Chapter 9 – Control Forces and Moments

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9.8 Control Moment Gyroscope
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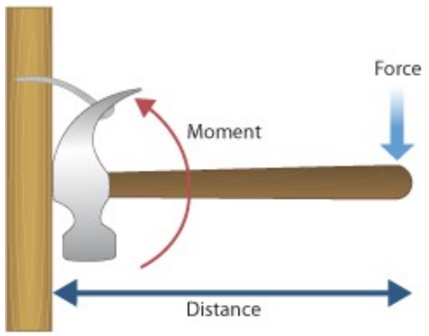
The overall goal of Chapter 9 is to present mathematical models for the control forces and moments acting on ships, floating structures, underwater vehicles and USVs.



Chapter Goals

- Be able to model thrusters, propellers, control surfaces and propulsion systems and add the models to the equations of motion.
- Understand how KT and KQ curves are used to compute propeller thrust and torque.
- Be able to explain what a Bollard pull test is and relate this to the KT and KQ curves.
- Understand how a propeller in a slipstream produces lift and drag, and how these quantities relate to the propeller yaw moment needed to turn a vehicle or a ship.
- Understand how control surfaces are used to dive and turn underwater vehicles.
- Understand how fin stabilizers are used.
- Understand how control moment gyros (CMGs) can be used to control the attitude of a vehicle even at zero speed.
- Understand how a moving mass can be used to control the attitude of a vehicle.

9.1 Propellers as Thrust Devices

Fixed-Pitch (FP) Propellers

$$J_a = \frac{u_a}{nD}$$

$$u_a = (1 - w)\, u$$

only for steady state

J_a = open-water advance coefficient
D = propeller diameter
n = propeller revolution
u_a = advance speed

Thrust and torque

$$T = \rho D^4 K_T(J_a)\, |n|n$$
$$Q = \rho D^5 K_Q(J_a)\, |n|n$$

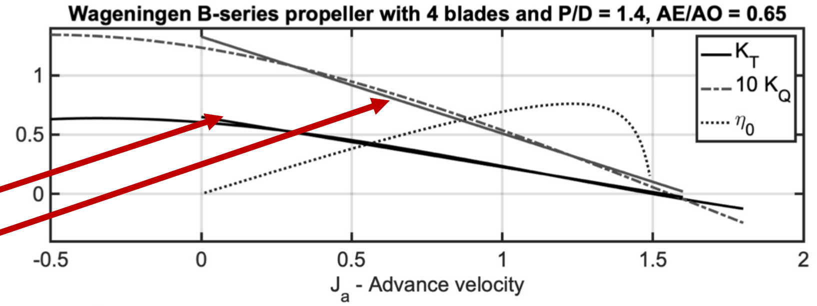
where

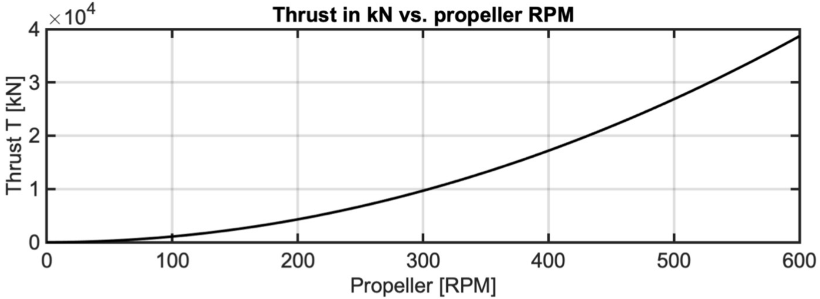
$$K_T = \frac{T}{\rho D^4 |n|n}, \qquad K_Q = \frac{Q}{\rho D^5 |n|n},$$

Linear approximations

$$K_T(J_a) \approx \alpha_1 - \alpha_2\, J_a$$
$$K_Q(J_a) \approx \beta_1 - \beta_2\, J_a$$

$$T = T_{|n|n}\, |n|n - T_{|n|u_a}\, |n|u_a$$
$$Q = Q_{|n|n}\, |n|n - Q_{|n|u_a}\, |n|u_a$$

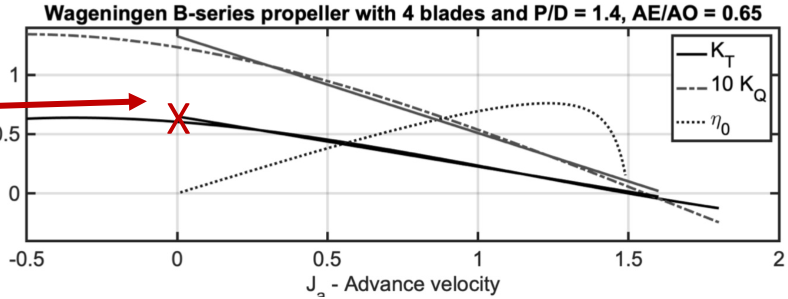




9.1 Propellers as Thrust Devices

For marine craft, it is common to approximate the thrust coefficients at $J_a = 0$ such that

$$F_y = \rho D^4 K_T(0)|n|n \approx T_{|n|n}\,|n|n$$



Bollard pull is a conventional measure of the pulling (or towing) power of a watercraft. It is defined as the force (in tons or kilonewtons (kN)) exerted by a vessel under full power, on a shore-mounted bollard through a tow-line.

The bollard pull of a vessel may be reported as two numbers:

1) The static or maximum bollard pull - the highest force measured
2) The steady or continuous bollard pull, the average of measurements over an interval of, for example, 10 minutes.

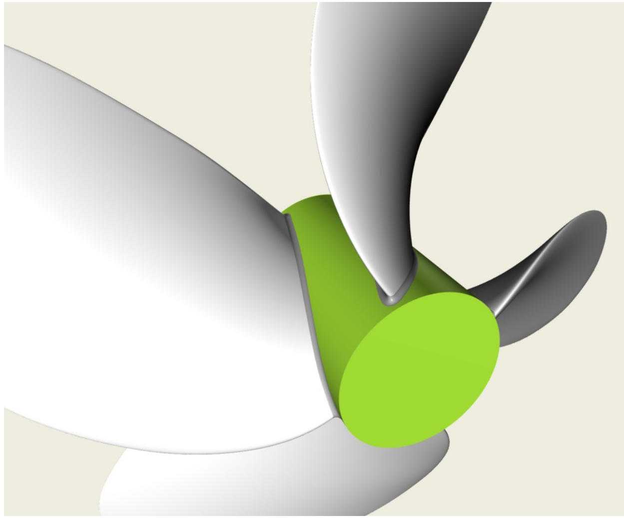
A bollard is a sturdy, short, vertical post. The term originally referred to a post on a ship or quay used principally for mooring boats.

From Wikipedia - the free encyclopedia

MSS Toolbox – Wageningen B-series Propellers

Matlab:
The Wageningen B-series of propellers were designed and tested at the Netherlands ship model basin in Wageningen (Barnitsas *et al.* 1981). The open-water characteristics of 120 propellers were tested and fitted to polynomials. The data set is available in the MSS toolbox as `WageningData.mat`. The propellers can be configured in Matlab using:

```
% see ExWageningen.m
rho = 1025;    % Density of water (kg/m^3)
D = 5;         % Propeller diameter (m)
PD = 1.4;      % pitch/diameter ratio
AEAO = 0.65;   % blade area ratio
z = 4;         % number of propeller blades


% Comput KT and KQ for advance velocites Ja
Ja = -0.8:0.01:1.8;
for i = 1:length(Ja)
    [KT(i), KQ(i)] = wageningen(Ja(i),PD,AEAO,z);
end

% Compute KT and KQ for Ja = 0 (Bollard pull)
[KT_0, KQ_0] = wageningen(0,PD,AEAO,z);

% Compute thrust [N]
n = 0:0.1:10;                          % propeller [RPS]
T = rho * D^4 * KT_0 * n .* abs(n);    % thrust [N]

% Fit KT and KQ data to straight lines
Jdata = 0:0.01:1.6;
for i = 1:length(Jdata)
    [KTdata(i), KQdata(i)] = wageningen(Jdata(i),PD,AEAO,z);
end

alpha = polyfit(Jdata,KTdata,1)     % KT = alpha(1)*Ja + alpha(2)
beta  = polyfit(Jdata,KQdata,1)     % KQ = beta(1) *Ja + beta(2)
```



http://www.wageningen-b-series-propeller.com

9.1 Propellers as Thrust Devices

Controllable-Pitch (CP) Propellers

CP propellers are screw-blade propellers where the blades can be turned under the control of a hydraulic servo system. CP propellers are used where maneuvering properties need to be improved, where a ship has equipment that requires constant shaft speed, or with most twin-screw ships.

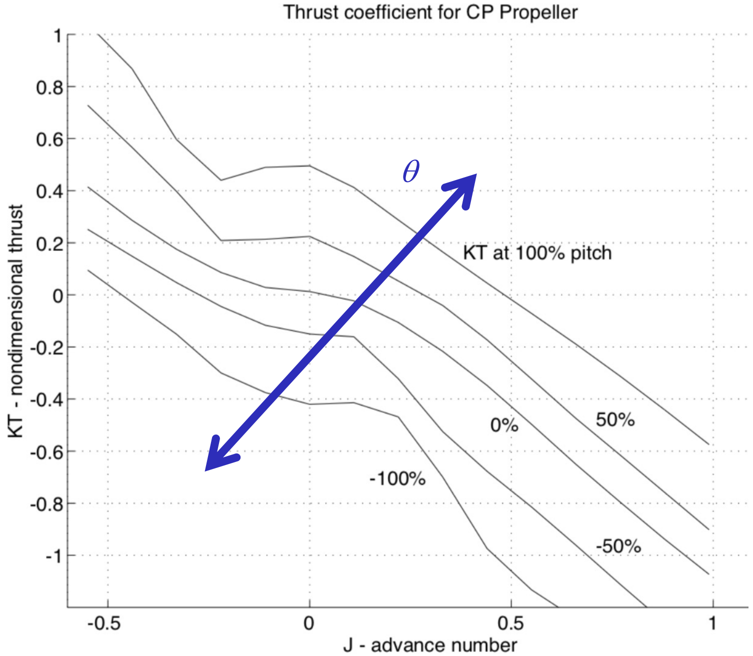
$$T = T_{|n|n}(\theta)\,|n|n - T_{|n|u_a}(\theta)\,|n|u_a$$
$$Q = Q_{|n|n}(\theta)\,|n|n - Q_{|n|u_a}(\theta)\,|n|u_a$$

The coefficients $T_{|n|n}(\theta)$, $T_{|n|ua}(\theta)$, $Q_{|n|n}(\theta)$ and $Q_{|n|ua}(\theta)$ are complex functions of the pitch angle θ.

