Chapter 5 – Seakeeping Models

The study of ship dynamics has traditionally been covered by two main theories: maneuvering and seakeeping.

- Maneuvering refers to the study of ship motion in the absence of wave excitation (calm water). The maneuvering equations of motion are derived in Chapter 6 under the assumption that the hydrodynamic potential coefficients and radiation-induced forces are frequency independent (constant).

- Seakeeping refers to the study of motion of marine craft on constant course and speed when there is wave excitation. This includes the trivial case of zero speed. In sea keeping analysis, a dissipative force known as fluid memory effects (Cummins 1962) is introduced due to frequency-dependent potential coefficients.

Cummins, W. E. (1962). The Impulse Response Function and Ship Motions. Techn. Report 1661. *David Taylor Model Basin. Hydrodynamics Laboratory*, USA.

Chapter Goals

- Understand seakeeping coordinates {s} and how they relate to BODY and NED
- Understand frequency-dependent hydrodynamic matrices and their application
- Application of Cummins equation to transform frequency-dependent matrices to the time domain

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\text{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

$$\boldsymbol{B}_{\text{total}}(\omega) = \boldsymbol{B}(\omega) + \boldsymbol{B}_V(\omega)$$

Computed using hydrodynamic seakeeping codes

- $\boldsymbol{A}(\omega)$ added mass matrix
- $\boldsymbol{B}(\omega)$ potential damping matrix
- $\boldsymbol{B}_V(\omega)$ viscous damping matrix
- $\boldsymbol{C}$ spring stiffness or hydrostatic matrix

Transformation from SEAKEEPING {s} to BODY axes {b}

$$\begin{array}{lr}
\text{Inertia forces:} & \boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}^*_{RB}\boldsymbol{\nu} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}^*_A\boldsymbol{\nu}_r \\
\text{Damping forces:} & +(\boldsymbol{D}_P + \boldsymbol{D}_V)\boldsymbol{\nu}_r + \boldsymbol{\mu}_r \\
\text{Restoring forces:} & +\boldsymbol{G}\boldsymbol{\eta} + \boldsymbol{g}_o \\
\text{Wind and wave forces:} & = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} \\
\text{Propulsion forces:} & +\boldsymbol{\tau}
\end{array}$$

Linear model used for control design and simulation

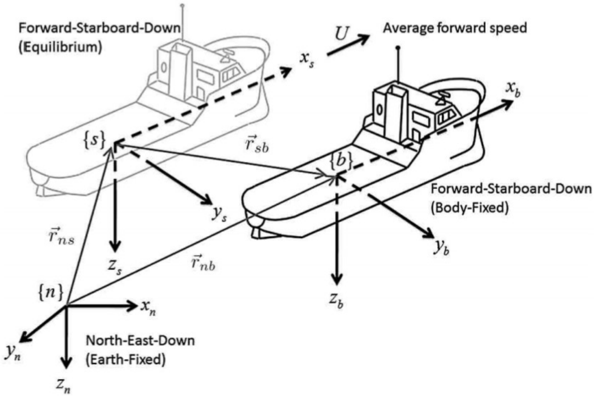
Chapter 5 - Seakeeping Models

Equations of Motion

Seakeeping theory is formulated in equilibrium (SEAKEEPING) axes {s} but it can be transformed to BODY axes {b} by including fluid memory effects represented by impulse response functions.

The transformation is done within a linear framework such that additional nonlinear damping must be added in the time-domain under the assumption of linear superposition.

Inertia forces:	$\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}^{*}\boldsymbol{\nu} + \boldsymbol{M}_{A}\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_{A}^{*}\boldsymbol{\nu}_r$
Damping forces:	$+(\boldsymbol{D}_P + \boldsymbol{D}_V)\boldsymbol{\nu}_r + \boldsymbol{\mu}_r$
Restoring forces:	$+\boldsymbol{G}\boldsymbol{\eta} + \boldsymbol{g}_o$
Wind and wave forces:	$= \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}}$
Propulsion forces:	$+\boldsymbol{\tau}$



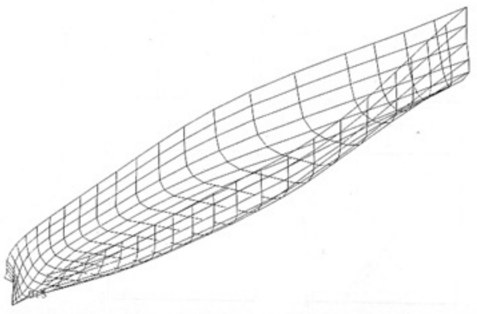
μ_r is an additional term representing the fluid memory effects.

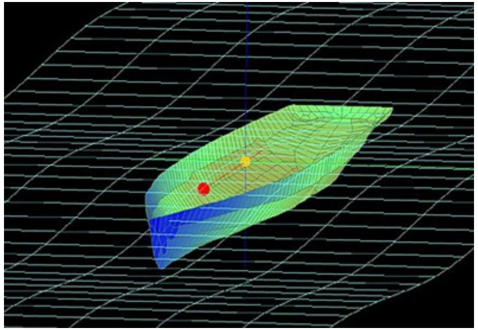
-In the absence of wave excitation, {s} coincides with {b}.
- Under the action of the waves, the hull is disturbed from its equilibrium and {s} oscillates, with respect to its equilibrium position.

5.1 Hydrodynamic Concepts and Potential Theory

Strip Theory (2-D Potential Theory)

For slender bodies, the motion of the fluid can be formulated as a 2-D problem. An accurate estimate of the hydrodynamic forces can be obtained by applying strip theory (*Newman 1977; Faltinsen 1990; Journee and Massie 2001*).

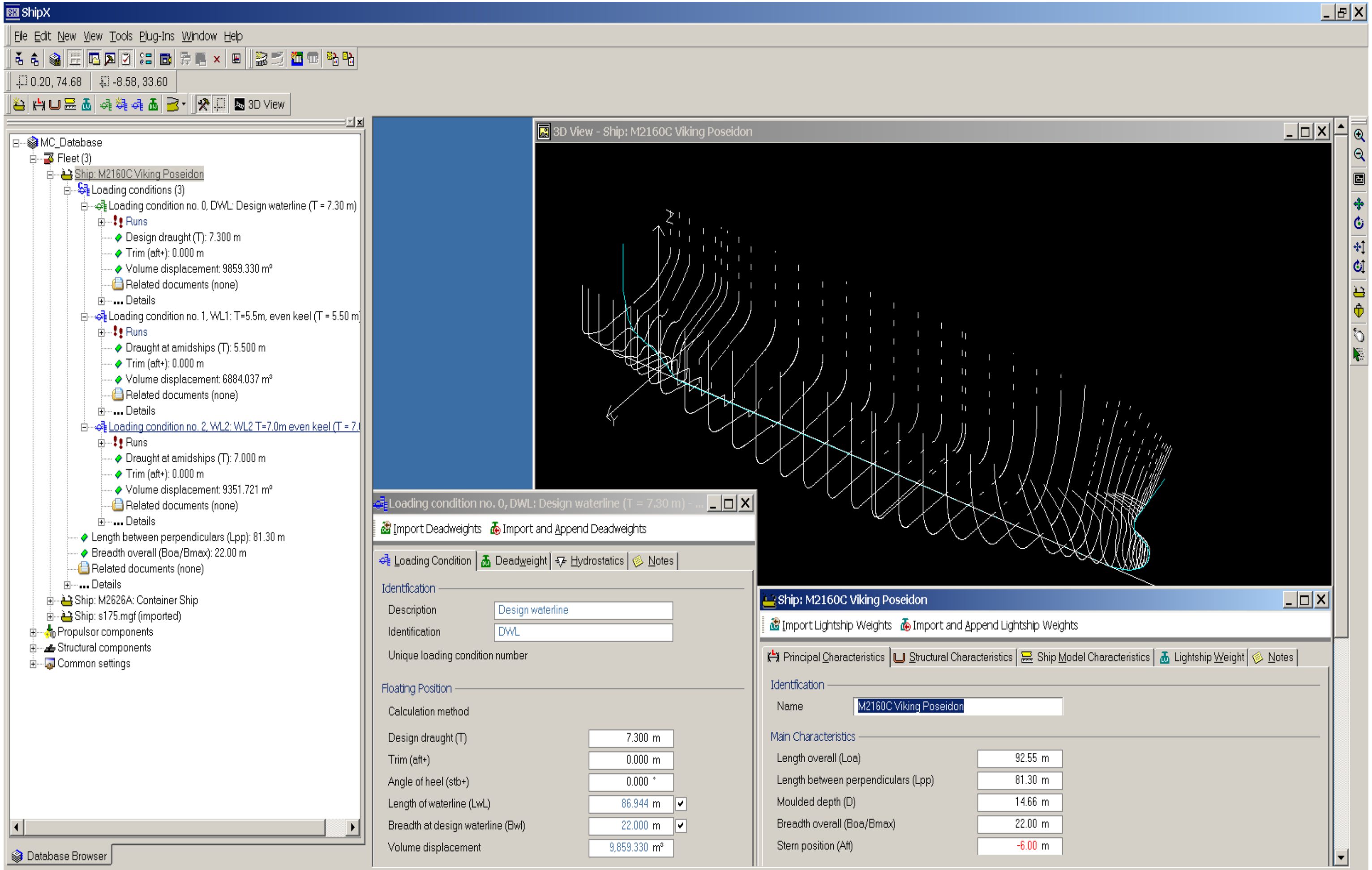




The 2-D theory takes into account that variation of the flow in the cross-directional plane is much larger than the variation in the longitudinal direction of the ship.

The principle of strip theory involves dividing the submerged part of the craft into a finite number of strips. Hence, 2-D hydrodynamic coefficients for added mass can be computed for each strip and then summed over the length of the body to yield the 3-D coefficients.

There exists more than 20 commercial strip theory programs and several opens source university codes.

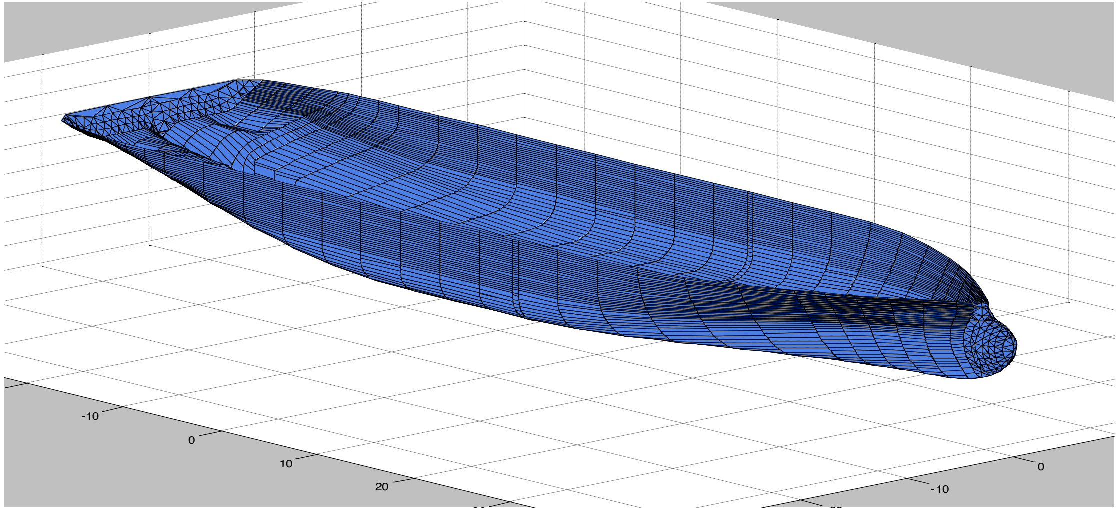


ShipX (VERES) - VEssel RESponse program is a Strip Theory Program which calculates wave-induced loads on and motions of mono-hulls and barges in deep to very shallow water. The program is based on the famous paper by Salvesen, Tuck and Faltinsen (1970). “Ship Motions and Sea Loads”. Transactions of SNAME.

5.1 Hydrodynamic Concepts and Potential Theory

Panel Methods (3-D Potential Theory)

For potential flows, the integrals over the fluid domain can be transformed to integrals over the boundaries of the fluid domain. This allows the application of panel or boundary element methods to solve the 3-D potential theory problem.



