

1. (a) 成立

(b) 成立

(c) 成立

(d) 不成立, 考虑下面两个关系.

R:	Chr	Int	S:	Chr	Int
	a	1		a	1
	c	3		b	2

$$\text{设 } \theta: \text{Chr} = a, \text{ 则 } |\sigma_{\theta}(R) \times S| = 2$$
$$|R \times \sigma_{\theta}(S)| = 1$$

所以命题不成立.

(e) 成立

(f) 不成立, 考虑(d)中提及的  $R$ ,  $S$  和  $\theta$ , 则

$$|\sigma_{\theta}(R) - S| = 0, |R - \sigma_{\theta}(S)| = 1$$

所以命题不成立.

(g) 若  $L_1 \subseteq L_2$ , 则成立; 否则考虑(d)中的  $R$ ,  $L_1: \{\text{Chr}\}$ ,  $L_2: \{\text{Int}\}$  则

$$\pi_{L_1}(\pi_{L_2}(R)) = \emptyset, \pi_{L_1}(R) = \{a, c\}$$

(h) 成立

(i) 不成立, 考虑下面两个关系

R:	Chr	Int	S:	Chr	Int
	a	1		a	1
	a	2		b	2

$$\text{设 } L: \text{Int}, \text{ 则 } \pi_L(R - S) = \{1, 2\}$$

$$\pi_L(R) - \pi_L(S) = \emptyset$$

所以命题不成立.

(j) 不成立, 考虑下面三个关系.

R: Chr	Int	S: Int	Bool	T: Chr	Str
a	0	0	True	a	"aa"
b	1	1	False	b	"bb"

$$\text{则 } |R \bowtie S| = 2, |R \bowtie (S \bowtie T)| = 4$$

所以命题不成立。

2. 表达式:  $\bigcap_k (\sigma_{\text{amt} > 1}(\pi_k; \text{count}(* \rightarrow \text{amt}(R))) \cup \bigcap_k (\sigma_{k = \text{null}}(R))$

若表达式为空, 则关系R上属性K满足实体完整性。上式左边检查是否有重复的主键, 右边检查是否有空的主键。

3. 表达式:  $\bigcap_F (\sigma_{F \neq \text{null}} S) - \bigcap_k(R)$

若表达式为空, 则R和S的实例满足参照完整性。该式首先选出非空外键, 减后若非空则存在不在R中的外键。

4. 证明: 设  $R(A_1, A_2, \dots, A_n), S(A_1, A_2, \dots, A_n)$ , 我们需要证明

$$R \bowtie S \subseteq R \cap S \text{ 且 } R \bowtie S \supseteq R \cap S.$$

(1) 证明  $R \bowtie S \subseteq R \cap S$ .

对  $\forall t \in R \bowtie S$ , 设  $t = (t_1, t_2, \dots, t_n)$ , 则

$\exists r = (r_1, r_2, \dots, r_n) \in R, s = (s_1, s_2, \dots, s_n) \in S, \text{ s.t. } \forall_{i=1, \dots, n} t_i = r_i = s_i$ ,

