20-6-18

Discrete Mathematics

* study of methods of distinct/ discrete/ unconnected objects / elements.
* Logic - deals with study of valid reasoning. Main purpose is to check whether a given statement is valid (or) not.
* Discrete mathematics deals with the study of discrete objects. ~~It is the~~ ~~most~~ Main Aim: To provide tools by which you can determine whether the given reasoning is valid..
* Main building blocks - propositions (or) sentences - Meaningful declarative sentence which is exclusively true (or) false.
* Interrogative, exclamatory, imperative are not propositions.
* Propositional variable - used to represent proposition.
* Truth value: value which indicates whether the given proposition is true (or) false. True - T (or) 1 False - F (or) 0.
* Negation of proposition: Let P be a proposition, negation of P is represented by $\neg P$ (or) $\sim P$ (or) $\bar{P}$ (or) P'. It is a proposition 'It is not P'
  - If P is true then $\neg P$ is false and vice versa..
* Truth Table - A table that gives the truth value of the given proposition.

P	$\neg P$
0	1
1	0

Connectives: Logical operator that connects 2 or more proposition. AND - $\wedge$ - conjunction operator.
OR - $\vee$ - disjunction "

Conjunction of P and Q - $P \wedge Q$

P	Q	$P \wedge Q$
0	0	0
0	1	0
1	0	0
1	1	1

P	Q	$P \oplus Q$
0	0	0
0	1	1
1	0	1
1	1	0

Disjunction of P and Q - $P \vee Q$

P	Q	$P \vee Q$
0	0	0
0	1	1
1	0	1
1	1	1

* Exclusive OR (XOR): Let P and Q be propositions. It is denoted by $P \oplus Q$.
* Conditional statement (Implication): Let P and Q be propositions. It is denoted by $P \to Q$. P - hypothesis (or) premises (or) antecedent
  Q - conclusion (or) consequence (or) precedent.

P	Q	$P \to Q$
0	0	1
0	1	1
1	0	0
1	1	1

* If P, Q.
* P is sufficient for Q
* Q, if P.
* A necessary condition for P is Q.
* P implies Q.
* Q whenever P
* Q follows from P

~~Converve~~ Converse	Inverse	Contrapositive
$Q \to P$	$\neg P \to \neg Q$	$\neg Q \to \neg P$

P	Q	$P \leftrightarrow Q$
0	0	1
0	1	0
1	0	0
1	1	1

* Biconditional statement (Bi-implication):
  P iff Q $P \leftrightarrow Q$

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① Prove that $p \to q \equiv \neg p \vee q$

p	q	$p \to q$	$\neg p$	$\neg p \vee q$
0	0	~~0~~ 1	1	1
0	1	1	1	1
1	0	0	0	0
1	1	1	0	1

② Construct truth table

a) $(p \leftrightarrow q) \leftrightarrow (r \leftrightarrow s) \equiv t$

p	q	r	s	$p \leftrightarrow q$	$r \leftrightarrow s$	t
0	0	0	0	1	~~0~~ 1	1
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	1	1
0	1	0	0	0	1	0
0	1	0	1	0	0	~~0~~ 1
0	1	1	0	0	0	~~0~~ 1
0	1	1	1	0	1	0
1	0	0	0	0	1	0
1	0	0	1	0	0	~~0~~ 1
1	0	1	0	0	0	~~0~~ 1
1	0	1	1	0	1	0
1	1	0	0	1	1	1
1	1	0	1	1	0	0
1	1	1	0	1	0	0
1	1	1	1	1	1	1

(b) $[(p \to q) \wedge (q \to r)] \to (p \to r) \equiv t$

p	q	r	$(p \to q)$	$(q \to r)$	$(p \to q) \wedge (q \to r)$	$(p \to r)$	t
0	0	0	1	~~0~~ 1	1	1	1
0	0	1	1	~~0~~ 1	~~0~~ 1	~~0~~ 1	1
0	1	0	~~0~~ 1	0	0	1	1
0	1	1	~~0~~ 1	1	~~0~~ 1	1	1
1	0	0	0	1	0	0	1
1	0	1	0	~~0~~ 1	~~0~~ 0	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

4) Show that:

(a) $\neg(p \vee q) \equiv \neg p \wedge \neg q$

p	q	$\neg(p \vee q)$	$\neg p \wedge \neg q$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	0	0

(b) $\neg(p \wedge q) \equiv \neg p \vee \neg q$

p	q	$\neg(p \wedge q)$	$\neg p \vee \neg q$
0	0	1	1
0	1	1	1
1	0	1	1
1	1	0	0

b) $(p \leftrightarrow \neg q) \leftrightarrow (q \leftrightarrow r) \equiv t$

p	q	r	$\neg q$	$p \leftrightarrow \neg q$	$q \leftrightarrow r$	t
0	0	0	1	0	1	0
0	0	1	1	0	0	1
0	1	0	0	~~0~~ 1	0	0
0	1	1	0	~~0~~ 1	1	1
1	0	0	1	1	1	1
1	0	1	1	1	0	0
1	1	0	0	0	0	1
1	1	1	0	0	1	0

3) S.T the following implication is tautology

(a) $[(p \vee q) \wedge (p \to r) \wedge (q \to r)] \to r \equiv t$

p	q	r	$p \vee q$	$\neg p \vee r$	$\neg q \vee r$	t	$\neg t \vee r$
0	0	0	0	1	1	0	1
0	0	1	0	1	1	0	1
0	1	0	1	1	0	0	1
0	1	1	1	1	1	1	1
1	0	0	1	0	1	0	1
1	0	1	1	1	1	1	1
1	1	0	1	0	0	0	1
1	1	1	1	1	1	1	1

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1) Prove the following implications without using truth table

1) $\neg p \to (p \to q)$

~~$\neg(\neg p) \vee (p \to q)$~~
~~$p \vee (\neg p \vee q)$~~

$\equiv \neg p \to (\neg p \vee q)$ $[p \to q \equiv \neg p \vee q]$

$\equiv \neg(\neg p) \vee (\neg p \vee q)$ $[p \to q \equiv \neg p \vee q]$

$\equiv p \vee (\neg p \vee q)$ (Double negation)

$\equiv (p \vee \neg p) \vee q$ (Associative law)

$\equiv T \vee q$ (Negation law)

$\equiv T$ (Domination law)

2) $[(p \wedge \neg(\neg p \vee q)) \vee (p \wedge q)] \to p$ is tautology

$\equiv [(p \wedge (p \wedge \neg q)) \vee (p \wedge q)] \to p$ (De Morgan's law & Double negation law)

$\equiv [((p \wedge p) \wedge \neg q) \vee (p \wedge q)] \to p$ (associative law)

$\equiv [(p \wedge \neg q) \vee (p \wedge q)] \to p$ (Idempotent law)

~~$\equiv (p \vee (p \wedge q)) \wedge (\neg q \vee (p \wedge q))] \to p$~~

~~(Distributive law)~~

~~$\equiv (p \wedge q) \wedge \neg q \vee (p \wedge q)] \to p$~~

(Absorptive law)

