Correlation Analysis

Course Taught at SUAD

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This presentation includes…

- Introduction to Relationship Analysis
- Correlation Analyses
- Measures of Correlations
  - Karl Pearson’s correlation coefficient
  - Charles Spearman’s correlation coefficient
  - Chi-square coefficient of correlation
  - Other types of correlations

Data for Relationship Analysis

Univariate population: The population consisting of only one variable.

Example:

Temperatur	20	30	21	18	23	45	52

Here, statistical measures suffice to find a relationship.

Bivariate population: Here, the data happen to be with two variables.

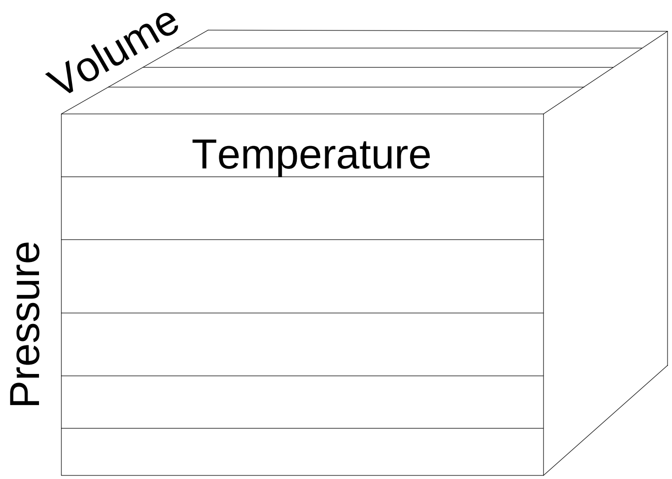
Example:

Pressure	1	1.1	… …	0.8
Temperatu	35	41		29

Data for Relationship Analysis

Multivariate population: If the data happen to be one more than two variable.
.

Example:



If we add another variable say viscosity in addition to Pressure, Volume or Temperature?

Measures of Relationship

In case of bivariate and multivariate populations, usually, we have to answer two types of questions:

Q1: Does there exist relation between two variables (in case of bivariate population) ?

- If yes, of what degree?

Q2: Is there any relationship between one variable in one side and two or more variables on the other side (in case of multivariate population)?

- If yes, of what degree and in which direction?

Measures of Relationship

In case of bivariate and multivariate populations, usually, we have to answer two types of questions:

Q1: Does there exist relation between two variables (in case of bivariate population) ?

Q2: Is there any relationship between one variable in one side and two or more variables on the other side (in case of multivariate population)?

Solution

?

Measures of Relationship

In case of bivariate and multivariate populations, usually, we have to answer two types of questions:

Q1: Does there exist relation between two variables (in case of bivariate population) ?

Q2: Is there any relationship between one variable in one side and two or more variables on the other side (in case of multivariate population)?

To find solutions to the above questions, two approaches are known.

- **Correlation Analysis**
- **Regression Analysis**

Correlation Analysis

Correlation Analysis

In statistics, the word correlation is used to denote some form of association between two variables.

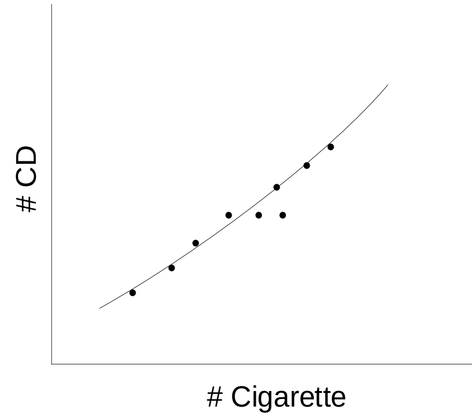
Example: Weight is correlated with height

Correlation Analysis

Do you find any correlation between X and Y as shown in the table?

Look at the table given below

No. of CD's sold in shop X	**25**	**30**	**35**	**42**	**48**	**52**	**56**
No. of cigarette sold in Y	5	7	9	10	11	11	12



Note

- In data analytics, a correlation analysis makes sense only when a relationship makes sense.
- Correlation does NOT imply causation.

Correlation Analysis

A	a_1	a_2	a_3	a_4	a_5	a_6	… …
B	b_1	b_2	b_3	b_4	b_5	b_6	… …

Correlation

Positive correlation

If the value of the attribute **A** increases with the increase in the value of the attribute **B** and vice-versa.

Negative correlation

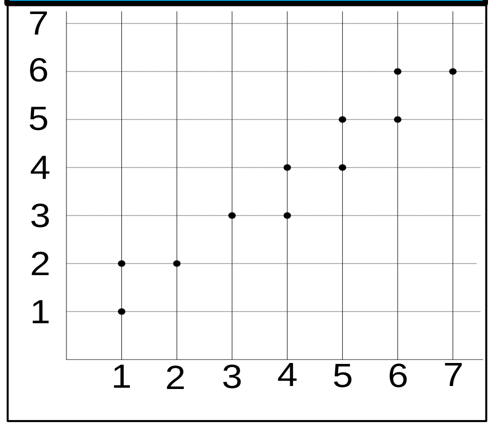
If the value of the attribute **A** decreases with the increase in the value of the attribute **B** and vice-versa.

Zero correlation

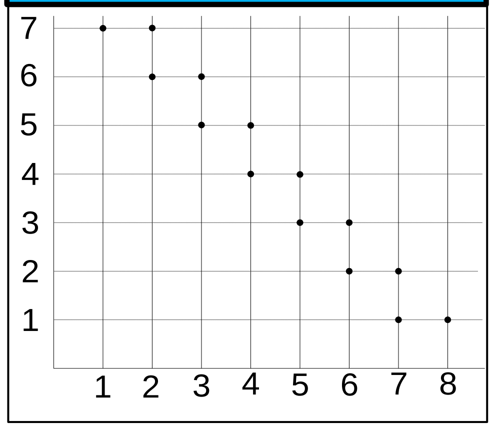
When the values of attribute **A** varies at random with **B** and vice-versa.

Correlation Analysis

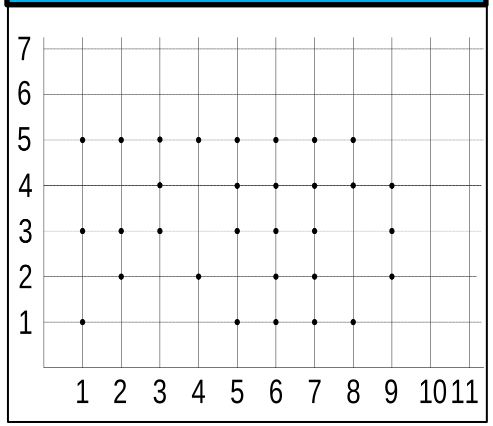
Positive correlation



Negative correlation



Zero correlation

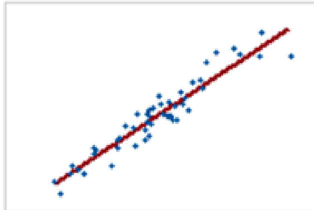


Form of Correlation

Concerning the form of a correlation, it could be linear, non-linear, or monotonic.

