1) $$\sum_{n=1}^{\infty} \frac{\cos nx}{n} = -\ln\left(2\sin\frac{x}{2}\right) \qquad (0 < x < 2\pi), \qquad \text{[Ch. 3, (14.1)]}$$

2) $$\sum_{n=1}^{\infty} \frac{\sin nx}{n} = \frac{\pi - x}{2} \qquad (0 < x < 2\pi), \qquad \text{[Ch. 1, (13.7)]}$$

3) $$\sum_{n=1}^{\infty} \frac{\cos nx}{n^2} = \frac{3x^2 - 6\pi x + 2\pi^2}{12} \qquad (0 \leqslant x \leqslant 2\pi), \qquad \text{[Ch. 1, (13.8)]}$$

4) $$\sum_{n=1}^{\infty} \frac{\sin nx}{n^2} = -\int_0^x \ln\left(2\sin\frac{x}{2}\right) dx \qquad (0 \leqslant x \leqslant 2\pi), \qquad \text{[See Sec. 11]}$$

5) $$\sum_{n=1}^{\infty} \frac{\cos nx}{n^3} = \int_0^x dx \int_0^x \ln\left(2\sin\frac{x}{2}\right) dx + \sum_{n=1}^{\infty} \frac{1}{n^3} \qquad (0 \leqslant x \leqslant 2\pi)$$

$$\left(\sum_{n=1}^{\infty} \frac{1}{n^3} = \frac{\pi^3}{25.79436\ldots} = 1.20205\ldots\right),$$

which is obtained by term by term integration of the preceding series,

6) $$\sum_{n=1}^{\infty} \frac{\sin nx}{n^3} = \frac{x^3 - 3\pi x^2 + 2\pi^2 x}{12} \qquad (0 \leqslant x \leqslant 2\pi), \qquad \text{[See Sec. 11]}$$

7) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos nx}{n} = \ln\left(2\cos\frac{x}{2}\right) \qquad (-\pi < x < \pi), \qquad \text{[Ch. 3, (14.2)]}$$

8) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n} = \frac{x}{2} \qquad (-\pi < x < \pi), \qquad \text{[Ch. 1, (13.9)]}$$

9) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos nx}{n^2} = \frac{\pi^2 - 3x^2}{12} \qquad (-\pi \leqslant x \leqslant \pi), \qquad \text{[Ch. 1, (13.10)]}$$

10) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n^2} = \int_0^x \ln\left(2\cos\frac{x}{2}\right) dx \qquad (-\pi \leqslant x \leqslant \pi),$$

obtained by term by term integration of the series 7),

11) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\cos nx}{n^3}$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n^3} - \int_0^x dx \int_0^x \ln\left(2\cos\frac{x}{2}\right) dx \qquad (-\pi \leqslant x \leqslant \pi),$$

obtained by term by term integration of the series 10),

12) $$\sum_{n=1}^{\infty} (-1)^{n+1} \frac{\sin nx}{n^3} = \frac{\pi^2 x - x^3}{12} \qquad (-\pi \leqslant x \leqslant \pi),$$

obtained by term by term integration of the series 9),

13) $$\sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{2n+1} = -\frac{1}{2}\ln\tan\frac{x}{2} \qquad (0 < x < \pi),$$

obtained by addition of the series 1) and 7),

14) $$\sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{2n+1} = \frac{\pi}{4} \qquad (0 < x < \pi), \qquad \text{[Ch. 1, (13.11)]}$$

15) $$\sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{(2n+1)^2} = \frac{\pi^2 - 2\pi x}{8} \qquad (0 \leqslant x \leqslant \pi), \qquad \text{[Ch. 1, (13.12)]}$$

16) $$\sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{(2n+1)^2} = -\frac{1}{2}\int_0^x \ln\tan\frac{x}{2}\, dx \qquad (0 \leqslant x \leqslant \pi),$$

obtained by term by term integration of the series 13),

17) $$\sum_{n=0}^{\infty} \frac{\cos(2n+1)x}{(2n+1)^3}$$

$$= \frac{1}{2}\int_0^x dx \int_0^x \ln\tan\frac{x}{2}\, dx + \sum_{n=0}^{\infty} \frac{1}{(2n+1)^3} \qquad (0 \leqslant x \leqslant \pi),$$

obtained by term by term integration of the series 16),

18) $$\sum_{n=0}^{\infty} \frac{\sin(2n+1)x}{(2n+1)^3} = \frac{\pi^2 x - \pi x^2}{8} \qquad (0 \leqslant x \leqslant \pi),$$

obtained by term by term integration of the series 15).

If in the formulas 13) through 18), we replace x by t and then set $t = \frac{1}{2}\pi - x$, we obtain the expansions

19) $$\sum_{n=0}^{\infty} (-1)^n \frac{\cos(2n+1)x}{2n+1} = \frac{\pi}{4} \qquad \left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right),$$

20) $$\sum_{n=0}^{\infty} (-1)^n \frac{\sin(2n+1)x}{2n+1} = -\frac{1}{2}\ln\tan\left(\frac{\pi}{4} - \frac{x}{2}\right)$$

$$\left(-\frac{\pi}{2} < x < \frac{\pi}{2}\right),$$

21) $$\sum_{n=0}^{\infty} (-1)^n \frac{\cos(2n+1)x}{(2n+1)^2} = -\frac{1}{2}\int_0^{\frac{1}{2}\pi - x} \ln\tan\frac{t}{2}\, dt$$

$$\left(-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}\right),$$

22) $$\sum_{n=0}^{\infty} (-1)^n \frac{\sin(2n+1)x}{(2n+1)^2} = \frac{\pi x}{4} \qquad \left(-\frac{\pi}{2} \leqslant x \leqslant \frac{\pi}{2}\right),$$