Power carried by a nonparaxial TM beam

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In paraxial optics, the power carried by an optical beam can be accurately calculated by means of the integral of the squared modulus of its electric field over a plane transverse to the propagation axis. However, for nonparaxial electromagnetic beams, it is more appropriate to define the power carried by the beam by the integral of the longitudinal component of its time-averaged Poynting vector over a plane transverse to the propagation axis. In this paper, the expression of the power carried by a high-aperture transverse magnetic (TM) beam of any order is determined. The general expression of the power carried by a TM beam, which also applies for a transverse electric (TE) beam, is given in terms of a modified Struve function of order equal to an integer plus one-half. © 2009 Optical Society of America

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1. INTRODUCTION

Since the generation of laser beams focused to a spot size comparable to the wavelength is nowadays achievable, the propagation of nonparaxial optical beams is a subject of growing interest. Such strongly focused beams have found applications in electron acceleration [1,2] and in high-resolution microscopy [3,4]. Several authors have developed expressions in order to describe electromagnetic beams beyond the paraxial approximation [5–8]. For many applications, it is relevant to determine the power carried by these nonparaxial electromagnetic beams. In paraxial optics, it is well known that the power carried by an optical beam is usually calculated by means of the integral of the squared modulus of its electric field over a plane transverse to the propagation axis. While this method is rather accurate for a well-collimated beam, it can be only approximately true for an optical beam with a higher angle of divergence. Moreover, if the beam is far from obeying the paraxial approximation, the definition of the power it carries must be reconsidered. In fact, for nonparaxial optical beams, it has been shown that the integral of the squared modulus of the electric field over a transverse plane is no longer equivalent to the power carried by the beam [9]. This can be essentially explained through the fact that the envelope of a nonparaxial beam varies more rapidly with the longitudinal coordinate than its paraxial counterpart. Furthermore, it is known that, for transverse magnetic (TM) and transverse electric (TE) beams, the integral of the squared modulus of the electric field is logarithmically divergent [10]. In the nonparaxial regime, it is more appropriate to define the power carried by an optical beam by the integral of the longitudinal component of its Poynting vector over a plane transverse to the propagation axis.

Quabis *et al.* have shown that smaller spot sizes can be reached with a radially polarized light instead of linearly polarized light [6,11]. Because of their remarkable focusing properties, radially polarized beams (a special case of TM beams that are cylindrically symmetric) are of considerable interest, for example, in high-resolution microscopy. This type of optical beam has been extensively investigated during recent years [12–16]. The TM_{01} beam (the TM beam of lowest order), whose electromagnetic fields have transverse components that are proportional to a Laguerre–Gaussian mode of order (0,1), is an important special case. When it is tightly focused, the TM_{01} beam exhibits a significant longitudinal electric field component that can be exploited in particle trapping and electron acceleration. More generally, a radially polarized beam of order p may be called a TM_{p1} beam, where $p=0,1,2,\ldots$ is the radial mode number. General TM beams may be designated as $\mathrm{TM}_{p,m+1}$ beams, where $m=0,1,2,\ldots$ is the angular mode number [17]. Similarly, general TE beams may be denoted as $\mathrm{TE}_{p,m+1}$ beams, and an azimuthally polarized beam of order p is a TE_{p1} beam. Lekner [10] and, more recently, Seshadri [18] have presented an expression for the power carried by the TM_{01} and the TE_{01} beams, but no expression seems to be available for beams of higher order. The aim of this paper is to provide a general closed-form expression for the power carried by nonparaxial $\mathrm{TM}_{p,m+1}$ and $\mathrm{TE}_{p,m+1}$ beams.