Linear Correlation

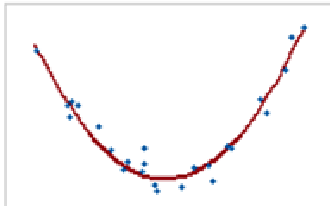
A correlation is linear when two variables change at constant rate.



Linear Correlation

Non-linear Correlation

In this case, the relationship between the variables graph as a curved pattern parabola, hyperbola ... etc).



Non-linear Correlation

Form of Correlation

Concerning the form of a correlation , it could be linear, non-linear, or **monotonic.**

Monotonicity of a function

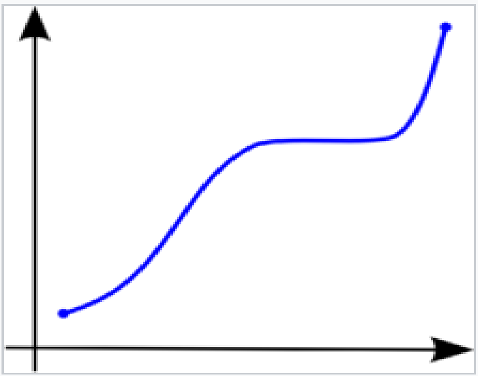


Figure 1. A monotonically increasing function.

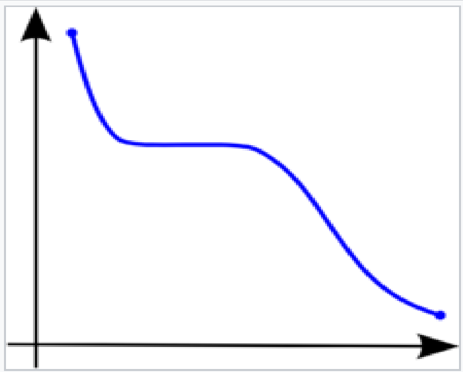


Figure 2. A monotonically decreasing function

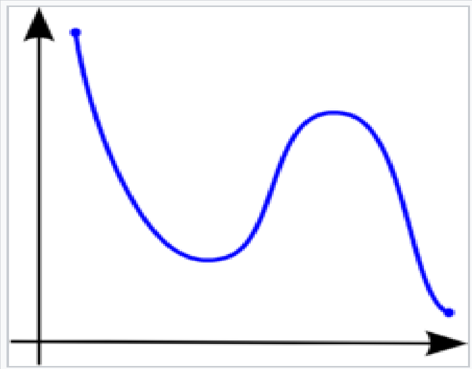


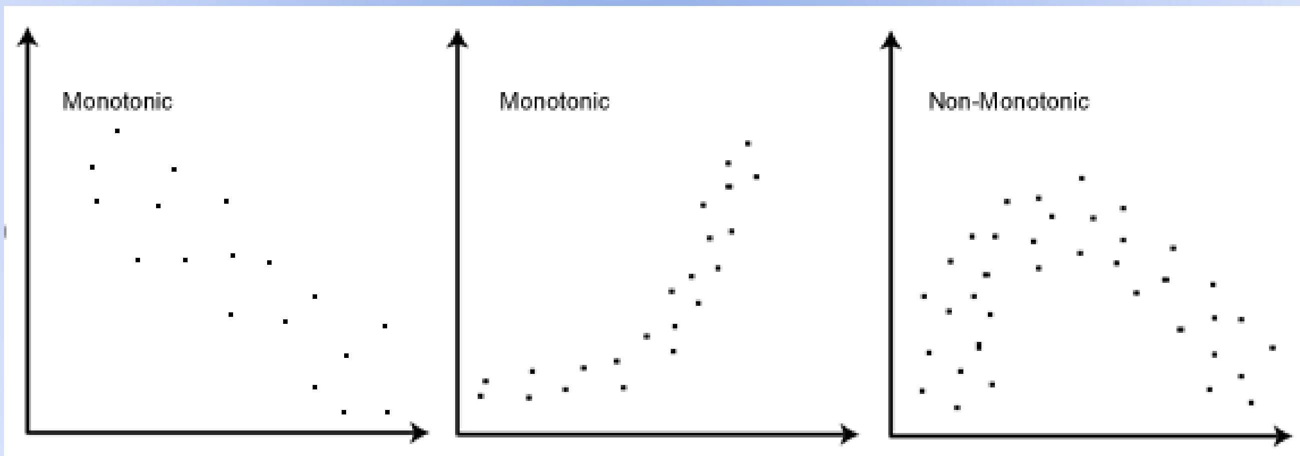
Figure 3. A function that is not monotonic

Form of Correlation

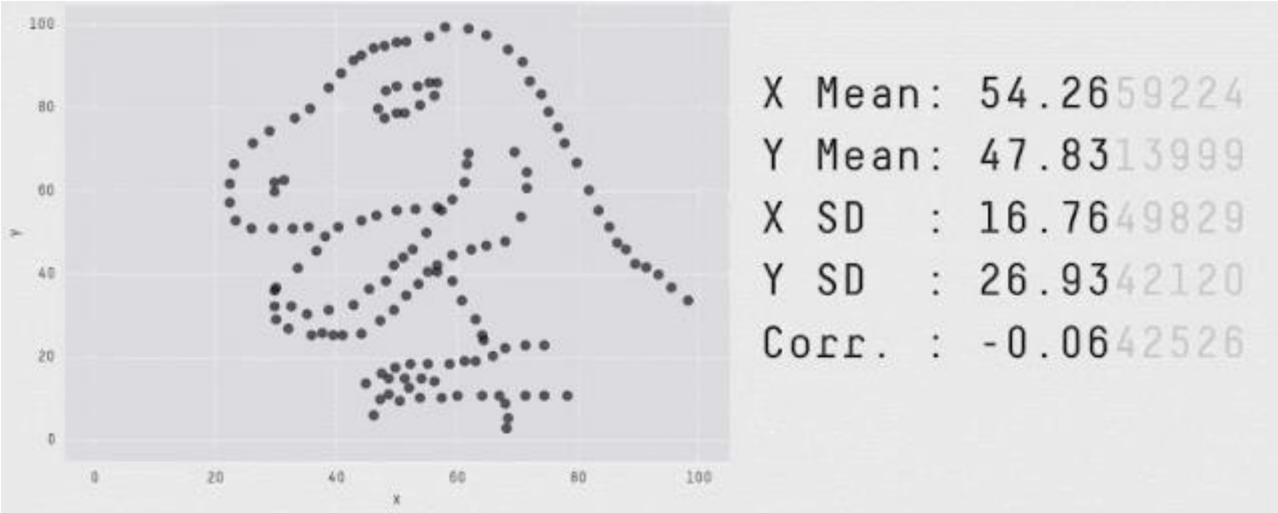
Concerning the form of a correlation , it could be linear, non-linear, or **monotonic** :

Monotonic and non-monotonic relations

Monotonic correlation: In a monotonic relationship, the variables tend to move in the same relative direction or opposite direction, but not necessarily at a constant rate.



