

N3Results

C. T. Kelley

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Here is an example from [3] and [4], [7], [5], and [6].

This notebook reproduces the results from the paper

C. T. Kelley, *Newton's Method in Three Precisions*, 2023 [8].

The example is the Chandrasekhar H-equation [2].

All computations were done in Julia [1].

We use the software from [7], [5], and [6]. Our nonlinear solvers from [6] work well with our data structures for iterative refinement. The Julia module with the data structures we use for IR is **src/N3Results**.

The results are in Table 2 and Table 3.

The first computation generates Table 2. This is the ill-conditioned example from [4] and is the well conditioned example from the paper. The function that creates the examples is the Julia function `htest1` from the file `H_equation_examples.jl`. We also put a plot in this notebook.

```
[6]: htest1(4096,.99)
```

```
Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large  4.05010e-01
Try increasing maxit and checking your function and
      Jacobian for bugs.
Give the history array a look to see what's happening.
```

```
Norm increased: 1.013279e-6, 1.013279e-6, 1.00000e-06
```

```
\begin{tabular}{lllllll}
```

n&	F64&	F32&	F16&	IR 32-16&	IR-GM \&	\hline
0 &	1.000e+00 &	1.000e+00 &	1.000e+00 &	1.000e+00 \&		
1 &	2.289e-01 &	2.289e-01 &	5.065e-01 &	2.289e-01 \&		
2 &	3.934e-02 &	3.934e-02 &	2.958e-01 &	3.934e-02 \&		
3 &	2.737e-03 &	2.737e-03 &	1.890e-01 &	2.737e-03 \&		
4 &	1.767e-05 &	1.767e-05 &	1.255e-01 &	1.767e-05 \&		
5 &	7.486e-10 &	7.516e-10 &	8.518e-02 &	7.552e-10 \&		
6 &	&	&	6.068e-02 &	&	\&	
7 &	&	&	4.240e-02 &	&	\&	
8 &	&	&	3.195e-02 &	&	\&	
9 &	&	&	2.280e-02 &	&	\&	
10 &	&	&	1.713e-02 &	&	\&	

```

\hline
\end{tabular}
[1.47022e-10 1.28427e-10 5.97601e-02]
[6 6 11 6 6]
2.09064e-06