~~$\equiv [p \wedge (q \wedge \neg q) \vee (p \wedge q)] \to p$~~

(Associative law)

$\equiv [p \wedge [\neg q \vee q]] \to p$ (Distributive law)

$\equiv (p \wedge T) \to p$ (Negation law)

$\equiv p \to p$ (Domination law)

$\equiv \neg p \vee p$ $[p \to q \equiv \neg p \vee q]$

$\equiv T$ (Negation law)

3) $\neg p \leftrightarrow q \equiv p \leftrightarrow \neg q$

LHS

$\equiv \neg p \leftrightarrow q \equiv (\neg(\neg p) \vee q) \wedge (\neg p \vee \neg q)$

$[p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)]$

$\equiv (p \vee q) \wedge (\neg p \vee \neg q)$ (Double negation law)

$\equiv (q \vee p) \wedge (\neg q \vee \neg p)$ (commutative law)

$\equiv (\neg(\neg q) \vee p) \wedge (\neg q \vee \neg p)$ (Double negation law)

$\equiv (\neg q \to p) \wedge (p \to \neg q)$ $[\neg p \vee q \equiv p \to q]$

$\equiv \neg q \leftrightarrow p$ $[(p \to q) \wedge (q \to p) \equiv p \leftrightarrow q]$

$\equiv p \leftrightarrow \neg q \equiv$ RHS

4) $p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$

LHS $\equiv (p \to q) \wedge (q \to p)$ [Biconditional definition]

$\equiv (\neg p \vee q) \wedge (\neg q \vee p)$ $[p \to q \equiv \neg p \vee q]$

$\equiv (p \vee \neg q) \wedge (q \vee \neg p)$ [Commutative law]

$\equiv (\neg p \to \neg q) \wedge (\neg q \to \neg p)$ $[p \to q \equiv \neg p \vee q]$

$\equiv \neg p \leftrightarrow \neg q$ [Biconditional definition]

5) $\neg(p \oplus q) \equiv p \leftrightarrow q$

$p \oplus q = \neg((p \wedge q) \vee (\neg p \wedge \neg q))$

LHS $\equiv \neg(\neg((p \wedge q) \vee (\neg p \wedge \neg q)))$ (Xor sum definition)

$\equiv (p \wedge q) \vee (\neg p \wedge \neg q)$ (Double negation law)

$\equiv \neg(\neg p \vee \neg q) \vee \neg(p \vee q)$ (De morgan law)

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$\equiv \neg((\neg p \vee \neg q) \wedge (p \vee q))$ (De morgan's law)

$\equiv \neg((p \rightarrow \neg q) \wedge (\neg q \rightarrow p))$

$[p \rightarrow q \equiv \neg p \vee q]$

$\equiv \neg(p \leftrightarrow \neg q)$ [Bidirectional definition]

RHS $\equiv$

$p \leftrightarrow q \equiv (\neg p \vee q) \wedge (p \vee \neg q)$

[Bidirectional definition]

$\equiv$

$(\neg p \wedge p) \vee (q \wedge p) \vee$
$(q \wedge \neg q) \vee (\neg p \wedge \neg q)$

(Distributive law)

$\equiv (0) \vee (q \wedge p) \vee (0) \vee (\neg p \wedge \neg q)$ (Negation law)

$\equiv \neg(\neg(q \wedge p) \wedge \neg(\neg p \wedge \neg q))$

$\equiv \neg((\neg p \vee \neg q) \wedge (p \vee q))$

$\equiv \neg(p \leftrightarrow \neg q)$ ✓

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(6) $\neg r \wedge (q \rightarrow \neg(p \wedge \neg r))$

$\equiv (\neg p \wedge \neg r) \vee ((p \wedge \neg q) \wedge \neg r)$

$\neg r \wedge (q \rightarrow \neg(p \wedge \neg r))$

LHS $\equiv \neg r \wedge (\neg q \vee \neg(p \wedge \neg r))$

$(p \rightarrow q \equiv \neg p \vee q)$

$\equiv \neg r \wedge (\neg q \vee (\neg p \vee r))$ (De morgan's law)

$\equiv \neg r \wedge (\neg p \vee (\neg q \vee r))$ (commutative law)

$\equiv (\neg r \wedge \neg p) \vee (\neg r \wedge (\neg q \vee r))$ (Distributive law)

$\equiv (\neg p \wedge \neg r) \vee ((\neg r \wedge \neg q) \vee (\neg r \wedge r))$ (Distributive law)

$\equiv (\neg p \wedge \neg r) \vee ((\neg r \wedge \neg q) \vee 0)$ (Negation law)

$\equiv (\neg p \wedge \neg r) \vee (\neg r \wedge \neg q)$

25-6-18

Determine whether the following system specification is consistent (or) inconsistent. (a)

1) "The diagnostic message is stored in the buffer (or) it is retransmitted".
2) "The diagnostic message is not stored in the buffer".
3) "If the diagnostic message is stored in the buffered, then it is retransmitted".

p: The diagnostic message is stored in the buffer

q: The diagnostic message is ~~stored~~ retransmitted.

1) $p \vee q$ p = false q = true ⇒ ① is true
2) $\neg p$ Assume ② is true, ⇒ p = false
3) $p \rightarrow q$ p = false q = true ⇒ ③ is true
4) "The system message is not retransmitted".
4) $\neg q$

(b)

1) "Nobody is ever wrong".
2) "George is wrong"
3) "$2+2=4$"

The given system is inconsistent as ① is false if ② is true and vice versa.

Predicates and Quantifiers

Predicate: Property that is affirmed (or) denied / having (or) not having / hold or does not hold.

Part of a sentence that attributes a property to the subject variable (or argument)

x is greater than 7.
y is not greater than 7

Subject: x and y

Property: Greater (or) not greater

$P(x) \rightarrow$ Propositional function
Variable x having property P.

① Let $P(x)$ denote the statement, "$x+2$ is a odd integer", then find the truth value of $P(2)$, $P(3)$ and $\neg P(x)$.

$P(2) = F$
$P(3) = T$
$\neg P(x) =$ "$x+2$ is not a odd integer".

② Let $Q(x,y)$ denote the statement, "$y+2$, $x+y$, $x-2y$ are even integers", find the truth values of $Q(-2,4)$, $Q(3,-1)$, $\neg Q(5,7)$

$Q(-2,4) = T$
$Q(3,-1) = F$
$\neg Q(5,7) = T$

Precondition and post condition

① void write sqrt (double x)

Pre condition : $x \geq 0$
Post condition : Square root of x

② temp := x
x := y
y := temp

Pre condition : $x=a; y=b$
Post condition : $x=b; y=a$

Quantifiers $\rightarrow$ Symbols used in the process of quantification
$\downarrow$
Method of creating a proposition from $p(x)$

(i) Universal Quantifier : $\forall x p(x)$
(ii) Existential Quantifier : $\exists x p(x)$

$\hookrightarrow$ For all x such that $p(x) \equiv p(x_1) \wedge p(x_2) \wedge \ldots \wedge p(x_n)$
$\hookrightarrow$ There exists atleast one x such that $p(x) \equiv p(x_1) \vee p(x_2) \vee \ldots \vee p(x_n)$

Negation of quantifiers :
(i) $\neg [\forall x p(x)] \equiv \exists x \neg p(x)$
(ii) $\neg [\exists x p(x)] \equiv \forall x \neg p(x)$

Prove that ~~negation of~~
$\neg [\exists x p(x)] \equiv \forall x \neg p(x)$

All students are having laptop
x : students.
P : Possessing laptop
$p(x)$: Student having laptop
Given : $\forall x p(x)$

LHS:
$\exists x p(x) \equiv$ There exists atleast one student having laptop.
$\neg [\exists x p(x)] \equiv$ All students do not have laptop.

