac{(y-1.684)(y-3.732)(y-3.825)}{(3.779-3.684)(2.779-3.732)(3.779-3.825)} \cdot 54$$
$$+ \frac{(y-3.684)(y-3.732)(y-3.779)}{(3.825-3.684)(3.825-3.732)(3.825-3.779)} \cdot 56$$

For $y = 3.756$,

$$x = -2.9619 + 28.3198 + 31.3478$$
$$+ -3.6897$$
$$= 53.016 \text{ (appx)}$$

7. $$\frac{x^2 + 6x - 1}{(x^2-1)(x-4)(x-6)} = \frac{x^2 + 6x - 1}{(x-1)(x+1)(x-4)(x-6)}$$

Ans. Let $f(x) = x^2 + 6x - 1$

x	-1	1	4	6
y	-6	6	39	71.

Lagrange's intp. formula gives,

$$f(x) = \frac{(x-1)(x-4)(x-6)}{(-1-1)(-1-4)(-1-6)} \cdot (-6) + \frac{(x+1)(x-4)(x-6)}{(1+1)(1-4)(1-6)} \cdot (6)$$
$$+ \frac{(x+1)(x-1)(x-6)}{(4+1)(4-1)(4-6)} \cdot 39 + \frac{(x+1)(x-1)(x-4)}{(6+1)(6-1)(6-4)} \cdot (71)$$

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$$= \frac{(x-1)(x-4)(x-6)}{}\left(\frac{8}{35}\right) + \frac{(x+1)(x-4)(x-6)}{5}$$

$$+ \frac{(x+1)(x-1)(x-6)}{}\left(\frac{-13}{10}\right) + \frac{(x+1)(x-1)(x-4)}{}\left(\frac{71}{70}\right)$$

dividing by $(x+1)(x-1)(x-4)(x-6)$

$$\frac{x^2+6x-1}{(x+1)(x-1)(x-4)(x-6)} = \frac{3}{35(x+1)} + \frac{1}{5(x-1)} + \frac{-17}{10(x-4)}$$

$$+ \frac{71}{70(x-6)}$$

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8. P.T n^{th} order DD of a polynomial of n^{th} degree are constant.

Ans. Consider a 1 degree polynomial,

$$f_1(x) = a_1 x + a_0$$

$$[x_0\ x_1] = \frac{f(x_1) - f(x_0)}{x_1 - x_0} = \frac{a_1x_1 + a_0 - a_1x_0 - a_0}{x_1 - x_0}$$

$$= \frac{a_1(x_1 - x_0)}{(x_1 - x_0)} = a_1$$

∴ Statement true for $n = 1$

Consider, $f_2(x) = a_2x^2 + a_1x + a_0$

$$[x_0\ x_1\ x_2] = \frac{[x_1, x_2] - [x_0, x_1]}{x_2 - x_0}$$

$$= \frac{\frac{f(x_2) - f(x_1)}{x_2 - x_1} - \frac{f(x_1) - f(x_0)}{x_1 - x_0}}{x_2 - x_0}$$

$$= \frac{\frac{a_2x_2^2 + a_1x_2 + \cancel{a_0} - a_2x_1^2 - a_1x_1 - \cancel{a_0}}{x_2 - x_1} - \left(\frac{a_2x_1^2 + a_1x_1 + \cancel{a_0} - a_2x_0^2 - a_1x_0 - \cancel{a_0}}{x_1 - x_0}\right.}{x_2 - x_0}$$

$$= \frac{\frac{a_2(x_2^2 - x_1^2) + a_1(x_2 - x_1)}{(x_2 - x_1)} - \frac{a_2(x_1^2 - x_0^2) + a_1(}{(x_1 - x_0)}}{x_2 - x_0}$$

$$= \frac{a_2(x_2 + x_1) + \cancel{a_1} - a_2(x_1 + x_0) - \cancel{a_1}}{x_2 - x_0}$$

$$= \frac{a_2x_2 + \cancel{a_2x_1} - \cancel{a_2x_1} - a_2x_0}{x_2 - x_0}$$

$$= \frac{a_2(x_2 - x_0)}{(x_2 - x_0)} = a_2 \text{ (constant)}$$

Slly for $n = 3$, we get

$$[x_0\ x_1\ x_2\ x_3] = a_3$$

On generalizing we have,

for $f(x) = a_nx^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \dots + a_1x + a_0$

the n^{th} order DD $= a_n$ (const).

