MATH -260 Course @ Sorbonne

Day 1: Multivariate Data Analysis*

STATISTICS + MACHINE LEARNING + DATA SCIENCE

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Course for BSc Mathematics (Spe. Data Science) Students.

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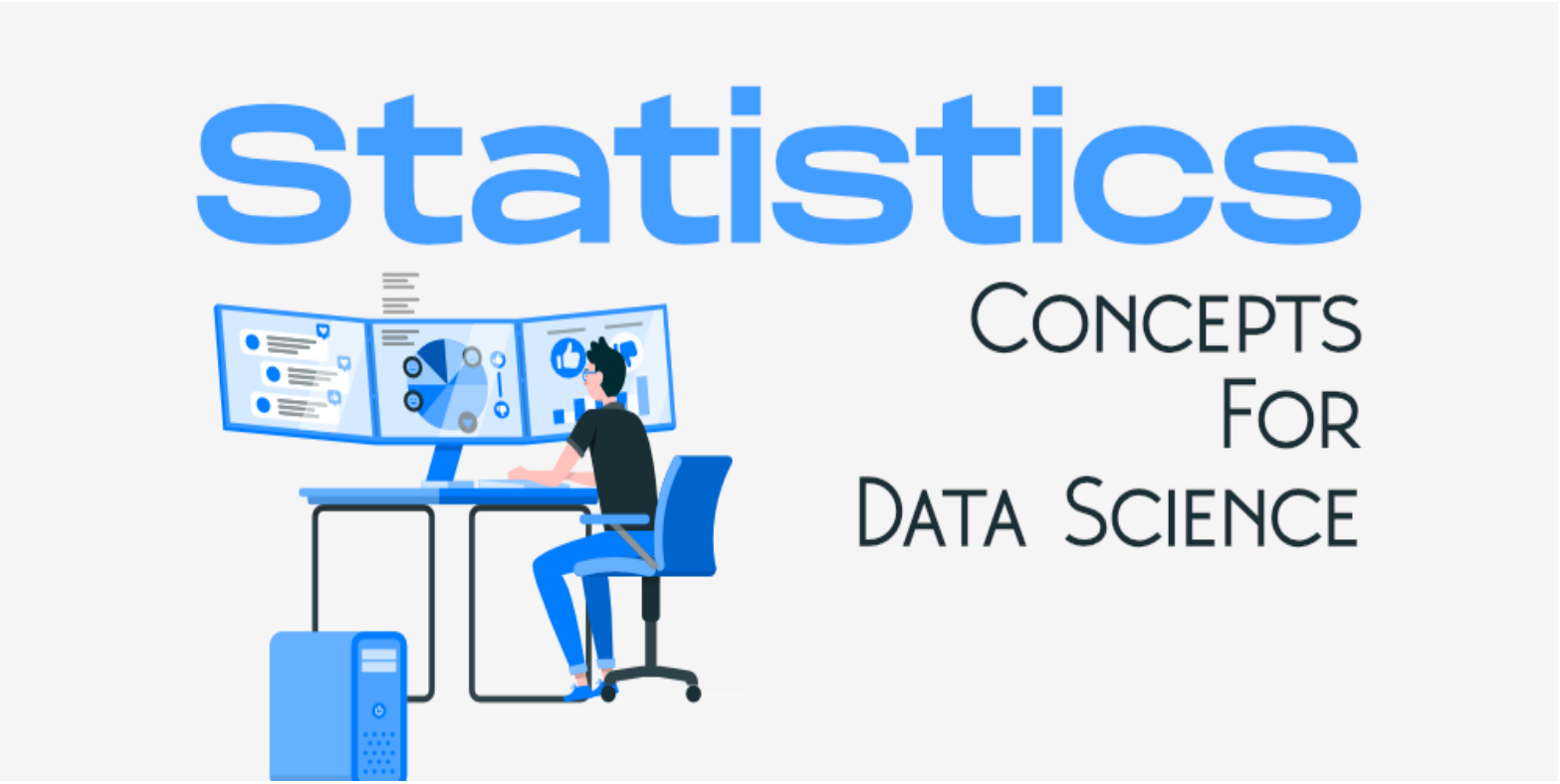
* In 1962, John Tukey described a field called "data analysis" which resembles modern data science.

Course

"It is very easy to be a teacher, but very difficult to be a student.

A good student has to learn many concepts, perform in examinations, loyal to his / her teacher and others."



Why this course?

Let data drive decisions, not the **Highest Paid Person's Opinion**.

Course Prerequisites

Basic Statistics:

- Descriptive Statistics
- Estimation and Hypothesis Testing
- Analysis of Variance (ANOVA)

Programming Skills:

- Basics of R & RStudio
- Basic statistics using R
- Basics of Python
- Basic statistics using Python

Probability Theory:

- Introductory Probability Theory
- Probability Distributions
- Sampling Distributions
- Linear Algebra & Calculus

Course Syllabus

Regression Analysis:

- Linear Regression
- Nonlinear Regression
- Shrinkage Methods
- Logistic Regression & GLM
- Time Series Forecasting

Multivariate Analysis:

- SVD & PCA
- Factor Analysis
- LDA & QDA
- Correspondence Analysis
- Market Basket Analysis

Coding & Projects:

- Implementations in RStudio (mostly) & Python
- Kaggle Competition (20%)
- Project Presentation (20%)
- Mid-term (20% and Final Exam (40%)

What is Data?

"Data is the new oil. It's valuable, but if unrefined it cannot really be used. It has to be changed into gas, plastic, chemicals, etc. to create a valuable entity that drives profitable activity; so must data be broken down and analyzed for it to have value."

- Clive Humby, UK Mathematician and Architect of Tesco's Clubcard.

Example:

City	Morning temperature	Evening temperature
Austin	62	90.7
Boston	41	48.0
Chicago	51	57.2
Denver	45	52.5

Role of data: Present

Role of data: Present

"It is a capital mistake to theorize before one has data. Insensibly one begins to twist facts to suit theories, instead of theories to suit facts."

- Sherlock Holmes, in A Scandal in Bohemia.

The World is Data Rich

Astronomy

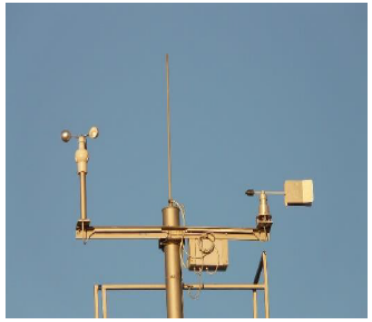
Social Networks

Healthcare

Banking

Genomics

Weather measurements



Recent Trends and Buzzwords

1. Statistics is the study of the collection, analysis, interpretation, presentation, and organization of data.

2. Data science is the study of the generalizable extraction of knowledge from data, yet the keyword is science.

3. Machine learning is the sub-field of computer science that gives computers the ability to learn without being explicitly programmed.

4. Artificial Intelligence research is defined as the study of intelligent agents: any device that perceives its environment and takes actions that maximize its chance of success at some goal.

5. Big data is an all-encompassing term for any collection of data sets so large and complex that it becomes difficult to process using traditional data processing applications.

Topic 1 : Statistics

Statistics: View Points

"Statistics is the universal tool of inductive inference, research in natural and social sciences, and technological applications. Statistics must have a clearly defined purpose, one aspect of which is scientific advance and the other, human welfare and national development"

- Professor P C Mahalanobis.

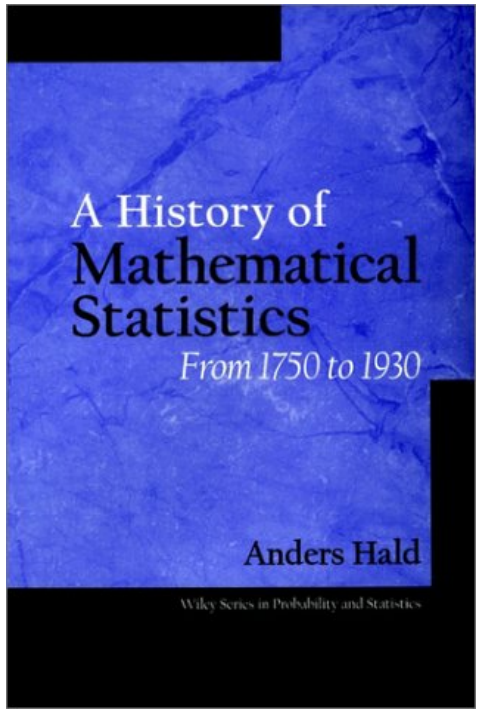
"All knowledge is, in final analysis, History.
All sciences are, in the abstract, Mathematics.
All judgements are, in their rationale, Statistics."

