

N3Results

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Here is an example from [1] and [2], [3], [4], and [5].

This notebook reproduces the results from the paper

C. T. Kelley, *Newton's Method in Three Precisions*, 2023 [6].

The example is the Chandrasekhar H-equation [7].

All computations were done in Julia [8].

We use the software from [3], [4], and [5]. Our nonlinear solvers from [5] work well with our data structures for iterative refinement. The Julia module with the data structures we use for IR is

The results are in Table 2 and Table 3.

The first computation generates Table 2. This is the ill-conditioned example from [2] and is the well conditioned example from the paper. The function that creates the examples is the Julia function `htest1` from the file `H_equation_examples.jl`. We also put a plot in this notebook.

[6]: `htest1(4096,.99)`

```
Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large  4.05010e-01
Try increasing maxit and checking your function and
      Jacobian for bugs.
Give the history array a look to see what's happening.
```

```
Norm increased: 1.013279e-6, 1.013279e-6, 1.00000e-06
```

```
\begin{tabular}{lllllll}
n&      F64&      F32&      F16&  IR 32-16&  IR-GM \\ \hline
0 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & \\
1 & 2.289e-01 & 2.289e-01 & 5.065e-01 & 2.289e-01 & 2.289e-01 & \\
2 & 3.934e-02 & 3.934e-02 & 2.958e-01 & 3.934e-02 & 3.934e-02 & \\
3 & 2.737e-03 & 2.737e-03 & 1.890e-01 & 2.737e-03 & 2.737e-03 & \\
4 & 1.767e-05 & 1.767e-05 & 1.255e-01 & 1.767e-05 & 1.767e-05 & \\
5 & 7.486e-10 & 7.516e-10 & 8.518e-02 & 7.552e-10 & 7.502e-10 & \\
6 & & & 6.068e-02 & & & \\
7 & & & 4.240e-02 & & & \\
8 & & & 3.195e-02 & & & \\
9 & & & 2.280e-02 & & & \\
10 & & & 1.713e-02 & & & \\
\hline\end{tabular}
```

```

\end{tabular}
[1.47022e-10 1.28427e-10 5.97601e-02]
[6 6 11 6 6]
2.09064e-06

```

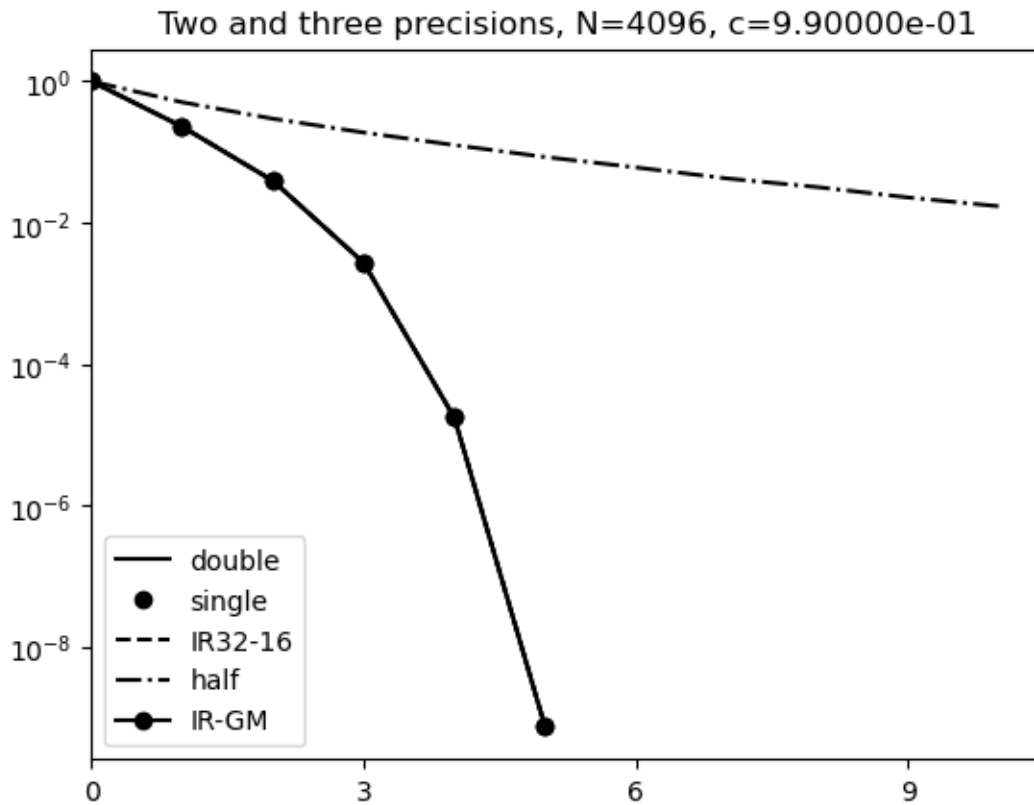


Table 3 is a very ill-conditioned example.

```
[7]: htest1(4096,.9999)
```

```

Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large 7.07016e-01
Try increasing maxit and checking your function and
      Jacobian for bugs.
Give the history array a look to see what's happening.