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10. $f(1) + f(2) + f(3) = 25$

$f(4) = 29$

$f(5) + f(6) = 113$ }

$f(7) = ?$

Ans. $\cancel{\sum_1^3 f(x) \neq x}$ $\sum_1^3 f(x) = 25 = F(3)$

$$\sum_1^4 f(x) = F(4) = 25 + 29 = 54$$

$$\sum_1^6 f(x) = F(6) = 54 + 113 = 167$$

$$f(7) = F(7) - F(6)$$

$$= \sum_1^7 f(x) - F(6)$$

X	$Y = F(X)$
3	25
4	54
6	167

~~mmm~~ intp. with uneq

$\Delta = E - 1$
$E = \Delta + 1$

$\therefore$

$$Y = F(X) = \frac{(X-4)(X-6)}{(3-4)(3-6)} \times 25 + \frac{(X-3)(X-6)}{(4-3)(4-6)} \times 54 + \frac{(X-3)(X-4)}{(6-3)(6-4)} \times 167$$

$$\therefore F(7) = 25 + (-108) + 334$$
$$= 251.$$

$$\therefore f(7) = 251 - 167$$
$$= 84.$$

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12. $\log 2 = 0·3010$, $\log 3 = 0·4771$, $\log 7 = 0·8451$
$\log 13 = 1·1139$, $\log 19 = 1·2788$, $\log 37 = 1·5682$
$\log 37·2$?

Ans. Let $f(x) = \log x$

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
2	0·301					
		0·1761				
3	0·4771		−0·01682			
		0·092		0·0011		
					−0·0000526	
7	0·8451		−0·00472			0·00000136
		0·0448		0·000205		
13	1·1139		−0·00144		−0·000005	
		0·02748		0·000032		
19	1·2788		−0·000475			
		0·01607				
37	1·5682					

$f(x) = 0·301 + (x-2)(0$
$+ (x-2)(x-3)(x-7)·(0·0011)$
$+ (x-2)(x-3)(x-7)(x-13)·(-0·0000526)$
$+ (x-2)(x-3)(x-7)(x-13)(x-19)·(0·00000136)$
$+$

$x = 37·2$
$f(x) = 1·7416$ (appx)

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11. P.T
~~$u_1 x + u_2 x^2 + u_3 x^3 + \cdots\cdots$~~
~~$= \frac{x}{1-x} u_1 + \frac{x^2}{(1-x)^2} \Delta u_2 + \frac{x^3}{(1-x)^3} \cdot \Delta^2 u_1 + \cdots\cdots$~~

~~$\because \Delta = E - 1$~~
~~$\Rightarrow E = \Delta + 1$~~
~~$E^n f(x) = f(x + nh)$~~
~~$E^1 f(x) = f(x+h)$, $E^2 f(x) = f(x+2h)$~~

~~Let the tabular pts for $f(x)$ be~~

x	x_0	x_1	x_2	x_3	x_4	----
$f(x)$	f_0	f_1	f_2	f_3	f_4	

~~$E^0 = f(x) = f_0 = u_0$~~
~~$E^1 = f(x+h) = f_1 = u_1$~~
~~$E^2 = f(x+2h) = f_2 = u_2 \cdots$~~

~~$u_1 x + u_2 x^2 + u_3 x^3 + \cdots$~~
~~$= E^1 x + E^2 x^2 + E^3 x^3 + \cdots\cdots$~~

