N3Presults

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Here is an example from [3], [4], [7], [5], [6], and [8]

This notebook reproduces the results from the paper

C. T. Kelley, *Newton's Method in Three Precisions*, 2023.

The example is the Chandrasekhar H-equation [2]

The computations were done in Julia [1]

We use the software from [7], [5], and [6]. Our nonlinear solvers from [6] work well with our data structures for iterative refinement. The Julia module with the data structures we use for IR is `src/N3Presults.jl`

The results are in Table 2 and Table 3.

The first computation generates Table 2. This is the ill-conditioned example from [4] and is the well conditioned example from the paper. The function that creates the examples is the Julia function `htest1` from the file `H_equation_examples.jl`. We also put a plot in this notebook.

[2]:
```
htest1(4096,.99);
```

```
Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large  4.05010e-01
Try increasing maxit and checking your function and
                 Jacobian for bugs.
Give the history array a look to see what's happening.

\begin{tabular}{llllll}
 n&        F64&        F32&        F16&   IR 32-16&      IR-GM \\ \hline
 0 &  1.000e+00 &  1.000e+00 &  1.000e+00 &  1.000e+00 &  1.000e+00  \\
 1 &  2.289e-01 &  2.289e-01 &  5.065e-01 &  2.289e-01 &  2.289e-01  \\
 2 &  3.934e-02 &  3.934e-02 &  2.958e-01 &  3.934e-02 &  3.934e-02  \\
 3 &  2.737e-03 &  2.737e-03 &  1.890e-01 &  2.737e-03 &  2.737e-03  \\
 4 &  1.767e-05 &  1.767e-05 &  1.255e-01 &  1.767e-05 &  1.767e-05  \\
 5 &  7.486e-10 &  7.536e-10 &  8.518e-02 &  7.538e-10 &  7.506e-10  \\
 6 &            &            &  6.068e-02 &            &             \\
 7 &            &            &  4.240e-02 &            &             \\
 8 &            &            &  3.195e-02 &            &             \\
 9 &            &            &  2.280e-02 &            &             \\
10 &            &            &  1.713e-02 &            &             \\
\hline
```

```
\end{tabular}
[1.62595e-10 1.50632e-10 5.97601e-02]
[6 6 11 6 6]
1.75821e-06
```

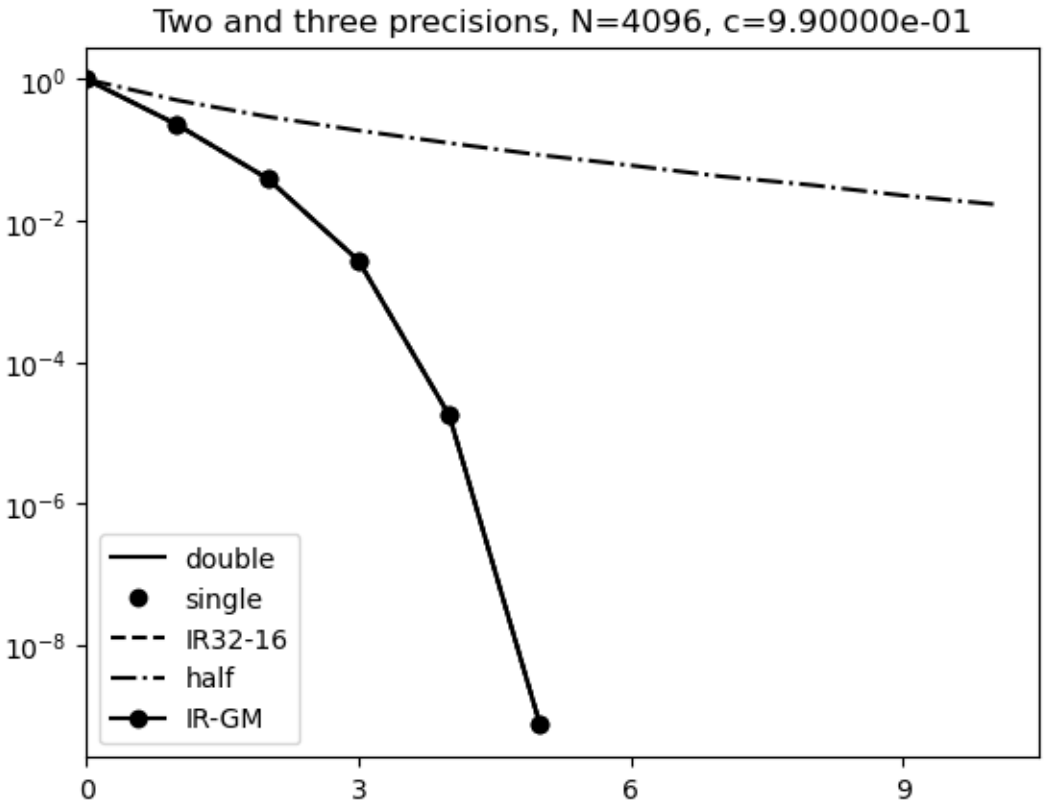


Table 3 is a very ill-conditioned example.

