Heurísticas para *t*-admissibilidade com abordagens de redes complexas

Universidade Federal Fluminense

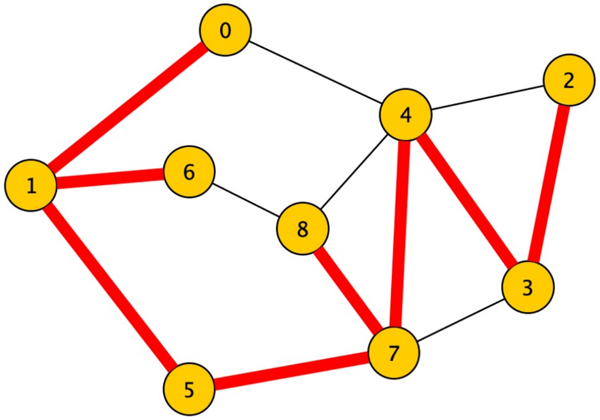
Carlos Santos
Anderson Zudio
Leandro Santiago
Luís Cunha

t-admissibilidade - definição

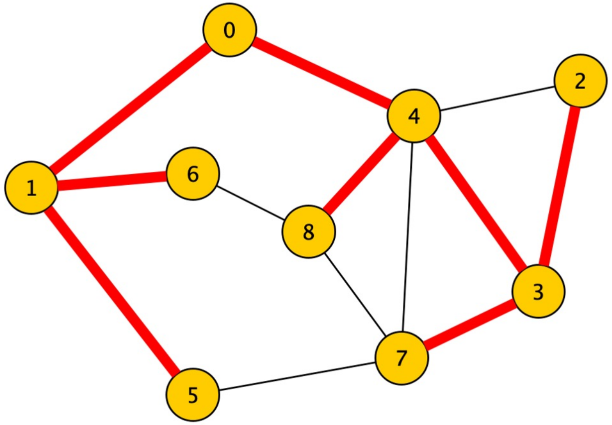
- O problema da **t-admissibilidade** visa decidir se um grafo G possui uma árvore geradora T no qual a **maior distância** em **T** entre dois vértices adjacentes de G é no **máximo t**. (**fator de extensão**)

- **O menor t** para o qual o grafo é **t-admissível**, chamamos de **índice de extensão**.

t-admissibilidade - exemplo

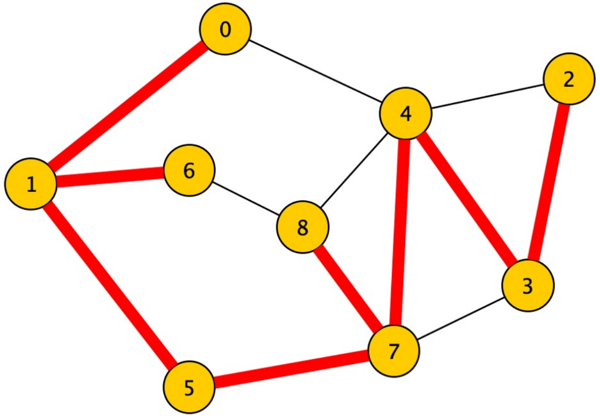


d(0,4) = 4
d(6,8) = 4
d(4,8) = 2
d(3,7) = 2
d(2,4) = 2

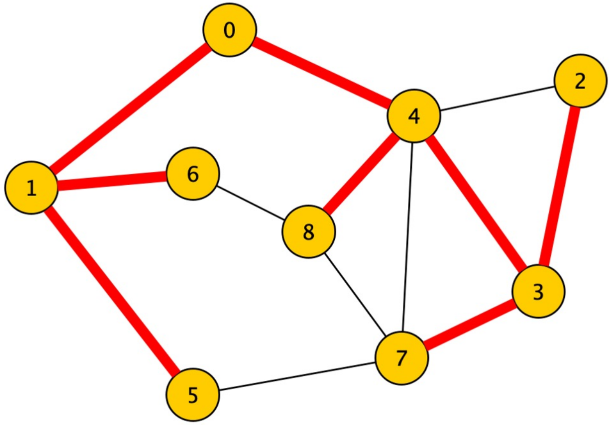


d(5,7) = 5
d(6,8) = 4
d(7,8) = 3
d(4,7) = 2
d(2,4) = 2

t-admissibilidade - exemplo



d(0,4) = 4
d(6,8) = 4



d(5,7) = 5

Índice de extensão = 4

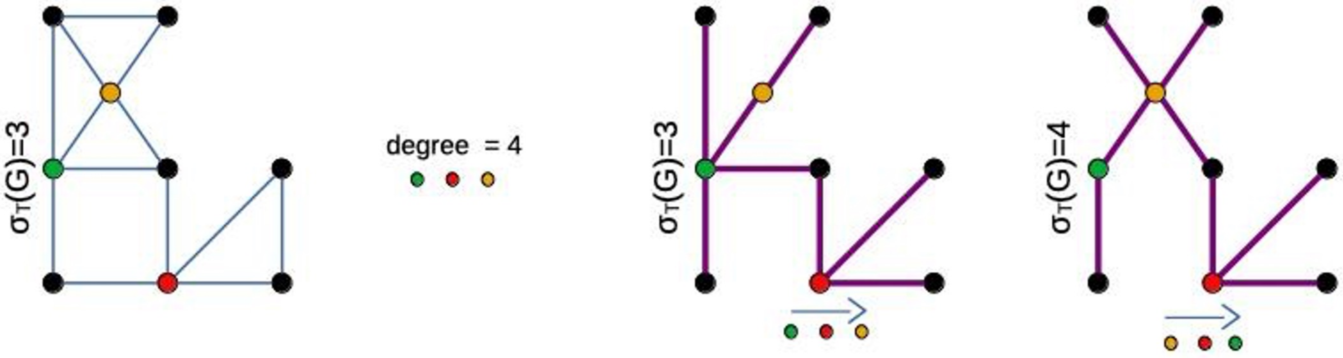
COMPLEXIDADE COMPUTACIONAL

	Complexidade
t = 2	**Polinomial**
t = 3	**Em aberto**
t ≥ 4	**NP-Completo**

1. **Cai and Corneil** https://doi.org/10.1137/S0895480192237403

HEURÍSTICAS

Strategies for generating tree spanners: Algorithms, heuristics and optimal graph classes[2]



