

# Math: Vectors

# Gilbert Strang, Professor of Mathematics at MIT:

## Too Much Calculus

Calculus I, Calculus II, Calculus III—what an imbalance in our teaching! All the rest of mathematics is overwhelmed by calculus. The next course might be differential equations (more derivatives), and the previous course is probably pre-calculus. I really think it is our job to adjust this balance, we cannot expect others to do it. We know the central role of linear algebra. It is much more than a random math course, its applications touch many more students than calculus. We are in a digital world now.

# What is a vector?

## Vector

From Wikipedia, the free encyclopedia

*For the default skin on the English Wikipedia, see [Wikipedia:Vector](#).*

**Vector** may refer to:

## Biology [[edit](#)]

- Vector (epidemiology)**, an agent that carries and transmits an infectious pathogen
- Vector (molecular biology)**, a DNA molecule used as a vehicle to artificially introduce foreign DNA into a cell
  - Cloning vector**, a small piece of DNA into which a foreign DNA fragment is inserted
  - Shuttle vector**, a plasmid constructed so that it can propagate in two different hosts
  - Viral vector**, a tool commonly used by molecular biologists to deliver genes into cells

## Business [[edit](#)]

- Vector Informatik**, a software tool and components provider for developers
- Vector Engineering**, multinational engineering, procurement, construction and management
- Vector Graphic**, an early manufacturer of 8-bit microcomputers
- Vector Group**, a publicly traded holding company, focusing on tobacco and alcohol
- Vector Limited**, a New Zealand gas and electricity supplier
- Vector Marketing**, a multi-level knife marketing company
- Vector Motors**, an automobile manufacturer
- Vector Launch**, an American space technology company, 2016–2019
- State Research Center of Virology and Biotechnology VECTOR**, a biological research center in Novosibirsk

## Computer science [[edit](#)]

- A one-dimensional **array data structure**
- Vector (C++)**, a type in the C++ Standard Template Library
- Vector (malware)**, approach used, or vulnerability exploited, in attacking a computer
- Distance-vector routing protocol**, a class of routing protocols
- Dope vector**, a data structure used to store information about an array
- Feature vector**, an n-dimensional vector of numerical features that represent an object
- Initialization vector**, a fixed-size input to a cryptographic primitive
- Interrupt vector**, the location in memory of an interrupt handling routine
- Vector clock**, an algorithm
- Vector game**, any video game that uses a vector graphics display

- Interrupt vector**, the location in memory of an interrupt handling routine
- Vector clock**, an algorithm
- Vector game**, any video game that uses a vector graphics display
- Vector graphics**, images defined by geometric primitives as opposed to bitmaps
- Vector monitor**, a display device used for early computers
- Vector processor**, a computer processor which works on arrays of several numbers
- Vector space model**, an algebraic model for representing text documents

## Literature [[edit](#)]

- Vector** (magazine)*, the critical journal of the British Science Fiction Association
- Vectors: Journal of Culture and Technology in a Dynamic Vernacular**, an online journal
- The Vector** (newspaper)*, a student-run newspaper of the New Jersey Institute of Technology
- Vector** (novel)*, a 1999 novel by Robin Cook
- Vector Prime***, a 1999 *Star Wars* novel written by R. A. Salvatore
- Vector 13***, a comic strip in the *2000 AD* anthology

## Entertainment [[edit](#)]

### Characters and fictional elements [[edit](#)]

- Vector** (*Battle Angel Alita*)*, a character in the *Battle Angel Alita* manga series
- Vector** (comics)*, a character in the Marvel Comics Universe
- Vector** (G.I. Joe)*, a fictional vehicle from the G.I. Joe Battleforce 2000 toy line
- Vector the Crocodile***, a character in the *Sonic the Hedgehog* video game series
- Septima Vector, a *Hogwarts professor in the Harry Potter universe*
- Vector, a Barian Emperor from *Yu-Gi-Oh! Zexal*
- Vector class, a beginner racing class in *Wipeout* video game series
- Victor "Vector" Perkins, the main antagonist of the 2010 animated film *Despicable Me*
- Vector, a character in the video game *Resident Evil: Operation Raccoon City*
- Vector, an invisible arm telepathically controlled as a weapon by Dionell in the series *Star Trek: Voyager*
- Vector, a robot in "Greetings from Earth" (1979), an episode of the *Battlestar Galactica*
- Vector, a *location in the video game Final Fantasy VI*
- Vector Industries, an *organization in the Xenosaga video game series*
- Vector manipulation, an ability held by the character "Accelerator" in the series *A Certain Magical Index*

