Computational Algebraic Topology: Lecture 6

Alberto Paoluzzi

March 24, 2017

Simplicial complexes & Delaunay triangulations

1. Delaunay triangulations
2. `Scipy.spatial` package in Python
3. Julia interface to Python
4. ‘Facet and extrusion operations in Simple_X^n
5. References

Delaunay triangulations

Delaunay triangulation

In mathematics and computational geometry, a Delaunay triangulation for a set P of points in $\mathbb{E}^2$ is a triangulation $\mathcal{T}(P)$ such that no point in P is inside the circumcircle of any triangle in $\mathcal{T}(P)$

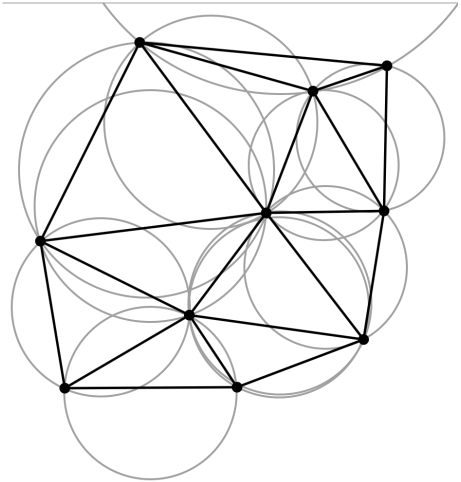
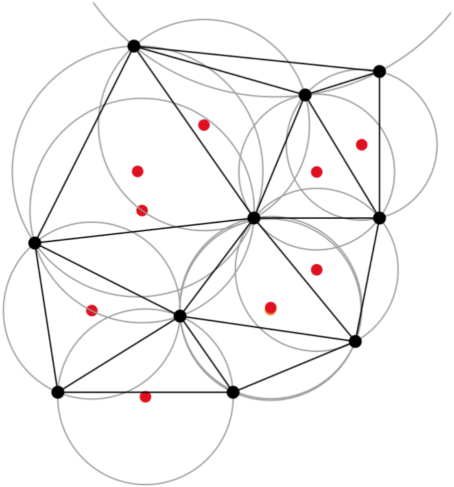


Figure 1: Delaunay triangulation

Dual vertices

The Delaunay triangulation of a discrete point set P in general position corresponds to the dual graph of the Voronoi diagram for P



Voronoi complex

- a Voronoi complex is a partitioning of a plane into a cellular complex based on distance to points in a discrete set P
- The regions are called Voronoi cells. The Voronoi diagram of a set of points is dual to its Delaunay triangulation.
- The 2-cells are convex

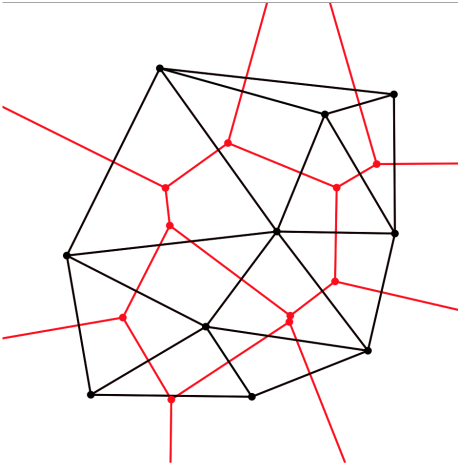


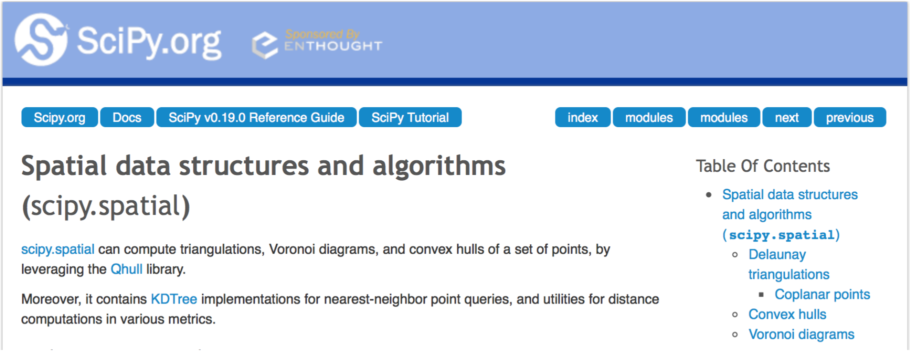
Figure 3: Delaunay triangulation

Properties of Delaunay triangulations (from Wikipedia)

- Delaunay triangulations maximize the minimum angle of all the angles of the triangles in the triangulation; they tend to avoid sliver triangles.
- For a set of points on the same line there is no Delaunay triangulation (the notion of triangulation is degenerate for this case).
- For four or more points on the same circle (e.g., the vertices of a rectangle) the Delaunay triangulation is not unique:
  - each of the two possible triangulations that split the quadrangle into two triangles satisfies the "Delaunay condition", i.e., the requirement that the circumcircles of all triangles have empty interiors.
- By considering circumscribed spheres, the notion of Delaunay triangulation extends to three and higher dimensions.

