Chapter 10 - Environmental Forces and Moments

Chapter Goals

- Understand the principle for linear superposition of wind and wave forces
- Understand the equations of relative motion for ocean currents
- Be able to compute the wind loads on a marine craft using wind coefficient look-up tables and projected areas found from general arrangement drawings as required by the MSS functions: *Blendermann94* and *Isherwood72.*
- Understand how statistical descriptions can be used to define the sea state in terms of parameters such as wave period, wind speed, etc. for different regions on the Earth.
- Explain what we mean with regular and irregular waves, wave spectrum and directional wave spectrum.
- Be able to compute and simulate the wave elevation of regular and irregular waves for a given sea state and a wave spectrum.
- Understand the three methods for simulation and computation of wave loads:
  - Motion RAOs
  - Force RAOs
  - Simplified transfer functions for WF motions
- Be able to explain what we mean by first- and second-order wave loads and how these relate to hydrodynamic data computed in Wamit and ShipX.
- Be able to simulate ocean currents (magnitude and direction) in 2D and 3D and transform the results to current velocities.

Chapter 10 - Environmental Forces and Moments

Superposition of Wind and Wave Disturbances

For control systems design it is common to assume the principle of superposition when considering wind and wave forces:

$$\boldsymbol{M}\dot{\boldsymbol{\nu}} + \boldsymbol{C}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{D}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) + \boldsymbol{g}_0 = \underbrace{\boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}}}_{\boldsymbol{w}} + \boldsymbol{\tau}$$

Ocean Currents

The effect due to ocean currents is simulated by introducing the relative velocity vector:

$$\underbrace{\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu}}_{\text{rigid-body terms}} + \underbrace{\boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r}_{\text{hydrodynamic terms}} + \underbrace{\boldsymbol{g}(\boldsymbol{\eta}) + \boldsymbol{g}_0}_{\text{hydrostatic terms}} = \boldsymbol{\tau} + \boldsymbol{w}$$

$$\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c$$

$$\boldsymbol{\nu}_c = \begin{bmatrix} u_c \\ v_c \\ w_c \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

10.1 Wind Forces and Moments on Marine Craft at Rest

Generalized force using area-based wind coefficients:

$$
\begin{aligned}
X_{\text{wind}} &= qC_X(\gamma_w)A_{Fw} \\
Y_{\text{wind}} &= qC_Y(\gamma_w)A_{Lw} \\
Z_{\text{wind}} &= qC_Z(\gamma_w)A_{Fw} \\
K_{\text{wind}} &= qC_K(\gamma_w)A_{Lw}H_{L_w} \\
M_{\text{wind}} &= qC_M(\gamma_w)A_{Fw}H_{F_w} \\
N_{\text{wind}} &= qC_N(\gamma_w)A_{Lw}L_{oa}
\end{aligned}
$$

$$\gamma_w = \psi - \beta_{V_w} - \pi$$

wind angle of attack

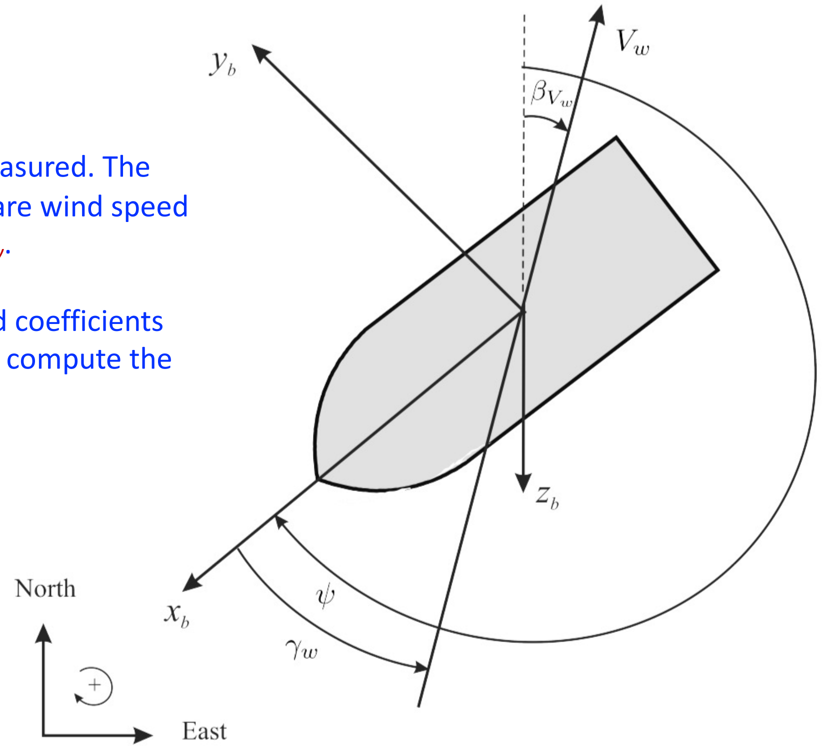
Dynamic pressure

$$q = \frac{1}{2}\rho_a V_w^2$$

ρ_a = air density
A_{Fw} = frontal projected area
A_{Lw} = lateral projected area
H_{Fw} = centroid of A_{Fw} above the water line
H_{Lw} = centroid of A_{Lw} above the water line
L_{oa} = length over all

Wind loads cannot be measured. The standard measurements are wind speed and direction, V_w and β_{Vw}.

Hence, you need the wind coefficients C_X, C_Y, C_Z, C_K, C_M and C_N to compute the wind loads.



10.1 Wind Forces and Moments on Marine Craft at Rest

The wind speed is usually specified in terms of Beaufort numbers (Price and Bishop 1974)

Beaufort number	Description of wind	Wind speed (knots)
0	Calm	0–1
1	Light air	2–3
2	Light breeze	4–7
3	Gentle breeze	8–11
4	Moderate breeze	12–16
5	Fresh breeze	17–21
6	Strong breeze	22–27
7	Moderate gale	28–33
8	Fresh gale	34–40
9	Strong gale	41–48
10	Whole gale	49–56
11	Storm	57–65
12	Hurricane	More than 65

1 knots = 0.51 m/s

Air density as a function of temperature

°C	Air density, ρ_a (kg/m^3)
−10	1.342
−5	1.317
0	1.292
5	1.269
10	1.247
15	1.225
20	1.204
25	1.184
30	1.165

Experiments with Ship Models in a Wind Tunnel

Force Technology: https://forcetechnology.com/no/alla-faciliteter/wind-tunnel-facility

Youtube: https://www.youtube.com/watch?time_continue=1&v=NyiaMRSGPg8&feature=emb_logo

10.1 Wind Forces and Moments on Marine Craft at Rest

Wind Coefficient Approximation for Symmetrical Ships

For ships that are symmetrical with respect to the xz- and yz-planes, the wind coefficients for horizontal-plane motions can be approximated by

$$
\begin{aligned}
C_X(\gamma_w) &\approx -c_x \cos(\gamma_w) \\
C_Y(\gamma_w) &\approx c_y \sin(\gamma_w) \\
C_N(\gamma_w) &\approx c_n \sin(2\gamma_w)
\end{aligned}
$$

which are convenient formulae for computer simulations.

Experiments indicate that:

$$
\begin{aligned}
c_x &\in [0.50, 0.90] \\
c_y &\in [0.70, 0.95] \\
c_n &\in [0.05, 0.20]
\end{aligned}
$$

These values should be used with care.

10.1 Wind Forces and Moments on Moving Marine Craft

Generalized force in terms of relative angle of attack γ_{rw} and relative wind speed V_{rw}

$$\boldsymbol{\tau}_{\text{wind}} = \frac{1}{2}\rho_a V_{rw}^2 \begin{bmatrix} C_X(\gamma_{rw})A_{Fw} \\ C_Y(\gamma_{rw})A_{Lw} \\ C_Z(\gamma_{rw})A_{Fw} \\ C_K(\gamma_{rw})A_{Lw}H_{L_w} \\ C_M(\gamma_{rw})A_{Fw}H_{F_w} \\ C_N(\gamma_{rw})A_{Lw}L_{oa} \end{bmatrix}$$

$$V_{rw} = \sqrt{u_{rw}^2 + v_{rw}^2}$$
$$\gamma_{rw} = -atan2(v_{rw}, u_{rw})$$

$$u_{rw} = u - u_w \qquad u_w = V_w \cos(\beta_{V_w} - \psi)$$
$$v_{rw} = v - v_w \qquad v_w = V_w \sin(\beta_{V_w} - \psi)$$

The wind speed V_w and its direction β_{Vw} can be measured by a wind sensor (anemometer)

The wind coefficients (look-up tables) are computed numerically or by experiments in wind tunnels.

10.1 Wind Coefficients Based on Flow over a Helmholtz-Kirchhoff Plate

Reference: Blendermann (1986, 1994)

The load functions are parameterized in terms of four primary wind load parameters:

$$C_X(\gamma_w) = -\underbrace{\mathrm{CD}_l \frac{A_{L_w}}{A_{F_w}}}_{\mathrm{CD}_{lAF}} \frac{\cos(\gamma_w)}{1 - \frac{\delta}{2}\left(1 - \frac{\mathrm{CD}_l}{\mathrm{CD}_t}\right)\sin^2(2\gamma_w)}$$

$$C_Y(\gamma_w) = \mathrm{CD}_t \frac{\sin(\gamma_w)}{1 - \frac{\delta}{2}\left(1 - \frac{\mathrm{CD}_l}{\mathrm{CD}_t}\right)\sin^2(2\gamma_w)}$$

$$C_K(\gamma_w) = \kappa C_Y(\gamma_w)$$

$$C_N(\gamma_w) = \left[\frac{s_L}{L_{oa}} - 0.18\left(\gamma_w - \frac{\pi}{2}\right)\right] C_Y(\gamma_w)$$

$$\mathrm{CD}_l = \mathrm{CD}_{l_{AF}}(\gamma_w)\frac{A_{F_w}}{A_{L_w}}$$

Type of vessel	CD_t	$CD_{l_{AF}}(0)$	$CD_{l_{AF}}(\pi)$	δ	κ
1. Car carrier	0.95	0.55	0.60	0.80	1.2
2. Cargo vessel, loaded	0.85	0.65	0.55	0.40	1.7
3. Cargo vessel, container on deck	0.85	0.55	0.50	0.40	1.4
4. Container ship, loaded	0.90	0.55	0.55	0.40	1.4
5. Destroyer	0.85	0.60	0.65	0.65	1.1
6. Diving support vessel	0.90	0.60	0.80	0.55	1.7
7. Drilling vessel	1.00	0.70–1.00	0.75–1.10	0.10	1.7
8. Ferry	0.90	0.45	0.50	0.80	1.1
9. Fishing vessel	0.95	0.70	0.70	0.40	1.1
10. Liquefied natural gas tanker	0.70	0.60	0.65	0.50	1.1
11. Offshore supply vessel	0.90	0.55	0.80	0.55	1.2
12. Passenger liner	0.90	0.40	0.40	0.80	1.2
13. Research vessel	0.85	0.55	0.65	0.60	1.4
14. Speed boat	0.90	0.55	0.60	0.60	1.1
15. Tanker, loaded	0.70	0.90	0.55	0.40	3.1
16. Tanker, in ballast	0.70	0.75	0.55	0.40	2.2
17. Tender	0.85	0.55	0.55	0.65	1.1

Matlab MSS toolbox

```
[w_wind,CX,CY,CK,CN] = blendermann94(gamma_r,V_r,AFw,ALw,sH,sL,Loa,vessel_no)
```

MSS Toolbox

```
gam_deg = (0:10:180)';
gamma_r = (pi/180)*gam_deg;
V_r = 20;

% ****************************************************************
%%  CASE 1 Blendermann (1994)
% ****************************************************************
CDt         = 0.85;
CDl_AF{1} = 0.55;     % gamma_r = 0
CDl_AF{2} = 0.65;     % gamma_r = pi
delta       = 0.60;
kappa       = 1.4;
AFw         = 160.7;
ALw         = 434.8;
Loa         = 55.0;
sL          = 1.48;
sH          = 5.1;
vessel_no = 13;

[tau_w,CX,CY,CK,CN] = blendermann94(gamma_r,V_r,AFw,ALw,sH,sL,Loa,vessel_no);
```

From table of ships

Lookalike ship

```
% vessel_no =
%   1. Car carrier
%   2. Cargo vessel, loaded
%   3. Cargo vessel, container on deck
%   4. Container ship, loaded
%   5. Destroyer
%   6. Diving support vessel
%   7. Drilling vessel
%   8. Ferry
%   9. Fishing vessel
%   10. Liquified natural gas tanker
%   11. Offshore supply vessel
%   12. Passenger liner
%   13. Research vessel
%   14. Speed boat
%   15. Tanker, loaded
%   16. Tanker, in ballast
%   17. Tender
```

The ship data are computed using a general arrangement drawing (manually or digitally)

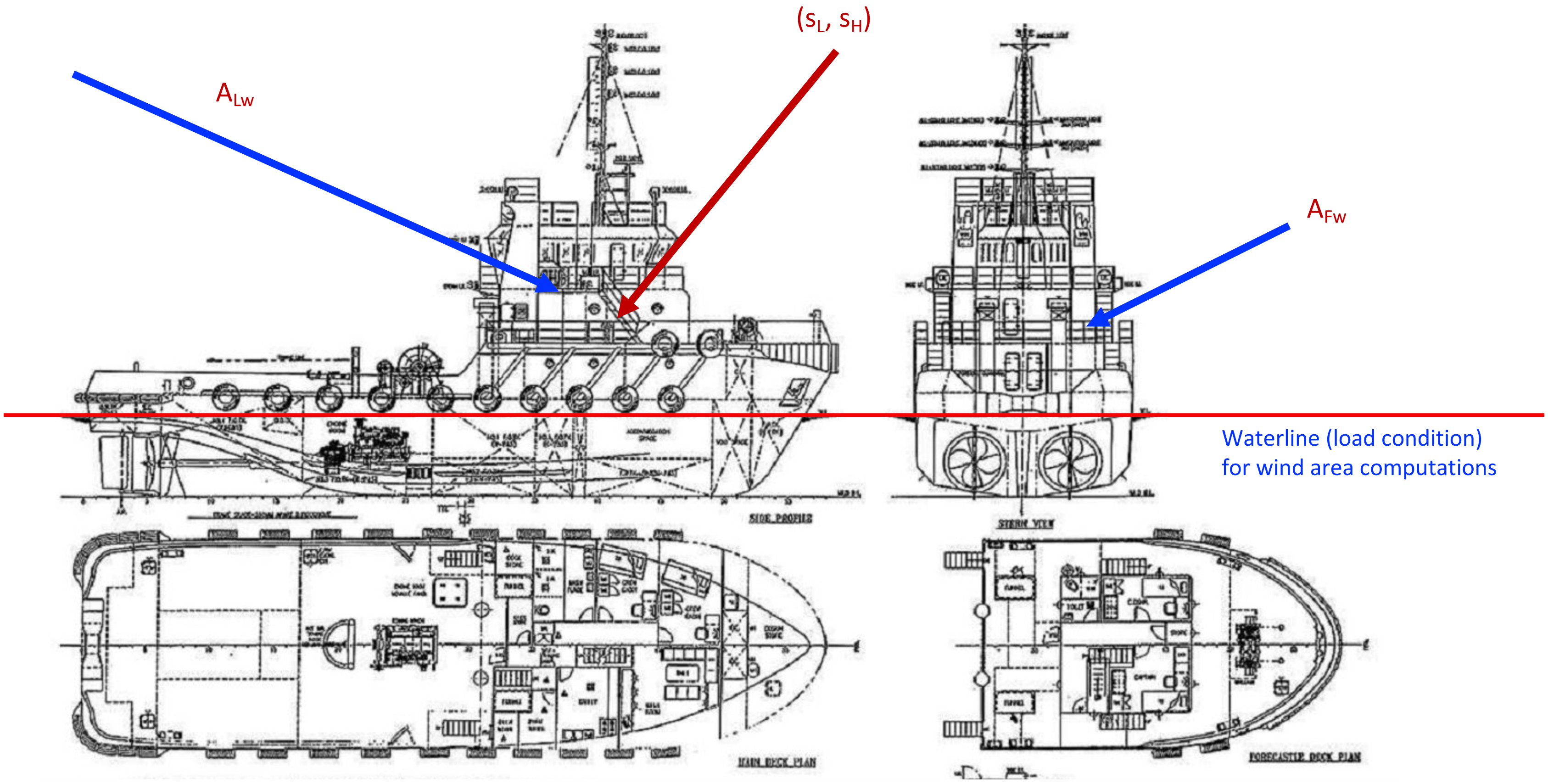
A_{Fw} = Frontal projected area

A_{Lw} = Lateral (transverse) projected area

Loa = Length overall

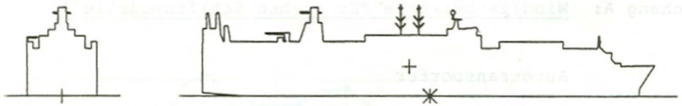
(s_L, s_H) = x_b and y_b coordinates of the centroid of A_{Lw}