3-D panelization of a supply vessel

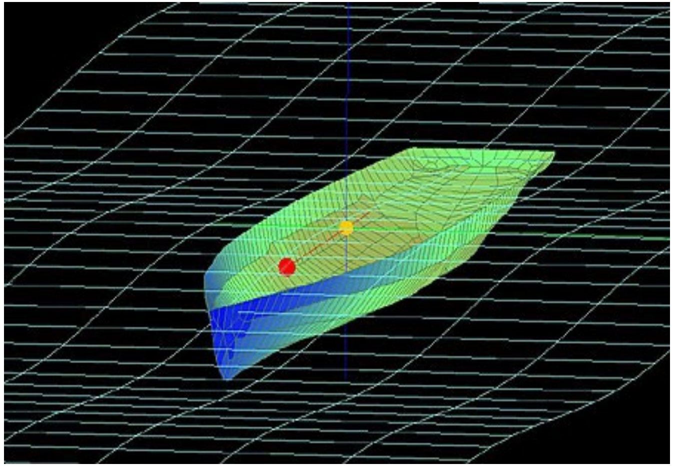
Panel methods divide the surface of the ship and the surrounding water into discrete elements (panels). On each of these elements, a distribution of sources and sinks is defined which fulfill the Laplace equation.

Commercial code: WAMIT (www.wamit.com)

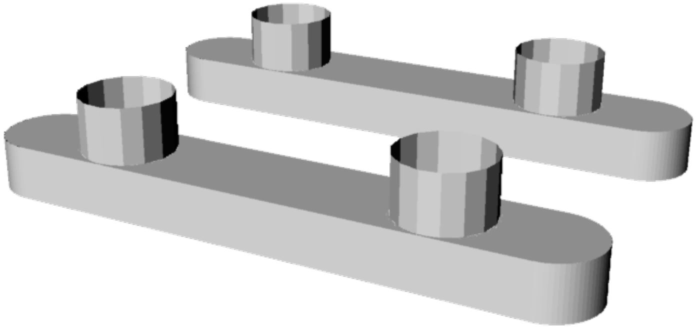
WAMIT

WAMIT® is the most advanced set of tools available for analyzing wave interactions with offshore platforms and other structures or vessels.

WAMIT® was developed by Professor Newman and coworkers at MIT in 1987, and it has gained widespread recognition for its ability to analyze the complex structures with a high degree of accuracy and efficiency.



Panelization of semi-submersible using WAMIT user supplied tools



Over the past 30 years WAMIT has been licensed to more than 100 industrial and research organizations worldwide.

5.1 Hydrodynamic Concepts and Potential Theory

Potential theory programs typically compute:

- Frequency-dependent added mass, **A**(ω)
- Potential damping coefficients, ***B***(ω)
- Restoring terms, ***C***
- 1st- and 2nd-order wave-induced forces and motions (amplitudes and phases) for given wave directions and frequencies
- … and much more

However, potential theory codes give you no viscous damping terms

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\mathrm{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\mathrm{exc}}$$

$$\boldsymbol{B}_{\mathrm{total}}(\omega) = \boldsymbol{B}(\omega) + \boldsymbol{B}_V(\omega)$$

You must add viscous terms manually (Section 5.3)

One special feature of WAMIT is that the program solves a boundary value problem for zero and infinite added mass. These boundary values are useful when computing the retardation functions describing the fluid memory effects.

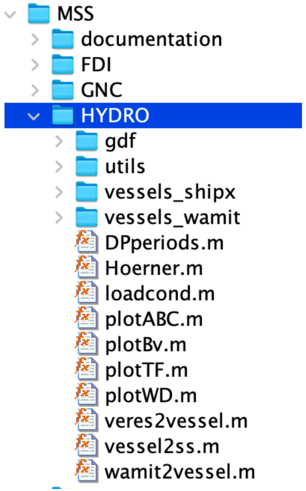
Processing of hydrodynamic data

Matlab MSS Toolbox: https://github.com/cybergalactic/MSS

The toolbox reads output data files generated by the hydrodynamic programs:

- **ShipX** – www.sintef.no/en/software/shipx/
- **WAMIT** – www.wamit.com

and processes the data for use in Matlab/Simulink.



MSS Toolbox

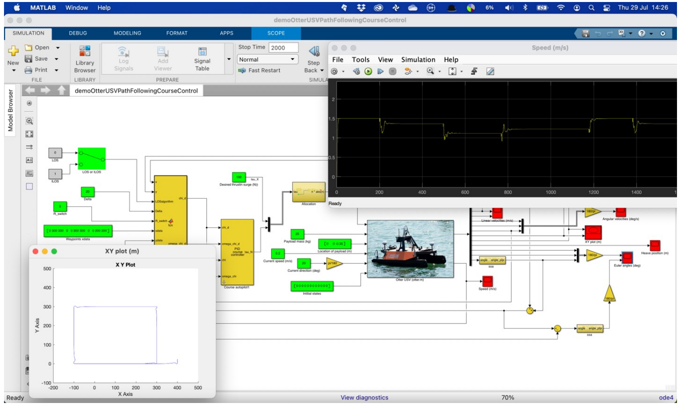
1. Download the Matlab MSS Toolbox from GitHub: https://github.com/cybergalactic/MSS
2. Copy the contents of the directory MSS/ to your computer and "add the path with subfolders" to Matlab.
3. Type >> help mss

The PDF file ‘MSS Quick Reference.pdf’ lists and explains the MSS commands.

Examples and demo files are located under the catalogues:

/MSS/mssExamples/	Textbook m-file examples (Fossen 2021)
/MSS/mssDemos/	GNC m-file demos
/MSS/SIMULINK/mssSimulinkDemos/	Simulink demos
/MSS/SIMULINK/mssWamitShipXTemplates/	Simulink templates for simulation of WAMIT and ShipX vessel and RAO data

The Simulink library is loaded by typing: >> mssSimulink



MSS Hydro

- MSS
  - documentation
  - FDI
  - GNC
  - HYDRO
    - gdf
    - utils
    - vessels_shipx
    - vessels_wamit
    - DPperiods.m
    - Hoerner.m
    - loadcond.m
    - plotABC.m
    - plotBv.m
    - plotTF.m
    - plotWD.m
    - veres2vessel.m
    - vessel2ss.m
    - wamit2vessel.m

ShipX and WAMIT data sets in the toolbox

- vessels_shipx
  - s175
  - supply
- vessels_wamit
  - fpso
  - offshorewind
  - semisub
  - tanker

MSS Hydro

- MSS
  - documentation
    - Configuration guide for ShipX and Wamit vessels models
      - README.txt
      - ShipX and Wamit configuration guide.pdf
    - FDI Identification of seakeeping models from frequency response data
      - FDI Tutorial.pdf
      - PerezFossen MIC–29(1) 2008.pdf
      - PerezFossen MIC–29(3) 2008.pdf
      - README.tex
    - Ship roll motion control
    - Tutoiral on modelling and simulation of marine craft
      - Powerpoint
      - M1_intro.pdf
      - M2_Hydrodynamics.pdf
      - M3_RB_dynamics.pdf
      - M4_maneuvering models.pdf
      - M5_disturbances.pdf
      - M6_FD_Seakeeping models.pdf
      - M7_time_domain_models_short_II.pdf
      - M8_man_in_seaway.pdf
      - M9_models_n_motion_control.pdf
      - M10_HydroSW.pdf
      - README.tex
    - Textbooks.pdf

- MSS
  - documentation
  - FDI
  - GNC
  - HYDRO
    - gdf
    - utils
    - vessels_shipx
    - vessels_wamit
    - DPperiods.m
    - Hoerner.m
    - loadcond.m
    - plotABC.m
    - plotBv.m
    - plotTF.m
    - plotWD.m
    - veres2vessel.m
    - vessel2ss.m
    - wamit2vessel.m

Load WAMIT data into Matlab vessel structure

```
vessel = wamit2vessel(filename) reads data from WAMIT output files
and store the data in vessel.mat using the MSS vessel struture. The Wamit
GDF-file must be defined in GLOBAL COORDINATES, i.e. origin [Lpp/2 B/2 WL],
see the Wamit manual. The axes are transformed from Wamit axes to Fossen
(2002) axes.

Example:

>> vessel = wamit2vessel('tanker')
>> vessel = wamit2vessel('tanker',10,246,46)
>> vessel = wamit2vessel('tanker',10,246,46,'1111')

Inputs:
   filename (without extension) reads and processes the following
                    WAMIT files:
                      *.1     added mass and damping
                      *.2     force RAOs from Haskind (vessel.forceRAO)
                      *.3     force RAOs from diffraction (vessel.forceRAO)
                      *.4     motion RAOs (vessel.motionRAO)
                      *.8     wave drift data from momentum (vessel.driftfrc)
                      *.9     wave drift data from pressure (vessel.driftfrc)
                      *.frc   rigid body mass parameters
                      *.out   output file
```

MSS Hydro

- MSS
  - documentation
  - FDI
  - GNC
  - HYDRO
    - gdf
    - utils
    - vessels_shipx
    - vessels_wamit
    - DPperiods.m
    - Hoerner.m
    - loadcond.m
    - plotABC.m
    - plotBv.m
    - plotTF.m
    - plotWD.m
    - veres2vessel.m
    - vessel2ss.m
    - wamit2vessel.m

Load ShipX data into Matlab vessel structure

```
veres2vessel (MSS Hydro)

vessel = veres2vessel(filename, disp_flag) reads data from the
ShipX (Veres) output files *.re1, *.re2, *.re7, *.re8, and *.hyd and
store the data in vesselname.mat using the MSS vessel struture. Examples:

>> veres2vessel('input')
>> veres2vessel('input','1111')

where input are the Veres output data file name.
--------------------------------------------------------------------------
Inputs:
  filename: *  (without extension) reads and processes the following
                ShipX (Veres) files:
                   *.re1  motion RAOs (vessel.motionRAO)
                   *.re2  wave drift data (vessel.driftfrc)
                   *.re7  added mass, damping, restoring forces
                   *.re8  force RAOsu (vessel.forceRAO)

   plot_flag (optionally): '1000' plot A and B matrices
                                      '0100' plot force RAOs
                           '0010' plot motion RAOs
                           '0001' plot wave drift forces
                           '0000' NO PLOT
                           '1111' PLOT ALL
--------------------------------------------------------------------------
```

MSS Hydro

```
>> load tanker
>> vessel
vessel =
  struct with fields:

          main: [1×1 struct]
    velocities: 0
      headings: [1×36 double]
           MRB: [6×6 double]
             C: [6×6×60 double]
         freqs: [1×60 double]
             A: [6×6×60 double]
             B: [6×6×60 double]
     motionRAO: [1×1 struct]
      forceRAO: [1×1 struct]
      driftfrc: [1×1 struct]
            Bv: [6×6 double]
```

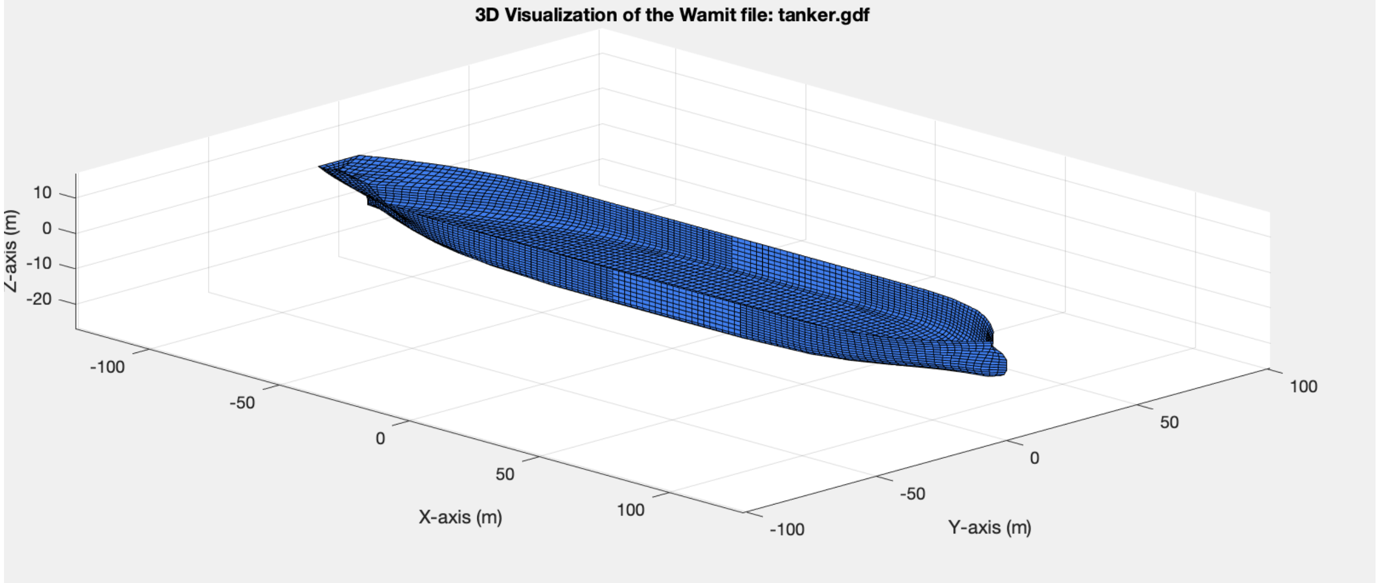
Viscous damping is added manually (see Section 5.3)

```
>> vessel.main
ans =
  struct with fields:

     name: 'tanker'
        T: 10
        B: 46
      Lpp: 246
        m: 94620210
      rho: 1025
      k44: 17.0200
      k55: 63.9600
      k66: 66.4200
        g: 9.8066
    nabla: 9.2312e+04
       CB: [3.9259 0 5.1845]
     GM_T: 9.9823
     GM_L: 470.9226
       CG: [3.9300 0 12.5000]
>> vessel.A(:,:,1:2)
ans(:,:,1) =
   1.0e+11 *
    0.0000         0   -0.0000         0   -0.0128         0
         0    0.0005         0    0.0030         0    0.0029
   -0.0000         0    0.0039         0    0.0096         0
         0    0.0030         0    0.1103         0   -0.0419
   -0.0128         0    0.0096         0    9.0021         0
         0    0.0029         0   -0.0420         0    1.7383
ans(:,:,2) =
   1.0e+11 *
    0.0000         0   -0.0000         0   -0.0128         0
         0    0.0005         0    0.0030         0    0.0029
   -0.0000         0    0.0040         0    0.0099         0
         0    0.0030         0    0.1103         0   -0.0419
   -0.0128         0    0.0099         0    9.0263         0
         0    0.0029         0   -0.0420         0    1.7407
```