Scatter Plot tells stories

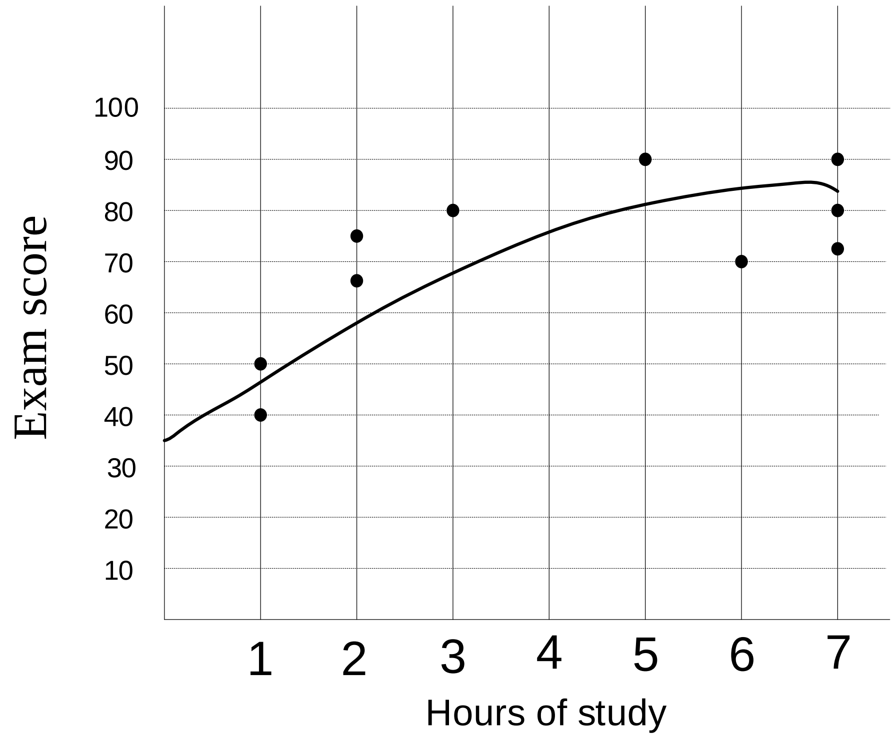


- In 1973, a famous statistician, Francis Anscombe, demonstrated how important it is to visualize the data. The concept got extended later to create Datasaurus Dozen.
- It is a collection of 12 scatterplots with the same means, standard deviations, and correlation coefficient for X and Y (up to 2 decimal places).
- However, the shape of the data is very different from each other. Therefore, the scatterplots tell very different stories about the behavior and interrelationships of X and Y.
- Data available at https://cran.r-project.org/web/packages/datasauRus/vignettes/Datasaurus.html

Correlation Analysis

We need to measure the degree of correlation between two attributes.

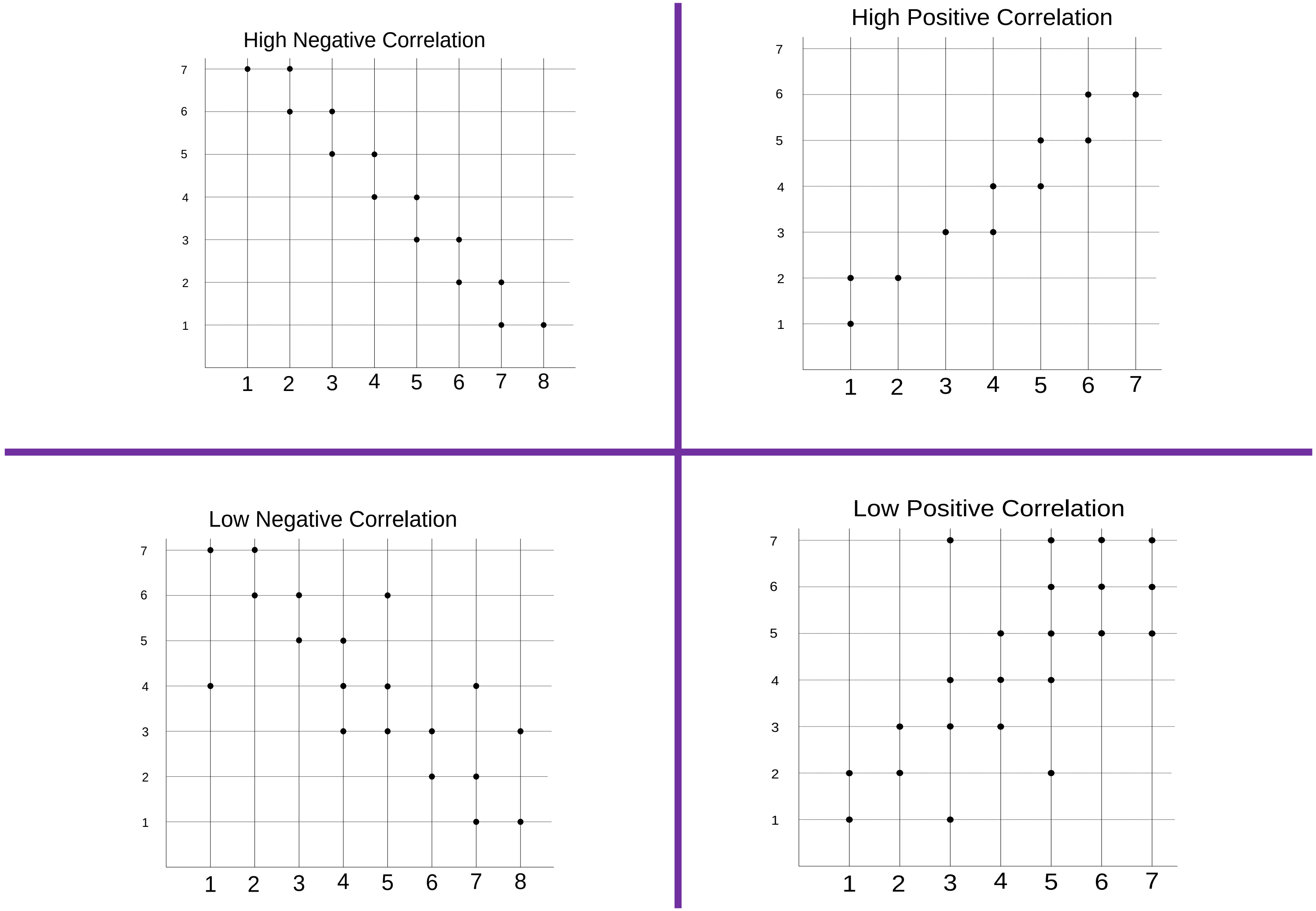
Hours Study	***Exam Score***
3	80
5	90
2	75
6	80
7	90
1	50
2	65
7	85
1	40
7	100