- Professor C R Rao.

- Role of Statistics:
  1. Making inference from samples
  2. Development of new methods for complex data sets
  3. Quantification of uncertainty and variability
- Two Views of Statistics:
  1. Statistics as a Mathematical Science
  2. Statistics as a Data Science

Statistics: View Points

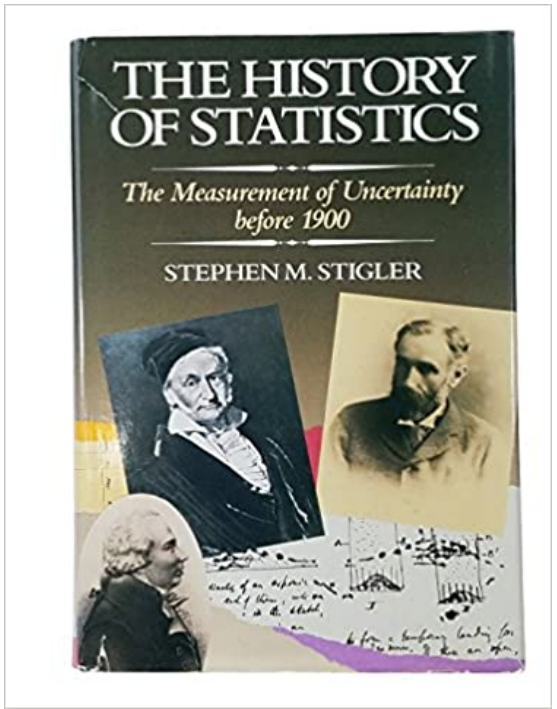
The science of Statistics is essentially a branch of Applied Mathematics, and may be regarded as mathematics applied to observational data. As in other mathematical studies, the same formula is equally relevant to widely different groups of subject-matter. Consequently the unity of the different applications has usually been overlooked, the more naturally because the development of the underlying mathematical theory has been much neglected. Statistics may be regarded as (i) the study of populations, (ii) as the study of variation, (iii) as the study of methods of the reduction of data.



History of Statistics : Pre 1900

- Probability and its application to Gambling & Astronomy

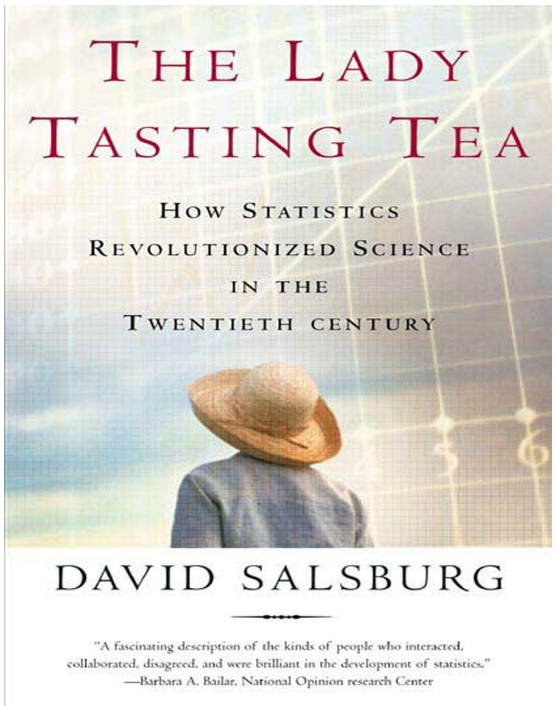
  - Jacob Bernoulli (Prob. distribution, Law of large numbers, etc.)
  - Pierre-Simon Laplace (Double exponential, Transformation, etc.)
  - Thomas Bayes (Bayes' theorem, etc.)
  - Gauss and Legendre (Least Square Method)
  - Francis Galton (Correlation & Regression)
  - Karl Pearson (χ^2 test, distribution, etc.)
  - and many others.



History of Statistics : 1900 - 1980

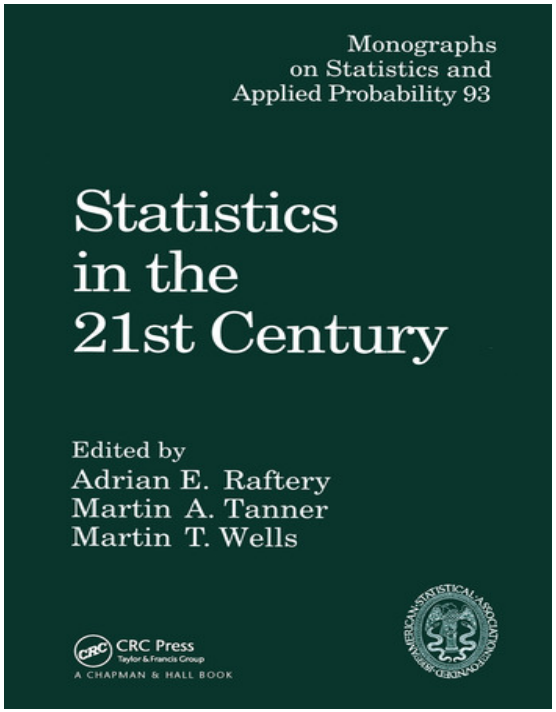
- Development of Statistics and its application to Agriculture, Economics, Geology, Medical, Technology, Clinical Trials, etc.

  - Ronald Fisher (Discriminant analysis, Likelihood, ANOVA & DOE, etc.)
  - Jerzy Neyman, Egon Sharpe Pearson & Wald (Decision theory, Optimality, etc.)
  - Lehmann, Hotelling, Anderson & Tukey (Multivariate & Inferential Statistics, etc.)
  - Box, Cox, Jenkin and Blackwell (Time Series)
  - Shewhart, Deming, Taguchi & Juran (SQC)
  - Efron, Breiman, Friedman, Cramer (Modern Statistical Tools)
  - PC Mahalanobis (Mahalanobis Distance), C. R. Rao (Linear Models, Multivariate Analysis, Orthogonal arrays, etc.), etc.



History of Statistics : Upto 1980

- Parametric Models : One Sample, two sample, linear models, survival data, Estimation, Testing of Hypothesis.
- Probability distributions were believed to generate data (e.g., Gaussian, Logistic, Poisson, Exponential, etc.).
- Semiparametric & Nonparametric Models : Dropping assumptions on population, dependence and errors.
- Emphases on Optimality in various ways : Bayes optimality, Decision theory, minimax and unbiasedness.
- Exact distributional (t, F) approaches and asymptotic methods (samples size $\to \infty$ viewed as approximation).



Statistics : 1980 - Present

- Data : Large bodies of data with complex data structures are generated from computers, sensors, manufacturing industries, etc.
- Models : Non/Semiparametric models but in complex probability spaces / high-dimensional functional spaces (e.g., deep neural net, reinforcement learning, decision trees, etc.).
- Emphases : Making predictions, causation, algorithmic convergence.
- Data are necessary and at the core of Statistical Learning, Data Science & Machine Learning.
- Statistics : Not only has strong interactions with Probability but also other parts of Data Science (Machine Learning, Artificial Intelligence, etc.).

Statistics : Present (2021)

- Probability : Has moved to the center of Mathematics and having strong interactions with Statistical Physics and Theoretical Computer Science.
- Statistics : Not only has strong interactions with Probability but also other parts of Data Science (Machine Learning, Artificial Intelligence, etc.).
- Computational : Computing skills are essential, construction of fast training algorithms and computation time.
- Applications : Strong interactions with substantive fields in all areas. Applications of statistical methods in almost all the fields are evident. Statistics became a key technology driven by data (**“Data is the new oil"**).

Statistics: Past Vs. Present

- Traditional Problems in Applied Statistics:

  - Well formulated question that we would like to answer.
  - Expensive to gather data and/or expensive to do computation.
  - Create specially designed experiments to collect high quality data.

- Current Situation : Information Revolution

  - Improvements in computers and data storage devices.
  - Powerful data capturing devices.
  - Lots of data with potentially valuable information available.

What is the Difference?

- Data characteristics:

  - Size
  - Dimensionality
  - Complexity
  - Messy
  - Secondary sources

- Focus on generalization performance :

  - Prediction on new data
  - Action in new circumstances
  - Complex models needed for good generalization

- Computational considerations :

  - Large scale and complex systems

Topic 2 : Data Science

What is Data Science?

