

# A Two-Stage Charging Station Allocation Model for EV Taxi Fleet Considering Interdependence Between the Networks of Transportation and Power Distribution

Shixin Liu      Lusha Wang      Jian Hu      Zhi Zhou

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## Abstract

abstract-text

## Nomenclature

### Sets

$I$	Set of TN nodes
$N$	Set of PDN nodes
$N^F$	Set of PDN root nodes
$E$	Set of PDN lines
$E^+$	$E \cup \{0'0\}$ where $0'0$ is a dummy station
$J$	Set of opening locations of EVCS's
$J^+$	$J \cup \{0\}$ where $0$ is a dummy station
$K$	Set of possible PDN lines to expand
$\Omega$	Set of all scenarios

### Parameters

$A_j$	Fixed cost of building a charging station at location $j$ .
$C_j$	Cost of building a unity of capacity at location $j$
$\bar{D}$	Largest charging demand over all scenarios.
$L$	Cost of a unity substation expansion.
$G_{mn}$	Cost of each PDN line $mn$ .
$H_0$	Unit penalty of an uncharged car.

$H$	Unit penalty of an unsatisfied car.
$U_{ij}(\omega)$	Utility of EV owners at region $i$ choosing station $j$ for charging at scenario $\omega$ .
$U^{min}(\omega)$	Threshold utility value below which EV owners do not make charging at scenario $\omega$ .
$D_i(\omega)$	Demand at TN node $i$ at scenario at scenario $\omega$ .
$P_j(\omega)$	Price of charging unity at charging station $j$ at scenario $\omega$ .
$T_{ij}(\omega)$	Travelling time from TN node $i$ to charging station $j$ at scenario at scenario $\omega$ .
$P_n^{load}(\omega)$	Real load at PDN node $n$ at scenario $\omega$ if it is positive; otherwise real flow generated from electrical supplies.
$Q_n^{load}(\omega)$	Reactive load at PDN node $n$ at scenario $\omega$ if it is positive; otherwise real flow generated from electrical supplies.
$P_{mn}^{Max}$	Capacity of allowed real power flow at PDN line $mn$ .
$Q_{mn}^{Max}$	Capacity of allowed reactive power flow at PDN line $mn$ .
$V^{max}$	Upper bound of voltage.
$V^{min}$	Lower bound of voltage.
$V^{Max}$	Square of the upper bound of voltage, $V^{Max} = (V^{max})^2$
$V^{Min}$	Square of the lower bound of voltage, $V^{Min} = (V^{min})^2$
$R_{mn}$	Resistance of PDN line $mn$ .
$X_{mn}$	Reactance of PDN line $mn$ .
$W_n$	Real power needed for each unit of charging load.

### Decision Variables - First Stage

$z_j$	Fixed cost of building a charging station at location $j$ .
$x_j$	Charging capacity at station $j$ .
$u_0'$	Exceed real power as substation expansion capacity.
$u_{mn}^k$	Binary variable indicating whether the $k$ line should be added to PDN line $mn$ .

### Decision Variables - Second Stage

$s_j$	Unsatisfied demands at station $j$ .
$y_{ij}$	Charging demand flow from traffic node $i$ to station $j$ .
$p_{mn}$	Real power flow passing PDN line $mn$ .
$q_{mn}$	Reactive power flow passing PDN line $mn$ .
$V_m$	Voltage at node $m$ .

$v_m$	Lineariaztion Variable, $v_m = V_m^2$
$r_{mn}^k$	Lineariaztion Variable, $r_{mn}^k = u_{mn}^k(v_m - v_n)$ .

### Notations

$\omega$	Random variable with sample space $\Omega$ .
$\mathbf{z}$	Vector of all $z_j$ .
$\mathbf{x}$	Vector of all $x_j$ .
$\mathbf{u}$	Vector of all $u_{mn}^k$ and $u_{0'0}$ .

## 1 Introduction

With the advances of power electronics and battery technologies, Plug-in electric vehicles (PEVs) have made rapid development and become an attractive option in the transportation industry [8]. PEVs are regarded as a promising solution to relieve global environment issues for its low emission and high energy efficiency properties and are expected to increase in the next few years [12]. With the growing PEV penetration, more public charging stations (CS) are required to provide fast charging for PEVs on the road with more convenient charging services, which can further promote the adoption of PEVs.

The optimal CS placement is determined in [6] where PEVs are able to access a CS within its driving capacity anywhere in the city. The human factors rather than technological ones is the focus. However, the aggregated charging power from CS will have a great impact on distribution system operation, since it can change the load profile and result in overloading, low-quality voltage or increased losses in the system[2]. [7, 21] investigate the charging stations planning considering the operation of power distribution network (PDN). The optimal sites of PEV CS are identified with environmental factors and PEV CS service radius in the first step and the optimal size is determined with the minimization of total cost in the second step[7]. An optimal design of charging station and battery swap station based on life cycle cost is developed in [21] and a comparison of charging/swap stations is made. The joint planning of CS and PDN assets such as distribution lines, substation and other distributed energy resources with the aim of minimizing upgrading cost is discussed in [1, 9, 14].

Although a detailed formulation of PDN is considered in the above studies, the transportation network (TN) is ignored and leads to less efficient CS planning. The interdependency of TN and

PDN brought by PEV behaviors indicates that it is necessary to consider TN and PDN simultaneously in PEV CS planning problem. Optimal planning of PEV CS in coupled TN and PDN has been widely discussed in literature [13, 4, 16, 17, 10, 19, 18, 5, 20]. The TN, PDN and PEV owner's driving behavior are taken into account in [13] with the objective of maximizing charging service ability and minimizing total power loss and voltage deviation. A user equilibrium based traffic assignment model is incorporated in [16] to capture the traffic flow pattern and the objective is to minimize overall annual investment cost and energy losses and maximize annual captured traffic flow at the same time. A capacitated-flow refueling location model is proposed in [19] which utilizes OD traffic flows to estimate PEV charging demand and queueing theory to model CS service abilities on TN. The mixed-integer linear programming (MILP) model is solved by deterministic branch-and-bound methods. A service performance metric for PEV CS is proposed in [18] which measures the probability that charging demands arriving in a given time interval can be directly fulfilled without extra waiting time. Dynamic OD traffic flow on TN is used to model time-varying PEV charging demand and a stochastic mixed-integer SOCP is developed and is solved by the branch-and-cut method using an off-the-shelf solver.

The above literature address charging station planning in coupled TN and PDN, but system expansion is ignored. [15] proposes a comprehensive planning model which determines the optimal expansion strategies for both TN and PDN, including new CS sites and sizes, charging spots, TN lanes and PDN lines. An unconstrained traffic assignment model is used to capture the steady state distribution of traffic flows and linearized DistFlow is used to describe PDN operating conditions and an equivalent MILP is formulated. [3] presents an enhanced coordinated planning of multiple facilities in TN and PDN, including power lines, transportation roads, energy storage systems and fast CSs. To calculate the optimal solution, they introduce the applications of linear optimization theory including KKT conditions, the big M method, and a linear expression of power loss to transform the nonlinear planning problem into a mixed-integer quadratically constrained programming (MIQCP) formulation, which is solved by commercial solvers. However, both [15] and [3] focus on the planning stage without tackling the uncertain system operation stage.

To our understanding, there is no literature which deals with PEV CS planning problem in coupled TN and PDN with system expansion and uncertain system operation condition at the same time. Furthermore, the problem size is relatively small in the literature due to the computation

complexity, which greatly limits the application of the planning model in real world. Consequently, this paper aims to propose a comprehensive PEV CS planning problem with consideration of PEV owners' utility, traffic flow in TN, as well as PDN expansion in a two-stage model, where the location and size of EVCS and PDN expansion strategies are determined in the first stage, the uncertain operation condition is considered in the second stage. The model is able to solve big-size planning problem in real world.

## 2 Problem Description

The electrification of taxi fleet will overturn the relationship between transportation network(TN) and power distribution network(PDN) and make them interdependent. In the past, the transportation of traditional taxi does not require large electricity support, so it does not have direct impacts on the PDN. Also, the usage of power in a traditional city does not have direct impacts on transportation of vehicle as well. In contrast, electrical taxis will choose a charging station where the travel time is short and charging price is low, and thus the charging station will have more burden on PDN during the charging period. The upper limit of charging capacity of a station is determined by PDN. Therefore, if too many electrical taxis choose the same station where PDN cannot support, there will make a long waiting time or traffic congestion. Therefore, there is a strong interdependency between TN and PDN under the electrification of taxi fleet, which will also cause problems which are not considered in the past.