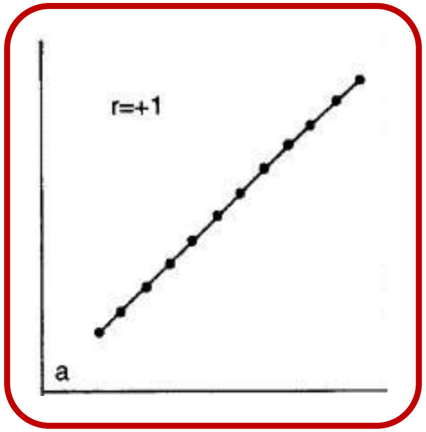
Correlation Coefficient

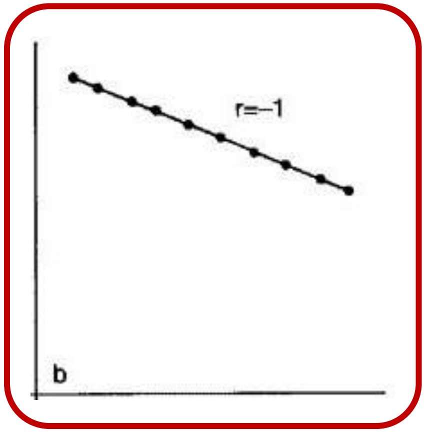
- **Correlation coefficient is used to measure the degree of association.**
- It is usually denoted by r.
- The value of r lies between +1 and -1.
- Positive values of r indicates positive correlation between two variables, whereas, negative values of r indicate negative correlation.
- $r = +1$ implies **perfect positive correlation**, and otherwise.
- The value of r nearer to +1 or -1 indicates **high degree of correlation** between the two variables.
- $r = 0$ implies, there is **no correlation**

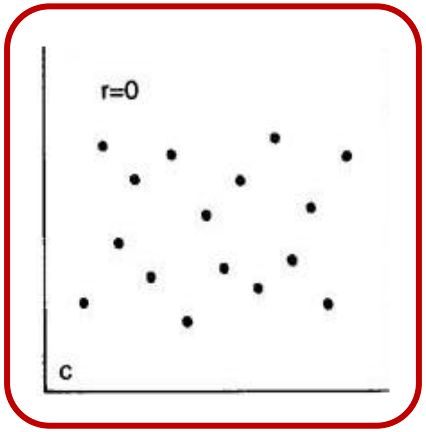
Correlation Coefficient

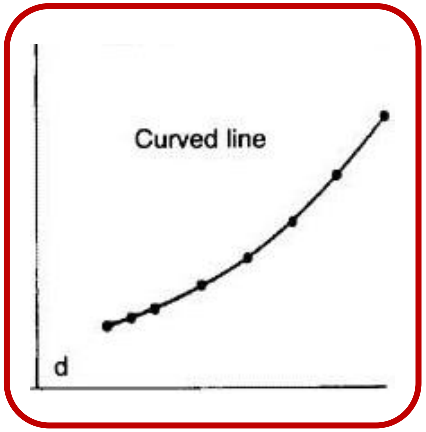


Correlation Coefficient

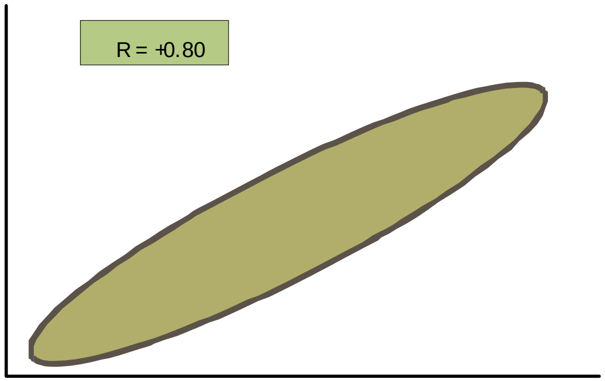


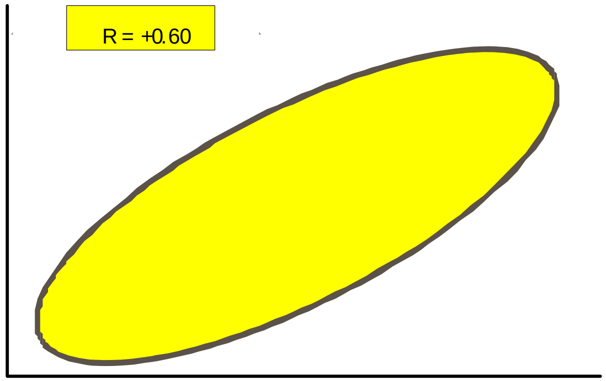


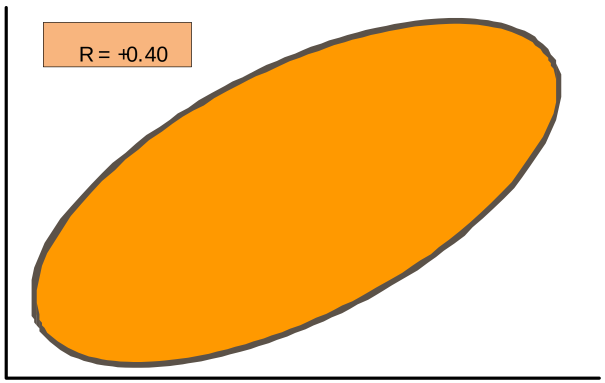


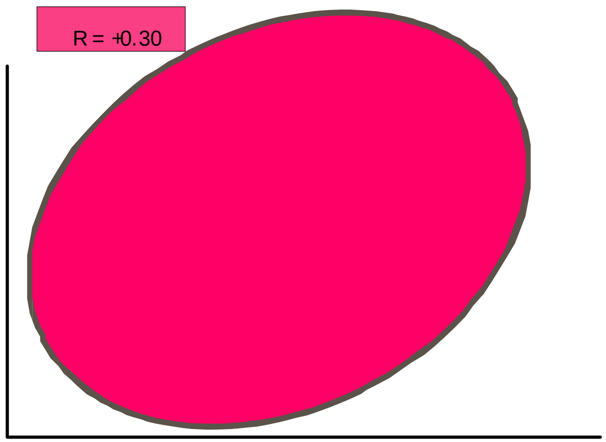


Correlation Coefficient









Measuring Correlation Coefficients

Three methods to measure the correlation coefficients

Karl Pearson's coefficient

Find correlation coefficient between two **numerical** attributes

Charles Spearman's coefficient

Find correlation coefficient between two **ordinal** attributes

Chi-square coefficient of correlation

Find correlation coefficient between two **nominal** attributes

Pearson’s Correlation Analysis

Karl Pearson's Correlation Analysis

This is also called Pearson's Product Moment Correlation

Definition : Karl Pearson's correlation coefficient

Let us consider two attributes are X and Y.

