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# A Two-Stage Charging Station Allocation Model for EV Taxi Fleet Considering Interdependence Between the Networks of Transportation and Power Distribution

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*Key words*:

## Nomenclature

### Sets

| | |
|---|---|
| $I$ | Set of TN nodes |
| $N$ | Set of PDN nodes |
| $N^F$ | Set of PDN root nodes |
| $E$ | Set of PDN lines |
| $E^+$ | $E \cup \{0'0\}$ where $0'0$ is a dummy station |
| $J$ | Set of opening locations of EVCS's |
| $J^+$ | $J \cup \{0\}$ where 0 is a dummy station |
| $K$ | Set of possible PDN lines to expand |
| $\Omega$ | Set of all scenarios |

### Parameters

$A_j$ Fixed cost of building a charging station at location $j$.

$C_j$ Cost of building a unity of capacity at location $j$

$\overline{D}$ Largest charging demand over all scenarios.

$L$ Cost of a unity substation expansion.

$G_{mn}$ Cost of each PDN line $mn$.

$H_0$ Unit penalty of an uncharged car.

$H$ Unit penalty of an unsatisfied car.

$U_{ij}(\omega)$ Utility of EV owners at region $i$ choosing station $j$ for charging at scenario $\omega$.

$U^{min}(\omega)$ Threshold utility value below which EV owners do not make charging at scenario $\omega$.

$D_i(\omega)$ Demand at TN node $i$ at scenario at scenario $\omega$.

$P_j(\omega)$ Price of charging unity at charging station $j$ at scenario $\omega$.

$T_{ij}(\omega)$ Travelling time from TN node $i$ to charging station $j$ at scenario at scenario $\omega$.

$P_n^{load}(\omega)$ Real load at PDN node n at scenario $\omega$ if it is positive; otherwise real flow generated from electrical supplies.

$Q_n^{load}(\omega)$ Reactive load at PDN node n at scenario $\omega$ if it is positive; otherwise real flow generated from electrical supplies.

$P_{mn}^{Max}$ Capacity of allowed real power flow at PDN line $mn$.

$Q_{mn}^{Max}$ Capacity of allowed reactive power flow at PDN line $mn$.

$V^{max}$ Upper bound of voltage.

$V^{min}$ Lower bound of voltage.

$V^{Max}$ Square of the upper bound of voltage, $V^{Max} = (V^{max})^2$

$V^{Min}$ Square of the lower bound of voltage, $V^{Min} = (V^{min})^2$

$R_{mn}$ Resistance of PDN line $mn$.

$X_{mn}$ Reactance of PDN line $mn$.

$W_n$ Real power needed for each unit of charging load.

**Decision Variables - First Stage**

$z_j$ Fixed cost of building a charging station at location $j$.

$x_j$ Charging capacity at station $j$.

$u_{0'0}$ Exceed real power as substation expansion capacity.

$u^k_{mn}$ Binary variable indicating whether the $k$ line should be added to PDN line $mn$.

**Decision Variables - Second Stage**

$s_j$ Unsatisfied demands at station $j$.

$y_{ij}$ Charging demand flow from traffic node $i$ to station $j$.

$p_{mn}$ Real power flow passing PDN line $mn$.

$q_{mn}$ Reactive power flow passing PDN line $mn$.

$V_m$ Voltage at node $m$.

$v_m$ Lineariaztion Variable, $v_m = V_m^2$

$r^k_{mn}$ Lineariaztion Variable, $r^k_{mn} = u^k_{mn}(v_m - v_n)$.

**Notations**

$\omega$ Random variable with sample space $\Omega$.

$\mathbf{z}$ Vector of all $z_j$.

$\mathbf{x}$ Vector of all $x_j$.

$\mathbf{u}$ Vector of all $u^k_{mn}$ and $u_{0'0}$.

## 1. Introduction

With the advances of power electronics and battery technologies, Plug-in electric vehicles (PEVs) have made rapid development and become an attractive option in the transportation industry [Martinez et al., 2016]. PEVs are regarded as a promising solution to relieve global environment issues for its low emission and high energy efficiency properties and are expected to increase in the next few years [Shaukat et al., 2018]. With the growing PEV penetration, more public charging stations (CS) are required to provide fast charging for PEVs on the road with more convenient charging services, which can further promote the adoption of PEVs.

