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B.Tech. First Assessment – August 2016

Third Semester

Computer Science and Engineering

15MAT201 Discrete Mathematics

Time: Two hours Maximum: 50 Marks

Answer all questions

PART A $(\mathbf{10 \times 2 = 20\ Marks})$

1. Using truth table, show that $p \to q \Leftrightarrow \neg p \vee q$.

2. Write down the inverse and contrapositive of the statement "Whenever you get a speeding ticket, you are driving over 65 miles per hour."

3. Express the statement "No student in CSE class likes Ice – cream" using quantifiers. Negate thesamein simple English.

4. Find the truth values of the statements $\forall x\, p(x)$ and $\exists x\, \neg p(x)$ for the propositional function $p(x): x + 3 = 2x$, where x is a real number. Justify your answer.

5. Suppose that the domain of the propositional function $p(x)$ consists of $-5, -3, -1, 1, 3$ and 5. Then express the following statements using only negations, disjunctions and conjunctions$(i)\ \exists x\ [(x \geq 0) \wedge p(x)] (ii)\ \forall x\ [(x \neq 1) \to p(x)]$.

6. Write down a recursive algorithm for finding the value of $n!$, where$n$is a non - negative integer.

7. Let A and B be two non – empty sets with finite number of elements m and n respectively and $f: A \to B$be a bijection.Then show that $m = n$.

8. Let $f, g: \mathbb{Z} \to \mathbb{Z}$be twofunctions,where $\mathbb{Z}$ is the set of all integers, defined by $f(x) = 3x + 2$and $g(x) = 4x - 5$. Find $fog$and$gof$where $o$denotes the composition of two functions.Is$fog = gof$?

9. What is the minimum number of elementsthat must be selected from the set $\{1,2,3,4,5,6\}$ to guarantee that at least one pair of these numbers add up to 7 ?Explain.

10. In how many different ways can the letters of the word 'OPTICAL' be arranged so that the vowels always come together?

PART B $(\mathbf{6 \times 5 = 30\ Marks})$

11. Show that $\neg[p \vee (\neg p \wedge q)]$and $\neg p \wedge \neg q$ are logically equivalent by developing a series of logical equivalences with the specificationof corresponding laws.

12. Prove that$\sqrt{3}$ is irrational by the method of contradiction.

13. Using the following true statements and rules of inference, locate the treasure hidden in the estate. “If the house is next to a lake, then the treasure is not in the kitchen ”, “If the tree in front of yard is a mango tree, then the treasure is in the kitchen” , “The house is next to a lake”, “The tree in the front yard is a mango tree or the treasure is placed behind the mirror in the wall of the bed room”, and “If the tree in the back yard is an oak tree, then the treasure is in the drawing room”.

14. Show that $\sum_{i=1}^{n} \frac{1}{2^i} = \frac{2^n - 1}{2^n}$ by mathematical induction.

15. Let $f, g: \mathbb{R} \to \mathbb{R}$ be two functions, where $\mathbb{R}$ is the set of all real numbers. Let f be bijective. Show that if $g = 2f(x) + 3$, then g is also bijective.

16. Suppose 7 students are staying in a hall in a hostel and they are allotted 7 beds. Among them, Parvin does not want a bed next to Ankur because Ankur snores. Then, in how many ways can you allot the beds?

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