General Arrangement Drawing



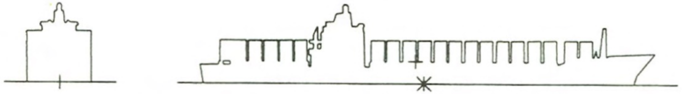
Blendermann (1986, 1994)

Car carrier

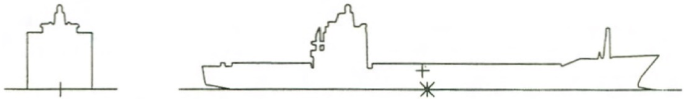


LAENGE PP	LPP M	180.00	SCHWERPUNKT VON AL VOR ✳	SL M	-8.98
LAENGE OA	LOA M	195.00	SCHWERPUNKT VON AL UEBER WL	SH M	12.56
BREITE IN DER WL	BWL M	30.00	MITTLERE HOEHE VON AL	HM M	23.51
LATERALFLAECHE	AL M2	4584.16	TIEFGANG VORN	TV M	8.50
FRONTFLAECHE	AF M2	811.68	TIEFGANG HINTEN	TH M	8.50

Container ship

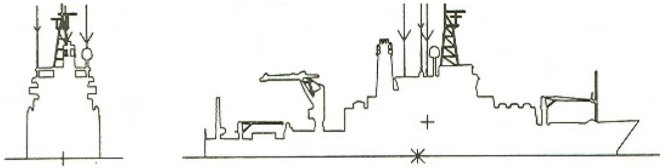


LAENGE PP	LPP M	194.51	SCHWERPUNKT VON AL VOR ✳	SL M	-3.87
LAENGE OA	LOA M	210.75	SCHWERPUNKT VON AL UEBER WL	SH M	10.08
BREITE IN DER WL	BWL M	30.50	MITTLERE HOEHE VON AL	HM M	17.80
LATERALFLAECHE	AL M2	3751.11	TIEFGANG VORN	TV M	11.60
FRONTFLAECHE	AF M2	801.97	TIEFGANG HINTEN	TH M	11.60



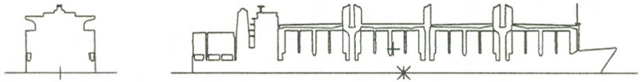
LAENGE PP	LPP M	194.51	SCHWERPUNKT VON AL VOR ✳	SL M	-2.14
LAENGE OA	LOA M	210.75	SCHWERPUNKT VON AL UEBER WL	SH M	8.67
BREITE IN DER WL	BWL M	30.50	MITTLERE HOEHE VON AL	HM M	13.98
LATERALFLAECHE	AL M2	2947.30	TIEFGANG VORN	TV M	9.60
FRONTFLAECHE	AF M2	857.06	TIEFGANG HINTEN	TH M	9.60

Research vessel



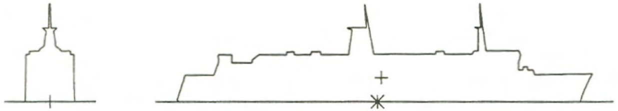
LAENGE PP	LPP M	90.00	SCHWERPUNKT VON AL VOR ✳	SL M	2.18
LAENGE OA	LOA M	97.50	SCHWERPUNKT VON AL UEBER WL	SH M	7.61
BREITE IN DER WL	BWL M	16.50	MITTLERE HOEHE VON AL	HM M	11.22
LATERALFLAECHE	AL M2	1093.83	TIEFGANG VORN	TV M	4.80
FRONTFLAECHE	AF M2	320.18	TIEFGANG HINTEN	TH M	4.80

Cargo ship



LAENGE PP	LPP M	187.30	SCHWERPUNKT VON AL VOR ✳	SL M	-4.92
LAENGE OA	LOA M	198.25	SCHWERPUNKT VON AL UEBER WL	SH M	11.07
BREITE IN DER WL	BWL M	32.20	MITTLERE HOEHE VON AL	HM M	18.61
LATERALFLAECHE	AL M2	3689.73	TIEFGANG VORN	TV M	12.20
FRONTFLAECHE	AF M2	826.32	TIEFGANG HINTEN	TH M	12.20

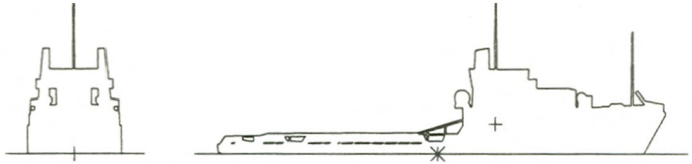
Ferry



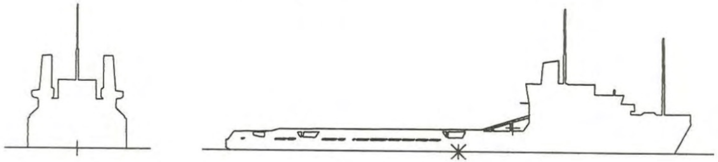
LAENGE PP	LPP M	139.60	SCHWERPUNKT VON AL VOR ✳	SL M	1.36
LAENGE OA	LOA M	143.90	SCHWERPUNKT VON AL UEBER WL	SH M	8.15
BREITE IN DER WL	BWL M	17.35	MITTLERE HOEHE VON AL	HM M	15.25
LATERALFLAECHE	AL M2	2125.75	TIEFGANG VORN	TV M	5.90
FRONTFLAECHE	AF M2	325.33	TIEFGANG HINTEN	TH M	5.90

Blendermann (1986, 1994)

Supply vessel



LAENGE PP	LPP M	58.35	SCHWERPUNKT VON AL VOR ✳	SL M	7.95
LAENGE OA	LOA M	61.95	SCHWERPUNKT VON AL UEBER WL	SH M	4.08
BREITE IN DER WL	BWL M	13.00	MITTLERE HOEHE VON AL	HM M	5.76
LATERALFLAECHE	AL M2	336.80	TIEFGANG VORN	TV M	4.85
FRONTFLAECHE	AF M2	137.48	TIEFGANG HINTEN	TH M	4.85



LAENGE PP	LPP M	58.35	SCHWERPUNKT VON AL VOR ✳	SL M	7.12
LAENGE OA	LOA M	61.00	SCHWERPUNKT VON AL UEBER WL	SH M	3.14
BREITE IN DER WL	BWL M	13.00	MITTLERE HOEHE VON AL	HM M	4.46
LATERALFLAECHE	AL M2	260.33	TIEFGANG VORN	TV M	4.85
FRONTFLAECHE	AF M2	110.12	TIEFGANG HINTEN	TH M	4.85

Tanker

LAENGE PP	LPP M	336.00	SCHWERPUNKT VON AL VOR ✳	SL M	-24.45
LAENGE OA	LOA M	351.40	SCHWERPUNKT VON AL UEBER WL	SH M	6.83
BREITE IN DER WL	BWL M	55.40	MITTLERE HOEHE VON AL	HM M	9.68
LATERALFLAECHE	AL M2	3401.47	TIEFGANG VORN	TV M	23.50
FRONTFLAECHE	AF M2	1131.79	TIEFGANG HINTEN	TH M	23.50

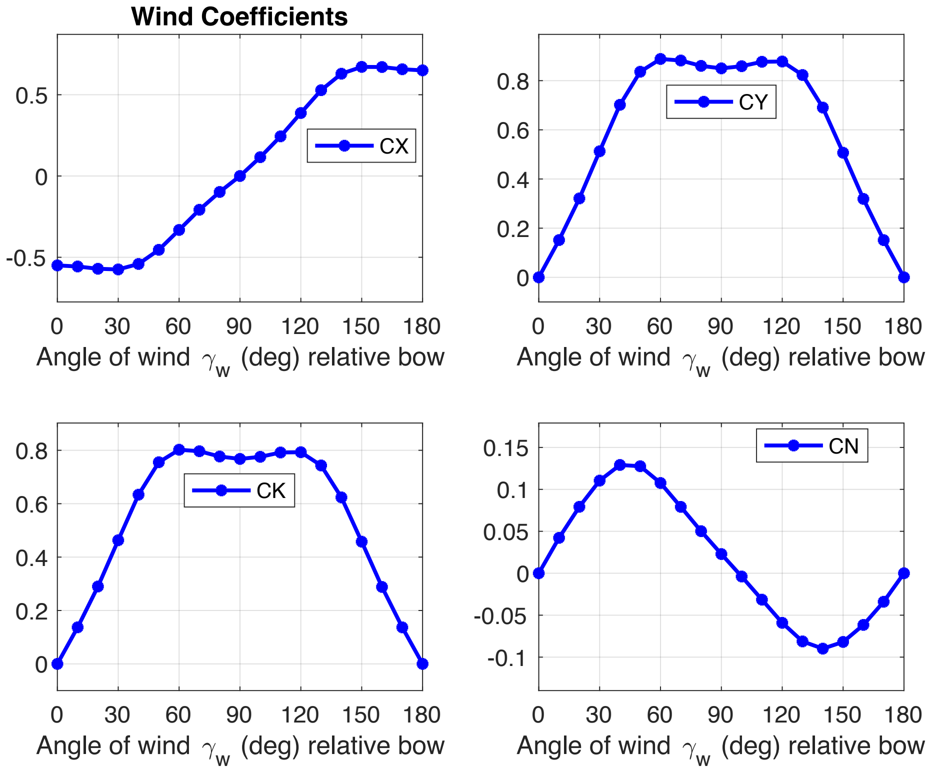
MSS Toolbox

Research vessel #13

$$\boldsymbol{\tau}_{\text{wind}} = \frac{1}{2}\rho_a V_{rw}^2 \begin{bmatrix} C_X(\gamma_{rw})A_{Fw} \\ C_Y(\gamma_{rw})A_{Lw} \\ \cancel{C_Z(\gamma_{rw})A_{Fw}} \\ C_K(\gamma_{rw})A_{Lw}H_{L_w} \\ \cancel{C_M(\gamma_{rw})A_{Fw}H_{F_w}} \\ C_N(\gamma_{rw})A_{Lw}L_{oa} \end{bmatrix}$$

```
CDt         = 0.85;
CDl_AF{1} = 0.55;      % gamma_r = 0
CDl_AF{2} = 0.65;      % gamma_r = pi
delta       = 0.60;
kappa       = 1.4;
AFw         = 160.7;
ALw         = 434.8;
Loa         = 55.0;
sL          = 1.48;
sH          = 5.1;
vessel_no = 13;

[tau_w,CX,CY,CK,CN] = blendermann94(gamma_r,V_r,AFw,ALw,sH,sL,Loa,vessel_no);
```

Ship data



10.1 Wind Coefficients for Merchant Ships (Isherwood 1972)

Isherwood (1972) has derived a set of wind coefficients by using multiple regression techniques to fit experimental data of merchant ships. The wind coefficients are parameterized in terms of eight parameters.

L_{oa}	length overall
B	beam
A_{L_w}	lateral projected area
A_{T_w}	transverse projected area
A_{SS}	lateral projected area of superstructure
S	length of perimeter of lateral projection of model excluding water line and slender bodies such as masts and ventilators
C	distance from bow of centroid of lateral projected area
M	number of distinct groups of masts or king posts seen in lateral projection; king posts close against the bridge front are not included

Typical merchant ships are:

- Bulk carriers for transport of raw materials
- Container ships
- Tankers
- Ferries and cruise ships

From regression analyses it was concluded that the measured data were best fitted to the following three equations:

$$C_X = -\left(A_0 + A_1 \frac{2A_L}{L^2} + A_2 \frac{2A_T}{B^2} + A_3 \frac{L}{B} + A_4 \frac{S}{L} + A_5 \frac{C}{L} + A_6 M\right)$$

$$C_Y = B_0 + B_1 \frac{2A_L}{L^2} + B_2 \frac{2A_T}{B^2} + B_3 \frac{L}{B} + B_4 \frac{S}{L} + B_5 \frac{C}{L} + B_6 \frac{A_{SS}}{A_L}$$

$$C_N = C_0 + C_1 \frac{2A_L}{L^2} + C_2 \frac{2A_T}{B^2} + C_3 \frac{L}{B} + C_4 \frac{S}{L} + C_5 \frac{C}{L}$$

Matlab MSS toolbox

```
[w_wind,CX,CY,CN] = isherwood72(gamma_r,V_r,Loa,B,AFw,ALw,A_SS,S,C,M)
```

10.1 Wind Coefficients for Merchant Ships (Isherwood 1972)

γ_w (deg)	A_0	A_1	A_2	A_3	A_4	A_5	A_6	S.E.
0	2.152	−5.00	0.243	−0.164	–	–	–	0.086
10	1.714	−3.33	0.145	−0.121	–	–	–	0.104
20	1.818	−3.97	0.211	−0.143	–	–	0.033	0.096
30	1.965	−4.81	0.243	−0.154	–	–	0.041	0.117
40	2.333	−5.99	0.247	−0.190	–	–	0.042	0.115
50	1.726	−6.54	0.189	−0.173	0.348	–	0.048	0.109
60	0.913	−4.68	–	−0.104	0.482	–	0.052	0.082
70	0.457	−2.88	–	−0.068	0.346	–	0.043	0.077
80	0.341	−0.91	–	−0.031	–	–	0.032	0.090
90	0.355	–	–	–	−0.247	–	0.018	0.094
100	0.601	–	–	–	−0.372	–	−0.020	0.096
110	0.651	1.29	–	–	−0.582	–	−0.031	0.090
120	0.564	2.54	–	–	−0.748	–	−0.024	0.100
130	−0.142	3.58	–	0.047	−0.700	–	−0.028	0.105
140	−0.677	3.64	–	0.069	−0.529	–	−0.032	0.123
150	−0.723	3.14	–	0.064	−0.475	–	−0.032	0.128
160	−2.148	2.56	–	0.081	–	1.27	−0.027	0.123
170	−2.707	3.97	−0.175	0.126	–	1.81	–	0.115
180	−2.529	3.76	−0.174	0.128	–	1.55	–	0.112
							Mean S.E.	0.103