RHS:
$\neg p(x) \equiv$ A Student not having laptop
$\forall x \neg p(x) \equiv$ All students are not having laptop.

Express each of the following statements using quantifiers (i) Form the negation of the statement so that no negation is in the left (ii) Express the negation in english sentence.

1) All dogs have fleas.
2) There is a rabbit that knows calculus.
3) That Koala can climb.
4) No monkey can speak english.
5) There is a pig that can swim and catch fish.

1) x : dog
p : having fleas
$p(x)$: dogs having fleas
$\forall x p(x)$ (i) $\exists x \neg p(x)$ (ii) There exists atleast one dog that doesn't have fleas.

2) x : rabbit.
p : Knowing calculus
$p(x)$: A rabbit knowing calculus
$\exists x p(x)$ (i) $\forall x \neg p(x)$ (ii) All rabbits do not know calculus.

3)

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Rules of inference

A conclusion reached on the basis of evidence and reasoning.

Theorem: statement that has to be proved/has proof.

Proof: set of statement to form an argument in order to prove the theorem.

Validity of the statement:

Rules of inference: To check the validity of statement/argument.

Theorem: A statement that has to be proved (or) that can be shown true.

Proof: A sequence of proposition (or) statement that form an argument for showing that the theorem is true.

Argument: sequence of statements/propositions.

Premises/hypothesis: All, except the final proposition of the argument.

The final proposition is called conclusion/consequence. An argument is said to be valid if the truth of all its premises imply that the conclusion is true.

Demonstrate the validity of the arguments.

1. $p \to r$
   $\neg p \to q$
   $q \to s$
   $\therefore \neg r \to s$

Sol:

Steps	Reasons
1) $p \to r$	Premises
2) $\neg p \to q$	Premises
3) $q \to s$	Premises
4) $\neg p \to s$	2,3 chain rule
5) $\neg r \to \neg p$	1, contrapositive
6) $\neg s \to s$	5,4 chain rule

2) $p \to q$
   $q \to (r \wedge s)$
   $\neg r \vee (\neg t \vee u)$
   $p \wedge t$
   $\therefore u$

Steps	Reasons
~~1) $p \to q$~~	~~Premises~~
~~2) $q \to (r \wedge s)$~~	~~Premises~~
~~3) $p \to (r \wedge s)$~~	~~chain rule~~
~~4) $\neg(r \wedge s) \to \neg p$~~	~~Contrapositive~~
~~5) $p \wedge t$~~	~~premises~~
~~6)~~	

Hypothetical syllogism - Chain rule

Steps	Reasons
1) $p \to q$	Premises
2) $q \to (r \wedge s)$	Premises
3) $p \to (r \wedge s)$	1,2 chainrule
4) $\neg p \vee (r \wedge s)$	conditional definition
5) $p \wedge t$	premises
6) $(p \wedge t) \vee (\neg p \vee (r \wedge s))$	$T \vee T \equiv T$
7) $(p \vee \neg p \vee (r \wedge s)) \wedge (t \vee \neg p \vee (r \wedge s))$	Distributive law
8) $(1 \vee (r \wedge s)) \wedge (t \vee \neg p \vee (r \wedge s))$	Negation law
$(r \wedge s) \wedge (t \vee \neg p \vee (r \wedge s))$	

3) $\neg p \leftrightarrow q$
   $q \to r$
   $\neg r$
   $\therefore p$

Steps	Reasons
1) $\neg p \leftrightarrow q$	Premises
2) $(\neg p \to q) \wedge (q \to \neg p)$	1, biconditional decomposition
3) $\neg p \to q$	(2), simplification
4) $q \to r$	Premise
5) $\neg p \to r$	3,4 chain rule
6) $\neg r$	premise
~~7) $p \vee r$~~	~~5, conditional definition~~
~~8) $(p \vee r) \wedge (\neg r)$~~	~~$T \vee T \equiv T$~~
~~9) $p \vee (r \vee \neg r)$~~	
7) $\neg r \to p$	(5), contrapositive
	Premises
8) p	(7)(8) Modus ponens

(2) Steps	Reasons
(1) $p \wedge t$	Premise
(2) p	(1), simplification
(3) $p \to q$	Premise
(4) q	(2),(3) Modus ponens
(5) $q \to (r \wedge s)$	Premise
(6) $r \wedge s$	(4),(5) Modus ponens

⑦ r ⑥, simplification

(OR) 27-6-18

Steps	Reason
① $r \to t$	Premise
② $\neg t$	Premise
③ $\neg r$	1,2 Modus tollens
④ $\neg r \vee \neg s$	3, Addition
⑤ $\neg(r \wedge s)$	4, De Morgan's
⑥ $(\neg p \vee \neg q) \to r \wedge s$	Premise
⑦ $\neg(\neg p \vee \neg q)$	5,6 Modus tollens
⑧ $p \wedge q$	~~7, Simplification~~ De Morgan's
⑨ p	8, Simplification

1) If the band could not play rock music or the refreshments were not delivered on time, then the new year's party have been cancelled and george would have been angry. If the party were cancelled, then refunds would have had to be made. No refunds were made. Therefore, the band could play rock music.

p: Band could play rock music

q: refrereshments are ~~made~~ delivered on time

r: New year party is cancelled

s: george is angry

t: refunds were made.