Many approaches have been demonstrated to produce TM or TE beams in laboratory. Radially polarized beams may be generated directly from a laser by inserting in the laser cavity axially symmetric optical elements with suitable polarization selectivity; such elements include a conical reflector used as a resonator mirror, a conical Brewster window, and a birefringent c-cut laser crystal [19]. Another method consists of introducing into a laser cavity a polarization-selective diffractive element with axially symmetric groove structure [20]. A TM_{01} beam can be generated interferometrically, outside the resonator, with a Mach–Zehnder interferometer that allows the coherent superposition of two orthogonally polarized Laguerre–Gaussian beams of order (0,1) of different parity with the same beam waist [21]. Other techniques to produce radially polarized beams involve, for instance, a polarization converter consisting of four half-wave plates, one in each quadrant [22]. TM_{p1} beams of higher order p can be generated when Laguerre–Gaussian beams of order $(p,1)$ interact coherently with each other with the help of a scheme involving a modified Sagnac interferometer [23]. Another technique to generate a single-mode beam of TM_{11} profile was based on the use of a polarization-selective grating mirror [24]. More recently, single-transverse-mode operation leading to higher-order TM_{p1} beams was achieved in a Nd:YAG laser with an annularly modulated-reflectivity photonic crystal coupler [25]. Finally, $TM_{p,m+1}$ beams for which $m \neq 0$ could be produced with the help of spiral phase plates [26] or spatial light modulators [27]. Specifically, a Mach–Zehnder interferometer used with a spiral phase plate placed in one of its arms can be exploited to construct interferometrically an optical beam exhibiting nodal lines.

The paper is arranged as follows. In Section 2, we give the electromagnetic field components of a nonparaxial TM optical beam that will be used in the following analysis. In Section 3, the power carried by an electromagnetic beam is defined and the general approach to determine it in the case of a TM beam is presented. Finally, in Section 4, the resulting expression for the power carried is discussed. In this paper, the case of the power carried by a TM_{p1} beam for which $m=0$ is considered in detail. The general expression for the power carried by a $TM_{p,m+1}$ beam is also presented; one may derive the general result by following an approach similar to that presented in this paper.

2. ELECTROMAGNETIC FIELDS OF A TM BEAM

When a beam is strongly focused, the vectorial character of light becomes crucial to accurately describe such a nonparaxial beam. Furthermore, the expressions of its electromagnetic fields must satisfy Maxwell's equations beyond the paraxial approximation. Many approaches have been exploited to obtain these expressions for a TM beam, such as the Richards–Wolf theory [13,28], the method of Lax *et al.* [7,29] and the complex-source/sink model [12,17]. The last mentioned method is used in the present paper. The complex source-point method is known to be a useful mathematical technique to convert a spherical wave into a nonparaxial Gaussian beam [30,31]. This method consists of assuming the source of the wave to be located at an imaginary distance along the propagation axis, which is herein taken as the z axis. However, the phasor obtained with the help of the complex source-point method exhibits an annular singularity in the plane of the beam waist. The removal of this nonphysical singularity may be accomplished by combining a sink to the source, leading to the complex-source/sink method [32,33]. The wave obtained with the complex-source/sink method may be viewed as a superposition of an outgoing beam produced by the source located at a given imaginary distance along the z axis and an incoming beam absorbed by the sink located at the same position. This optical wave, consisting of a superposition of two counterpropagating beams, results in a standing-wave component near the $z=0$ plane. Therefore, producing such an interference would require a focusing element that subtends a solid angle greater that 2π, such as a parabolic mirror of large extent or a 4Pi microscope. Nevertheless, such a singularity-free phasor describes a nonparaxial, physically realizable optical beam.

The transverse components of the electromagnetic fields of a TM_{p1} beam in free space, found in the context of the complex-source/sink method, are given by [17]

$$E_x = E_o \frac{K_{p,0}}{K_{p,1}} \tilde{V}^e_{p,1}, \tag{1a}$$

$$E_y = E_o \frac{K_{p,0}}{K_{p,1}} \tilde{V}^o_{p,1}, \tag{1b}$$

$$H_x = -H_o \frac{K_{p,0}}{K_{p,1}} \tilde{U}^o_{p,1}, \tag{1c}$$

$$H_y = H_o \frac{K_{p,0}}{K_{p,1}} \tilde{U}^e_{p,1}, \tag{1d}$$

where E_o is a constant amplitude, $H_o = E_o/\eta_o$ (η_o is the intrinsic impedance of free space), and $K_{p,m}$ is a normalization constant (which may be arbitrarily taken to be unity). Here a time dependence $\exp(j\omega t)$ is assumed, where ω is the angular frequency. The integral representation of the functions $\tilde{U}^e_{p,m}$ and $\tilde{V}^e_{p,m}$ may be written as [34]

$$\tilde{U}^e_{p,m} = K_{p,m} \frac{2a}{p!} \left(\frac{a}{2k} \right)^{p+m/2} \exp(-ka)\cos(m\varphi) \times \int_0^\infty k_r^{2p+m} \frac{\cos[k_z(z+ja)]}{k_z} J_m(k_r r) k_r \mathrm{d}k_r, \tag{2a}$$