```

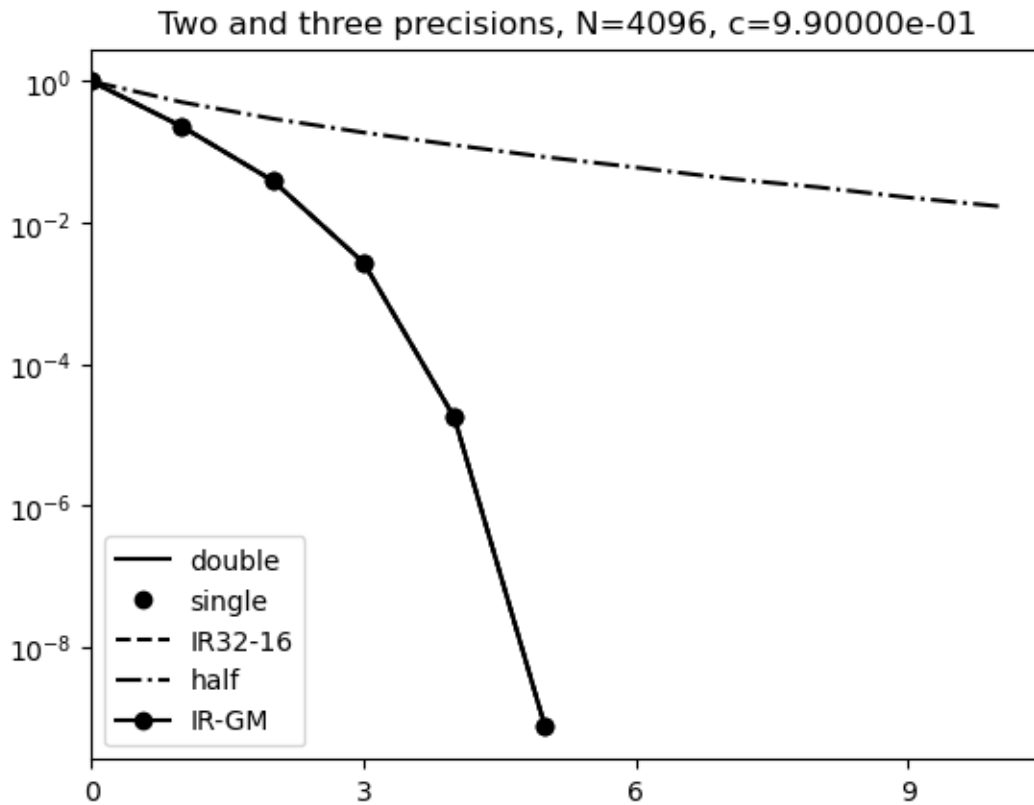


Table 3 is a very ill-conditioned example.

```
[7]: htest1(4096,.9999)
```

```

Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large 7.07016e-01
Try increasing maxit and checking your function and
      Jacobian for bugs.
Give the history array a look to see what's happening.

```

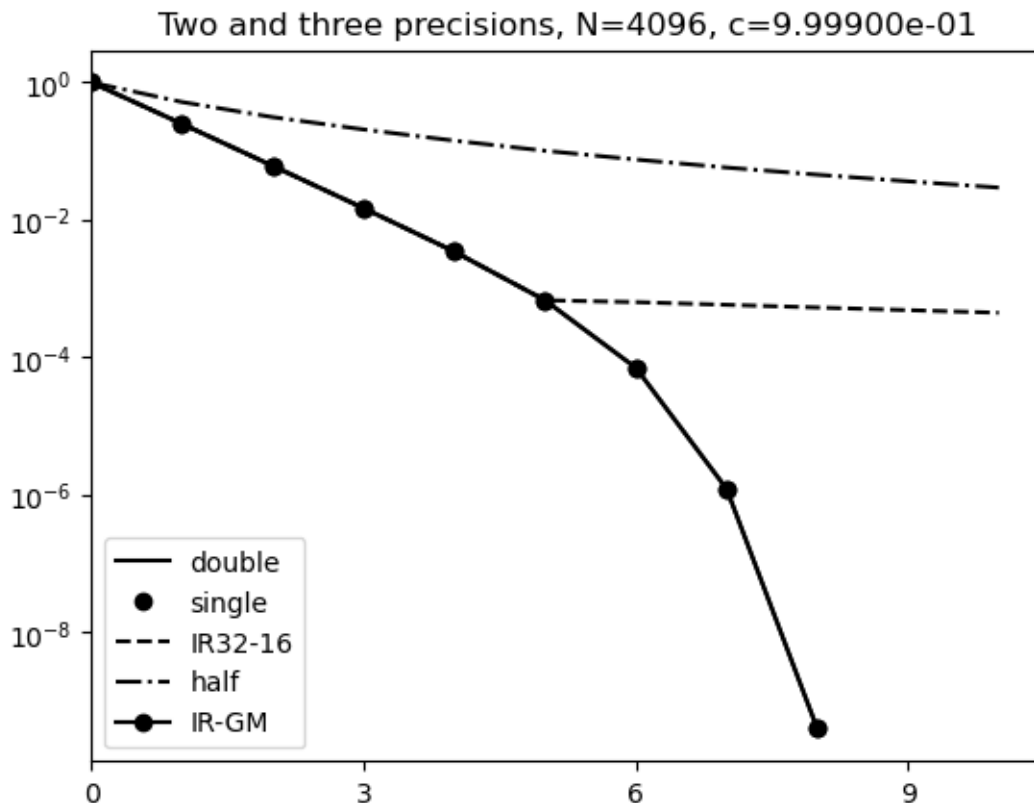
```

Norm increased: 1.3113022e-6, 1.3113022e-6, 1.00000e-06
Norm increased: 3.516674e-6, 3.516674e-6, 1.00000e-06
Norm increased: 0.60077906, 0.6, 1.00000e-06
Norm increased: 0.00028830767, 0.00028008223, 1.00000e-06
Norm increased: 0.7379273, 0.7238793, 1.00000e-06
Norm increased: 0.90214574, 0.8763197, 1.00000e-06

