

# N3Presults

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Here is an example from [3], [4], [7], [5], [6], and [8]

This notebook reproduces the results from the paper

C. T. Kelley, *Newton's Method in Three Precisions*, 2023.

The example is the Chandrasekhar H-equation [2]

The computations were done in Julia [1]

We use the software from [7], [5], and [6]. Our nonlinear solvers from [6] work well with our data structures for iterative refinement. The Julia module with the data structures we use for IR is `src/N3Presults.jl`

The results are in Table 2 and Table 3.

The first computation generates Table 2. This is the ill-conditioned example from [4] and is the well conditioned example from the paper. The function that creates the examples is the Julia function `htest1` from the file `H_equation_examples.jl`. We also put a plot in this notebook.

```
[2]: htest1(4096,.99);
```

```
Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large  4.05010e-01
Try increasing maxit and checking your function and
      Jacobian for bugs.
Give the history array a look to see what's happening.
```

```
\begin{tabular}{lllllll}
n&      F64&      F32&      F16&  IR 32-16&  IR-GM & \\\hline
0 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & 1.000e+00 & \\\
1 & 2.289e-01 & 2.289e-01 & 5.065e-01 & 2.289e-01 & 2.289e-01 & \\\
2 & 3.934e-02 & 3.934e-02 & 2.958e-01 & 3.934e-02 & 3.934e-02 & \\\
3 & 2.737e-03 & 2.737e-03 & 1.890e-01 & 2.737e-03 & 2.737e-03 & \\\
4 & 1.767e-05 & 1.767e-05 & 1.255e-01 & 1.767e-05 & 1.767e-05 & \\\
5 & 7.486e-10 & 7.536e-10 & 8.518e-02 & 7.538e-10 & 7.506e-10 & \\\
6 &          &          & 6.068e-02 &          &          & \\\
7 &          &          & 4.240e-02 &          &          & \\\
8 &          &          & 3.195e-02 &          &          & \\\
9 &          &          & 2.280e-02 &          &          & \\\
10 &          &          & 1.713e-02 &          &          & \\\hline
```

```

\end{tabular}
[1.62595e-10 1.50632e-10 5.97601e-02]
[6 6 11 6 6]
1.75821e-06

```

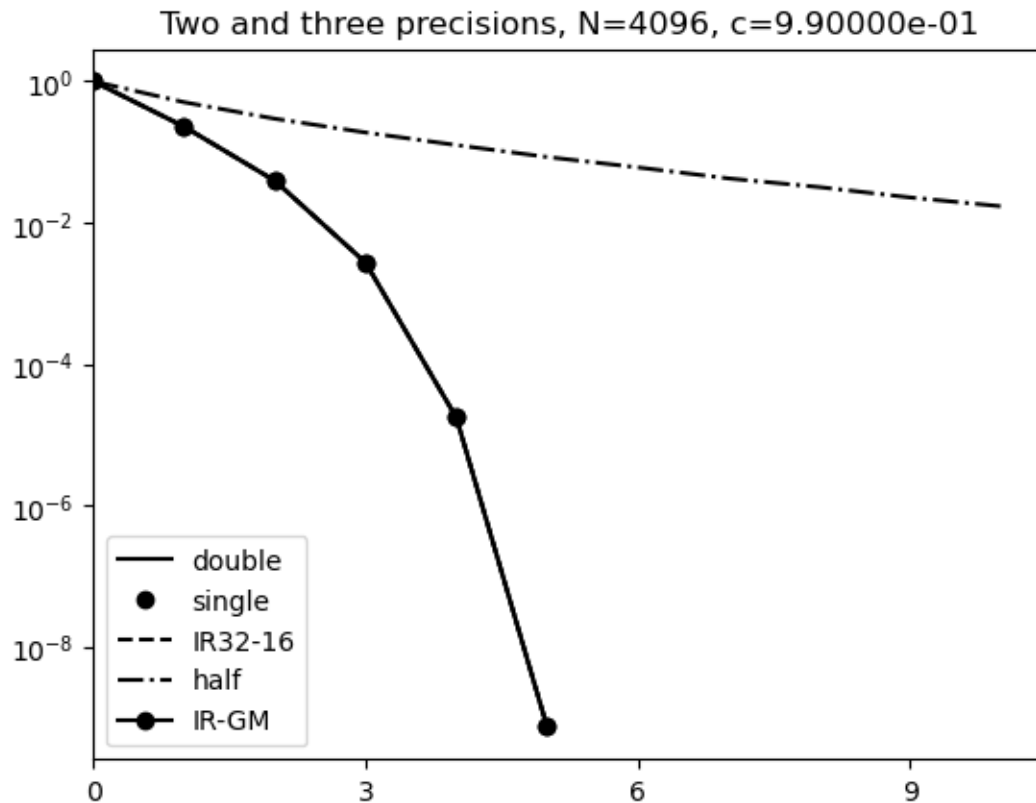


Table 3 is a very ill-conditioned example.

```
[3]: htest1(4096,.9999);
```

```

Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large 7.07016e-01
Try increasing maxit and checking your function and
      Jacobian for bugs.
Give the history array a look to see what's happening.