$$\tilde{V}^e_{p,m} = -jK_{p,m} \frac{2a}{p!} \left(\frac{a}{2k} \right)^{p+m/2} \frac{\exp(-ka)}{k} \cos(m\varphi) \times \int_0^\infty k_r^{2p+m} \sin[k_z(z+ja)] J_m(k_r r) k_r \mathrm{d}k_r, \tag{2b}$$

where $J_m(\cdot)$ is the Bessel function of the first kind of order m, φ is the azimuthal angle, r is the coordinate transverse to the propagation axis, $k=\omega/c$ is the wave number (c being the speed of light in vacuum), $k_z=(k^2-k_r^2)^{1/2}$, and a is a real constant that characterizes the divergence of the beam. For a known wave number k, parameter a is related to the waist spot size w_o, the Rayleigh range $z_R \equiv \frac{1}{2}kw_o^2$, or the angle of divergence $\delta \equiv \arctan(w_o/z_R)$ of the beam by $a = w_o[1+(\frac{1}{2}kw_o)^2]^{1/2} = z_R[1+2/(kz_R)]^{1/2} = 2/(k \sin\delta \tan\delta)$. Odd modes $\tilde{U}^o_{p,m}$ and $\tilde{V}^o_{p,m}$ can be obtained from Eqs. (2) simply by replacing $\cos(m\varphi)$ by $\sin(m\varphi)$. It should be noted that the functions given by Eqs. (2) are exact solutions to the Helmholtz equation, and they reduce to the well-known elegant Laguerre–Gaussian beams in the paraxial limit, i.e. for $ka \gg 1$ [34]. Moreover, the field components given by Eqs. (1) are rigorous solutions to Maxwell's equations in free space [17].

3. POWER CARRIED BY A TM BEAM

The average optical power carried by the electromagnetic beam is given by the integral of the longitudinal component of its time-averaged Poynting vector $\mathbf{S}=\frac{1}{2}\,\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}$ over a plane transverse to the propagation axis, i.e.,

$$P \equiv \int_0^{2\pi}\int_0^{\infty} \mathbf{S}\cdot\hat{\mathbf{a}}_z r\mathrm{d}r\mathrm{d}\varphi = \frac{1}{2}\int_0^{2\pi}\int_0^{\infty} \mathrm{Re}\{E_xH_y^* - E_yH_x^*\} r\mathrm{d}r\mathrm{d}\varphi. \tag{3}$$

Here, the asterisk denotes the complex conjugate and Re stands for real part. Substituting Eq. (1) in Eq. (3) gives the following expression for the power $P_{p,1}$ carried by the TM_{p1} beam:

$$P_{p,1} = \frac{|E_o|^2}{2\eta_o}\frac{|K_{p,0}|^2}{|K_{p,1}|^2}\int_0^{2\pi}\int_0^{\infty} \mathrm{Re}\{\tilde{V}^e_{p,1}\tilde{U}^{*e}_{p,1} + \tilde{V}^o_{p,1}\tilde{U}^{*o}_{p,1}\} r\mathrm{d}r\mathrm{d}\varphi. \tag{4}$$

To evaluate $P_{p,1}$, one has to substitute Eqs. (2a) and (2b) with $m=1$ in Eq. (4), which leads to a four-dimensional integral. Using the fact that $\cos^2\varphi+\sin^2\varphi=1$, Eq. (4) can be written explicitly as

$$P_{p,1} = \frac{a|E_o|^2|K_{p,0}|^2}{2^{2p}(p!)^2\eta_o}\left(\frac{a}{k}\right)^{2p+2} \times\exp(-2ka)\mathrm{Re}\left\{\int_0^{2\pi}\int_0^{\infty}\int_0^{\infty} k_r^{2p+1}k_r'^{2p+1} \times\frac{\cos[k_z^*(z-ja)]\sin[k_z'(z+ja)]}{jk_z^*} \times\int_0^{\infty} J_1(k_r r)J_1(k_r' r) r\mathrm{d}r k_r k_r'\mathrm{d}k_r\mathrm{d}k_r'\mathrm{d}\varphi\right\}, \tag{5}$$