## 2. Problem Description

The successful rollout of the electrification of taxi fleet is highly dependent upon the affordability, availability, quality and resilience of the services that our nation's critical infrastructures can provide. However, the majority of the existing U.S. power grid was built in the early 1930s, making it more than 80 years old[][]. This aging and overburdened power grid infrastructure is under great pressure to meet the rising demand of EV charging load[]. Similarly, the current U.S. road transportation infrastructure was largely built during the post-World War II boom that mainly supports traditional petroleum fuel vehicles. In addition, the emerging EVs will change the traffic tempo-spatial pattern in the transportation networks, which essentially leads to uncertain charging load to the power grid systems. As a result, the existing U.S. critical infrastructure does not meet the growing electricity and transportation charging demand for EV of the 21st century, which is an ominous sign of a looming economic crisis. Besides, the emerging technologies (e.g., Smart Grid and intelligent transportation systems) can barely make much of a difference if the fundamental infrastructure is not ready yet.

The above argument indicates an uncontroversial fact that to satisfy the emerging charging demand of electric taxis, it is necessary to improve or replace our existing infrastructure. Since electric taxis cannot charge at private charging spot, we need to build charging stations in transportation network(TN). Furthermore, we know the existing infrastructure cannot support a large station, we need to make expansion on power distribution network(PDN) where it is necessary. However, one major issue of making a such improvement is that the old infrastructure construction is lack of consideration of electrification of taxis which emerges after almost one century. Especially, TN and PDN in the same city are geographically interdependent. Therefore, TN and PDN has a mapping structure which was determined when the old infrastructure was constructed several decades ago. Such a old correspondence structure becomes a main technical barrier when we try to enhance the TN and PDN to satisfy the EV charging demand. For example, we consider a south residential area in TN with high density of population, and the electricity is supported by a PDN whose root is located far from it in the north of the TN where population is less. Such a structure designed could be due to the high cost rooted at a high population area. The high population typically attracts the large number of taxi. Thus, large amount of EV taxis will concentrate into the area, which makes the EV charging demand very high, so we need a large charging station in that area of TN. However, the building charging station in TN and making expansion on PDN is interdependent. On one hand, we want to build a station in TN where the EVs can reach best satisfaction. On the other hand, we want to choose a location where we most easily to make expansion to satisfy the large demand. The main problem is that these two objectives actually conflict with each other. In the above example, to reach the best satisfaction, we want to build the station as closed to the south as possible. On the contrary, to make expansion easily, we want the station as closed to the north as possible. In addition, if we build a too small station, that station will attract EVs to take a chance to charge there, which will lead to a long waiting time or even traffic congestion, which can be a more serious issue than unsatisfied charging demand. Therefore, we have to make trade off between satisfaction and expansion. Thus, loss of satisfaction is inevitable, which could lead to a large proportion of unsatisfied demand. Therefore, we want to know the key factors or restriction of loss of satisfaction. There are two possible cases. First, we cannot build a large station due to the physical limitation of expansion line. Second, we are allow to make expansion physically to build a large station but we decide not to do so because of the high cost in TN station building or PDN expansion. To sum up, we can consider that the main problem is due to physical feasibility and economical efficiency caused by the interdependency of TN and PDN.

The above issue is a circumstance that it is not physically feasible or economical efficient to build large stations reaching a ideal level of EV charging demand. However, it is not always realistic to abandon the existing infrastructure and rebuild a new one, so we need solutions allowing us keep the existing structure TN and PDN. If we want to figure out a solution to build a larger station, we are considering solutions from supply perspective. From supply persective, possible solutions include increasing the physical feasibility or decreasing related costs. For example, to increase physical feasibility, we can increase the possible number of each power distribution lines able to be added for between two nodes in PDN to allow more power flows. We can also try to reduce cost during construction of charging stations in TN, or we can save cost during making expansion on PDN, for example, using cheaper expansion lines. However, if we have already reached the limitation with current condition of all related infrastructure, we can still make improvement from demand perspective by reallocating the EV charging demand by allowing autonomous vehicle. Human drivers will choose the station with the highest satisfaction considering the travel time and price of the charging. Therefore, typically, human drivers will choose adjacent stations to charge, which leads to the structural conflicts as above analysis. However, if we implement autonomous taxis, system can allocate the station to charge for each vehicle. There are two potential benefits of this. First, we can stop an EV going to take a chance to go to charge to avoid complains about waiting and traffic congestion. Second, if we can allocate the EVs, we can build a small station when PDN expansion is hard, and guide the remaining EVs to go to other large stations where PDN expansion is easier. We expect autonomous vehicle will play a significant role in solving the problem due to interdependency.