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[2] Couto e colaboradores https://doi.org/10.1016/j.ipl.2022.106265

MEDIDAS DE CENTRALIDADE

Degree centrality

$$D_c(v)=\sum_{u=1}^{n} A_{uv}, u\neq v$$

Leverage centrality

$$L_c(v)=\frac{1}{d(v)}\cdot\sum_{v_j\in N(v)}\frac{d(v)-d(v_j)}{d(v)+d(v_j)}$$

Closeness centrality

$$C_c(v)=\frac{1}{\sum_{u\in V(G)\setminus v} d(u,v)}$$

CONTRIBUIÇÃO

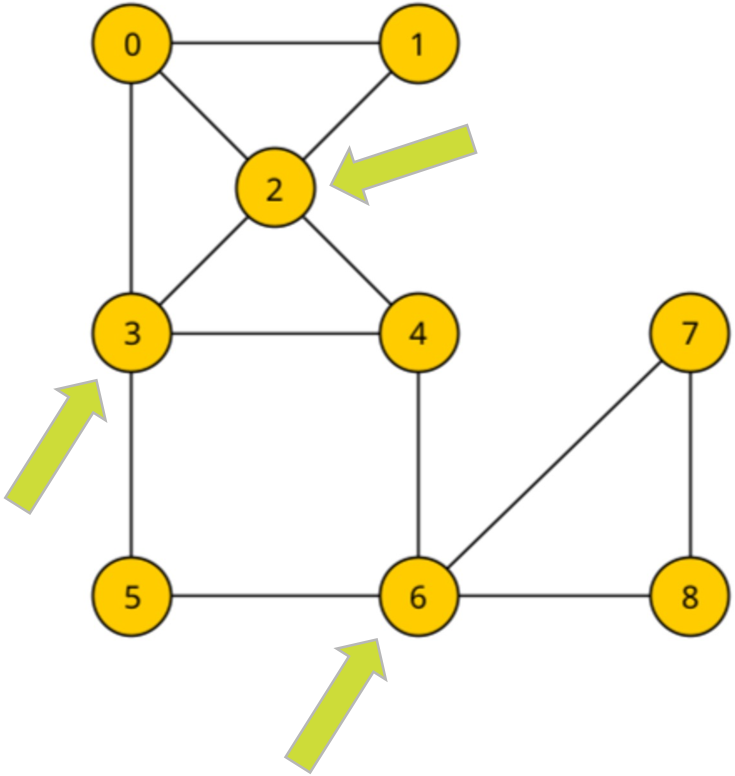
- Utilizar novas medidas de centralidade para o problema de empate
- Construção de 4 heurísticas (2 adaptadas e 2 novas)
- Análise de qualidade das heurísticas

INSTRUÇÕES PARA HEURÍSTICA 1

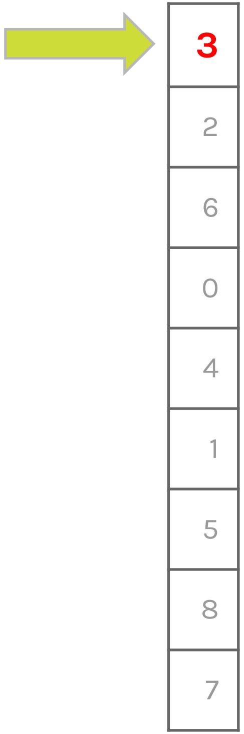
- Classificar os vértices por

  + degree
  + closeness
  - leverage

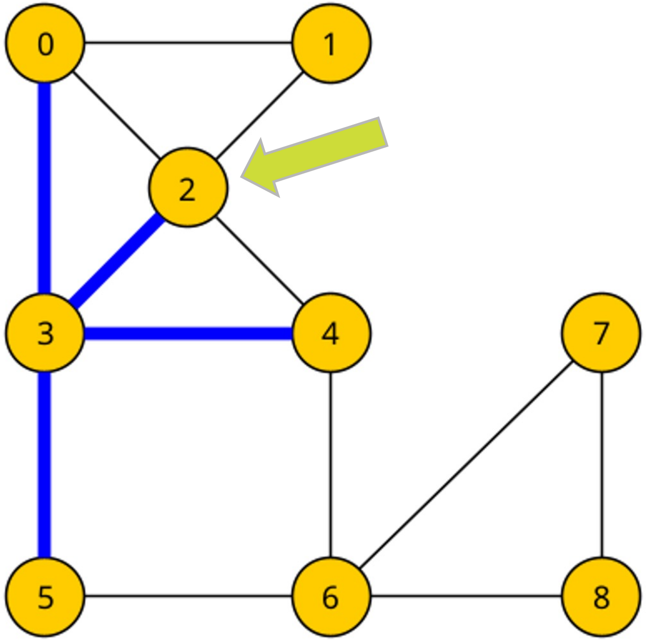
v	d(v)	Clos.	Lev.
3	**4**	**0,0434**	**0,1547**
2	4	0,0434	0,1547
6	4	0,0434	0,2857
0	3	0,0370	-0,0285
4	3	0,0454	-0,1428
1	2	0,0344	-0,0266
5	2	0,0416	-0,3333
8	2	0,0344	-0,1666
7	2	0,0344	-0,1666