```

Norm increased: 0.84842783, 0.8370482, 1.00000e-06
 Norm increased: 0.9719259, 0.9010718, 1.00000e-06
 Norm increased: 0.5126856, 0.50187343, 1.00000e-06
 Maximum iterations (maxit) of 10 exceeded
 Convergence failure: residual norm too large 8.53620e-03
 Try increasing maxit and checking your function and
 Jacobian for bugs.
 Give the history array a look to see what's happening.

```
\begin{tabular}{lllllll}
n& F64& F32& F16& IR 32-16& IR-GM & \hline
0 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & \\
1 & 2.494e-01 & 2.494e-01 & 5.182e-01 & 2.494e-01 & 2.494e-01 & \\
2 & 6.093e-02 & 6.093e-02 & 3.123e-01 & 6.093e-02 & 6.093e-02 & \\
3 & 1.480e-02 & 1.480e-02 & 2.067e-01 & 1.480e-02 & 1.480e-02 & \\
4 & 3.454e-03 & 3.454e-03 & 1.421e-01 & 7.500e-03 & 3.454e-03 & \\
5 & 6.762e-04 & 6.762e-04 & 1.012e-01 & 1.756e-03 & 6.762e-04 & \\
6 & 7.049e-05 & 7.049e-05 & 7.552e-02 & 1.009e-03 & 7.049e-05 & \\
7 & 1.223e-06 & 1.223e-06 & 5.773e-02 & 8.345e-04 & 1.223e-06 & \\
8 & 3.947e-10 & 3.952e-10 & 4.543e-02 & 7.244e-04 & 3.955e-10 & \\
9 & & & 3.639e-02 & 6.789e-04 & & \\
10 & & & 2.949e-02 & 3.560e-04 & & \\
\hline
\end{tabular}
[9.69122e-03 1.03566e-10 2.82749e-01]
[9 9 11 11 9]
1.15802e-06
```



References

- [1] J. BEZANSON, A. EDELMAN, S. KARPINSKI, AND V. B. SHAH, *Julia: A fresh approach to numerical computing*, SIAM Review, 59 (2017), pp. 65–98.
- [2] S. CHANDRASEKHAR, *Radiative Transfer*, Dover, New York, 1960.
- [3] C. T. KELLEY, *Iterative Methods for Linear and Nonlinear Equations*, no. 16 in Frontiers in Applied Mathematics, SIAM, Philadelphia, 1995.
- [4] C. T. KELLEY, *Newton’s method in mixed precision*, SIAM Review, 64 (2022), pp. 191–211, <https://doi.org/10.1137/20M1342902>.
- [5] C. T. KELLEY, *Notebook for Solving Nonlinear Equations with Iterative Methods: Solvers and Examples in Julia*. <https://github.com/ctkelley/NotebookSIAMFANL>, 2022, <https://doi.org/10.5281/zenodo.4284687>, <https://github.com/ctkelley/NotebookSIAMFANL>. IJulia Notebook.
- [6] C. T. KELLEY, *SIAMFANLEquations.jl*. <https://github.com/ctkelley/SIAMFANLEquations.jl>, 2022, <https://doi.org/10.5281/zenodo.4284807>, <https://github.com/ctkelley/SIAMFANLEquations.jl>. Julia Package.
- [7] C. T. KELLEY, *Solving Nonlinear Equations with Iterative Methods: Solvers and Examples in Julia*, no. 20 in Fundamentals of Algorithms, SIAM, Philadelphia, 2022.

- [8] C. T. KELLEY, *Newton's method in three precisions*, 2023, <https://arxiv.org/abs/2307.16051>.