Computational topology: Lecture 16

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May 10, 2019

1. Surface extraction from medical imaging
2. Terminology
3. Computational pipeline

Surface extraction from medical imaging

Topologically exact models of microvessels between neurons

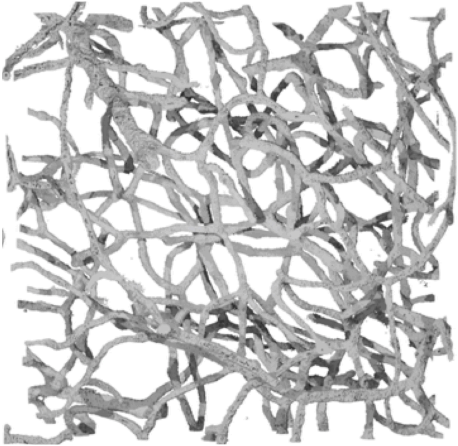

Figure 1: Some examples

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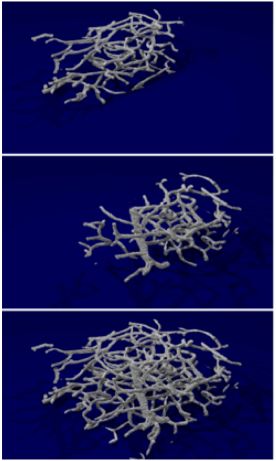

Figure 2: Some examples

Marching-cubes (Siggraph, 1987)

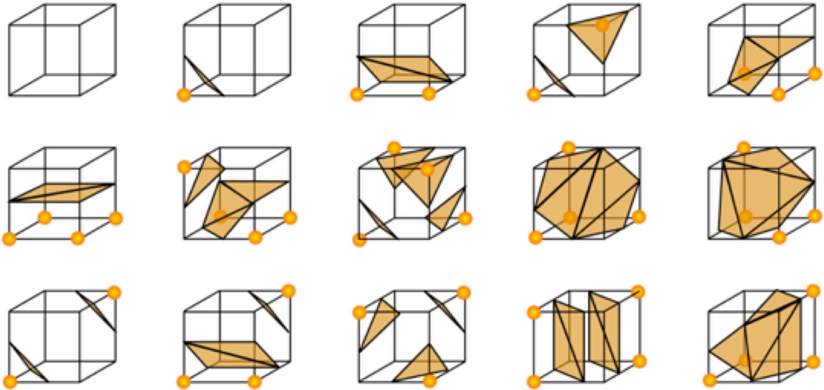

Figure 3: Find the 0-surface (or any iso-surface) of a discrete 3D field

Progress of high-resolution imaging technology (1987)

William E. Lorensen & Harvey E. Cline, "Marching Cubes: a High Resolution 3D Surface Construction Algorithm", Siggraph, 1987

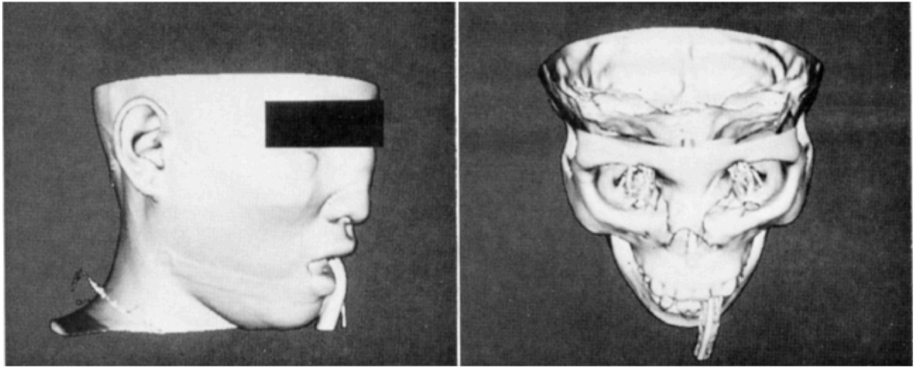

Figure 4: $260 \times 260 \times 93 CT images.$

The 93 axial slices are 1.5 mm thick, with pixel dimensions of 0.8 mm

Progress of high-resolution imaging technology (2015)

Nobel Prize in Chemistry 2014 awarded to Betzig, Moerner & Hell for "development of super-resolved fluorescence microscopy," which brings "optical microscopy into the nanodimension"