Linear approximation

$$K_T(J_a) \approx (\alpha_1 - \alpha_2 J_a)\,\theta$$

$$T \approx T_{\theta|n|n}\,\theta\,|n|n - T_{\theta|n|u_a}\,\theta|n|u_a$$



9.2 Podded Propulsion Units

A podded propulsion unit consists of a FP propeller mounted on a steerable gondola (pod), which also contains the electric motor driving the propeller. In a podded unit, an electric motor is mounted inside the propulsion unit and the propeller is connected directly to the motor shaft.

http://www.kongsberg.com

$$T = \rho D^4 K_T(J_a)|n|n \approx T_{|n|n}\,|n|n - T_{|n|u_a}\,(1-w)\,|n|u_r$$

$$\boldsymbol{\tau} = (1-t)\,T_{|n|n}\,|n|n \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \\ l_x\,\sin(\alpha) - l_y\,\cos(\alpha) \end{bmatrix} - \boldsymbol{d}_{\text{loss}}(n,\alpha)u_r$$

$$\boldsymbol{d}_{\text{loss}}(n,\alpha) = (1-t)(1-w)\,T_{|n|u_a}\,|n| \begin{bmatrix} \cos(\alpha) \\ \sin(\alpha) \\ l_x\,\sin(\alpha) - l_y\,\cos(\alpha) \end{bmatrix}$$

α = azimuth angle
n = RPM

http://www.wartsila.com

Matrix-vector form

$$\boldsymbol{M}\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + (\boldsymbol{D}(\boldsymbol{\nu}_r) + \boldsymbol{D}_{\text{loss}}(n,\alpha))\,\boldsymbol{\nu}_r = \boldsymbol{B}\boldsymbol{u} + \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}}$$

$$\boldsymbol{\tau} = \boldsymbol{B}\boldsymbol{u} - \boldsymbol{D}_{\text{loss}}(n,\alpha)\boldsymbol{\nu}_r$$

$$\boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ -l_y & l_x \end{bmatrix} \qquad \boldsymbol{D}_{\text{loss}}(n,\alpha) = \begin{bmatrix} \boldsymbol{d}_{\text{loss}}(n,\alpha) & \boldsymbol{0}_{3\times 2} \end{bmatrix}$$

Control allocation

$$u_1 = (1-t)\,T_{|n|n}\,|n|n\,\cos(\alpha)$$
$$u_2 = (1-t)\,T_{|n|n}\,|n|n\,\sin(\alpha)$$

$$|n|n = \frac{1}{(1-t)\,T_{|n|n}}\sqrt{u_1^2 + u_2^2}$$
$$\alpha = \text{atan2}(u_2, u_1)$$

9.2 Prime Mover System

A modern ship-speed propulsion system needs to include models for the rotational shaft dynamics, engine dynamics and governor. The dynamics of the prime mover and its control system is tightly coupled to the speed dynamics of the ship via the propeller thrust T and torque Q

$$
\begin{aligned}
I_m \dot{n} &= Q_m - Q - Q_f \\
(m - X_{\dot{u}})\, \dot{u}_r &= X_{|u|u}\, |u_r| u_r + (1 - t)\, T + d_u
\end{aligned}
$$

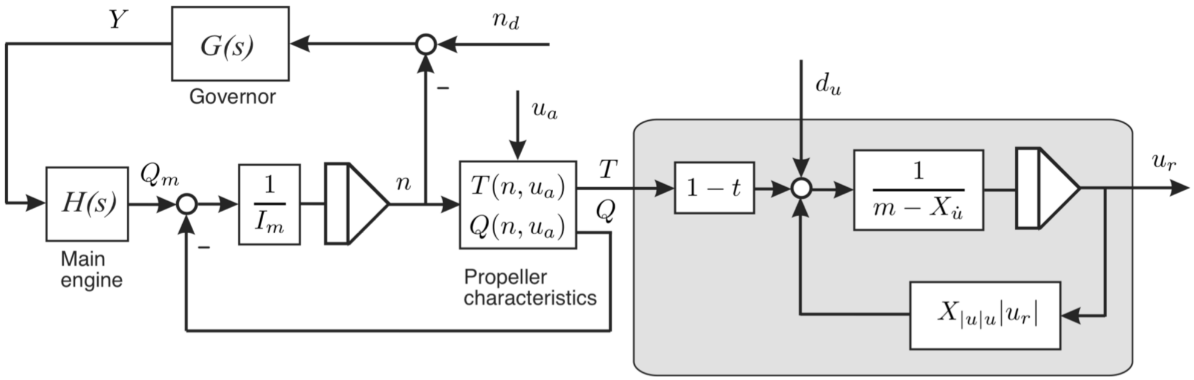
n	shaft speed (rad/s)
I_m	inertia of the rotating parts including the propeller and added inertia of the water (kg m^2)
Q	propeller torque (Nm)
Q_m	produced torque developed by the diesel engine (Nm)
Q_f	friction torque (Nm)
u_r	relative surge velocity (m/s)
d_u	unmodeled dynamics and external disturbances(N)

Simplified model of the main engine

$$
H(s) = \frac{Q_m}{Y}(s) = \frac{K}{Ts+1} e^{-\tau s}
$$

Governor

Speed limiter or controller used to measure and regulate the speed of the engine.



9.3 USV and Underwater Vehicle Propulsion Systems

Small USVs and underwater vehicles use propellers for propulsion, maneuvering, attitude control and dynamic positioning.

$$\boxed{J_m \dot{n} + K_n\, n = Q_m - Q}$$

$$T = T_{|n|n}\, |n|n - T_{|n|u_a}\, |n|u_a \overset{u_a=0}{=} T_{|n|n}\, |n|n$$
$$Q = Q_{|n|n}\, |n|n - Q_{|n|u_a}\, |n|u_a \overset{u_a=0}{=} Q_{|n|n}\, |n|n$$

$$T = \rho D^4 K_T(J_a)\, |n|n$$
$$Q = \rho D^5 K_Q(J_a)\, |n|n$$

Maritime Robotics Otter USV

DC motor

$$L_a \frac{d}{dt} i_m = -R_a i_m - K_m n + V_m$$
$$Q_m = K_m i_m$$

$$\frac{L_a}{R_a} \frac{d}{dt} i_m \approx 0$$

$$0 = -R_a i_m - K_m n + V_m$$
$$J_m \dot{n} = K_m i_m - Q$$

$$\boxed{J_m \dot{n} + \frac{K_m^2}{R_a} n = \frac{K_m}{R_a} V_m - Q}$$

9.3 Motor Armature Current Control

DC motor

$$\boxed{J_m \dot{n} + \frac{K_m^2}{R_a} n = \frac{K_m}{R_a} V_m - Q}$$

Torque/thrust ratio

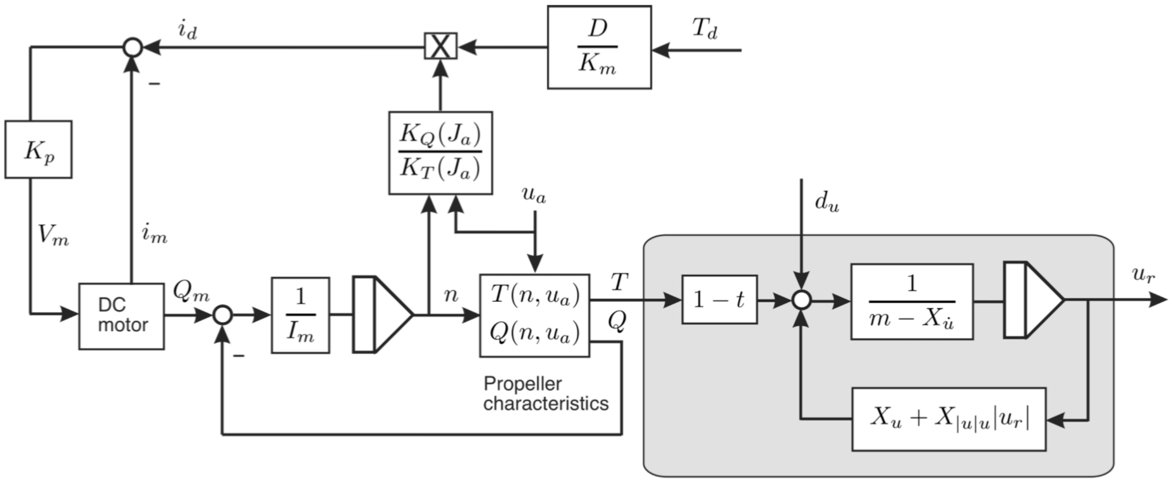
$$Q = D \frac{K_Q(J_a)}{K_T(J_a)} T$$

P controller for armature current

$$V_m = K_p(i_d - i_m), \qquad K_p > 0$$

$$Q_d = K_m i_d$$

$$i_d = \frac{D}{K_m} \frac{K_Q(J_a)}{K_T(J_a)} T_d$$



9.3 Motor RPM Control

DC motor

$$\boxed{J_m \dot{n} + \frac{K_m^2}{R_a} n = \frac{K_m}{R_a} V_m - Q}$$

$$T_d = \rho D^4 K_T(0) |n_d| n_d$$

P controller for propeller revolution

$$V_m = K_p (n_d - n), \qquad K_p > 0$$

$$n_d = \text{sgn}(T_d) \sqrt{\frac{|T_d|}{\rho D^4 K_T(0)}}$$

Video: Maritime Robotics Otter USV
http://www.maritimerobotics.com

9.4 Thrusters

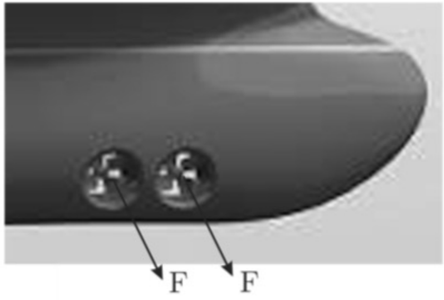
- **Tunnel thruster –** Propeller installed in a transverse tunnel, producing a transverse force. Both bow and stern thrusters are used to dynamically position a ship or a boat.
- **Externally mounted thrusters –** Instead of a tunnel thruster, marine craft may have externally mounted thrusters. They are usually used by craft where it is impossible or undesirable to install a tunnel thruster, due to hull shape or outfitting.
- **Azimuth thruster –** Ship propellers placed in pods that can be rotated on the horizontal plane, making a rudder unnecessary. This is also referred to as thrust vectoring since both the magnitude and direction of the force can be controlled.
- **CRP thruster –** An azimuthing thruster equipped with twin contra-rotating propellers.
- **Jet thruster –** A pump arranged to take suction from beneath or close to the keel and to discharge to either side, to develop port or starboard thrust, or in many cases through 360°.

