Roll No.: _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _ _

Amrita Vishwa Vidyapeetham

B.Tech. Second Assessment Examinations – September 2017

Third Semester

Computer Science and Engineering

15MAT201 Discrete Mathematics

[Time: Two hours Maximum: 50 Marks]

Answer all the questions

PART – A **(11 × 2 = 22)**

1. Let R_1={(1, 2), (2, 1), (2, 2), (2, 3), (3, 1)} and R_2 ={(1, 2), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}. Find the matrices that represent $R_1 \cup R_2$ and $R_2 \circ R_1$.
2. Determine whether the relation R on the set of all real number is symmetric and transitive, where R= { (x, y) / $x\,y = 0$}.
3. How many reflexive relations are there with a set of n elements? Justify your answer.
4. Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time. Write the initial conditions?
5. Find $f(n)$ when $n = 2^k$, where f satisfies the recurrence relation $f(n) = f(n/2) + 1$ with $f(1) = 1$.
6. Find the generating function for the finite sequence 5, 5, 5, 5, 5.
7. Prove that the relation $R = \{(x, y) / x^2 = y^2,\ \text{where } x, y \in \boldsymbol{Z}\}$ is an equivalence relation on a set of integers $\boldsymbol{Z}$.
8. Let R be the relation $\{(a, b) / a \text{ divides } b\}$ on the set of integers. What is the symmetric closure of R?
9. A survey of households in a country reveals that 96% have at least one television set, 98% have telephone service, and 95% have telephone service and at least one television set. What percentage of households in the country has neither telephone service not a television set?
10. Find the error in the proof of the following theorem.
    Theorem: Let R be a relation on a set A that is symmetric and transitive. Then R is reflexive.
    Proof: Let $a \in A$. Take an element $b \in A$ such that $(a, b) \in R$. Because R is symmetric, we also have $(b, a) \in R$. Now using the transitive property, we can conclude that $(a, a) \in R$ because $(a, b) \in R$ and $(b, a) \in R$.
11. Show that the relation R on a set A is symmetric if and only if $R = R^{-1}$, where R^{-1} is the inverse relation of R.

PART – B **(4 × 7 = 28)**

12. Solve the recurrence relation $S_n - 3S_{n-1} - 4\,S_{n-2} = n^2 + n$, $S_0=1$ and $S_1=3$.

13. Using the generating function, solve the recurrence relation $P_n - 7P_{n-1} + 12\,P_{n-2} = 0$, $P_0=3$ and $P_1=5$.

14. Write the Warshall's algorithm. Using the algorithm find the transitive closure of the relation {(1, 2), (2, 1), (2, 3), (3, 4), (4, 1)} on a set A = {1, 2, 3, 4}.

15. Prove that the relation R on a set A is transitive if and only if $R^n \subseteq \mathrm{R}$ for $n = 1,2,3,...$

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