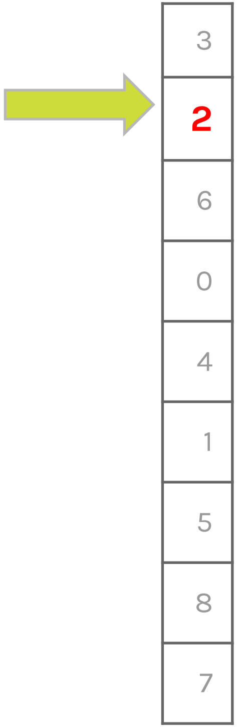
INSTRUÇÕES PARA
HEURÍSTICA 1

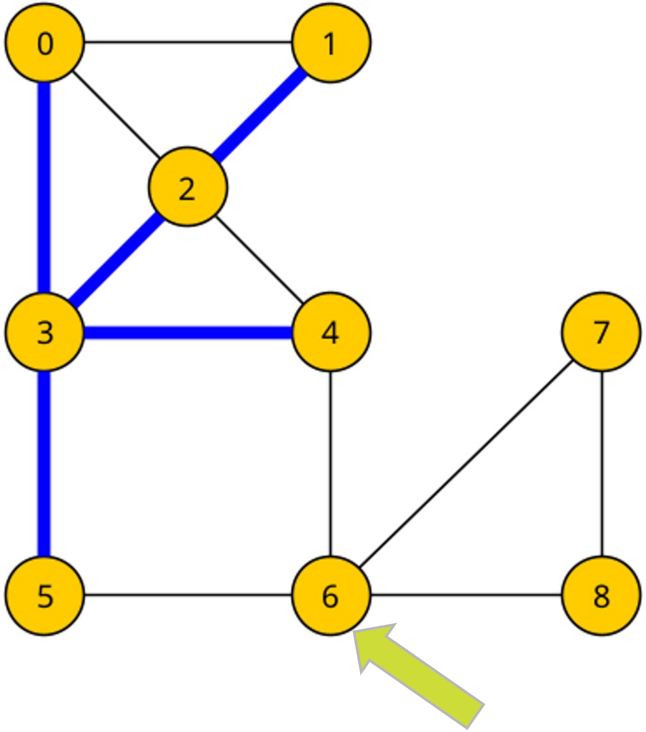


3
2
6
0
4
1
5
8
7



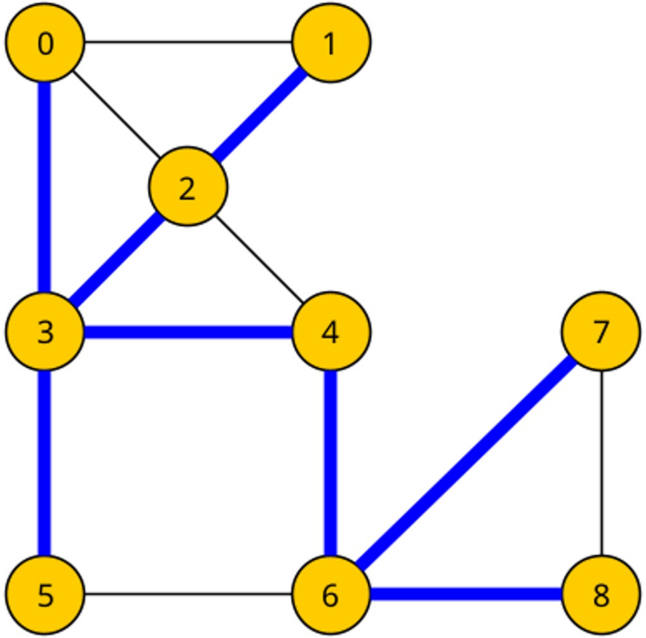
INSTRUÇÕES PARA
HEURÍSTICA 1


3
2
6
0
4
1
5
8
7



INSTRUÇÕES PARA
HEURÍSTICA 1

3
2
6
0
4
1
5
8
7



INSTRUÇÕES PARA HEURÍSTICA 2

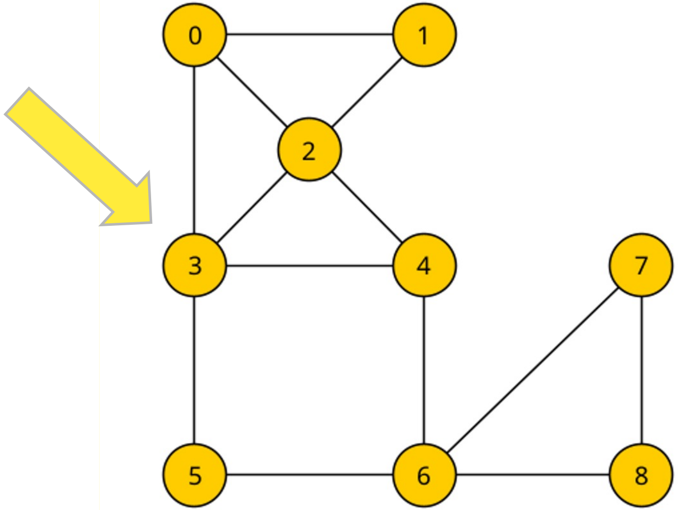
- Classificar os vértices por

  \+ degree

  \+ closeness

  \- leverage

v	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2



INSTRUÇÕES PARA HEURÍSTICA 2

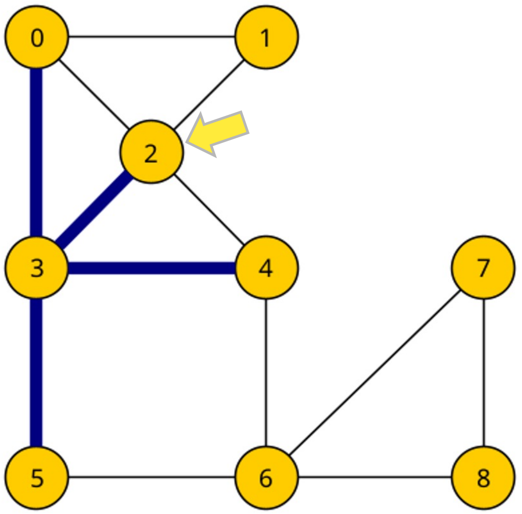
v	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2