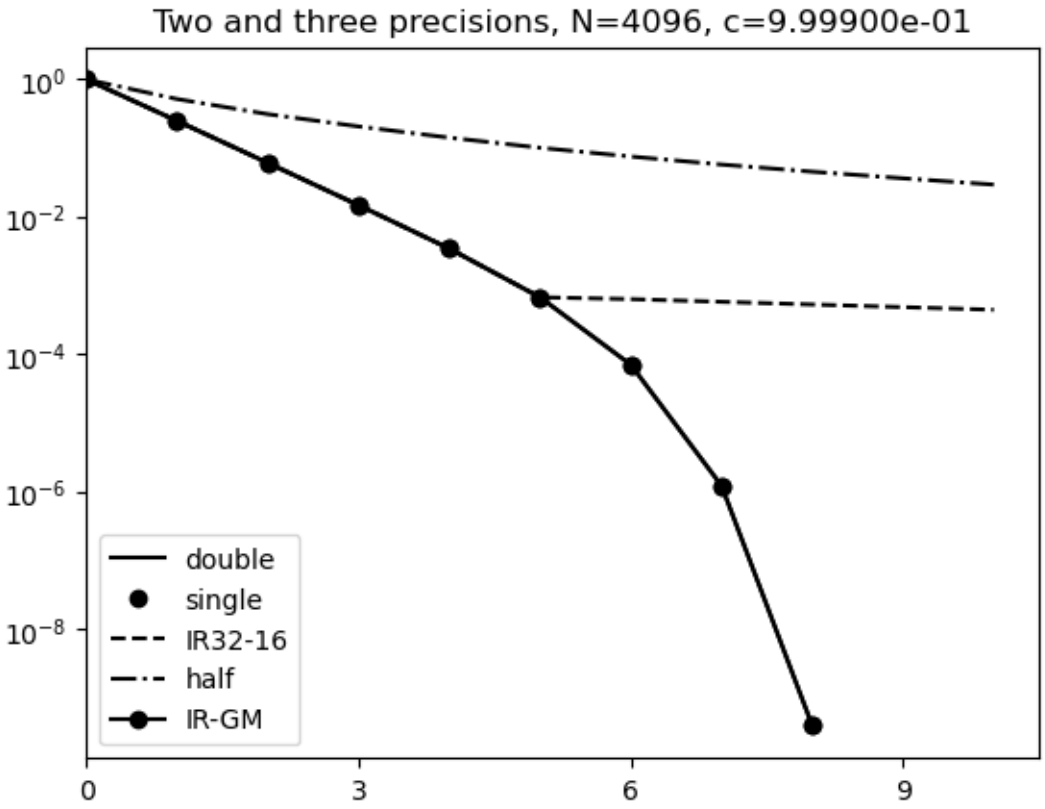
[3]:
```
htest1(4096,.9999);
```

```
Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large  7.07016e-01
Try increasing maxit and checking your function and
                 Jacobian for bugs.
Give the history array a look to see what's happening.

Maximum iterations (maxit) of 10 exceeded
Convergence failure: residual norm too large  1.06833e-02
Try increasing maxit and checking your function and
                 Jacobian for bugs.
Give the history array a look to see what's happening.

\begin{tabular}{llllll}
```

```
 n&        F64&        F32&        F16&   IR 32-16&      IR-GM \\ \hline
 0 &  1.000e+00 &  1.000e+00 &  1.000e+00 &  1.000e+00 &  1.000e+00  \\
 1 &  2.494e-01 &  2.494e-01 &  5.182e-01 &  2.494e-01 &  2.494e-01  \\
 2 &  6.093e-02 &  6.093e-02 &  3.123e-01 &  6.093e-02 &  6.093e-02  \\
 3 &  1.480e-02 &  1.480e-02 &  2.067e-01 &  1.480e-02 &  1.480e-02  \\
 4 &  3.454e-03 &  3.454e-03 &  1.421e-01 &  3.455e-03 &  3.454e-03  \\
 5 &  6.762e-04 &  6.762e-04 &  1.012e-01 &  6.766e-04 &  6.762e-04  \\
 6 &  7.049e-05 &  7.049e-05 &  7.552e-02 &  6.360e-04 &  7.049e-05  \\
 7 &  1.223e-06 &  1.223e-06 &  5.773e-02 &  5.811e-04 &  1.223e-06  \\
 8 &  3.947e-10 &  3.957e-10 &  4.543e-02 &  5.312e-04 &  3.952e-10  \\
 9 &            &            &  3.639e-02 &  4.862e-04 &             \\
10 &            &            &  2.949e-02 &  4.456e-04 &             \\
\hline
\end{tabular}
[1.07201e-02 1.58618e-10 2.82749e-01]
[9 9 11 11 9]
1.02072e-06
```



References

[1] J. Bezanson, A. Edelman, S. Karpinski, and V. B. Shah, *Julia: A fresh approach to*

numerical computing, SIAM Review, 59 (2017), pp. 65–98.

[2] S. Chandrasekhar, *Radiative Transfer*, Dover, New York, 1960.

[3] C. T. Kelley, *Iterative Methods for Linear and Nonlinear Equations*, no. 16 in Frontiers in Applied Mathematics, SIAM, Philadelphia, 1995.

[4] C. T. Kelley, *Newton's method in mixed precision*, SIAM Review, 64 (2022), pp. 191–211, https://doi.org/10.1137/20M1342902.

[5] C. T. Kelley, *Notebook for Solving Nonlinear Equations with Iterative Methods: Solvers and Examples in Julia*. https://github.com/ctkelley/NotebookSIAMFANL, 2022, https://doi.org/10.5281/zenodo.4284687, https://github.com/ctkelley/NotebookSIAMFANL. IJulia Notebook.

[6] C. T. Kelley, *SIAMFANLEquations.jl*. https://github.com/ctkelley/SIAMFANLEquations.jl, 2022, https://doi.org/10.5281/zenodo.4284807, https://github.com/ctkelley/SIAMFANLEquations.jl. Julia Package.

[7] C. T. Kelley, *Solving Nonlinear Equations with Iterative Methods: Solvers and Examples in Julia*, no. 20 in Fundamentals of Algorithms, SIAM, Philadelphia, 2022.

[8] C. T. Kelley, *Newton's method in three precisions*, 2023, https://arxiv.org/abs/2307.16051. to appear in Pacific Journal of Optimization.