```

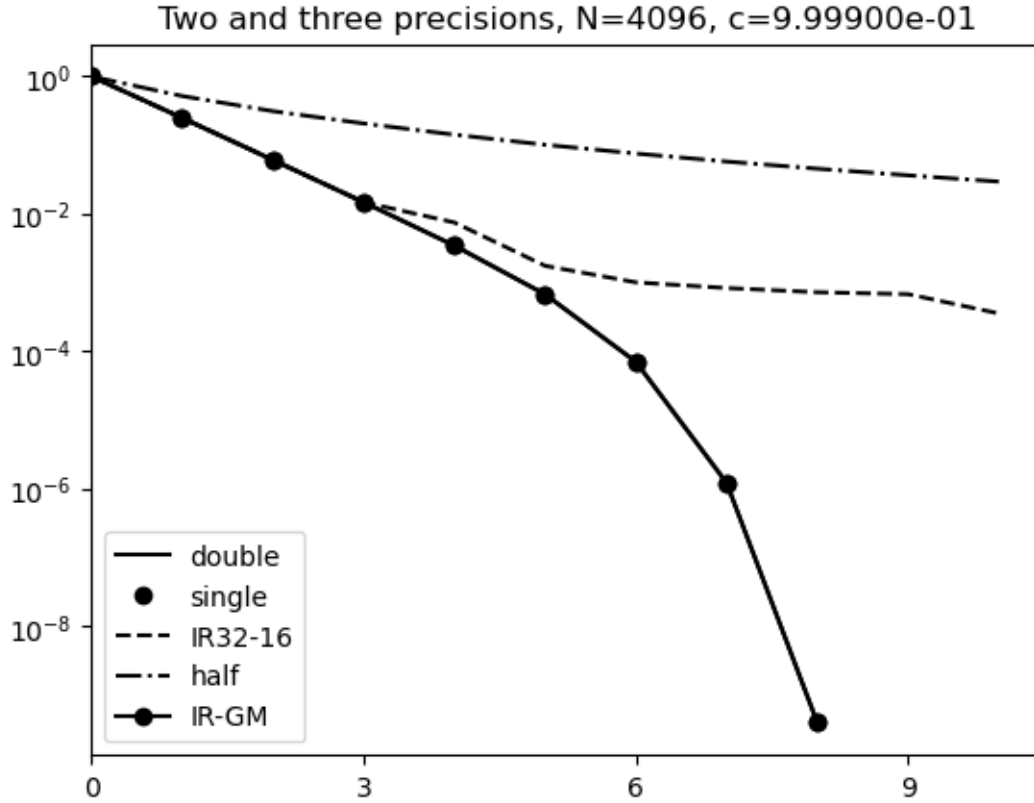
```

Norm increased: 1.3113022e-6, 1.3113022e-6, 1.00000e-06
Norm increased: 3.516674e-6, 3.516674e-6, 1.00000e-06
Norm increased: 0.60077906, 0.6, 1.00000e-06
Norm increased: 0.00028830767, 0.00028008223, 1.00000e-06
Norm increased: 0.7379273, 0.7238793, 1.00000e-06
Norm increased: 0.90214574, 0.8763197, 1.00000e-06
Norm increased: 0.84842783, 0.8370482, 1.00000e-06

```

Norm increased: 0.9719259, 0.9010718, 1.00000e-06
 Norm increased: 0.5126856, 0.50187343, 1.00000e-06
 Maximum iterations (maxit) of 10 exceeded
 Convergence failure: residual norm too large 8.53620e-03
 Try increasing maxit and checking your function and
 Jacobian for bugs.
 Give the history array a look to see what's happening.

```
\begin{tabular}{lllllll}
n&      F64&      F32&      F16&  IR 32-16&  IR-GM  \\ \hline
0 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & \\
1 & 2.494e-01 & 2.494e-01 & 5.182e-01 & 2.494e-01 & 2.494e-01 & \\
2 & 6.093e-02 & 6.093e-02 & 3.123e-01 & 6.093e-02 & 6.093e-02 & \\
3 & 1.480e-02 & 1.480e-02 & 2.067e-01 & 1.480e-02 & 1.480e-02 & \\
4 & 3.454e-03 & 3.454e-03 & 1.421e-01 & 7.500e-03 & 3.454e-03 & \\
5 & 6.762e-04 & 6.762e-04 & 1.012e-01 & 1.756e-03 & 6.762e-04 & \\
6 & 7.049e-05 & 7.049e-05 & 7.552e-02 & 1.009e-03 & 7.049e-05 & \\
7 & 1.223e-06 & 1.223e-06 & 5.773e-02 & 8.345e-04 & 1.223e-06 & \\
8 & 3.947e-10 & 3.952e-10 & 4.543e-02 & 7.244e-04 & 3.955e-10 & \\
9 &          &          & 3.639e-02 & 6.789e-04 &          & \\
10 &          &          & 2.949e-02 & 3.560e-04 &          & \\
\hline
\end{tabular}
[9.69122e-03 1.03566e-10 2.82749e-01]
[9 9 11 11 9]
1.15802e-06
```



References

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- [5] C. T. KELLEY, *SIAMFANLEquations.jl*. <https://github.com/ctkelley/SIAMFANLEquations.jl>, 2022, doi:10.5281/zenodo.4284807, <https://github.com/ctkelley/SIAMFANLEquations.jl>. Julia Package.
- [6] C. T. KELLEY, *Newton's method in three precisions*, 2023, arXiv:2307.16051.
- [7] S. CHANDRASEKHAR, *Radiative Transfer*, Dover, New York, 1960.

- [8] J. BEZANSON, A. EDELMAN, S. KARPINSKI, AND V. B. SHAH, *Julia: A fresh approach to numerical computing*, SIAM Review, 59 (2017), pp. 65–98.