"Data is the new oil. It's valuable, but if unrefined it cannot really be used. It has to be changed into gas, plastic, chemicals, etc to create a valuable entity that drives profitable activity; so must data be broken down, analyzed for it to have value."

- Clive Humby, UK Mathematician and Architect of Tesco's Clubcard.

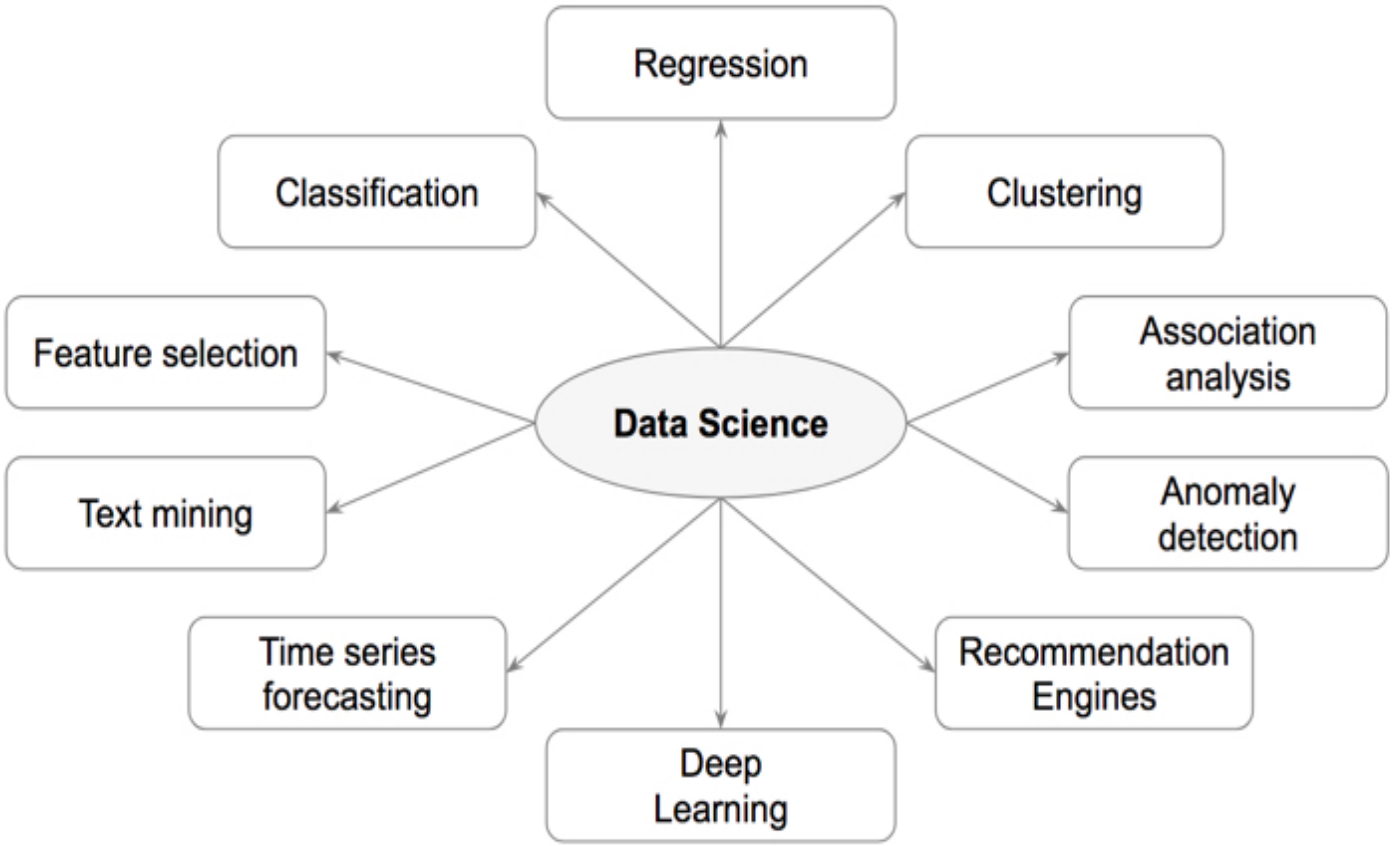
"Everybody needs data literacy, because data is everywhere. It's the new currency, it's the language of the business. We need to be able to speak that."

- MIT Sloan School of Management

What is Data Science?

- *Interdisciplinary field about scientific methods, processes, and systems to extract knowledge or insights from complex data in various forms, either structured or unstructured.*
- *A concept to unify Statistics, Data Analysis and their related methods in order to "understand and analyze actual phenomena" with data.*
- *Employs techniques and theories drawn from many fields within the broad areas of Mathematics, Statistics, Information Science, and Computer Science, in particular from the subdomains of Machine learning, classification, cluster analysis, data mining, databases, and visualization.*
- *Fourth paradigm of Science (empirical, theoretical, computational and data-driven)*

Types of Data Science?



Data in Data Analytics

Basic Definitions:

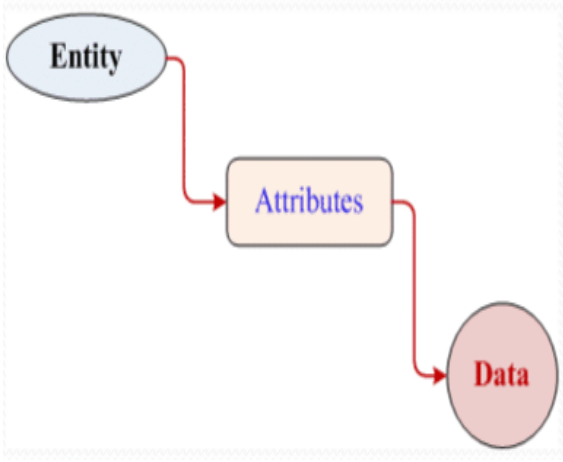
Entity: A particular thing is called entity or object.

Attribute: An attribute is a measurable or observable property of an entity.

Data: A measurement of an attribute is called data.

Note: Data defines an entity and Computer can manage all type of data (e.g., audio, video, text, etc.). In general, there are many types of data that can be used to measure the properties of an entity.

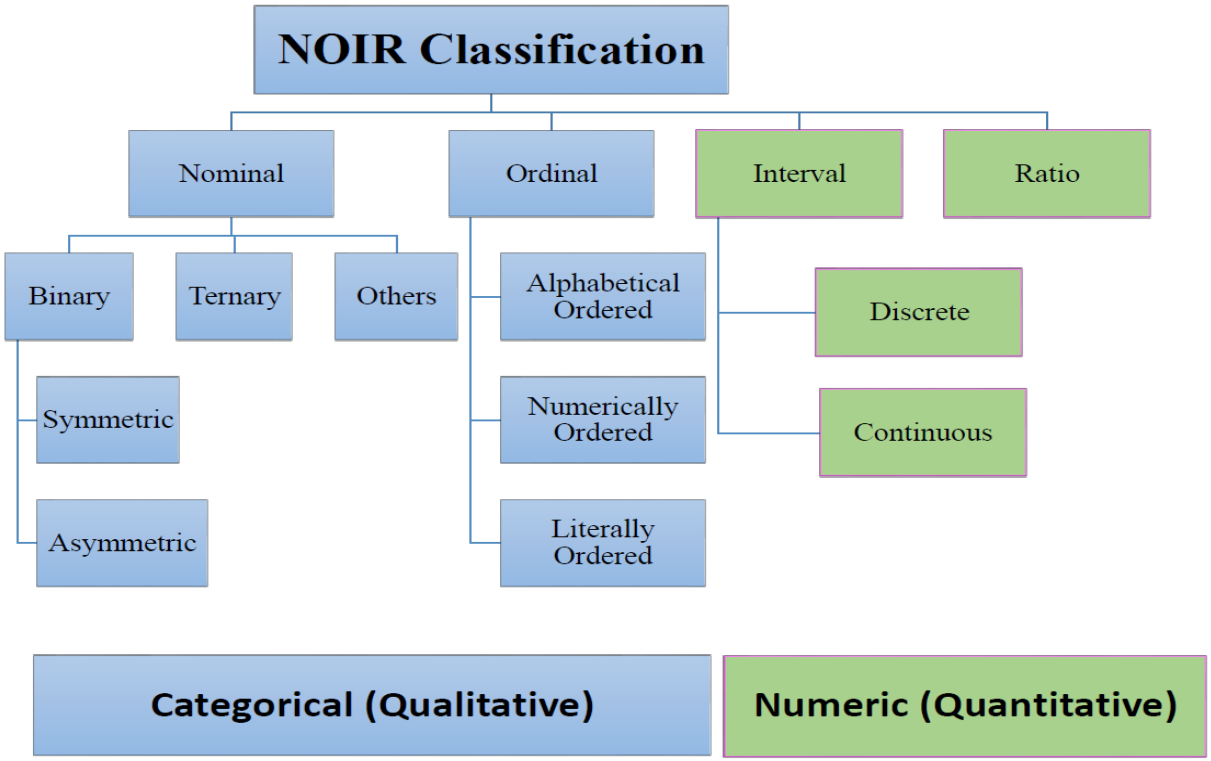
Scale: A good understanding of data scales (also called scales of measurement) is important. Depending on the scales of measurement, different techniques are followed to derive hitherto unknown knowledge in the form of patterns, associations, anomalies or similarities from a volume of data.