$(\neg p \vee \neg q) \to (r \wedge s)$

$r \to t$

$\neg t$

$\therefore p$

Steps	Premises.
① $\neg t$	Premise
② $r \to t$	Premise
③ $\neg t \to \neg r$	Contrapositive
④ $\neg r$	①③ Modus ponens
⑤ $(\neg p \vee \neg q) \to (r \wedge s)$	Premise
⑥ $(\neg p \vee \neg q) \to F$	$r \equiv F \Rightarrow r \wedge s \equiv F$ From ④
⑦ $T \to \neg(\neg p \vee \neg q)$	contrapositive
⑧ $\neg(\neg p \vee \neg q)$	⑦ Simplification
⑨ $p \wedge q$	De morgan's law
⑩ p	simplification

(2)

p: It rains

q: It is foggy

r: Life saving demonstration will go on.

s: Sailing race will be held.

t: Trophy was awarded

$(\neg p \vee \neg q) \to (s \wedge r)$

$s \to t$

$\neg t$

$\therefore p$

Steps	Reason
① $s \to t$	Premise
② $\neg t$	Premise
③ $\neg s$	①,② Modus tollens
④ $\neg s \vee \neg r$	Addition rule
⑤ $\neg(s \wedge r)$	De Morgan's rule
⑥ $(\neg p \vee \neg q) \to (s \wedge r)$	Premise
⑦ $\neg(\neg p \vee \neg q)$	⑤,⑥ Modus tollens
⑧ $\neg(\neg(p \wedge q))$	De Morgan's law
⑨ $p \wedge q$	Double negation
⑩ p	⑨ Simplification

Rules of Inference for Quantified statement

$\dfrac{\forall x\, p(x)}{\therefore p(c)}$ universal instantiation

$\dfrac{p(c)}{\therefore \forall x\, p(x)}$ universal generalisation

$\boxed{\begin{array}{l} \exists x\, p(x) \\ \hline \therefore p(C) \end{array}}$ Existential Instantiation

$\boxed{\begin{array}{l} p(c) \\ \hline \therefore \exists x\, p(x) \end{array}}$ Existential generalisation

Domain - Universe of discourse

$$\begin{array}{l} \forall x\,[p(x) \to q(x)] \\ \forall x\,[q(x) \to r(x)] \\ \hline \therefore \forall x\,[p(x) \to r(x)] \end{array}$$

①	$\forall x\,[p(x) \to q(x)]$	premise
②	$p(c) \to q(c)$	universal instantiation
③	$\forall x\,[q(x) \to r(x)]$	premise
④	$q(c) \to r(c)$	universal instantiation
⑤	$p(c) \to r(c)$	2,4 Chain rule
⑥	$\forall x\,[p(x) \to r(x)]$	Universal generalisation

② $$\begin{array}{l} \forall x\,[p(x) \vee q(x)] \\ \forall x\,[(\neg p(x) \wedge q(x)) \to r(x)] \\ \hline \therefore \forall x\,[\neg r(x) \to p(x)] \end{array}$$

Steps		Reason
①	$\forall x\,[p(x) \vee q(x)]$	Premise
②	$p(c) \vee q(c)$	universal instantiation
③	$\forall x\,[(\neg p(x) \wedge q(x)) \to r(x)]$	Premise
④	$(\neg p(c) \wedge q(c)) \to r(c)$	universal instantiation
⑤	$\neg r(c) \to \neg(\neg p(c) \wedge q(c))$	Contrapositive
⑥	$\neg r(c) \to (p(c) \vee \neg q(c))$	De Morgan's
⑦	$\neg r(c) \to (p(c) \vee \neg q(c))$	
⑧	$\neg r(c) \to (p(c) \vee \neg q(c)) \vee (p(c) \vee q(c))$	2,2 Addition Rule
⑨	$r(c) \vee (p(c) \vee \neg q(c))$	

6-th hour

Consider the universe of all triangles in the plane in triangle abc there is no pair of angles.

3) universal discourse: {All triangles in the plane}

$x \in$ "

Let $\Delta abc = c$

$p(x)$: x is having pair of angles of equal measure.

$q(x)$: x is having two sides of equal length.

$r(x)$: x is isoceles.

$$\begin{array}{l} \neg p(c) \\ \forall x\,[q(x) \to r(x)] \\ \forall x\,[r(x) \to p(x)] \\ \hline \therefore \neg q(c) \end{array}$$

Steps		Reasons
①	$\forall x\,[q(x) \to r(x)]$	Premisee
②	$q(c) \to r(c)$	universal instantiation
③	$\forall x\,[r(x) \to p(x)]$	Premisee
④	$r(c) \to p(c)$	universal instantiation
⑤	$q(c) \to p(c)$	Hypothetical 2,4 syllogism
~~⑥~~	~~$\neg p(c) \to \neg q(c)$~~	~~Contrapositive~~
⑥	$\neg p(c)$	Premise
⑦	$\neg q(c)$	5,6 Modus tollens.

④ Universe of discourse: {All students in a particular college}

$x \in$ { " }

Let William be 'e'.

$p(x)$: x is a junior

$q(x)$: x is a senior

$r(x)$: x is enrolled in phy. ed class

$$\begin{array}{l} \forall x\, \neg((p(x) \vee q(x)) \wedge r(x)) \\ r(e) \\ \hline \neg q(e) \end{array}$$

steps

① $\forall x \neg((p(x) \vee q(x)) \wedge r(x))$ premise

② $\neg((p(c) \vee q(c)) \wedge r(c))$ Universal instantiation

③ $\neg(p(c) \vee q(c)) \vee \neg r(c)$ De morgan's law

④ $r(c)$ premise

⑤ $\neg(p(c) \vee q(c)) \vee (\neg r(c) \vee r(c))$ Addition rule

⑥ $\neg(p(c) \vee q(c))$ negation law

⑦ $\neg p(c) \wedge \neg q(c)$ De morgan's law

⑧ $\neg q(c)$ ⑦ simplification

Tom is a math major, but not computer science major.

c : Tom

$p(x)$: x is a math major

$q(c)$: x is a computer science major

$p(c) \wedge \neg q(c)$

p: John is healthy

q : John is wealthy

r : John is wise

$p \wedge q \wedge \neg r$

$\neg p \wedge q \wedge r$

$\neg(p \vee q \vee r)$

(e) p : ride the rollercoaster

q : under 4 feet tall

r : older than 16 years

$(\neg q \vee r) \rightarrow \neg p$

(f) (i) $(r \wedge \neg q)$

(ii) $(p \wedge q \wedge r)$

(iii) $p \rightarrow r$

(iv) $p \wedge \neg q \wedge r$

(v) $(p \wedge q) \rightarrow r$

(vi) $r \leftrightarrow (p \vee q)$

3)(i) If we go fishing, then we are on a vacation - converse

If we do not go fishing, then we are not on a vacation - contrapositive

~~If we do not go fishing, then we are not on a vacation~~

~~con~~

If we are not on a vacation then we do not go for vacation - inverse

(ii) $p \rightarrow q$

$\neg p \rightarrow \neg q$

$q \rightarrow p$

$\neg q \rightarrow \neg p$

(iii) $\neg p \rightarrow q$

$p \rightarrow \neg q$

$q \rightarrow \neg p$

$\neg q \rightarrow p$

(iv) $q \rightarrow \neg p$

$\neg q \rightarrow p$

$\neg p \rightarrow q$

$p \rightarrow \neg q$

(v) $p \rightarrow \neg q$

$\neg p \rightarrow q$

$\neg q \rightarrow p$

$q \rightarrow \neg p$

Practice Problems (H2)

1) Tom is a computer science major, but not a math major.

