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B.Tech. Second Assessment – Oct/Nov. 2016

Third Semester

Computer Science and Engineering

15MAT201 Discrete Mathematics

Time: Two hours | Maximum: 50 Marks

Answer all questions

Part A **(5 x 2 = 10 Marks)**

1. Find the sum of the first six elements of the sequence defined by the recurrence relation: $a_n = 2a_{n-1} - 2a_{n-2} + n^3,\ a_0 = 0,\ a_1 = 3/2$

2. Let D_n denote the determinant of the matrix 'A' of order 'n'. Find a recurrence relation for D_n where the matrix is given by

$$A = \begin{bmatrix} b & b & 0 & 0 & \dots & \dots & 0 \\ b & b & b & 0 & \dots & \dots & 0 \\ 0 & b & b & b & 0 & \dots & 0 \\ \dots & \dots. & \dots & \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & \dots & 0 & 0 & b & b & b \\ 0 & 0 & 0 & \dots & 0 & b & b \end{bmatrix}$$

3. Find $f(n)$ when $n = 5^k$, where f satisfies the recurrence relation
$f(n) = 2f(n/5) + 6,\ f(1) = 1.$

4. If R is the relation defined on the set $A = \{1, 2, 3, 4\}$ such that
$R = \{(1,1), (1,3), (1,4), (2,2)(4,4)\}$, then which of the following is/are correct?
(a) R is reflexive (b) R is symmetric(c) R is anti-symmetric (d) R is transitive

5. If $R$and S are relations on the set $A = \{a, b, c, d\}$, find, M_{RoS} given that

$$M_R = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \text{and } M_S = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$$

Part A **(5 x 8 = 40 Marks)**

6. Solve the following recurrence relation using the method of generating functions:

$$3a_{n+2} = 11a_{n+1} - 10a_n + 2^n,\ a_0 = -3,\ a_1 = 5$$

7. Find the number of integers those are divisible by at least one of 4, 9 or 12 in the interval [2253, 5129] using the principle of inclusion-exclusion.

8. Let R be a relation on the set of integers defined as :

$R = \{(a,b) / a - b \text{ is a non-negative rational number}\}$ where a and b are integers.

Verify whether R defines a partial order for the set of integers. Is this partial order a total order? Justify.

9. Let A be a nonempty set and $\mathscr{P}(A)$ denotes the collection of all subsets of A. Let R_B denotes the relation defined on $\mathscr{P}(A)$ with respect to the given (fixed) subset B of A as:

$R_B = \{(X,Y) / X \subseteq A, Y \subseteq A \text{ and } B \cap X = B \cap Y\}$.

i) Verify whether R_B is an equivalence relation on $\mathscr{P}(A)$

ii) If $A = \{1, 2, 3, 4, 5\}$, $B = \{1, 2, 3\}$ and $X = \{1, 3, 5\}$, find $[X]$, the equivalence class of X.

10. Using Warshall's algorithm find the transitive closure of the relation:

$R = \{(a,a), (a,c), (a,d), (c,c)(d,d)\}$ defined on set $A = \{a, b, c, d, e\}$.

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