NAME	AGE	GENDER	SALARY	EMPLOYER
:				
:				
ABCD	34	F	40000	XYZ
:				
:				

NOIR: Scales of Measurement

- The NOIR scale is the fundamental building block on which the extended data types are built.
- Further, nominal (Blood groups, Attendance) and ordinal (Shirt size) are collectively referred to as categorical or qualitative data. Whereas, interval (weight, temperature) and ratio (Sound intensity in Decibel) data are collectively referred to as quantitative or numeric data.



Data Cube

Concept of data cube:

A multidimensional data model views data in the form of a cube. A data cube is characterized with two things:

- Dimension: The perspective or entities with respect to which an organization wants to keep record.
- Fact: The actual values in the record.

Example: Rainfall data of Meteorological Department.

- Time (Year, Season, Month, Week, Day, etc.)
- Location (Country, Region, State, etc.)

2-D view of rainfall data

- In this 2-D representation, the rainfall for “North-East” region are shown with respect to different months for a period of years...

Reagion: North-East

Year \ Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
2005												
2006												
2007												
2008												
2009												
2010												

View of 3-D rainfall data

- Suppose, we want to represent data according to times (Year, Month) as well as regions of a country say East, West, North, North-East, etc.

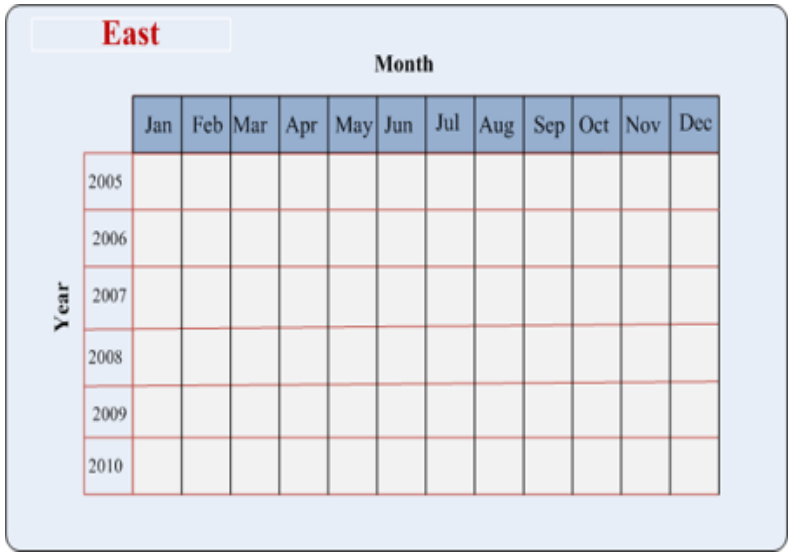


Figure: 2-D view of rainfall data

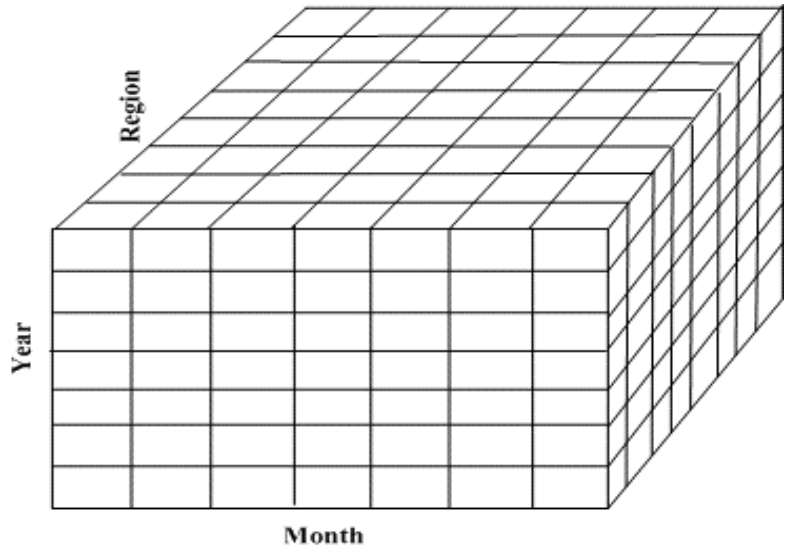
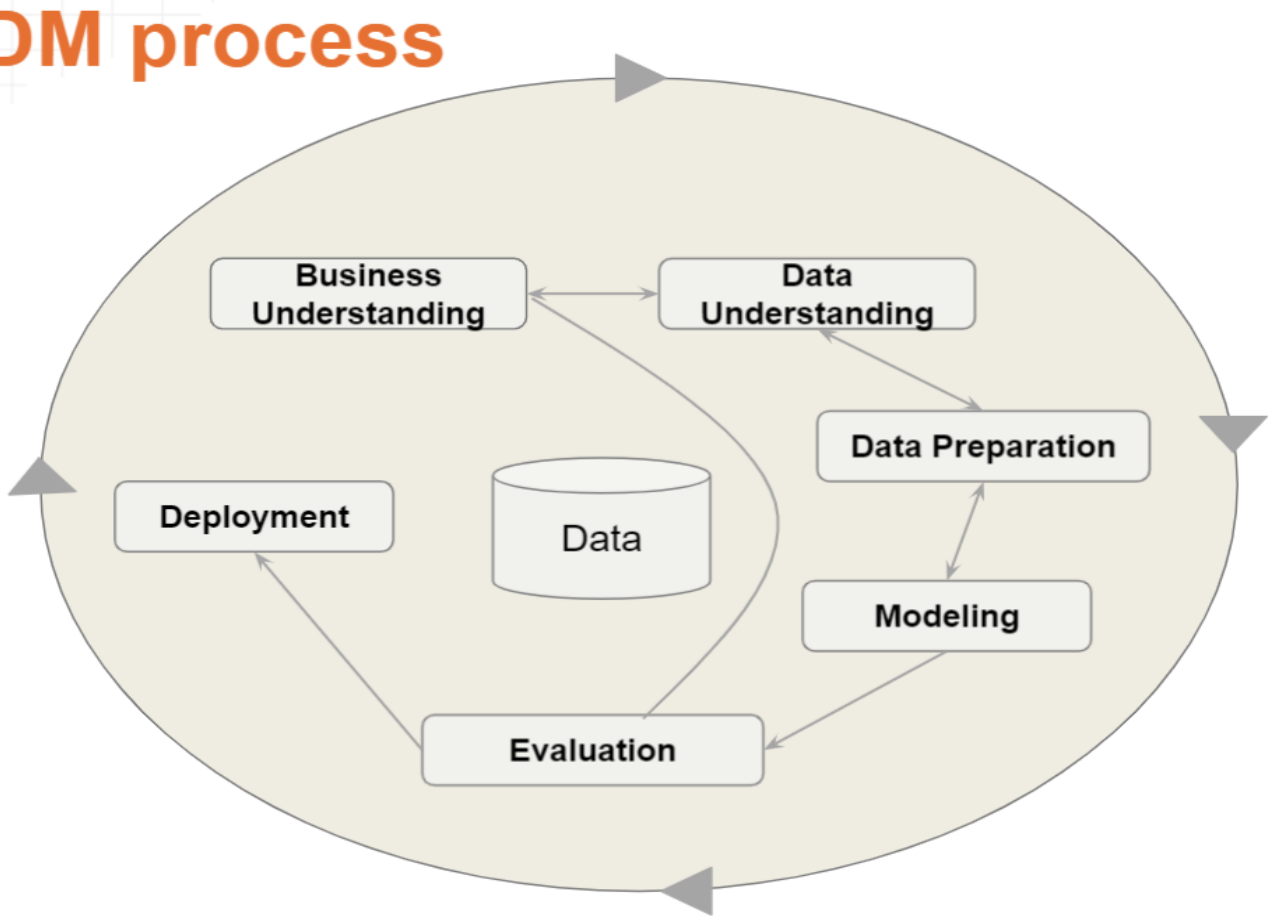