2) (a) $(p \wedge q) \wedge \neg r$ (i)

$\neg p \wedge (q \wedge r)$ (ii)

$\neg(p \vee q \vee r)$ (iii)

(b) $(r \wedge s) \rightarrow \neg(p \vee q)$

(c) $(p \wedge q) \vee (r \wedge s)$

(d) $(q \wedge \neg r) \rightarrow p$

4) i) $\forall x\, p(x)$

x → dog p → have fleas

p(x) - dogs have fleas.

ii) $\exists x\, p(x)$

x → rabbit, p → knows calculus.

p(x) → rabbit knows calculation

iii) $\forall_x p(x)$

x → Koala, p → Can Climb

p(x) → Koala can climb.

iv) $\exists x\, p(x)$

x → everyone, p → has an Internet connection

p(x) → Everyone has an Internet Connection.

v) $\forall x \neg p(x)$

x → monkey, p → speak french.

p(x) → monkey can speak french.

vi) $\exists x [p(x) \wedge q(x)]$ $\forall x[$

x → pig, p → can swim, q → catch fish.

p(x) → pig can swim, q(x) → pig catch fish.

vii) $\exists x . p(x)$ $\neg(\forall x p(x))$ $\forall x \neg p(x)$

x → students, p → not asking doubts

p(x) → students not asking doubts.

viii) $\exists x\, p(x)$ $\neg(\exists x p(x))$ $\forall x \neg p(x)$

x → Students, p → asking doubts

p(x) → students are asking doubts

3. (viii) p: Positive integer is a prime

q: It has divisor other than 1 and itself.

$p \leftrightarrow \neg q$

$q \leftrightarrow \neg p$

p	q	$p \leftrightarrow q$	$\neg q \leftrightarrow \neg p$
0	0	1	1
0	1	0	0
1	0	0	0
1	1	1	1

$\boxed{3\text{-}7\text{-}18}$

Methods of Proof.

* To solve Mathematical theorems
* Applications in C.S

↓

→ To verify comp. prog. are correct.
→ Establishing the O.S is secure.
→ System specification are consistent.
→ Making inference in AI.

Types of proof

Direct proof

Inductive proof
(a) Proof by contraposition
(b) Proof by contradiction

(i) Direct proof: $p \to q$ is true, assume p is true, then prove q is true.

(ii) Proof by contraposition:

$p \to q \equiv \neg q \to \neg p$

Assume $\neg q$ is true, then prove $\neg p$ is true (or) Assume q is false, then prove p is false.

(iii) Proof by contradication: Assume p is true and q is false, then arrive at a contradiction for q

(iv) Vacous proof: $p \to q$ is true based on the fact that p is false.

(v) Trivial proof: $p \to q$ is true based on the fact that q is true.

Proof by equivalence (or) Equivalent statement.

P_1, P_2, P_3

$P_1 \to P_2$

$P_2 \to P_3$

$P_3 \to P_1$

~~Direct proof:~~

(i) By using direct method

(ii) indirect Method

prove that, If 'm' is an even integer, then 'm+7' is an odd integer.

p: m is an even integer.

q: 'm+7' is an odd integer.

(i) Direct method: Let us assume P is true.

⇒ m is an even integer.

~~$\Rightarrow m+7$ is an odd inte~~

$\Rightarrow m = 2k \; (k \in I)$

To prove: $m+7$ is an odd integer.

Consider, $m+7 = 2k+7$

$= 2k+6+1$

$= 2(k+3)+1$

Since $2(k+3)$ is even, if 1 is added to it, it becomes an odd integer.

$\therefore m+7$ is an odd integer.

(ii) Proof by contraposition:

Assume $\neg q$ is true $\Rightarrow$ $m+7$ is an even integer

To prove: $\neg p$ is true $\Rightarrow$ m is an odd integer.

Proof:

$m+7 = 2k, \; k \in I$

$\Rightarrow m = 2k-7$

$\Rightarrow m = 2(k-4)+1$

Since, $2(k-4)$ is even, if 1 is added to an even integer, it becomes odd integer.

$\therefore m$ is an odd integer

$\Rightarrow \neg p$ is true.

(iii) Proof by contradiction:

Assume p is true and q is false $\Rightarrow$ m is an even integer and $m+7$ is also an even integer.

$m = 2k, \; k \in I \rightarrow ①$

$m+7 = 2l, \; l \in I \rightarrow ②$

Substituting ① in ② gives,

$2k+7 = 2l$

$\Rightarrow 2(l-k) = 7$

Since, l and k are integers and $2(l-k)$ is even, it can not be equated to odd integer 7. Hence, there does not exist l and k such ...

For all integer k, l, if k and l are both odd, then their product kl is also odd.

p : k and l are odd.

q : kl is odd.

Assume p is true

$\Rightarrow k$ and l are odd.

$\Rightarrow k = 2m+1$, $l = 2n+1$, where $m, n \in I$

$\Rightarrow k \cdot l = (2m+1)(2n+1)$

$= 4mn + 2(m+n) + 1$

$= 2(2mn+m+n)+1$

Since $2(2mn+m+n)$ is even, adding 1 to it makes it an odd integer.

$\therefore$ product of k and l is odd $\Rightarrow q$ is true.

For any two integers m, n

$m^2 = n^2$ iff $m = \pm n$

$m^2 = n^2$

$\Leftrightarrow m^2 - n^2 = 0$

$\Leftrightarrow (m+n)(m-n) = 0$

$\Leftrightarrow m = n, \; m = -n$

Prove that $\sqrt{2}$ is irrational.

Let us assume that $\sqrt{2}$ is rational.

Then, $\sqrt{2} = \frac{p}{q}$, $q \neq 0$, $p, q \in I$, $\gcd(p,q) = 1$

$\Rightarrow 2 = \frac{p^2}{q^2} \rightarrow ①$

$\Rightarrow p^2 = 2q^2 \Rightarrow 2 \mid p^2$

$\Rightarrow 2 \mid p \Rightarrow p = 2a \rightarrow ②$, $a \in I$

Substitute ② in ①,

$4a^2 = 2q^2$

$2a^2 = q^2$

$\Rightarrow 2 \mid q^2$

iii) $\forall_x p(x)$

$x \to$ Koala, $p \to$ Can climb

$p(x) \to$ Koala can climb.

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$p(x) \to$ students not asking doubts.

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$p(x) \to$ students are asking doubts

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$\boxed{3-7-18}$

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$\Rightarrow$ m is an even integer.

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~~$\Rightarrow m+7$ is an odd integer~~

$\Rightarrow m = 2k \ (k \in I)$

<u>To prove:</u> $m+7$ is an odd integer.

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$\therefore m+7$ is an odd integer.

(ii) Proof by contraposition:

Assume $\neg q$ is true $\Rightarrow$ $m+7$ is an even integer

To prove: $\neg p$ is true $\Rightarrow$ m is an odd integer.

Proof:

$m+7 = 2k, \ k \in I$

$\Rightarrow m = 2k-7$

$\Rightarrow m = 2(k-4)+1$

Since, $2(k-4)$ is even, if 1 is added to an even integer, it becomes odd integer.

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Since, l and k are integers and $2(l-k)$ is even, it can not be equated to odd integer 7. Hence, there does not exist l and k such that p is true and q is false.

Hence, q is true when p is true.

For all integer k, l, if k and l are both odd, then their product kl is also odd.

p : k and l are odd.

q : kl is odd.