故  $t = r = s, t \in R \cap S$ , 所以  $R \bowtie S \subseteq R \cap S$

(2) 证明  $R \bowtie S \supseteq R \cap S$

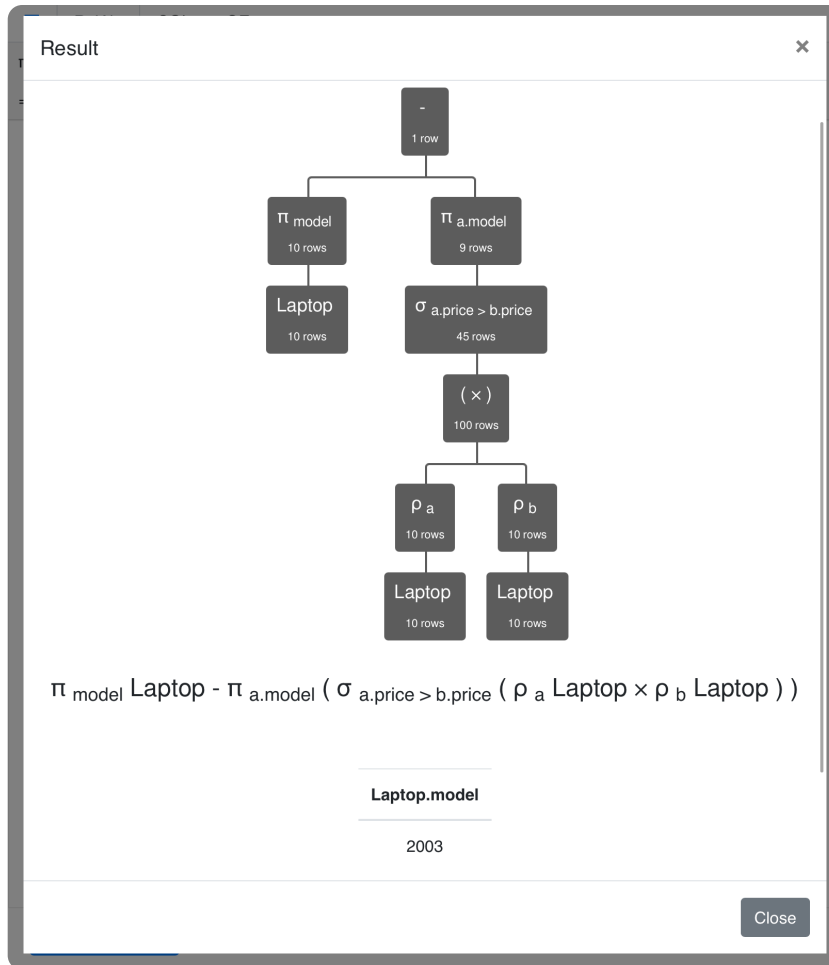
对  $\forall t \in R \cap S$ , 设  $t = (t_1, t_2, \dots, t_n)$ , 则

$\exists r = (r_1, r_2, \dots, r_n) \in R, s = (s_1, s_2, \dots, s_n) \in S, \text{ s.t. } t = r = s$ .

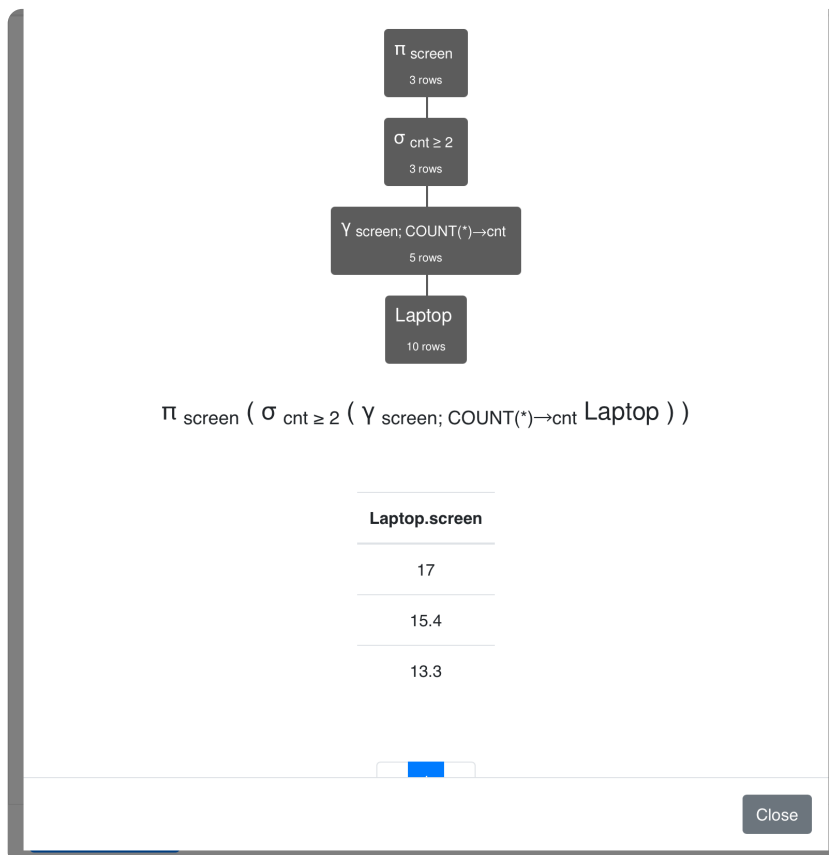
故  $\forall_{i=1, \dots, n} t_i = r_i = s_i$ , 所以  $t \in R \bowtie S$ , 所以  $R \bowtie S \supseteq R \cap S$ .