Scipy.spatial package in Python

Spatial data structures and algorithms (scipy.spatial)



Spatial data structures and algorithms (scipy.spatial)

scipy.spatial can compute triangulations, Voronoi diagrams, and convex hulls of a set of points, by leveraging the Qhull library.

Moreover, it contains KDTree implementations for nearest-neighbor point queries, and utilities for distance computations in various metrics.

Table Of Contents

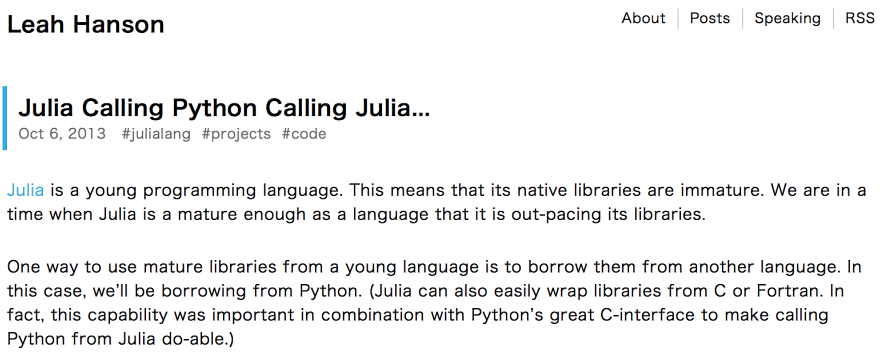
- Spatial data structures and algorithms (`scipy.spatial`)
  - Delaunay triangulations
    - Coplanar points
  - Convex hulls
  - Voronoi diagrams

Assignment !!

: Follow step by step the tutorial . . .

Julia interface to Python

Julia Calling Python Calling Julia. . .



Leah Hanson

About | Posts | Speaking | RSS

Julia Calling Python Calling Julia...

Oct 6, 2013 #julialang #projects #code

Julia is a young programming language. This means that its native libraries are immature. We are in a time when Julia is a mature enough as a language that it is out-pacing its libraries.

One way to use mature libraries from a young language is to borrow them from another language. In this case, we'll be borrowing from Python. (Julia can also easily wrap libraries from C or Fortran. In fact, this capability was important in combination with Python's great C-interface to make calling Python from Julia do-able.)

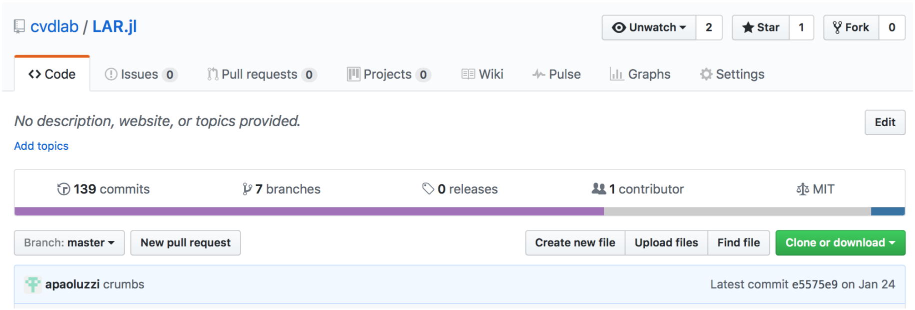
Using Python Libraries from Julia

```
using PyCall
@pyimport pylab

x = linspace(0,2*pi,1000);
y = sin(3*x + 4*cos(2*x));

pylab.plot(x, y; color="red", linewidth=2.0, linestyle="--")
pylab.show()
```

Using Larlib from Julia


cvdlab / LAR.jl

No description, website, or topics provided.