Assume p is true

$\Rightarrow k$ and l are odd.

$\Rightarrow k = 2m+1$ & $l = 2n+1$, where $m, n \in I$

$\Rightarrow k \cdot l = (2m+1)(2n+1)$

$= 4mn + 2(m+n) + 1$

$= 2(2mn+m+n)+1$

Since $2(2mn+m+n)$ is even, adding 1 to it makes it an odd integer.

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$m^2 = n^2$ iff $m = \pm n$

$m^2 = n^2$

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Let us assume that $\sqrt{2}$ is rational.

Then, $\sqrt{2} = \frac{p}{q}, \ q \neq 0, \ p, q \in I$, $\gcd(p,q) = 1$

$\Rightarrow 2 = \frac{p^2}{q^2} \rightarrow ①$

$\Rightarrow p^2 = 2q^2 \Rightarrow 2 | p^2$

$\Rightarrow 2 | p \Rightarrow p = 2a \rightarrow ②$, $a \in I$

Substitute ② in ①,

$4a^2 = 2q^2$

$2a^2 = q^2$

$\Rightarrow 2 | q^2$

$\Rightarrow 2 | q \Rightarrow q = 2b, \ b \in I$

$\gcd(p,q) = \gcd(2a, 2b) \geq 2 \neq 1$

which contradicts $\gcd(p,q) = 1$

$\boxed{4\text{-}7\text{-}18}$

5) S.T the statements are equivalent.

P_1 : n is an even integer

P_2 : $n-1$ is an odd integer

P_3 : n^2 is an even integer

(i) To prove: $P_1 \rightarrow P_2$

Let us assume P_1 is true,

$\Rightarrow n$ is an even integer

$\Rightarrow n = 2k, \ k \in I$

$\Rightarrow n-1 = 2k - 1 + 2 - 2$

~~$\Rightarrow n-1 = 2(k+1) - 3$~~

Since, $2k$ is even, subtracting 1 from it makes it odd.

$\therefore n-1$ is an odd integer

$\Rightarrow P_2$ is true.

$\Rightarrow P_1 \rightarrow P_2$ is true.

(ii) To prove: $P_2 \rightarrow P_3$

Let us assume P_2 is true,

$\Rightarrow n-1$ is an odd integer

$\Rightarrow n-1 = 2k+1, \ k \in I$

$\Rightarrow n = 2k+2$

$= 2(k+1)$

Consider $n^2 = (2k+2)^2$

$= 4k^2 + 4 + 8k$

$= 2(2k^2 + 2 + 4k)$

Since, n^2 is in the form of '$2l$', where $l = (2k^2+2+4k)$, n^2 is an even integer.

$\Rightarrow P_3$ is true.

$\Rightarrow P_2 \rightarrow P_3$.

(iii) To prove: $P_3 \rightarrow P_1$

Contrapositive: $\neg P_1 \rightarrow \neg P_3$

Assume $\neg P_1$

$\Rightarrow n$ is an odd integer

$\Rightarrow n = 2k+1, \ k \in I$

Show that atleast 4 of any 22 days must fall on the same day of the week.

Ans:

Atmost 3 of the 22 days fall on the same day of the week.

Since there are 7 days in a week, the maximum no. of days will be $7 \times 3 = 21$ days, contradicting to the fact that there are 22 days.

Hence, atleast 4 of any 22 days must fall on the same day of the week.

UNIT - 2

Induction

- Principles of mathematical induction (Incomplete induction)
- Principles of Strong induction (Complete induction)

To prove: $P(n)$: propositional fn.

$P(n)$ is true for all +ve integer 'n'

Basic step:

To prove: $P(c)$ is true, $c \in \mathbb{N}$

Inductive step:

Assume $P(k)$ is true, show that $P(k+1)$ is true, $k \in I$.

(i.e) $p(K) \rightarrow p(K+1),\ K \in I$

Strong induction:

$P(n)$: propositional fn.

To prove: $p(n)$ is true.

Basis step: To prove $p(c)$ is true. $c \in N$

Induction step:

Assume $p(1), p(2), \dots, p(K)$ is true.

Claim: $p(K+1)$ is true.

$[[p(1) \wedge p(2) \wedge \dots \wedge p(K)] \rightarrow p(K+1)],\ K \in I$

Show that

$P(n): 1^2+3^2+5^2+\dots+(2n-1)^2 = \frac{(n)(2n-1)(2n+1)}{3}$

Basis step:

claim: $p(1)$ is true.

$$P(1) \equiv 1^2 = \frac{(1)(2(1)-1)(2(1)+1)}{3}$$

$$= \frac{(1)(1)(3)}{3} = 1$$

$\therefore$ $p(1)$ is true.

Inductive step:

Assume $p(K)$ is true.

$$p(K) \equiv \frac{K(2K-1)(2K+1)}{3} \rightarrow ①$$

Claim: $p(K+1)$ is true.

$$P(K+1) \equiv 1^2+3^2+5^2+\dots+(2K-1)^2 + (2K+1)^2$$

$$\equiv p(K) + (2K+1)^2$$

$$\equiv \frac{K(2K-1)(2K+1)}{3} + (2K+1)^2 \quad \text{(From ①)}$$

$$\cancel{\equiv \frac{(2K^2-K)(2K+1)}{3} + 4K^2+}$$

$$\equiv (2K+1)\left(\frac{K(2K-1)}{3} + 2K+1\right)$$

$$\equiv (2K+1)\left(\frac{K(2K-1)+6K+3}{3}\right)$$

$$\equiv (2K+1)\left(\frac{2K^2-K+6K+3}{3}\right)$$

$$\equiv \frac{(2K+1)}{3}(2K^2+5K+3)$$

$$\equiv \frac{(2K+1)(K+1)(2K+3)}{3}$$

$\Rightarrow p(K+1)$ is true.

② $P(n) \equiv 1+2+3+\dots+n = \frac{n(n+1)}{2}$.

Basic step:

To prove: $p(1)$ is true.

(i.e) $p(1) \equiv 1$

Proof:

$$p(1) \equiv 1 = \frac{(1)(1+1)}{2}$$

$$\equiv 1 = \frac{(1)(2)}{2}$$

$$\equiv 1 = 1$$

$\therefore$ $p(1)$ is true.

Inductive step:

Assume $p(K)$ is true.

(i.e) $p(K) \equiv 1+2+3+\dots+K = \frac{K(K+1)}{2}$

To prove: $p(K+1)$ is true.

(i.e) $p(K+1) \equiv 1+2+3+\dots+K+K+1 \equiv \frac{(K+1)(K+2)}{2}$

Proof:

$$LHS = 1+2+3+\dots+K+K+1$$

$$= p(K) + K+1$$

$$= \frac{K(K+1)}{2} + (K+1) \quad (\because p(K) \text{ is true})$$

$$= (K+1)\left(\frac{K}{2}+1\right)$$

$$= \frac{(K+1)(K+2)}{2}$$

$$= RHS$$

$\therefore$ $p(K+1)$ is true.