### Games [[edit](#)]

- Vector** (game)*, a 1970 board game
- Vector** (video game)*, a 2010 platform video game
- Vectorman***, a 1995 game for Sega Genesis platform

### Music [[edit](#)]

- Vector** (band), a 1980s New Wave rock band
- Vector** (Haken album)*, 2018 album by British progressive metal band Haken
- Vector (rapper)** (born 1983), Nigerian rap artist
- Vector Lovers**, moniker used by British electronic music producer Martin Wheeler
- "Vectors", a song by the band [Area 11](#)

### Other entertainment [[edit](#)]

- Vector, an entertainment robot by the company Anki

## Mathematics and physics [[edit](#)]

- Vector (mathematics and physics)**
  - Euclidean vector**, a geometric object that has magnitude (or length) and direction
  - Row and column vectors**, matrices consisting of a single column or row
  - Vector space**, a collection of objects called vectors, which may be added together and multiplied by scalars
  - Vector field**, assignment of a vector to each point in a subset of space

## Transportation [[edit](#)]

### Vehicles [[edit](#)]

#### Aircraft [[edit](#)]

- Aerodyne Systems Vector**, an ultralight aircraft
- Hall Vector 1**, a glider

#### Nautical vessels [[edit](#)]

- MT Vector**, a Philippine tanker ship
- CCGS Vector**, a 1967 hydrographic survey vessel in the Canadian Coast Guard

#### Automotive vehicles [[edit](#)]

- Pinzgauer High-Mobility All-Terrain Vehicle (ATV)** (Vector), a military patrol vehicle
- VECTOR**, a Dutch light utility vehicle
- Vector, a variant of the **Saab 9-3** automobile
- Vector Motors** sports cars
  - Vector M12**, a vehicle designed by Vector Motors based on the Lamborghini Diablo
  - Vector W2**, a concept car created by Vector Motors
  - Vector W8**, a production car produced by Vector Motors
  - Vector WX-3**, a prototype automobile created by Vector Motors in 1992
  - Vector SRV8**, a prototype automobile created by Vector Motors in 1999

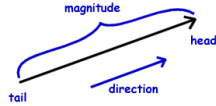
### Other transportation [[edit](#)]

- Lift vector**, an upwards force acting on an aircraft

# What is a vector?

A Physicist might say:

“A vector has a direction and a size (magnitude)”



$$\begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$(\vec{v})$$

$$[x \ y \ z]$$

$$(\vec{v}^T)$$

A Computer Scientist might say:

“A vector is just a list of numbers (but the order matters)”

```
my_vector = np.array((x,y,z))
```



```
np.array((x,z,y))
```

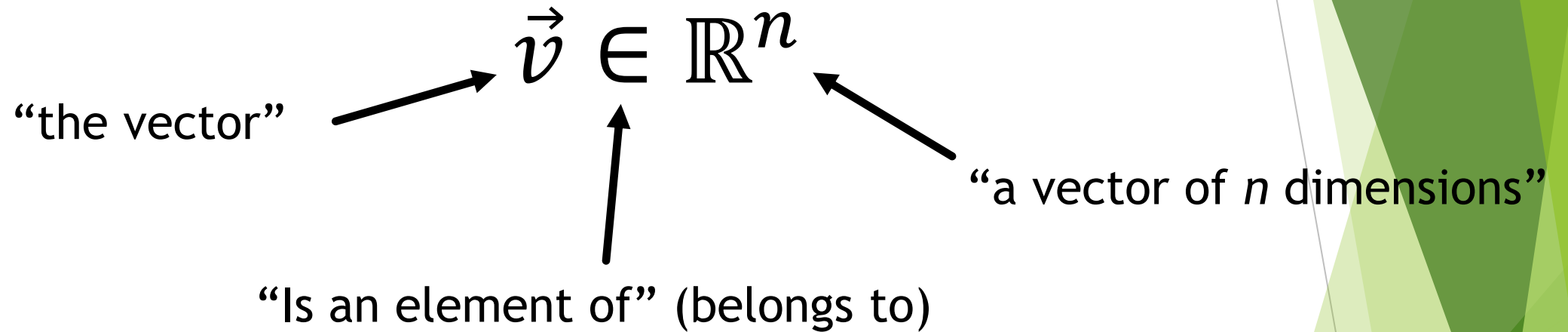
A Mathematician might say:

“A vector is an element of a vector space”

$$\vec{v} \in \mathbb{R}^n$$

(Thanks, mathematicians...)

Hold on, what does  $\vec{v} \in \mathbb{R}^n$  mean?



*You  $\in$  The Human Species*

*Pluto  $\notin$  The Solar System's Planets*

In our physics example:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} \in \mathbb{R}^3, \text{ where } x, y \text{ and } z \in \mathbb{R}$$

# What is a **vector**? Physicist:

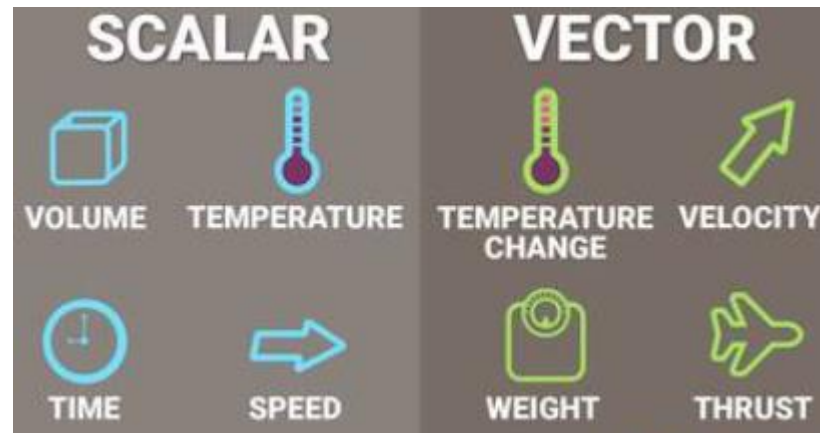
To a physicist, a vector is an arrow in space.

## Scalar: has a magnitude

- Temperature
- Volume
- Speed
- Energy

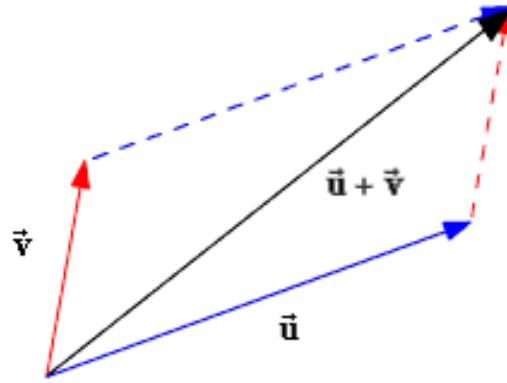
## Vector: has a magnitude AND a direction

- Velocity
- Force
- Acceleration
- Displacement



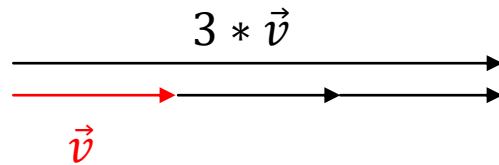
# What can we do with **vectors**?

We can add them:



$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} + \begin{bmatrix} d \\ e \\ f \end{bmatrix} = \begin{bmatrix} a + d \\ b + e \\ c + f \end{bmatrix}$$

We can scale them:



$$s * \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} s * a \\ s * b \\ s * c \end{bmatrix}$$