The generalized force in 6 DOFs corresponding to the thrust vector $\boldsymbol{f}_t^b = [F_x,\ F_y,\ F_z]^\top$ expressed in $\{b\}$ is

$$\boldsymbol{\tau} = \begin{bmatrix} \boldsymbol{f}_t^b \\ \boldsymbol{r}_{tb}^b \times \boldsymbol{f}_t^b \end{bmatrix} = \begin{bmatrix} F_x \\ F_y \\ F_z \\ l_y F_z - l_z F_y \\ l_z F_x - l_x F_z \\ l_x F_y - l_y F_x \end{bmatrix} \quad \underset{\Longrightarrow}{\text{surge, sway and yaw}} \quad \boldsymbol{\tau} = \begin{bmatrix} F_x \\ F_y \\ l_x F_y - l_y F_x \end{bmatrix}$$

where $\boldsymbol{r}_{tb}^b = [l_x,\ l_y,\ l_z]^\top$ is a vector of thruster lever arms with respect to the CO.

9.4 Tunnel Thrusters



The figure shows two tunnel thrusters in the the bow of a ship

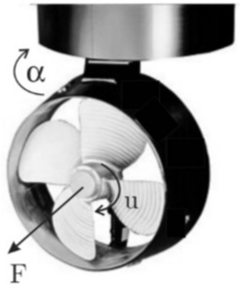
Highly effective at low speeds (< 2 m/s) but not in transit

For tunnel thrusters the advance speed u_a will be small. Hence, $J_a \approx 0$ (Bollard pull) and the propeller produces a transverse force, which gives the generalized force

$$\boxed{\boldsymbol{\tau} = [0,\ F_y,\ 0,\ 0,\ 0,\ l_x F_y]^\top}$$

$$\begin{aligned}\text{FP propeller:} \quad & F_y = \rho D^4 K_T(0)|n|n \approx T_{|n|n}\,|n|n \\ \text{CP propeller:} \quad & F_y \approx T_{\theta|n|n}\,\theta\,|n|n\end{aligned}$$

9.4 Azimuth Thrusters



Azimuth thruster

Azimuth thrusters can be rotated an angle α about the z axis and produce two force components (F_x, F_y) in the horizontal plane.

They are usually mounted under the hull of the craft and the most sophisticated units are retractable. Azimuth thrusters are frequently used in DP systems since they can produce forces in different directions.

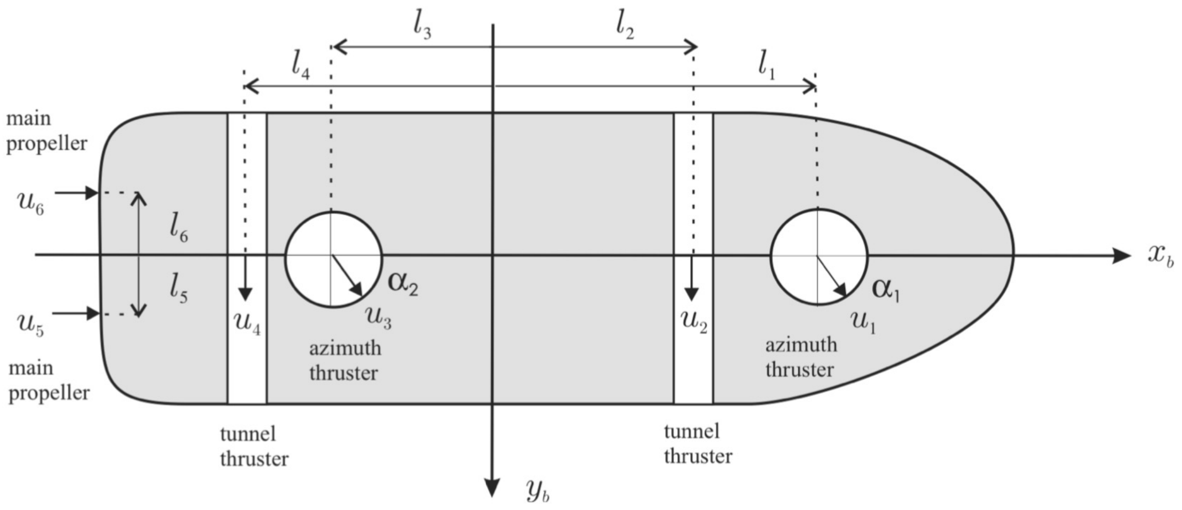
For azimuth thrusters the advance speed u_a will be small. Hence, $J_a \approx 0$ (Bollard pull) and the generalized force becomes

$$\boxed{\boldsymbol{\tau} = [F\cos(\alpha),\ F\sin(\alpha),\ 0,\ 0,\ 0,\ l_x F\sin(\alpha) - l_y F\cos(\alpha)]^\top}$$

$$\text{FP propeller:} \quad F = \rho D^4 K_T(0)|n|n \approx T_{|n|n}\,|n|n$$

$$\text{CP propeller:} \quad F \approx T_{\theta|n|n}\,\theta\,|n|n$$

9.4 Thrust Configuration Matrix for a DP Vessel



Tunnel thrusters and main propellers

$$F_i = \rho D_i^4 K_{T_i}(0)|n_i|n_i := K_i u_i \qquad u_i := |n_i|n_i$$

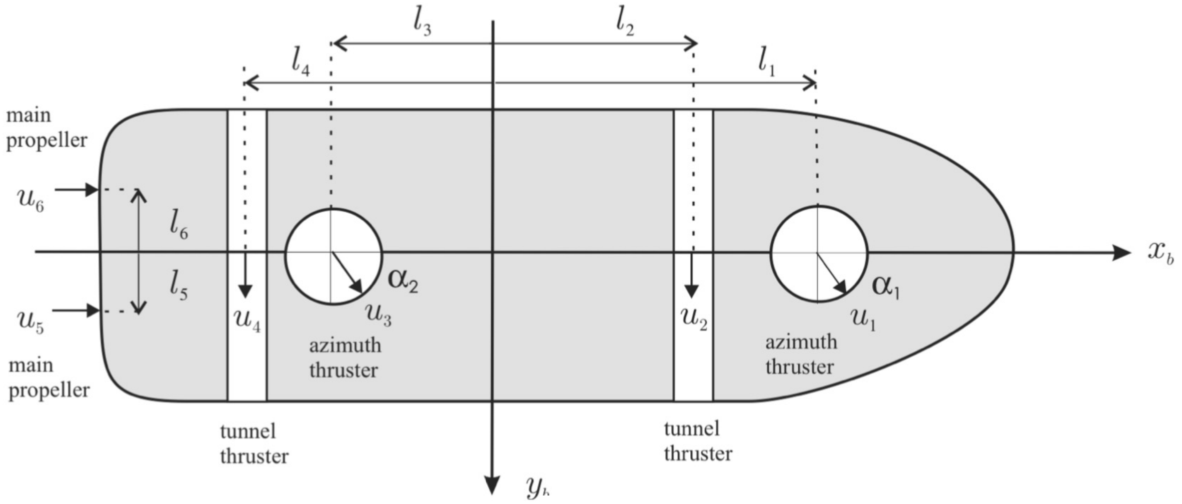
$$K_i := \rho D_i^4 K_{T_i}(0)$$

Azimuth thrusters (two virtual forces)

$$F_{ix} = K_i u_{ix} \qquad u_{ix} := u_i \cos(\alpha_i)$$

$$F_{iy} = K_i u_{iy} \qquad u_{iy} := u_i \sin(\alpha_i)$$

9.4 Thrust Configuration Matrix for a DP Vessel



$$\boldsymbol{\tau} = \boldsymbol{T}\boldsymbol{K}\boldsymbol{u}$$

$$\Updownarrow$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & l_1 & l_2 & 0 & l_3 & l_4 & -l_5 & -l_6 \end{bmatrix} \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_6 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_2 \\ u_{3x} \\ u_{3y} \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}$$

9.4 Thrust Configuration Matrix for a DP Vessel

$$\boldsymbol{\tau} = \boldsymbol{T}\boldsymbol{K}\boldsymbol{u}$$

$$\Updownarrow$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_6 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & l_1 & l_2 & 0 & l_3 & l_4 & -l_5 & -l_6 \end{bmatrix} \begin{bmatrix} K_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & K_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & K_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & K_3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & K_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & K_4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & K_5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & K_6 \end{bmatrix} \begin{bmatrix} u_{1x} \\ u_{1y} \\ u_2 \\ u_{3x} \\ u_{3y} \\ u_4 \\ u_5 \\ u_6 \end{bmatrix}$$

Control allocation

$$\begin{aligned} \boldsymbol{u} &= \boldsymbol{B}^{\dagger}\,\boldsymbol{\tau} \\ &= \boldsymbol{K}^{-1}\boldsymbol{T}^{\dagger}\,\boldsymbol{\tau} \end{aligned}$$