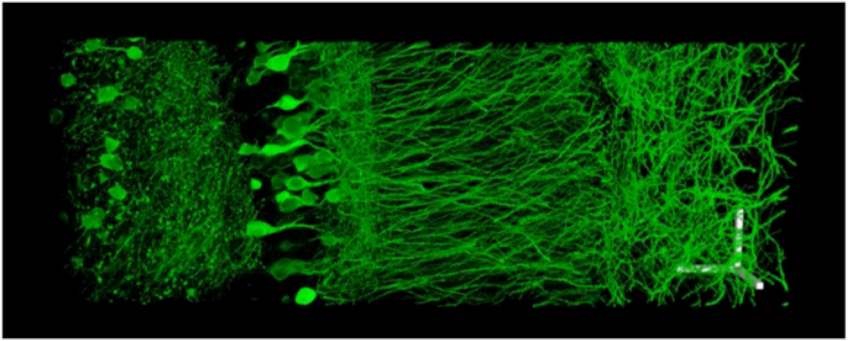

Figure 3: Volume rendering (i.e. dynamic images) of a portion of hippocampus showing neurons and synapses

Source: Fei Chen, Paul W. Tillberg, Edward S. Boyden. Expansion microscopy. Science, January 15 2015; DOI: 10.1126/science.1260088

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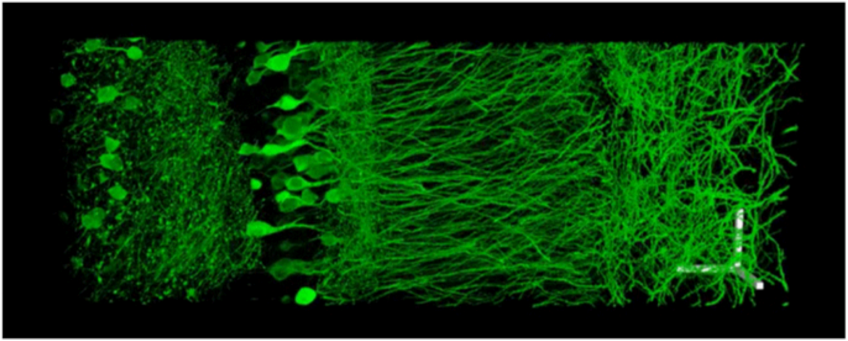

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LAR examples (1/2)

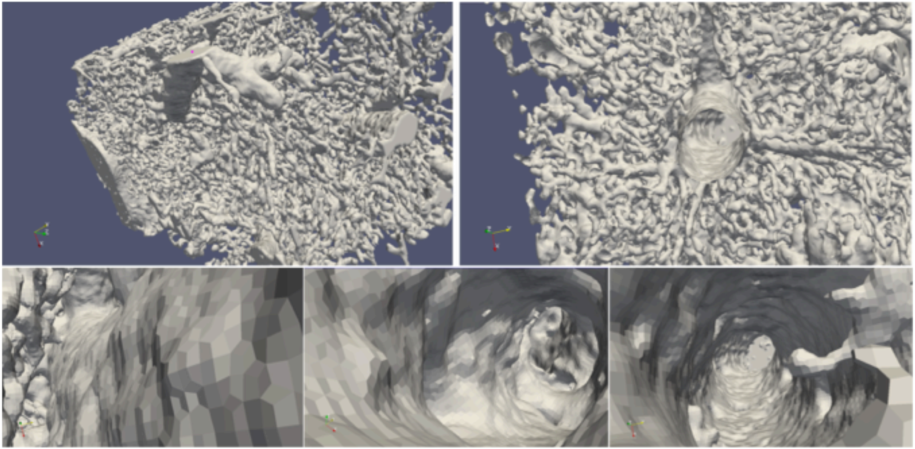


Figure 7: Paoluzzi, DiCarlo, Furiani, Jirik, CAD models from medical images using LAR, Computer-Aided Design and Applications, 2016

LAR examples (2/2)