The Karl Pearson's coefficient of correlation is denoted by r^* and is defined as

$$r^* = \frac{cov(X,Y)}{\sigma_x \sigma_y} = \frac{\sum(X_i - \bar{X})(Y_i - \bar{Y})}{\sqrt{\sum(X_i - \bar{X})^2}\sqrt{\sum(Y_i - \bar{Y})^2}}$$

where

X_i = i – th value of X – variable, $\bar{X}$ = mean of X

Y_i = i – th value of Y – variable, $\bar{Y}$ = mean of Y

n = number of pairs of observation of X and Y

$\text{cov}(X, Y)$ = covariance of X *and* Y, σ_X = SD of X, σ_Y = SD of Y

Karl Pearson's Coefficient of Correlation

Example : Correlation of Gestational Age and Birth Weight

A small study is conducted involving 17 infants to investigate the association between gestational age at birth, measured in weeks, and birth weight, measured in grams.

Infant ID #	Gestational Age (wks)	Birth Weight (gm)
1	34.7	1895
2	36.0	2030
3	29.3	1440
4	40.1	2835
5	35.7	3090
6	42.4	3827
7	40.3	3260
8	37.3	2690
9	40.9	3285
10	38.3	2920
11	38.5	3430
12	41.4	3657
13	39.7	3685
14	39.7	3345
15	41.1	3260
16	38.0	2680
17	38.7	2005

Karl Pearson's coefficient of Correlation

Example : Correlation of Gestational Age and Birth Weight

A small study is conducted involving 17 infants to investigate the association between gestational age at birth, measured in weeks, and birth weight, measured in grams.

We wish to estimate the association between gestational age and infant birth weight.

Birth weight → Dependent variable

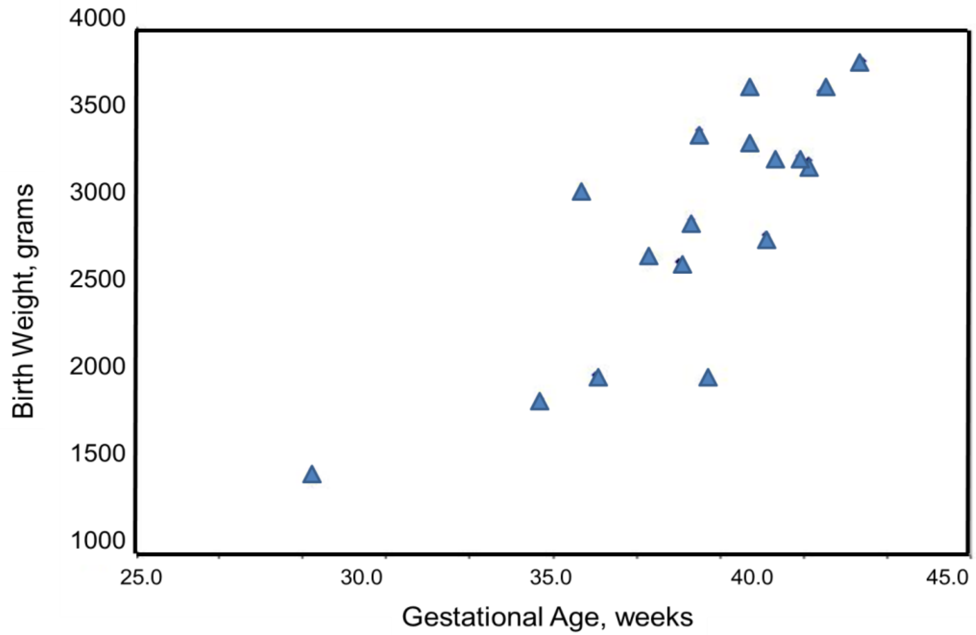
Gestational age → Independent variable

Thus

$$Y = birth\ weight \text{ and}$$

$$X = gestational\ age$$

The data are displayed in the scatter diagram.



Karl Pearson's coefficient of Correlation

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14	39.7	3345
15	41.1	3260
16	38.0	2680
17	38.7	2005

For the given data

- $\bar{X} = \frac{\sum X}{n} = \frac{652.1}{17} = \mathbf{38.4}$
- $\bar{Y} = \frac{\sum Y}{n} = \frac{49334}{17} = \mathbf{2902}$
- $S_x^2 = \frac{\sum(X-\bar{X})^2}{n-1} = \frac{159.45}{16} = \mathbf{9.97}$
- $S_y^2 = \frac{\sum(Y-\bar{Y})^2}{n-1} = \frac{7767660}{16} = \mathbf{485578.8}$
- $r^* = \frac{\sum(X_i-\bar{X})(Y_i-\bar{Y})}{\sqrt{\sum(X_i-\bar{X})^2}\sqrt{\sum(Y_i-\bar{Y})^2}} = \mathbf{0.82}$

Conclusion: The sample's correlation coefficient indicates a strong positive correlation between Gestational Age and Birth Weight.

Significance Test

Definition : Karl Pearson's correlation coefficient

- Say we have an n sized sample data with two variables x and y.
- The sample correlation coefficient (r) between x and y is known
- The population correlation coefficient ρ between x and y is unknown
- **Goal:** We want to make an inference about the value of ρ based on r

$$\text{Null hypothesis } H_0\text{: } \rho = r$$

$$\text{Alternative hypothesis } H_1\text{: } \rho \neq r$$

Karl Pearson's Coefficient of Correlation

Significance Test

- To test whether the association is merely apparent, and might have arisen by chance use the *t* test in the following calculation

$$t = r\sqrt{\frac{n-2}{1-r^2}}$$

- Here, the number of pair of observation is 17. Hence,

$$t = 0.82\sqrt{\frac{17-2}{1-0.82^2}} = 1.44$$

- Consulting the t-test table, at degrees of freedom 15 and for $\alpha = 0.05$, we find that t = 1.753.
- Thus, the value of Pearson's correlation coefficient in this case indicates that we fail to reject the null hypothesis.

Rank Correlation Analysis

Charles Spearman's Correlation Coefficient

This correlation measurement is also called Rank correlation

- This technique is applicable to determine the degree of correlation between two variables in case of ordinal data.
- We can assign rank to the different values of a variable with ordinal data type.