RPM from control inputs

$$u_i := |n_i|n_i$$

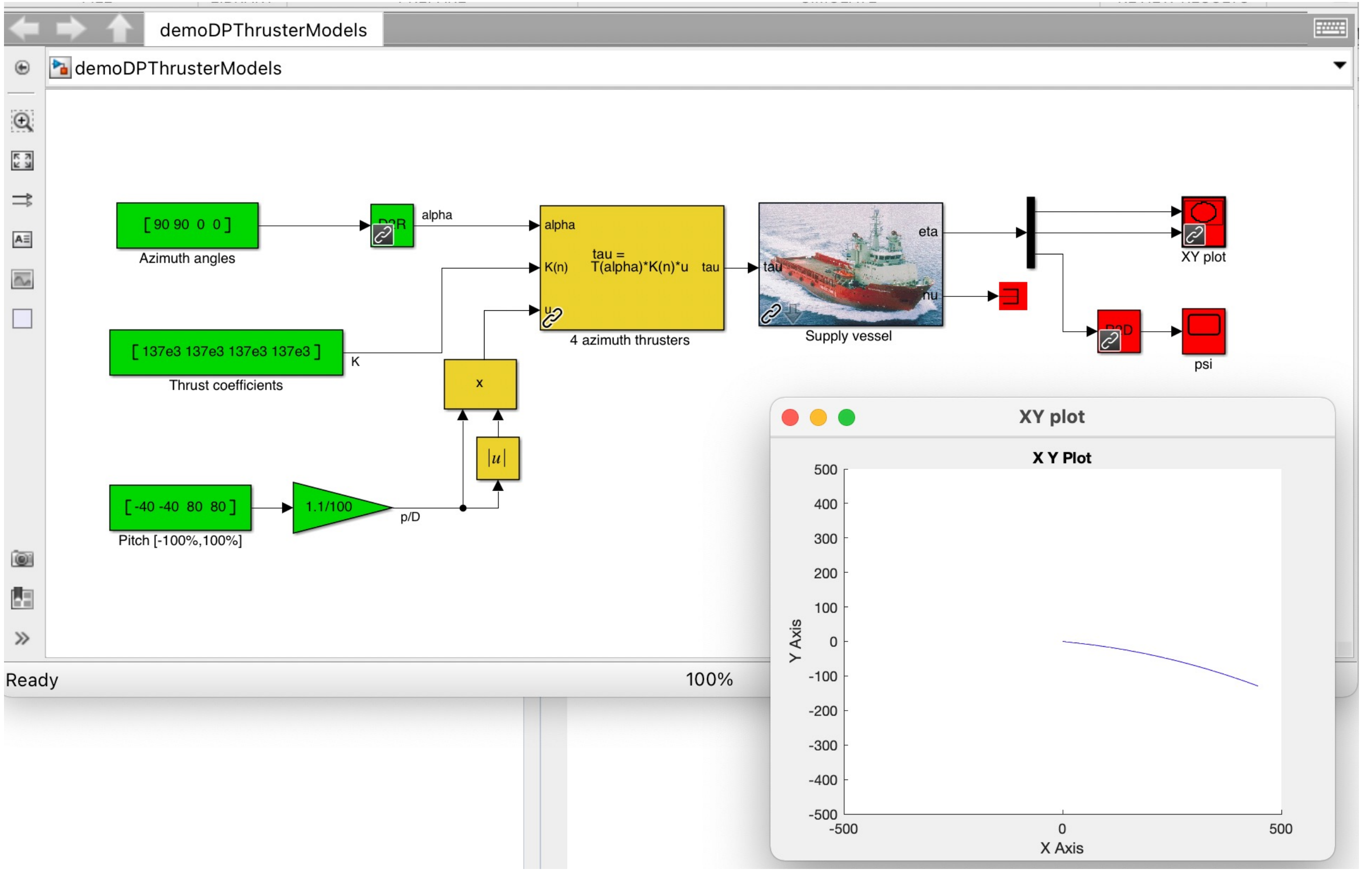
$$n_i = \mathrm{sgn}(u_i)\sqrt{|u_i|}$$

Azimuth angle from control inputs

$$u_1 = \sqrt{u_{1x}^2 + u_{1y}^2}, \qquad \alpha_1 = \mathrm{atan2}(u_{1y}, u_{1x})$$

$$u_3 = \sqrt{u_{3x}^2 + u_{3y}^2}, \qquad \alpha_3 = \mathrm{atan2}(u_{3y}, u_{3x})$$

MSS Toolbox: DP Ship with Azimuth Thrusters



9.5 Rudder in Propeller Slipstream

Rudder normal force

$$F_N = \frac{1}{2}\rho U_R^2 A_R C_N \sin(\alpha_R)$$

U_R = rudder inflow speed
α_R = effective rudder angle

$$\boxed{C_N = \frac{6.13\Lambda}{\Lambda + 2.25}}$$

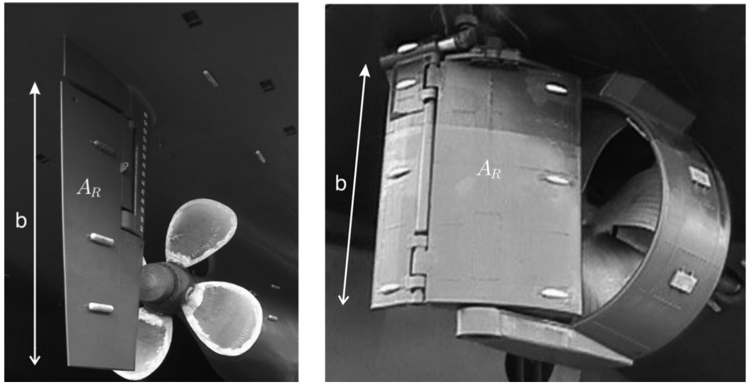
$$U_R = \sqrt{u_R^2 + v_R^2}$$

$$\alpha_R = \delta - \tan^{-1}\left(\frac{v_R}{u_R}\right) \approx \delta - \beta_R$$

Aspect ratio

$$\Lambda = \frac{b^2}{A_R}$$

b is taken as the length of the rudder and A_R is the rudder area



Left: Conventional stern rudder for a ship. Right: High-lift flap rudder consisting of two or more sections which move relative to each other as helm is applied.

$$X_R = -(1 - t_R) F_N \sin(\delta)$$
$$Y_R = -(1 + a_H) F_N \cos(\delta)$$
$$N_R = -(x_R + a_H x_H) F_N \cos(\delta)$$

Small-angle approximation

$$\alpha_R \approx \delta$$
$$U_R \approx u_R$$

$$u_R = \varepsilon u (1 - w_P)\sqrt{\eta\left(1 + \kappa\left(\sqrt{1 + \frac{8K_T}{\pi J_a^2}} - 1\right)\right)^2 + (1 - \eta)}$$

9.5 Rudder in Propeller Slipstream

Generalized force (surge, sway and yaw)

$$X_R = -(1 - t_R)\left(\frac{1}{2}\rho U_R^2 A_r C_N\right)\sin^2(\delta)$$

$$Y_R = -(1 + a_H)\left(\frac{1}{2}\rho U_R^2 A_r C_N\right)\frac{1}{2}\sin(2\delta)$$

$$N_R = -(x_R + a_H x_H)\left(\frac{1}{2}\rho U_R^2 A_r C_N\right)\frac{1}{2}\sin(2\delta)$$

The surge force increases resistance during turning, while the sway force tries to move the ship sideways.

The yaw moment is used by the autopilot to turn the ship

$$\boldsymbol{\tau}_R = \begin{bmatrix} -\frac{1}{2}(1 - t_R)\rho U_R^2 A_r C_N \sin^2(\delta) \\ -\frac{1}{4}(1 + a_H)\rho U_R^2 A_r C_N \sin(2\delta) \\ -\frac{1}{4}(x_R + a_H x_H)\rho U_R^2 A_r C_N \sin(2\delta) \end{bmatrix} \approx \begin{bmatrix} -X_{\delta\delta}\,\delta^2 \\ -Y_\delta\,\delta \\ -N_\delta\,\delta \end{bmatrix}$$

Rudder coefficients

$$X_{\delta\delta} = \frac{1}{2}(1 - t_R)\rho U_R^2 A_R C_N > 0$$

$$Y_\delta = \frac{1}{2}(1 + a_H)\rho U_R^2 A_R C_N > 0$$

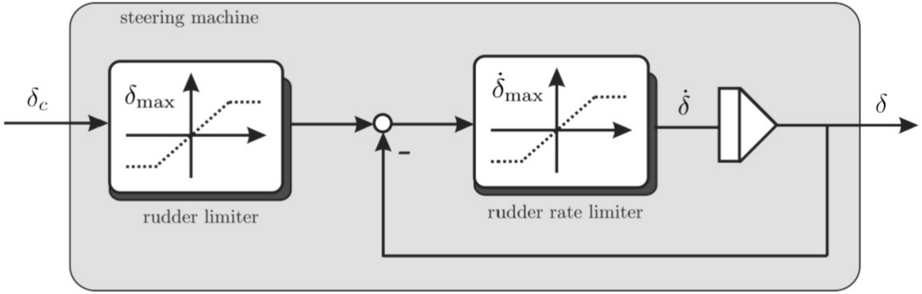
$$N_\delta = \frac{1}{2}(x_R + a_H x_H)\rho U_R^2 A_R C_N < 0$$

$$U_R \approx u_R$$

$$u_R = \varepsilon u(1 - w_P)\sqrt{\eta\left(1 + \kappa\left(\sqrt{1 + \frac{8K_T}{\pi J_a^2}} - 1\right)\right)^2 + (1 - \eta)}$$

9.5 Steering Machine Dynamics

The steering machine, hydraulic or electric, is controlled by a feedback control system which ensures that the rudder angle δ is close to the commanded rudder angle δ_c.

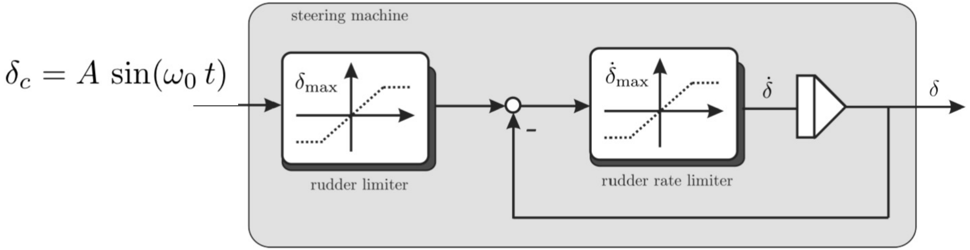


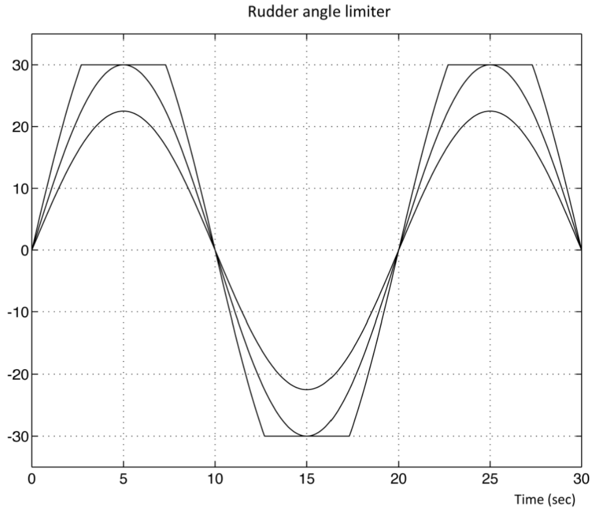
The steering machine is a nonlinear system with two important physical limitations: the maximum rudder angle and rudder rate.

In computer simulations and when designing autopilots, (Van Amerongen 1982) suggests using a simplified representation of the steering machine where the maximum rudder angle δ_{max} and rudder rate $\dot{\delta}_{max}$ are specified

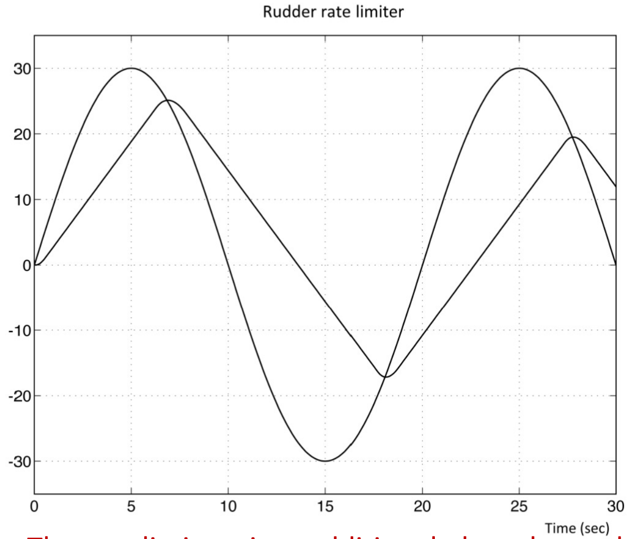
$$\delta_{\max} = 35^\circ; \qquad 2.3\ (\mathrm{deg/s}) \leq \dot{\delta}_{\max}$$

9.5 Influence of Rudder Limiters





The angle limiter gives reduced performance



The rate limiter gives additional phase lag and reduced stability margins

9.6 Fin Stabilizators

Fin stabilizers are primarily used for roll damping. They provide considerable damping if the speed of the ship is not too low.