MSS Hydro

```
>> plot_wamitgdf('tanker')
```

```
>> plot_wamitgdf(filename,colorcode,figno,1);
----------------------------------------------------------------------------
Inputs:
   filename:    Low-order Wamit GDF file without extension
   colorcode:   'r','g','b','c','m','y','w','k' or colormaps [0 0 1] etc.
   figno:       figure number
   waterline:   optional flag used to plot free surface points
   half:        optional, use 1 to plot GDF data (no mirror)
                         for Veres data. Octopus data always has mirror

Output:
   gdf_data:    Table dimension N x 12 of Wamit panels where N is the
                number of panels given by 3x4=12 points on each line
```



Next Step: Transform the Hydrodynamic Model from SEAKEEPING to BODY

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\text{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

$$\boldsymbol{B}_{\text{total}}(\omega) = \boldsymbol{B}(\omega) + \boldsymbol{B}_V(\omega)$$

- $\boldsymbol{A}(\omega)$ added mass matrix
- $\boldsymbol{B}(\omega)$ potential damping matrix
- $\boldsymbol{B}_V(\omega)$ viscous damping matrix
- $\boldsymbol{C}$ spring stiffness or hydrostatic matrix

Computed using hydrodynamic seakeeping codes

Transformation from SEAKEEPING) axes {s} to BODY axes {b}
Using kinematic transformation (Sections 5.2-5.3)

$$\begin{array}{lr}
\text{Inertia forces:} & \boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}^*_{RB}\boldsymbol{\nu} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}^*_A\boldsymbol{\nu}_r \\
\text{Damping forces:} & +(\boldsymbol{D}_P + \boldsymbol{D}_V)\boldsymbol{\nu}_r + \boldsymbol{\mu}_r \\
\text{Restoring forces:} & +\boldsymbol{G}\boldsymbol{\eta} + \boldsymbol{g}_o \\
\text{Wind and wave forces:} & = \quad \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} \\
\text{Propulsion forces:} & +\boldsymbol{\tau}
\end{array}$$

Linear model used for control design and simulation

5.2 Seakeeping and Maneuvering Kinematics

Seakeeping Theory (Perturbation Coordinates)

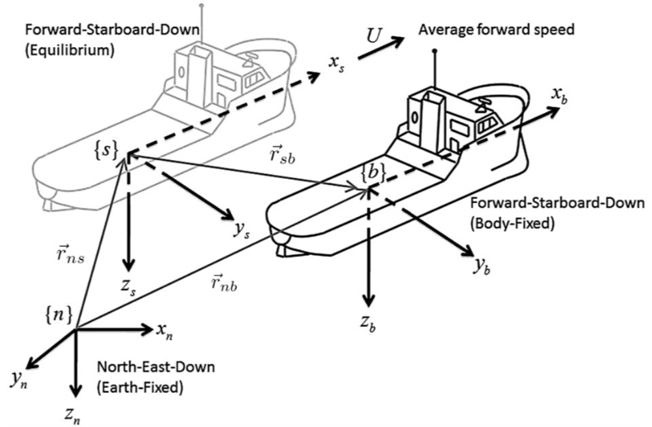
The SEAKEEPING reference frame {s} is not fixed to the craft; it is fixed to the equilibrium state

$$\boldsymbol{\zeta} = [\xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6]^T$$

$$\boldsymbol{\xi} = \delta\boldsymbol{\eta}$$

$$\dot{\boldsymbol{\xi}} = \delta\dot{\boldsymbol{\eta}} = \delta\boldsymbol{\nu}$$

$$\boldsymbol{\Theta}_{sb} = [\xi_4, \xi_5, \xi_6]^\top = [\delta\phi, \delta\theta, \delta\psi]^\top$$



-In the absence of wave excitation, {s} coincides with {b}.
- Under the action of the waves, the hull is disturbed from its equilibrium and {s} oscillates, with respect to its equilibrium position.

Transformation between {b} and {s}

$$\delta\boldsymbol{\nu} \approx \boldsymbol{\nu} + U(\mathbf{L}\delta\boldsymbol{\eta} - \mathbf{e}_1)$$
$$\delta\dot{\boldsymbol{\nu}} \approx \dot{\boldsymbol{\nu}} + U\mathbf{L}\boldsymbol{\nu}$$

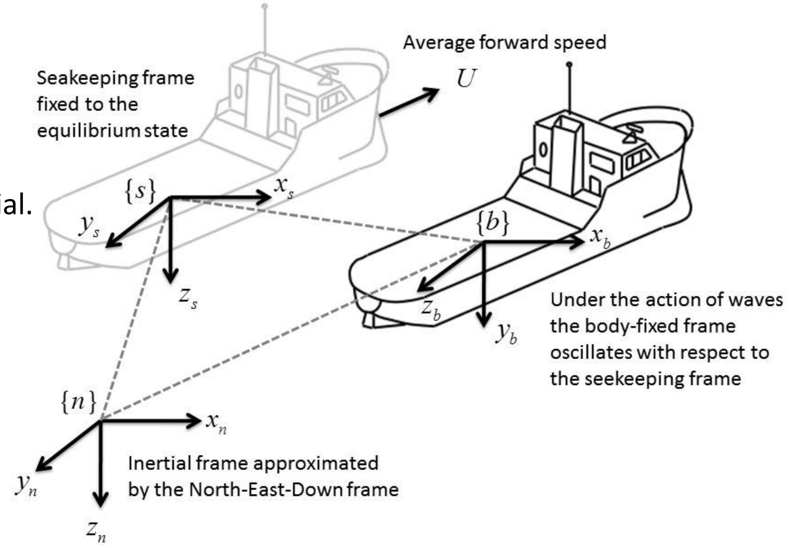
$$\mathbf{e}_1 = [1,0,0,0,0,0]^\top$$

$$\mathbf{L} := \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{v}_{ns}^n = [U\cos\bar{\psi},\ U\sin\bar{\psi},\ 0]^\top$$
$$\boldsymbol{\omega}_{ns}^n = [0,0,0]^\top$$
$$\boldsymbol{\Theta}_{ns} = [0,0,\bar{\psi}]^\top$$

$$\begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \bar{\psi} \end{bmatrix} + \begin{bmatrix} \delta\phi \\ \delta\theta \\ \delta\psi \end{bmatrix}$$

5.3 Cummins Equation



Seakeeping Analysis

The seakeeping equations of motion are assumed to be inertial.

Equations of Motion

$$\boldsymbol{\xi} = \delta\boldsymbol{\eta} = [\delta x, \delta y, \delta z, \delta\phi, \delta\theta, \delta\psi]^\top$$

$$\mathbf{M}_{RB}\ddot{\boldsymbol{\xi}} = \boldsymbol{\tau}_{\text{hyd}} + \boldsymbol{\tau}_{\text{hs}} + \boldsymbol{\tau}_{\text{exc}}$$

Cummins (1962) showed that the radiation-induced hydrodynamic forces in an ideal fluid can be related to frequency-dependent added mass **A**(ω) and potential damping **B**(ω).

$$(\boldsymbol{M}_{RB} + \boldsymbol{A}(\infty))\ddot{\boldsymbol{\xi}} + \int_0^t \bar{\boldsymbol{K}}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)\mathrm{d}\tau + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

$$\boldsymbol{\tau}_{\text{hyd}} = -\bar{\mathbf{A}}\ddot{\boldsymbol{\xi}} - \int_0^t \bar{\mathbf{K}}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau$$

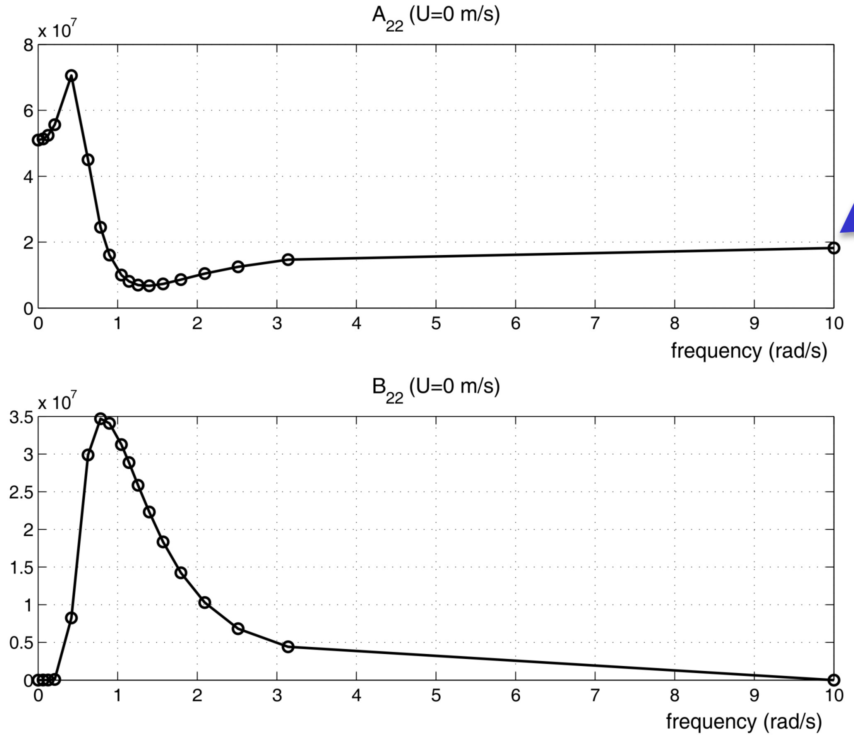
$$\bar{\mathbf{A}} = \mathbf{A}(\infty)$$

$$\bar{\mathbf{K}}(t) = \frac{2}{\pi}\int_0^\infty \mathbf{B}(\omega)\cos(\omega t)d\omega$$

Cummins, W. E. (1962). The Impulse Response Function and Ship Motions. Technical Report 1661. David Taylor Model Basin. Hydrodynamics Laboratory, USA.