The successful rollout of the electrification of taxi fleet is highly dependent upon the affordability, availability, quality and resilience of the services that our nation's critical infrastructures can provide. However, the majority of the existing U.S. power grid was built in the early 1930s, making it more than 80 years old[1]. This aging and overburdened power grid infrastructure is under great pressure to meet the rising demand of EV charging load[2]. Similarly, the current U.S. road transportation infrastructure was largely built during the post-World War II boom that mainly supports traditional petroleum fuel vehicles. In addition, the emerging EVs will change the traffic tempo-spatial pattern in the transportation networks, which essentially leads to uncertain charging load to the power grid systems. As a result, the existing U.S. critical infrastructure does not meet the growing electricity and transportation charging demand for EV of the 21st century, which is

an ominous sign of a looming economic crisis. Besides, the emerging technologies (e.g., Smart Grid and intelligent transportation systems) can barely make much of a difference if the fundamental infrastructure is not ready yet.

The above argument indicates an uncontroversial fact that to satisfy the emerging charging demand of electric taxis, it is necessary to improve or replace our existing infrastructure. However, one major issue of making a such improvement is that the infrastructure was construct almost one century before, so the construction did consider the interdependency between TN and PDN due to electric vehicle. More specifically, TN and PDN has a mapping structure which was determined in the past. Such a old structure becomes a main technical barrier when we try to enhance the TN and PDN to satisfy the EV charging demand.

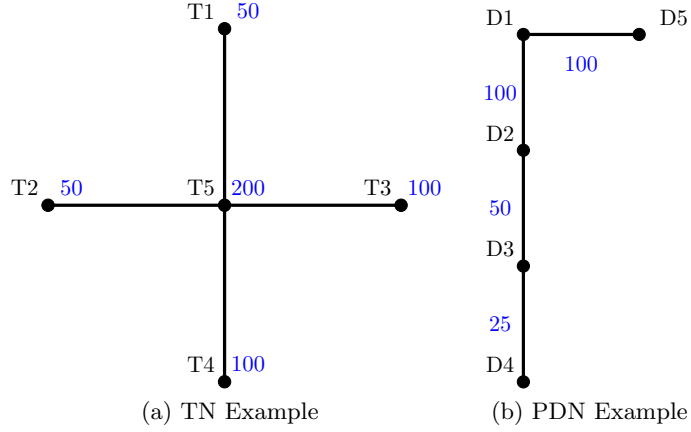


Figure 1: 1. The number are corresponding, for example T1 maps to D1. 2. Blue numbers in TN indicate the number of taxi needs to charge. 3. Blue number in PDN indicate the number of additional EV taxis satisfied if the line is expanded

For example, we consider the above TN and PDN. We assume there are 4 residential uptown areas and a commercial downtown area in the center of the city. The main branch of PDN is rooted at the North uptown with smaller population and surrounds the city ended at south uptown with larger population. In the past, without electrical taxis, these two networks work well independently but the power usage is closed to the upper limit of capacity of PDN, so we need expansion on PDN. Now when we need to decide whether to build a station at T4, we meet a dilemma. To satisfy 100 charging demand, we need 4 expansion lines at D3-D4, which is usually physically difficult or economically expensive. Alternatively, if we can sacrifice some satisfaction and build a larger station at downtown with only one extra expansion line at D1-D5. However, if the demand at

T2 T3 also come to T5, it will make T5 overburdened, so some demand will be unsatisfied. Such burden cannot be release by build a small stations at uptown, because if we build a small stations, there will be too much local demand taking chance to charge at the small local station. As a result, long waiting time and traffic congestion will be a more serious problem.

The above example shows that the building charging station in TN and making expansion on PDN is interdependent. On one hand, we want to build a station in TN where the EVs can reach best satisfaction. On the other hand, we want to choose a location where we most easily to make expansion to satisfy the large demand. The main problem is that these two objectives actually conflict with each other. In the above example, to reach the best satisfaction, we want to build the station as closed to the south as possible. On the contrary, to make expansion easily, we want the station as closed to the north as possible. In addition, if we build a too small station, that station will attract EVs to take a chance to charge there, which will lead to a long waiting time or even traffic congestion, which can be a more serious issue than unsatisfied charging demand. Therefore, we have to make trade off between satisfaction and expansion. Thus, loss of satisfaction is inevitable, which could lead to a large proportion of unsatisfied demand.

Therefore, we want to know the key factors or restriction of loss of satisfaction. There are two possible cases. First, we cannot build a large station due to the physical limitation of expansion line. Second, we are allow to make expansion physically to build a large station but we decide not to do so because of the high cost in TN station building or PDN expansion. To sum up, we can consider that the main problem is due to physical feasibility and economical efficiency caused by the interdependency of TN and PDN.

The above issue is a circumstance that it is not physically feasible or economical efficient to build large stations reaching a ideal level of EV charging demand. However, it is not always realistic to abandon the existing infrastructure and rebuild a new one, so we need solutions allowing us keep the existing structure TN and PDN. If we want to figure out a solution to build a larger station, we are considering solutions from supply perspective. From supply persective, possible solutions include increasing the physical feasibility or decreasing related costs. For example, to increase physical feasibility, we can increase the possible number of each power distribution lines able to be added for between two nodes in PDN to allow more power flows. We can also try to reduce cost during construction of charging stations in TN, or we can save cost during making expansion on PDN,

for example, using cheaper expansion lines. However, if we have already reached the limitation with current condition of all related infrastructure, we can still make improvement from demand perspective by reallocating the EV charging demand by allowing autonomous vehicle. Human drivers will choose the station with the highest satisfaction considering the travel time and price of the charging. Therefore, typically, human drivers will choose adjacent stations to charge, which leads to the structural conflicts as above analysis. However, if we implement autonomous taxis, system can allocate the station to charge for each vehicle. There are two potential benefits of this. First, we can stop an EV going to take a chance to go to charge to avoid complains about waiting and traffic congestion. Second, if we can allocate the EVs, we can build a small station when PDN expansion is hard, and guide the remaining EVs to go to other large stations where PDN expansion is easier. We expect autonomous vehicle will play a significant role in solving the problem due to interdependency.

In our paper, we want to build a model which is able to catch the interdependency between PDN and TN, which includes traffic time and price of charging impacts on the choice of charging location, the injected power load TN from charging station in TN, and upper bound of charging station after corresponding PDN expansion. Furthermore, as a transportation tool and electricity carrier, EVs can be charged at any charging facility and anytime, so we want to consider EVs demand as uncertain demand. Besides, since the retail electricity price and travel time may also have an impact on the customer behavior, we will also consider them as uncertain factors. Therefore, we choose two-stage stochastic model which enable us to separate long-term planning stage and short-term operational stage, which enable us not only to make optimal long-term decision but also to operate and guide the charging route efficiently.

More specifically, we want to build a model to address the following four questions:

- (a) How many EV fast charging stations should be built?
- (b) Where and how large should each station be built?
- (c) How is the existing PDN expanded to satisfy the potential charging loads?
- (d) How are the constructed EVCS used to serve EV charging demands?

Questions (a)-(c) considers the long-term planning strategy of building EVCS and expanding



PDN. On this basis, question (d) is to seek the optimal policy to operate the constructed charging stations to serve EV charging demand during a short-term period. This feature of the hierarchical structure motives us to specify a two-stage stochastic programming model, of which the first here-and-now stage is to study questions (a)-(c) and the second wait-and-see one is to answer question (d).