The disadvantage with additional fins is increased hull resistance and high costs associated with the installation, since at least two new hydraulic systems must be installed.

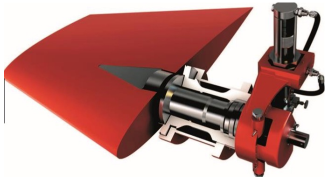
Retractable fins are popular, since they are inside the hull when not in use (no additional drag). It should be noted that fins are not effective at low speed and that they cause underwater noise in addition to drag.

$$F_{\text{drag}} = \frac{1}{2}\rho V_r^2 A_F C_D(\alpha_F)$$

$$F_{\text{lift}} = \frac{1}{2}\rho V_r^2 A_F C_L(\alpha_F)$$

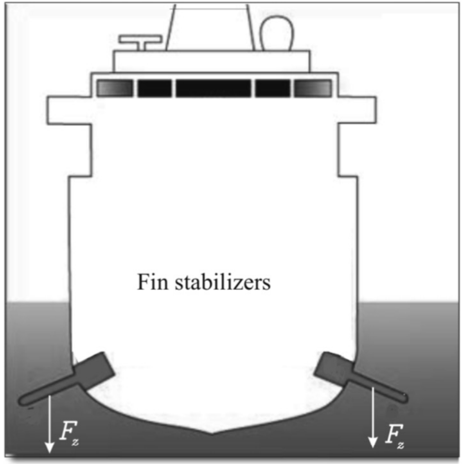
$$\begin{bmatrix} F_x \\ F_z \end{bmatrix} = \begin{bmatrix} \cos(\alpha_F) & -\sin(\alpha_F) \\ \sin(\alpha_F) & \cos(\alpha_F) \end{bmatrix} \begin{bmatrix} -F_{\text{drag}} \\ -F_{\text{lift}} \end{bmatrix}$$

$$\boxed{\tau_4 = 2\, l_y F_z}$$



http://www.wartsila.com

http://www.fincanteri.com



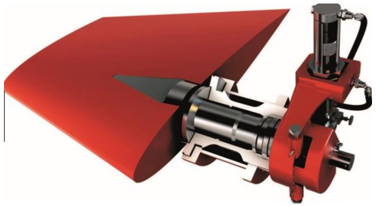
9.6 Fin Stabilizators

Consider a ship with a pair of symmetrical fin stabilizers. If the two control signals are chosen equal, the roll moment becomes

$$\boxed{\tau_4 = 2\, l_y F_z}$$

$$\boxed{F_z = -F_{\text{drag}} \sin(\alpha_F) - F_{\text{lift}} \cos(\alpha_F)}$$

$$C_L(\alpha_F) \approx C_{L_0} + C_{L_\alpha} \alpha_F \qquad C_{L_0} = C_{D_0} = 0$$
$$C_D(\alpha_F) \approx C_{D_0} + C_{D_\alpha} \alpha_F$$



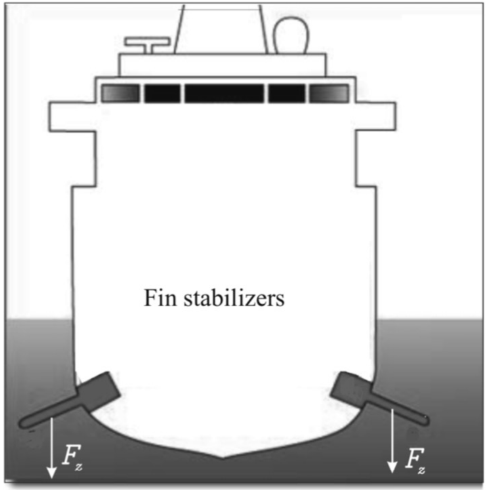
http://www.wartsila.com

Small-angle approximation

$$\sin(\alpha_F) \approx \alpha_F \qquad \alpha_F^2 \text{ can be neglected}$$

$$F_z \approx -\frac{1}{2}\rho V_r^2 A_F \left(C_{D_\alpha} \alpha_F^2 + C_{L_\alpha} \alpha_F\right)$$

$$\tau_4 = -l_y \rho A_F C_{L_\alpha} V_r^2 \alpha_F$$

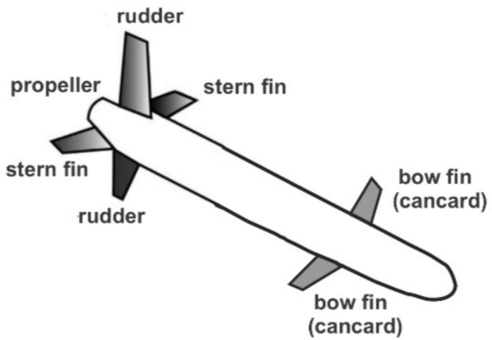


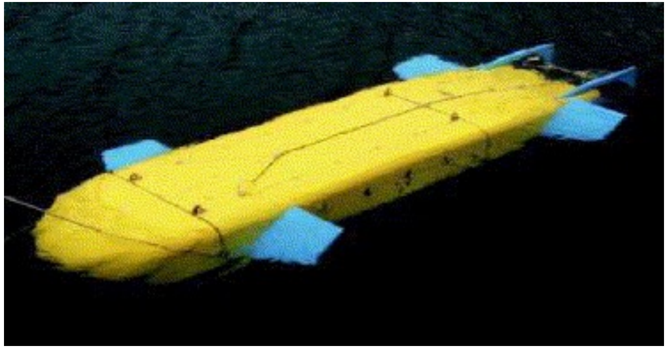
9.7 Underwater Vehicle Control Surfaces

Control surfaces are hydrodynamic devices allowing an operator or autopilot to control the velocity and attitude of the craft. They are used on underwater vehicles such as submarines, torpedoes and AUVs.

The primary control surfaces are:

- **Rudders** – stern vertical surfaces used for turning
- **Dive planes** – bow and stern horizontal surfaces used for depth control





INFANTE-AUV. Picture courtesy of Dynamic Systems and Ocean Robotics Laboratory (DSOR), Instituto Superior Tecnico de Lisboa, Portugal.

Additional dive planes can be located in the front of the vehicle similar to canard wings on an aircraft.
Cancards increase the lift and pitch response of the vehicle but the downside is additional drag and fuel consumption.

9.7 Underwater Vehicle Control Surfaces

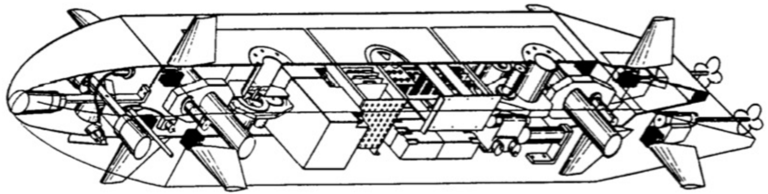
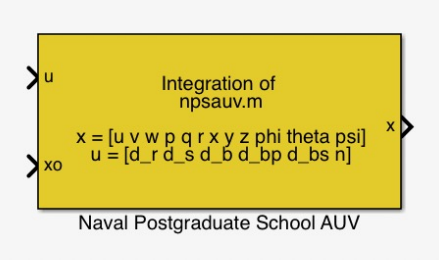


Figure 9.20: Schematic drawing of the NPS II AUV, length $L = 5.3$ m and weight $W = 53.4$ kN, showing the vertical rudder, propellers and dive planes (Healey and Lienard 1993).



mssSImulink library block for numerical integration of the m-file function npsauv

Matlab:

```
function [xdot,U] = npsauv(x,ui)
% States: x  = [ u v w p q r x y z phi theta psi ]'
% Inputs: ui = [ delta_r delta_s delta_b delta_bp delta_bs n ]'
% where
%   delta_r  = rudder angle                    (rad)
%   delta_s  = port and starboard stern plane  (rad)
%   delta_b  = top and bottom bow plane        (rad)
%   delta_bp = port bow plane                  (rad)
%   delta_bs = starboard bow plane             (rad)
%   n        = propeller shaft speed           (rpm)
```

9.7 Rudder

The rudder can be deflected an angle δ_R, which will force the vehicle to turn.
The rudder forces are function of the rudder lift and drag coefficients $C_L(\delta_R)$ and $C_D(\delta_R)$

$$F_{\text{drag}} = \frac{1}{2}\rho V_r^2 A_R C_D(\delta_R) \qquad C_L(\delta_R) \approx C_{L_0} + C_{L_\delta}\delta_R$$

$$F_{\text{lift}} = \frac{1}{2}\rho V_r^2 A_R C_L(\delta_R) \qquad C_D(\delta_R) \approx C_{D_0} + C_{D_\delta}\delta_R$$

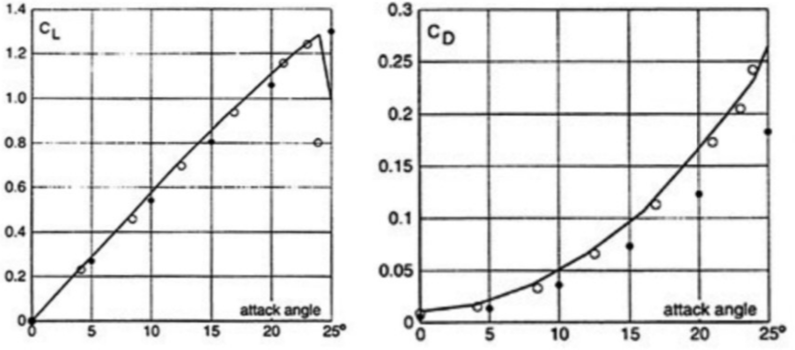


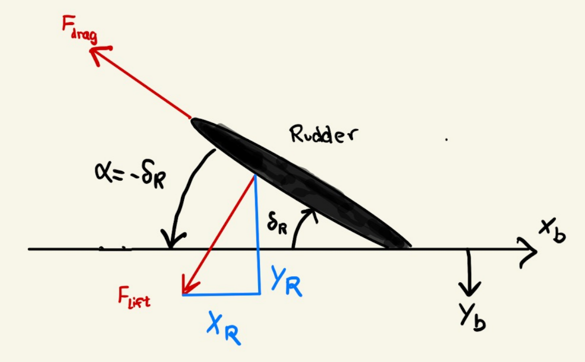
Figure 9.21: Experimental rudder lift and drag curves (circles) compared to Rans calculations (solid lines) (Söding 1990).