$\Rightarrow p(K) \rightarrow p(K+1)$ is true.

③ $p(n) \equiv 1\cdot3+2\cdot4+\dots+n(n+2) = \frac{n(n+1)(2n+7)}{6}$

Basic step:

To prove: $p(1)$ is true.

(i.e) $p(1) \equiv 1$.

Proof:

$$p(1) \equiv 1\cdot3 = \frac{(1)(1+1)(2(1)+7)}{6}$$

$$3 = \frac{(1)(2)(9)}{6} = 3$$

$\therefore$ p(1) is true.

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Induction step:

Assume p(k) is true.

(i.e) $p(k) \equiv 1\cdot3+2\cdot4+\dots+(k)(k+2) = \frac{(k)(k+1)(2k+7)}{6}$

To prove:

p(k+1) is true.

(i.e) $p(k+1) \equiv 1\cdot3+2\cdot4+\dots+(k)(k+2)+(k+1)(k+3) = \frac{(k+1)(k+2)(2(k+1)+7)}{6}$

Proof:

$LHS = 1\cdot3+2\cdot4+\dots+(k)(k+2)+(k+1)(k+3)$

$= P(k) + (k+1)(k+3)$

$= \frac{(k)(k+1)(2k+7)}{6} + (k+1)(k+3)$

$= \frac{(k+1)}{6}\left((k)(2k+7)+6(k+3)\right)$

$= \frac{(k+1)}{6}(2k^2+7k+6k+18)$

$= \frac{(k+1)}{6}(2k^2+9k+4k+18)$

$= \frac{(k+1)}{6}(2k(k+2)+9(k+2))$

$= \frac{(k+1)(k+2)(2k+9)}{6}$

$= RHS$.

$\therefore$ p(k+1) is true.

$\Rightarrow p(k) \to p(k+1)$ is true

$\therefore$ p(n) is true $\forall n \in N$

④. S.T $\sum_{i=1}^{n} \frac{1}{i(i+1)} = \frac{n}{n+1}$

⑤ S.T $a_n \le 3^n \ \forall n \in N$ where $a_0=1$; $a_1=2$; $a_2=3$ and $a_n = a_{n-1}+a_{n-2}+a_{n-3}$

⑥ $P(n): 4n < n^2-7 \ \forall n \ge 6$

④ $n=1$ $\frac{1}{1(1+1)} = \frac{1}{1+1}$ $\frac{1}{2}=\frac{1}{2}$ ✓

$n=k$ $\sum_{i=1}^{k} \frac{1}{i(i+1)} = \frac{k}{k+1}$

$n=k+1$ $\sum_{i=1}^{k+1} \frac{1}{i(i+1)}$

$= \frac{k}{k+1} + \frac{1}{(k+1)(k+2)}$

$= \frac{k(k+2)+1}{(k+1)(k+2)}$

$= \frac{k^2+2k+1}{(k+1)(k+2)}$

$= \frac{(k+1)^2}{(k+1)(k+2)}$

$= \frac{k+1}{k+2}$ ✓

⑤ $a_n \le 3^n$

$a_0 = 1$ $3^0 = 1$

$a_0 \le 3^0$

$1 \le 1$ ✓

$n=k$ $a_k \le 3^k$

$n=k+1$ $a_{k+1} \le 3^{k+1}$

$a_{k+1} = a_k + a_{k-1} + a_{k-2}$

$\le 3^k + 3^{k-1} + 3^{k-2}$

$a_k + a_{k-1} + a_{k-2} \le 3^{k+1}$

$a_{k+1} - a_{k-1} - a_{k-2} \le a_k$

$a_{k+1} - a_{k-1} - a_{k-2} \le 3^k$

$3^k + 3^{k-1} + 3^{k-2} \le 3^{k+1}$

$3^k \le 3^{k+1}$ ✓

Strong induction

Show that any +ve int $n \ge 2$ is either a prime (or) product of primes.

Basis step: claim: P(2) is true

$P(2) = 2$

P(2) is true

Inductive step: Assume p(j) is true $2 \le j \le k$, $k \in N-\{1\}$

$\Rightarrow$ p(2), p(3), ..., p(k) is true.

Claim: $p(2) \wedge p(3) \wedge \dots \wedge p(k) \to p(k+1)$

case(i) If k+1 is prime, then p(k+1) is true.

case(ii) If k+1 is composite, then k+1 can be written as product of two pos. int as by p and q, where $2 \le p \le k$ and

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$2 \le q \le k$

by inductive hypothesis

$\Rightarrow$ p can be written as either prime (or) as a product of prime.

similarily, q can be written as either prime (or) as a product of prime.

$\Rightarrow$ pq can be the product of primes.

$\Rightarrow$ $p(k+1)$ is true.

Suppose we can reach first and second rungs of an infinite ladder, and we know that if we can reach a rung, then we can reach two rungs higher. Prove that we can reach every rung by using strong induction.

Sol:

$p(n)$: we can reach the nth rung of the ladder.

Basis step:

~~Assume~~ we have reached first and second rung $\Rightarrow$ $p(1)$, $p(2)$ are true.

Inductive step:

Assume $p(1), p(2), \ldots, p(k)$ are true.

Claim: $p(k+1)$ is true.

Since ~~we have reap~~ $p(k)$ is true

$\Rightarrow$ we have reached $k-1$th rung.

If $k-1$th rung is reached, then we can reach two rungs higher (i.e) $k-1+2 = k+1$ $\Rightarrow$ $p(k+1)$ is true.

Binary representation theorem:

~~$k-2) k+1 k$~~

Every +ve int n can be expressed as $n = c_r 2^r + c_{r-1} 2^{r-1} + \ldots + c_2 2^2 + c_1 2^1 + c_0 2^0$

Basis step:

$n=1$; $r=0$; $c_0=1$

$p(1) = c_0 2^0 = 1$

$\Rightarrow$ $p(1)$ is true.

$k-1 = \frac{k}{2}$

$2k-2 = k$

$k=2$

Inductive step:

Assume upto $p(k-1)$ is true from $p(1)$

claim: $p(k)$ is true.

Case 1: k is even

$\Rightarrow$ $\frac{k}{2}$ can be a positive integer less than k.

$$\frac{k}{2} = c_r 2^r + c_{r-1} 2^{r-1} + \ldots + c_1 2^1 + c_0 2^0$$

$$k = c_r 2^{r+1} + c_{r-1} 2^r + \ldots + c_1 2^2 + c_0 2^1$$

(ii) k is odd

$\Rightarrow$ $\frac{k-1}{2}$ is an +ve integer.

$$\frac{k-1}{2} = c_r 2^r + c_{r-1} 2^{r-1} + \ldots + c_2 2^2 + c_1 2^1 + c_0$$

$\boxed{10-7-18}$ Basis of counting

① Product rule: m and n $\Rightarrow$ $m \times n$

② Sum rule: m or n $\Rightarrow$ $m+n$

③ Pigeon-hole principle:

k: boxes

k+1: pigeons

Definitely one box will contain more than one pigeon.

N objects placed in k boxes.

