形式语言与自动机 作业四

cycleke

1 第一题

Design a PDA (Diagram) to accept each of the following languages. You may accept either by final state or by empty stack, whichever is more convenient.

1.1 (a)

The set of all strings of 0's and 1's such that no prefix has more 1’ s than 0's.

解 1.1 此 PDA 使用终结状态方式。

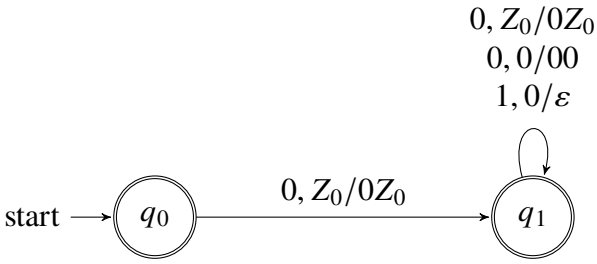

图 1: 第一题 (a)

1.2 (b)

The set of all strings of 0's and 1's with twice as many 0's as 1's.

解 1.2 此 PDA 使用空栈方式。

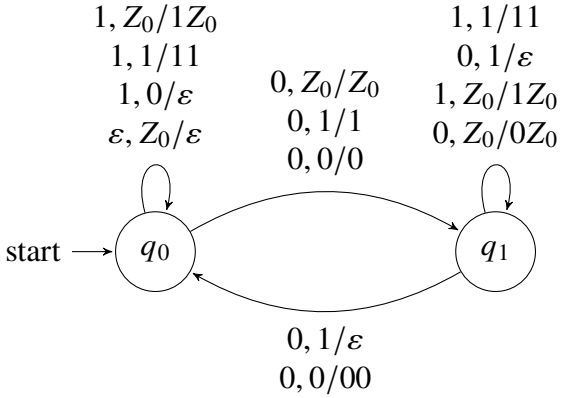

图 2: 第一题 (b)

其中 q_0 表示没有 0 与栈最上方的 1 匹配，而 q_1 表示有一个 0 与栈最上方的 1 匹配。

1.3 (c)

$\{a^i b^j c^k | i \neq j \ or \ j \neq k\}$.

解 1.3 此 PDA 使用终结状态方式。

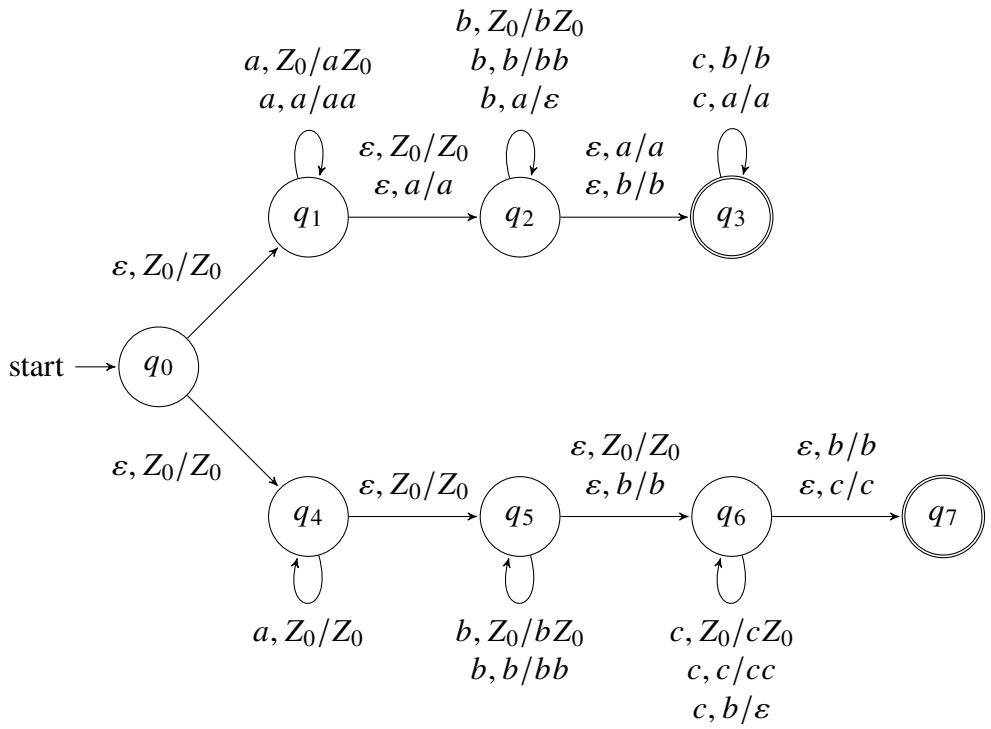

图 3: 第一题 (c)

2 第二题

Let $\Sigma = \{0, 1\}$. Suppose w is a non-null string of even length so that w can be written as $uxyv$ with x, y in Σ and $|u| = |v|$. Then we will say that xy is the middle of w. For example, in the string 00110011 we have 10 as its middle. Let $L \subseteq \Sigma^*$ be given by: w is in L if and only if it is of non-null string of even length and its middle is 00 or 11.