with $k_z'=(k^2-k_r'^2)^{1/2}$. Note that we have to take the complex conjugate of k_z because it is a complex quantity; in fact it is real for $k_r<k$, whereas it is purely imaginary for $k_r>k$. Using the result $\int_0^\infty J_n(k_r r)J_n(k_r' r)r\mathrm{d}r=\delta(k_r-k_r')/k_r$ [35], the integration over r in Eq. (5) can be performed. Then, making use of the sampling property of the Dirac delta function $\delta(\cdot)$ as well as the definition $\int_0^\infty \delta(k_r-k_r')\mathrm{d}k_r'=1$, the integration with respect to the dummy variable k_r' is carried out with the result that

$$P_{p,1} = \frac{a|E_o|^2|K_{p,0}|^2}{2^{2p}(p!)^2\eta_o}\left(\frac{a}{k}\right)^{2p+2}\exp(-2ka)\mathrm{Re}\left\{\int_0^{2\pi}\int_0^{\infty} k_r^{4p+2} \times\frac{\cos[k_z^*(z-ja)]\sin[k_z(z+ja)]}{jk_z^*}k_r\mathrm{d}k_r\mathrm{d}\varphi\right\}. \tag{6}$$

We use the identity $\cos[k_z^*(z-ja)]\sin[k_z(z+ja)] =\frac{1}{2}\sin[2\,\mathrm{Re}(k_z)z]+\frac{1}{2}j\sinh[2\,\mathrm{Re}(k_z)a]$ to find the real part in Eq. (6). Since k_z has no real part for $k_r>k$, the integration with respect to k_r has then to be carried out only over the interval $0<k_r<k$. Physically, the reason that the contribution to $P_{p,1}$ vanishes for $k<k_r<\infty$ is that evanescent waves do not transport power. After having taken the real part, Eq. (6) reduces to

$$P_{p,1} = \frac{a|E_o|^2|K_{p,0}|^2}{2^{2p+1}(p!)^2\eta_o}\left(\frac{a}{k}\right)^{2p+2} \times\exp(-2ka)\int_0^{2\pi}\int_0^{k} k_r^{4p+2}\frac{\sinh(2k_z a)}{k_z}k_r\mathrm{d}k_r\mathrm{d}\varphi. \tag{7}$$

Note that Eq. (7) is independent of coordinate z. Setting $k_r=k\sin\theta$ and therefore $k_z=k\cos\theta$, Eq. (7) becomes

$$P_{p,1} = \frac{a|E_o|^2|K_{p,0}|^2(ka)^{2p+2}}{2^{2p+1}(p!)^2k\,\eta_o} \times\exp(-2ka)\int_0^{2\pi}\int_0^{\pi/2} \sin^{4p+3}\theta\sinh(2ka\cos\theta)\mathrm{d}\theta\mathrm{d}\varphi. \tag{8}$$

Because k_r is the transverse component of the wave vector and k_z is its longitudinal component, the angle θ can be interpreted as the angle of inclination of the wave vector with respect to the propagation axis. This means that the angles θ and φ are the angular spherical coordinates and $\mathrm{d}\Omega=\sin\theta\mathrm{d}\theta\mathrm{d}\varphi$ is an element of solid angle. Accordingly, the function

$$\Phi_{p,1}(\theta,\varphi) \equiv \frac{a|E_o|^2|K_{p,0}|^2(ka)^{2p+2}}{2^{2p+1}(p!)^2k\,\eta_o} \times\exp(-2ka)\sin^{4p+2}\theta\sinh(2ka\cos\theta) \tag{9}$$

may be viewed as the radiation intensity, defined as the time-averaged power flow per unit solid angle in a specified direction (θ,φ) [18].