5.3 Frequency-Dependent Potential Coefficients

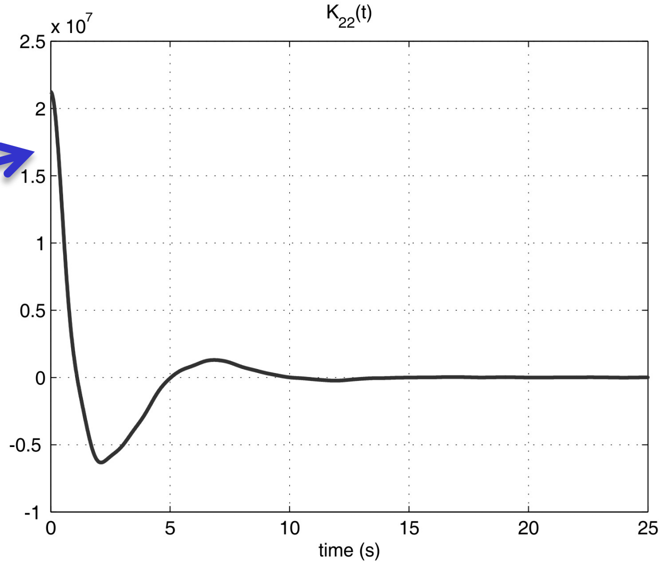
Frequency-dependent added mass $A_{22}(\omega)$ and potential damping $B_{22}(\omega)$ in sway



$$\bar{\mathbf{A}} = \mathbf{A}(\infty)$$

5.3 Retardation Functions and Fluid Memory Effects

$$\boldsymbol{\tau}_{\text{hyd}} = -\bar{\mathbf{A}}\dot{\boldsymbol{\xi}} - \int_0^t \bar{\mathbf{K}}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau$$



Matrix of retardation functions

$$\bar{\mathbf{K}}(t) = \frac{2}{\pi}\int_0^{\infty} \mathbf{B}(\omega)\cos(\omega t)d\omega$$

Cummins Equation

If linear restoring forces $\boldsymbol{\tau}_{\text{hs}} = -\boldsymbol{C}\xi$ are included in the model, this results in the time-domain model

$$(\mathbf{M}_{RB} + \mathbf{A}(\infty))\ddot{\boldsymbol{\xi}} + \int_0^t \bar{\mathbf{K}}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau + \mathbf{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

The fluid memory effects can be replaced by a state-space model to avoid the integral, see Section 5.5.

In mathematics, an integro-differential equation is an equation that involves both integrals and derivatives of a function.

5.3 Forced Oscillations

Wave excitation

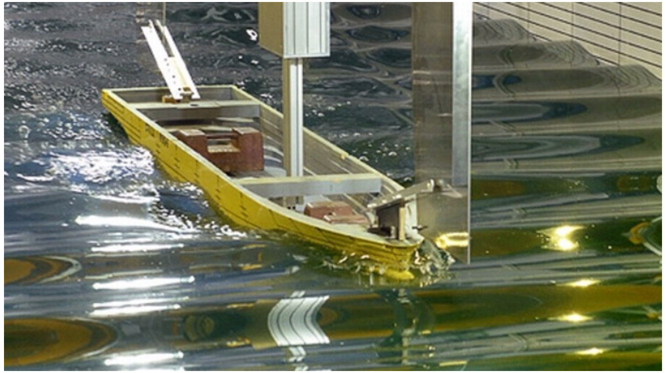
$$\boldsymbol{M}_{RB}\ddot{\boldsymbol{\xi}} = \boldsymbol{\tau}_{\text{hyd}} + \boldsymbol{\tau}_{\text{hs}} + \boldsymbol{\tau}_{\text{exc}}$$

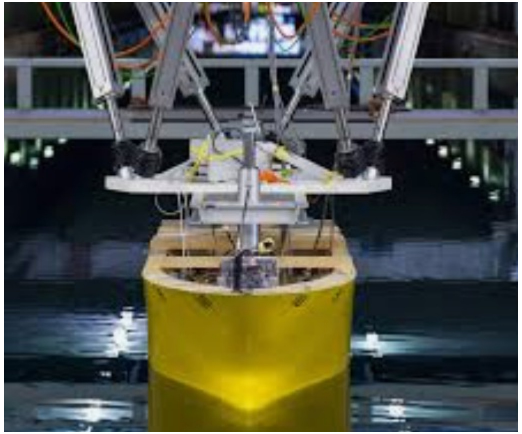
$$\boldsymbol{\tau}_{\text{exc}} = \boldsymbol{f}\cos(\omega t)$$

In an experimental setup with a restrained scale model, it is possible to vary the wave excitation frequency ω and the amplitudes f_i for $i = 1, .., 6$ of the excitation force. Hence, by measuring the position and attitude vector ξ_i the response of the second-order system can be fitted to a linear model (e.g. by least squares) for each frequency ω.

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\text{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

This gives two frequency-dependent model matrices $\boldsymbol{A}(\omega)$ and $\boldsymbol{B}_{\text{total}}(\omega)$





5.3 Frequency-Dependent Hydrodynamic Coefficients

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\text{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

$$\boldsymbol{B}_{\text{total}}(\omega) = \boldsymbol{B}(\omega) + \boldsymbol{B}_V(\omega)$$

- $\boldsymbol{A}(\omega)$ added mass matrix
- $\boldsymbol{B}(\omega)$ potential damping matrix
- $\boldsymbol{B}_V(\omega)$ viscous damping matrix
- $\boldsymbol{C}$ spring stiffness or hydrostatic matrix

$$\boldsymbol{\tau}_{\text{hyd}} = \underbrace{-\boldsymbol{A}(\omega)\ddot{\boldsymbol{\xi}} - \boldsymbol{B}(\omega)\dot{\boldsymbol{\xi}}}_{\text{radiation force}} - \underbrace{\boldsymbol{B}_V(\omega)\dot{\boldsymbol{\xi}}}_{\text{viscous damping force}}$$

$$\boldsymbol{\tau}_{\text{hs}} = \underbrace{-\boldsymbol{C}\boldsymbol{\xi}}_{\text{restoring force}}$$

The matrices $\boldsymbol{A}(\omega)$, $\boldsymbol{B}_{\text{total}}(\omega)$ and $\boldsymbol{C}$ represent a **hydrodynamic mass–damper–spring system** which varies with the frequency of the forced oscillation.

Definition 5.1 (Added Mass)
Hydrodynamic added mass can be seen as a virtual mass added to a system because an accelerating or decelerating body must move some volume of the surrounding fluid as it moves through it. Moreover, the object and fluid cannot occupy the same physical space simultaneously.

The added mass matrix $\boldsymbol{A}(\omega)$ should not be understood as additional mass due to a finite amount of water that is dragged with the vessel.

5.3 Incorrect Mixture of Time and Frequency in Hydrodynamic Models

Hydrodynamic mass—damper—spring system

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\text{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

This model is rooted deeply in the literature of hydrodynamics and this false time-domain model has been discussed eloquently in the literature since the model mixes time and frequency incorrectly (abuse of notation). Hence, it is not an ODE.

An Ordinary Differential Equation (ODE) is an equation with a function and one or more of its derivatives. It is a function of only one independent variable, typically the time.

Consequently, the correct approach is to integrate Cummins time-domain model to find the solution for all frequencies

$$(\boldsymbol{M}_{RB} + \boldsymbol{A}(\infty))\ddot{\boldsymbol{\xi}} + \int_0^t \bar{\boldsymbol{K}}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)\mathrm{d}\tau + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

If the matrices $\boldsymbol{A}(\omega)$ and $\boldsymbol{B}_{\text{total}}(\omega)$ are computed at one and only one constant frequency (e.g., the zero frequency which is used in maneuvering theory) the hydrodynamic mass damper-spring-system will be an ODE, which can be solved by numerical integration.

5.3 Viscous Damping

When running seakeeping codes, it is important to include an external viscous damping matrix $\boldsymbol{B}_V(\omega)$ to obtain accurate estimates of the vessel responses.

$\boldsymbol{B}_V(\omega)$ can be chosen as a diagonal matrix

$$\boldsymbol{B}_V(\omega) = \mathrm{diag}\left\{ \beta_1 \mathrm{e}^{-\alpha\omega} + N_{\mathrm{ITTC}}(A_1),\ \beta_2 \mathrm{e}^{-\alpha\omega},\ \beta_3,\ \beta_{\mathrm{IKEDA}}(\omega),\ \beta_5,\ \beta_6 \mathrm{e}^{-\alpha\omega} \right\}$$

where

$\alpha > 0$ is the exponential rate
$\beta_i > 0$ ($i = 1, 2, 6$) are linear viscous skin friction coefficients in *surge, sway* and *yaw*
$N_{\mathrm{ITTC}}(A_1)$ is equivalent linear surge resistance as a function of the surge velocity amplitude A_1
$\beta_{\mathrm{IKEDA}}(\omega)$ is frequency-dependent roll damping based on the theory of Ikeda et al. (1976)
(other models for viscous roll damping can also be used)

One useful property of the exponential function is that *linear skin friction* only affects low-frequency motions.

Ikeda, Y., K. Komatsu, Y. Himeno and N. Tanaka (1976). On Roll Damping Force of Ship: Effects of Friction of Hull and Normal Force of Bilge Keels. *Journal of the Kansai Society of Naval Architects* 142, 54–66.

5.3 Viscous Damping (cont.)

$$\boldsymbol{B}_V(\omega) = \text{diag}\left\{ \beta_1 e^{-\alpha\omega} + N_{\text{ITTC}}(A_1),\ \beta_2 e^{-\alpha\omega},\ \beta_3,\ \beta_{\text{IKEDA}}(\omega),\ \beta_5,\ \beta_6 e^{-\alpha\omega} \right\}$$

Surge, sway and yaw

Specify the time constant T_1, T_2, and T_6 satisfying

$$T_1 = \frac{m + A_{11}(0)}{B_{v,11}(0)}, \quad T_2 = \frac{m + A_{22}(0)}{B_{v,22}(0)}, \quad T_6 = \frac{I_z + A_{66}(0)}{B_{v,66}(0)}$$

This gives

$$\boxed{\beta_1 = \frac{m + A_{11}(0)}{T_1}, \quad \beta_2 = \frac{m + A_{22}(0)}{T_2}, \quad \beta_6 = \frac{I_z + A_{66}(0)}{T_6}}$$

Heave and pitch

Specify a percentage increase in damping, e.g., $\beta_3 = p\, B_{33}(\omega_3)$ where $p > 0$.
Then solve for β_3 and β_5 in

$$\boxed{2\zeta_3\omega_3 = \frac{B_{33}(\omega_3) + \beta_3}{m + A_{33}(\omega_3)}} \qquad \boxed{2\zeta_5\omega_5 = \frac{B_{55}(\omega_5) + \beta_5}{I_y + A_{55}(\omega_5)}}$$

5.3 Quadratic Surge Resistance

Quadratic surge resistance can be approximated using a describing function
(Like the equivalent linearization method)

$y = N(A)u$ Sinusoidal input $u = A\sin(\omega t)$

For static linearities, displaying no dependence upon the derivatives, the describing function for the particular odd polynomial nonlinearity

$$y = c_1 x + c_2 x|x| + c_3 x_3^3$$

is (Gelb and Vander Velde 1968)

$$N(A) = c_1 + \frac{8A}{3\pi} c_2 + \frac{3A^2}{4} c_3$$

Quadratic damping approximation

$$X = -X_{|u|u}|u|u \approx N_{\mathrm{ITTC}}(A_1)u$$

$$\boxed{N_{\mathrm{ITTC}}(A_1) = -\frac{8A_1}{3\pi} X_{|u|u}}$$

This assumes that the surge velocity is harmonic with amplitude A_1, i.e.

$u = A_1 \cos(\omega t)$

For a ship moving at high speed, the amplitudes A_2 and A_6 will be much smaller than A_1. Hence, it is common to neglect these terms in seakeeping analysis.

Gelb, A. and W. E. Vander Velde (1968). Multiple-Input Describing Functions and Nonlinear System Design. McGraw-Hill Book Company.