## Mathematicians give us 7 rules:

<u>Associativity</u> of addition	$\mathbf{u} + (\mathbf{v} + \mathbf{w}) = (\mathbf{u} + \mathbf{v}) + \mathbf{w}$
Commutativity of addition	$\mathbf{u} + \mathbf{v} = \mathbf{v} + \mathbf{u}$
Identity element of addition	There exists an element $\mathbf{0} \in V$ , called the <i>zero vector</i> , such that $\mathbf{v} + \mathbf{0} = \mathbf{v}$ for all $\mathbf{v} \in V$ .
Inverse elements of addition	For every $\mathbf{v} \in V$ , there exists an element $-\mathbf{v} \in V$ , called the <i>additive inverse</i> of $\mathbf{v}$ , such that $\mathbf{v} + (-\mathbf{v}) = \mathbf{0}$ .
Compatibility of scalar multiplication with field multiplication	$a(b\mathbf{v}) = (ab)\mathbf{v}$ <sup>[nb 2]</sup>
Identity element of scalar multiplication	$1\mathbf{v} = \mathbf{v}$ , where 1 denotes the <i>multiplicative identity</i> in $F$ .
Distributivity of scalar multiplication with respect to vector addition	$a(\mathbf{u} + \mathbf{v}) = a\mathbf{u} + a\mathbf{v}$
Distributivity of scalar multiplication with respect to field addition	$(a + b)\mathbf{v} = a\mathbf{v} + b\mathbf{v}$

But basically, vectors can be scaled and added in any order.



Let's try some examples:

$$\begin{bmatrix} 1 \\ 3 \\ 7 \end{bmatrix} + \begin{bmatrix} 10 \\ -1 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \\ 7 \end{bmatrix}$$

$$\begin{bmatrix} 10 \\ -1 \\ 0 \\ 4 \end{bmatrix} + \begin{bmatrix} 1 \\ 3 \\ 2 \\ 4 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} 11 \\ 2 \\ 2 \\ 8 \end{bmatrix}$$

$$3 * \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} 3 \\ 12 \\ -6 \end{bmatrix}$$

$$2 * \begin{bmatrix} 2 \\ 4 \\ \pi \end{bmatrix} + \begin{bmatrix} 12 \\ -2 \\ 7 \\ -1 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} \text{ Not Permitted!}$$

$$4 * \begin{bmatrix} 9 \\ .5 \\ 1 \end{bmatrix} + (-2) * \begin{bmatrix} 7 \\ -2 \\ 0 \end{bmatrix} = \begin{bmatrix} ? \\ ? \\ ? \end{bmatrix} = \begin{bmatrix} 22 \\ 6 \\ 4 \end{bmatrix}$$

# What is a **vector**? Computer Scientist: (Football Players as **vectors**?!)



Denver Bronco Vectors:

**['name', height, weight]**

Receivers:

- Jeudy = ['jeudy', 70, 200]
- Sutton = ['sutton', 73, 210]
- Fant = ['fant', 78, 265]
- Hamler = ['hamler', 71, 190]
- Patrick = ['patrick', 78, 275]