### 3 Mathematical Model and Solution Method

#### 3.1 The Two-Stage Model

first stage of the model is to minimize long-term cost of three components: transportation network, power distribution network, and long-term charging satisfaction. For TN incurred cost, we decide whether we need to build a charging station at candidates ( $J$ ) with capacity of the stations. Since we are considering large charging demand, we treat the charging demand as continuous variable, and thus the capacity of station is also reasonable to be considered as continuous number. For PDN, there two different expansions, substation expansion and expansion lines. Substation expansion is considered as a continuous

$$\min \sum_{j \in J} A_j z_j + \sum_{j \in J} C_j x_j + L u_{0'0} + \sum_{(m,n) \in E} G_{mn} \sum_{k \in K} u_{mn}^k + \mathbb{E}[\Pi(\mathbf{z}, \mathbf{x}, \mathbf{u}, \omega)] \quad (1)$$

$$\begin{aligned} \text{s.t.} \quad & x_j \leq \overline{D} z_j, & j \in J \\ & z_j \in \{0, 1\}, & j \in J \\ & x_j \geq 0, & j \in J \\ & u_{0'0} \geq 0 \end{aligned} \quad (2)$$

The objective of the first-stage minimizes the costs incurred from the selection of locations and sizes of EVCS's, the expansion of PDN, and the penalty  $\Pi(x, r, u, \omega)$  for unsatisfied demand, on average, in the second-stage.

In the second stage, the PDN acts as a supplier which needs to best satisfy the charging demands of EV's in TN. The charging demands are generated at certain regions in TN and each EV is decentralized. Owners are able to makes charging decision to maximize its individual utility (see [27]). We characterize the preference as the following exponential utility function of charging cost and traffic time:

$$U_{ij}(\omega) = \exp(-bp_j(\omega) - ct_{ij}(\omega))$$

where  $U_{ij}(\omega)$  is the utility value of EV in region  $i$  charging at station  $j$  at scenario  $\omega$  and the charging cost  $p_j(\omega)$  and traffic time  $t_{ij}(\omega)$  both varies in scenario. We set a dummy station ( $j = 0$ ) for drivers decide not to charge their EV's, and let  $\overline{D}$  which is a given threshold.

The penalty term serves as the objective function of the second-stage model as follows:

$$\Pi(z, x, u, \omega) = \min H_0 s_0 + \sum_{j \in J} H s_j - \sum_{i \in I} \sum_{j \in J} G U_{ij}(\omega) y_{ij} \quad (3)$$

The three terms describe our different treatments on three types of EV charging demands:

- $H_0 s_0$ : The negative utility of unsatisfied EV charging demands due to geographical limitation. In this case, the demand voluntarily quit charging. This term serves as the degree of encouragement of building stations for EV.
- $\sum_{j \in J} H s_j$ : The penalty of unsatisfied EV charging demands due to EV charging station capacity limitation. In this case, the demand involuntarily fails to charge.
- $\sum_{i \in I} \sum_{j \in J} G U_{ij}(\omega) y_{ij}$ : The utility of satisfied EV charging demands.
- $H > H_0$ : The penalty for involuntarily unsatisfied demand is larger than the penalty for voluntarily unsatisfied demand.

Now the largest utility value tells the station that an owner prefers to go. Hence, if we have decided the locations  $z_j$  and sizes  $x_j$  of charging stations, the user-choice assignment model [27] is formulated as follows:

$$\sum_{i \in I} y_{ij} - s_i \leq x_j, \quad j \in J^+ \quad (4)$$

$$s_j \leq \bar{D} z_j, \quad j \in J^+ \quad (5)$$

$$U_{ij} y_{ij} \geq y_{ij} \max(U_{ik}(\omega) - \Delta U, U^{min}) - D_i(\omega)(1 - z_k), \quad i \in I, j \in J^+, \quad (6)$$

$$\sum_{j \in J^+} y_{ij} = D_i(\omega), \quad i \in I(\omega) \quad (7)$$

We want for each station, the satisfied demand cannot exceed the capacity of the station, so we let constraint (4) depicts capacity requirement on each built EVCS. Constraint (5) indicates we do not allow unsatisfied demand at a location without charging station. Constraint (7) is for assigning

charging demands to different EVCS's. Constraint (6) ensures EV owners select EVCS that they prefer, and we want illustrate it with the follow example:

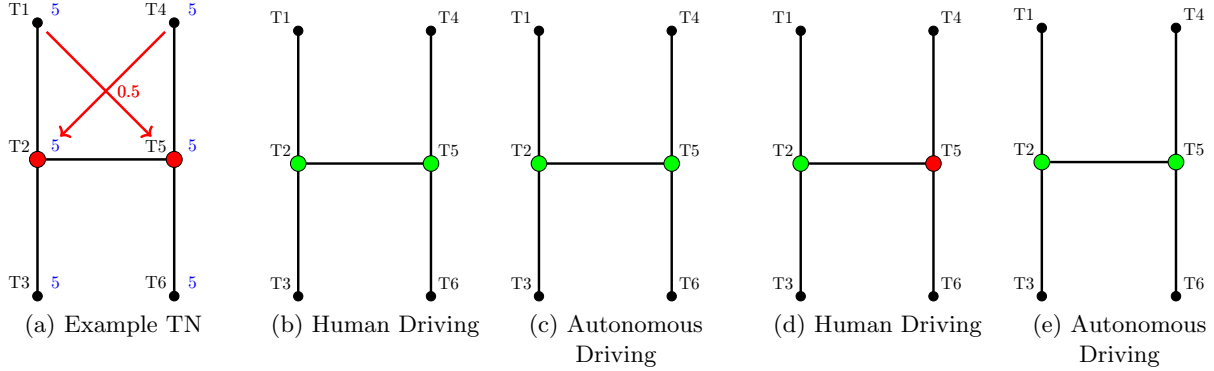


Figure 2: Strong PDN (b, c ), Weak PDN (d, e)

In the above network, we consider two cases. The first case supposes we have a strong PDN, which allow us to build whatever large station we need. In contrast, the second case supposes we can only build large station at T2, and the capacity at T5 is upper bounded by 5. This constrain is particularly important for us to allow we use  $\Delta U$  in the constraint to control the degree of freedom the demand is willing to sacrifice but to contribute the system. For example, we suppose human driving are completely selfish ( $\Delta U = 0$ ), and the autonomous driving are completely selfless ( $\Delta U = 1$ ). Under certain parameters, the above results in the Figure 2 could happen. The results for strong PDN have no difference between human driving and automous driving, and we build two stations at T2 and T5 both with capacity 15. However, we assume PDN can only support T5 with capacity 5. Thus, for human driving, if we build still build T5, the demand generated at T4 and T6 will flow to T5, which becomes penalty of unsatisfied EV. To avoid large penalty, we want to to sacrifice utility and let the demand at right side to move further to T2. On the contrary, for autonomous driving, even though we cannot build a large station at T5, we can still build a small, because we can allocate where the demand at T4 and T6 flow to. In this way, at least 5 demand can have better utility and the rest can still at least keep the original choice.

We now integrate the EV charging demand into the PDN model as follows:

$$\sum_{m:(m,n) \in E^+} p_{mn} - \sum_{m:(n,m) \in E^+} p_{nm} = P_n^{load}(\omega) + W_n \left( \sum_{i \in I} y_{in} - s_n \right), \quad n \in N \quad (8)$$

$$\sum_{m:(m,n) \in E^+} q_{mn} - \sum_{m:(n,m) \in E^+} q_{nm} = Q_n^{load}(\omega), \quad n \in N \quad (9)$$

$$V_m^2 - V_n^2 \geq \frac{2R_{mn}p_{mn} + 2X_{mn}q_{mn}}{1 + \sum_{k \in K} ku_{mn}^k}, \quad (m, n) \in E \quad (10)$$

$$V_n = 1 \text{ p.u.} \quad n \in N^F \quad (11)$$

$$V^{min} \leq V_n \leq V^{max}, \quad n \in N \quad (12)$$

$$0 \leq p_{mn} \leq P_{mn}^{Max} \left( 1 + \sum_{k \in K} ku_{mn}^k \right) \quad (m, n) \in E \quad (13)$$

$$0 \leq p_{0'0} \leq P_{0'0}^{Max} + u_{0'0} \quad (14)$$

$$0 \leq q_{mn} \leq Q_{mn}^{Max} \left( 1 + \sum_{k \in K} ku_{mn}^k \right) \quad (m, n) \in E \quad (15)$$

$$0 \leq q_{0'0} \leq Q_{0'0}^{Max} + u_{0'0} \quad (16)$$

Constraints (8)-(9) formulates the power flow to satisfy all loads including EV charging  $(\sum_{i \in I} y_{in} - s_n)$ . Note that Constraints (4)-(5) also take in account the supplies from the renewable energy with a negative  $P_n^{Load}(\omega)$ . Constraints (10)-(12) are voltage requirement to ensure charging quality. Constraints (13)-(16) specify the capacity restriction of power flows.