To determine the power carried by the beam, the two integrals in Eq. (8) have to be performed. Since the integrand in Eq. (8) is independent of φ, the integration with respect to φ yields 2π. To solve analytically the integration along θ, one can make use of the modified Struve function $\mathbf{L}_\nu(x)$, whose integral representation is given by Eq. (A2) in Appendix A. Setting $\nu=2p+3/2$ in Eq. (A2) and substituting the result in Eq. (8), we find

$$P_{p,1} = \frac{\pi|E_o|^2|K_{p,0}|^2(2p+1)!}{2^{2p+1}(p!)^2\eta_o} \times\frac{(\pi ka^3)^{1/2}\exp(-2ka)}{k}\mathbf{L}_{2p+3/2}(2ka). \tag{10}$$

Eq. (10) gives the power carried by a TM_{p1} (radially polarized) beam, but this result can be generalized to azimuthally varying $\mathrm{TM}_{p,m+1}$ beams, where m is the angular mode number. Following the same approach, it can be shown that, using Eqs. (4) of [17] with Eqs. (2a) and (2b), the power $P_{p,m+1}$ carried by a $\mathrm{TM}_{p,m+1}$ beam is given by

$$P_{p,m+1} = \frac{(1+\delta_{0,m})\pi|E_o|^2|K_{p,m}|^2(2p+m+1)!}{2^{2p+m+2}(p!)^2\eta_o} \times \frac{(\pi ka^3)^{1/2}\exp(-2ka)}{k}\mathbf{L}_{2p+m+3/2}(2ka), \quad (11)$$

where $\delta_{0,m}$ is the Kronecker delta, which is 1 if $m=0$ and 0 if $m\neq 0$. Equation (11) is the main result of this paper. If $m=0$, Eq. (11) reduces to Eq. (10), as expected. As a special case, we can express the power carried by a strongly focused TM_{01} beam by setting $p=m=0$ in Eq. (11). With the help of Eq. (A4b), one finds explicitly

$$P_{0,1} = \frac{\pi|E_o|^2|K_{0,0}|^2}{4\eta_o k^2}\exp(-2ka)[2ka\sinh(2ka) - \cosh(2ka) + 1 - 2(ka)^2]. \quad (12)$$

It should be recalled that the expressions for the electromagnetic fields of TE beams can be easily found from those of TM beams by means of the duality transformation $\mathbf{E}\to\eta_o\mathbf{H}$ and $\mathbf{H}\to-\mathbf{E}/\eta_o$. Since $\frac{1}{2}\mathrm{Re}\{\mathbf{E}\times\mathbf{H}^*\}=\frac{1}{2}\mathrm{Re}\{\mathbf{E}^*\times\mathbf{H}\}$, the power carried by a $\mathrm{TE}_{p,m+1}$ beam is the same as the power carried by a $\mathrm{TM}_{p,m+1}$ beam, i.e., it is also given by Eq. (11).

4. DISCUSSION

We consider the important case of a paraxial beam. The asymptotic form of the modified Struve function [see Eq. (A5)] may be written $\mathbf{L}_\nu(x)\sim\exp(x)/(2\pi x)^{1/2}$ as $x\gg 1$. Therefore, the power $P^{(0)}_{p,m+1}$ carried by a paraxial $\mathrm{TM}_{p,m+1}$ beam is obtained from Eq. (11) in the limiting case $ka \gg 1$, that is,

$$P^{(0)}_{p,m+1} \equiv \lim_{ka\gg 1} P_{p,m+1} = (1+\delta_{0,m}) \times\left(\frac{\pi a}{2k}\right)\frac{|E_o|^2|K_{p,m}|^2(2p+m+1)!}{2^{2p+m+2}(p!)^2\eta_o}. \quad (13)$$

The superscript (0) on $P_{p,m+1}$ denotes that the power is valid in the paraxial limit. The ratio of the power carried by a nonparaxial beam with respect to the one carried by its paraxial counterpart is

$$\frac{P_{p,m+1}}{P^{(0)}_{p,m+1}} = 2(\pi ka)^{1/2}\exp(-2ka)\mathbf{L}_{2p+m+3/2}(2ka). \quad (14)$$

Figure 1 shows the power carried by a $\mathrm{TM}_{p,m+1}$ beam as given by Eq. (14) as a function of the parameter ka for different values of p and m.

It is apparent from Fig. 1 that there is no power carried by the beam when the parameter ka is zero. Mathematically, this is due to the fact that the modified Struve function vanishes when its argument is zero. Physically, it is due to the fact that the phasor of the beam is made of two contributions: the first one is a wave that radiates outward from the plane of the beam waist ($z=0$) and the second one is a wave that is absorbed in the plane of the beam waist. As mentioned in Section 2, the complex-source/sink method gives a model for nonparaxial beams that consists of two counterpropagating beams of different amplitudes. The addition of these two contributions leads to a traveling beam superposed on a counterpropagating traveling beam of weaker amplitude. On the one hand, the maximum amplitude of the counterpropagating beam is very weak when the angle of divergence of the beam is reasonably small, and the power of the beam is thus mainly carried in the positive z direction. On the other hand, when ka is small, the beam can be viewed as the superposition of an outgoing and an incoming quasi-spherical wave of comparable amplitude, leading to a nearly zero net power carried in the z direction.