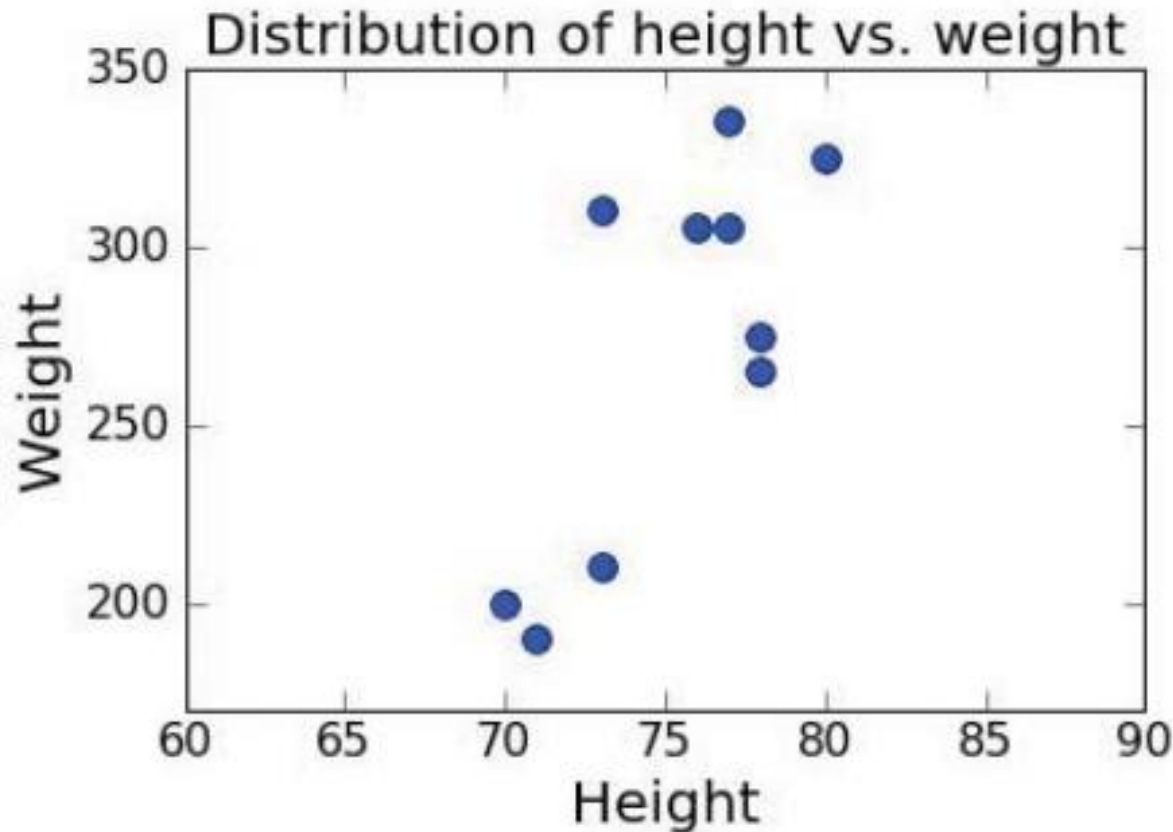
Linemen:

- Glasgow = ['glasgow', 77, 335]
- Bolles = ['bolles', 80, 325]
- Cushenberry = ['cushenberry', 73, 310]
- Risner = ['risner', 77, 305]
- Muti = ['muti', 76, 305]

# What is a **vector**? Computer Scientist:

Denver Bronco Vectors: ['name', height, weight]

## Unlabeled Data



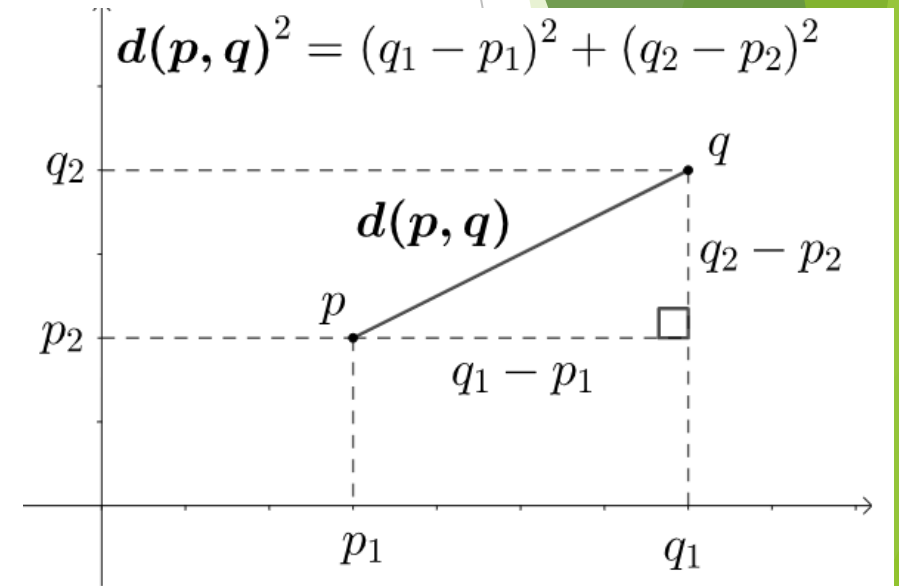
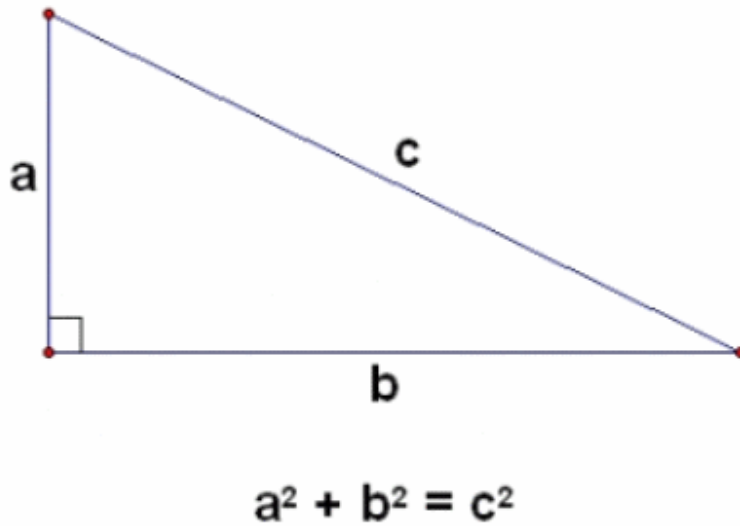
# How can we measure distance?