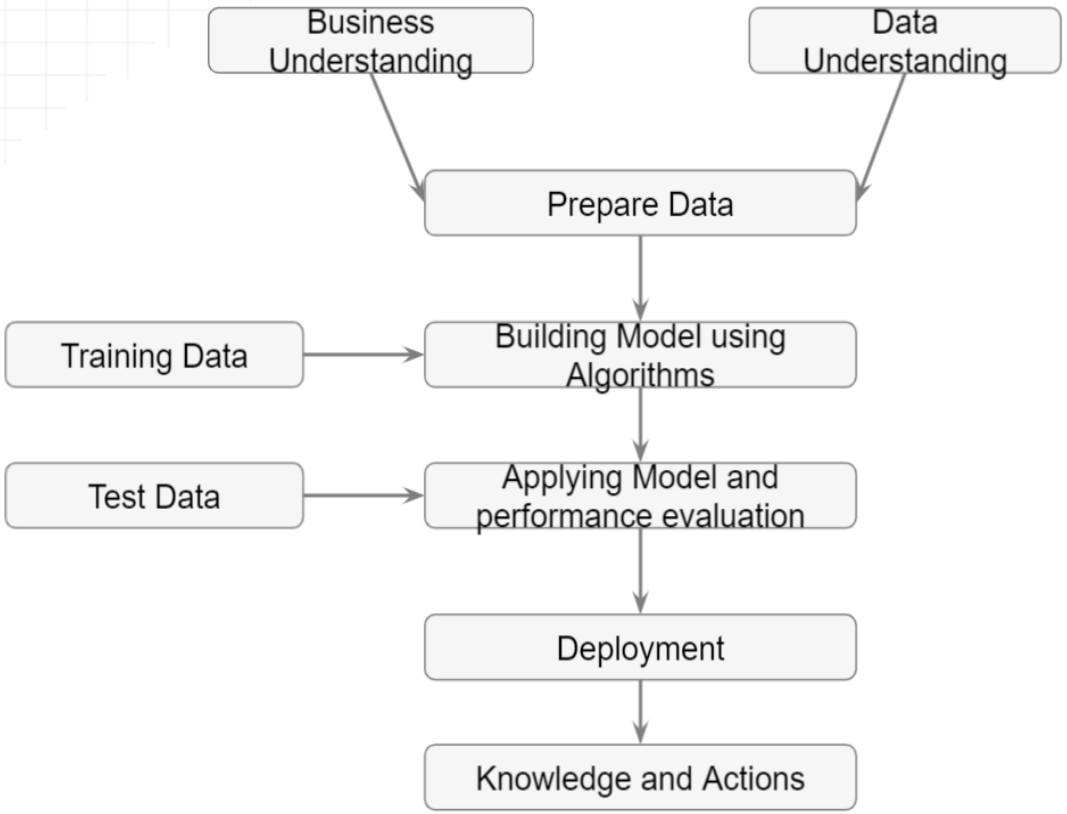
Figure: 3-D view of rainfall data

Data Mining Process



Data Mining Process

Process



1. Prior Knowledge
2. Preparation
3. Modeling
4. Application
5. Knowledge

Gaining information on

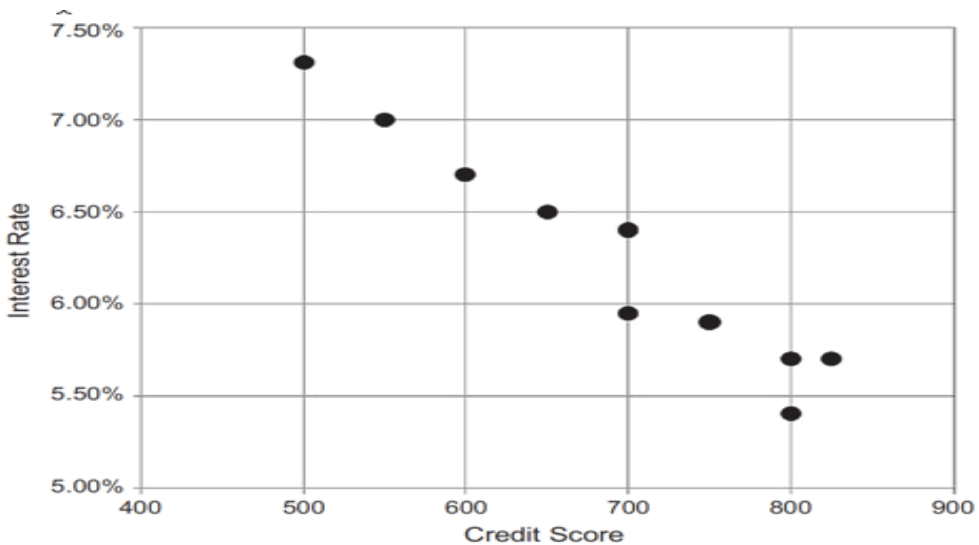
- Objective of the problem.
- Subject area of the problem.
- Data.

Table 2.1 Data Set

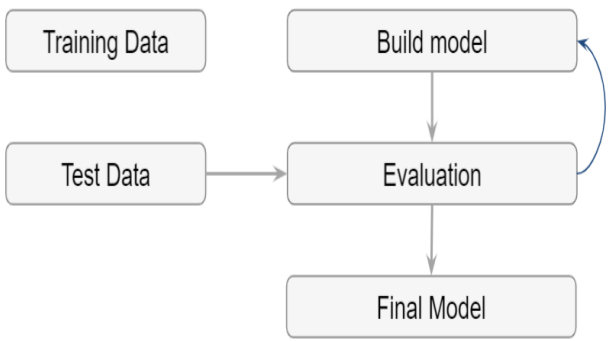
Borrower ID	Credit Score	Interest Rate
01	500	7.31%
02	600	6.70%
03	700	5.95%
04	700	6.40%
05	800	5.40%
06	800	5.70%
07	750	5.90%
08	550	7.00%
09	650	6.50%
10	825	5.70%

Gaining information on

- Data Exploration and Data quality.
- Handling missing values and Outliers.
- Data type conversion.
- Transformation, Feature selection and Sampling.



3. Modeling



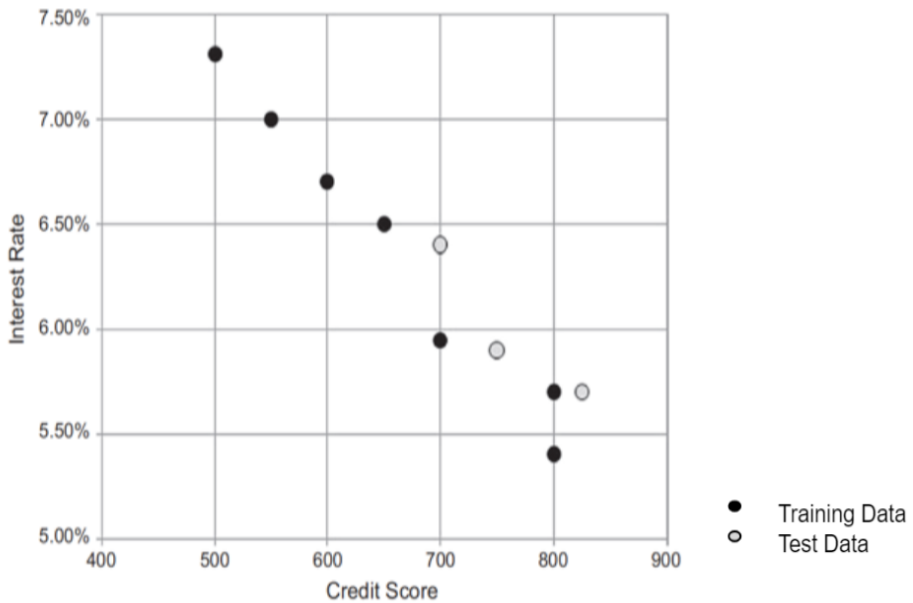
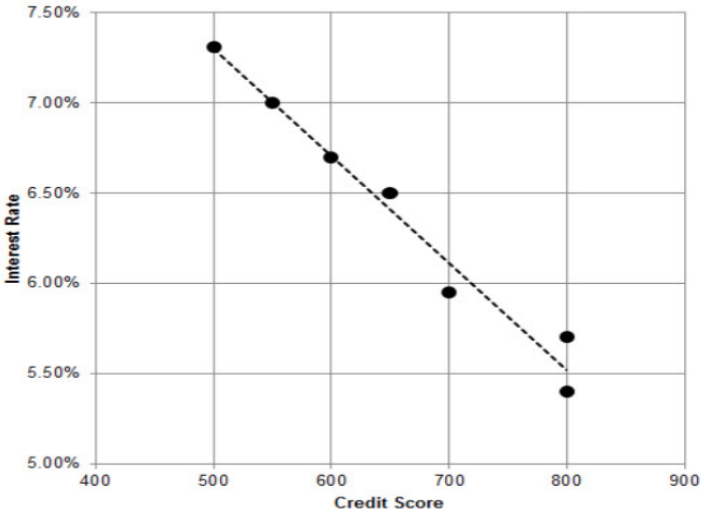


Figure: Splitting data into training and test data sets (right).