Example

Height: [VS S N T VT]
5 4 3 2 1

T – shirt: [XXL XL L S VS]
1 2 3 4 5

Rank assigned

Charles Spearman's Correlation Coefficient

Definition: Charles Spearman's correlation coefficient

The rank correlation can be defined as

$$r_s = 1 - \frac{6 \sum d_i^2}{n(n^2 - 1)}$$

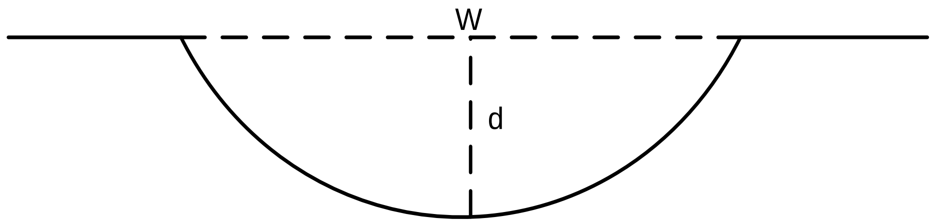
where d_i = Difference between ranks of i^{th} pair of the two variables

n = Number of pairs of observations

- The Spearman's coefficient is often used as a statistical methods to aid either providing or disproving a hypothesis.

Charles Spearman’s Coefficient of Correlation

Example: The hypothesis that the depth of a river **does not progressively increase** further from the bank.



A sample of size 10 is collected to test the hypothesis, using Spearman’s correlation coefficient.

Sample#	*Width in m*	*Depth in m*
1	0	0
2	50	10
3	150	28
4	200	42
5	250	59
6	300	51
7	350	73
8	400	85
9	450	104
10	500	96

Charles Spearman’s Coefficient of Correlation

Step 1: Assign rank to each data. It is customary to assign rank 1 to the largest data, and 2 to next largest and so on.

Note: If there are two or more samples with the same value, the mean rank should be used.

$Data$	20	25	25	25	30
$Assign\ rank$	5	4	3	2	1
$Final\ rank$	5	3	3	3	1

Charles Spearman's Coefficient of Correlation

Step 2: The contingency table will look like

Sample	*Width*	*Width r*	*Depth*	*Depth r*	d	d^2
1	0	10	0	10	0	0
2	50	9	10	9	0	0
3	150	8	28	8	0	0
4	200	7	42	7	0	0
5	250	6	59	5	1	1
6	300	5	51	6	-1	1
7	350	4	73	4	0	0
8	400	3	85	3	0	0
9	450	2	104	1	1	1
10	500	1	96	2	-1	1

$$\sum d^2 = 4$$

$$r_s = 1 - \frac{6\sum d_i^2}{n(n^2-1)} = 1 - \frac{6 \times 4}{10 \times 99}$$

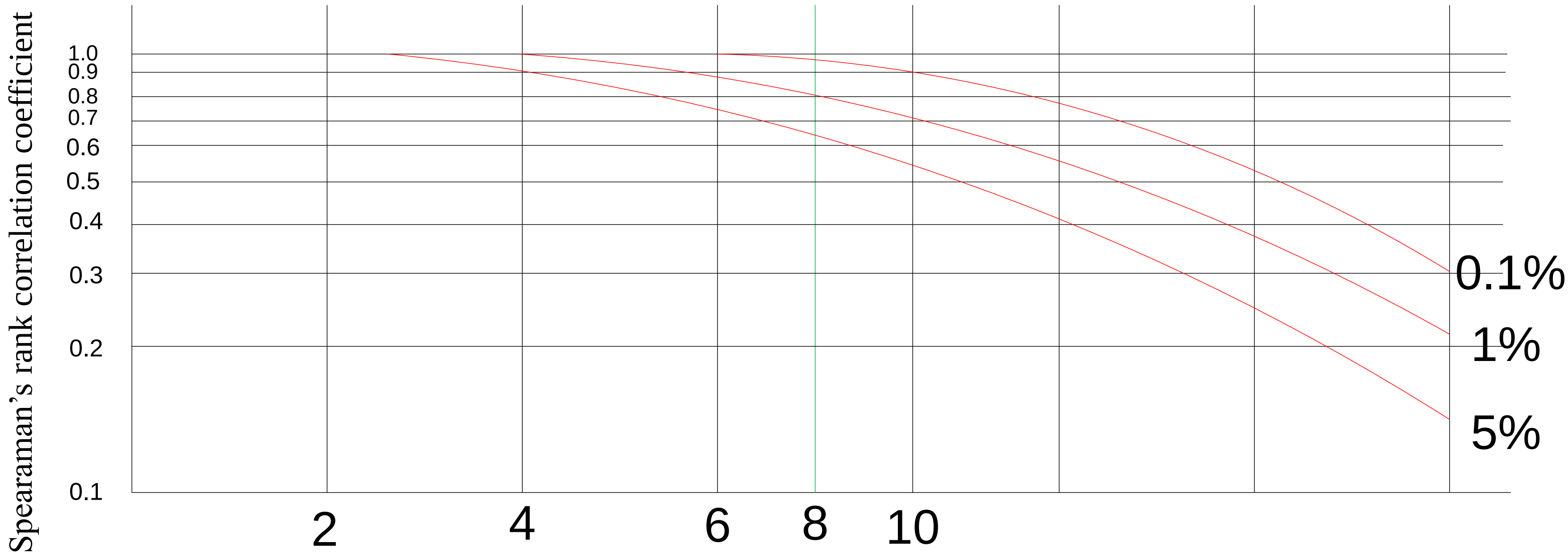
$$r_s = 0.9757$$

Charles Spearman's Coefficient of Correlation

Step 3: To see, if this r_s value is significant, the Spearman's rank significance table (or graph) must be consulted.

Note: The degrees of freedom for the sample $= \text{n} - 2 = 8$

Assume, the significance level $= 0.1\%$



Charles Spearman's Coefficient of Correlation

Step 4: Final conclusion

From the graph, we see that $r_s = 0.9757$ lies above the line at 8 and 0.1% significance level. Hence, there is a greater than 99% chance that the relationship is significant (i.e., not random) and hence the hypothesis should be rejected.

Thus, we can reject the hypothesis and conclude that in this case, depth of a river **progressively increases** the further the distance from the river bank.

χ^2 Correlation Analysis

Chi-Squared Test of Correlation

- This method is also alternatively termed as Pearson's χ^2–test or simply χ^2 -test
- This method is applicable to categorical (discrete) data only.

  - Suppose, two attributes A and B with categorical values

$$A = a_1, a_2, a_3, \ldots.., a_m \quad \text{and}$$
$$B = b_1, b_2, b_3, \ldots.., b_n$$

    having m and n distinct values.

A	a_1	a_2	a_3	a_1	a_5	a_1	… …
B	b_1	b_2	b_3	b_1	b_5	b_1	… …

Between whom we are to find the correlation relationship.