γ_w (deg)	B_0	B_1	B_2	B_3	B_4	B_5	B_6	S.E.
10	0.096	0.22	–	–	–	–	–	0.015
20	0.176	0.71	–	–	–	–	–	0.023
30	0.225	1.38	–	0.023	–	−0.29	–	0.030
40	0.329	1.82	–	0.043	–	−0.59	–	0.054
50	1.164	1.26	0.121	–	−0.242	−0.95	–	0.055
60	1.163	0.96	0.101	–	−0.177	−0.88	–	0.049
70	0.916	0.53	0.069	–	–	−0.65	–	0.047
80	0.844	0.55	0.082	–	–	−0.54	–	0.046
90	0.889	–	0.138	–	–	−0.66	–	0.051
100	0.799	–	0.155	–	–	−0.55	–	0.050
110	0.797	–	0.151	–	–	−0.55	–	0.049
120	0.996	–	0.184	–	−0.212	−0.66	0.34	0.047
130	1.014	–	0.191	–	−0.280	−0.69	0.44	0.051
140	0.784	–	0.166	–	−0.209	−0.53	0.38	0.060
150	0.536	–	0.176	−0.029	−0.163	–	0.27	0.055
160	0.251	–	0.106	−0.022	–	–	–	0.036
170	0.125	–	0.046	−0.012	–	–	–	0.022
							Mean S.E.	0.044

Isherwood (1972) regression tables for merchant ships

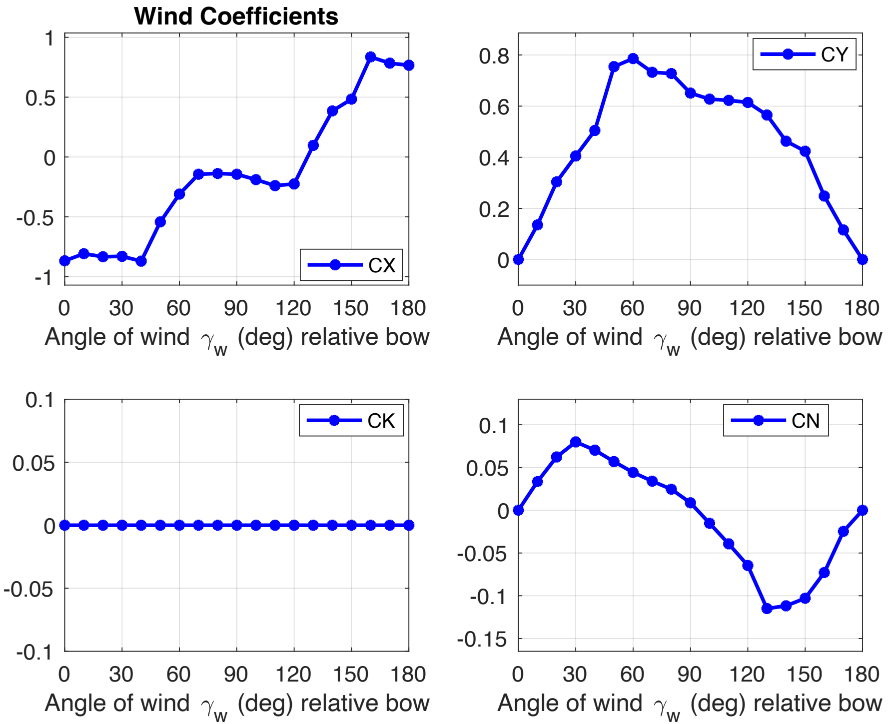
Used by the isherwood72.m MSS Matlab function

γ_w (deg)	C_0	C_1	C_2	C_3	C_4	C_5	S.E.
10	0.0596	0.061	–	–	–	−0.074	0.0048
20	0.1106	0.204	–	–	–	−0.170	0.0074
30	0.2258	0.245	–	–	–	−0.380	0.0105
40	0.2017	0.457	–	0.0067	–	−0.472	0.0137
50	0.1759	0.573	–	0.0118	–	−0.523	0.0149
60	0.1925	0.480	–	0.0115	–	−0.546	0.0133
70	0.2133	0.315	–	0.0081	–	−0.526	0.0125
80	0.1827	0.254	–	0.0053	–	−0.443	0.0123
90	0.2627	–	–	–	–	−0.508	0.0141
100	0.2102	–	−0.0195	–	0.0335	−0.492	0.0146
110	0.1567	–	−0.0258	–	0.0497	−0.457	0.0163
120	0.0801	–	−0.0311	–	0.0740	−0.396	0.0179
130	−0.0189	–	−0.0488	0.0101	0.1128	−0.420	0.0166
140	0.0256	–	−0.0422	0.0100	0.0889	−0.463	0.0162
150	0.0552	–	−0.0381	0.0109	0.0689	−0.476	0.0141
160	0.0881	–	−0.0306	0.0091	0.0366	−0.415	0.0105
170	0.0851	–	−0.0122	0.0025	–	−0.220	0.0057
						Mean S.E.	0.0127

MSS Toolbox

Merchant ship with masts/king posts

$$
\boldsymbol{\tau}_{\text{wind}} = \frac{1}{2}\rho_a V_{rw}^2 \begin{bmatrix} C_X(\gamma_{rw})A_{Fw} \\ C_Y(\gamma_{rw})A_{Lw} \\ \cancel{C_Z(\gamma_{rw})A_{Fw}} \\ \cancel{C_K(\gamma_{rw})A_{Lw}H_{Lw}} \\ \cancel{C_M(\gamma_{rw})A_{Fw}H_{Fw}} \\ C_N(\gamma_{rw})A_{Lw}L_{oa} \end{bmatrix}
$$



Ship data

```
Loa       =  100;  % length overall (m)
B         =   30;  % beam (m)
ALw       = 900;   % lateral projected area (m^2)
AFw       = 300;   % transverse projected area (m^2)
A_SS      = 100;   % lateral projected area of superstructure (m^2)
S         = 100;   % length of perimeter of lateral projection of model (m)
                   % excluding waterline and slender bodies such as masts and ventilators (m)
C         = 50;    % distance from bow of centroid of lateral projected area (m)
M         = 2;     % number of distinct groups of masts or king posts seen in lateral
                   % projection; king posts close against the bridge front are not included

[tau_wind,CX,CY,CN] = isherwood72(gamma_r,V_r,Loa,B,ALw,AFw,A_SS,S,C,M);
CK = 0*CX;
```

10.1 Wind Coefficients – Other Useful Methods and References

Wind Resistance of Very Large Crude Carriers (OCIMF, 1977)

Wind loads on very large crude carriers (VLCCs) in the range 150 000 to 500 000 dwt can be computed by applying the results of (OCIMF 1977).

Wind Resistance of Large Tankers and Medium Sized Ships

For wind resistance on large tankers in the 100 000 to 500 000 dwt class the reader is advised to consult Van Berlekom et al. (1974).

Medium sized ships of the order 600 to 50 000 dwt is discussed by Wagner (1967).

Wind Resistance of Moored Ships and Floating Structures

Wind loads on moored ships are discussed by De Kat and Wichers (1991) while an excellent reference for huge pontoon type floating structures is Kitamura et al. (1997).

10.2 Wave Forces and Moments

A marine control system can be simulated under influence of wave-induced forces by separating the 1st-order and 2nd-order effects:

$$M\dot{\nu} + C(\nu)\nu + D(\nu)\nu + g(\eta) + g_0 = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau$$

- **1st-order wave-induced forces (wave-frequency motion)**
  Zero-mean oscillatory motions

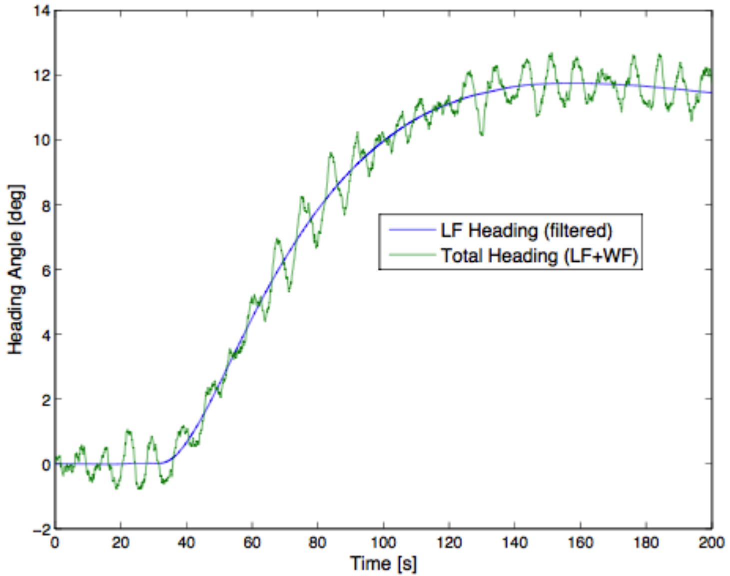
- **2nd-order wave-induced forces (wave drift forces)**
  Nonzero slowly-varying component

$$\boxed{\tau_{wave} = \tau_{wave1} + \tau_{wave2}}$$

Wave forces are observed as a mean slowly-varying component referred to as Low- Frequency (LF) motions and an oscillatory component called Wave-Frequency (WF) motions which must be compensated for by the feedback control system

- 1st-order wave-induced forces are usually removed by using a wave filter to filter out the WF component (to be designed later)

- 2nd-order wave-induced forces (drift) are removed by using integral action in the control law.



LF and WF heading angles

RAO = Response Amplitude Operator

10.2 Wave Forces and Moments

Wave Response Models:

1. **Force RAOs**
2. **Motion RAOs**
3. **Linear state-space models (WF models)**

The first two methods require that the RAO tables are computed using a hydrodynamic program (the RAO tables depend on the vessel's geometry)

Sea state → **Wave spectrum** → **Vessel response**

Input — RAO response output

For Methods 1 and 2, the vessel response is simulated using three blocks in a cascaded structure

White noise → **State-space model** (•LF vessel motions •WF motions) → **Vessel response**

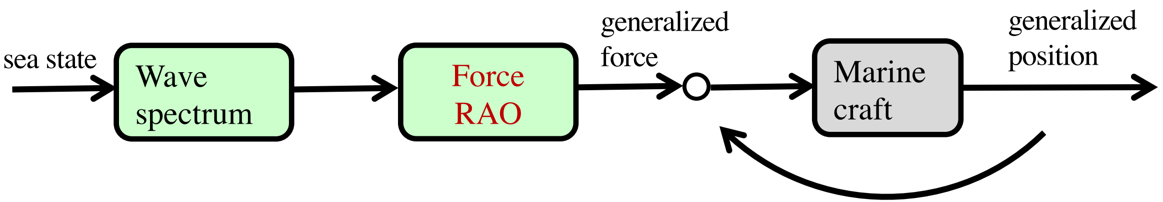
Input — State-space model output

Method 3 is a state-space model driven by white noise (no sea state description needed)

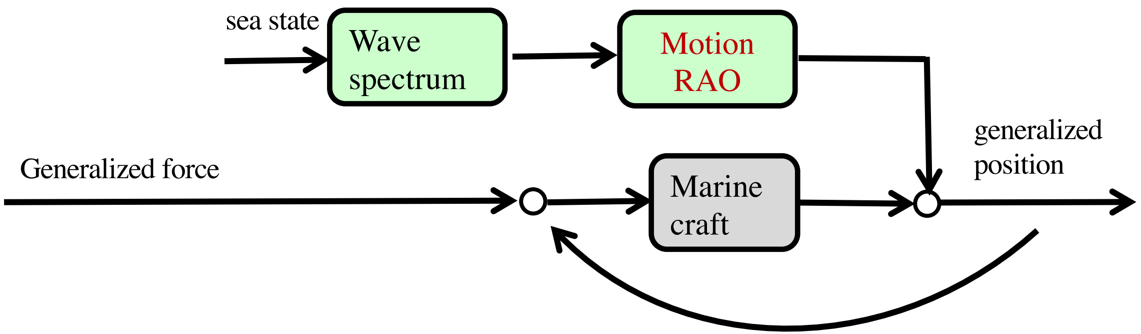
The last method is frequently used due to simplicity, but it is only intended for testing of robustness and performance of control systems, that is closed-loop analysis.

10.2 Wave Forces and Moments

1. Force RAOs

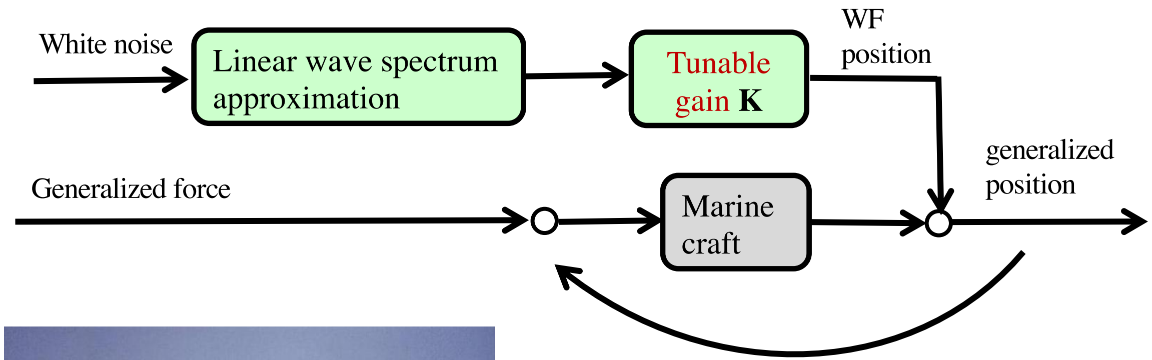


2. Motion RAOs



10.2 Wave Forces and Moments

3. Linear State-Space Models (WF models)



10.2 Sea State Descriptions

For marine craft, the sea can be characterized by the following wave spectrum parameters:

- The significant wave height H_s

  The mean wave height of the one third highest waves, also denoted as $H_{1/3}$

- One of the following wave periods:
  - The average wave period, T_1
  - Average zero-crossing wave period, T_z
  - Peak period, T_p (this is equivalent to the modal period, T_0)

$$T_z = 0.710 T_0 = 0.921 T_1$$

Sea state code	Description of sea	Wave height observed (m)	Percentage probability: World wide	North Atlantic	Northern North Atlantic
0	Calm (glassy)	0			
1	Calm (rippled)	0–0.1	11.2486	8.3103	6.0616
2	Smooth (wavelets)	0.1–0.5			
3	Slight	0.5–1.25	31.6851	28.1996	21.5683
4	Moderate	1.25–2.5	40.1944	42.0273	40.9915
5	Rough	2.5–4.0	12.8005	15.4435	21.2383
6	Very rough	4.0–6.0	3.0253	4.2938	7.0101
7	High	6.0–9.0	0.9263	1.4968	2.6931
8	Very high	9.0–14.0	0.1190	0.2263	0.4346
9	Phenomenal	Over 14.0	0.0009	0.0016	0.0035

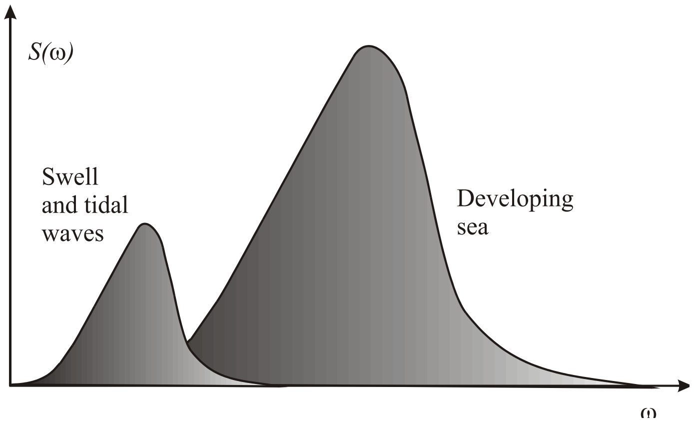
10.2 Wave Spectra

The process of wave generation due to wind starts with small wavelets appearing on the water surface. This increases the drag force, which in turn allows short waves to grow. These short waves continue to grow until they finally break, and their energy is dissipated.