MSS Hydro

- MSS
  - documentation
  - FDI
  - GNC
  - HYDRO
    - gdf
    - utils
    - vessels_shipx
    - vessels_wamit
    - DPperiods.m
    - Hoerner.m
    - loadcond.m
    - plotABC.m
    - plotBv.m
    - plotTF.m
    - plotWD.m
    - veres2vessel.m
    - vessel2ss.m
    - wamit2vessel.m

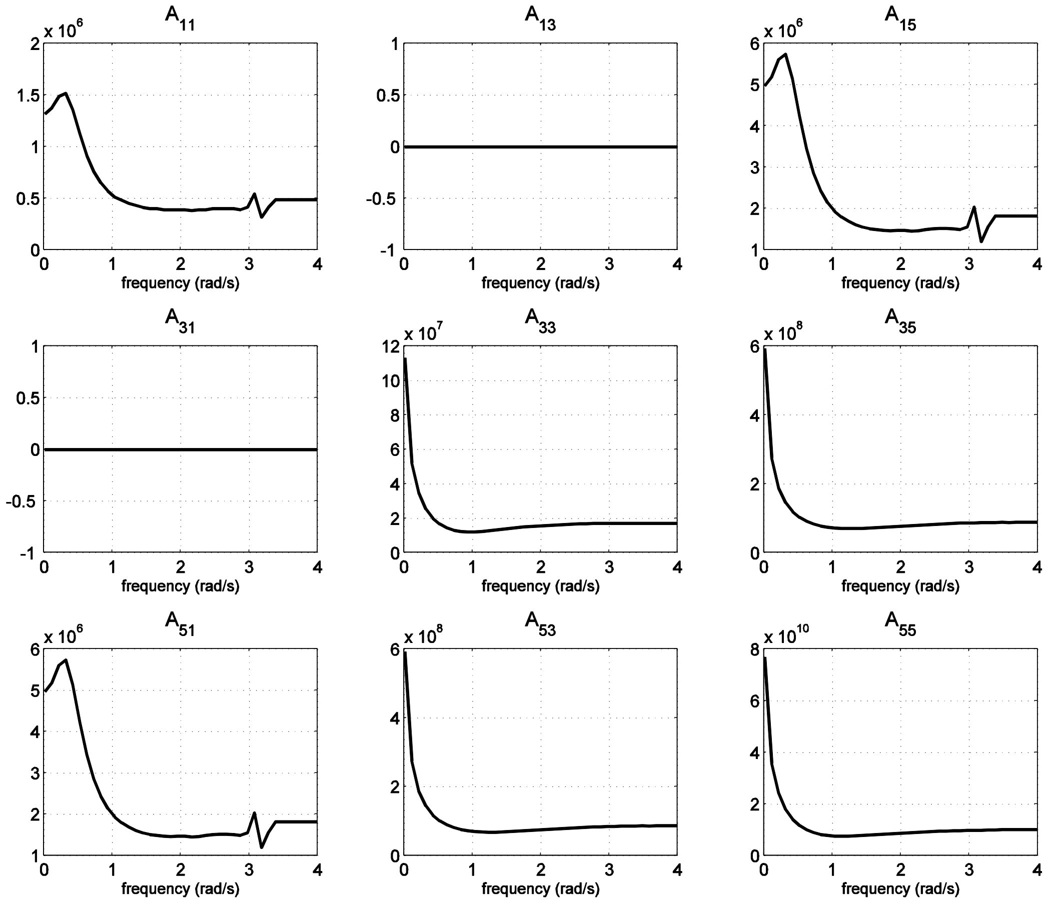
Plots the hydrodynamic matrices ***A, B, C*** and $\boldsymbol{B}_v$ as a function of frequency.

```
>> load tanker
>> vessel
vessel =
  struct with fields:

         main: [1×1 struct]
   velocities: 0
     headings: [1×36 double]
          MRB: [6×6 double]
            C: [6×6×60 double]
        freqs: [1×60 double]
            A: [6×6×60 double]
            B: [6×6×60 double]
    motionRAO: [1×1 struct]
     forceRAO: [1×1 struct]
     driftfrc: [1×1 struct]
           Bv: [6×6 double]
```

```
>> plotABC(vessel,mtrx) plots all elements Aij,Bij,Cij versus
   frequency and speed

Inputs:
  vessel:  MSS vessel structure generated from a data file: *
  (file name without extension) by using the following calls:

  ShipX (VERES), MARINTEK :  vessel = veres2vessel(*)
  WAMIT                   :  vessel = wamit2vessel(*)

  mtrx = 'A'    added mass
         'B'    potential + viscous damping
         'C'    restoring forces

 i, j (optionally):    matrix element
 velno (optionally):   speed number
```

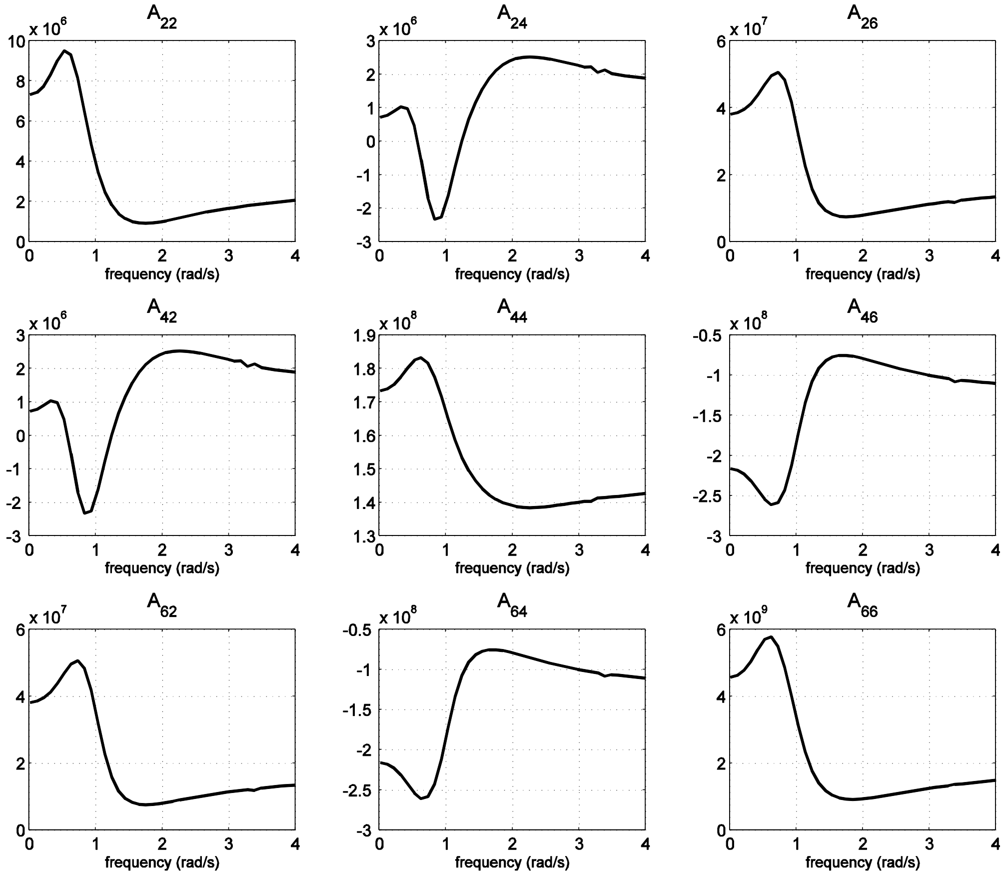
5.3 Potential Coefficients

Longitudinal added mass coefficients as a function of frequency.



5.3 Potential Coefficients (cont.)

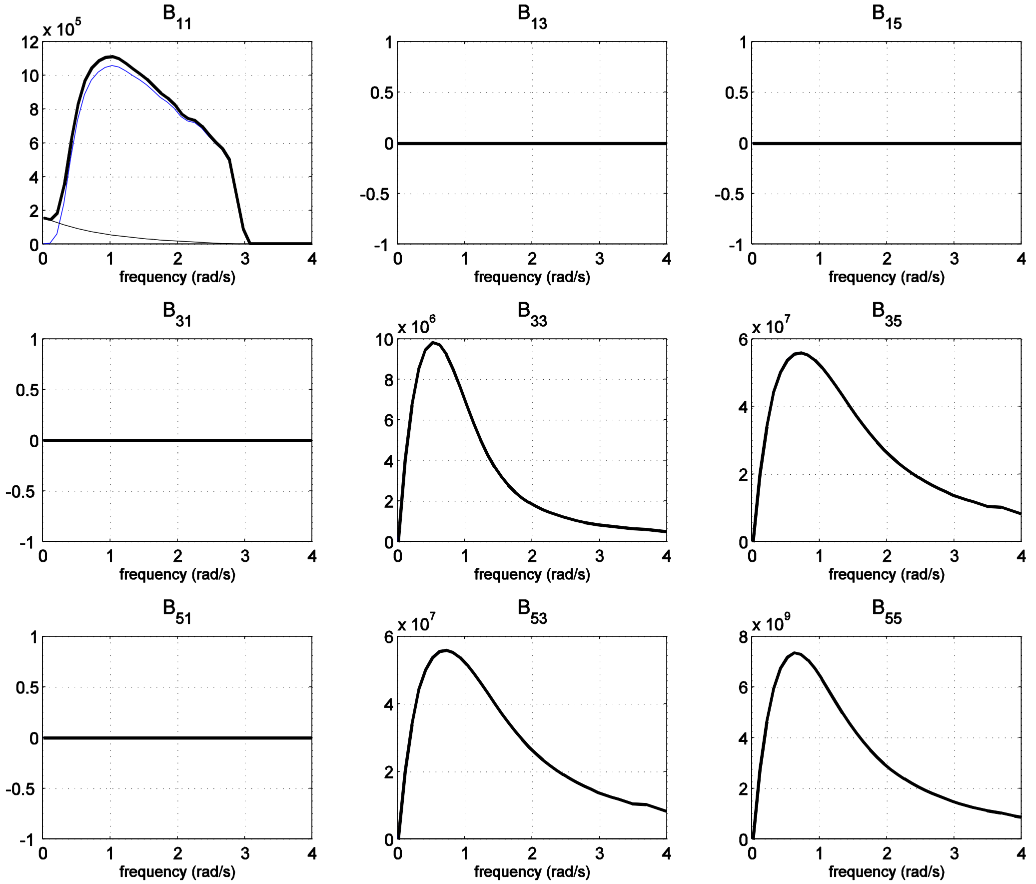
Lateral added mass coefficients as a function of frequency.



5.3 Potential Coefficients (cont.)

Longitudinal potential damping coefficients as a function of frequency.

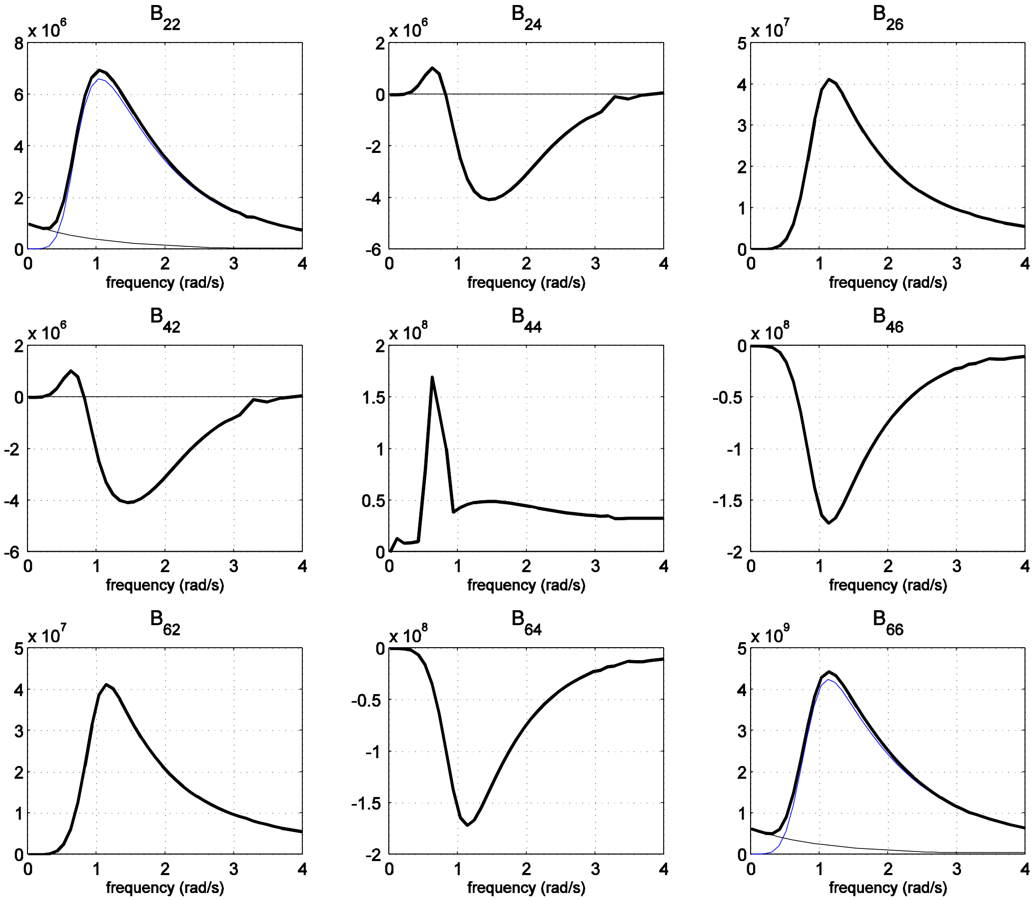
Exponential decaying viscous damping is included for $\boldsymbol{B}_{11}$.