3. Modeling



$$y = 0.1 + \frac{6}{100,000}x$$

Table 2.5 Evaluation of Test Data Set

Borrower	Credit Score (X)	Interest Rate (Y)	Model Predicted (Y)	Model Error
04	700	6.40%	6.11%	-0.29%
07	750	5.90%	5.81%	-0.09%
10	825	5.70%	5.37%	-0.33%

Figure: Evaluation of test dataset (right).

Application and Knowledge

4. Application:

- Product readiness.
- Technical integration.
- Model response time.
- Remodeling.
- Assimilation.

5. Knowledge:

- Posterior knowledge.

Objectives of Data Exploration:

- Understanding data.
- Data preparation and Data mining tasks.
- Interpreting data mining results.

Data exploration

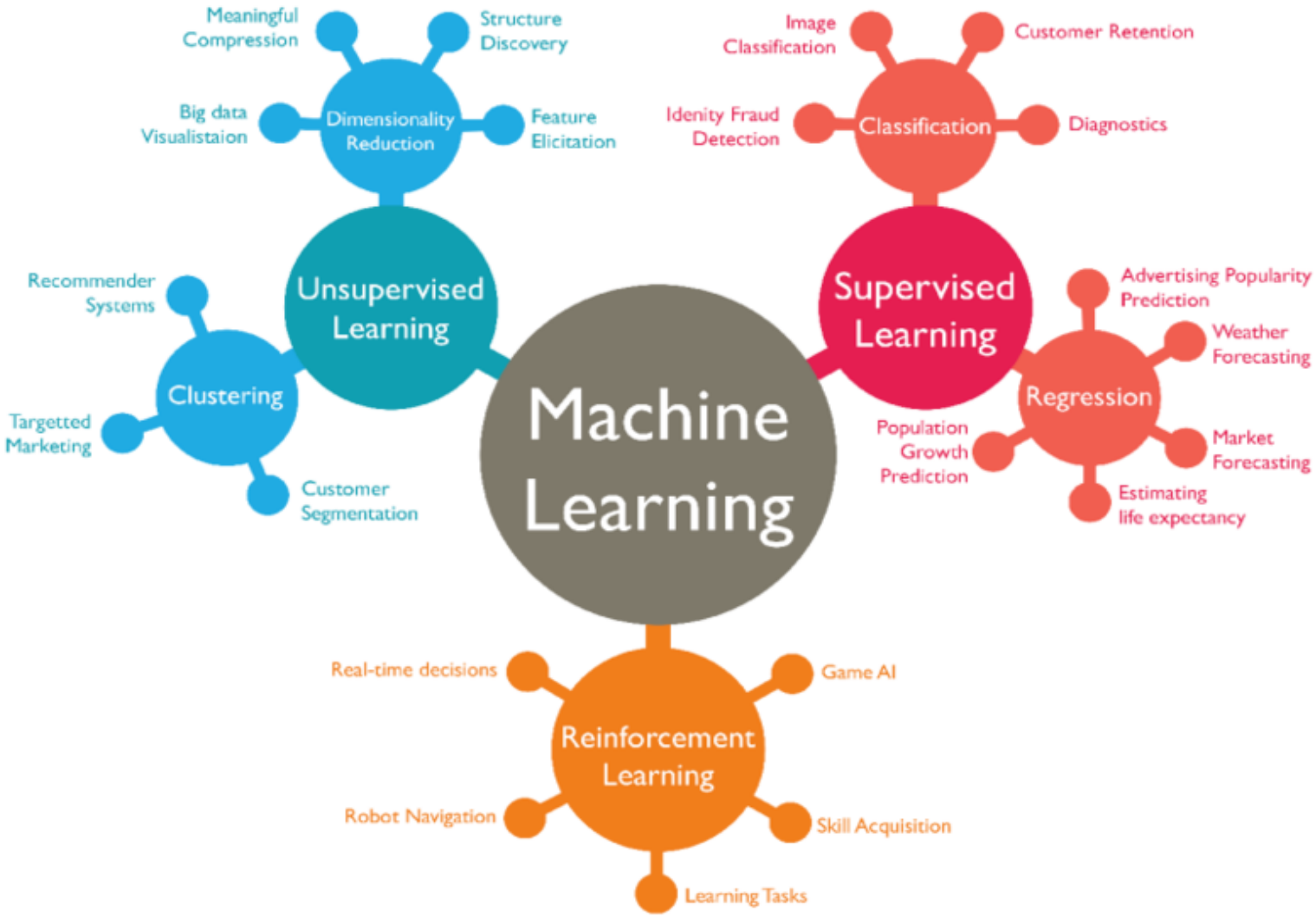
Roadmap:

- Organize the data set.
- Find the central point for each attribute (central tendency).
- Understand the spread of the attributes (dispersion).
- Visualize the distribution of each attribute (shapes).
- Pivot the data.
- Watch out for outliers.
- Understanding the relationship between attributes.
- Visualize the relationship between attributes.
- Visualization high dimensional data sets.

Topic 3 : Machine Learning

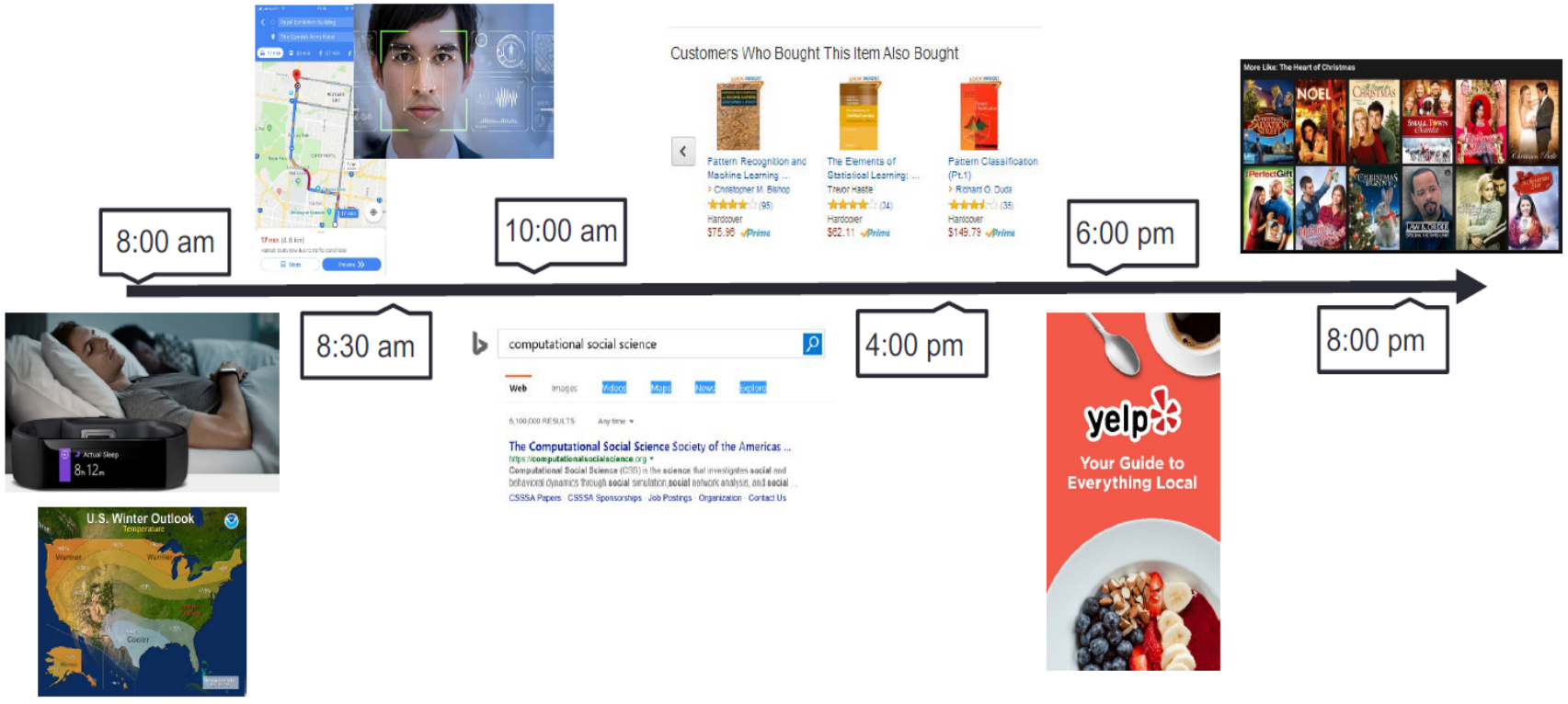
What is Machine Learning?