In the above model, we particularly care about the following factors:

- Satisfied Cars(SAT#): the number of satisfied cars,  $(\sum_{i \in I} y_{in} - s_n)$
- Unsatisfied Cars(UNSAT#): the number of unsatisfied cars,  $(s_n)$
- Satisfaction Score(SATScore): the number quantifies satisfaction of charging,  $(\Pi(z, x, u, \omega))$
- TN Costs(TN\$): all costs related to TN,  $(\sum_{j \in J} A_j z_j + \sum_{j \in J} C_j x_j)$
- PDN Costs(PDN\$): all costs related to PDN,  $(\sum_{(m,n) \in E} G_{mn} \sum_{k \in K} ku_{mn}^k)$

### 3.2 Reformulation and Solution Method

The aforementioned model aggregates TN and PDN. Either is a huge and complex system consisting of millions of subnetworks and individual agents. Moreover, the uncertainties of the EV charging loads bring a large number of scenarios into the model and further increase the complexity. We develop a fast algorithm based on the Bender decomposition method. Since the original problem

is nonlinear and has high dimensional constraints, we perform linearization and reformulation to shrink the feasible region to speed up the solving process.

### 3.2.1 Linearization

We want to first linearize constrain (10). This is the only voltage related constraint, so we want to replace the squared voltage with voltage-square ( $v_m = V_m^2$ ), and we want to introduce a new variable  $r_{mn}^k = \sum_k u_{mn}^k (v_m - v_n)$ . Therefore, by simple calculation, we have:

$$\begin{aligned} (v_m - v_n) + \sum_k u_{mn}^k (v_m - v_n) &\geq 2R_{mn}p_{mn} + 2X_{mn}q_{mn} \\ (v_m - v_n) + \sum_k r_{mn}^k &\geq 2R_{mn}p_{mn} + 2X_{mn}q_{mn} \end{aligned}$$

Since  $u_{mn}^k$  is binary, we want  $r_{mn}^k$  to be 0 if  $u_{mn}^k = 0$ , and to be  $(v_m - v_n)$  otherwise. Therefore, we introduce the following to linear constraints.

$$\begin{aligned} 0 \leq r_{mn}^k &\leq V^{Max} u_{mn}^k, & (m, n) \in E, k \in K \\ 0 \leq (v_m - v_n) - r_{mn}^k &\leq V^{Max} (1 - u_{mn}^k), & (m, n) \in E, k \in K \end{aligned}$$

Therefore, the original problem becomes a mixed-integer linear programming problem.

### 3.2.2 Treatments on Integer Programming

We notice some integer variables which can be further transformed in order to have a faster speed. First, we want to avoid multiple solution due to multiple expansion line choice. We replace the original  $u_{mn}^k$  with  $\tilde{u}_{mn}^k$  which indicates we make  $k$  expansion lines for PDN line  $mn$ . To keep the format consistent, we still let  $\sum_k \tilde{u}_{mn}^k \leq 1$  to allow only one choice for each PDN line.

We notice that constraint (6) is 3 dimensional constraint, so we need it to be as tight as possible. This function of this constraint is to let certain location generated demand choose acceptable locations to charge. Such function can be perform by an indicator function which is much tighter than original one as the following:

$$\mathbb{1}\{(U_{ij}(\omega) - \max(U_{ik}(\omega) - \Delta U, U^{min})) < 0\} y_{ij} \leq D_i(\omega)(1 - z_k)$$

In summary, we put our full reformulated model here. The first stage problem is reformulated as:

$$\min \sum_{j \in J} A_j z_j + \sum_{j \in J} C_j x_j + L u_{0'0} + \sum_{(m,n) \in E} G_{mn} \sum_{k \in K} k \tilde{u}_{mn}^k + \mathbb{E}[\Pi(\mathbf{z}, \mathbf{x}, \tilde{\mathbf{u}}, \omega)] \quad (17)$$

s.t.

$$x_j \leq \overline{D} z_j, \quad j \in J \quad (18)$$

$$\sum_{k \in K} \tilde{u}_{mn}^k \leq 1, \quad (m, n) \in E, \quad (19)$$

$$\tilde{u}_{mn}^k \in \{0, 1\}, \quad (m, n) \in E, \quad k \in K \quad (20)$$

$$z_j \in \{0, 1\}, \quad j \in J \quad (21)$$

$$x_j \geq 0, \quad j \in J \quad (22)$$

$$u_{0'0} \geq 0 \quad (23)$$

The second stage problem is reformulated as:

$$\min H_0 s_0 + \sum_{j \in J} H s_j - \sum_{i \in I} \sum_{j \in J} G U_{ij}(\omega) y_{ij}$$

s.t.

$$\sum_{i \in I} y_{ij} - s_i \leq x_j, \quad j \in J^+ \quad (24)$$

$$s_j \leq \overline{D} z_j, \quad j \in J^+ \quad (25)$$

$$\mathbb{1}\{(U_{ij}(\omega) - \max(U_{ik}(\omega) - \Delta U, U^{min})) < 0\} y_{ij} \leq D_i(\omega)(1 - z_k), \quad i \in I, j \in J^+, \quad k \in J^+ \quad (26)$$

$$\sum_{j \in J^+} y_{ij} = D_i(\omega), \quad i \in I \quad (27)$$

$$\sum_{m: (m,n) \in E^+} p_{mn} - \sum_{m: (n,m) \in E^+} p_{nm} = P_n^{load}(\omega) + W_n \left( \sum_{i \in I} y_{in} - s_n \right), \quad n \in N \quad (28)$$

$$\sum_{m:(m,n) \in E^+} q_{mn} - \sum_{m:(n,m) \in E^+} q_{nm} = Q_n^{load}(\omega), \quad n \in N \quad (29)$$

$$v_m - v_n + \sum_{k \in K} k r_{mn}^k \geq 2R_{mn}p_{mn} + 2X_{mn}q_{mn}, \quad (m, n) \in E \quad (30)$$

$$0 \leq r_{mn}^k \leq V^{Max} \tilde{u}_{mn}^k, \quad (m, n) \in E, \quad k \in K \quad (31)$$

$$0 \leq (v_m - v_n) - r_{mn}^k \leq V^{Max}(1 - \tilde{u}_{mn}^k), \quad (m, n) \in E, \quad k \in K \quad (32)$$

$$v_n = 1 \text{ p.u.} \quad n \in N^F \quad (33)$$

$$V^{Min} \leq v_n \leq V^{Max}, \quad n \in N \quad (34)$$

$$0 \leq p_{mn} \leq P_{mn}^{Max} \left( 1 + \sum_{k \in K} k \tilde{u}_{mn}^k \right) \quad (m, n) \in E \quad (35)$$

$$0 \leq p_{0'0} \leq P_{0'0}^{Max} + u_{0'0} \quad (36)$$

$$0 \leq q_{mn} \leq Q_{mn}^{Max} \left( 1 + \sum_{k \in K} k \tilde{u}_{mn}^k \right) \quad (m, n) \in E \quad (37)$$

$$0 \leq q_{0'0} \leq Q_{0'0}^{Max} + u_{0'0} \quad (38)$$

## 4 Numerical Analysis

In our numerical analysis, we design a common structure on two existing networks and create a reasonable default case. We want to first explore how the existence of interdependence impacts the default case. After noticing the interdependence, we changed economical and physical factors to explore potential improve from supply perspective. Lastly, by allowing autonomous vehicle, we explore the solution from both demand and supply perspectives.

### 4.1 Case Overview and Default Case Settings

We test our model with two widely used benchmark networks in literatures, IEEE 33-node test feeder(PDN)[] coupled with Sioux Falls transportation network(TN)[]. We want to design a structure for a city with old infrastructure. In TN, we consider two classes, downtown as commercial area and uptown as residential area based on both geographic location and size of EV charging de-



mand. In both classes, the EV charging demand is larger in east area than west area, and larger in southern area than north northern area. In Sioux Falls transportation network, we let T4, T5, T10, T11, T14, T15 be commercial node and the rest as residential nodes. In PDN, we assume that the northern uptown has relatively small number of residents so it is relatively economically efficient to constructed the PDN root node at the northern uptown without considering EV charging demand in the past and make the long branches surround the city from the north to the south. Two shorter branches are specially support the downtown commercial area. In IEEE 33-node test feeder, we set D2 - D22 and D3 - D25 to be two lines specially support to downtown, and the longest branch D1-D18 to be the line surrounding the entire city and the second longest line is the line surrounding north uptown. In the past, the PDN and TN work independently, the structure of PDN is able to support the traditional electric demand. However, this structure could cause some problems when making PDN expansion for southern uptown. On one hand, from TN perspective, we tend to build some large stations in southern uptown area because there are large number of residents. On the other hand, from PDN perspective, the southern uptown are at the end of the longest PDN branch which is physically difficult to be expanded or costly to make expansion. Beside this principle problem, there are also some detailed problem on trade off under this conflicted situation caused by interdependence. We want to first set up a default case in order to adjust parameters to see the interdependence and potential solution explicitly.