You're near the Apple Store by Central Park.  
I want to meet you at High Line Park.  
How far will you travel? It depends...

# ...Are you a bird?

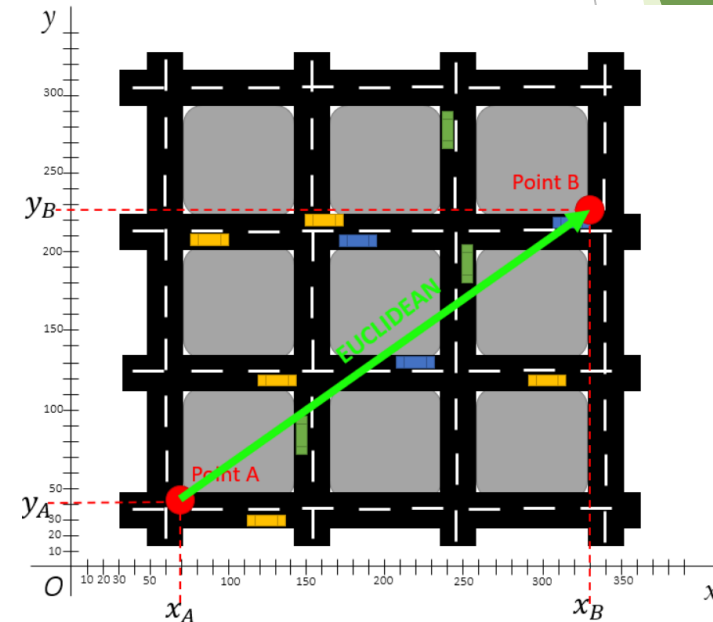
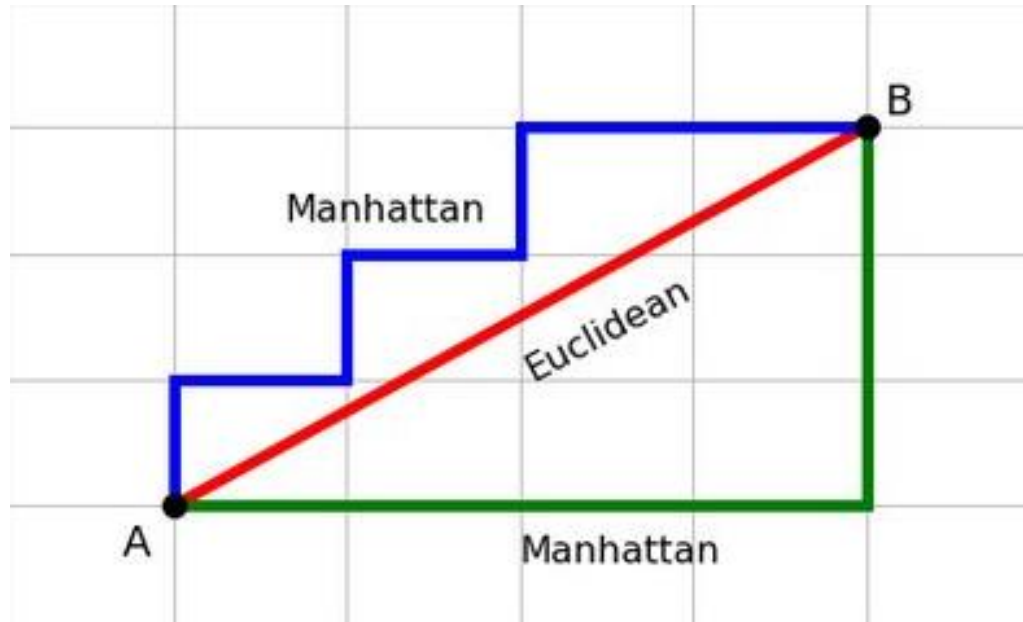
Euclidean Distance OR the L2 Norm OR Pythagoras!