χ^2 –Test Methodology

Contingency Table

Given a data set, it is customary to draw a contingency table, whose structure is given below.

	b_1	b_2	-----	b_j	------	b_n	**Row Total**
a_1							
a_2							
⋮							
a_i							
⋮							
a_m							
Column Total							**Grand Total**

χ^2 –Test Methodology

Entry into Contingency Table: Observed Frequency

In contingency table, an entry O_{ij} denotes the event that attribute A takes on value a_i and attribute B takes on value b_j (i.e., $A = a_i$, $B = b_j$).

A	a_i	a_2	a_3	a_i	a_5	a_i	… …
B	b_j	b_2	b_3	b_j	b_5	b_j	… …

	b_1	b_2	-----	b_j	------	b_n	Row Total
a_1							
a_2							
⋮							
a_i				O_{ij}			
⋮							
a_m							
Column Total							Grand Total

χ^2 –Test Methodology

Entry into Contingency Table: Expected Frequency

In contingency table, an entry e_{ij} denotes the expected frequency, which can be calculated as

$$e_{ij} = \frac{Count(A = a_i) \times Count(B = b_j)}{Grand\ Total} = \frac{A_i \times B_j}{N}$$

	b_1	b_2	-----	b_j	------	b_n	Row Total
a_1							
a_2							
⋮							
a_i				e_{ij}			A_i
⋮							
a_m							
Column Total				B_j			N

A	B
...	...
ai	bj
...	...
ai	bj
...	...
...	...
ai	bj
...	...
...	...
...	...
...	...

χ^2 – Test

Definition: χ^2-Value

The $\boldsymbol{\chi}^2$ value (also known as the Pearson's $\boldsymbol{\chi}^2$ test) can be computes as

$$\boldsymbol{\chi}^2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\left(o_{ij} - e_{ij}\right)^2}{e_{ij}}$$

where o_{ij} is the **o**bserved frequency

e_{ij} is the **e**xpected frequency

χ^2 – Test

- The cell that contribute the most to the $\boldsymbol{\chi}^2$ value are those whose actual count is very different from the expected.

- The $\boldsymbol{\chi}^2$ statistics tests the hypothesis that A and B are independent. The test is based on a significance level, with $(n\text{-}1) \times (m\text{-}1)$ degrees of freedom., with a contingency table of size $n \times m$

- If the hypothesis can be rejected, then we say that A and B are statistically related or associated.

χ^2 – Test

Example 3: Survey on Gender versus Hobby.

- Suppose, a survey was conducted among a population of size 1500. In this survey, gender of each person and their hobby as either “book” or “computer” was noted. The survey result obtained in a table like the following.

GENDER	HOBBY
....	....
....	....
M	Book
F	Computer
....	....
....	....
....	....

- We have to find if there is any association between Gender and Hobby of a people, that is, we are to test whether “gender” and “hobby” are correlated.

χ^2 –Test

Example : Survey on Gender versus Hobby.

From the survey table, the **observed frequency** are counted and entered into the contingency table, which is shown below.

GENDER	HOBBY
....	....
....	....
M	Book
F	Computer
....	....

		GENDER		
		Male	Female	Total
HOBBY	Book			
	Computer			
	Total			

χ^2 – Test

Example: Survey on Gender versus Hobby.

- From the survey table, the observed frequency are counted and entered into the contingency table, which is shown below.

		GENDER		
		Male	Female	Total
HOBBY	Book	250	200	450
	Computer	50	1000	1050
	Total	300	1200	**1500**

χ^2 – Test

Example: Survey on Gender versus Hobby.

- From the survey table, the expected frequency are counted and entered into the contingency table, which is shown below.

HOBBY	GENDER		
	Male	Female	Total
Book	90	360	450
Computer	210	840	1050
Total	300	1200	**1500**

χ^2 – Test

- Using equation for χ^2 computation, we get

$$\chi^2 = \frac{(250-90)^2}{90} + \frac{(50-210)^2}{210} + \frac{(200-360)^2}{360} + \frac{(1000-840)^2}{840}$$
$$= 507.93$$

- This value needs to be compared with the tabulated value of χ^2 (available in any standard book on statistics) with 1 degree of freedom (for a table of $m \times n$, the degrees of freedom is $(m-1)\times(n-1)$; here $m = 2$, $n = 2$).

- For 1 degree of freedom, the χ^2 value needed to reject the hypothesis at the 0.01 significance level is 10.828. Since our computed value is above this, we reject the hypothesis that "Gender" and "Hobby" are independent and hence, conclude that the two attributes are *strongly correlated* for the given group of people.

Significance Test for χ^2 -Test

Cramer's V Test

- For χ^2 –test, the most commonly used test to measure the strength of the relation is Cramer's V test. The test takes the following form:

$$V = \sqrt{\frac{\chi^2/n}{(k-1)}}$$

- Here, n is the number of total observation, and k is the number of rows or columns, whichever is less.
- For the example case, n = 1500 and k = 2. Hence, V = 0.58.
- Thus, it is neither weak nor a strong correlation; this implies that **Gender** and **Hobby** are related with the degree of correlation 0.58

More on Correlation Analysis

Other Types of Correlation

- Binary variable to binary variable correlation
  - **Tetrachoric correlation**
- Nominal/ categorical valued variable to binary variable correlation
  - **Cramer's V correlation**
- Continuous variable to binary variable correlation
  - **Point-biserial correlation**

Tetrachoric correlation

- **Tetrachoric correlation** is a measure of the association between two binary variables, that is, variables that can only take on two values like "yes" and "no" or "good" and "bad."

- Suppose, we have the following 2×2 table with two variables, x and y, that both take on two values:

Here
a = Total count for $x = 0$ and $y = 0$
b = Total count for $x = 0$ and $y = 1$
c = Total count for $x = 1$ and $y = 0$
d = Total count for $x = 1$ and $y = 1$

	$y = 0$	$y = 1$
$x = 0$	a	b
$x = 1$	c	d

$$r_t = cos\left(\frac{180}{1+\sqrt{\frac{b*c}{a*d}}}\right)$$

Tetrachoric correlation: Example

- **Example:**

Suppose, we want to know whether or not gender is associated with political party preference so we take a simple random sample of 47 voters and survey them on their political party preference.