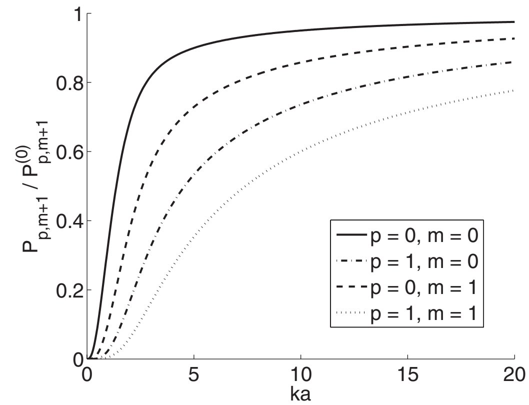

Fig. 1. Power carried by a $\mathrm{TM}_{p,m+1}$ beam normalized with the power of its paraxial counterpart as a function of the parameter ka for the first values of p and m.

In conclusion, we have obtained an expression for the power carried by TM and TE beams of any order, which involves a modified Struve function of order equal to an integer plus one-half. Since the power is a finite quantity for all values of ka, it confirms that the functions $\tilde{U}^e_{p,m}$ and $\tilde{V}^e_{p,m}$ are good candidates to describe physically realizable tightly focused beams.

APPENDIX A: MODIFIED STRUVE FUNCTIONS

The modified Struve function of order ν is defined by the following series expansion [36]:

$$\mathbf{L}_\nu(x) = \sum_{s=0}^{\infty}\frac{1}{\Gamma\left(s+\frac{3}{2}\right)\Gamma\left(\nu+s+\frac{3}{2}\right)}\left(\frac{x}{2}\right)^{2s+\nu+1}, \quad (A1)$$

where $\Gamma(\cdot)$ is the gamma function. The integral representation of the modified Struve function is given by

$$\mathbf{L}_\nu(x) = \frac{2(x/2)^\nu}{\sqrt{\pi}\Gamma\left(\nu+\frac{1}{2}\right)}\int_0^{\pi/2}\sin^{2\nu}\theta\sinh(x\cos\theta)\mathrm{d}\theta. \quad (A2)$$

When the order is equal to an integer plus one-half, the modified Struve function reduces to a finite sum of elementary functions:

$$\mathbf{L}_{n+1/2}(x) = I_{-n-1/2}(x) - \left(\frac{2}{\pi x}\right)^{1/2} \sum_{m=0}^{n} \frac{(-1)^m (2m-1)!!}{(2n-2m)!!} x^{n-2m}, \tag{A3}$$

where $n=0,1,2,\ldots$ and $I_{-n-1/2}(x)$ is the modified Bessel function of the first kind of order $-n-1/2$, which is a combination of hyperbolic sine and hyperbolic cosine. Here are some special cases of Eq. (A3):

$$\mathbf{L}_{1/2}(x) = (2\pi x)^{-1/2}(2\cosh x - 2), \tag{A4a}$$

$$\mathbf{L}_{3/2}(x) = (2\pi x^3)^{-1/2}(2x \sinh x - 2\cosh x + 2 - x^2), \tag{A4b}$$

$$\mathbf{L}_{5/2}(x) = (2\pi x^5)^{-1/2}\left(-6 + x^2 - \frac{1}{4}x^4 + 2(x^2+3)\cosh x - 6x \sinh x\right). \tag{A4c}$$

Asymptotically (i.e., for $x \gg 1$), the modified Struve function of order ν may be approximated by

$$\lim_{x \gg 1} \mathbf{L}_\nu(x) = \frac{\exp(x)}{(2\pi x)^{1/2}} - \frac{1}{\pi^{1/2}\Gamma\left(\nu + \frac{1}{2}\right)} \left(\frac{x}{2}\right)^{\nu-1}. \tag{A5}$$

If the second term of Eq. (A5) is neglected with respect to the first one, it can be seen that the modified Struve function does not depend on its order if its argument is sufficiently large.

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