Table 1: TN and PDN nodes corresponding map

TN	1	2	4	5	10	11	13	14	15	16	20
PDN	2	30	4	26	19	23	18	24	21	7	11

In Fig.1 (a), the red colored nodes are the candidates for building EV charging stations in TN, which is marked correspondingly in Fig.1 (b). In our model, we want to analyze the marginal effects of changing factors in TN and PDN to study their interdependence and the impacts of autonomous vehicles. We first set up a default case with data following recent papers.

In our default case, we want to use data commonly used in literatures. The charging demand we take the original traffic flow data as bases for EV charging demand and the demand is higher closed to the center. In order to reflect the change of the demand during a regular day, we simulated 108 scenarios from 24 hours pattern for our EV charging demand. We consider fixed cost of building a

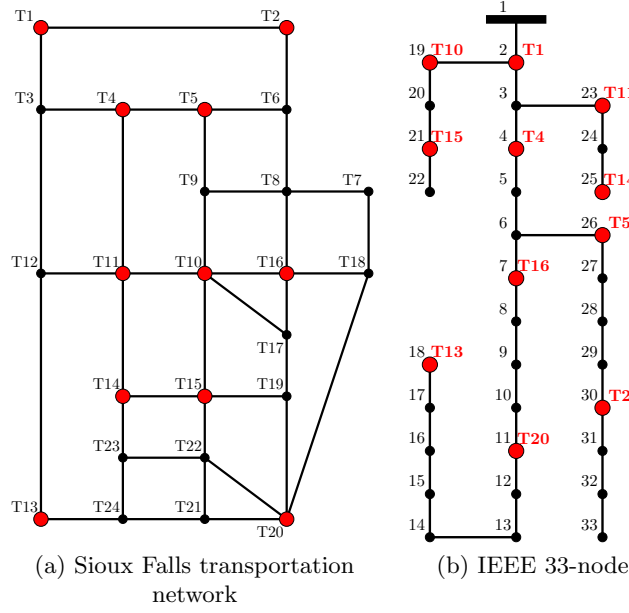


Figure 3: Test Networks

station and their according capacity cost as TN related cost. The default fixed cost of TN related cost increasing from in residential area to center of the city. We consider expansion line cost and substation cost as PDN cost. The per unit expansion line cost is fixed for each additional line, and the substation cost depends on the extra power flower we need for the network. We fixed the the power needed for each unit of charging load for all nodes. In order to analyze the marginal effects, we use introduce two multipliers  $\alpha$  and  $\beta$  on the cost of TN and PDN. The multiplier  $\alpha$  is on both fixed cost of charging station and the capacity cost, and  $\beta$  is on unit cost of distribution lines. The default values are  $\alpha = 1$  and  $\beta = 1$ . In our default case, we consider all vehicles are driven by human who always choose the closest station to charge. Lastly, we set the maximum number of line we are able to expand for each PDN line ( $K$ ) as 2. We will stated the reason for this later in the analysis.

We solved our cases using iOptimize Solver based on Benders decomposition method via CPLEX 12.90. All simulation and solving processes are implemented on a computer with Intel i7-1065G7 CPU and 16-GB RAM. The default case result is as follows:

#### DATA GENERATION MISSING

For example, in our default case, the number of satisfied cars is 384, which means over 108 scenarios, on average, 384 cars get charging services. The number of unsatisfied cars is 84, which

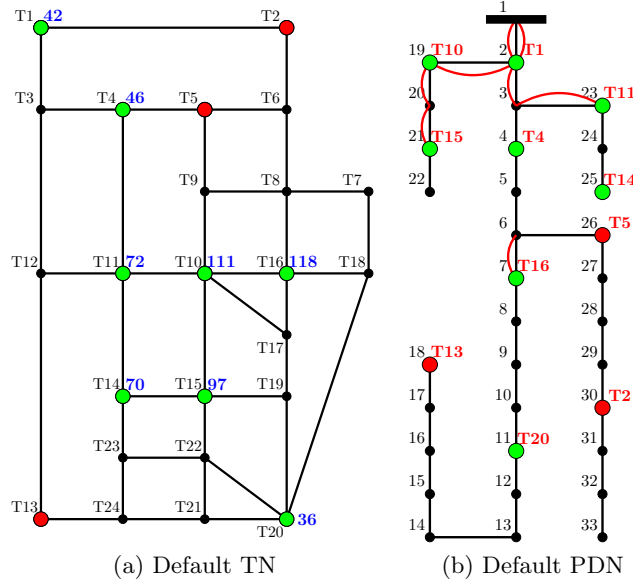


Figure 4: Default Decision Results: The green nodes in TN indicate we decide to build a charging station, and we also marked the corresponding PDN nodes as green; The red nodes are the candidate nodes we decide not to build stations; The blue numbers in the TN indicate the capacity of the charging stations.

means on average, 84 cars cannot get charging services, including the cars too far from the closest station to charge and the cars actually go to the stations but are not able to charge due to the limited capacity of the stations. The satisfaction score is 5015, which takes account of both positive effects of the cars able to charge and the negative effects of the cars not able to charge. The TN Cost is \$4977K including the fixed costs of the selected candidates and the capacity costs. The PDN Cost is \$4564K including substation costs and expanding line costs.

## 4.2 Existing Interdependence Impacts

The result of default case shows we still have large amount of EV charging demand is unsatisfied. We believe the unsatisfied demand is due to interdependence between TN and PDN. In general, from TN perspective, we want to build large stations at southern and eastern uptown because the building cost is low and there will be high EV charging demand, and we want to avoid building stations at northern and western downtown because the demand is low and building cost is high. On the contrary, from PDN perspective, we want to avoid building large stations at southern and eastern uptown because there are located at the downstream and end of the longest branches.

Instead, we want to build large station at northwestern area and the nodes on two special branches because it is physically and economically easy to expand. Therefore, the bottleneck could be caused by either TN factors or PDN factors, which depends on the parameter given. When a factor of a network changes, it will make impacts on the decision for both networks. We want to first explore the interdependent properties of the default case.

In order to see the optimal decision solely from TN and PDN, we design two cases. We set all TN related cost including building fixed cost and station capacity cost to be zero, which is denoted by "FreeTN". We set the current PDN to be large enough to allow us to build any large size charging stations if needed, which is denoted by "FreePDN". The FreeTN case reflects the decision solely from PDN perspective, and the FreePDN case reflects the decision solely from TN perspective. In addition, we also set a case "FreeAll" by removing all constrains, so the result of it just reflect the worst case of the EV charging demand on the station. We want to compare the result with default case first and find some properties in our designed structure. The results are as follows:

Table 2: TN Decision of Extreme Cases

TN	1	2	4	5	10	11	13	14	15	16	20
Default	42	0	46	0	110	72	0	70	97	118	37
FreeTN	50	45	90	74	194	113	0	92	150	0	80
FreePDN	35	0	0	84	134	76	106	0	162	148	76
FreeAll	30	44	62	92	168	58	108	104	162	182	92

Table 3: PDN Decision of Extreme Cases

PDN Line	1-2	2-3	3-4	6-7	10-11	2-19	19-20	20-21	3-23
Default	2	1	0	1	0	1	1	1	1
Free TN	2	1	0	0	1	2	2	2	1
Free PDN	-	-	-	-	-	-	-	-	-

Table 4: Factors of Extreme Cases

Factors	SAT#	UNSAT#	SATScore	TN\$	PDN\$
Default	384	84	7147	4977	4864
Free TN	419	49	8028	-	6215
Free PDN	459	9	9813	5821	-

Comparing the result of FreeTN with default case, we notice after all TN related cost removed, the expansion strategy in PDN changed. On one hand, the eastern downtown has large size of demand, so when building station is free, we have more economical incentive to build station there.