A developing sea, or storm, starts with high frequencies creating a spectrum with a peak at a relative high frequency.

A storm which has lasted for a long time is said to create a fully developed sea.

After the wind has stopped, a low -frequency decaying sea or swell is formed. These long waves form a wave spectrum with a low peak frequency.



If the swell from one storm interacts with the waves from another storm, a wave spectrum with two-peak frequencies may be observed.

In addition, tidal waves will generate a peak at a low frequency. Hence, the resulting wave spectrum might be quite complicated in cases where the weather changes rapidly

10.2 Wave Spectra

Neumann Spectrum (1952)
One-parameter spectrum

$$S(\omega) = C\omega^{-6} \exp\left(-2g^2\omega^{-2}V^{-2}\right)$$

$$\lim_{\omega \gg 1} S(\omega) = \alpha\, g^2\, \omega^{-5}$$

Bretschneider Spectrum (1959)
Two-parameter spectrum

$$S(\omega) = 1.25\frac{\omega_0^4 H_s^2}{4}\omega^{-5} \exp\left[-1.25\left(\omega_0/\omega\right)^4\right]$$

Pierson-Moskowitz Spectrum (1964)
Two-parameter spectrum

$$S(\omega) = A\omega^{-5}\, \exp\left(-B\omega^{-4}\right)$$

$$A = 8.1 \times 10^{-3}\, g^2 = \text{constant}$$

$$B = 0.74\left(\frac{g}{V_{19.5}}\right)^4 = \frac{3.11}{H_s^2}$$

Matlab:
The Bretschneider and PM spectra are implemented in the MSS toolbox as wave spectra 1 and 2

```
S = wavespec(1,[A,B],w,1);
S = wavespec(2,V20,w,1);
```

where `A` and `B` are the spectrum parameters, `V20` is wind speed at 20 m height and `w` is the wave frequency vector.

10.2 Wave Spectra

Modified Pierson-Moskowitz (MPM) Spectrum (1964)

For prediction of responses of marine craft and offshore structures in open sea, the International Ship and Offshore Structures Congress and the International Towing Tank Conference (ITTC), have recommended the use of a modified version of the PM-spectrum where

$$S(\omega) = A\omega^{-5} \exp\left(-B\omega^{-4}\right) \qquad A = \frac{4\pi^3 H_s^2}{T_z^4}, \qquad B = \frac{16\pi^3}{T_z^4} \qquad H_s = \frac{2.06}{g^2} V_{19.5}^2$$

Matlab:
The modified PM spectrum is implemented in the MSS toolbox as wave spectra 3 to 5

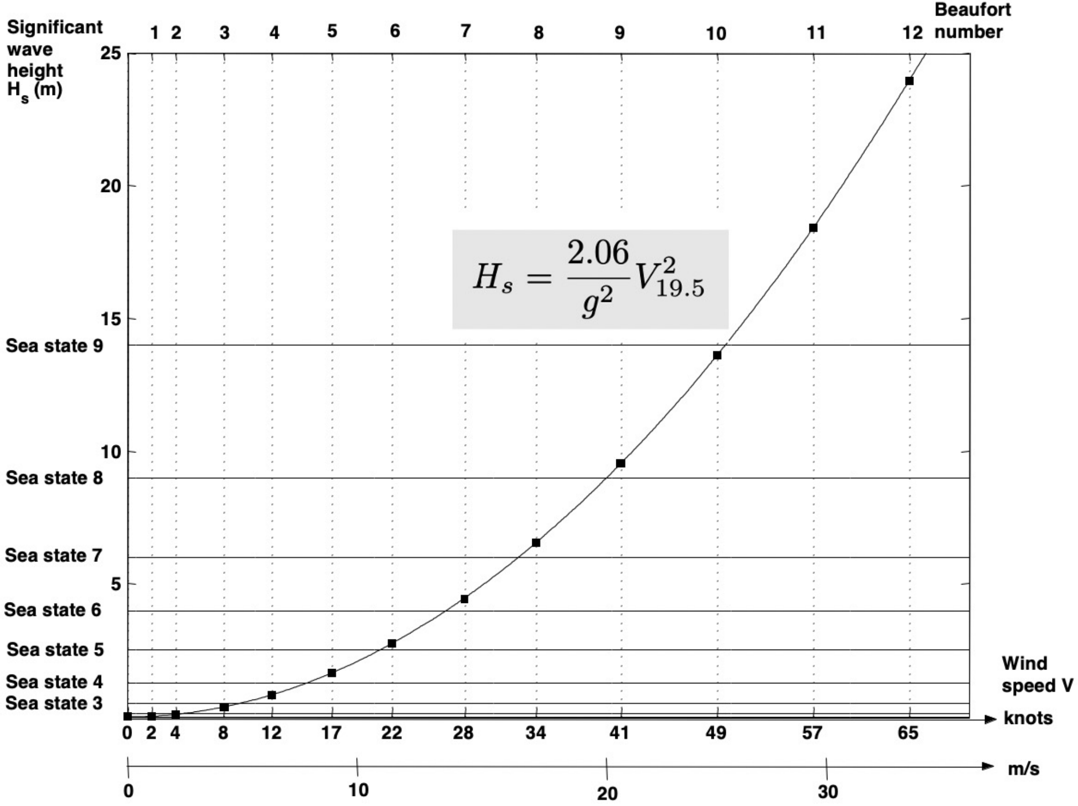
```
S = wavespec(3,[Hs,T0],w,1);
S = wavespec(4,[Hs,T1],w,1);
S = wavespec(5,[Hs,Tz],w,1);
```

where `Hs` is the significant wave height, `T0`, `T1` and `Tz` are the peak, average and average zero-crossing wave periods, respectively, while `w` is the wave frequency vector.

- The average wave period, T_1
- Average zero-crossing wave period, T_z
- Peak period, T_p (this is equivalent to the modal period, T_o)

$$T_z = 0.710 T_0 = 0.921 T_1$$

10.2 Wave Spectra



Significant wave height as a function of wind speed

Why 19.5 meters above the surface level?

This is the height of the anemometers on the weather ships used by Pierson and Moskowitz in 1964.

10.2 Wave Spectra

JONSWAP Spectrum

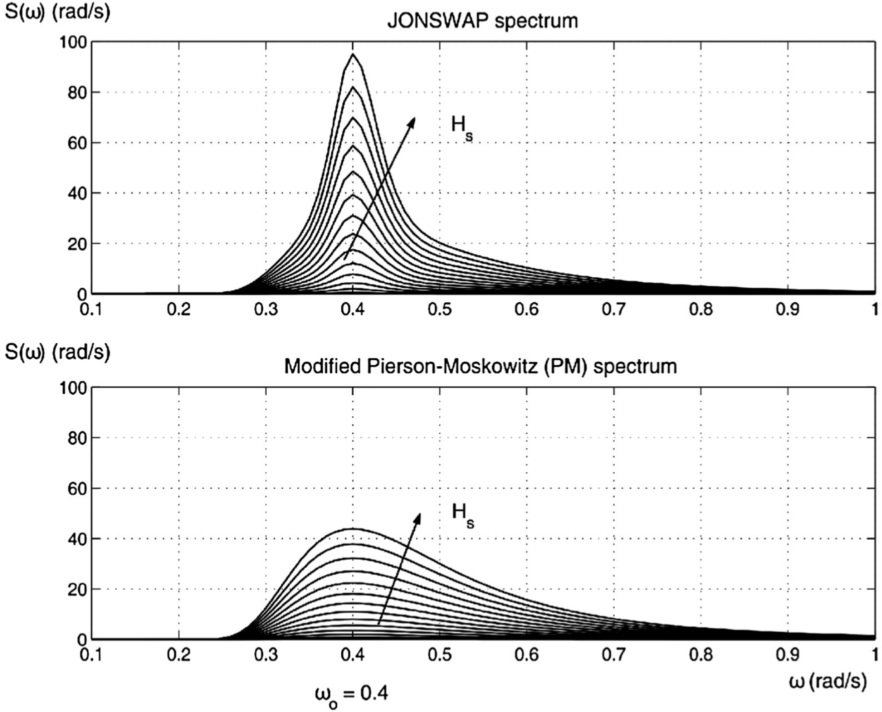
In 1968-1969 an extensive measurement program was carried out in the North Sea, between the island Sylt in Germany and Iceland. The measurement program is known as the Joint North Sea Wave Project (JONSWAP) and the results from these investigations have been adopted as an ITTC standard in 1984.

$$S(\omega) = 155 \frac{H_s^2}{T_1^4} \omega^{-5} \exp\left(\frac{-944}{T_1^4}\omega^{-4}\right)\gamma^Y$$

Peak enhancement factor γ^Y

$$Y = \exp\left[-\left(\frac{0.191\omega T_1 - 1}{\sqrt{2}\,\sigma}\right)^2\right]$$

$$\sigma = \begin{cases} 0.07 & for \quad \omega \leq 5.24/T_1 \\ 0.09 & for \quad \omega > 5.24/T_1 \end{cases}$$



Matlab:
The JONSWAP spectrum is included in the MSS toolbox as wave spectra 6 and 7

```
S = wavespec(6,[V10,fetch],w,1);
S = wavespec(7,[Hs,w0,gamma],w,1);
```

where `V10` is the wind speed at 10 m height, `Hs` is the significant wave height, `w0` is peak frequency and `w` is the wave frequency vector.

10.2 Wave Spectra

Torsethaugen Spectrum

The Torsethaugen spectrum (1996) is an empirical, two-peaked spectrum. Includes the effect of swell (low-frequency peak) and newly developed waves (high-frequency peak).

The spectrum is developed using curve fitting of experimental data from the North Sea.

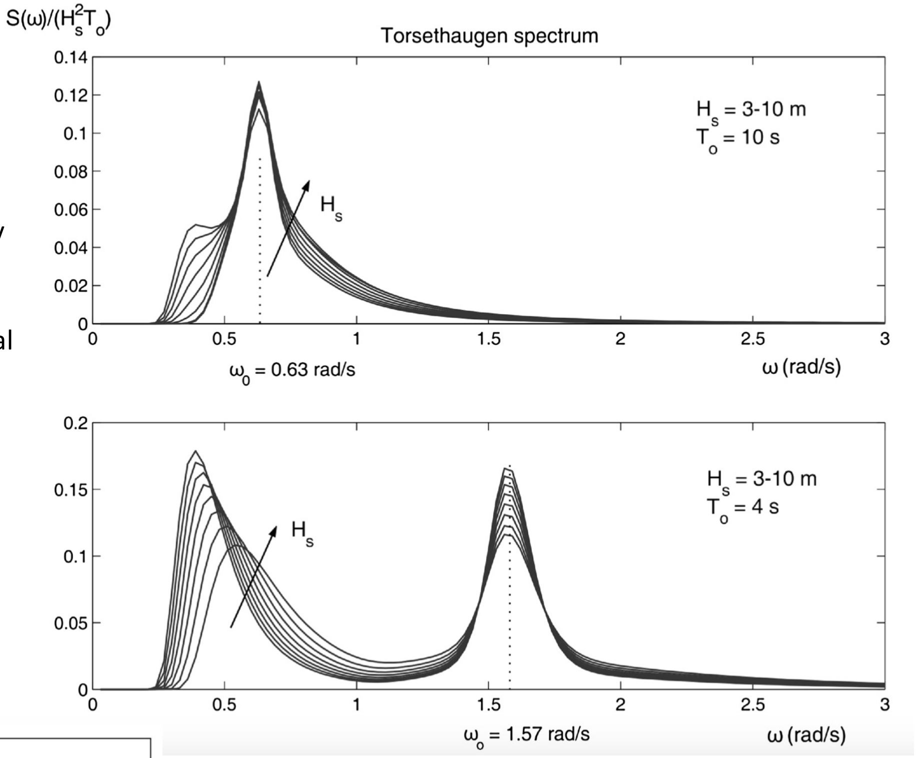


Figure: Upper plot shows only one peak at ω_0 = 0.63 rad/s representing swell and developing sea while the lower plot shows low-frequency swell and newly developing sea with peak frequency ω_0 = 1.57 rad/s.

Matlab:
The Torsethaugen spectrum is included in the MSS toolbox as wave spectrum 7

```
S = wavespec(7,[Hs,w0],w,1);
```

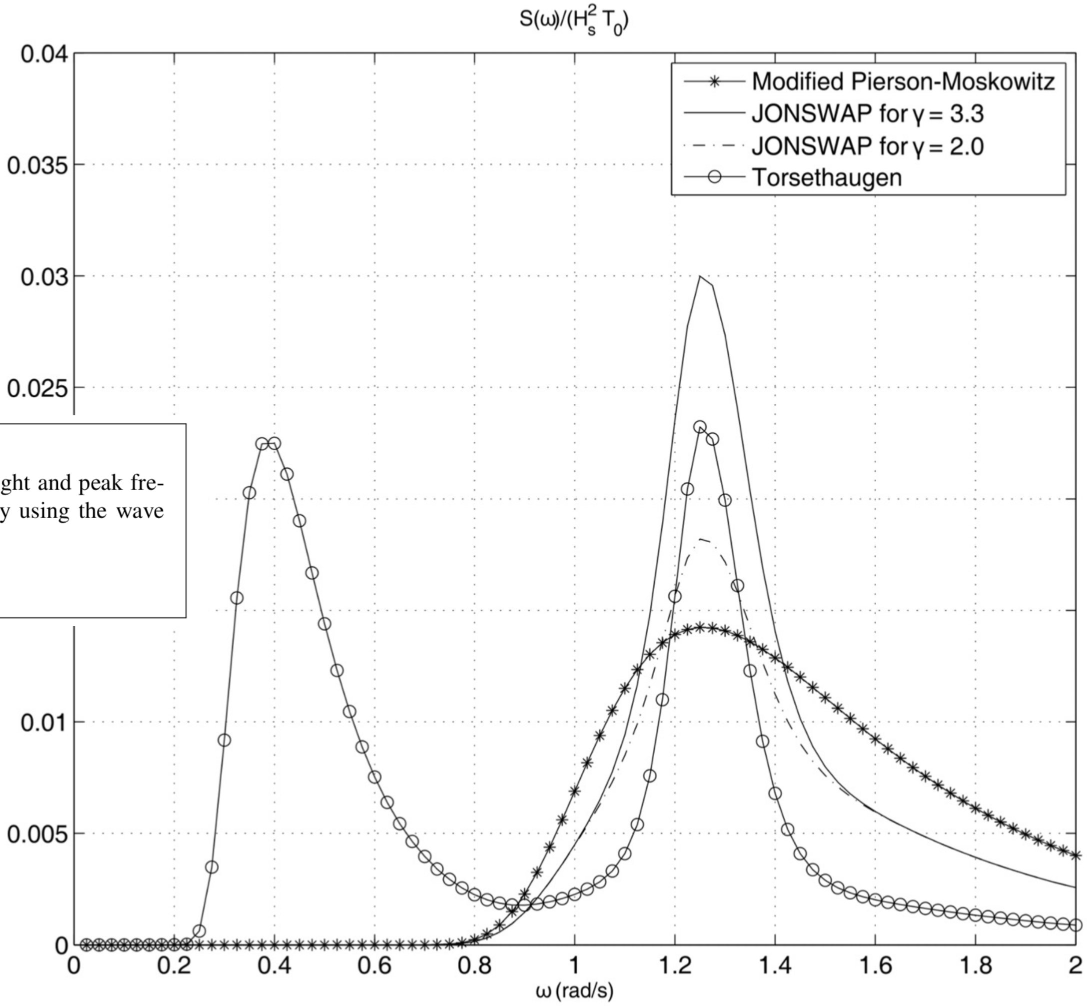
where `Hs` is the significant wave height, `w0` is peak frequency and `w` is the wave frequency vector.