5.3 Potential Coefficients (cont.)

Lateral potential damping coefficients as a function of frequency.

Exponential decaying viscous damping is included for B_{22} and B_{66} while viscous IKEDA damping is included in B_{44}.



5.3 Response Amplitude Operator (RAO)

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\mathrm{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\mathrm{exc}}$$

Laplace

$$\mathcal{L}\{\ddot{\boldsymbol{\xi}}(t)\} = s^2\boldsymbol{\xi}(s) \qquad \mathcal{L}\{\dot{\boldsymbol{\xi}}\} = s\boldsymbol{\xi}(s) \qquad s = \mathrm{j}\omega$$

$$\left(-\omega^2[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)] + \mathrm{j}\omega\boldsymbol{B}_{\mathrm{total}}(\omega) + \boldsymbol{C}\right)\boldsymbol{\xi}(\mathrm{j}\omega) = \boldsymbol{\tau}_{\mathrm{exc}}(\mathrm{j}\omega)$$

Harmonic excitations

$$\boldsymbol{\tau}_{\mathrm{exc}}(\mathrm{j}\omega) = F_i\zeta \text{ for } i = 1, 2, ..., 6$$

Regular wave

$$\zeta = \zeta_a e^{\,\mathrm{j}\omega t}$$

$$\longrightarrow \quad \xi_i = \bar{\xi}_i e^{\,\mathrm{j}\omega t}$$

RAO from wave amplitude ζ_a to response ξ_i

$$\mathrm{RAO}_i(\omega) = \frac{\bar{\xi}_i}{\zeta_a} = \frac{F_i}{C_{ii} - \omega^2[M_{RB,ii} + A_{ii}(\omega)] + \mathrm{j}B_{\mathrm{total},ii}(\omega)\omega}$$

Amplitude and phase

$$|\mathrm{RAO}_i(\omega)| = \frac{F_i}{\sqrt{\left(C_{ii} - \omega^2[M_{RB,ii} + A_{ii}(\omega)]\right)^2 + \left(\omega B_{\mathrm{total},ii}(\omega)\right)^2}}$$

$$\angle\mathrm{RAO}_i(\omega) = \mathrm{atan}\left(\frac{\omega B_{\mathrm{total},ii}(\omega)}{C_{ii} - \omega^2[M_{RB,ii} + A_{ii}(\omega)]}\right)$$

Note the similarity to **Bode magnitude and phase plots**, for which magnitude is logarithmic and given in decibels while phase is plotted in degrees using a common logarithmic frequency axis.

5.4 Time-Domain Models including Fluid Memory Effects

Linear SEAKEEPING model based on Cummins equation

Linear theory where frequency-dependent matrices are replaced by fluid memory effects

↓

Linear equations of motion expressed in BODY

Transform from SEAKEEPING to BODY coordinates using the linear kinematic transformation

↓

Unified maneuvering and seakeeping model expressed in BODY

Replace the linear kinematic and Coriolis terms with their nonlinear counterparts. Add nonlinear viscous damping and maneuvering coefficients

5.4 Cummins Equation in SEAKEEPING Coordinates

Cummins (1962) Equation

$$(\mathbf{M}_{RB} + \bar{\mathbf{A}})\ddot{\boldsymbol{\xi}} + \int_{-\infty}^{t} \bar{\mathbf{K}}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau + \bar{\mathbf{C}}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \delta\boldsymbol{\tau}$$

The Ogilvie (1964) Transformation gives

$$\bar{\mathbf{A}} = \mathbf{A}(\infty)$$

$$\bar{\mathbf{K}}(t) = \frac{2}{\pi}\int_{0}^{\infty} \mathbf{B}_{\text{total}}(\omega)\cos(\omega t)d\omega$$

From a numerical point of view is it better to integrate the difference

$$\mathbf{K}(t) = \frac{2}{\pi}\int_{0}^{\infty} [\mathbf{B}_{\text{total}}(\omega) - \mathbf{B}_{\text{total}}(\infty)]\cos(\omega t)d\omega$$

This can be don by rewriting Cummins equation as

$$[\mathbf{M}_{RB} + \mathbf{A}(\infty)]\ddot{\boldsymbol{\xi}} + \mathbf{B}_{\text{total}}(\infty)\dot{\boldsymbol{\xi}} + \int_{0}^{t} \mathbf{K}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau + \mathbf{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \delta\boldsymbol{\tau}$$

Cummins, W. E. (1962). The Impulse Response Function and Ship Motions. Techn. Report 1661. David Taylor Model Basin. Hydrodynamics Laboratory, USA.
Ogilvie, T. F. (1964). Recent Progress Towards the Understanding and Prediction of Ship Motions. 5th Symposium on Naval Hydrodynamics, pp. 3–79.

5.4 Linear Time-Domain Seakeeping Equations in BODY Coordinates

$$[\mathbf{M}_{RB} + \mathbf{A}(\infty)]\ddot{\boldsymbol{\xi}} + \mathbf{B}_{\text{total}}(\infty)\dot{\boldsymbol{\xi}} + \int_0^t \mathbf{K}(t-\tau)\dot{\boldsymbol{\xi}}(\tau)d\tau + \mathbf{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \delta\boldsymbol{\tau}$$

It is possible to transform the time-domain representation of Cummins equation from {s} to {b} using the kinematic relationships

$$\delta\boldsymbol{\nu} \approx \boldsymbol{\nu} + U(\mathbf{L}\delta\boldsymbol{\eta} - \mathbf{e}_1)$$
$$\delta\dot{\boldsymbol{\nu}} \approx \dot{\boldsymbol{\nu}} + U\mathbf{L}\boldsymbol{\nu}$$

$$\boldsymbol{\xi} = \delta\boldsymbol{\eta}$$
$$\dot{\boldsymbol{\xi}} = \delta\boldsymbol{\nu}$$

This gives:

$$[\mathbf{M}_{RB} + \mathbf{A}(\infty)][\dot{\boldsymbol{\nu}} + U\mathbf{L}\boldsymbol{\nu}] + \mathbf{B}_{\text{total}}(\infty)[\boldsymbol{\nu} + U(\mathbf{L}\delta\boldsymbol{\eta} - \mathbf{e}_1)] + \int_0^t \mathbf{K}(t-\tau)\delta\boldsymbol{\nu}(\tau)d\tau + \mathbf{C}\delta\boldsymbol{\eta} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + (\boldsymbol{\tau} - \bar{\boldsymbol{\tau}})$$

The steady-state control force $\boldsymbol{\tau}$ needed to obtain the forward speed U when $\boldsymbol{\tau}_{\text{wind}} = \boldsymbol{\tau}_{\text{wave}} = \mathbf{0}$ and $\delta\boldsymbol{\eta} = \mathbf{0}$ is

$$\bar{\boldsymbol{\tau}} = \mathbf{B}_{\text{total}}(\infty)U\mathbf{e}_1$$

Hence,

$$[\mathbf{M}_{RB} + \mathbf{A}(\infty)][\dot{\boldsymbol{\nu}} + U\mathbf{L}\boldsymbol{\nu}] + \mathbf{B}_{\text{total}}(\infty)[\boldsymbol{\nu} + U\mathbf{L}\delta\boldsymbol{\eta}] + \int_0^t \mathbf{K}(t-\tau)\delta\boldsymbol{\nu}(\tau)d\tau + \mathbf{C}\delta\boldsymbol{\eta} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau}$$

5.4 Linear Time-Domain Seakeeping Equations in BODY Coordinates

$$[\mathbf{M}_{RB} + \mathbf{A}(\infty)][\dot{\boldsymbol{\nu}} + U\mathbf{L}\boldsymbol{\nu}] + \mathbf{B}_{\text{total}}(\infty)[\boldsymbol{\nu} + U\mathbf{L}\delta\boldsymbol{\eta}] + \int_0^t \mathbf{K}(t-\tau)\delta\boldsymbol{\nu}(\tau)d\tau + \mathbf{C}\delta\boldsymbol{\eta} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau}$$

When computing the damping and retardation functions, it is common to neglect the influence of $\delta\boldsymbol{\eta}$ on the forward speed such that

$$\delta\boldsymbol{\nu} \approx \boldsymbol{\nu} + U(\mathbf{L}\delta\boldsymbol{\eta} - \mathbf{e}_1) \approx \boldsymbol{\nu} - U\mathbf{e}_1$$

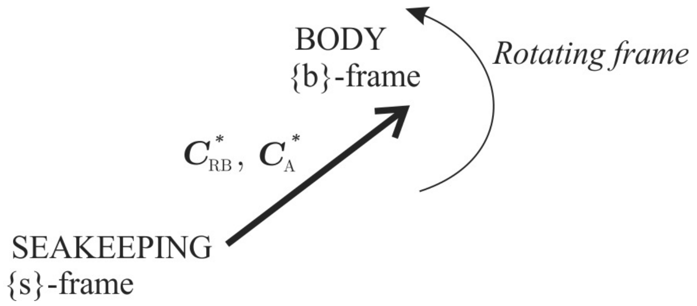
Finally, let use replace $\boldsymbol{\nu}$ by the relative velocity $\boldsymbol{\nu}_r$ to include ocean currents and define $\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A$ such that

$$\mathbf{M}\dot{\boldsymbol{\nu}} + \mathbf{C}_{RB}^{*}\boldsymbol{\nu} + \mathbf{C}_{A}^{*}\boldsymbol{\nu}_r + \mathbf{D}\boldsymbol{\nu}_r + \int_0^t \mathbf{K}(t-\tau)[\boldsymbol{\nu}(\tau) - U\mathbf{e}_1]d\tau + \mathbf{G}\boldsymbol{\eta} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau}$$

where

$$\begin{aligned}
\mathbf{M}_A &= \mathbf{A}(\infty) \\
\mathbf{C}_A^{*} &= U\mathbf{A}(\infty)\mathbf{L} \\
\mathbf{C}_{RB}^{*} &= U\mathbf{M}_{RB}\mathbf{L} \\
\mathbf{D} &= \mathbf{B}_{\text{total}}(\infty) \\
\mathbf{G} &= \mathbf{C}
\end{aligned}$$

Linear Coriolis and centripetal forces due to a rotation of {b} about {s}



5.4 Linear Time-Domain Seakeeping Equations in BODY Coordinates

Fluid Memory Effects

The integral in the following equation represents the fluid memory effects

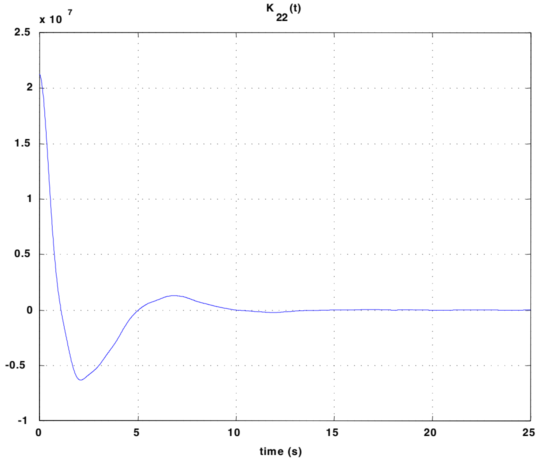
$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}^{*}_{RB}\boldsymbol{\nu} + \boldsymbol{C}^{*}_{A}\boldsymbol{\nu} + \boldsymbol{D}\boldsymbol{\nu} + \int_0^t \boldsymbol{K}(t-\tau)[\boldsymbol{\nu}(\tau) - U\boldsymbol{e}_1]\mathrm{d}\tau + \boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau}$$

$$\boldsymbol{\mu} := \int_0^t \mathbf{K}(t-\tau)\underbrace{[\boldsymbol{\nu}(\tau) - U\mathbf{e}_1]}_{\delta\boldsymbol{\nu}}\, d\tau$$

Approximated by a state-space model

$$\boldsymbol{\mu} = \mathbf{H}(s)[\boldsymbol{\nu} - U\mathbf{e}_1]$$

$$\dot{\mathbf{x}} = \mathbf{A}_r\mathbf{x} + \mathbf{B}_r\delta\boldsymbol{\nu}$$
$$\boldsymbol{\mu} = \mathbf{C}_r\mathbf{x}$$



Impulse response function

$$\mathbf{K}(t) = \frac{2}{\pi}\int_0^{\infty}[\mathbf{B}(\omega) - \mathbf{B}(\infty)]\cos(\omega t)d\omega$$

5.4 Nonlinear Unified Seakeeping and Maneuvering Model with Fluid Memory Effects

Linear Seakeeping Equations (BODY coordinates)

$$
\begin{aligned}
&\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}^{*}\boldsymbol{\nu} + \boldsymbol{C}_{A}^{*}\boldsymbol{\nu} + \boldsymbol{D}\boldsymbol{\nu} \\
&\qquad + \int_0^t \boldsymbol{K}(t-\tau)[\boldsymbol{\nu}(\tau) - U\boldsymbol{e}_1]\mathrm{d}\tau + \boldsymbol{G}\boldsymbol{\eta} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau}
\end{aligned}
$$

Unified Nonlinear Seakeeping and Maneuvering Model

- Add ocean currents (relative velocity)
- Use nonlinear kinematics
- Replace linear Coriolis and centripetal forces with their nonlinear counterparts
- Include maneuvering coefficients in a nonlinear damping matrix (linear superposition)

$$
\begin{aligned}
\dot{\boldsymbol{\eta}} &= \boldsymbol{J}_{\Theta}(\boldsymbol{\eta})\boldsymbol{\nu} \\
\boldsymbol{M}\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{\mu}_r + \boldsymbol{G}\boldsymbol{\eta} &= \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau}
\end{aligned}
$$

$$
\boldsymbol{\mu}_r := \int_0^t \boldsymbol{K}(t-\tau)\underbrace{[\boldsymbol{\nu}_r(\tau) - U_r\boldsymbol{e}_1]}_{\delta\boldsymbol{\nu}_r}\mathrm{d}\tau
$$

Fossen, T. I. (2005). Nonlinear Unified State-Space Model for Ship Maneuvering and Control in a Seaway. *International Journal of Bifurcation and Chaos* 15(9), 2717–2746. Also in the Proceedings of the ENOC'05 (Plenary Talk), Eindhoven, Netherlands, August 2005, pp. 43–70.