On the other hand, the eastern downtown branch is closed to root node in PDN, so Correspondingly expansion for eastern downtown becomes more economically efficient. We can see we make more expansion for all lines on that branch, and thus we can build a larger station in eastern down. Furthermore, if we have large enough station at eastern downtown, making expansion for eastern uptown becomes less economically efficient, and we give up expansion for line D6-D7. Instead of making expansion for east uptown, we make expansion for southeastern uptown now. We also notice that compared with default case, we build two more stations at T2 and T5 without any extra expansion. Therefore, we know fixed cost is an important factors in northern area stations because the demand is low.

Comparing the result of FreePDN with default case, the station building strategy is also changed. On one hand, the southern eastern uptown is at downstream of the longest branch, so when don't need to make expansion to build any large size of stations, we have more incentive to build larger stations in southern and eastern uptown. On the other hand, the building large station in downtown is expensive, so if we can move the demand from downtown to uptown without too much loss of satisfaction, we will give up a downtown station and move the demand to uptown. As we can see, we build larger size station in eastern uptown at T16, and in southern uptown at T13. We give up building station at southeastern downtown at T14. On the contrary, even though we can move the demand from eastern downtown to eastern uptown and saving cost, we still keep the station at eastern down because we will lose too much satisfaction if we move it to uptown.

Based on the results and analysis above, we can summarize some properties of the structure of the two networks. The first priority of building station is eastern downtown at T10 and T15, because even though building cost is high, the demand is also high and it is economically efficient on PDN expansion. Western downtown is also important but we would like to move the station of southwestern downtown to uptown if PDN could support. Eastern uptown station, T16, is very closed to east downtown, so it acts as an auxiliary candidate to support the downtown demand when needed. The southeastern uptown station T20 connects eastern area, so whenever possible, we are willing to make a larger station at T20. Since the northern area has less demand and also closed to root node, they does not play a crucial role in the interdependence. Lastly, we can see the satisfied cars increase significantly in the case of FreeTN and FreePDN, which means we still can improve the satisfaction if we can change physical and economical factors. Therefore, we need

to explore more realistic cases.

### 4.3 Interdependence and Solution on Supply

In the last section, we explore the extreme cases by removing consideration of the economical and physical factors in two networks. In this section we want separate the important factors and explore more intermediate results. In the meantime, we how the bottleneck will change when different factors of current interdependent networks changes. We want to first change the physical feasibility because in reality, such factors is typically fixed. If the capacity of a station is limited by PDN where we do not make expansion not for physical reason allow but for economical efficiency, we treat such unsatisfied demand is caused by economical efficiency. We want to explore if physically reason is not constrained, how economic factors in PDN and TN will influence the optimal decision. When we change the cost in either network, we notice a similar impacts, for example, the expansion strategy for eastern downtown. The reason for that is when we make the decision on building strategy, the economical efficiency we consider is not balancing between TN and PDN but balancing the total cost of two networks for such station and its contribution on total satisfaction. Therefore, we will see a hierarchic pattern on the decisions when we changes the factors.

#### 4.3.1 Changing Maxmium number of Expansion line

We want to change the physical feasibility by changing the number of lines we are able to expand. By allowing more number of expansion line, we can increase the physical feasibility of PDN, and we still apply cost on expansion. Therefore, we expect at some point increase the number of line will not increase the satisfaction because we are meeting the bottleneck of physical feasibility. In our model the set of possible lines is denoted by  $K$ . For convenience, we use  $K$  to denote the number of lines possible to expand. We test our cases by increasing  $K$  from 0 to 3, and the results are as follows:

From the Figure 4, if we do not expand the PDN, our current PDN can only support some small stations. It also indicates that even though we cannot build a large station, we still need some small stations in the important areas. We slightly increase the physical feasibility by increase  $K$  to 1, we notice we can have much larger stations than the station supported by the current PDN. We are able to build two more stations in southwestern downtown at T14 and in north uptown at T1. In

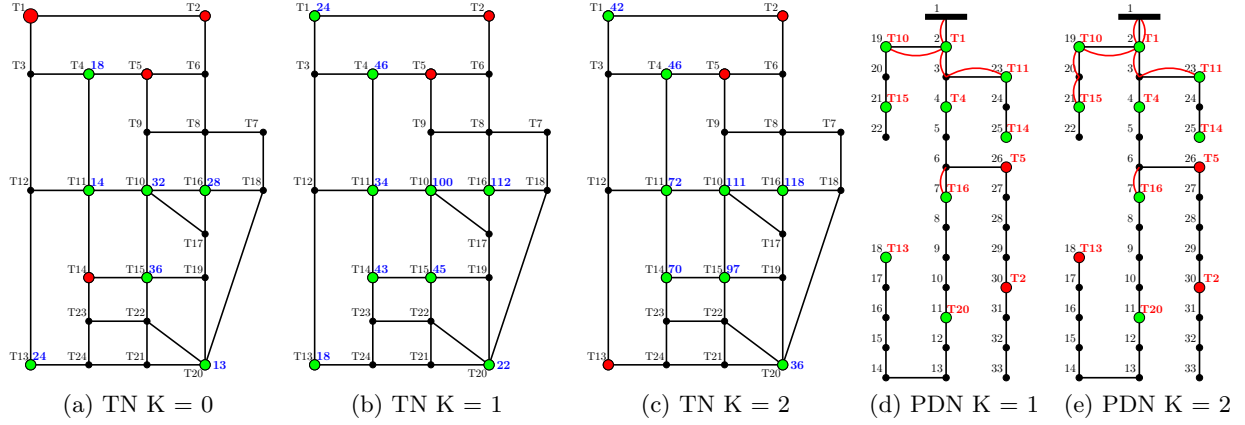


Figure 5: Decision Result of Changing K: The yellow node in (c) means the closed station compared with default case

our default case,  $K$  is 2, and we notice compared with the case of  $K = 1$ , we do not build station at T13. This reason is that T13 is located at the end of the deepest branch of PDN, which makes it both economically and physically to expand. When we are able to make more expansion, even though the southwestern downtown needs higher building cost, it still has high demand and easily to expand from PDN perspective. Furthermore, the total demand in entire region of southwest is not very large, when we can have a large station in the downtown, we do not need a small station in the uptown. Lastly, we notice the strategy remain the same when we increase  $K$  to 3, which is the case we mention that we do not make expansion is not because of physical feasibility but the economical efficiency.

Table 5: TN Decision of Changing K

TN	T1	T2	T4	T5	T10	T11	T13	T14	T15	T16	T20
$K = 0$	0	0	18	0	32	14	24	0	36	28	12
$K = 1$	24	0	46	0	100	34	18	43	45	112	22
$K=2$ (Default)	42	0	46	0	110	72	0	70	97	118	37
$K = 3$	42	0	46	0	110	72	0	70	97	118	37

Table 6: Factors of Changing K

Factors	SAT#	UNSAT#	SATScore	TN\$	PDN\$
$K=0$	119	349	-10950	2547	0
$K = 1$	306	162	741	4419	2787
$K = 2$ (Default)	384	84	5016	4977	4564
$K = 3$	384	84	5016	4977	4564

From Table 6, we notice the satisfied cars and satisfaction core significantly increase when we

increase  $K$  from 0 to 1. However, the increment of the second expansion line are much smaller. Therefore, the first expansion line is more economically efficient than the second one. As stated before, with the current PDN the best expansion strategy still cannot support most of demand. We have already reach the bottleneck of physical feasibility. Therefore, we want to explore potential solution if we can change costs.