Show that L is a context free language by constructing a (non-deterministic) push-down automaton that accepts L.

解 2.1 此 PDA 使用空栈方式。

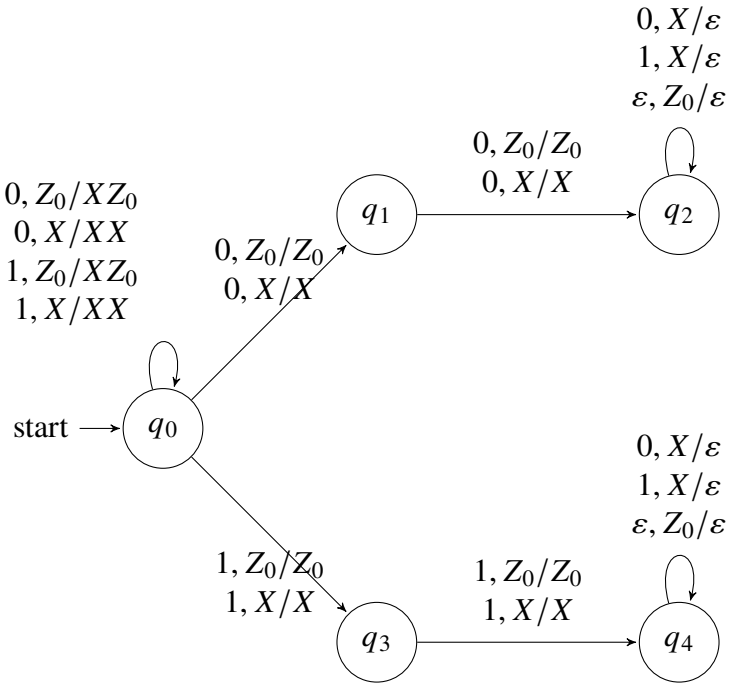

图 4: 第二题

我们使用 X 字符来计数，而如果中间为 00 或 11，那么就可以通过 $q1, q2$ 或 $q3, q4$ 来清空栈。当且仅当 00 或 11 为中心时，其两端的字符数才会相等，这样计数字符 X 才会清空。

3 第三题

Convert the PDA $P = (\{p, q\}, \{O, 1\}, \{X, Z_0\}, , q, Z_0)$to a CFG, if δ is given by:
(1) $\delta(q, 1, Z_0) = \{(q, XZ_0)\}$.
(2) $\delta(q, 1, X) = \{(q, XX)\}$.
(3) $\delta(q, 0, X) = \{(p, X)\}$.
(4) $\delta(q, , Z_0) = \{(q, \varepsilon)\}$.
(5) $\delta(p, 1, X) = \{(p, \varepsilon)\}$.
(6) $\delta(p, 0, Z_0) = \{(q, Z_0)\}$.

解 3.1 我们构造 CFG $G = (V, \{0, 1\}, P, S)$，其中 $V = \{[qXp] | q, p \in \{p, q\}, X \in \{0, 1\}\} \cup \{S\}$。而又有 δ 函数，我们可以得出

- $$S \to [qZ_0q] | [qZ_0p]$$
- $\delta(q, 1, Z_0) = \{(q, XZ_0)\}$

$$[qZ_0q] \to 1[qXq][qZ_0q] | 1[qXp][pZ_0q]$$
$$[qZ_0p] \to 1[qXq][qZ_0p] | 1[qXp][pZ_0p]$$

- $\delta(q, 1, X) = \{(q, XX)\}$

$$[qZ_0q] \to 1[qXq][qXq]|1[qXp][pXq]$$
$$[qZ_0p] \to 1[qXq][qXp]|1[qXp][qXp]$$

- $\delta(q, 0, X) = \{(p, X)\}$

$$[qXq] \to 0[pXq]$$
$$[qXp] \to 0[pXp]$$

- $\delta(q, , Z_0) = \{(q, \varepsilon)\}$

$$[qZq] \to \varepsilon$$

- $\delta(p, 1, X) = \{(p, \varepsilon)\}$

$$[pXp] \to 1$$

- $\delta(p, 1, X) = \{(p, \varepsilon)\}$

$$[pXp] \to 1$$

- $\delta(p, 0, Z_0) = \{(q, Z_0)\}$

$$[pZ_0q] \to 0[qZ_0q]$$

化简后有

$$S \to [qZ_0q]$$
$$[qZ_0q] \to 1[qXp][pZ_0q]|\varepsilon$$
$$[qXp] \to 1[qXp][pXp]|0[pXp]$$
$$[pZ_0q] \to 0[pZ_0q]$$
$$[pXp] \to 1$$