MSS Simulink Library, Models and Templates

- SIMULINK
  - figs
  - init
  - mssSimulinkDemos
    - demoCS2passiveObserverDP.slx
    - demoDPThrusterModels.slx
    - demoDSRVdepthControl.slx
    - demoKalmanWavefilterDP.slx
    - demoNavalVesselMano.slx
    - demoPanamaxContainerShip.slx
    - demoPassiveWavefilterAutopilot1.slx
    - demoPassiveWavefilterAutopilot2.slx
    - demoS175WindCurrentAutopilot.slx
    - demoSemisubDPsystem.slx
    - demoWaveElevation.slx
    - demoWaypointGuidance.slx
  - mssWamitShipXTemplates
    - cache
    - slprj
    - DP_ForceRAO.slx
    - DP_MotionRAO.slx
    - MAN_ForceRAO.slx
  - mssSimulink.slx

Simulink demo files for marine craft simulation, wave generation, state estimation, etc.

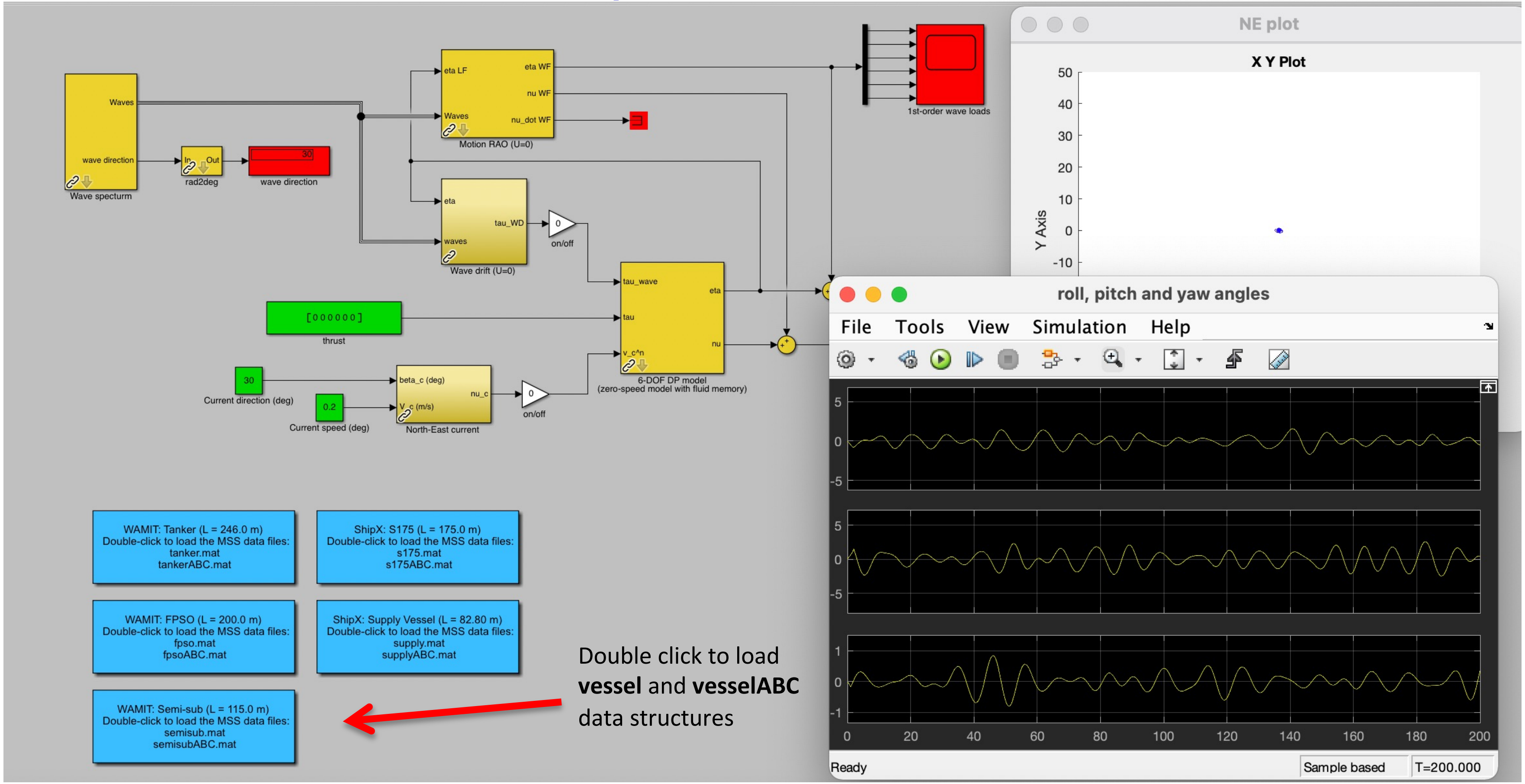
User editable Simulink templates for DP and maneuvering.

Motion and force RAOs are used in DP, while the unified maneuvering model can only be used with force RAOs

All models have <u>fluid memory effects</u>

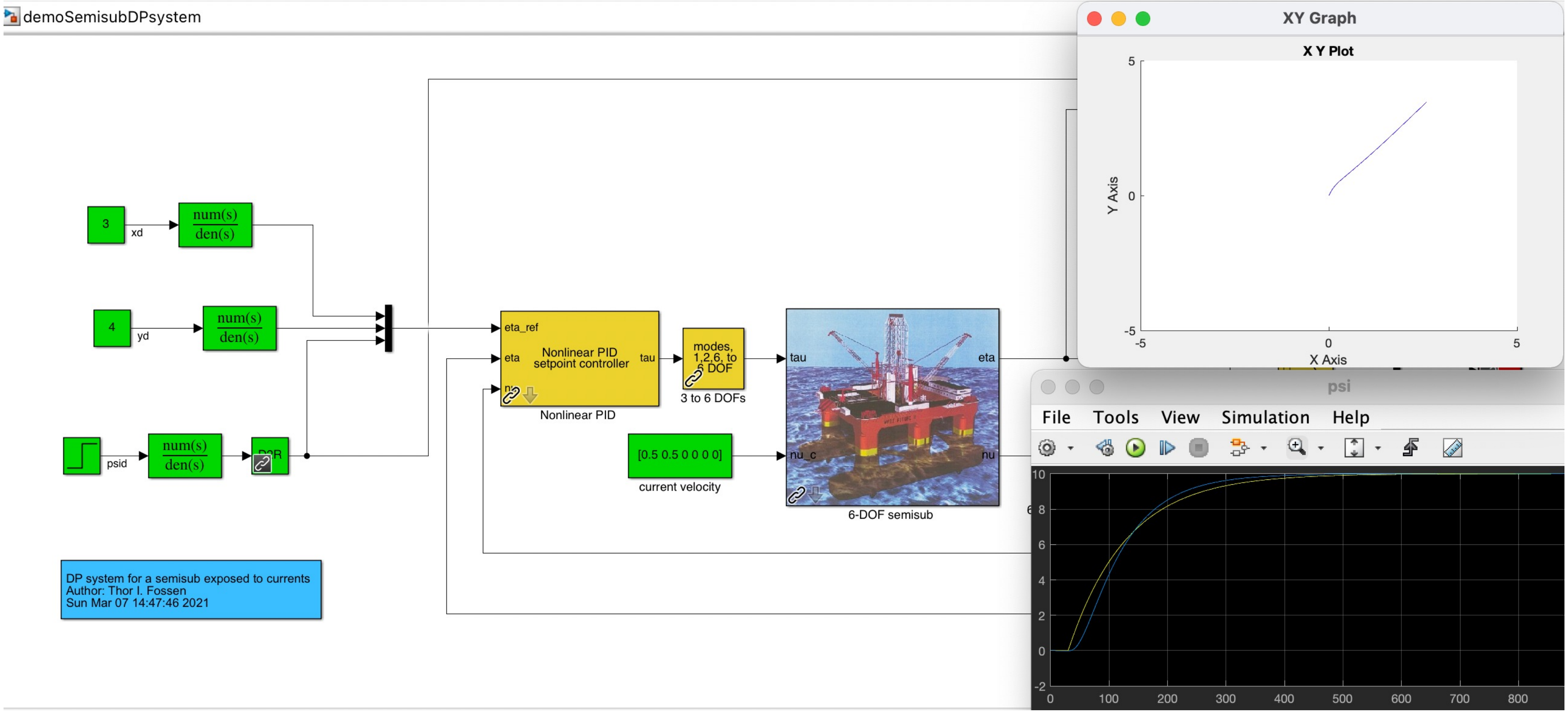
The MSS Simulink library is loaded by double-clicking **mssSimulink.slx**, which contains many Simulink blocks that can be used in own applications.