$$d(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=1}^n (q_i - p_i)^2}$$

# ...Or do you have to walk?

“Manhattan Distance” OR the L1 Norm OR Taxicab Geometry



$$d_1(\mathbf{p}, \mathbf{q}) = \|\mathbf{p} - \mathbf{q}\|_1 = \sum_{i=1}^n |p_i - q_i|$$

(There are a bunch of these)

Euclidean	$d(x, y) = \sqrt{\sum (x_i - y_i)^2}$
Squared Euclidean	$d(x, y) = \sum (x_i - y_i)^2$
Manhattan	$d(x, y) = \sum  x_i - y_i $
Canberra	$d(x, y) = \sum \frac{ x_i - y_i }{ x_i + y_i }$
Chebychev	$d(x, y) = \max( x_i - y_i )$
Bray Curtis	$d(x, y) = \frac{\sum  x_i - y_i }{\sum x_i + y_i}$
Cosine Correlation	$d(x, y) = \frac{\sum (x_i y_i)}{\sqrt{\sum (x_i)^2} \sqrt{\sum (y_i)^2}}$
Pearson Correlation	$d(x, y) = \frac{\sum (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum (y_i - \bar{y})^2} \sqrt{\sum (y_i - \bar{y})^2}}$
Uncentered Peason Correlation	$d(x, y) = \frac{\sum x_i y_i}{\sqrt{\sum (y_i - \bar{y})^2} \sqrt{\sum (y_i - \bar{y})^2}}$
Euclidean Nullweighted	Same as Euclidean, but only the indexes where both x and y have a value (not NULL) are used, and the result is weighted by the number of values calculated. Nulls must be replaced by the missing value calculator (in dataloader).

# Vectors: Snakes?

Features						Label
Name	Egg-laying	Scales	Poisonous	Cold-blooded	# legs	Reptile
Cobra	True	True	True	True	0	Yes
Rattlesnake	True	True	True	True	0	Yes
Boa constrictor	False	True	False	True	0	Yes
Chicken	True	True	False	False	2	No
Alligator	True	True	False	True	4	Yes
Dart frog	True	False	True	False	4	No
Salmon	True	True	False	True	0	No
Python	True	True	False	True	0	Yes

$Cobra = [1 \ 1 \ 1 \ 1 \ 0 \ 1]$   
 $Dart \ Frog = [1 \ 0 \ 1 \ 0 \ 4 \ 0]$



# What's the Euclidean Distance between: A Cobra and a Dart Frog?

$$d(\mathbf{p}, \mathbf{q}) = \sqrt{\sum_{i=1}^n (q_i - p_i)^2}$$

$$\begin{aligned} \text{Cobra} &= [1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 1] \\ \text{Dart Frog} &= [1 \quad 0 \quad 1 \quad 0 \quad 4 \quad 1] \end{aligned}$$