Grau do vértice no grafo

Número de vizinhos do vértice que estão na árvore

v	f(v) = $d_G(v) - A_{tree}(v)$
0	1
2	1
4	1
5	1

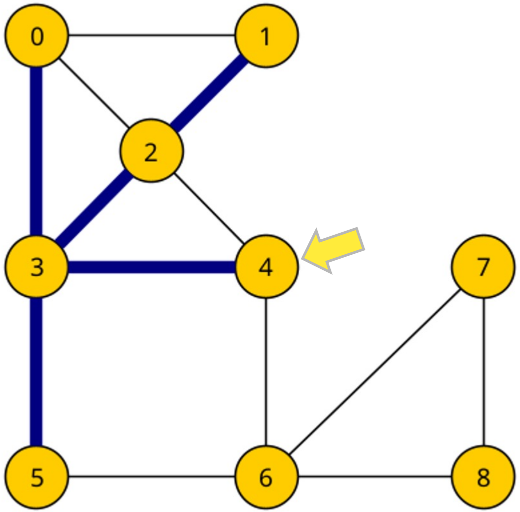
O vértice que tem a capacidade de contribuir com o maior número de novos vizinhos na árvore



INSTRUÇÕES PARA HEURÍSTICA 2

v	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2

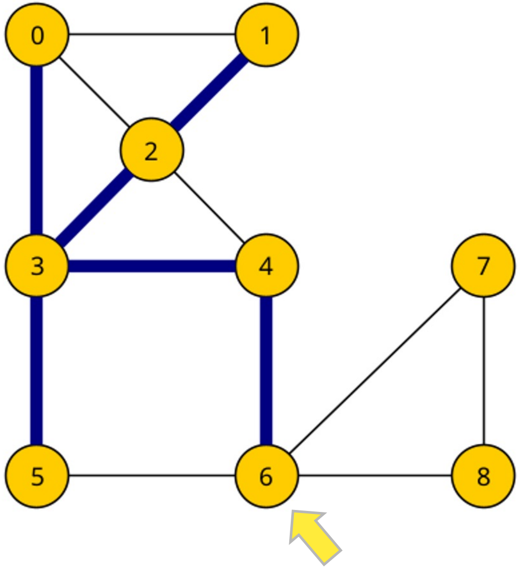
v	f(v) = $d_G(v) - A_{tree}(v)$
0	0
1	0
4	1
5	1



INSTRUÇÕES PARA
HEURÍSTICA 2

v	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2

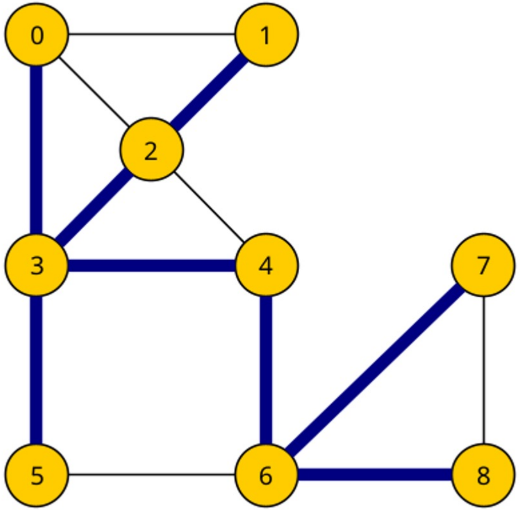
v	f(v) = $d_G(v) - A_{tree}(v)$
0	0
5	0
1	0
6	2



INSTRUÇÕES PARA HEURÍSTICA 2

v	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2

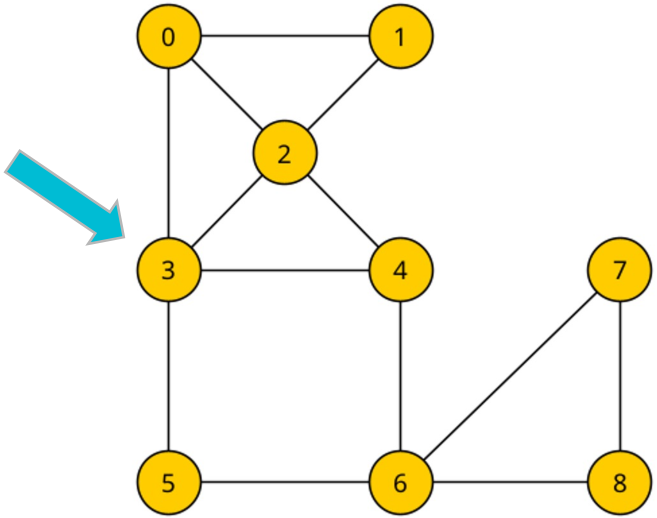
v	f(v) = $d_G(v) - A_{tree}(v)$
0	0
1	0
5	0
7	0
8	0



INSTRUÇÕES PARA
HEURÍSTICA 3

- Liste os vértices por
  + degree
  + closeness
  - leverage

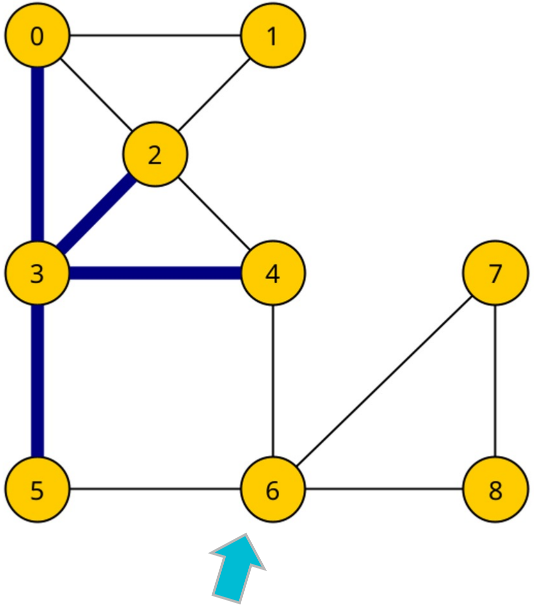
V'	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2



