Fast algebraic filtering of surfaces from 3D medical images with Julia

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Abstract

In this paper we introduce a novel algebraic LAR-SURF filter, well founded on algebraic topology methods, to extract and smooth the boundary surface of any subset of voxels arising from the segmentation of a 3D medical image. The input is defined as a *chain*, i.e. as a vector from a linear space of 3-chains, represented in coordinates as a sparse Boolean vector. The output is produced as the result of the mapping via the linear boundary operator $\partial_3 : C_3 \to C_2$ between linear spaces of 3- and 2-chains. In particular, when the input set of voxels is either not (4-)connected, or contains one or more empty regions inside, LAR-SURF generates a non connected set of closed surfaces, i.e. a set of 2-cycles—using the language of algebraic topology. The only data structures used by this approach are sparse arrays with one or two indices, i.e. sparse vectors and matrices. This work is based on LAR (Linear Algebraic Representation) methods, and is implemented in Julia language, natively supporting parallel computing on hybrid architectures.

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