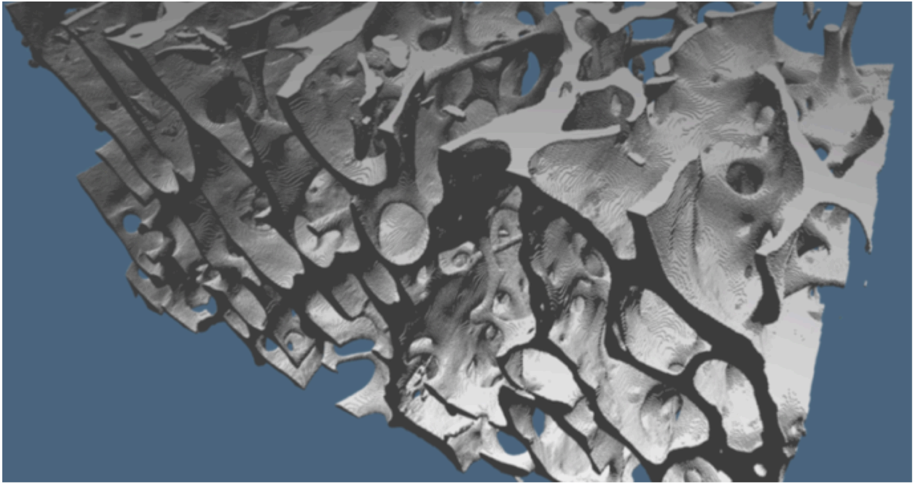


Figure 8: Paoluzzi, DiCarlo, Furiani, Jirik, CAD models from medical images using LAR, Computer-Aided Design and Applications, 2016

Terminology

Medical imaging devices

Radiography using X-rays or gamma rays to view the internal form of an object

Magnetic resonance imaging MRI scanner or NMR (nuclear magnetic resonance) scanner uses powerful magnets to polarize and excite hydrogen nuclei (i.e., single protons) of water molecules* in human tissue

Nuclear medicine Gamma cameras and PET scanners are used (e.g. scintigraphy) to detect regions of biologic activity that may be associated with a disease

Ultrasound uses high frequency broadband sound waves in the megahertz range, that are reflected by tissue to varying degrees to produce (up to 3D) images

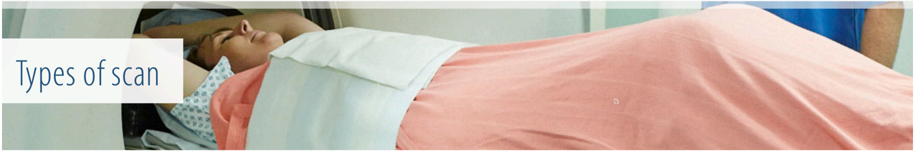
Elastography Elastography maps the elastic properties of soft tissue, as elasticity can discern healthy from unhealthy tissue for specific organs/growths.

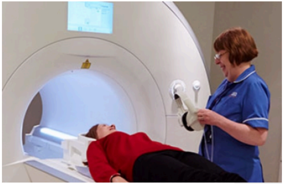
Tomography X-ray computed tomography (CT), or Computed Axial Tomography (CAT) scans produces a 2D image of the structures in a thin section of the body. Positron emission tomography (PET) also used in conjunction with computed tomography, PET-CT, and magnetic resonance imaging PET-MRI.

Echocardiography When ultrasound is used to image the heart it is referred to as an echocardiogram. Echocardiography allows detailed structures of the heart

Medical imaging

Digital Imaging and Communications in Medicine (DICOM) is the standard for the communication and management of medical imaging information and related data.





Medical imaging standard

Medical imaging creates visual representations of the interior of a body for clinical analysis and medical intervention, as well as visual representation of the function of some organs or tissues (physiology)

DICOM®

Digital Imaging and Communications in Medicine

ABOUT DICOM® STANDARD ACTIVITY USING DICOM® RESOURCES CONFERENCES

DICOM Data format

DICOM® — Digital Imaging and Communications in Medicine — is the international standard for medical images and related information

- It defines the formats for medical images that can be exchanged with the data and quality necessary for clinical use

DICOM groups information into data sets

- The file of a chest x-ray image, for example, actually contains the patient ID within the file, so that the image can never be separated from this information by mistake
- This is similar to the way that image formats such as JPEG can also have embedded tags to identify and otherwise describe the image

DICOM Data object

A DICOM data object consists of a number of attributes, including items such as name, ID, etc., and also one special attribute containing the image pixel data (i.e

- The main object has no "header" as such, being merely a list of attributes, including the pixel data
- A single DICOM object can have only one attribute containing pixel data
- For many modalities, this corresponds to a single image
- The attribute may contain multiple "frames", allowing storage of cine loops or other multi-frame data
- Another example is NM data, where an NM image, by definition, is a multi-dimensional multi-frame image
- Three- or four-dimensional data can be encapsulated in a single DICOM object
- Pixel data can be compressed using a variety of standards, including JPEG, Lossless JPEG, JPEG 2000, and RLE (Run-length encoding)
- LZW (zip) compression can be used for the whole data set (not just the pixel data), but this has rarely been implemented.