INSTRUÇÕES PARA
HEURÍSTICA 3

V	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2

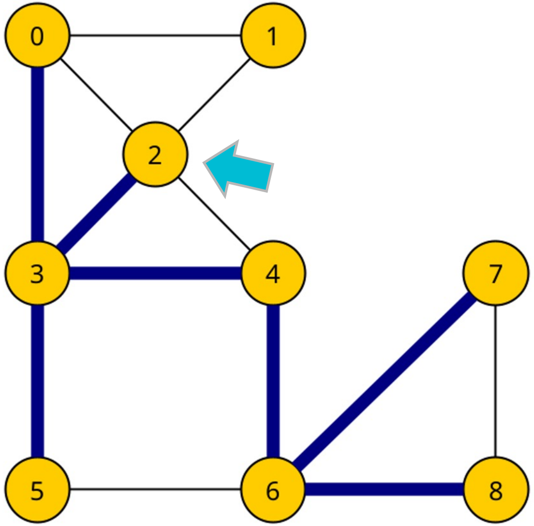
v	f(v) = $d_G(v) - A_{tree}(v)$
1	0
0	1
2	1
4	1
5	1
6	2
7	2
8	2



INSTRUÇÕES PARA
HEURÍSTICA 3

v	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2

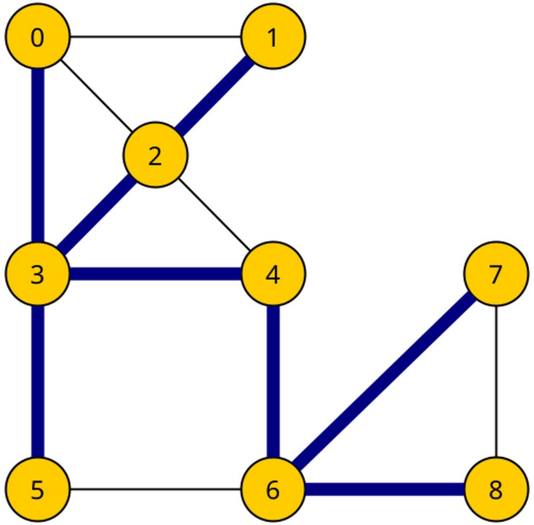
v	f(v) = $d_G(v) - A_{tree}(v)$
1	0
4	0
5	0
7	0
8	0
0	1
2	1



INSTRUÇÕES PARA
HEURÍSTICA 3

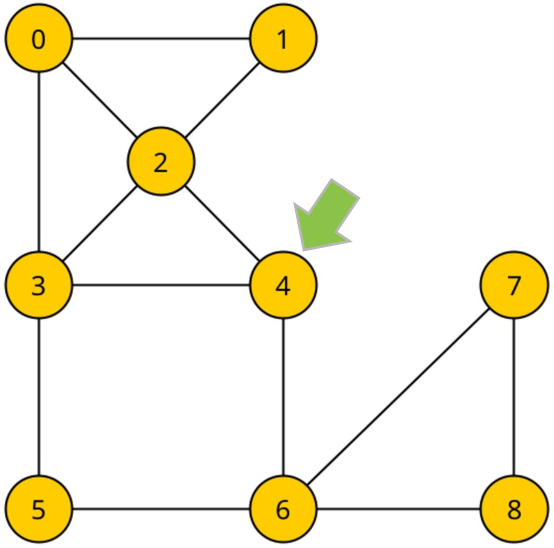
v	d(v)
3	4
2	4
6	4
0	3
4	3
1	2
5	2
8	2
7	2

v	f(v) = $d_G(v) - A_{tree}(v)$
1	0
4	0
5	0
7	0
8	0
0	0
2	0

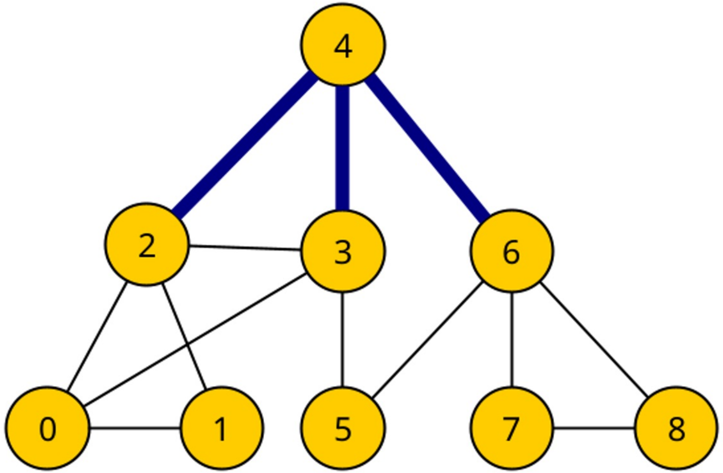


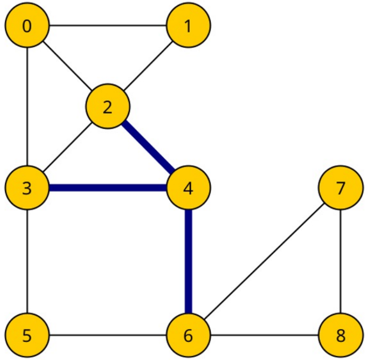
INSTRUÇÕES PARA
HEURÍSTICA 4

v	Closeness	Leverage
0	0,0370	-0,0285
1	0,0344	-0,0266
2	0,0434	0,1547
3	0,0434	0,1547
4	**0,0454**	**-0,1428**
5	0,0416	-0,3333
6	0,0434	0,2857
7	0,0344	-0,1666
8	0,0344	-0,1666

