Binomial splitting

When to stop iterative image reconstruction?

Structured illumination microscopy with noise-controlled image reconstructions

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Binomial on *Poisson* stays ***Poisson***!

$$P_{Poiss.}(n_A + n_B|\mu) = \frac{e^{-\mu}\mu^{n_A+n_B}}{(n_A+n_B)!}$$

μ : expected number of photons

$n_A + n_B$: measured number of photons

$$P_{Binom.}(n_A, n_B|n_A + n_B, p) = \frac{(n_A+n_B)!}{n_A!n_B!} \cdot p^{n_A} \cdot (1-p)^{n_B}$$

p : probabiltiy for drawing n_A photons

1 - p : probabiltiy for drawing n_B photons

Draw binomially from Poisson distributed data:

$$P_{Binom.} \cdot P_{Poiss.} = \frac{\cancel{(n_A+n_B)!}}{n_A!n_B!} \cdot p^{n_A} \cdot (1-p)^{n_B} \cdot \frac{e^{-\mu}\mu^{n_A+n_B}}{\cancel{(n_A+n_B)!}}$$

$$= e^{-\mu} \cdot \frac{\mu^{n_A}p^{n_A}}{n_A!} \cdot \frac{\mu^{n_B}(1-p)^{n_B}}{n_B!}$$

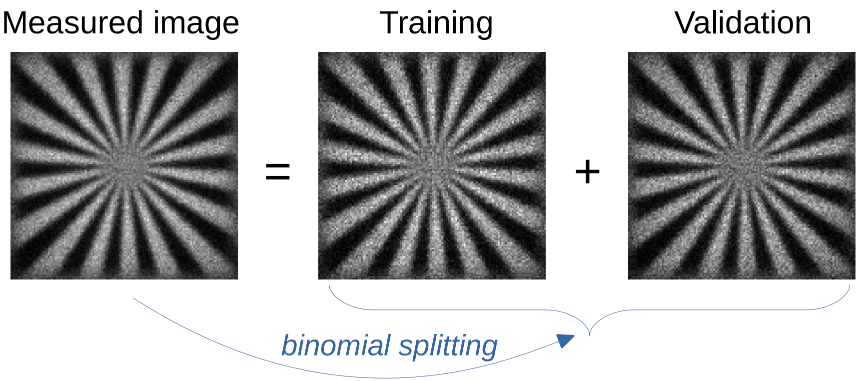
Modify exponential term:

$$e^{-\mu} = e^{-\mu} \cdot \frac{e^{-\mu p}}{e^{-\mu p}} = e^{-\mu p} \cdot e^{-\mu(1-p)}$$

Express as two Poisson distributions:

$$\begin{aligned} P_{Binom.} \cdot P_{Poiss.} &= \frac{e^{-\mu p}[\mu p]^{n_A}}{n_A!} \cdot \frac{e^{-\mu(1-p)}[\mu(1-p)]^{n_B}}{n_B!} = \\ &= P_{Poiss.}(n_A|p\mu) \cdot P_{Poiss.}(n_B|(1-p)\mu) \end{aligned}$$

Split image data into *training* and *validation* set



- Both sub-images are *Poisson* distributed and stochastically independent!
- Perform the reconstruction on the training data, which maximizes the log-likelihood L_T.
- In each iteration validate the performance by calculating L_V.
- When L_V decreases, the image reconstruction should be stopped.

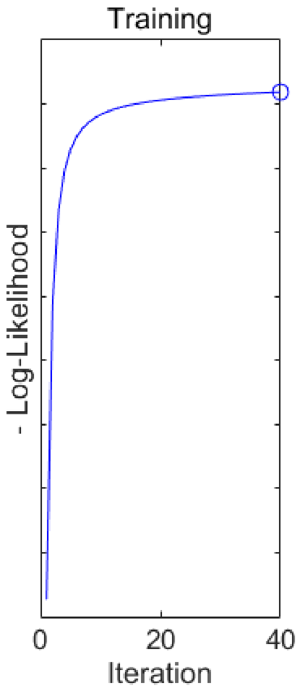
Negative log-likelihood:

$$P = \prod_i P_i \quad \rightarrow \quad L = -\ln P = -\sum_i P_i$$

$$L = -\sum_i (\mu_i - n_i \ln \mu_i)$$

Normalized cross-correlation:

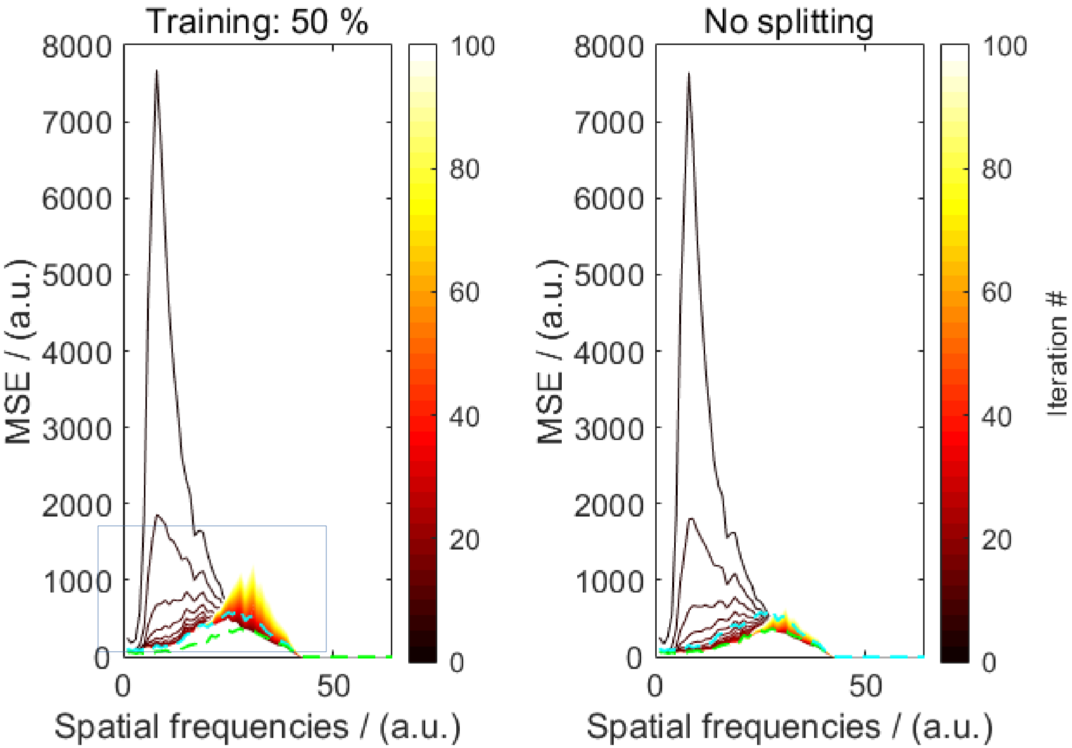
$$NCC = \frac{\frac{1}{N} \cdot \sum \left[(\mathrm{est} - \bar{\mathrm{est}}) \cdot (\mathrm{obj} - \bar{\mathrm{obj}}) \right]}{\sigma(est) \cdot \sigma(obj)}$$

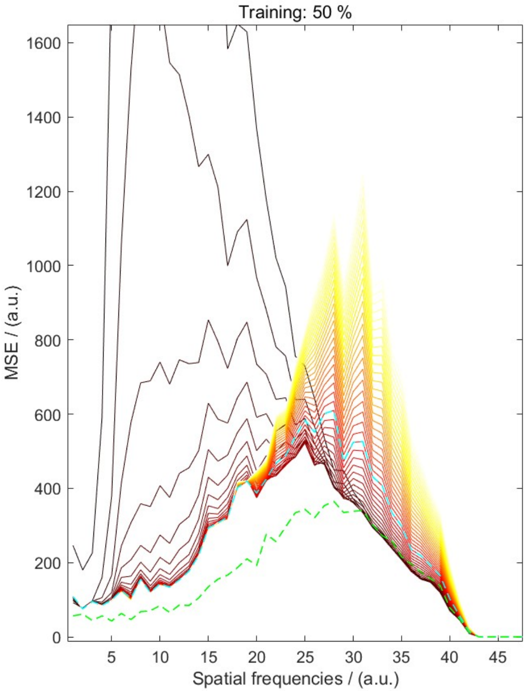


Lets look at the error in *Fourier* space

Mean-square error vs. spatial frequencies shows that “binomial stopping” predicts point when over fitting starts.

