frontal.lib

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1 Public procedures

These procedures are available for other libraries and the end user to use.

1.1 fcodim

Given a frontal map germ $f\colon (N,x) \to (Z,y)$, compute the frontal codimension of f. For plane curves, if we write the equation for the image of f as $g = 0$, we can use the formula

$$\operatorname{codim}_{\mathscr{F}_e} f = \tau(g) - \delta(g) - \operatorname{mult}(g) + 1$$

from [2].

For the general case, let $g = 0$ be the equation for the image of f and (f_t) be a frontal disentanglement of f (see [1]) with equation $G = 0$ in the image. A conjecture by Nuño-Ballesteros states that

$$\operatorname{codim}_{\mathscr{F}_e} f = \dim \frac{J(G) + (G)}{J_{rel}(G) + (g)} \otimes \mathbb{C}\{t\},$$

where $J(G)$ is the Jacobian ideal of G and $J_{rel}(G)$ is the *relative* Jacobian ideal, wherein the partial derivatives with respect to the parameter t are ignored.

2 Static procedures

These procedures are only available for this library, and are declared with the `static` type. To make them available, simply remove the `static` type in code.

2.1 wdeg

Given a weighted homogeneous polynomial $g \in \mathbb{K}[x_1, \ldots, x_n]$, there exist by definition $w_1, \ldots, w_n, d \geq 0$ such that

$$g((\lambda x_1)^{w_1}, \ldots, (\lambda x_n)^{w_n}) = \lambda^d g(x_1^{w_1}, \ldots, x_n^{w_n}).$$

The values $w_1, \ldots, w_n$ are known as the *weights* of g, and d, as the *weighted homogeneous degree* of g. This procedure takes a weighted homogeneous polynomial g and returns

1. the weights of g;
2. the weighted homogeneous degree of g.

References

[1] C. Muñoz-Cabello, J. J. Nuño-Ballesteros, and R. Oset Sinha. Singularities of frontal surfaces, 2022.

[2] C. Muñoz-Cabello, J. J. Nuño-Ballesteros, and R. Oset Sinha. Deformations of corank 1 frontals, 2023.