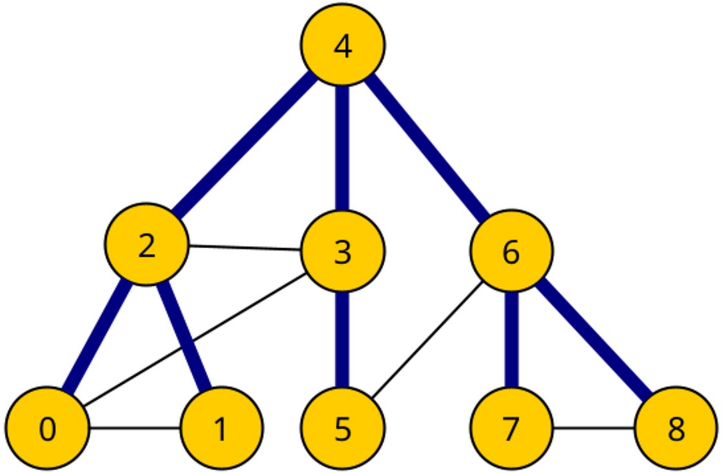


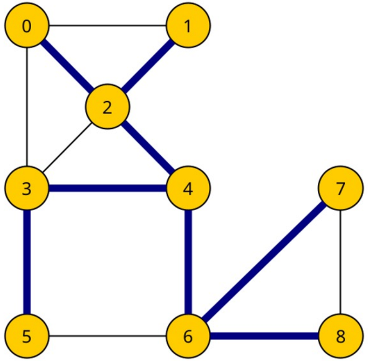
INSTRUÇÕES PARA HEURÍSTICA 4





INSTRUÇÕES PARA
HEURÍSTICA 4

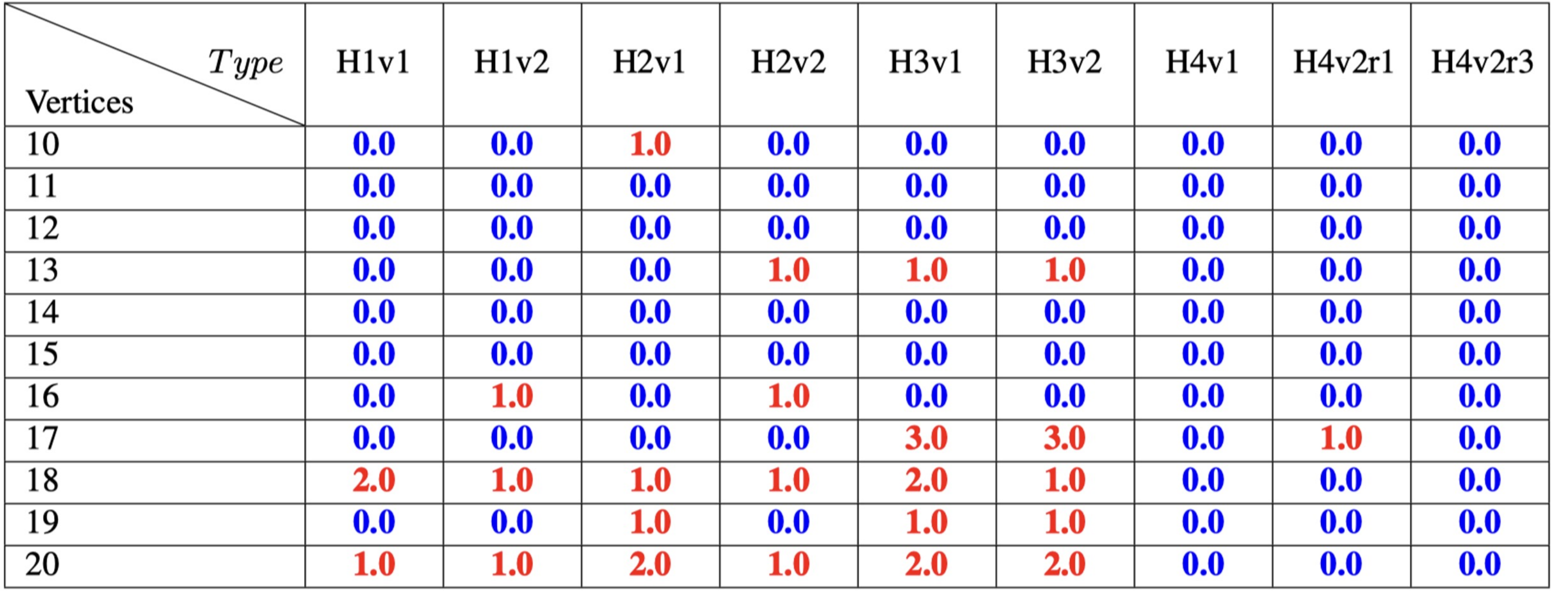




EXPERIMENTOS

- 11 grafos aleatórios de 10 a 20 vértices com máximo de 34 arestas.
- 400 grafos distribuídos de 100 a 1000 vértices das classes Bipartido, Erdos, Watts e Barabási.

QUALIDADE DAS HEURÍSTICAS



Type / Vertices	H1v1	H1v2	H2v1	H2v2	H3v1	H3v2	H4v1	H4v2r1	H4v2r3
10	0.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0	0.0
11	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
12	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
13	0.0	0.0	0.0	1.0	1.0	1.0	0.0	0.0	0.0
14	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
15	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
16	0.0	1.0	0.0	1.0	0.0	0.0	0.0	0.0	0.0
17	0.0	0.0	0.0	0.0	3.0	3.0	0.0	1.0	0.0
18	2.0	1.0	1.0	1.0	2.0	1.0	0.0	0.0	0.0
19	0.0	0.0	1.0	0.0	1.0	1.0	0.0	0.0	0.0
20	1.0	1.0	2.0	1.0	2.0	2.0	0.0	0.0	0.0

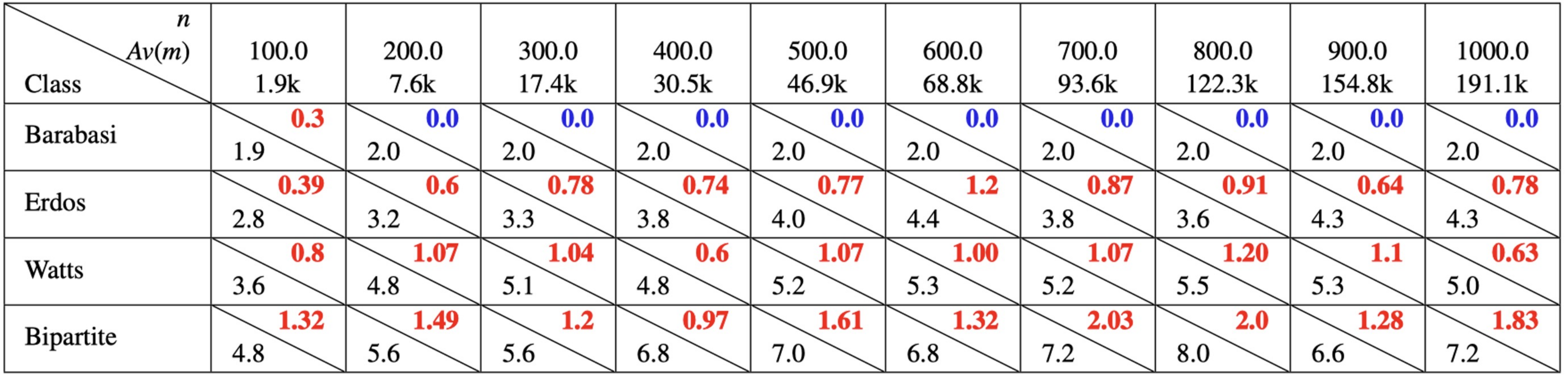
	DC	DC CC LC	DC	DC CC LC	DC	DC CC LC	DC CC LC	CC LC	CC LC

