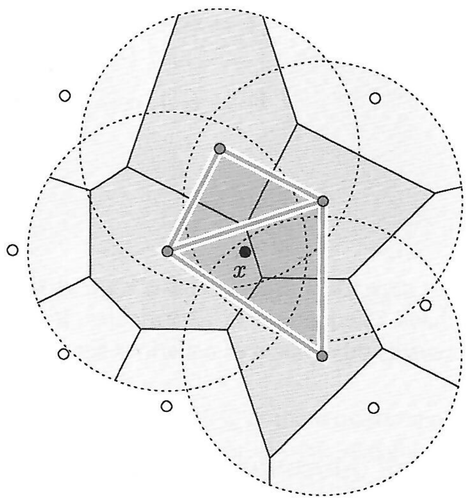

Fig. 7.4 The Voronoi diagram of 11 sites and the full subcomplex of the alpha complex defined by the four sites whose disks contain the point x

$$\operatorname{area}\mathbb{U}(\alpha) = \sum_{\sigma \in A} (-1)^{\dim \sigma} \cdot \operatorname{area}\left(\bigcap_{s \in \sigma} D_s(\alpha) \right).$$

In the plane, A has at most a constant times $n = |S|$ simplices, and each simplex corresponds to an independent collection of disks. We thus have a formula with few and simple terms. The formula generalizes to higher dimensions, and to collections of balls with different radii [3]. The summation can also be done over all simplices of the Delaunay triangulation, and while this sum contains redundant terms, the result is still correct [4].

References

1. Bryant R, Edelsbrunner H, Koehl P, Levitt M (2004) The area derivative of a space-filling diagram. Discrete Comput Geom 32:293–308
2. Kratky KW (1978) The area of intersection of n equal circular disks. J Phys A: Math Gen 11:1017–1024
3. Edelsbrunner H (1995) The union of balls and its dual shape. Discrete Comput Geom 13:415–440
4. Naiman DQ, Wynn HP (1992) Inclusion-exclusion Bonferroni identities and inequalities for discrete tube-like problems via Euler characteristics. Ann Statist 20:43–76