Approximations

Streamlined $\boxed{F_{\text{drag}} \approx 0.}$

Small angle $\sin(\delta_R) \approx \delta_R \quad \cos(\delta_R) \approx 1$

$\boxed{C_L(\delta_R) \approx C_{L_\delta}\delta_R}$ $\boxed{F_{\text{lift}} \approx \frac{1}{2}\rho V_r^2 A_R C_{L_\delta}\delta_R}$



Rudder forces

$$\begin{bmatrix} X_R \\ Y_R \end{bmatrix} = \begin{bmatrix} \cos(-\delta_R) & -\sin(-\delta_R) \\ \sin(-\delta_R) & \cos(-\delta_R) \end{bmatrix} \begin{bmatrix} -F_{\text{drag}} \\ -F_{\text{lift}} \end{bmatrix}$$

$$X_R = -\frac{1}{2}\rho U_R^2 A_r C_{L_\delta}\,\delta_R^2$$

$$Y_R = -\frac{1}{2}\rho U_R^2 A_r C_{L_\delta}\,\delta_R$$

The rotation matrix uses the rudder angle of attack: $\alpha = -\delta_R$ (rotation from FLOW to BODY axes)

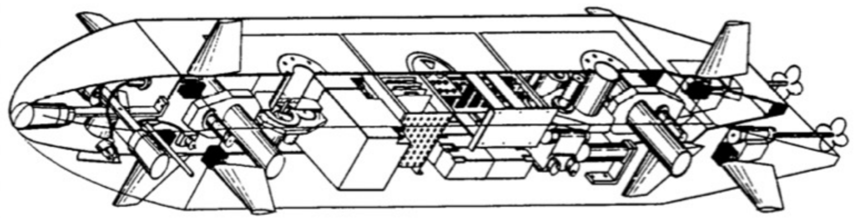
9.7 Rudder

Twin-rudder system

Two stern rudders located at $\mathbf{r}_{bR1} = [x_{1R}, y_{1R}, z_{1R}]^\top$ and $\mathbf{r}_{bR2} = [x_{2R}, y_{2R}, z_{2R}]^\top$

where

$$x_{1R} = x_{2R}$$
$$y_{1R} = -y_{2R}$$
$$z_{1R} = z_{2R}$$



Healey and Lienhard (1993)

$$\boldsymbol{\tau}_R \overset{\substack{\text{single}\\\text{rudder}}}{=} \begin{bmatrix} X_R \\ Y_R \\ 0 \\ -z_R Y_R \\ z_R X_R \\ x_R Y_R \end{bmatrix}, \quad \boldsymbol{\tau}_R \overset{\substack{\text{twin}\\\text{rudders}}}{=} \begin{bmatrix} X_{1R} + X_{2R} \\ Y_{1R} + Y_{2R} \\ 0 \\ -z_{1R} Y_{1R} - z_{2R} Y_{2R} \\ z_{1R} X_{1R} + z_{2R} X_{2R} \\ x_{1R} Y_{1R} - y_{1R} X_{1R} + x_{2R} Y_{2R} - y_{2R} X_{2R} \end{bmatrix}$$

NTNU

HYDROID KONGSBERG COMPANY

KONGSBERG

https://www.youtube.com/watch?v=lmz3H17N6YM

The REMUS (Remote Environmental Monitoring UnitS) series AUVs (REMUS 100, REMUS 600, REMUS 6000, REMUS M3V) are manufactured by Hydroid Inc. a wholly owned subsidiary of Kongsberg Maritime. The series are designed to be low cost, they have shared control software and electronic subsystems and can be operated from a laptop computer.

9.7 Dive Planes

Dive planes are control surfaces, usually located at the stern of an underwater vehicle. They control the vehicle's pitch angle, and therefore the angle of attack and lift of the dive plane. For aircraft dive planes are called elevators.

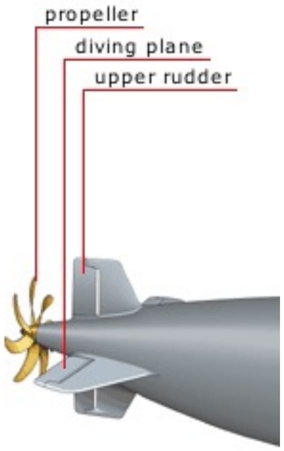
Consider a vehicle which is controller by stern dive planes δ_S and bow dive planes δ_B . Assume that the dive planes are streamlined to produce high lift with minimum drag such that drag can be neglected. Hence,

$$F_{\mathrm{S,lift}} = -\frac{1}{2}\rho V_r^2 A_S C_{L_\delta} \delta_S, \qquad F_{\mathrm{B,lift}} = -\frac{1}{2}\rho V_r^2 A_B C_{L_\delta} \delta_B$$

$$X_S = -\frac{1}{2}\rho V_r^2 A_S C_{L_\delta} \delta_S^2, \qquad X_B = -\frac{1}{2}\rho V_r^2 A_B C_{L_\delta} \delta_B^2$$

$$Z_S = -\frac{1}{2}\rho V_r^2 A_S C_{L_\delta} \delta_S \qquad Z_B = -\frac{1}{2}\rho V_r^2 A_B C_{L_\delta} \delta_B$$

$$\boldsymbol{\tau}_S = \begin{bmatrix} X_S \\ 0 \\ Z_S \\ y_S Z_S \\ z_S X_S - x_S Z_S \\ -y_S X_S \end{bmatrix}, \qquad \boldsymbol{\tau}_B = \begin{bmatrix} X_B \\ 0 \\ Z_B \\ y_B Z_B \\ z_B X_B - x_B Z_B \\ -y_B X_B \end{bmatrix}$$



9.8 Control Moment Gyroscope

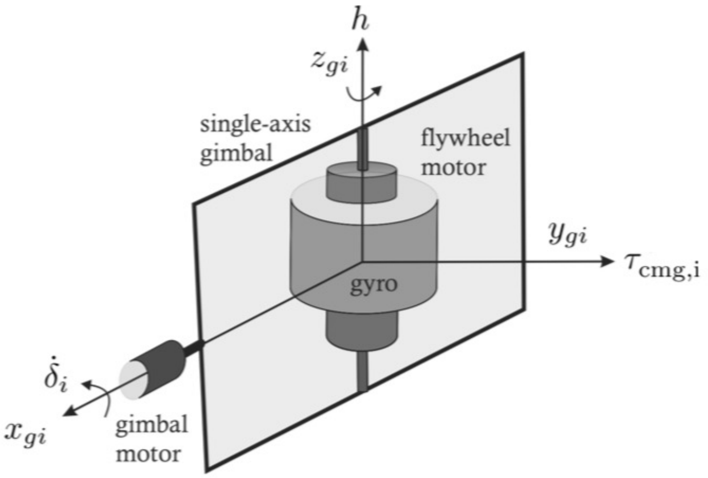
A control moment gyroscope (CMG) is a device, which is used in *spacecraft, ship* and *underwater vehicle* attitude control systems.

A CMG consists of a spinning rotor and one or more motorized gimbals that tilt the rotor's angular momentum. As the rotor tilts, the changing angular momentum causes a gyroscopic torque that rotates the craft.

Applications:

- Ship roll gyrostabilizer
- CMGs for underwater vehicles

Unlike rudder and fins, the gyroscope does not rely on the forward speed of the vehicle to generate roll, pitch and yaw stabilizing moments for attitude control. Hence, CMG systems can be used during stationkeeping.



Single CMG gimbal unit: The flywheel rotates with constant angular rate and the angular rate of the gimbal change the flywheel's angular momentum *h*. This produces a gyroscopic torque.

9.8 Ship Roll Gyrostabilizer

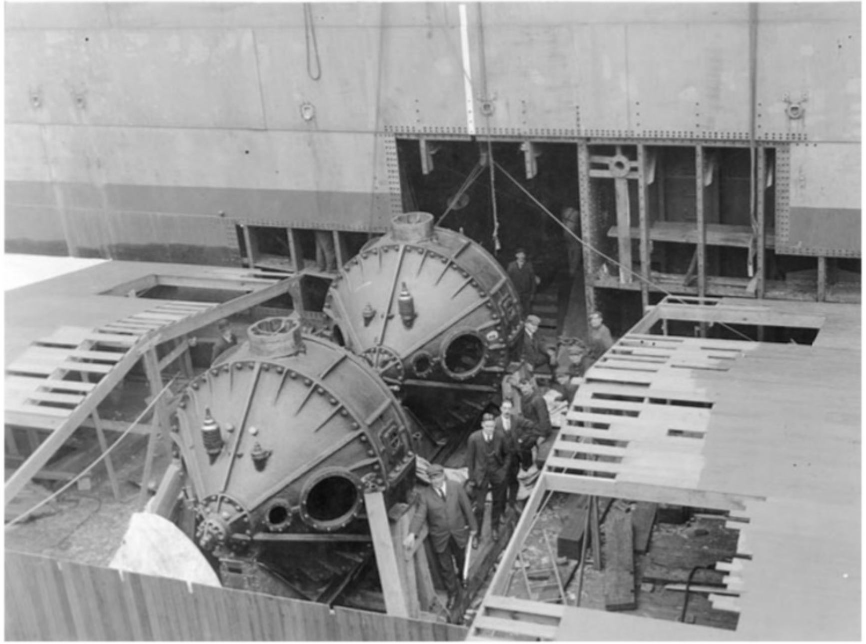
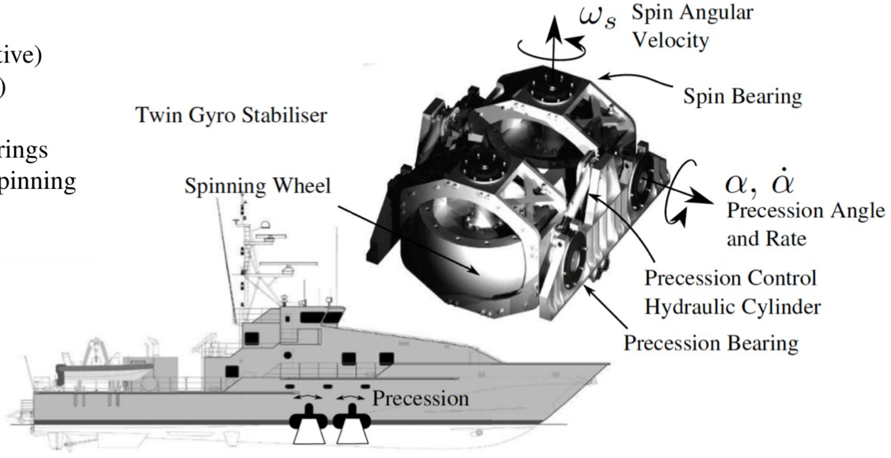


Figure 9.22: Two 25-ton roll-stabilizing gyroscopes being installed on the transport USS Henderson during construction in 1917, the first large ship to use gyroscopic stabilization. Online Library of Historical Images, US Navy Historical Center, Dept. of the Navy, Washington D.C.