#### 4.3.2 Changing PDN Cost

Now, we want to adjust PDN cost by changing the cost of each expansion line. If the cost of expansion line reduce, we have more incentive to make expansion in PDN, which allow us to build larger stations. If the cost of expansion line increase, we only want to make expansion for the east downtown. The results are as follows:

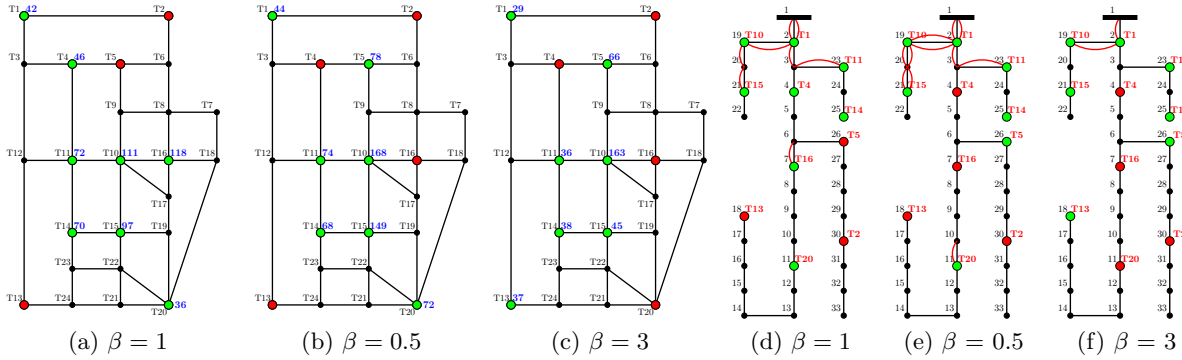


Figure 6: PDN Decision Result of Changing  $\beta$

From the Figure 5, we want to make extra expansion for eastern downtown. The reason is that when the cost of expansion reduce, the second line becomes more economically efficient, especially for the line on the eastern downtown branches. Therefore, we made extra expansion on that branch. With large enough station in eastern downtown at T10, making expansion for the eastern uptown at T16 becomes less economical efficient, so we move our expansion to southern uptown at T20. From the result, we also notice that when we can only make PDN expansion for eastern downtown, we choose to build station in southwestern uptown at T13 instead of southeastern uptown at T20. The reason is that when we cannot make expansion for west downtown, the unsatisfied cars in west area will be high. In contrast, since we made expansion for eastern area, we have less unsatisfied cars, so a strategy balancing west and east is more appropriate.



Table 7: TN Decision of Changing  $\beta$ 

TN	T1	T2	T4	T5	T10	T11	T13	T14	T15	T16	T20
$\beta = 0.5$	44	0	0	78	168	74	0	68	149	0	72
$\beta = 1$ (Default)	42	0	46	0	110	72	0	70	97	118	37
$\beta = 3$	29	0	0	66	163	36	37	38	45	0	0

Table 8: Factors of Changing  $\beta$ 

Factors	SAT#	UNSAT#	SATScore	TN\$	PDN\$
$\beta = 0.5$	407	60	5924	5504	3824
$\beta = 1$ (Default)	384	84	5016	4977	4564
$\beta = 3$	287	180	-1269	4420	3315

In Table 8, we note that we can only satisfy slightly more demand with lower PDN cost. There are still more than 10 percents demand are unsatisfied. The satisfaction score only increases from 5016 to 5925. However, if the cost of PDN is higher, the satisfaction will significantly drop to negative number. Therefore, we still are not satisfied with result of the lower cost on PDN. Therefore, we want to explore if the TN cost can resolve the issue, and we change the TN cost.

#### 4.3.3 Changing TN Cost

We want to adjust TN cost by changing the cost of both fixed cost of building charging stations and the cost of capacity of charging stations. When capacity cost decreases, we tend to build larger stations. When fixed cost reduces, we have more incentive to build more stations, and vice versa.

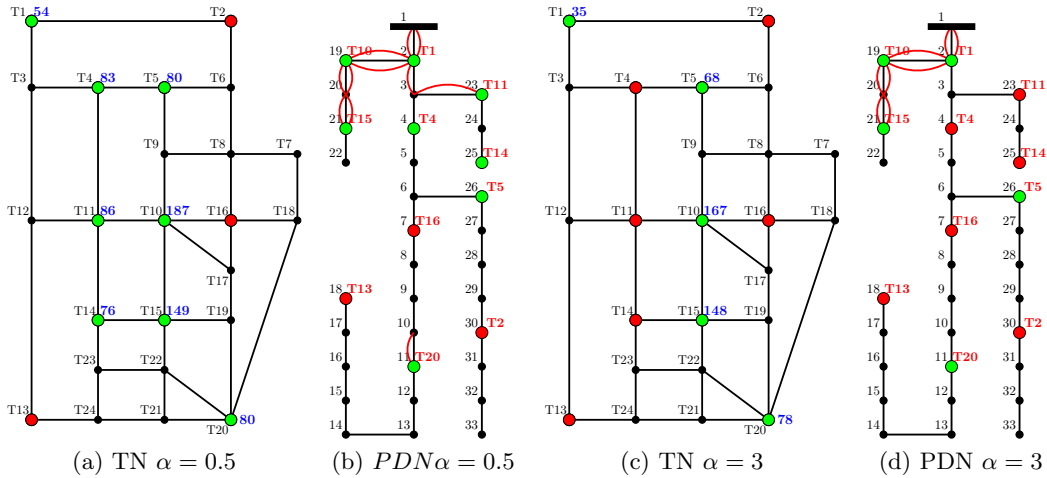


Figure 7: Decision Result of Changing K: The yellow node in (c) means the closed station compared with default case

From the result above, comparing the case of smaller cost of TN with the default case, we can notice solution is more closed to the case of FreeTN. It has exactly the same PDN expansion strategy as FreeTN. The reasoning is also the same as we analyzed in the above section. For the case of larger cost of TN, we see we only retain one station at each area. Since the fixed cost is high, we only want to build relatively large stations with respect to the corresponding amount of demand. Since the demand of east downtown is high, we need to make more expansion on the branch of east downtown in PDN.

Table 9: Building Capacity for TN

TN	T1	T2	T4	T5	T10	T11	T13	T14	T15	T16	T20
$\alpha = 0.5$	54	0	83	80	187	86	0	76	149	0	80
Default	42	0	46	0	110	72	0	70	97	118	37
$\alpha = 3$	35	0	0	68	167	0	0	0	148	0	78

Table 10: Expansion Strategy for PDN

Factors	SAT#	UNSAT#	SATScore	TN\$	PDN\$
$\alpha = 0.5$	417	51	6624	3241	4985
$\alpha = 1(\text{Default})$	384	84	5016	4977	4564
$\alpha = 3$	349	119	1729	12826	3558

In Table 10, we can see the satisfied cars with lower cost is already closed to FreeTN. However, the proportion of satisfied cars is still lower than 90 percents, which means we should still consider more solution to improve it.

Before moving forward, we want to summarize the analysis of cases above. If we can expand two lines like our default case, the first priority to make improvement is the eastern downtown. It is because from both TN and PDN perspective, large stations in eastern downtown are economically efficient. The west downtown also has high priority but still lower than the east. The next level of importance is the decision on eastern uptown station at T16 and southwestern uptown station at T13. The common features of them in TN is they are both closed to the downtown in their area, so they can function as auxiliary station for downtown. In contrast, the southeastern uptown is relative special, because we already have some station in east area. Especially, a large station in southeastern down at T15 makes us consider the southeastern downtown more independently. However, the priority of southeast uptown is still low. Therefore, only after we satisfied the most of demand in eastern and western downtown area, we will consider a larger station in southeastern

uptown at T20. In addition, those three uptown stations are all located in the downstream of the D1-D18 branches, so the expansion for them are limited. Therefore, if we have large station in downtown, making building stations at T13 and T16 unnecessary, we will put all the resource to build a large station at T20.

#### 4.3.4 Changing Demand Pattern

In all previous cases, we consider the demand pattern as given. In order to know whether the structural properties of the decisions are due to the structural of two networks or the demand pattern, we tried a uniformly distributed demand case. The results are as follows:

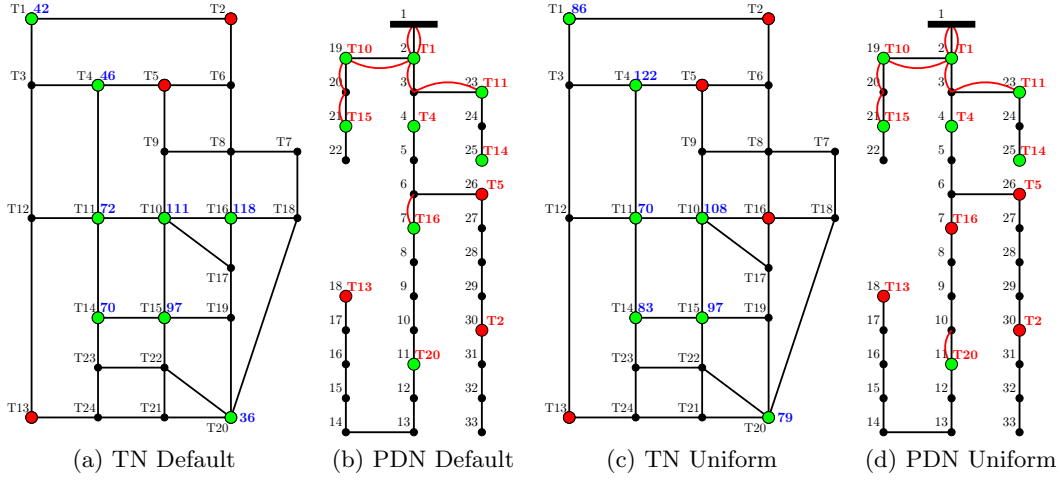


Figure 8: Decision Result of Changing K: The yellow node in (c) means the closed station compared with default case

The decision result are similar in both cases. The only difference is we have more incentive to build large station in uptown areas because of the higher demand. Therefore, the result in previous cases is mainly determined by the structural properties of TN and PDN.