DP Motion RAO Template

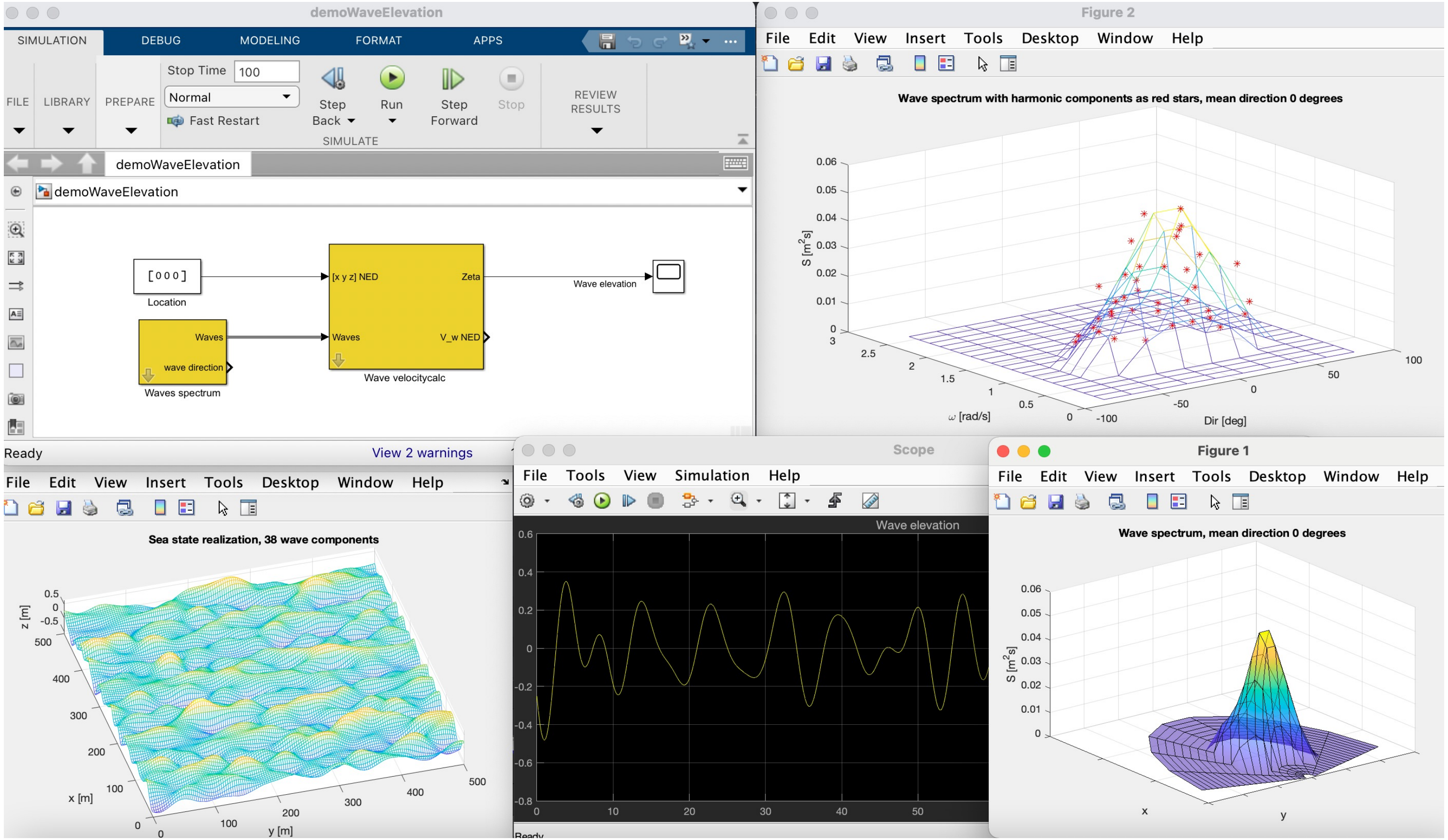


Double click to load **vessel** and **vesselABC** data structures

Simulink Demo: DP System for 6-DOF Semisubmersible



Simulink Demo: Wave Spectrum to Wave Elevation



5.5 Case Study: Identification of Fluid Memory Effects

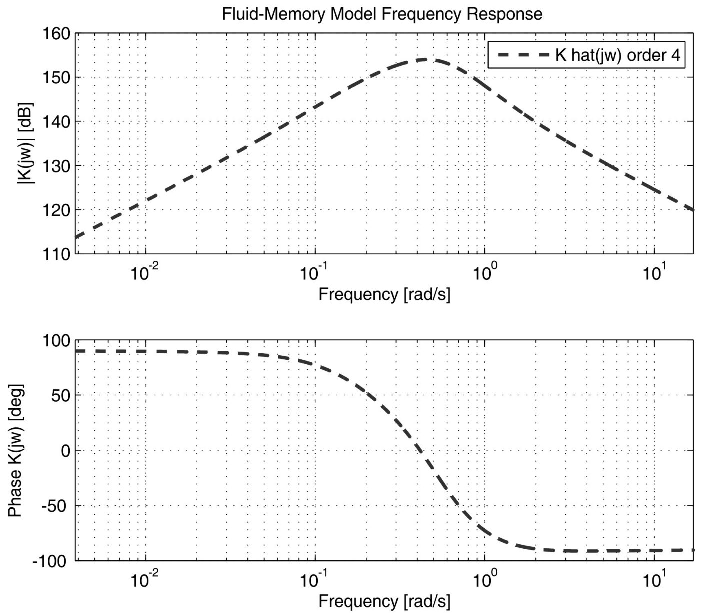
The fluid memory effects can be approximated using frequency-domain identification. The main tool for this is the **MSS FDI toolbox** (Perez and Fossen 2009)

When using the frequency-domain approach, the property that the mapping: $\delta\boldsymbol{\nu}_r \rightarrow \boldsymbol{\mu}_r$ has <u>relative degree one</u> is exploited. Hence, the fluid memory effects **μ** can be approximated by a matrix **H**(s) containing relative degree one transfer functions:

$$\boxed{\boldsymbol{\mu}_r = \boldsymbol{H}(s)\delta\boldsymbol{\nu}_r}$$

$$h_{ij}(s) = \frac{p_r s^r + p_{r-1}s^{r-1} + \ldots + p_0}{s^n + q_{n-1}s^{n-1} + \ldots + q_0}$$

$$r = n - 1, \quad n \geq 2$$



State-space model:

$$\mathbf{H}(s) = \mathbf{C}_r(s\mathbf{I} - \mathbf{A}_r)^{-1}\mathbf{B}_r$$

$$\dot{\boldsymbol{x}} = \boldsymbol{A}_r\boldsymbol{x} + \boldsymbol{B}_r\delta\boldsymbol{\nu}_r$$
$$\boldsymbol{\mu}_r = \boldsymbol{C}_r\boldsymbol{x}$$

Perez, T. and T. I. Fossen (2009). A Matlab Toolbox for Parametric Identification of Radiation-Force Models of Ships and Offshore Structures. Modeling, Identification and Control 30(1), 1–15.

5.5 Frequency-Domain Identification using the MSS FDI Toolbox

Matlab:
The fluid memory transfer function (5.115) can be computed using the MSS toolbox, which includes the FDI toolbox for frequency-domain identification (Perez and Fossen 2009). The toolbox includes two demo files for the cases where infinite added mass is unknown (2-D strip theory codes) or computed by the hydrodynamic code, for instance the 3-D code by WAMIT.

Example 5.2 (Computation of Fluid Memory Effects)
Consider the FPSO data set in the MSS toolbox and assume that the infinite-frequency added mass matrix is unknown. Hence, we can estimate the fluid transfer function $h_{33}(s)$ by using the following Matlab code

```
load fpso
Dof = [3,3];                    % Use coupling 3-3 heave-heave
Nf = length(vessel.freqs);
w = vessel.freqs(1:Nf-1)';
Ainf = vessel.A(Dof(1),Dof(2),Nf)  % Ainf computed by WAMIT
Ainf =
   1.7283e+08

A = reshape(vessel.A(Dof(1),Dof(2),1:Nf-1),1,length(w))';
B = reshape(vessel.B(Dof(1),Dof(2),1:Nf-1),1,length(w))';
```

A fourth-order transfer function of relative degree 1 is found by using the following options (see Perez and Fossen (2009) for a more detailed explanation).

```
FDIopt.OrdMax = 20;
FDIopt.AinfFlag = 0;
FDIopt.Method = 2;
FDIopt.Iterations = 20;
FDIopt.PlotFlag = 0;
FDIopt.LogLin = 1;
FDIopt.wsFactor = 0.1;
FDIopt.wminFactor = 0.1;
FDIopt.wmaxFactor = 5;

[Krad,Ainf_hat] = FDIRadMod(W,A,0,B,FDIopt,Dof)

Krad =

      1.647e07 s^3 + 2.358e07 s^2 + 2.122e06 s
  --------------------------------------------------
  s^4 + 1.253 s^3 + 0.7452 s^2 + 0.2012 s + 0.01686

Ainf_hat =
   1.7265e+08
```

The state-space model (5.119) is obtained by calling

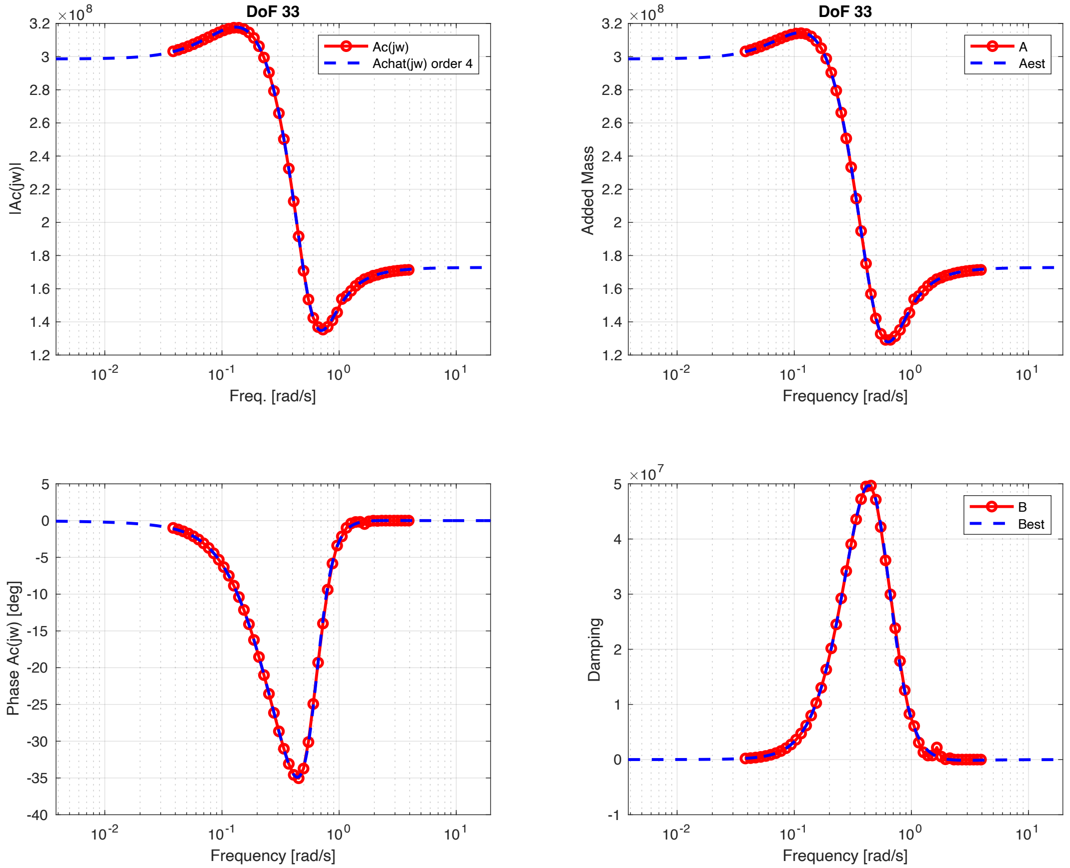
```
[num,den] = tfdata(Krad,'v');
[A_r,B_r,C_r,D_r] = tf2ss(num,den)

A_r =
   -1.2529   -0.7452   -0.2012   -0.0169
    1.0000         0         0         0
         0    1.0000         0         0
         0         0    1.0000         0
B_r =
     1
     0
     0
     0
C_r =
   1.0e+07 *
    1.6472    2.3582    0.2122         0
D_r =
     0
```

5.5 Frequency-Domain Identification using the MSS FDI Toolbox

FPSO identification results for $h_{33}(s)$ without using the infinite added mass $A_{33}(\infty)$. The left-hand-side plots show the complex coefficient

$$\tilde{A}(j\omega) \triangleq \frac{B_{ik}(\omega)}{j\omega} + A_{ik}(\omega)$$

and its estimate while added mass $A(j\omega)$ and potential damping $B(j\omega)$ are plotted on the right-hand-side.



Chapter Goals - Revisited

- Understand seakeeping coordinates {s} and how they relate to BODY and NED
- Understand frequency-dependent hydrodynamic matrices and their application
- Application of Cummins equation to transform frequency-dependent matrices to the time domain

Computed using hydrodynamic seakeeping codes

- $\boldsymbol{A}(\omega)$ added mass matrix
- $\boldsymbol{B}(\omega)$ potential damping matrix
- $\boldsymbol{B}_V(\omega)$ viscous damping matrix
- $\boldsymbol{C}$ spring stiffness or hydrostatic matrix

$$[\boldsymbol{M}_{RB} + \boldsymbol{A}(\omega)]\ddot{\boldsymbol{\xi}} + \boldsymbol{B}_{\text{total}}(\omega)\dot{\boldsymbol{\xi}} + \boldsymbol{C}\boldsymbol{\xi} = \boldsymbol{\tau}_{\text{exc}}$$

$$\boldsymbol{B}_{\text{total}}(\omega) = \boldsymbol{B}(\omega) + \boldsymbol{B}_V(\omega)$$

Transformation from SEAKEEPING {s} to BODY axes {b}

$$\begin{array}{lr} \text{Inertia forces:} & \boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}^{*}\boldsymbol{\nu} + \boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_A^{*}\boldsymbol{\nu}_r \\ \text{Damping forces:} & +(\boldsymbol{D}_P + \boldsymbol{D}_V)\boldsymbol{\nu}_r + \boldsymbol{\mu}_r \\ \text{Restoring forces:} & +\boldsymbol{G}\boldsymbol{\eta} + \boldsymbol{g}_o \\ \text{Wind and wave forces:} & = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} \\ \text{Propulsion forces:} & +\boldsymbol{\tau} \end{array}$$

Linear model used for control design and simulation