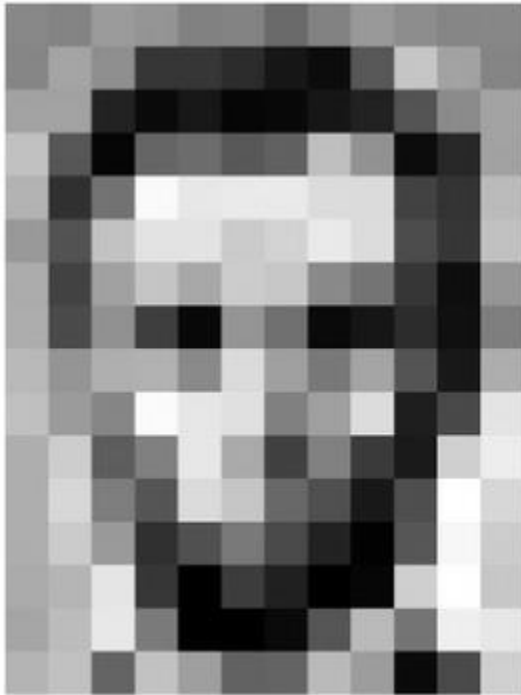
$$d(\text{Cobra}, \text{Boa Constrictor}) = \begin{bmatrix} (1-0)^2 \\ (1-1)^2 \\ (1-0)^2 \\ (1-1)^2 \\ (0-0)^2 \\ (1-1)^2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \longrightarrow \sqrt{\Sigma} \longrightarrow 1.41$$

$$d(\text{Cobra}, \text{Dart Frog}) = \begin{bmatrix} (1-1)^2 \\ (1-0)^2 \\ (1-1)^2 \\ (1-0)^2 \\ (4-0)^2 \\ (1-0)^2 \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \\ 16 \\ 1 \end{bmatrix} \longrightarrow \sqrt{\Sigma} \longrightarrow 4.36$$

Can we Represent a **Photo** as a **Vector**?

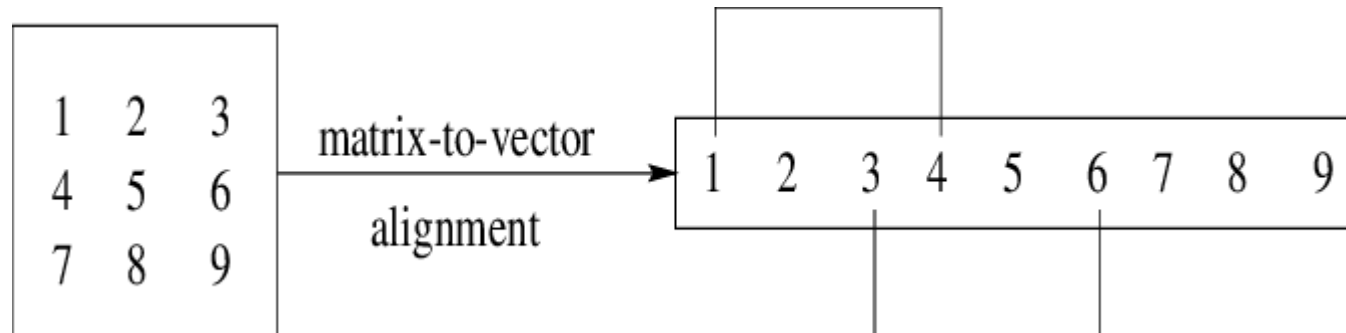


# Can we Represent a Photo as a Vector?



157	153	174	168	160	162	129	151	172	161	155	156
155	182	163	74	75	62	93	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
205	109	5	124	131	111	120	204	165	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	106	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218

157	153	174	168	160	162	129	151	172	161	155	156
155	182	163	74	75	62	93	17	110	210	180	154
180	180	50	14	34	6	10	33	48	106	159	181
205	109	5	124	131	111	120	204	165	15	56	180
194	68	137	251	237	239	239	228	227	87	71	201
172	106	207	233	233	214	220	239	228	98	74	206
188	88	179	209	185	215	211	158	139	75	20	169
189	97	165	84	10	168	134	11	31	62	22	148
199	168	191	193	158	227	178	143	182	106	36	190
205	174	155	252	236	231	149	178	228	43	95	234
190	216	116	149	236	187	86	150	79	38	218	241
190	224	147	108	227	210	127	102	36	101	255	224
190	214	173	66	103	143	96	50	2	109	249	215
187	196	235	75	1	81	47	0	6	217	255	211
183	202	237	145	0	0	12	108	200	138	243	236
195	206	123	207	177	121	123	200	175	13	96	218



Optional  
(Useful  
graphic?)