Compared with default case, we have almost the same number of satisfied cars in uniform case. However, the total satisfaction is low. The reason is that our designed PDN has two special branch for downtown where the demand should be high, but when the demand becomes diversified, these two special line has no any advantage, which making the overall performance becomes worse. Thus, we have tried all possible solutions from supply perspective, but we have not reach a high level satisfaction. Therefore, we want to consider solution from demand perspective.

Table 11: Building Capacity for TN

TN	T1	T2	T4	T5	T10	T11	T13	T14	T15	T16	T20
Default	42	0	46	0	110	72	0	70	97	118	37
Uniform	86	0	122	0	108	70	0	83	97	0	79

Table 12: Expansion Strategy for PDN

Factors	SAT#	UNSAT#	SATScore	TN\$	PDN\$
Default	384	84	5016	4977	4564
Uniform	385	83	3883	5103	4689

#### 4.4 Interdependence and Solution on Demand

From the above results and analysis, we know the main problem of the interdependent TN and PDN is that the uptowns with high EV charging demand are located at the downstream of PDN. We want to allow autonomous vehicle to allocated the demand and see whether the demand can be allocated to the station where PDN expansion is easier. We expect if the physical condition is not a restriction, the system with autonomous vehicle will have more potential, so we may want to invest more on the system. If the system is constrained by physical limitation, we expect similar level of satisfaction but less investment.

##### 4.4.1 ChangingK

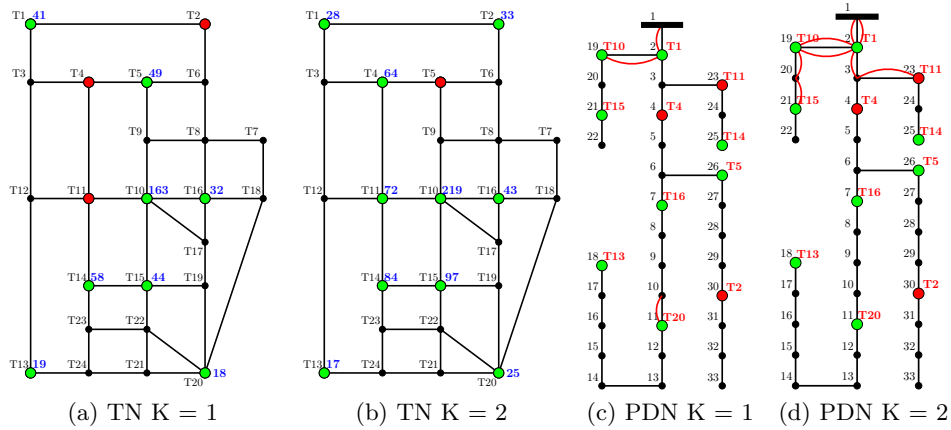


Figure 9: Decision Result of Changing K: The yellow node in (c) means the closed station compared with default case

We want compare the above with the result of traditional manual driving. If we can only make one expansion line, we closed stations at west and northwest downtown, and remove expansion line

at D2-D3, D3-D23 and D6-D7. while we have less stations and make less expansion, we still reach slightly higher satisfaction. In contrast, if we can make two expansion lines, we open an extra station at T2, and make more expansion lines for eastern downtown. As a result, we can reach a much higher level of satisfaction.

Table 13: Factors of Changing K

Factors	SAT#	UNSAT#	SATScore	TN\$	PDN\$
K=0	119	349	-10841	2592	0
K = 1	311	157	454	4485	2187
K = 2	447	21	8365	6317	4932

From the table 13, we know when physical factors is constrained, reallocating demand does not help much on the satisfaction, but it can save our investment. Whereas, if physical restriction is removed, we want to invest more to satisfy more charging demand with reallocation of autonomous taxis. Such a improvement is larger than any improvement done by change the supply costs. Therefore, we want to further explore, whether we still need to change supply factors to make more improve.

#### 4.4.2 Changing PDN cost

If the PDN line cost is reduced to half the same treatment as from supply perspective, there is no any improvement on satisfied demand. And we further reduce it to one-fifth, there is just a tiny improvement via larger station at eastern uptown. Therefore, there is little charging demand restricted by the economical efficiency of PDN. Compared with the result three times PDN cost only from supply perspective, we are more willing to invest more to make expansion.

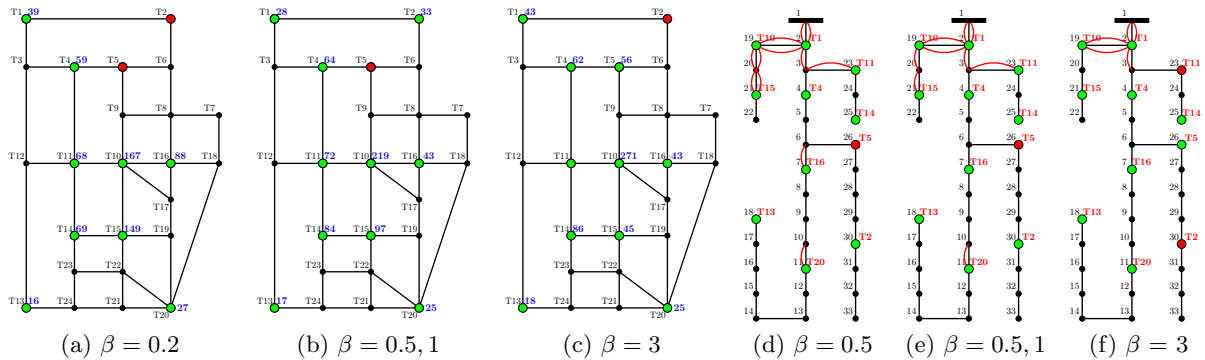


Figure 10: PDN Decision Result of Changing  $\beta$

Table 14: Factors of Changeing  $\beta$

Factors	SAT#	UNSAT#	SATScore	TN\$	PDN\$
$\beta = 0.2$	448	20	8676	5875	3546
$\beta = 0.5$	447	21	8364	6317	4032
$\beta = 3$	428	40	6695	6275	5618

## A Numerical Analysis Data

In our default case, we want to use data commonly used in literatures. The charging demand we take the original traffic flow data as bases for EV charging demand and the demand is higher closed to the center. In order to reflect the change of the demand during a regular day, we simulated 108 scenarios from 24 hours pattern for our EV charging demand. The default fixed cost of building a station in residential area is  $A_i = \$1.63 \times 10^5$  and the capacity cost is  $C_j = \$3,160$ . The base cost per-unit cost of a distribution line is  $G_{ij} = \$3 \times 10^5$ , and the cost of substation capacity expansion cost is  $L = \$5,000/kVA$ . For each demand to power flow,  $W_n = 7.7$ [11]. In order to analyze the marginal effects, we use introduce two multipliers  $\alpha$  and  $\beta$  on the cost of TN and PDN. The multiplier  $\alpha$  is on both fixed cost of charging station and the capacity cost, and  $\beta$  is on unit cost of distribution lines. The default values are  $\alpha = 1$  and  $\beta = 1$ . In our default case, we consider all vehicles are driven by human who always choose the closest station to charge. Lastly, we set the maximum number of line we are able to expand for each PDN line ( $K$ ) as 2. We will stated the reason for this later in the analysis.

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