Matrices

A matrix is a 2-dimensional table of numbers

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}$$

A size of a matrix is defined by the number of its rows and its columns.
Above A is a $m \times n$ matrix. An element in a matrix is referenced by

$$A(\underset{\text{row}}{i}, \underset{\text{column}}{j}) = a_{ij}$$

Square matrix

A matrix is called a square matrix, if the number of rows is same as the number of columns

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix} \quad 3 \times 3 \text{ square matrix}$$

$$\begin{bmatrix} 10 & 11 \\ 0 & -5 \end{bmatrix} \quad 2 \times 2 \text{ square matrix}$$

Vectors

A column vector is a $n \times 1$ matrix, i.e. it has only one column.

$$\underset{4 \times 1}{\begin{bmatrix} 3 \\ 1 \\ 4 \\ 0 \end{bmatrix}} \quad \underset{2 \times 1}{\begin{bmatrix} 2 \\ 1 \end{bmatrix}} \quad \underset{3 \times 1}{\begin{bmatrix} 3 \\ 2 \\ 1 \end{bmatrix}}$$

A row vector is a $1 \times n$ matrix, i.e. it has only one row.

$$\underset{1 \times 3}{\begin{bmatrix} 1 & 2 & 3 \end{bmatrix}} \quad \underset{1 \times 2}{\begin{bmatrix} -1 & 2 \end{bmatrix}}$$

Zero matrix O

A matrix is called a zero matrix, if all of its elements are zero, regardless of its matrix dimensions.

$$O = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \quad 2 \times 3$$

$$O = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad 3 \times 1$$

Identity matrix I

An identity matrix is such that

$$IA = AI = A$$

for any matrix A.
Identity matrix is defined by

$$I(i,j) = \begin{cases} 1 & ; \ i = j \\ 0 & ; \ i \neq j \end{cases}$$

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Scalar multiplication

A matrix can be multiplied by a scalar (= real number).

$$\boxed{(\lambda A)(i,j) = \lambda A(i,j)}$$

$$3 \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} = \begin{bmatrix} 3 \cdot 1 \\ 3 \cdot 2 \\ 3 \cdot (-1) \end{bmatrix} = \begin{bmatrix} 3 \\ 6 \\ -3 \end{bmatrix}$$

$$2 \begin{bmatrix} 5 & 10 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 \cdot 5 & 2 \cdot 10 \\ 2 \cdot 0 & 2 \cdot 1 \end{bmatrix} = \begin{bmatrix} 10 & 20 \\ 0 & 2 \end{bmatrix}$$

Sum

Matrix sum is calculated elementwise.

$$\boxed{(A+B)(i,j) = A(i,j) + B(i,j)}$$

Matrices must be of the same size.

$$\begin{bmatrix} 2 & 1 \\ 0 & 5 \end{bmatrix} + \begin{bmatrix} -1 & 5 \\ 0 & 3 \end{bmatrix} = \begin{bmatrix} 2-1 & 1+5 \\ 0+0 & 5+3 \end{bmatrix} = \begin{bmatrix} 1 & 6 \\ 0 & 8 \end{bmatrix}$$

Matrix Product

Matrix product AB is defined, if the number of columns in A is the same as the number of rows in B.

$$\underset{m \times p}{A} \cdot \underset{p \times n}{B} = \underset{m \times n}{C}$$

must be the same

It is defined by

$$\boxed{(AB)(i,j) = \sum_{k=1}^{p} A(i,k) B(k,j)}$$