10.2 Wave Spectra

Wave spectra plotted for same wave height and peak frequency.

Matlab:
The different wave spectra when plotted for the same wave height and peak frequency are shown in Figure 10.11. The plots are generated by using the wave demo option in the MSS toolbox:

```
gncdemo;
```



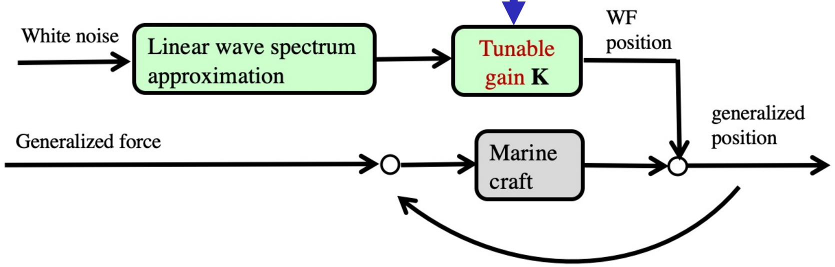
10.2 Second-Order Wave Transfer Function Approximation

$$\sigma = \sqrt{\frac{A}{\omega_0^5} \exp\left(-\frac{B}{\omega_0^4}\right)}$$

Transfer function

$$h(s) = \frac{\xi}{w}(s) = \frac{K_w s}{s^2 + 2\lambda\omega_0\, s + \omega_0^2}$$

$$K_w = 2\lambda\,\omega_0\,\sigma$$



Power spectral density

$$h(\mathrm{j}\omega) = \frac{\mathrm{j}\,2(\lambda\,\omega_0\,\sigma)\,\omega}{(\omega_0^2 - \omega^2) + \mathrm{j}\,2\lambda\,\omega_0\,\omega}$$

$$P_{\xi\xi}(\omega) = |h(\mathrm{j}\omega)|^2\; P_{ww}(\omega) = |h(\mathrm{j}\omega)|^2$$

Matlab:
Power spectral density function:

```
function Pyy = Slin(lambda,w)
% Pyy = Slin(lambda,w) 2nd-order linear PSD function
% w = wave spectrum frequency (rad/s)
% lambda = relative damping factor

global sigma wo
Pyy = 4*(lambda*wo*sigma)^2*w.^2 ./ ( (wo^2-w.^2).^2 + ...
     4*(lambda*wo.*w).^2 );
```

$$P_{\xi\xi}(\omega_0) = S(\omega_0)$$

$$\Updownarrow$$

$$\sigma^2 = \max_{0<\omega<\infty} S(\omega)$$

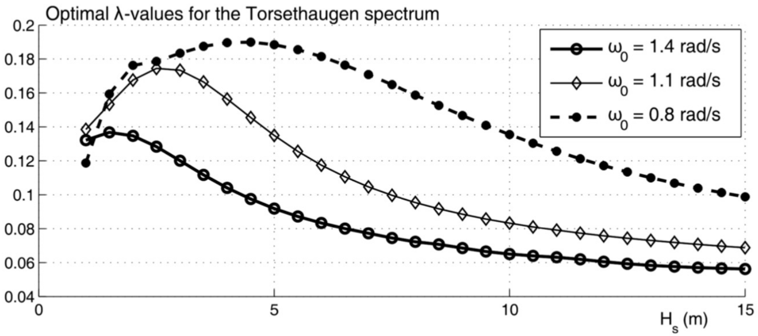
10.2 Nonlinear Least-Squares Optimization of Linear Spectra

Matlab:

Nonlinear least-squares:

```
% Script for plotting the nonlinear least-squares fit
% see ExLinspec.m
global sigma wo

wo = 0.8; Hs = 10; wmax = 3;
To = 2*pi/wo;
w = (0.0001:0.01:wmax)';

% Modified PM
subplot(311)
S = wavespec(3,[Hs,To],w,1);  sigma = sqrt(max(S));
lambda = lsqcurvefit('Slin', 0.1, w, S)
hold on; plot(w,Slin(lambda,w),'linewidth',2); hold off;
legend('Modified PM spectrum','Linear approximation')
```

```
% JONSWAP
subplot(312)
S = wavespec(7,[Hs,wo,3.3],w,1);   sigma = sqrt(max(S));
lambda = lsqcurvefit('Slin', 0.1, w, S)
hold on; plot(w,Slin(lambda,w),'linewidth',2); hold off;
legend('JONSWAP spectrum','Linear approximation')

% Torsethaugen (only one peak is fitted)
subplot(313)
S = wavespec(8,[Hs,wo],w,1);   sigma = sqrt(max(S));
lambda = lsqcurvefit('Slin', 0.1, w, S)
hold on; plot(w,Slin(lambda,w),'linewidth',2); hold off;
legend('Torsethaugen spectrum','Linear approximation')
```

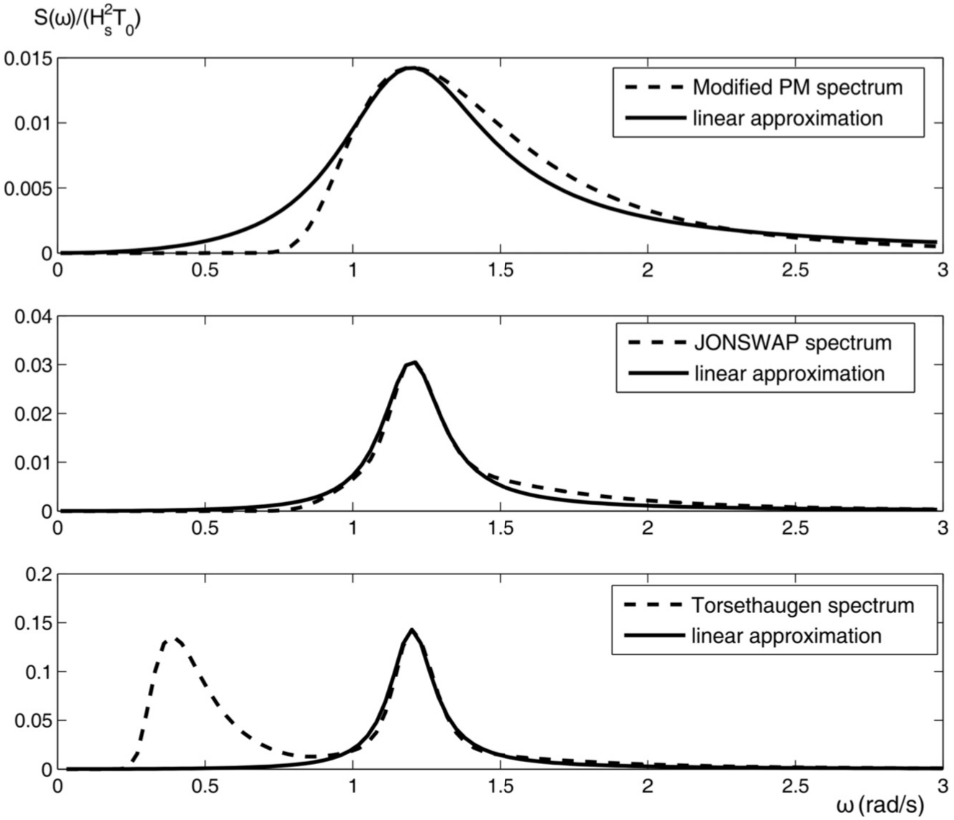


We can fit the nonlinear spectra to a linear model and find the optimal relative damping factor λ for varying frequencies ω_0

$$h(s) = \frac{\xi}{w}(s) = \frac{K_w s}{s^2 + 2\lambda\omega_0\, s + \omega_0^2}$$

ω_0	0.5	0.8	1.1	1.4	*Recommended value*
λ *(MPM)*	0.2565	0.2573	0.2588	0.2606	0.26
λ *(JONSWAP)*	0.1017	0.1017	0.1017	0.1017	0.10

10.2 Nonlinear Least-Squares Optimization of Linear Spectra



The nonlinear spectra compared with the transfer function model (linear approximation).

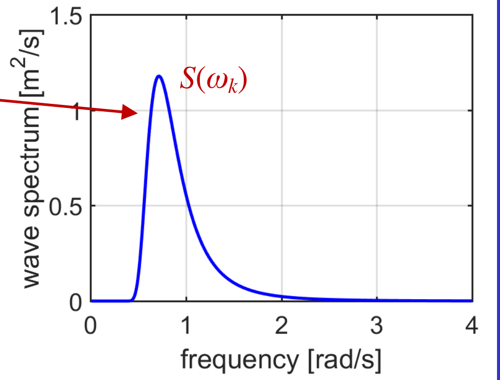
Quite good results are obtained.

One peak of the Torsethaugen spectrum is obliviously missed (lower plot) by the linear approximation. However, we can use two transfer functions to correct this if needed.

10.2 Wave Amplitude Response Model

The relationship between the wave spectrum $S(\omega_k)$ and the wave amplitude A_k for a wave component k is (Faltinsen 1990)

$$\frac{1}{2}A_k^2 = S(\omega_k)\Delta\omega$$

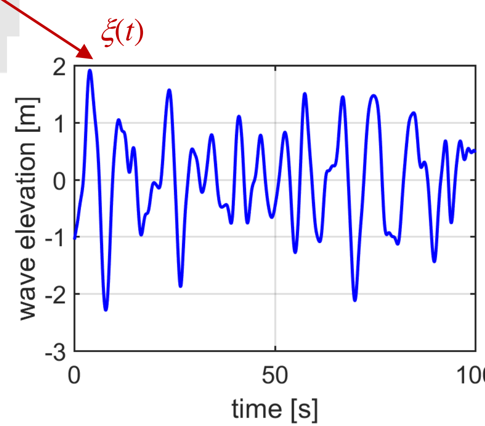


where $\Delta\omega$ is a constant difference between the frequencies.
This relationship can be used to compute wave-induced responses in the time domain.

Long-Crested Irregular Sea

The wave elevation of a long-crested irregular sea in the origin of {s} under the assumption of zero speed can be written as the sum of N harmonic components.

$$\begin{aligned}\xi &= \sum_{k=1}^{N} A_k \cos(\omega_k t + \epsilon_k) \\ &= \sum_{k=1}^{N} \sqrt{2S(\omega_k)\Delta\omega}\, \cos(\omega_k t + \epsilon_k)\end{aligned}$$



Here ε_k is the random phase angle of wave component number k. Since this expression repeats itself after a time $2\pi/\Delta\omega$ many wave components N are needed. However, a practical way to avoid this is to choose ω_k randomly in the interval

$$\left[\omega_k - \frac{\Delta\omega}{2}, \omega_k + \frac{\Delta\omega}{2}\right]$$

10.2 Wave Amplitude Response Model

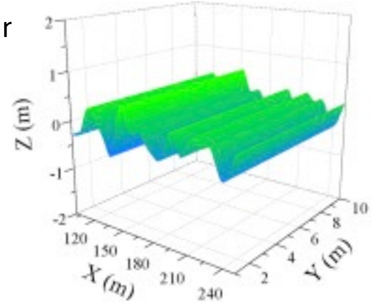
Short-Crested Irregular Sea

The most likely situation encountered at sea is short-crested or confused waves. This is observed as irregularities along the wave crest at right angles to the direction of the wind.

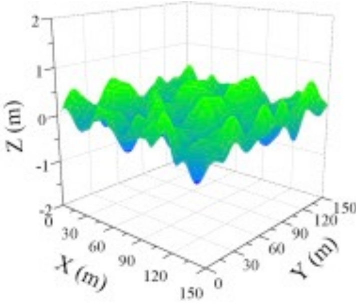
Short-crested irregular sea can be modeled by a 2-D wave spectrur

$$\boxed{S_M(\omega, \mu) = S(\omega) M(\mu)}$$

where $\mu = 0$ corresponds to the main wave propagation direction while a nonzero μ will spread the energy over a certain angle contained within $[-\pi/2, \pi/2]$ from the wind direction.



(a) Long-crested wave



(b) Short-crested wave

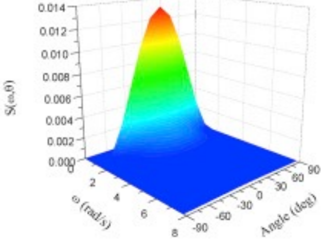
Spreading function

$$M(\mu) = \begin{cases} \frac{2}{\pi} \cos^2(\mu), & -\pi/2 \leq \mu \leq \pi/2 \\ 0, & \text{elsewhere} \end{cases}$$

$$\int_{-\pi/2}^{\pi/2} M(\mu) = 1$$

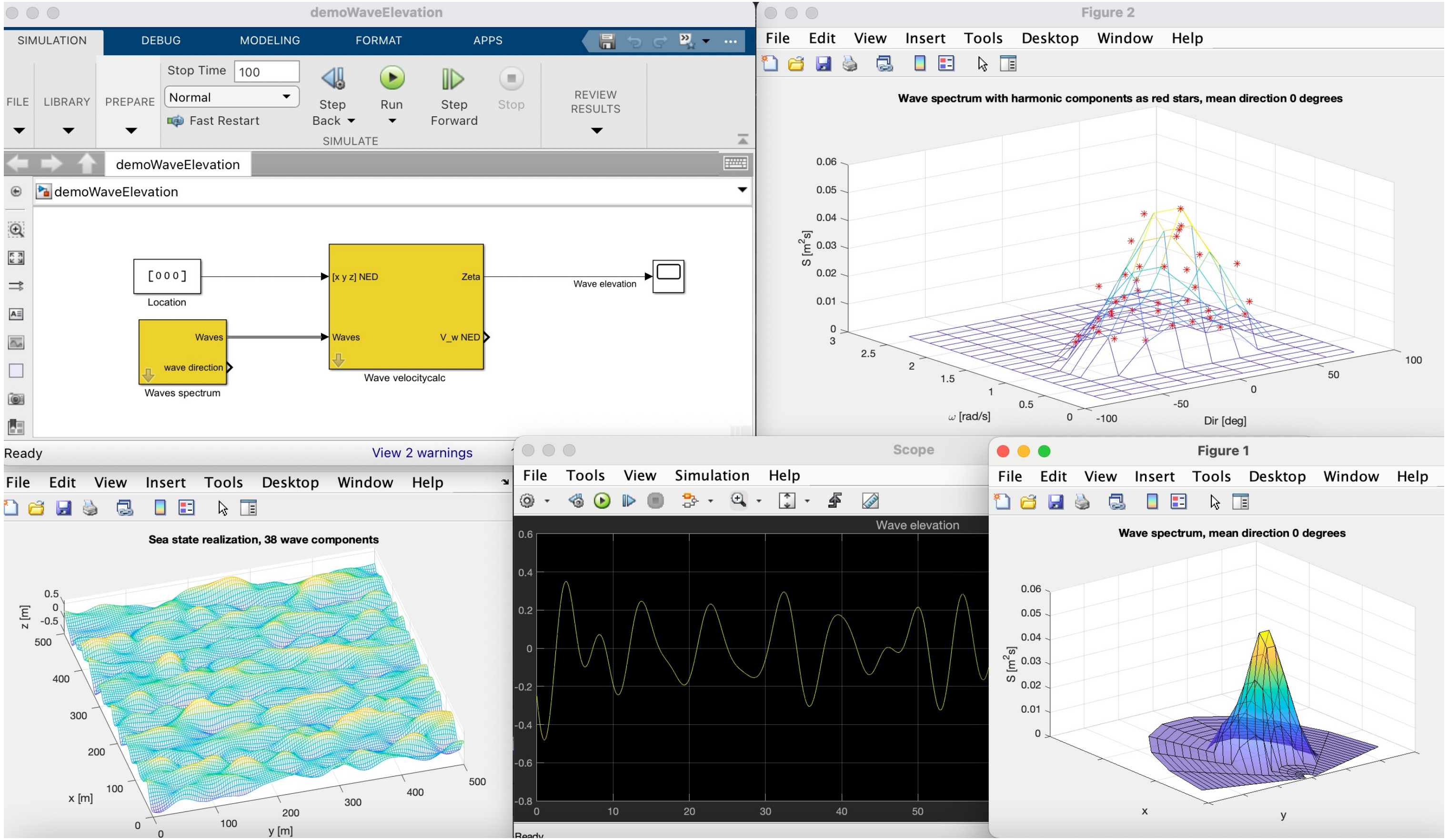
Wave elevation

$$\boxed{\xi = \sum_{k=1}^{N} \sum_{i=1}^{M} \sqrt{2 S_M(\omega_k, \mu_i - \beta) \Delta\omega \Delta\mu} \cos(\omega_k t + \epsilon_{i,k})}$$