综上所述,  $R \bowtie S = R \cap S$

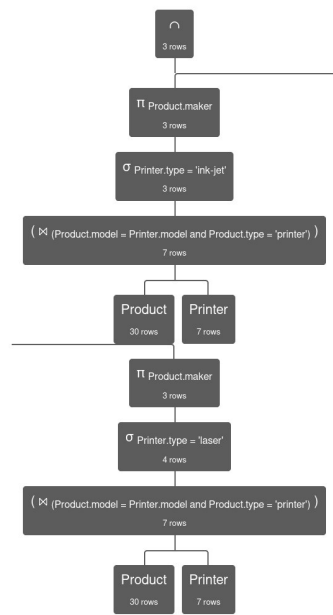
5. (a)



(b)



(C)


$$\begin{aligned} & \pi \text{ Product.maker } ( \sigma \text{ Printer.type} = \text{'ink-jet'} ( ( \text{Product} ) \bowtie \\ & (\text{Product.model} = \text{Printer.model and Product.type} = \text{'printer'} ) ( \text{Printer} ) ) ) \\ & \cap \pi \text{ Product.maker } ( \sigma \text{ Printer.type} = \text{'laser'} ( ( \text{Product} ) \bowtie \\ & (\text{Product.model} = \text{Printer.model and Product.type} = \text{'printer'} ) ( \text{Printer} ) ) ) \end{aligned}$$

Product.maker
'D'
'E'
'H'

b. (a)  $\{t[\text{model}] \mid t \in \text{Laptop} \wedge \forall u \in \text{Laptop} (t[\text{price}] \leq u[\text{price}])\}$   
 (b)  $\{t[\text{screen}] \mid t \in \text{Laptop} \wedge \exists u \in \text{Laptop} (t[\text{model}] \neq u[\text{model}] \wedge t[\text{screen}] = u[\text{screen}])\}$   
 (c)  $\{t[\text{maker}] \mid t \in \text{Product} \wedge t[\text{type}] = \text{"printer"} \wedge \exists u \in \text{Printer} (t[\text{model}] = u[\text{model}] \wedge u[\text{type}] = \text{"ink-jet"}) \wedge \exists u \in \text{Printer} (t[\text{model}] = u[\text{model}] \wedge u[\text{type}] = \text{"laser"})\}$

$$7. (a) \{ (a) \mid \exists b, c, d, e, f ((a, b, c, d, e, f) \in L_{\text{aptop}} \wedge \forall a', b', c', d', e', f' ((a', b', c', d', e', f') \in L_{\text{aptop}} \wedge f \leq f')) \}$$

$$(b) \{ (e) \mid \exists a, b, c, d, f \, ( (a, b, c, d, e, f) \in L_{\text{aptop}} \wedge \exists a', b', c', d', e', f' \, ( (a', b', c', d', e', f') \in L_{\text{aptop}} \wedge a \neq a' \wedge e = e' ) ) \}$$

$$(c) \{ (a) \mid \exists b, d, f \ ( (a, b, \text{"printer"}) \in \text{Product} \wedge (b, d, \text{"ink-jet"}, f) \in \text{Printer} ) \\ \cap \exists b', d', f' \ ( (a, b', \text{"printer"}) \in \text{Product} \wedge (b', d', \text{"laser"}, f') \in \text{Printer} ) \}$$