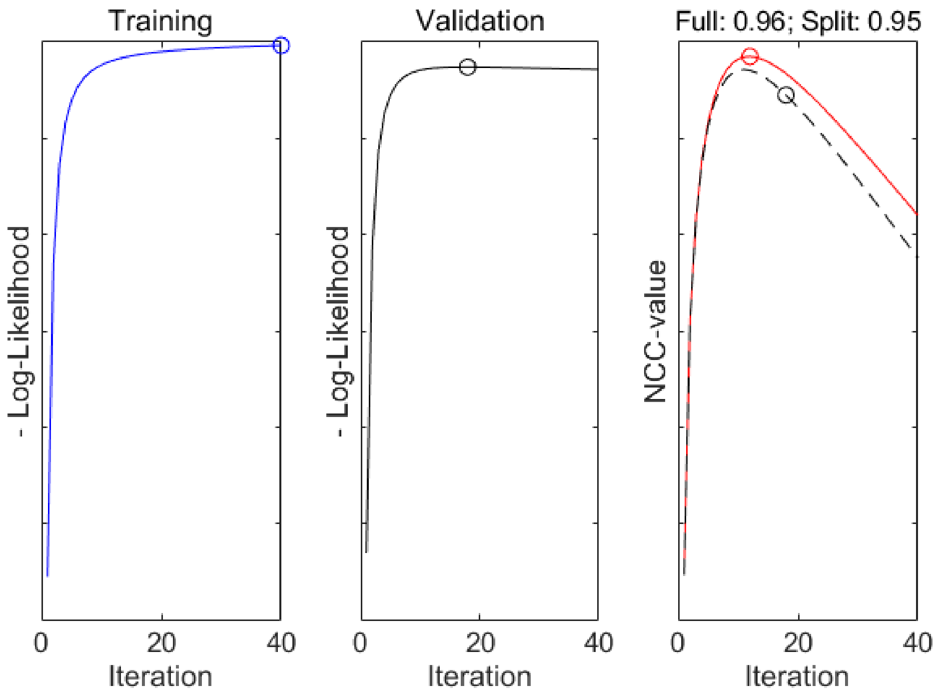
Cyan line : binomial stopping criterion   Green line : NCC criterion (only computable with groundtruth object)

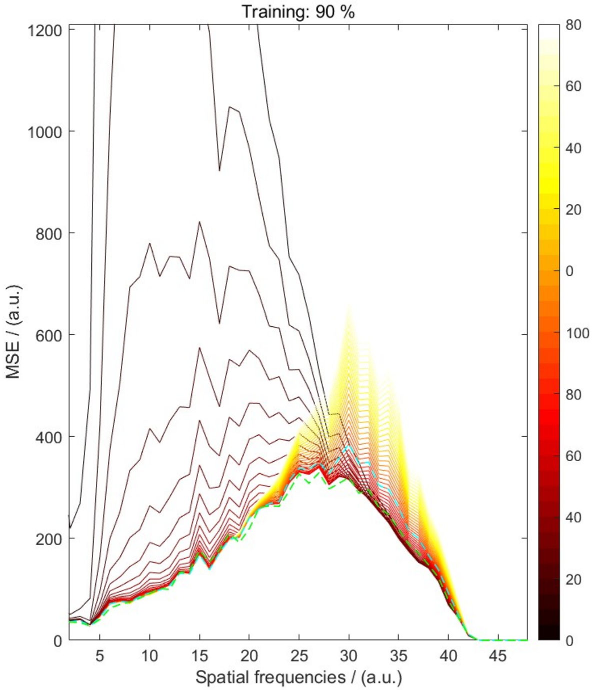




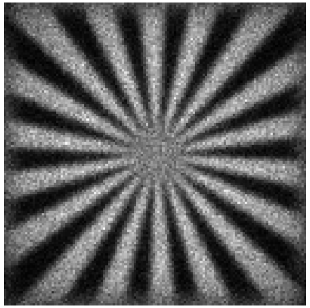
Make *non*-equal splitting to improve reconstruction quality

If we set p = 0.90 it seem that we can get (almost) the same reconstruction quality as in the optimum case, only we actually know when to stop.

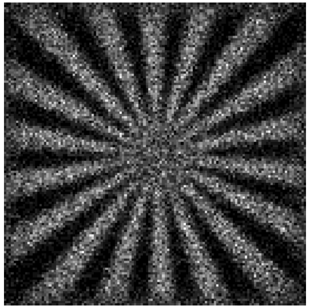




Training



Validation



Still to Do?

- Look at different types of objects (sparse, random, ...) and see how new stopping criterion performs.
- Always good to verify with experimental data?
- Can we use both sub-images to infere which spatial frequencies we can trust and which not?
- ...
- Make a github repo
- Readout noise
- Data from Craig
- ...
- Poissonian ont op of Poissonian
- Splitting factor vs. Stopping criterion
- Spatial splitting vs. temporal splitting