Computational pipeline

LAR-surf terminology

dD Image ($d \in \{2, 3\}$)

d-Block or Brick (cuboidal portion of Image) 3D image portions as 3-cell complexes: (a) image portion seen exploded; (b) divide et impera paradigm; (c) surface assembling by removal of double 2-cells, via a sort-based MapReduce algorithm

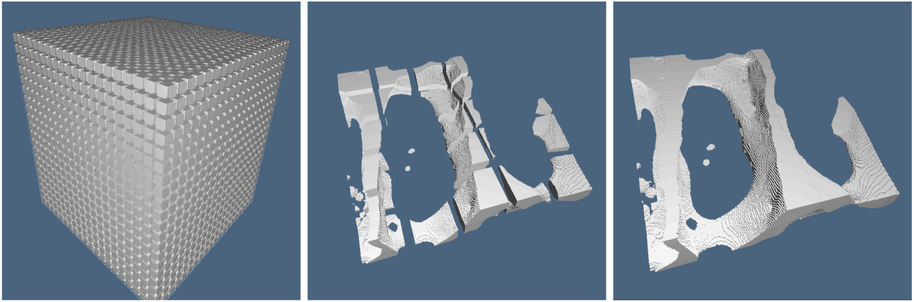


Figure 9: 3-Block or 3-brick of image

d-Block or Brick

B is a function of its element of the lowest block coordinates $i, j, k \in \mathbb{N}$ and of block dimension n: [B(i,j,k,n) := Image([in:in+n, jn:jn+n, kn:kn+n])]

block sides may not correspond to image edges

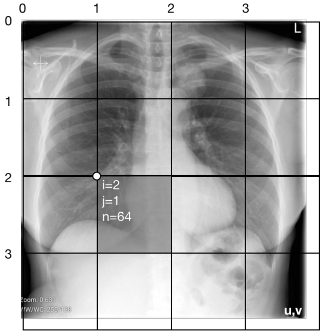


Figure 10: A possible block partition of a radiologic image. The evidenced 2D block, of size $n^d = 64^2$, is sliced by $B([2, 1, 64]) = Image([128 : 172], [64 : 128])$

Block chain ;-)

Map each image subspace $B(i, j, n)$ to the linear chain space C_2 of dimension $n \times n$, using coordinate vectors $c_{h,k} \in \mathbb{B}^{n\times n} := \{0, 1\}^{n\times n}$, where the basis element $c = c_{h,k} \in C_2$ is mapped via Cartesian-to-linear map to the Boolean vector

$$Image(h, k) \mapsto c_{h,k} := [0 \cdots 0\ 1\ 0 \cdots 0]^t \in \mathbb{B}^{n\times n}$$

for each $0 \le h, k \le n$, and where the (single) unit cell in position $nk + h \le n \times n$.

Each pixel (or voxel) in a block is seen as a basis vector in C_2, and each subset of image elements, as a vector $c \in C_2$, with many ones as the cardinality of the subset

Each $B(i, j, n)$ box maps to the unit vector $[1 \cdots\ 1\ \cdots 1]^t \in \mathbb{B}^{n\times n}$

Number of non-zeros in boundary matrix $[\partial_d]_{n^d}$

It is easy to see that the operator's matrix $[\partial_d]$ is very sparse

- $[\partial_d]_{n^d}$ contains $2d$ non-zero elements (ones) for each column, i.e.∼4 ones and 6 ones for 2D and 3D respectively
- the number of columns is n^d
- the number of rows is the number of 2-cells in $B(n^d)$

Question

How to compute the dimension (cardinality of basis) of $C_2(B(n^d))$?

Sparsity of boundary matrix $[\partial_d]$

We have $2d$ non-zero elements for each column, hence their total number is $2d\, n^d$.

The number of matrix element is $d\, n\, (1+n)^{d-1} \times n^d$, giving a ratio of:

$$\frac{\text{non-zero elements}}{\text{total elements}} = \frac{2d \times n^d}{d\, n\, (1+n)^{d-1} \times n^d} = \frac{2}{n+n^d}$$

Using sparse matrices in CSC (Compressed Sparse Column) format we get the storage size:

$$mem([\partial_d]_{n^d}) = 2 \times \#\text{nzero} + \#\text{columns} = 2 \times 2d\, n^d + n^d = (4d+1)n^d$$

Storage size of boundary matrix $[\partial_d]$

In conclusion:

- for block size $n = 64$, $[\partial_d]$ requires for 2D images $9 \times 64^2 = 36,864$ memory elements, and for 3D images $13 \times 64^3 = 3,407,872$ memory elements
- Counting the bytes for the standard implementation of a sparse binary matrix (1 byte for values and 8 bytes for indices) we get:
- $(18d + 8)n^d$ bytes, giving 176,KB for 2D and 15.872,MB for 3D

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