11. $U_1 x + U_2 x^2 + U_3 x^3 + \cdots\cdots$

$$= \frac{x}{1-x} \cdot U_1 + \frac{x^2}{(1-x)^2} \Delta U_1 + \frac{x^3}{(1-x)^3} \Delta^2 U_1 + \cdots$$

Let first term be U_1 $\quad f(x)$
and let $h=1$

Now, $E(U_1) = U_{1+h} = U_2$

$E(U_2) = U_{2+h} = U_3$

$\Rightarrow E(E(U_1)) = U_3$

$\Rightarrow U_3 = E^2(U_1)$

$U_4 = E^3(U_1)$

$U_5 = E^4(U_1) \ldots$

$U_1 x + U_2 x^2 + U_3 x^3 + \cdots\cdots$

$= U_1 x + E(U_1) x^2 + E^2(U_1) x^3 + E^3(U_1) \cdot x^4 + \cdots\cdots$

~~$= x + E x^2 E + E x^3 E + x^3$~~

$= (x + x^2 E + x^3 E^2 + x^4 E^3 + \cdots\cdots) U_1$

$= x(1 + xE + x^2 E^2 + x^3 E^3 + \cdots\cdots) U_1$

$= x\left(\frac{1}{1-xE}\right) \cdot U_1$ $\qquad [1-(\;)]^{-1}$

$= x\left(\frac{1}{1-x(\Delta+1)}\right) U_1$

$= \left\{\frac{x}{(1-x) - x\Delta}\right\} U_1$

$= \left\{\frac{1}{(1-x) - x\Delta}\right\} x U_1 = \left\{\frac{1}{(1-x)\left(1-\frac{x\Delta}{1-x}\right)}\right\} x U_1$

$$= \frac{x}{(1-x)} \left[1 - \frac{x\Delta}{1-x}\right]^{-1} \cdot U_1 \quad \text{——} \quad ①$$

②

$$(1+x)^{-n} = 1 + (-nx) + \frac{(-n)(-n-1)}{2!} x^2 + \frac{(-n)(-n-1)(-n-2)}{3!} x^3 + \cdots\cdots$$

$$\left(1 - \frac{x\Delta}{1-x}\right)^{-1} = 1 + (-1)\left(\frac{-x\Delta}{1-x}\right) + \frac{(-1)(-1-1)}{2!}\left(\frac{x\Delta}{1-x}\right)^2$$
$$+ \frac{(-1)(-1-1)(-1-2)}{3!}\left(\frac{-x\Delta}{1-x}\right)^3 + \frac{(-1)(-1-1)(-1-2)(-1-3)}{4!}\left(\frac{-x\Delta}{1-x}\right)^4$$
$$+ \cdots\cdots$$

$$= 1 + \left(\frac{x\Delta}{1-x}\right) + \left(\frac{x^2\Delta^2}{(1-x)^2}\right) + \frac{x^3\Delta^3}{(1-x)^3} + \frac{x^4\Delta^4}{(1-x)^4} + \cdots$$

① $\Rightarrow U_1 x + U_2 x^2 + U_3 x^3 + \cdots\cdots$

$$= \frac{x}{(1-x)}\left[1 + \frac{x\Delta}{1-x} + \frac{x^2\Delta^2}{(1-x)^2} + \frac{x^3\Delta^3}{(1-x)^3} + \cdots\cdots\right].$$

$$= \left[\frac{x}{(1-x)} + \frac{x^2\Delta}{(1-x)^2} + \frac{x^3\Delta^2}{(1-x)^3} + \frac{x^4\Delta^3}{(1-x)^4} + \cdots\right] U_1$$

$$= \left(\frac{x}{1-x}\right) \cdot U_1 + \frac{x^2}{(1-x)^2}\Delta U_1 + \frac{x^3\Delta^2 U_1}{(1-x)^3} + \frac{x^4\Delta^3 U_1}{(1-x)^4}$$

$+ \cdots\cdots$