139 commits · 7 branches · 0 releases · 1 contributor · MIT

apaoluzzi crumbs — Latest commit e5575e9 on Jan 24

Using Larlib from Julia

```
using LAR

jsonModel = """
    {"V" : [[5.0,0.0],[7.0,1.0],[9.0,0.0],[13.0,2.0],[15.0,4.0],[17.0,
    8.0],[14.0,9.0],[13.0,10.0],[11.0,11.0],[9.0,10.0],[5.0,9.0],[7.0,
    9.0],[3.0,8.0],[0.0,6.0],[2.0,3.0],[2.0,1.0],[8.0,3.0],[10.0,2.0],
    [13.0,4.0],[14.0,6.0],[13.0,7.0],[12.0,10.0],[11.0,9.0],[9.0,7.0],
    [7.0,7.0],[4.0,7.0],[2.0,6.0],[3.0,5.0],[4.0,2.0],[6.0,3.0],[11.0,
    4.0],[12.0,6.0],[12.0,7.0],[10.0,5.0],[8.0,5.0],[7.0,6.0],[5.0,5.0]],
    "FV" : [[0,1,16,28,29],[0,15,28],[1,2,17],[1,16,17,33],[2,3,17],
    [3,4,18,19],[3,17,18,30],[4,5,19],[5,6,19],[6,7,20,21,22,32],
    [6,19,20],[7,8,21],[8,9,21,22],[9,11,23,24],[9,22,23],
    [10,11,24,25],[10,12,25],[12,13,25,26],[13,14,27],[13,26,27],
    [14,15,28],[14,27,28,29,36],[16,29,34],[16,33,34],[17,30,33],
    [18,19,31],[18,30,31],[19,20,31,32],[22,23,32,33],[23,24,34,35],
    [23,33,34],[24,25,27,36],[24,35,36],[25,26,27],[29,34,35],
    [29,35,36],[30,31,32,33],[0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15]]}
""";
model = json2larmodel(jsonModel);
viewexploded(model.Verts,rebase(model.Lar.FV[1:end-1]))
```

Using Larlib from Julia

```
# Input of a LAR representation (2-complex in JSON format)
larDict = JSON.parse(jsonModel);
V = larDict["V"];
FV = larDict["FV"];

# extraction of facets (1-cells)
v,ev = p.larFacets((V,FV),2);

# visualization in Julia
viewexploded(v',(ev+1)')
viewLarIndices(v',(ev+1)')
```

Assignment !!

: look for `viewexploded` in `LAR.jl` and in `Larlib`

‘Facet and extrusion operations in $Simple_X^n$

Extraction of facets from a set of *d*-simplices

$$\partial\,\sigma^d = \sum_{k=0}^{d} (-1)^d \langle v_0, \ldots, v_{k-1}, v_{k+1}, \ldots, v_d \rangle$$

Implementation The `larSimplexFacets` function, for estraction of non-oriented $(d-1)$-facets of d-dimensional simplices, returns a list of d-tuples of integers, i.e. the input LAR representation of the topology of a cellular complex. The final steps are used to remove the duplicated facets, by transforming the sorted facets into a *set of strings*, so removing the duplicated elements.

⟨ Facets extraction from a set of simplices 8b ⟩ ≡

```
def larSimplexFacets(simplices):
    out = []
    d = len(simplices[0])
    for simplex in simplices:
        out += AA(sorted)([simplex[0:k]+simplex[k+1:d] for k in range(d)])
    out = set(AA(tuple)(out))
    return  sorted(out)
```

◇

Macro referenced in 10.

Extraction of facets from a set of d-simplices

```
def larSimplexFacets(simplices):
    """ Extraction of facets from a set of $d$-simplices"""
    out = []
    d = len(simplices[0])

    for simplex in simplices:
        out += AA(sorted)([simplex[0:k]+simplex[k+1:d] for k i

    out = set(AA(tuple)(out))
    return  sorted(out)
```

Assignment

In synthesis:

- Prepare a simplicial complex T using random point in Scipy
- Translate `larSimplexFacets` from Python to Julia
- Generate a Julia representation of 1-cells of T

Extrusion of a simplicial complex

```
def larExtrude1(model,pattern):
    V, FV = model
    d, m = len(FV[0]), len(pattern)
    coords = list(cumsum([0]+(AA(ABS)(pattern))))
    offset, outcells, rangelimit = len(V), [], d*m
    for cell in FV:
        tube = [v + k*offset for k in range(m+1) for v in cell]
        cellTube = [tube[k:k+d+1] for k in range(rangelimit)]
        outcells += [reshape(cellTube, newshape=(m,d,d+1)).tolist()]

    outcells = AA(CAT)(TRANS(outcells))
    cellGroups = [group for k,group in enumerate(outcells) if pattern[k]>0
    outVertices = [v+[z] for z in coords for v in V]
    outModel = outVertices, CAT(cellGroups)
    return outModel
```

Assignment

use Nuweb !!

1. Look for the meaning of `pattern` in `Larlib`
2. Translate in Julia
3. Generate

   two

   test examples

References