```

```

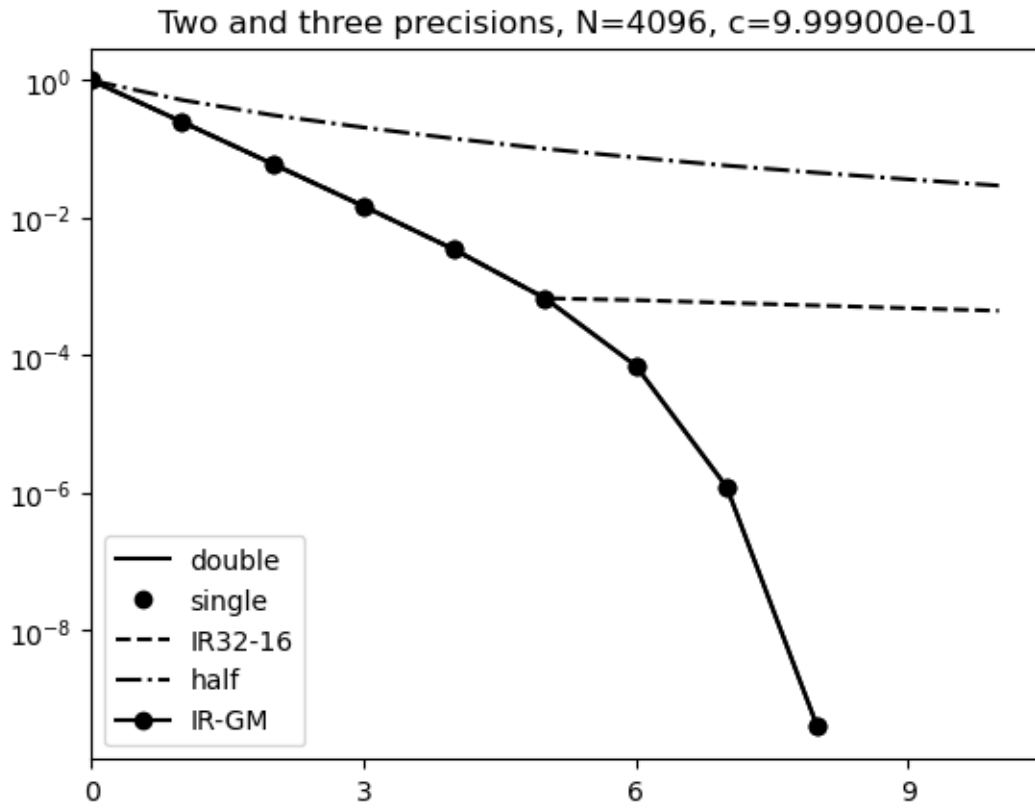
Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large 1.06833e-02
Try increasing maxit and checking your function and
      Jacobian for bugs.
Give the history array a look to see what's happening.

```

```
\begin{tabular}{l111111}
```

n	F64	F32	F16	IR 32-16	IR-GM
0	1.000e+00	1.000e+00	1.000e+00	1.000e+00	1.000e+00
1	2.494e-01	2.494e-01	5.182e-01	2.494e-01	2.494e-01
2	6.093e-02	6.093e-02	3.123e-01	6.093e-02	6.093e-02
3	1.480e-02	1.480e-02	2.067e-01	1.480e-02	1.480e-02
4	3.454e-03	3.454e-03	1.421e-01	3.455e-03	3.454e-03
5	6.762e-04	6.762e-04	1.012e-01	6.766e-04	6.762e-04
6	7.049e-05	7.049e-05	7.552e-02	6.360e-04	7.049e-05
7	1.223e-06	1.223e-06	5.773e-02	5.811e-04	1.223e-06
8	3.947e-10	3.957e-10	4.543e-02	5.312e-04	3.952e-10
9			3.639e-02	4.862e-04	
10			2.949e-02	4.456e-04	

[1.07201e-02 1.58618e-10 2.82749e-01]  
 [9 9 11 11 9]  
 1.02072e-06



## References

- [1] J. BEZANSON, A. EDELMAN, S. KARPINSKI, AND V. B. SHAH, *Julia: A fresh approach to*

- numerical computing*, SIAM Review, 59 (2017), pp. 65–98.
- [2] S. CHANDRASEKHAR, *Radiative Transfer*, Dover, New York, 1960.
  - [3] C. T. KELLEY, *Iterative Methods for Linear and Nonlinear Equations*, no. 16 in Frontiers in Applied Mathematics, SIAM, Philadelphia, 1995.
  - [4] C. T. KELLEY, *Newton’s method in mixed precision*, SIAM Review, 64 (2022), pp. 191–211, <https://doi.org/10.1137/20M1342902>.
  - [5] C. T. KELLEY, *Notebook for Solving Nonlinear Equations with Iterative Methods: Solvers and Examples in Julia*. <https://github.com/ctkelley/NotebookSIAMFANL>, 2022, <https://doi.org/10.5281/zenodo.4284687>, <https://github.com/ctkelley/NotebookSIAMFANL>. IJulia Notebook.
  - [6] C. T. KELLEY, *SIAMFANLEquations.jl*. <https://github.com/ctkelley/SIAMFANLEquations.jl>, 2022, <https://doi.org/10.5281/zenodo.4284807>, <https://github.com/ctkelley/SIAMFANLEquations.jl>. Julia Package.
  - [7] C. T. KELLEY, *Solving Nonlinear Equations with Iterative Methods: Solvers and Examples in Julia*, no. 20 in Fundamentals of Algorithms, SIAM, Philadelphia, 2022.
  - [8] C. T. KELLEY, *Newton’s method in three precisions*, 2023, <https://arxiv.org/abs/2307.16051>.