Machine learning is the field of study that gives computers the ability to learn without being explicitly programmed.



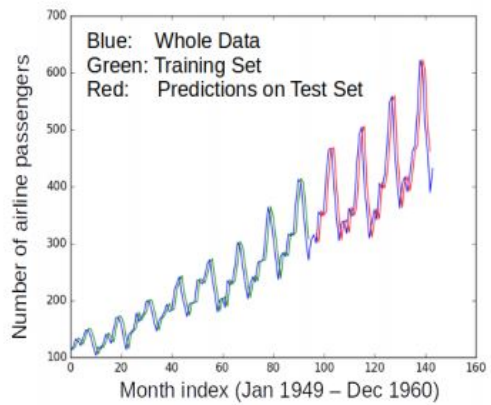
ML techniques are impacting our life

A day in our life with ML techniques...



Introduction to Machine Learning

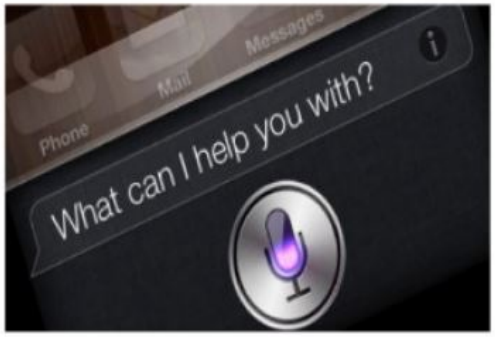
- Designing algorithms that ingest data and learn a model of the data.
- The learned model can be used to
  1. Detect patterns/structures/themes/trends etc. in the data
  2. Make predictions about future data and make decisions



- Modern ML algorithms are heavily "data-driven".
- Optimize a performance criterion using example data or past experience.

Machine Learning in the real-world

Broadly applicable in many domains (e.g., internet, robotics, healthcare and biology, computer vision, NLP, databases, computer systems, finance, etc.).




Predictive Policing

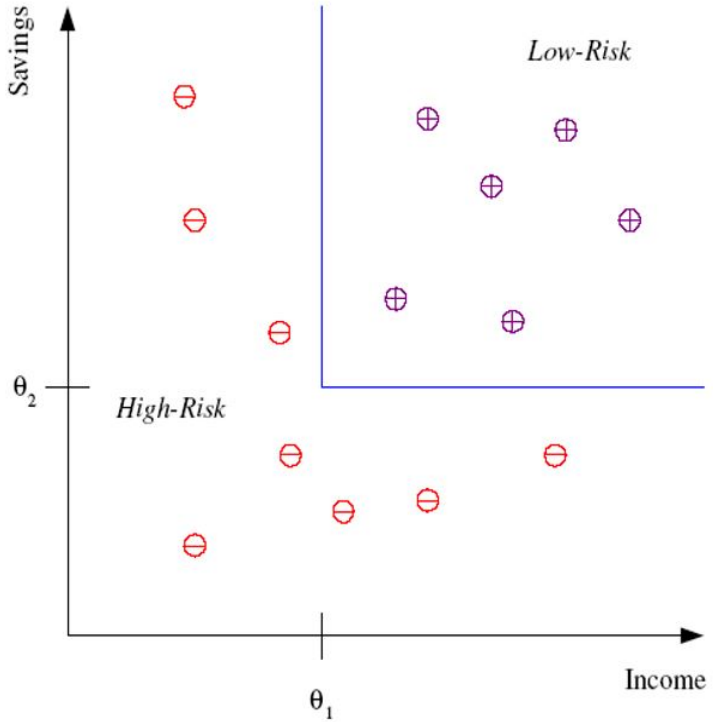

Online Fraud Detection

Development of Stat & ML Models

- Linear Regression (Galton, 1875).
- Linear Discriminant Analysis (R.A. Fisher, 1936).
- Logistic Regression (Berkson, JASA, 1944).
- k-Nearest Neighbor (Fix and Hodges, 1951).
- Parzen's Density Estimation (E Parzen, AMS, 1962)
- ARIMA Model (Box and Jenkins, 1970).
- Classification and Regression Tree (Breiman et al., 1984).
- Artificial Neural Network (Rumelhart et al., 1985).
- MARS (Friedman, 1991, Annals of Statistics).
- SVM (Cortes and Vapnik, Machine learning, 1995)
- Random forest (Breiman, 2001).
- Deep Convolutional Neural Nets (Krizhevsky, Sutskever, Hinton, NIPS 2012).
- Generative Adversarial Nets (Ian Goodfellow et al., NIPS 2014).
- Deep Learning (LeCun, Bengio, Hinton, Nature 2015).
- Bayesian Deep Neural Network (Yarin Gal, Islam, Zoubin Ghahramani, ICML 2017).

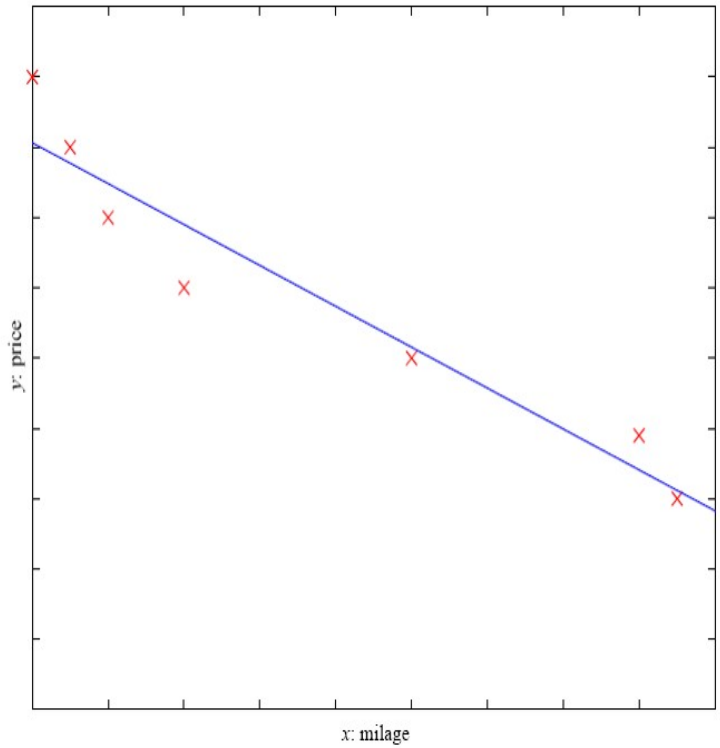
Supervised Learning: Classification

- Example: Credit scoring.
- Differentiating between low-risk and high-risk customers from their income and savings.
- Discriminant: IF Income $> \theta_1$ AND Savings $> \theta_2$ THEN low-risk ELSE high-risk.
- Classification: Learn a linear/nonlinear separator (the “model") using training data consisting of input-output pairs (each output is discrete-valued “label" of the corresponding input).
- Use it to predict the labels for new “test" inputs.
- Other Applications: Image Recognition, Spam Detection, Medical Diagnosis.