9.8 Ship Roll Gyrostabilizer

$$(I_x - K_{\dot{p}} + I_g)\,\dot{p} - K_p p + W\,\mathrm{GM}_T\,\phi = \tau_{\mathrm{wave}} + \tau_{\mathrm{wind}} - nK_g\cos(\alpha)\dot{\alpha}$$
$$I_g\ddot{\alpha} + D_g\dot{\alpha} + G_g\sin(\alpha) = K_g\cos(\alpha)\,p + \tau_p$$

I_x	ship moment of inertia
I_g	inertia of a single spinning wheel along the precession axis
$K_{\dot{p}}$	added moment of inertia coefficient caused by the ship hull (negative)
K_p	linear roll damping coefficient including viscous effects (negative)
K_g	spinning angular momentum, that is $K_g = I_{\mathrm{spin}}\,\omega_{\mathrm{spin}}$
D_g	damping coefficient associated with friction in the precession bearings
G_g	restoring coefficient associated with the mass distribution of the spinning wheel (pendulum effect)
τ_p	precession control torque



The Halcyon's twin gyrostabilizer (n = 2). Perez and Steinmann (2009).

9.8 Ship Roll Gyrostabilizer

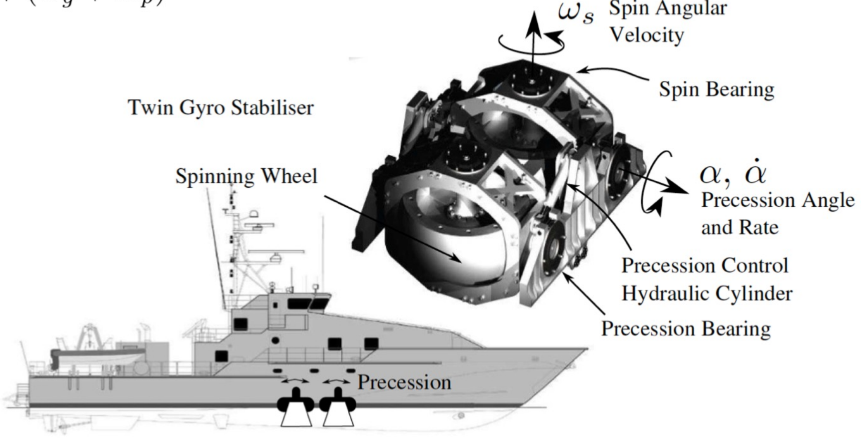
$$\begin{bmatrix} I_x - K_{\dot{p}} + I_g & 0 \\ 0 & I_g \end{bmatrix} \ddot{\boldsymbol{x}} + \begin{bmatrix} -K_p & nK_g \\ -K_g & D_g \end{bmatrix} \dot{\boldsymbol{x}} + \begin{bmatrix} W\,\mathrm{GM}_T & 0 \\ 0 & G_g \end{bmatrix} \boldsymbol{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} \tau_{\mathrm{wave}} + \tau_{\mathrm{wind}} \\ 0 \end{bmatrix}$$

where $\boldsymbol{x} = [\phi,\, \alpha]^\top$ and $u = \tau_p$

PD controller

$$\tau_p = -K_p\alpha - K_d\dot{\alpha} \quad \longrightarrow \quad \dot{\alpha} = \frac{K_g\, s}{s^2 I_g + (D_g + K_d)s + (G_g + K_p)}\, p \approx \kappa\, p$$

$$\boxed{\begin{aligned} \dot{\phi} &= p \\ (I_x - K_{\dot{p}})\,\dot{p} + (nK_g\kappa - K_p)\,p + W\,\mathrm{GM}_T\,\phi &= \tau_{\mathrm{wave}} + \tau_{\mathrm{wind}} \end{aligned}}$$



The Halcyon's twin gyrostabilizer (n = 2). Perez and Steinmann (2009).

9.8 Ship Roll Gyrostabilizer (cont.)

Seakeeper gyro stabilizers http://www.youtube.com/watch?v=f4vZtk1jZ5Q

9.8 Control Moment Gyros for Underwater Vehicles

Internal rotors (CMGs) can control 3-axis attitude of an underwater vehicle even at zero speed

For four equal gyros where δ_i (i = 1, 2, 3, 4) are the gimbal angles, the angular momentum of the CMG system is (Kurokawa 1997)

$$\boldsymbol{h}^b_{\mathrm{cmg}}(\boldsymbol{\delta}) = h \begin{bmatrix} -\cos(\beta)\sin(\delta_1) - \cos(\delta_2) + \cos(\beta)\sin(\delta_3) + \cos(\delta_4) \\ \cos(\delta_1) - \cos(\beta)\sin(\delta_2) - \cos((\delta_3)) + \cos(\beta)\sin(\delta_4) \\ \sin(\beta)(\sin(\delta_1) + \sin(\delta_2) + \sin(\delta_3) + \sin(\delta_4)) \end{bmatrix}$$

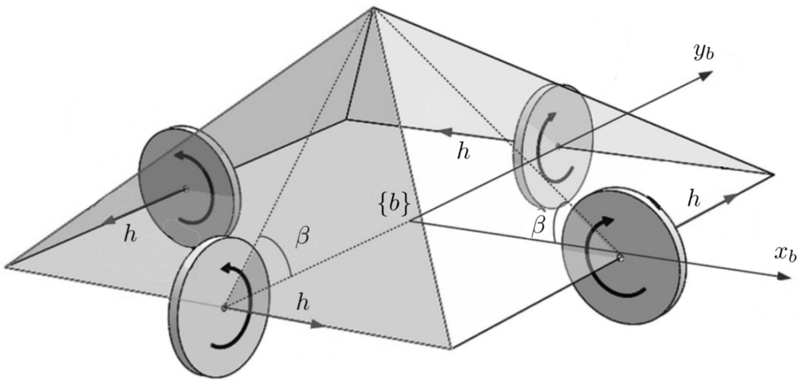


Figure 9.24: Four equal CMG units in a pyramid configuration with skew angle $\beta = \cos^{-1}(1/\sqrt{3})$. Each CMG produces a constant angular momentum h (Xua *et al.* 2019).

9.8 Control Moment Gyros for Underwater Vehicles

$$\boldsymbol{h}^b_{\text{cmg}}(\boldsymbol{\delta}) = h \begin{bmatrix} -\cos(\beta)\sin(\delta_1) - \cos(\delta_2) + \cos(\beta)\sin(\delta_3) + \cos(\delta_4) \\ \cos(\delta_1) - \cos(\beta)\sin(\delta_2) - \cos((\delta_3)) + \cos(\beta)\sin(\delta_4) \\ \sin(\beta)(\sin(\delta_1) + \sin(\delta_2) + \sin(\delta_3) + \sin(\delta_4)) \end{bmatrix}$$

$$\boxed{\dot{\boldsymbol{h}}^b_{\text{cmg}} = h\boldsymbol{A}(\boldsymbol{\delta})\dot{\boldsymbol{\delta}}}$$

$\boldsymbol{A}(\boldsymbol{\delta}) = \partial \boldsymbol{h}^b_{\text{cmg}}/\partial \boldsymbol{\delta} \in \mathbb{R}^{3\times4}$ is the Jacobian matrix

$$\boldsymbol{A}(\boldsymbol{\delta}) = \begin{bmatrix} -\cos(\beta)\cos(\delta_1) & \sin(\delta_2) & \cos(\beta)\cos(\delta_3) & -\sin(\delta_4) \\ -\sin(\delta_1) & -\cos(\beta)\cos(\delta_2) & \sin((\delta_3)) & \cos(\beta)\cos(\delta_4) \\ \sin(\beta)\cos(\delta_1) & \sin(\beta)\cos(\delta_2) & \sin(\beta)\cos(\delta_3) & \sin(\beta)\cos(\delta_4)) \end{bmatrix}$$

Angular momentum can be controlled by changing the CMG gimbal angular velocity vector

$$\dot{\boldsymbol{\delta}} = [\dot{\delta}_1, \dot{\delta}_2, \dot{\delta}_3, \dot{\delta}_4]^\top$$

This is not straightforward since the Jacobian can be singular. The damped-least squares method can be used for singularity avoidance (Nakamura and Hanafusa 1986) (Chiaverini 1993)

$$\dot{\boldsymbol{\delta}} = -\frac{1}{h}\boldsymbol{A}^\top(\boldsymbol{\delta})\left(\boldsymbol{A}(\boldsymbol{\delta})\boldsymbol{A}^\top(\boldsymbol{\delta}) + \lambda^2\boldsymbol{I}_3\right)^{-1}\boldsymbol{\tau}_{\text{cmg}}$$

$\lambda \geq 0$, damping
$\lambda = 0$, regular matrix inversion

9.8 Control Moment Gyros for Underwater Vehicles

AUV model including CMGs

$$\boldsymbol{M}(\boldsymbol{\delta})\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}(\boldsymbol{\delta},\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}\boldsymbol{\nu}_r + \boldsymbol{d}(\boldsymbol{\nu}_r) + \boldsymbol{g}(\boldsymbol{\eta}) = \begin{bmatrix} \boldsymbol{0}_{3\times 1} \\ \boldsymbol{\tau}_{\mathrm{cmg}} \end{bmatrix} + \boldsymbol{\tau}$$

$$\dot{\boldsymbol{h}}^b_{\mathrm{cmg}}(\boldsymbol{\delta}) = h\boldsymbol{A}(\boldsymbol{\delta})\dot{\boldsymbol{\delta}} = -\boldsymbol{\tau}_{\mathrm{cmg}}$$

Kinetic energy and Kirchhoff's equations

$$T = \frac{1}{2}\left[\boldsymbol{\nu}_1^\top, \boldsymbol{\nu}_2^\top\right] \begin{bmatrix} m\boldsymbol{I}_3 + \boldsymbol{A}_{11} & -m\boldsymbol{S}(\boldsymbol{r}_g^b) + \boldsymbol{A}_{12} \\ m\boldsymbol{S}(\boldsymbol{r}_g^b) + \boldsymbol{A}_{21} & \boldsymbol{I}_b^b + \boldsymbol{A}_{22} + \boldsymbol{I}^b_{\mathrm{cmg}}(\boldsymbol{\delta}) \end{bmatrix} \begin{bmatrix} \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \end{bmatrix}$$