DC Degree centrality

CC Closeness centrality

LC Leverage Centrality

RESULTADOS – Heurística 1



n *Av(m)* Class	100.0 1.9k	200.0 7.6k	300.0 17.4k	400.0 30.5k	500.0 46.9k	600.0 68.8k	700.0 93.6k	800.0 122.3k	900.0 154.8k	1000.0 191.1k
Barabasi	**0.3** 1.9	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0
Erdos	**0.39** 2.8	**0.6** 3.2	**0.78** 3.3	**0.74** 3.8	**0.77** 4.0	**1.2** 4.4	**0.87** 3.8	**0.91** 3.6	**0.64** 4.3	**0.78** 4.3
Watts	**0.8** 3.6	**1.07** 4.8	**1.04** 5.1	**0.6** 4.8	**1.07** 5.2	**1.00** 5.3	**1.07** 5.2	**1.20** 5.5	**1.1** 5.3	**0.63** 5.0
Bipartite	**1.32** 4.8	**1.49** 5.6	**1.2** 5.6	**0.97** 6.8	**1.61** 7.0	**1.32** 6.8	**2.03** 7.2	**2.0** 8.0	**1.28** 6.6	**1.83** 7.2

DC CC LC

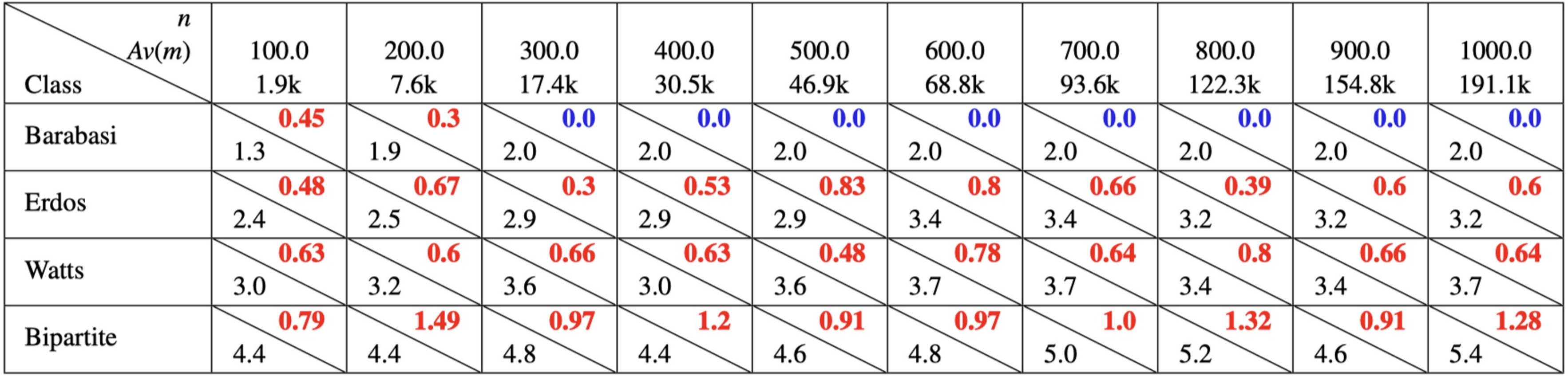
- Valores inferiores na célula representam a média entre o FE da heurística e o valor do limite inferior definido na literatura
- Valores superiores na célula representam o desvio padrão

DC Degree centrality

CC Closeness centrality

LC Leverage Centrality

RESULTADOS – Heurística 2



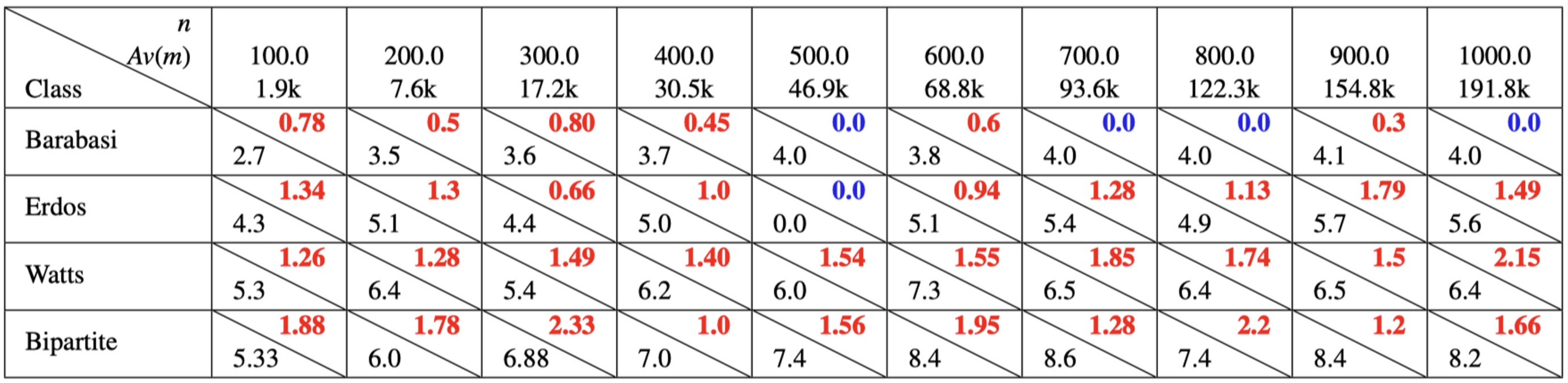
Class \ n Av(m)	100.0 1.9k	200.0 7.6k	300.0 17.4k	400.0 30.5k	500.0 46.9k	600.0 68.8k	700.0 93.6k	800.0 122.3k	900.0 154.8k	1000.0 191.1k
Barabasi	0.45 1.3	0.3 1.9	0.0 2.0	0.0 2.0	0.0 2.0	0.0 2.0	0.0 2.0	0.0 2.0	0.0 2.0	0.0 2.0
Erdos	0.48 2.4	0.67 2.5	0.3 2.9	0.53 2.9	0.83 2.9	0.8 3.4	0.66 3.4	0.39 3.2	0.6 3.2	0.6 3.2
Watts	0.63 3.0	0.6 3.2	0.66 3.6	0.63 3.0	0.48 3.6	0.78 3.7	0.64 3.7	0.8 3.4	0.66 3.4	0.64 3.7
Bipartite	0.79 4.4	1.49 4.4	0.97 4.8	1.2 4.4	0.91 4.6	0.97 4.8	1.0 5.0	1.32 5.2	0.91 4.6	1.28 5.4