(c) Directional spectrum

Simulink Demo: Wave Spectrum to Wave Elevation



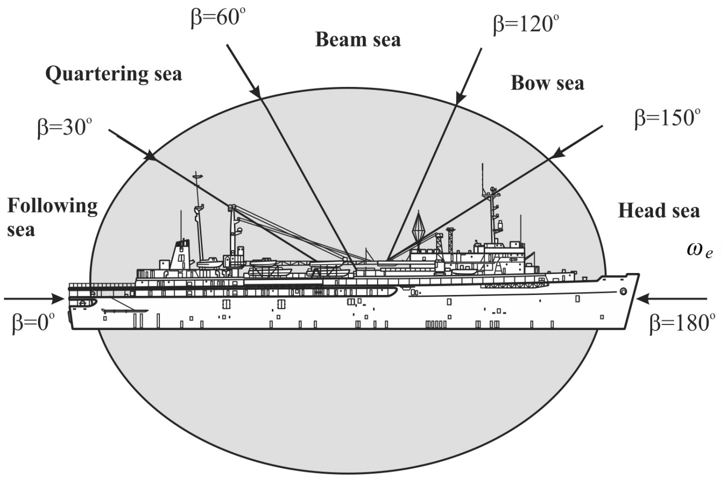
10.2 Wave Amplitude Response Model

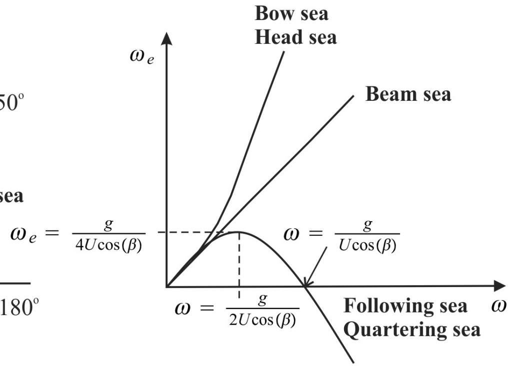
Extension to Forward Speed using the Frequency of Encounter

For a ship moving with forward speed U, the peak frequency of the spectrum ω_0 will be modified according to:

$$\omega_e(U,\omega_0,\beta) = \left|\omega_0 - \frac{\omega_0^2}{g} U\cos(\beta)\right|$$

ω_e is the frequency of encounter
β is the angle between heading and wave direction (not the sideslip angle!)



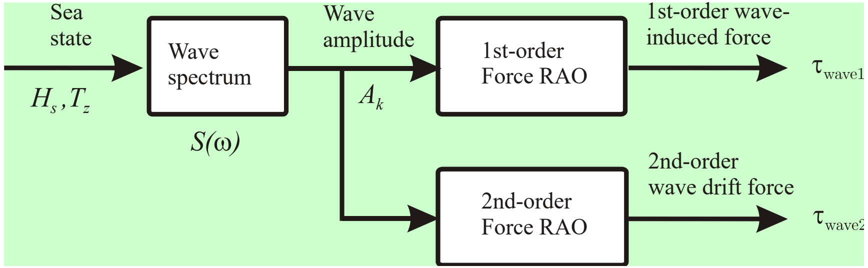


Wave Elevation using Frequency of Encounter

$$\xi = \sum_{k=1}^{N}\sum_{i=1}^{N} \sqrt{2S_M(\omega_k,\mu_i-\beta)\Delta\omega\Delta\mu}\,\cos\left(\omega_k t - \frac{\omega_k^2}{g}U\cos(\mu_i-\beta)\,t + \epsilon_k\right)$$

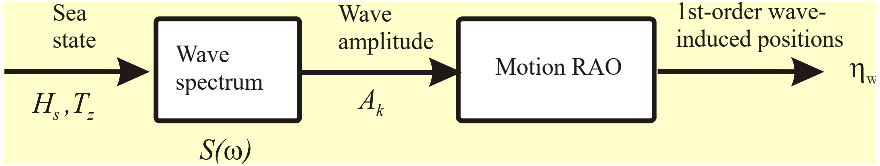
10.2 Wave Forces and Moments

1. Force RAOs



Generalized force

2. Motion RAOs



Generalized position

10.2 Wave Force Response Amplitude Operators

Normalized Force RAOs

The 1st- and 2nd-order wave forces for varying wave directions β_i and wave frequencies ω_k are

$$F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta_i) = \left| \frac{\tilde{\tau}_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta_i)}{\rho g A_k} \right| e^{j\angle \tilde{\tau}_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta_i)}$$

$$F_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \beta_i) = \left| \frac{\tilde{\tau}_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \beta_i)}{\rho g A_k^2} \right| e^{j\angle \tilde{\tau}_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \beta_i)}$$

complex variables

The Re-Im parts of the RAOs (ASCII file generated by the hydrodynamic code) are processed in the MSS toolbox to obtain the amplitudes and phases of the forces.

1st-order wave-induced forces

$$\left| F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta_i) \right| = \sqrt{(\texttt{ImRAO\_wave1\{dof\}(k,i)})^2 + (\texttt{ReRAO\_wave1\{dof\}(k,i)})^2}$$

$$\angle F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta_i) = \text{atan2}\,(\texttt{ImRAO\_wave1\{dof\}(k,i), ReRAO\_wave1\{dof\}(k,i)})$$

2nd-order wave-induced forces (wave drift)

$$\left| F_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \beta_i) \right| = \texttt{ReRAO\_wave2\{dof\}(k,i)}$$

$$\angle F_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \beta_i) = 0$$

Matlab structures containing the amplitudes and phases

10.2 Wave Force Response Amplitude Operators

Matlab:
The motion RAOs are processed in the MSS Hydro Matlab toolbox by using m-file commands:

```
wamit2vessel    % read and process WAMIT data
veres2vessel    % read and process ShipX (Veres) data
```

The data are represented in the workspace as Matlab structures:

```
vessel.forceRAO.w(k)                          % frequencies
vessel.forceRAO.amp{dof}(k,i,speed_no)        % amplitudes
vessel.forceRAO.phase{dof}(k,i,speed_no)      % phases
```

where `speed_no = 1` represents $U = 0$. For the mean drift forces only surge, sway and yaw are considered (`dof` $\in \{1, 2, 6\}$ where the third component corresponds to yaw)

```
vessel.driftfrc.w(k)                          % frequencies
vessel.driftfrc.amp{dof}(k,i,speed_no)        % amplitudes
```

It is possible to plot the force RAOs using

```
plotTF                      % plot transfer function
plotWD                      % plot wave drift
```

There are no phase signal for the drift force, only amplitude is needed to simulate drift

How to use amplitude and phase to simulate the wave forces in Matlab? →

10.2 Wave Force Response Amplitude Operators

Generalized wave-induced forces from force RAOs (no spreading function)

For the no spreading case, the encounter angle $\beta = \text{constant}$ such that

$$\begin{aligned}
\tau_{\text{wave1}}^{\{\text{dof}\}} &= \sum_{k=1}^{N} \rho g \left| F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta) \right| A_k \cos\left(\omega_e(U, \omega_k, \beta) t + \angle F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta) + \epsilon_k \right) \\
&= \sum_{k=1}^{N} \rho g \left| F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta) \right| \sqrt{2 S(\omega_k) \Delta\omega} \\
&\qquad \cos\left(\omega_e(U, \omega_k, \beta) t + \angle F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \beta) + \epsilon_k \right) \qquad (10.93)
\end{aligned}$$

$$\begin{aligned}
\tau_{\text{wave2}}^{\{\text{dof}\}} &= \sum_{k=1}^{N} \rho g \left| F_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \beta) \right| A_k^2 \\
&= \sum_{k=1}^{N} \rho g \left| F_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \beta) \right| 2 S(\omega_k) \Delta\omega \qquad (10.94)
\end{aligned}$$

where

$$\omega_e(U, \omega_k, \beta) = \omega_k - \frac{\omega_k^2}{g} U \cos(\beta) \qquad (10.95)$$

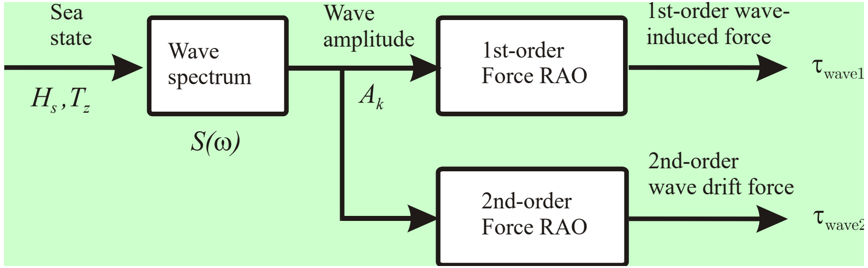
10.2 Wave Force Response Amplitude Operators

Generalized wave-induced forces from force RAOs (spreading function)

The more general case, where the spreading function (10.80) is included, can be simulated by using varying wave directions μ_i $(i = 1, .., M)$ and

$$\tau_{\text{wave1}}^{\{\text{dof}\}} = \sum_{k=1}^{N}\sum_{i=1}^{M} \rho g \left| F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \mu_i - \beta) \right| \sqrt{2S_M(\omega_k, \mu_i - \beta)\Delta\omega\Delta\mu}$$
$$\cos\left(\omega_e(U, \omega_k, \mu_i - \beta)t + \angle F_{\text{wave1}}^{\{\text{dof}\}}(\omega_k, \mu_i - \beta) + \epsilon_k\right) \quad (10.96)$$

$$\tau_{\text{wave2}}^{\{\text{dof}\}} = \sum_{k=1}^{N}\sum_{i=1}^{M} \rho g \left| F_{\text{wave2}}^{\{\text{dof}\}}(\omega_k, \mu_i - \beta) \right| 2S_M(\omega_k, \mu_i - \beta)\Delta\omega\Delta\mu \quad (10.97)$$



10.2 Motion Response Amplitude Operators

Generalized position (no spreading function)

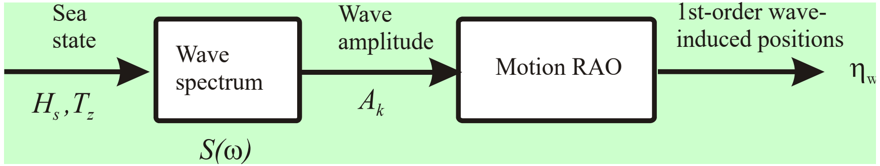
$$
\begin{aligned}
\eta_w^{\{\text{dof}\}} &= \sum_{k=1}^{N} \left| \eta_w^{\{\text{dof}\}}(\omega_k, \beta) \right| A_k \cos\left(\omega_e(U, \omega_k, \beta)t + \angle \eta_w^{\{\text{dof}\}}(\omega_k, \beta) + \epsilon_k \right) \\
&= \sum_{k=1}^{N} \left| \eta_w^{\{\text{dof}\}}(\omega_k, \beta) \right| \sqrt{2S(\omega_k)\Delta\omega} \cos\left(\omega_e(U, \omega_k, \beta)t + \angle \eta_w^{\{\text{dof}\}}(\omega_k, \beta) + \epsilon_k \right)
\end{aligned}
$$

where $|\eta_{w1}^{\{dof\}}(\omega_k, \beta_i)|$ and $\angle \eta_{w1}^{\{dof\}}(\omega_k, \beta_i)$ are the motion RAO amplitude and phase for frequency ω_k and wave direction β_i

Total Motion

$$\mathbf{y} = \boldsymbol{\eta} + \boldsymbol{\eta}_w$$

wave drift must be added manually in the equations of motion



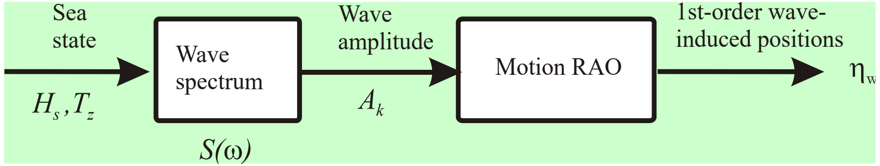
$$\frac{1}{2} A_k^2 = S(\omega_k)\Delta\omega$$

10.2 Motion Response Amplitude Operators

Generalized position (spreading function)

$$\eta_w^{\{\text{dof}\}} = \sum_{k=1}^{N}\sum_{i=1}^{M} \left|\eta_w^{\{\text{dof}\}}(\omega_k, \mu_i - \beta)\right| \sqrt{2S_M(\omega_k, \mu_i - \beta)\Delta\omega\Delta\mu}$$
$$\cos\left(\omega_e(U, \omega_k, \mu_i - \beta)t + \angle\eta_w^{\{\text{dof}\}}(\omega_k, \mu_i - \beta) + \epsilon_k\right) \quad (10.106)$$

where $|\eta_w^{\{\text{dof}\}}(\omega_k, \mu_i - \beta)|$ and $\angle\eta_w^{\{\text{dof}\}}(\omega_k, \mu_i - \beta)$ are the motion RAO amplitude and phase for frequency ω_k and encounter angle $\beta_i = \mu_i - \beta$. This expression does not contain the second-order wave-induced forces. Consequently, wave drift forces must be added manually, for instance by using the wave drift force RAO to compute $\tau_{\text{wave2}}^{\{\text{dof}\}}$.