There is atleast one box containing $\lceil N/k \rceil$ objects.

① ~~For the first person, he can be assigned 12 different offices. After~~

The procedure of assigning offices to these two employees consists of assigning an office to santhosh which can be done in 12 ways, then assigning office to patel different from office assigned to santhosh which can be done in 11 ways. Therefore, by product rule, there are $12 \times 11 = 132$ ways to assign office to the two employees.

The label of chairs with letters is 26 and label of chairs with numbers is 100. Totally there are $26 \times 100 = 2600$ ways to assign labels to chairs.

③ 24

$\underline{\phi}\ \underline{\phi}\ \underline{\phi}\ \underline{\phi}\ \underline{\phi}\ \underline{\phi}\ \underline{\phi}$

$2\ 2\ 2\ 2\ 2\ 2\ 2$

2^7

④ $2^7 = 128$

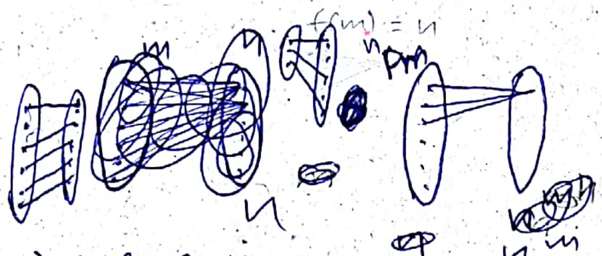
⑤ $26^3 \times 10^3$ (i) $\underline{26}\ \underline{26}\ \underline{26}\ \underline{10}\ \underline{10}\ \underline{10}$

(ii) $26 \times 25 \times 24 \times 10 \times 9 \times 8$

(iii) $26 \times 25 \times 24 \times 10^3$

(iv) $26 \times 26 \times 26 \times 10 \times 9 \times 8$

(v) $26^3 \times 10 \times 9 \times 8 + 26 \times 25 \times 24 \times 10^3$



8) Each element in the set may or may not be in the subset. Therefore, there are two possibilities for each elements. ∴ For each

How many diff 8 bit strings are there that begin and end with 1.

$\Rightarrow 2^6$

diff 8 bit strings are there that end with 0111.

$\Rightarrow 2^4$.

How many different 2 digit numbers can be made from digits 1,2,3,4,5,6,7,8,9,0

(i) rep allowed $= 9 \times 10$

(ii) rep not allowed $= 9 \times 9$

6 to 8   2 $26 + 10 = 36$

$\underline{10}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{36}\ \underline{36}\ \_\ \_\ \_$

$36^5 \times 10 + 36^6 \times 10 + 36^7 \times 10$

$36^5(10 + 360 + 36^2 \times 10)$

$(36^6 - 26^6)\ (36^8 - 26^8) + (36^7 - 26^7)$

draw heart/spade from ordinary deck of playing card

$^{52}C_{26}$

$52 \times \frac{26}{52} = 26$

11-7-18

13) Since there are 26 letters in english alphabet, therefore atleast

14) $\left\lceil \frac{102}{101} \right\rceil \geq 2$

102 students.

5

$\underline{5}\ \underline{4}$

$\underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \underline{1}$

$\frac{5!}{2!} = {}^5P_2$

16. Among 100 people, there are atleast $\left\lceil \frac{100}{12} \right\rceil \geq 9$ born in same month.

17. $N = 26$ $K = 5$

$\left\lceil \frac{N}{5} \right\rceil \geq 6$ $N = 26$

18. (i) $N = 27$

$\left\lceil \frac{27}{13} \right\rceil \geq 3$

$$\frac{{}^nP_r}{(r)!} = {}^nC_r$$

PRACTICE PROBLEMS

1) ${}^5P_3$ 2) ${}^{100}P_3$ 3) ${}^8P_3$

$5!$

4) ${}^7P_7$ 5) $6!$ 6) $7!$

7) ${}^4C_3$ 8) ${}^{52}C_{47}$,

9) ${}^{10}C_5$ 10) ${}^{30}C_6$ 11) ${}^nC_r$ 12) ${}^9C_3 \times {}^{11}C_4$

13) (i) ${}^{10}C_4$ (ii) ${}^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4$

(iii) ${}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10}$

(iv) ${}^{10}C_5 - {}^5C_5 = {}^{10}C_5$

16-7-18 Recursive definition

Basis step: $f(n) \rightarrow$ for smaller values

eg: $f(0), f(1)$

Recursive step:

$f(n) = f(n-1) + f(n-2)$

Find $f(1), f(2), f(3)$ if $f(n)$ is defined recursively by $f(0) = 3$, $f(n+1) = [f(n)]^2 - 2f(n) - 2$

$f(1) = f(0+1) = [f(0)]^2 - 2f(0) - 2$

$= 9 - 6 - 2 = 1$

$f(2) = f(1+1) = [1]^2 - 2(1) - 2$

$= 1 - 2 - 2 = -3$

$f(3) = f(2+1) = [f(2)]^2 - 2f(2) - 2$

$= 9 - 2(-3) - 2$

$= 9 + 6 - 2 = 13$

$f(n+1) = 3^{f(n)/3}$

$f(1) = 3^{3/3} = 3^1 = 3$

$f(2) = 3^{3/3} = 3^1 = 3$

$f(3) = 3^{3/3} = 3^1 = 3$

A recursive(or) inductive definition of a function consists of two steps

(i) Basis step : specify the value of function at 0(or) 1.

(ii) Recursive step: Give a rule for finding its value at an integer from its value at a smaller integer

Give a recursive formula or definition for computing n factorial for n being a non-negative integer.

Basis step:

$f(0) = 0! = 1$

Recursive step:

$f(n) = n * f(n-1)$

Write the recursive function for computing a^n, $n \in W$, $a \in R^+$

Basis step:

$f(0) = a^0 = 1$

When $n > 1$,

$f(1) = a^1 = a \times f(0) = a$

$f(2) = a^2 = a \times f(1) = a^2$

$f(3) = a^3 = a \times f(2) = a^3$

$\vdots$

$f(n) = a^n = a \times f(n-1)$ //.

Write the recursive formula for fibonacci series.

Basis step:

$f(0) = 0$

$f(1) = 1$

when $n = 2$,

$f(2) = 1 + 0 = f(1) + f(0)$

when $n = 3$;

$f(3) = 1 + 1 = f(2) + f(1)$

when $n = 4$,

$f(4) = 2 + 1 = f(3) + f(2)$

$\vdots$

$f(n) = f(n-1) + f(n-2)$

Validity-recursive formula :

(i) $f(0) = 0$

$f(n) = 2f(n-2)$, $n \geq 1$

(ii) $f(0) = 1$; $f(n) = f(n-1) - 1$, $n \geq 1$

(i) $f(1) = 2f(-1)$?

$f(2) = 2f(0) = 0$

$f(3) = 2f(1) = $? Not valid

(ii) $f(1) = f(0) - 1 = 0$

$f(2) = 0 - 1 = -1$

$f(3) = -1 - 1 = -2$ //.

Valid.