DC CC LC

- Valores inferiores na célula representam a média entre o FE da heurística e o valor do limite inferior definido na literatura
- Valores superiores na célula representam o desvio padrão

DC Degree centrality CC Closeness centrality LC Leverage Centrality

RESULTADOS – Heurística 3



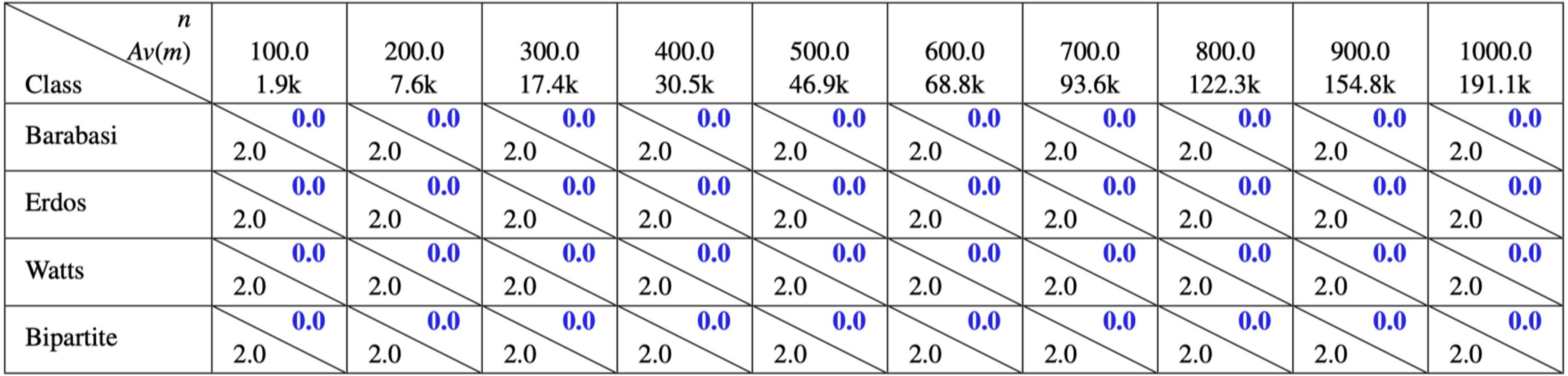
n *Av*(*m*) Class	100.0 1.9k	200.0 7.6k	300.0 17.2k	400.0 30.5k	500.0 46.9k	600.0 68.8k	700.0 93.6k	800.0 122.3k	900.0 154.8k	1000.0 191.8k
Barabasi	**0.78** 2.7	**0.5** 3.5	**0.80** 3.6	**0.45** 3.7	**0.0** 4.0	**0.6** 3.8	**0.0** 4.0	**0.0** 4.0	**0.3** 4.1	**0.0** 4.0
Erdos	**1.34** 4.3	**1.3** 5.1	**0.66** 4.4	**1.0** 5.0	**0.0** 0.0	**0.94** 5.1	**1.28** 5.4	**1.13** 4.9	**1.79** 5.7	**1.49** 5.6
Watts	**1.26** 5.3	**1.28** 6.4	**1.49** 5.4	**1.40** 6.2	**1.54** 6.0	**1.55** 7.3	**1.85** 6.5	**1.74** 6.4	**1.5** 6.5	**2.15** 6.4
Bipartite	**1.88** 5.33	**1.78** 6.0	**2.33** 6.88	**1.0** 7.0	**1.56** 7.4	**1.95** 8.4	**1.28** 8.6	**2.2** 7.4	**1.2** 8.4	**1.66** 8.2

DC CC LC

- Valores inferiores na célula representam a média entre o FE da heurística e o valor do limite inferior definido na literatura
- Valores superiores na célula representam o desvio padrão

DC Degree centrality   CC Closeness centrality   LC Leverage Centrality

RESULTADOS – Heurística 4



n *Av*(*m*) Class	100.0 1.9k	200.0 7.6k	300.0 17.4k	400.0 30.5k	500.0 46.9k	600.0 68.8k	700.0 93.6k	800.0 122.3k	900.0 154.8k	1000.0 191.1k
Barabasi	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0
Erdos	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0
Watts	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0
Bipartite	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0	**0.0** 2.0

DC CC LC

- Valores inferiores na célula representam a média entre o FE da heurística e o valor do limite inferior definido na literatura
- Valores superiores na célula representam o desvio padrão

DC Degree centrality CC Closeness centrality LC Leverage Centrality

CONCLUSÃO

- As medidas de centralidade aperfeiçoaram a seleção dos vértices
- A heurística 4 apresentou árvores com melhores soluções para o fator de extensão
- Avaliar as heurísticas com novas classes de grafos

☺

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OBRIGADO!

Perguntas?

@cthadeusantos & cthadeusantos@gmail.com

& carlosthadeu@id.uff.br