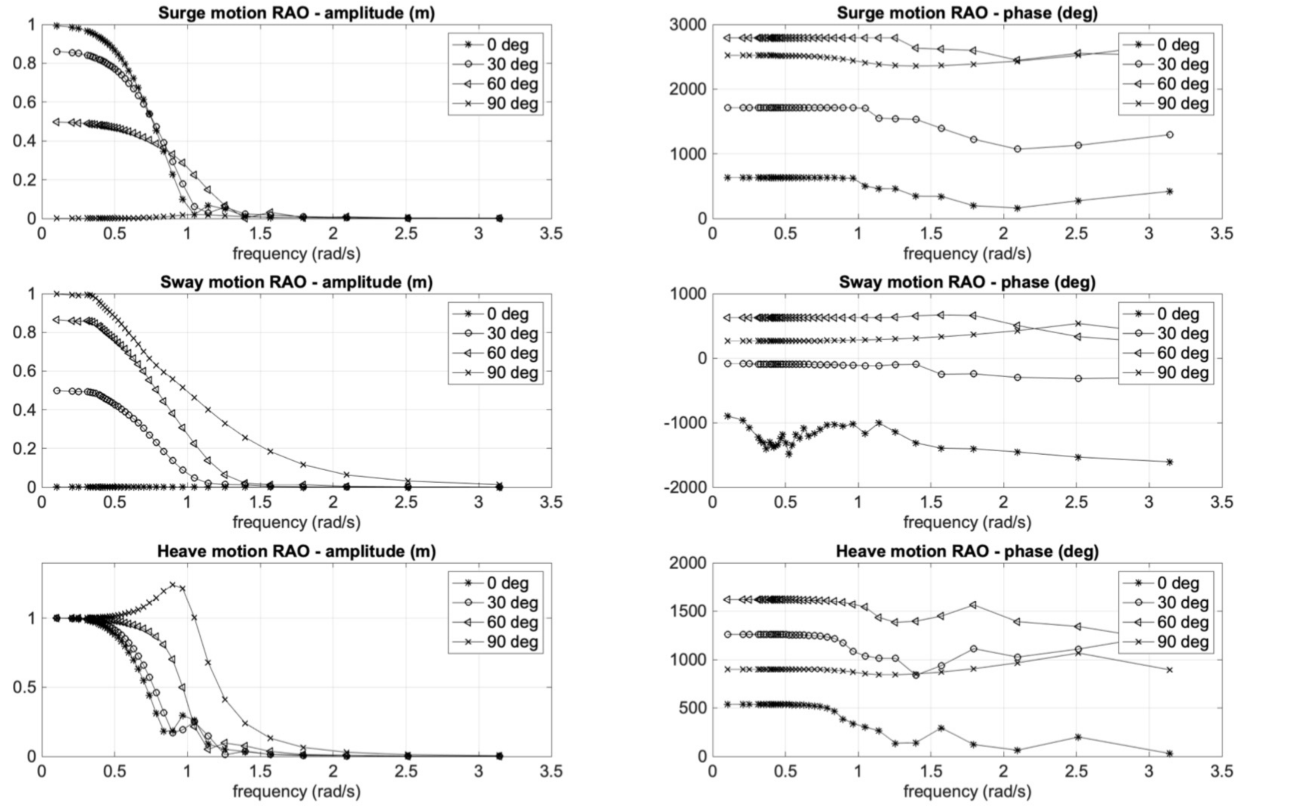
$$\frac{1}{2}A_k^2 = S(\omega_k)\Delta\omega$$

MSS Toolbox

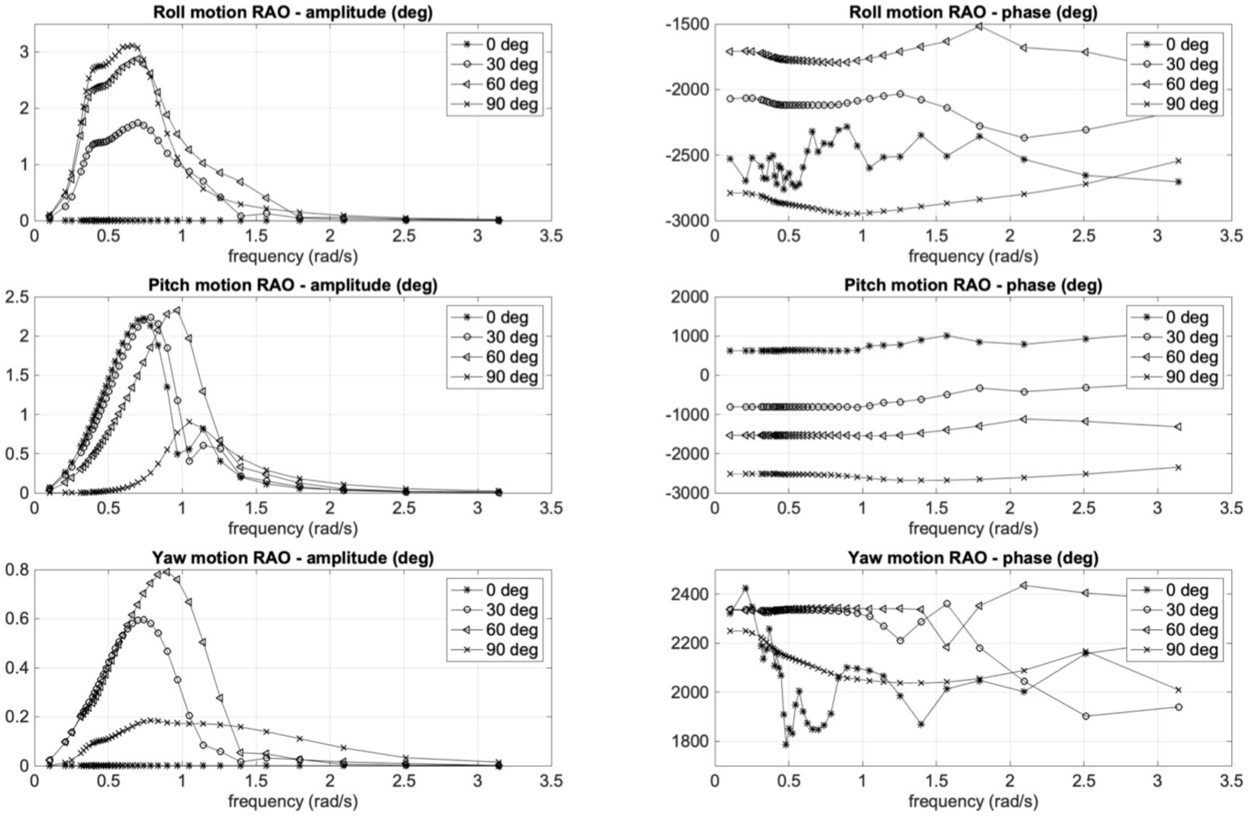
The MSS functions plotRAOamp and plotRAOphs plots the motion and force RAOs using the data in the Matlab vessel structure.

```
load supply;

for DOF = 1:6
    figure(DOF);

    % Plot motion RAO
    w     = vessel.motionRAO.w;
    amp   = vessel.motionRAO.amp;
    phase = vessel.motionRAO.phase;
    subplot(411); plotRAOamp(w,amp,DOF);
    subplot(412); plotRAOphs(w,phase,DOF);

    % Plot force RAO
    w     = vessel.forceRAO.w;
    amp   = vessel.forceRAO.amp;
    phase = vessel.forceRAO.phase;
    subplot(413); plotRAOamp(w,amp,DOF);
    subplot(414); plotRAOphs(w,phase,DOF);
end
```

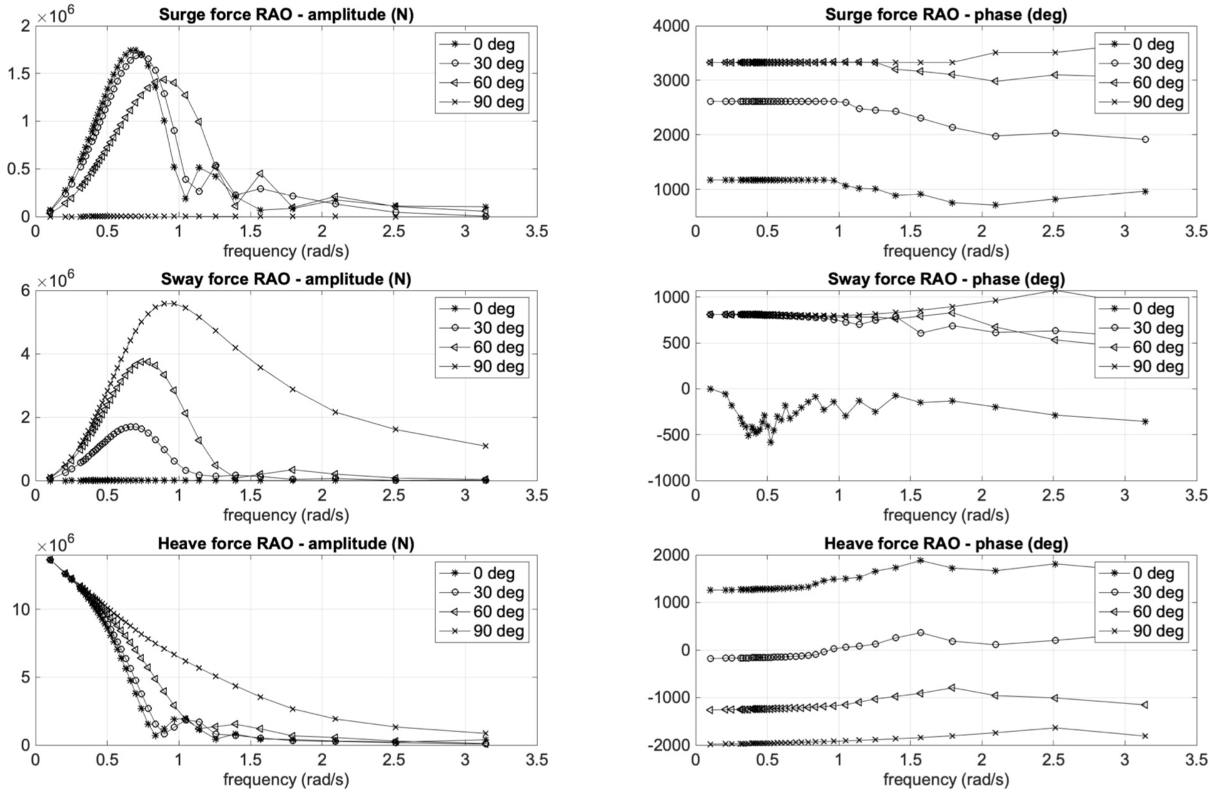
```
function plotRAOamp(w,amp,DOF)
    velno = 1;
    arg = amp{DOF}(:,:,velno);
    plot(w,arg(:,1),'-*k',w, arg(:,4),'-ko',w, ...
        arg(:,7),'-k<',w, arg(:,10),'-kx');
    legend('0 deg','30 deg','60 deg','90 deg');
    xlabel('wave encounter frequency (rad/s)'), grid;
end

function plotRAOphs(w,phase,DOF)
    velno = 1;
    phs = (180/pi)*unwrap(phase{DOF}(:,:,velno));
    plot(w,phs(:,1),'-*k',w, phs(:,4),'-ko',w, ...
        phs(:,7),'-k<',w, phs(:,10),'-kx');
    legend('0 deg','30 deg','60 deg','90 deg');
    xlabel('wave encounter frequency (rad/s)'), grid;
end
```



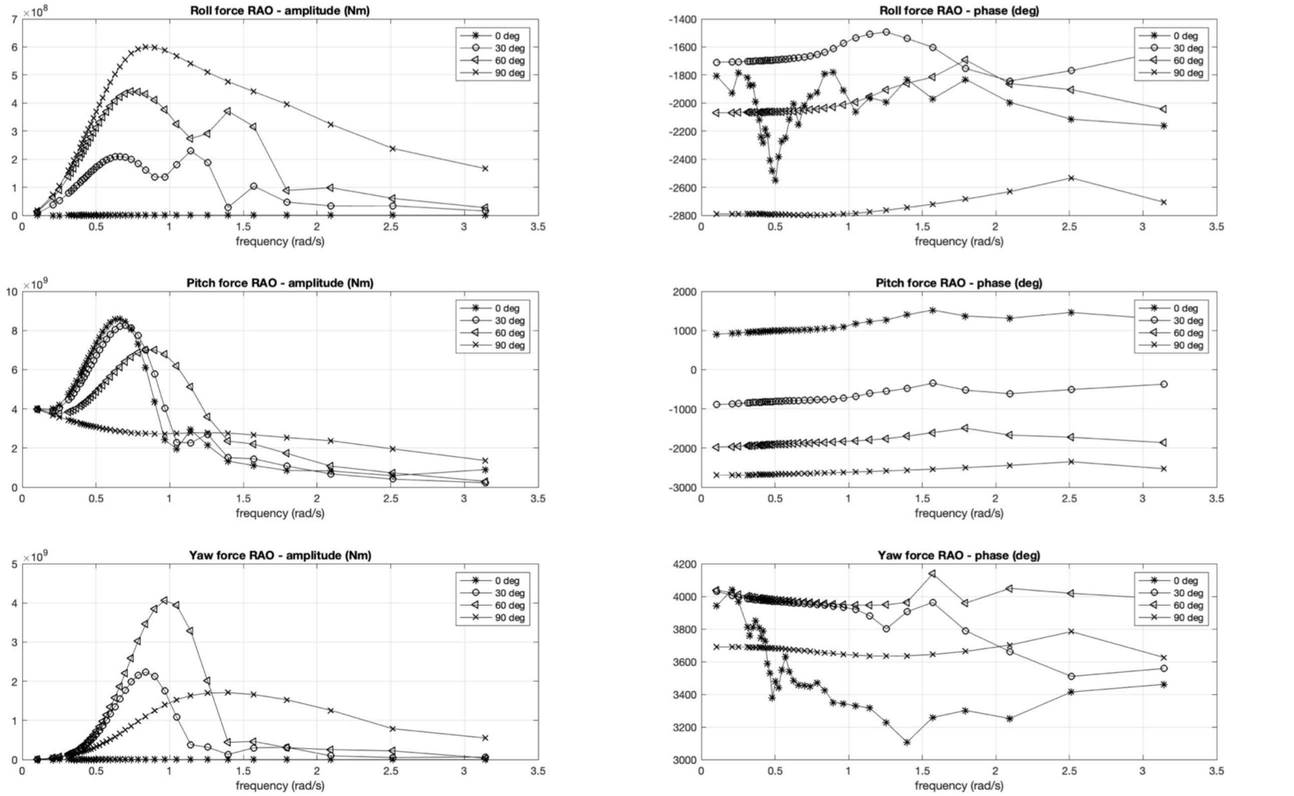
(a) Surge, sway and heave amplitudes (m) and phases (deg).



(b) Roll, pitch and yaw amplitudes (deg) and phases (deg)

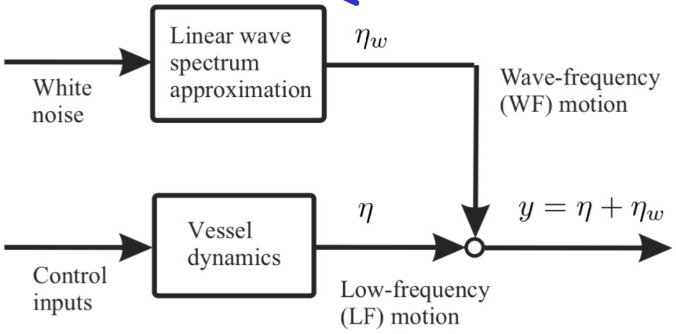


(a) Surge, sway and heave amplitudes (N) and phases (deg).



(b) Roll, pitch and yaw amplitudes (Nm) and phases (deg)

10.2 State-Space Models for Wave Response Simulation



$$\eta_{w,i}(s) = \frac{K_{w,i}\, s}{s^2 + 2\lambda\omega_e s + \omega_e^2} w_i(s)$$

$$\dot{\boldsymbol{x}}_{w,i} = \begin{bmatrix} 0 & 1 \\ -\omega_e^2 & -2\lambda\omega_e \end{bmatrix} \boldsymbol{x}_{w,i} + \begin{bmatrix} 0 \\ K_{w,i} \end{bmatrix} w_i$$

$$\eta_{w,i} = \begin{bmatrix} 0 & 1 \end{bmatrix} \boldsymbol{x}_{w,i}$$

A linear wave response approximation for *H*(s) is usually preferred by ship control systems engineers. Easy to use and easy to test the performance and robustness of the control system.