In our paper, we want to build a model which is able to catch the interdependency between PDN and TN, which includes traffic time and price of charging impacts on the choice of charging location, the injected power load TN from charging station in TN, and upper bound of charging station after corresponding PDN expansion. Furthermore, as a transportation tool and electricity carrier, EVs can be charged at any charging facility and anytime, so we want to consider EVs demand as uncertain demand. Besides, since the retail electricity price and travel time may also have an impact on the customer behavior, we will also consider them as uncertain factors. Therefore, we choose two-stage stochastic model which enable us to separate long-term planning stage and short-term operational stage, which enable us not only to make optimal long-term decision but also to operate and guide the charging route efficiently.

More specifically, we want to build a model to address the following four questions:

(a) How many EV fast charging stations should be built?

(b) Where and how large should each station be built?

(c) How is the existing PDN expanded to satisfy the potential charging loads?

(d) How are the constructed EVCS used to serve EV charging demands?

Questions (a)-(c) considers the long-term planning strategy of building EVCS and expanding PDN. On this basis, question (d) is to seek the optimal policy to operate the constructed charging stations to serve EV charging demand during a short-term period. This feature of the hierarchical structure motives us to specify a two-stage stochastic programming model, of which the first here-and-now stage is to study questions (a)-(c) and the second wait-and-see one is to answer question (d).

## 3. Numerical Analysis

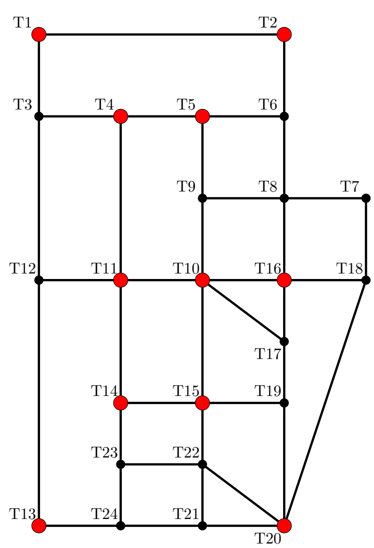
In our numerical analysis, we design a common structure on two existing networks and create a reasonable default case. We want to first explore how the existence of interdependence impacts the default case. After noticing the interdependence, we changed economical and physical factors to explore potential improve from supply perspective. Lastly, by allowing autonomous vehicle, we explore the solution from both demand and supply perspectives.

### 3.1. Case Overview and Default Case Settings

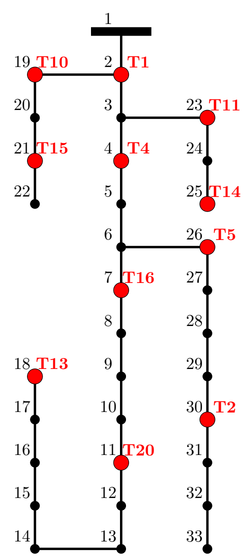
We test our model with two widely used benchmark networks in literatures, IEEE 33-node test feeder(PDN) coupled with Sioux Falls transportation network(TN). We want to design a structure for a city with old infrastructure. In TN, we consider two classes, downtown as commercial area and uptown as residential area based on both geographic location and size of EV charging demand. In both classes, the EV charging demand is larger in east area than west area, and larger in southern area than north northern area. In Sioux Falls transportation network, we let T4, T5,T10, T11, T14, T15 be commercial node and the rest as residential nodes. In PDN, we assume that the northern uptown has relatively small number of residents so it is relatively economically efficient to constructed the PDN root node at the northern uptown without considering EV charging demand in the past and make the long branches surround the city from the north to the south. Two shorter branches are specially support the downtown commercial area. In IEEE 33-node test feeder, we set D2 - D22 and D3 - D25 to be two lines specially support to downtown, and the longest branch D1-D18 to be the line surrounding the entire city and the second longest line is the line surrounding north uptown. In the past, the PDN and TN work independently, the structure of PDN is able to support the traditional electric demand. However, this structure could cause some problems when making PDN expansion for southern uptown. On one hand, from TN perspective, we tend to build some large stations in southern uptown area because there are large number of residents. On the other hand, from PDN perspective, the southern uptown are at the end of the longest PDN branch which is physically difficult to be expanded or costly to make expansion. Beside this principle problem, there are also some detailed problem on trade off under this conflicted situation caused by interdependence. We want to first set up a default case in order to adjust parameters to see the interdependence and potential solution explicitly.