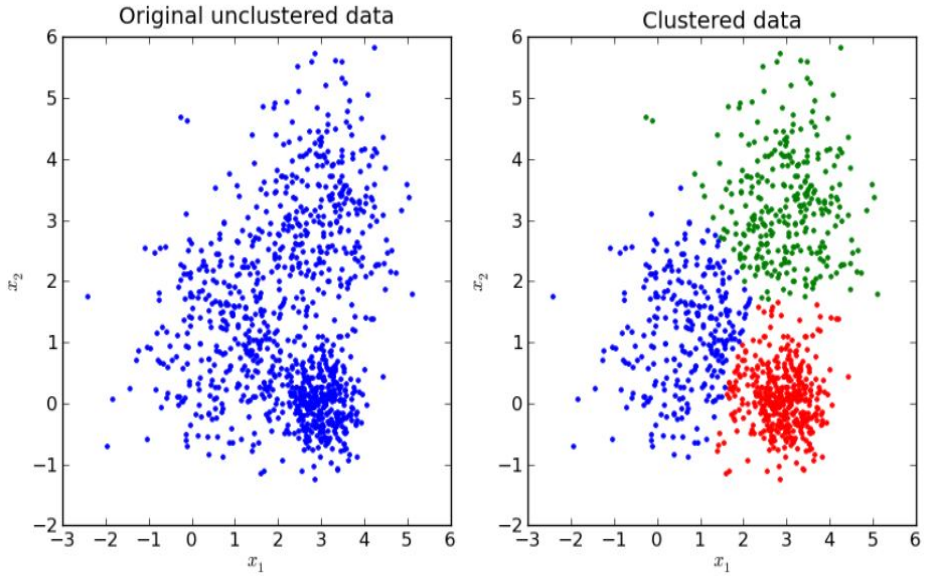
Supervised Learning: Regression

- Example: Price of a used car.
- X : car attributes; Y : price and $Y = f(X, \theta)$
- f( ) is the model and θ is the model parameters.
- Regression: Learn a line/curve (the "model") using training data consisting of Input-output pairs (each output is a real-valued number).
- Use it to predict the outputs for new "test" inputs.
- Other Applications: Price Estimation, Process Improvement, Weather Forecasting.

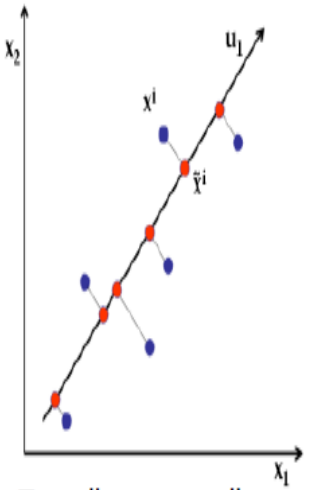


Unsupervised Learning: Clustering

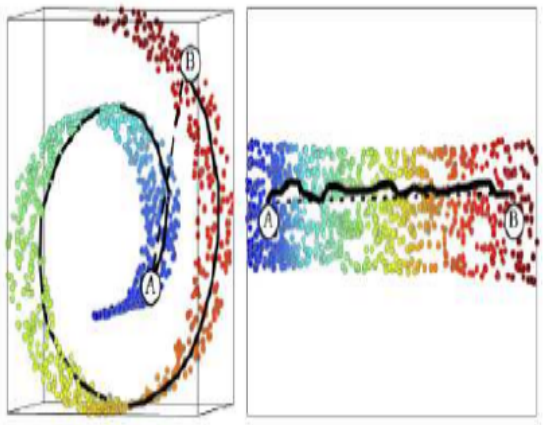
- Given: Training data in form of unlabeled instances $\{x_1, x_2, ..., x_N\}$
- Goal: Learn the intrinsic latent structure that summarizes/explains data
- Clustering: Learn the grouping structure for a given set of unlabeled inputs.
- Homogeneous groups as latent structure: Clustering
- Other Applications: Topic Modelling, Image Segmentation, Social Networking.



- Given: Training data in form of unlabeled instances $\{x_1, x_2, \ldots, x_N\}$
- Dimensionality Reduction: Learn a Low-dimensional representation for a given set of high-dimensional inputs
- Note: DR also comes in supervised flavors (supervised DR)
- Other Applications: facial recognition, computer vision and image compression.

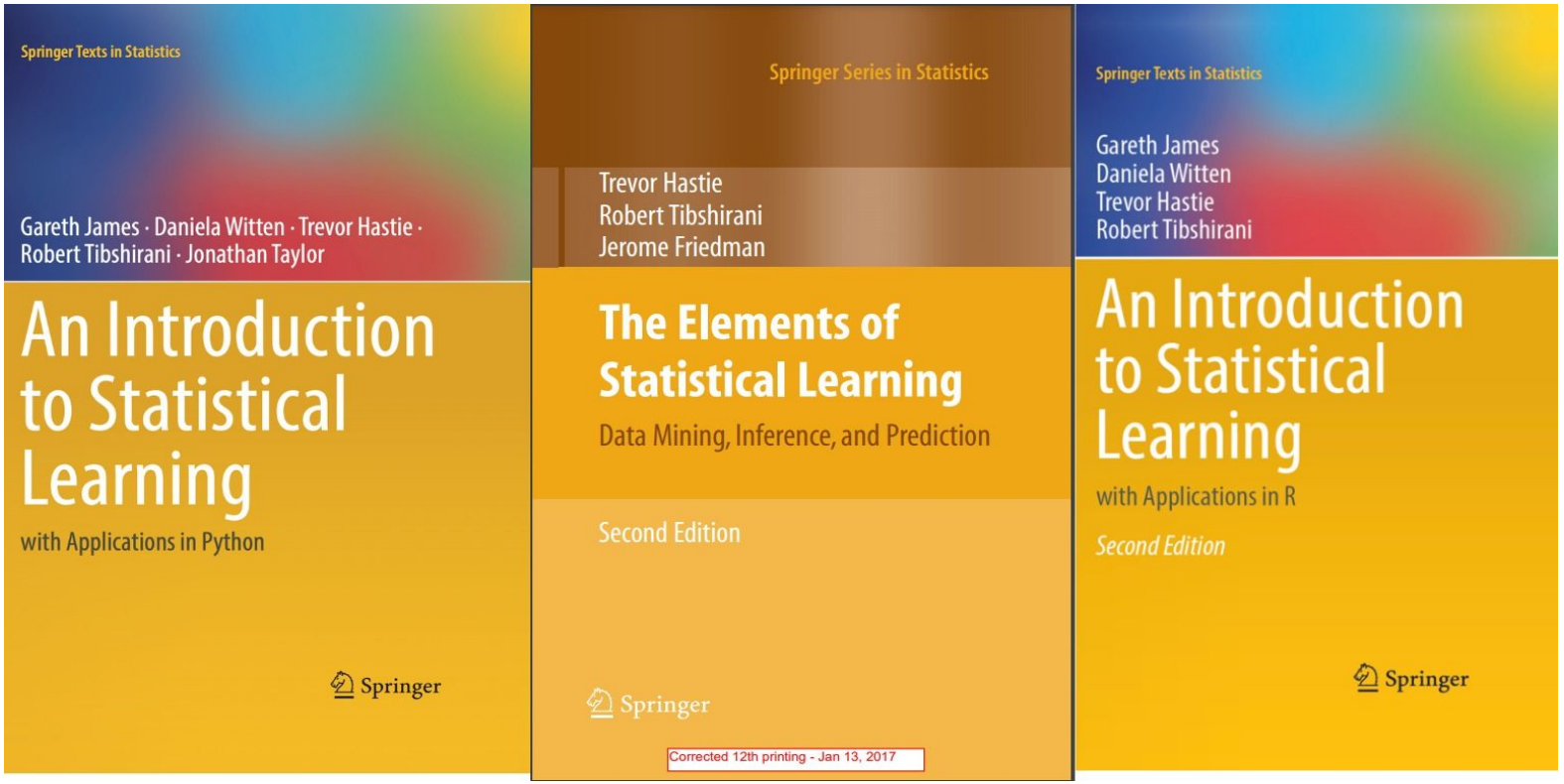


Two-dim to one-dim linear projection



Three-dim to two-dim nonlinear projection (a.k.a. manifold learning)

Textbooks


Springer Texts in Statistics
Gareth James · Daniela Witten · Trevor Hastie · Robert Tibshirani · Jonathan Taylor
An Introduction to Statistical Learning
with Applications in Python
Springer
Springer Series in Statistics
Trevor Hastie
Robert Tibshirani
Jerome Friedman
The Elements of Statistical Learning
Data Mining, Inference, and Prediction
Second Edition
Springer
Corrected 12th printing - Jan 13, 2017
Springer Texts in Statistics
Gareth James
Daniela Witten
Trevor Hastie
Robert Tibshirani
An Introduction to Statistical Learning
with Applications in R
Second Edition
Springer

Questions of the day

What type of data are involved in the following applications?

- Weather forecasting
- Mobile usage of all customers of a service provider
- Anomaly (e.g. fraud) detection in a bank organization
- Person categorization, that is, identifying a human
- Air traffic control in an airport
- Streaming data from all flying air crafts of Boeing

End of Session

Caution: *"Prediction is very difficult, especially if it's about the future"*
- Niels Bohr, Father of Quantum.