No hydrodynamic SW is needed to the price of a tunable gain $K_{w,i}$ which must be tuned manually in 6 DOFs to match the desired amplitudes of $\eta_{w,i}$

10.2 State-Space Models for Wave Response Simulation

LF model:
$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_\Theta(\boldsymbol{\eta})(\boldsymbol{\nu}_r + \boldsymbol{\nu}_c)$$
$$\boldsymbol{M}\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{g}(\boldsymbol{\eta}) + \boldsymbol{g}_0 = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave2}} + \boldsymbol{\tau}$$

WF models:
$$\boxed{\begin{aligned} \dot{\boldsymbol{x}}_{w,i} &= \boldsymbol{A}_{w,i}\,\boldsymbol{x}_{w,i} + \boldsymbol{E}_{w,i}\,w_i, \qquad (i = 1, 2, , \ldots 6) \\ \eta_{w,i} &= \boldsymbol{C}_{w,i}\,\boldsymbol{x}_{w,i} \end{aligned}}$$

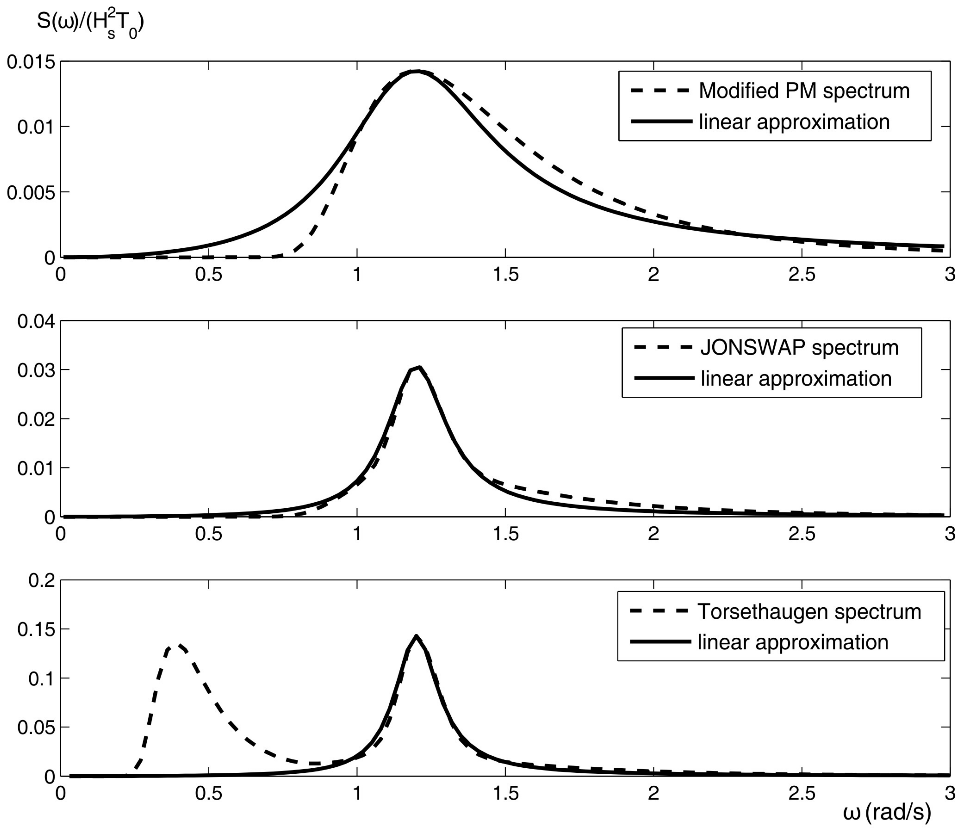
Wave drift:
$$\boxed{\begin{aligned} \dot{\boldsymbol{d}} &= \boldsymbol{w}_d \\ \boldsymbol{\tau}_{\text{wave2}} &= \boldsymbol{d} \end{aligned}}$$

Measurement:
$$\boldsymbol{y} = \boldsymbol{\eta} + \boldsymbol{\eta}_w + \boldsymbol{v}$$

State-space model using 6 transfer functions for the WF motions and Wiener processes (random walks) for generalized wave drift forces

For surface craft only 3 transfer functions (surge, sway and yaw) are needed to simulate WF motions (6 ODEs) and wave drift can be represented by 3 ODEs, giving a total of 9 ODEs.

10.2 State-Space Models for Wave Responses



10.3 Ocean Current Forces and Moments

Ocean current forces and moments can be included in the equations of motion by replacing the absolute velocity terms with relative velocity (only the hydrodynamic terms)

Equations of relative motion

$$\underbrace{\boldsymbol{M}_{RB}\dot{\boldsymbol{\nu}} + \boldsymbol{C}_{RB}(\boldsymbol{\nu})\boldsymbol{\nu} + \boldsymbol{g}(\boldsymbol{\eta}) + \boldsymbol{g}_0}_{\text{rigid-body and hydrostatic terms}}$$

$$+ \underbrace{\boldsymbol{M}_A\dot{\boldsymbol{\nu}}_r + \boldsymbol{C}_A(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r + \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r}_{\text{hydrodynamic terms}} = \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau}$$

$$\boldsymbol{\nu}_r = \boldsymbol{\nu} - \boldsymbol{\nu}_c$$

Definition 10.1 (Irrotational Constant Ocean Current)
An irrotational constant ocean current in $\{n\}$ is defined by

$$\dot{\boldsymbol{v}}_c^n = \dot{\boldsymbol{R}}(\boldsymbol{\Theta}_{nb})\boldsymbol{v}_c^b + \boldsymbol{R}(\boldsymbol{\Theta}_{nb})\dot{\boldsymbol{v}}_c^b := \boldsymbol{0}$$

where

$$\dot{\boldsymbol{R}}(\boldsymbol{\Theta}_{nb}) = \boldsymbol{R}(\boldsymbol{\Theta}_{nb})\boldsymbol{S}(\boldsymbol{\omega}_{nb}^b)$$

Consequently,

$$\dot{\boldsymbol{v}}_c^b = -\boldsymbol{S}(\boldsymbol{\omega}_{nb}^b)\boldsymbol{v}_c^b$$

$$\boldsymbol{\nu}_c = [\underbrace{u_c, v_c, w_c}_{\boldsymbol{v}_c^b}, 0, 0, 0]^\top$$

10.3 Ocean Current Forces and Moments

Property 10.1 (Irrotational Constant Ocean Currents)
If the Coriolis and centripetal matrix $C_{RB}(\nu_r)$ is parametrized independent of linear velocity $\nu_1 = [u,\ v,\ w]^\top$, for instance by using (3.63), and the ocean current is irrotational and constant (Definition 10.1), the rigid-body kinetics satisfies (Hegrenæs 2010)

$$M_{RB}\dot{\nu} + C_{RB}(\nu)\nu = M_{RB}\dot{\nu}_r + C_{RB}(\nu_r)\nu_r \tag{10.129}$$

with

$$\nu_r = \begin{bmatrix} v^b - v_c^b \\ \omega_{nb}^b \end{bmatrix} \tag{10.130}$$

Very useful result since it transforms the absolute velocity states to relative velocity states in the equations of motion

$$\underbrace{M_{RB}\dot{\nu} + C_{RB}(\nu)\nu + g(\eta) + g_0}_{\text{rigid-body and hydrostatic terms}}$$
$$+ \underbrace{M_A\dot{\nu}_r + C_A(\nu_r)\nu_r + D(\nu_r)\nu_r}_{\text{hydrodynamic terms}} = \tau_{\text{wind}} + \tau_{\text{wave}} + \tau$$

Proof. *Since the Coriolis and centripetal matrix represented by (3.63) is independent of linear velocity $\nu_1 = [u,\ v,\ w]^\top$, it follows that $C_{RB}(\nu_r) = C_{RB}(\nu)$. The property*

$$M_{RB}\dot{\nu}_c + C_{RB}(\nu_r)\nu_c = \mathbf{0} \tag{10.131}$$

is easily verified by expanding the matrices M_{RB} and $C_{RB}(\nu_r)$ and corresponding acceleration and velocity vectors according to

$$\begin{bmatrix} mI_3 & -mS(r_{bg}^b) \\ mS(r_{bg}^b) & I_b \end{bmatrix}\begin{bmatrix} -S(\omega_{nb}^b)v_c^b \\ 0_{3\times1} \end{bmatrix} + \begin{bmatrix} mS(\omega_{nb}^b) & -mS(\omega_{nb}^b)S(r_{bg}^b) \\ mS(r_{bg}^b)S(\omega_{nb}^b) & -S(I_b\omega_{nb}^b) \end{bmatrix}\begin{bmatrix} v_c^b \\ 0_{3\times1} \end{bmatrix} = \mathbf{0}$$

Finally, it follows that

$$\begin{aligned} M_{RB}\dot{\nu} + C_{RB}(\nu)\nu &= M_{RB}[\dot{\nu}_r + \dot{\nu}_c] + C_{RB}(\nu_r)[\nu_r + \nu_c] \\ &= M_{RB}\dot{\nu}_r + C_{RB}(\nu_r)\nu_r \end{aligned}$$

$$\begin{aligned} \dot{\eta} &= J_\Theta(\eta)\,(\nu_r + \nu_c) \\ \dot{\nu}_r &= M^{-1}\left(\tau_{\text{wind}} + \tau_{\text{wave}} + \tau - C(\nu_r)\nu_r - D(\nu_r)\nu_r - g(\eta) - g_0\right) \end{aligned}$$

$$\begin{aligned} M &= M_{RB} + M_A \\ C(\nu_r) &= C_{RB}(\nu_r) + C_A(\nu_r) \end{aligned}$$

10.3 Equations of Relative Motion

State-space model for relative velocity (6 DOF)

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_\Theta(\boldsymbol{\eta})\left[\boldsymbol{\nu}_r + \boldsymbol{\nu}_c\right]$$
$$\dot{\boldsymbol{\nu}}_r = \boldsymbol{M}^{-1}\left(\boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} + \boldsymbol{\tau} - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{g}(\boldsymbol{\eta}) - \boldsymbol{g}_0\right)$$

State-space model for absolute velocity (6 DOF)

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}_\Theta(\boldsymbol{\eta})\boldsymbol{\nu} \tag{10.141}$$
$$\dot{\boldsymbol{\nu}} = \begin{bmatrix} -\boldsymbol{S}(\boldsymbol{\omega}_{nb}^b)\boldsymbol{v}_c^b \\ \boldsymbol{0}_{3\times 1} \end{bmatrix} + \boldsymbol{M}^{-1}\left(\boldsymbol{\tau} + \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{g}(\boldsymbol{\eta}) - \boldsymbol{g}_0\right)$$

Surface craft (3 DOF)

$$\dot{\boldsymbol{\eta}} = \boldsymbol{R}(\psi)\boldsymbol{\nu}$$
$$\dot{\boldsymbol{\nu}} = \begin{bmatrix} r v_c \\ -r u_c \\ 0 \end{bmatrix} + \boldsymbol{M}^{-1}\left(\boldsymbol{\tau} + \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r\right)$$

Straight-line motion and stationkeeping (small turning rate, r = 0)

$$\dot{\boldsymbol{\nu}} = \boldsymbol{M}^{-1}\left(\boldsymbol{\tau} + \boldsymbol{\tau}_{\text{wind}} + \boldsymbol{\tau}_{\text{wave}} - \boldsymbol{C}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r - \boldsymbol{D}(\boldsymbol{\nu}_r)\boldsymbol{\nu}_r\right)$$

10.3 Current Velocity in NED and BODY

Current Speed and Direction

The current speed is denoted by V_c while its direction relative to the moving craft is conveniently expressed in terms of two angles: α_{Vc} and β_{Vc}

For computer simulations the current velocity can be generated by using a 1st-order Gauss-Markov Process

$$\dot{V}_c + \mu V_c = w \qquad V_{\min} \leq V_c(t) \leq V_{\max}$$

$$\boldsymbol{v}_c^n = \boldsymbol{R}_{y,\alpha_{V_c}}^\top \boldsymbol{R}_{z,-\beta_{V_c}}^\top \begin{bmatrix} V_c \\ 0 \\ 0 \end{bmatrix}$$

Current velocities expressed in NED

$$\boldsymbol{v}_c^n = \begin{bmatrix} V_c \cos(\alpha_{V_c}) \cos(\beta_{V_c}) \\ V_c \sin(\beta_{V_c}) \\ V_c \sin(\alpha_{V_c}) \cos(\beta_{V_c}) \end{bmatrix}$$

Current velocities expressed in BODY

$$\begin{bmatrix} u_c \\ v_c \\ w_c \end{bmatrix} = \boldsymbol{R}^\top(\boldsymbol{\Theta}_{nb}) \boldsymbol{v}_c^n$$

Surface craft (2-D)

$$V_c = \sqrt{u_c^2 + v_c^2} \qquad \boldsymbol{v}_c^n = \begin{bmatrix} V_c \cos(\beta_{V_c}) \\ V_c \sin(\beta_{V_c}) \\ 0 \end{bmatrix}$$

$$u_c = V_c \cos(\beta_{V_c} - \psi), \qquad v_c = V_c \sin(\beta_{V_c} - \psi)$$

10.3 2-D Irrotational Ocean Current Model

Linear maneuvering model using relative sway velocity

$$\begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{21} & m_{22} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{v} - \dot{v}_c \\ \dot{r} \\ \dot{\psi} \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & 0 \\ d_{21} & d_{22} & 0 \\ 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} v - v_c \\ r \\ \psi \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix} \delta + \begin{bmatrix} Y_{wind} \\ N_{wind} \\ 0 \end{bmatrix} + \begin{bmatrix} Y_{wave} \\ N_{wave} \\ 0 \end{bmatrix}$$

$$\begin{aligned} u_c &= V_c \cos(\beta_{V_c} - \psi) \\ v_c &= V_c \sin(\beta_{V_c} - \psi) \\ \dot{v}_c &= -r u_c \end{aligned}$$

Nonlinear state-space model where current speed is a state variable

$$\begin{bmatrix} \dot{v} \\ \dot{r} \\ \dot{\psi} \\ \dot{V}_c \end{bmatrix} = \begin{bmatrix} -rV_c \sin(\beta_{V_c} - \psi) \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} m_{11} & m_{12} & 0 & 0 \\ m_{21} & m_{22} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}^{-1} \left(\begin{bmatrix} b_1 \\ b_2 \\ 0 \\ 0 \end{bmatrix} \delta + \begin{bmatrix} Y_{wind} \\ N_{wind} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} Y_{wave} \\ N_{wave} \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} -d_{11} & -d_{12} & 0 & 0 \\ -d_{21} & -d_{22} & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -\mu \end{bmatrix} \begin{bmatrix} v \\ r \\ \psi \\ V_c \end{bmatrix} + \begin{bmatrix} d_{11} V_c \sin(\beta_{V_c} - \psi) \\ d_{21} V_c \sin(\beta_{V_c} - \psi) \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \end{bmatrix} w \right)$$

Note that the state-space model is nonlinear in ψ, V_c and β_{Vc} even though the maneuvering model is linear

Chapter Goals – Revisited

- Understand the principle for linear superposition of wind and wave forces
- Understand the equations of relative motion for ocean currents
- Be able to compute the wind loads on a marine craft using wind coefficient look-up tables and projected areas found from general arrangement drawings as required by the MSS functions: *Blendermann94* and *Isherwood72.*
- Understand how statistical descriptions can be used to define the sea state in terms of parameters such as wave period, wind speed, etc. for different regions on the Earth.
- Explain what we mean with regular and irregular waves, wave spectrum and directional wave spectrum.
- Be able to compute and simulate the wave elevation of regular and irregular waves for a given sea state and a wave spectrum.
- Understand the three methods for simulation and computation of wave loads:
  - Motion RAOs
  - Force RAOs
  - Simplified transfer functions for WF motions
- Be able to explain what we mean by first- and second-order wave loads and how these relate to hydrodynamic data computed in Wamit and ShipX.
- Be able to simulate ocean currents (magnitude and direction) in 2D and 3D and transform the results to current velocities.