**Table 1** TN and PDN nodes corresponding map

| TN | 1 | 2 | 4 | 5 | 10 | 11 | 13 | 14 | 15 | 16 | 20 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| PDN | 2 | 30 | 4 | 26 | 19 | 23 | 18 | 24 | 21 | 7 | 11 |




(a) Sioux Falls transportation network



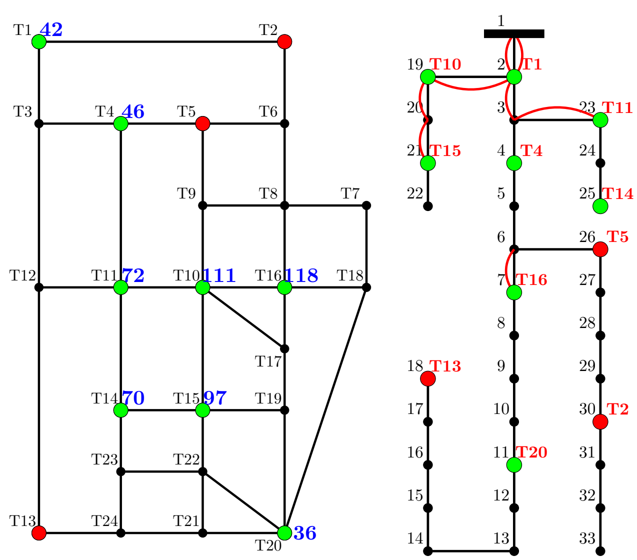

(b) IEEE 33-node

**Figure 1** Test Networks

In Fig.1 (a), the red colored nodes are the candidates for building EV charging stations in TN, which is marked correspondingly in Fig.1 (b). In our model, we want to analyze the marginal effects of changing factors in TN and PDN to study their interdependence and the impacts of autonomous vehicles. We first set up a default case with data following recent papers.

In our default case, we want to use data commonly used in literatures. The charging demand we take the original traffic flow data as bases for EV charging demand and the demand is higher closed to the center. In order to reflect the change of the demand during a regular day, we simulated 108 scenarios from 24 hours pattern for our EV charging demand. The default fixed cost of building a station in residential area is $A_i = \$1.63 \times 10^5$ and the capacity cost is $C_j = \$3,160$. The base cost per-unit cost of a distribution like is $G_{ij} = \$3 \times 10^5$, and the cost of substation capacity expansion cost is $L = \$5,000/kVA$. For each demand to power flow, $W_n = 7.7$Shareef et al. [2016]. In order to analyze the marginal effects, we use introduce two multipliers $\alpha$ and $\beta$ on the cost of TN and PDN. The multiplier $\alpha$ is on both fixed cost of charging station and the capacity cost, and $\beta$ is on unit cost of distribution lines. The default values are $\alpha = 1$ and $\beta = 1$. In our default case, we consider all vehicles are driven by human who always choose the closest station to charge. Lastly, we set the maximum number of line we are able to expand for each PDN line $(K)$ as 2. We will stated the reason for this later in the analysis.

We solved our cases using iOptimize Solver based on Benders decomposition method via CPLEX 12.90. All simulation and solving processes are implemented on a computer with Intel i7-1065G7 CPU and 16-GB RAM. The default case result is as follows:




(a) Default TN (b) Default PDN

**Figure 2** Default Decision Results: The green nodes in TN indicate we decide to build a charging station, and we also marked the corresponding PDN nodes as green; The red nodes are the candidate nodes we decide not to build stations; The blue numbers in the TN indicate the capacity of the charging stations.

For example, in our default case, the number of satisfied cars is 384, which means over 108 scenarios, on average, 384 cars get charging services. The number of unsatisfied cars is 84, which means on average, 84 cars cannot get charging services,including the cars too far from the closest station to charge and the cars actually go to the stations but are not able to charge due to the limited capacity of the stations. The satisfaction score is 5015, which takes account of both positive effects of the cars able to charge and the negative effects of the cars not able to charge. The TN Cost is $4977K including the fixed costs of the selected candidates and the capacity costs. The PDN Cost is $4564K including substation costs and expanding line costs.

### 3.2. Existing Interdependence Impacts

## References

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