	$y = party\ 1$	$y = party\ 2$
$x = male$	9	15
$x = female$	13	10

Here
$a = 9$ for $x = male$ and $y = party\ 1$
$b = 15$ for $x = male$ and $y = party\ 2$
$c = 13$ for $x = female$ and $y = party\ 1$
$d = 10$ for $x = female$ and $y = party\ 2$

Tetrachoric correlation: Example

$$r_t = cos\left(\frac{180}{1+\sqrt{\frac{b*c}{a*d}}}\right)$$

Here
a = 9 for $x = male$ and $y = party\ 1$
b = 15 for $x = male$ and $y = party\ 2$
c = 13 for $x = female$ and $y = party\ 1$
d = 10 for $x = female$ and $y = party\ 2$

	$y = party\ 1$	$y = party\ 2$
$x = male$	9	15
$x = female$	13	10

- $r_t = cos\left(\frac{180}{1+\sqrt{\frac{b*c}{a*d}}}\right)$

  $= cos\left(\frac{180}{1+\sqrt{\frac{15*13}{9*10}}}\right)$

  $= cos\left(\frac{180}{1+1.471}\right)$

  $= cos\left(\frac{180}{2.471}\right) = cos(72.84) = 0.29$
- Here, the coefficient of correlation between gender and political party preference is 0.29.
- This correlation is significantly low, which indicates that **there is a weak correlation between gender and preference of political party .**

Cramer's V correlation

Cramer's V correlation is used to measure the strength of association between two variables with nominal or categorical values.

Each variable can have two or more than two nominal or categorical values also.

Cramer's V correlation coefficient

$$r_{cv} = \sqrt{\frac{\frac{\chi 2}{n}}{\min(m-1, c-1)}}$$

$\chi 2$ = The Chi-square statistics
n = Total number of samples in the dataset
m = Number of classes of dependent variable
c= Number of columns in the dataset

Cramer's V correlation: Example

- **Example:**

Suppose, we want to know if there is any association between three different eye colors (blue, green and brown) and three regions (east, north and west). After surveying 50 random samples, the following data is obtained.

	Eye Color		
	Blue	Green	Brown
East	8	5	6
North	2	8	3
west	4	6	8

Cramer's V correlation: Example

	Eye Color			
	Blue	Green	Brown	**Row Total**
East	8	5	6	19
North	2	8	3	13
west	4	6	8	18
Column Total	14	19	17	**Grand total 50**

Step 1:

Here all the frequencies are called **observed frequency.**

Add all values row wise and column wise.
Here
row totals are 19,13,18
column totals are 14, 19, 17

Grand Total is 50

Cramer's V correlation: Example

	Eye Color			
	Blue	Green	Brown	**Row Total**
East	$\frac{19*14}{50} = 5.32$	$\frac{19*19}{50} = 7.22$	$\frac{19*17}{50} = 6.46$	19
North	$\frac{13*14}{50} = 3.64$	$\frac{13*19}{50} = 4.94$	$\frac{13*17}{50} = 4.42$	13
west	$\frac{18*14}{50} = 5.04$	$\frac{18*19}{50} = 6.84$	$\frac{18*17}{50} = 6.12$	18
Column Total	14	19	17	**Grand Total = 50**

Step 2:

Calculate expected frequencies for each cell.
Expected frequency

$$e_{ij} = \frac{(i^{th}\text{ row total})*(j^{th}\text{ column total})}{Grand\ total}$$

$$\chi^2 = \sum_{i=1}^{m}\sum_{j=1}^{n}\frac{\left(o_{ij} - e_{ij}\right)^2}{e_{ij}}$$

o_{ij} is the **o**bserved frequency

e_{ij} is the **e**xpected frequency

Cramer's V correlation: Example

Step 3:

Calculate $\chi^2 = \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{(o_{ij} - e_{ij})^2}{e_{ij}}$

Observed values	Eye Color		
	Blue	Green	Brown
East	8	5	6
North	2	8	3
west	4	6	8

Expected values	Eye Color		
	Blue	Green	Brown
East	5.32	7.22	6.46
North	3.64	4.94	4.42
west	5.04	6.84	6.12

$$\chi^2 = \frac{(8-5.32)^2}{5.32} + \frac{(5-7.22)^2}{7.22} + \frac{(6-6.46)^2}{6.46} + \frac{(2-3.64)^2}{3.64} + \frac{(8-4.94)^2}{4.94} + \frac{(3-4.42)^2}{4.42} + \frac{(4-5.04)^2}{5.04} + \frac{(6-6.84)^2}{6.84} + \frac{(8-6.12)^2}{6.12} = 6.35$$

Cramer's V correlation: Example

Step 4:

Putting the below given values to the equation

$\chi^2 = 6.35$
$n = 50$
$m = 3$
$c = 3$

$$r_{cv} = \sqrt{\frac{\frac{\chi 2}{n}}{\min(m-1, c-1)}} = \sqrt{\frac{\frac{6.35}{50}}{\min(3-1, 3-1)}} = \sqrt{\frac{0.127}{2}} = 0.25$$

The correlation between three different eye colors (blue, green and brown) and three regions (east, north and west) is 0.25

It means **eye color is weakly associated with the regions.**

Point-biserial correlation

Point-biserial correlation is a measure of the association between a continuous valued and a binary valued variable.

Point-biserial correlation coefficient

$$r_{pb} = \left|\frac{M_1 - M_0}{S_n}\right| \sqrt{p * q}$$

where,
M_1= mean of values in x_i, when y= 1.
M_0 = mean of values in x_i , when y= 0.
S_n = standard deviation of the attribute values x_i with a sample of size n.
p = Proportion of cases for y = 0
q = Proportion of cases for y = 1

Point-biserial correlation: Example

Example:

Suppose we want to know whether or not gender is associated with weekly expenditure of the students, where we take a simple random sample of 7 students and survey on them.

$x = expanditure$	$y = gender$
12	**1**
8	**1**
7	1
22	0
18	0
16	0
20	0

Step 1:

Here, 1= male and 0 = female

$$M_1 = \frac{(12+8+7)}{3} = 9$$

$$M_0 = \frac{(22+18+16+20)}{4} = 19$$

$$n = 7$$

Point-biserial correlation: Example

$x =$ *expanditure*	$y =$ *gender*
12	**1**
8	**1**
7	1
22	0
18	0
16	0
20	0

- **Step 2:**
- $p = \frac{Total\ number\ of\ male}{n} = \frac{3}{7} = 0.43$
- $q = \frac{Total\ number\ of\ female}{n} = \frac{4}{7} = 0.47$
- **Step 3:**
- $S_n = \sqrt{\frac{(x_i - \bar{x})^2}{n}} = 5.85$ where, $\bar{x} = \frac{12+8+7+22+18+16+20}{7} = 14.71$
- So,
- $r_{pb} = \left|\frac{M_1 - M_0}{S_n}\right| \sqrt{p * q} = \left|\frac{9-19}{14.71}\right| \sqrt{0.43 * 0.47} = 0.85$

- Here, the coefficient of correlation between gender and weekly expenditure of the students is 0.85.
- It means **gender is strongly associated with weekly expenditure of the students**.

Do Remember!

Figure 14-3. Another example of association or causation. (DILBERT © 2011 Scott Adams. Used by permission of UNIVERSAL UCLICK. All rights reserved.)