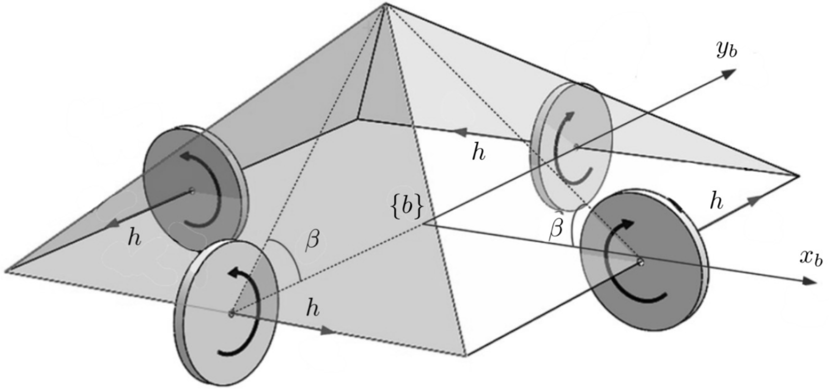
$$\boxed{\begin{aligned} \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial T}{\partial \boldsymbol{\nu}_1}\right) + \boldsymbol{S}(\boldsymbol{\nu}_2)\frac{\partial T}{\partial \boldsymbol{\nu}_1} &= \boldsymbol{\tau}_1 \\ \frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial T}{\partial \boldsymbol{\nu}_2}\right) + \boldsymbol{S}(\boldsymbol{\nu}_2)\frac{\partial T}{\partial \boldsymbol{\nu}_2} + \boldsymbol{S}(\boldsymbol{\nu}_1)\frac{\partial T}{\partial \boldsymbol{\nu}_1} &= \boldsymbol{\tau}_2 \end{aligned}}$$

$$\boldsymbol{M}(\boldsymbol{\delta}) = \begin{bmatrix} m\boldsymbol{I}_3 + \boldsymbol{A}_{11} & -m\boldsymbol{S}(\boldsymbol{r}_g^b) + \boldsymbol{A}_{12} \\ m\boldsymbol{S}(\boldsymbol{r}_g^b) + \boldsymbol{A}_{21} & \boldsymbol{I}_b^b + \boldsymbol{A}_{22} \end{bmatrix} + \boldsymbol{M}_{\mathrm{cmg}}(\boldsymbol{\delta})$$

$$\boldsymbol{C}(\boldsymbol{\delta},\boldsymbol{\nu}) = \begin{bmatrix} m\boldsymbol{S}(\boldsymbol{\nu}_2) & -m\boldsymbol{S}(\boldsymbol{\nu}_2)\boldsymbol{S}(\boldsymbol{r}_g^b) - \boldsymbol{S}(\boldsymbol{A}_{11}\boldsymbol{\nu}_1 + \boldsymbol{A}_{12}\boldsymbol{\nu}_2) \\ m\boldsymbol{S}(\boldsymbol{r}_g^b)\boldsymbol{S}(\boldsymbol{\nu}_2) - \boldsymbol{S}(\boldsymbol{A}_{11}\boldsymbol{\nu}_1 + \boldsymbol{A}_{12}\boldsymbol{\nu}_2) & \boldsymbol{S}(\boldsymbol{I}_b^b\boldsymbol{\nu}_2) - \boldsymbol{S}(\boldsymbol{A}_{21}\boldsymbol{\nu}_1 + \boldsymbol{A}_{22}\boldsymbol{\nu}_2) \end{bmatrix} + \boldsymbol{C}_{\mathrm{cmg}}(\boldsymbol{\delta})$$

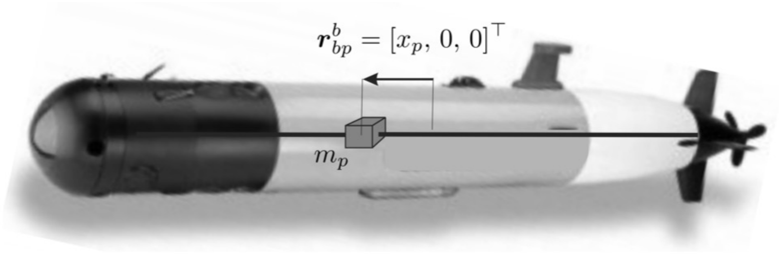
$$\boldsymbol{M}_{\mathrm{cmg}}(\boldsymbol{\delta}) = \begin{bmatrix} \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{I}^b_{\mathrm{cmg}}(\boldsymbol{\delta}) \end{bmatrix}$$

$$\boldsymbol{C}_{\mathrm{cmg}}(\boldsymbol{\delta}) = \begin{bmatrix} \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \dot{\boldsymbol{I}}^b_{\mathrm{cmg}}(\boldsymbol{\delta}) - \boldsymbol{S}(\boldsymbol{h}^b_{\mathrm{cmg}}(\boldsymbol{\delta})) \end{bmatrix}$$



9.9 Moving Mass Actuators

Internal moving mass actuators are promising alternatives to propellers and control surfaces. A moving mass system increases endurance and range by reducing the power consumption



$$m_{\text{total}} = m + m_p \qquad \boldsymbol{r}^b_{bg'} = \frac{m\,\boldsymbol{r}^b_{bg} + m_p\,\boldsymbol{r}^b_{bp}}{m_{\text{total}}}$$

Kinetic energy

$$T = \frac{1}{2}\left[(\boldsymbol{v}^b_{bp})^\top,\ \boldsymbol{\nu}_1^\top,\ \boldsymbol{\nu}_2^\top\right]\boldsymbol{M}(\boldsymbol{r}^b_{bp})\begin{bmatrix}\boldsymbol{v}^b_{bp}\\ \boldsymbol{\nu}_1\\ \boldsymbol{\nu}_2\end{bmatrix} \qquad \boldsymbol{M}(\boldsymbol{r}^b_{bp}) = \boldsymbol{M}_{RB}(\boldsymbol{r}^b_{bp}) + \boldsymbol{M}_A$$

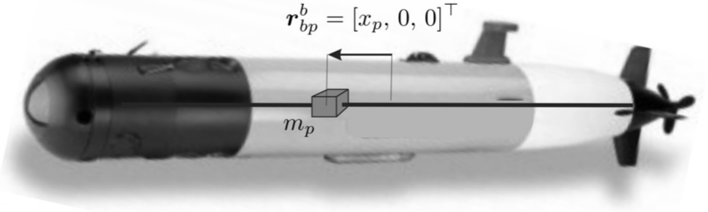
$$\boldsymbol{M}_{RB}(\boldsymbol{r}^b_{bp}) = \begin{bmatrix} m_p\boldsymbol{I}_3 & m_p\boldsymbol{I}_3 & -m_p\boldsymbol{S}(\boldsymbol{r}^b_{bp}) \\ m_p\boldsymbol{I}_3 & (m+m_p)\boldsymbol{I}_3 & -m_p\boldsymbol{S}(\boldsymbol{r}^b_{bp}) - m\boldsymbol{S}(\boldsymbol{r}^b_{bg}) \\ m_p\boldsymbol{S}(\boldsymbol{r}^b_{bp}) & m_p\boldsymbol{S}(\boldsymbol{r}^b_{bp}) + m\boldsymbol{S}(\boldsymbol{r}^b_{bg}) & \boldsymbol{I}^b_b - m_p\boldsymbol{S}^2(\boldsymbol{r}^b_{bp}) \end{bmatrix}$$

$$\boldsymbol{M}_A = \begin{bmatrix} \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} & \boldsymbol{0}_{3\times3} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{0}_{3\times3} & \boldsymbol{A}_{21} & \boldsymbol{A}_{22} \end{bmatrix}$$

9.9 Moving Mass

Linear and angular momentums

$$\mathcal{P}_p^b = \frac{\partial T}{\partial \boldsymbol{v}_{bp}^b}, \qquad \mathcal{P}^b = \frac{\partial T}{\partial \boldsymbol{\nu}_1}, \qquad \mathcal{H}^b = \frac{\partial T}{\partial \boldsymbol{\nu}_2}$$



$$\begin{bmatrix} \mathcal{P}_p^b \\ \mathcal{P}^b \\ \mathcal{H}^b \end{bmatrix} = \boldsymbol{M}(\boldsymbol{r}_{bp}^b) \begin{bmatrix} \boldsymbol{v}_{bp}^b \\ \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \end{bmatrix}$$

$$= \begin{bmatrix} m_p \boldsymbol{v}_{bp}^b + m_p \boldsymbol{\nu}_1 - m_p \boldsymbol{S}(\boldsymbol{r}_{bp}^b)\boldsymbol{\nu}_2 \\ m_p \boldsymbol{v}_{bp}^b + (\boldsymbol{A}_{11} + (m + m_p)\boldsymbol{I}_3)\boldsymbol{\nu}_1 \\ \qquad + \left(\boldsymbol{A}_{12} - m_p \boldsymbol{S}(\boldsymbol{r}_{bp}^b) - m\boldsymbol{S}(\boldsymbol{r}_{bg}^b)\right)\boldsymbol{\nu}_2 \\ m_p \boldsymbol{S}(\boldsymbol{r}_{bp}^b)\boldsymbol{v}_{bp}^b + \left(\boldsymbol{A}_{21} + m_p \boldsymbol{S}(\boldsymbol{r}_{bp}^b) + m\boldsymbol{S}(\boldsymbol{r}_{bg}^b)\right)\boldsymbol{\nu}_1 \\ \qquad + \left(\boldsymbol{A}_{22} + \boldsymbol{I}_b^b - m_p \boldsymbol{S}^2(\boldsymbol{r}_{bp}^b)\right)\boldsymbol{\nu}_2 \end{bmatrix}$$

AUV equations of motion including the moving mass m_p (follows from Kirchhoff's equations)

$$\begin{aligned} \dot{\mathcal{P}}_p^b &= -\boldsymbol{S}(\boldsymbol{\nu}_2)\mathcal{P}_p^b + \boldsymbol{R}^\top(\boldsymbol{\Theta}_{nb})\boldsymbol{f}_p^n + \boldsymbol{\tau}_p \\ \dot{\mathcal{P}}^b &= -\boldsymbol{S}(\boldsymbol{\nu}_2)\mathcal{P}^b \\ \dot{\mathcal{H}}^b &= -\boldsymbol{S}(\boldsymbol{\nu}_1)\mathcal{P}^b - \boldsymbol{S}(\boldsymbol{\nu}_2)\mathcal{H}^b + \boldsymbol{S}(\boldsymbol{r}_{bp}^b)\boldsymbol{R}^\top(\boldsymbol{\Theta}_{nb})\boldsymbol{f}_p^n + \boldsymbol{S}(\boldsymbol{r}_{bg}^b)\boldsymbol{R}^\top(\boldsymbol{\Theta}_{nb})\boldsymbol{f}_g^n \end{aligned}$$

$$\begin{bmatrix} \boldsymbol{v}_{bp}^b \\ \boldsymbol{\nu}_1 \\ \boldsymbol{\nu}_2 \end{bmatrix} = \boldsymbol{M}^{-1}(\boldsymbol{r}_{bp}^b) \begin{bmatrix} \mathcal{P}_p^b \\ \mathcal{P}^b \\ \mathcal{H}^b \end{bmatrix}$$

Chapter Goals - Revisited

- Be able to model thrusters, propellers, control surfaces and propulsion systems and add the models to the equations of motion.
- Understand how KT and KQ curves are used to compute propeller thrust and torque.
- Be able to explain what a Bollard pull test is and relate this to the KT and KQ curves.
- Understand how a propeller in a slipstream produces lift and drag, and how these quantities relate to the propeller yaw moment needed to turn a vehicle or a ship.
- Understand how control surfaces are used to dive and turn underwater vehicles.
- Understand how fin stabilizers are used.
- Understand how control moment gyros (CMGs) can be used to control the attitude of a vehicle even at zero speed.
- Understand how a moving mass can be used to control the attitude of a vehicle.