

1)

$$a) f(x, y) = xy^3 + 3x$$

$$\left[\frac{\partial}{\partial x} f(x, y) = y^3 + 3 \right.$$

$$\left. \frac{\partial}{\partial y} f(x, y) = x \cdot 3y^2 + 0 = 3xy^2 \right]$$

b)

$$f(x, y) = \ln \frac{x}{y}$$

$$x \cdot y^{-1} \quad \left[\frac{\partial}{\partial x} f(x, y) = \frac{1}{\frac{x}{y}} \cdot \frac{1}{y} = \frac{y}{x} \cdot \frac{1}{y} = \frac{1}{x} \right.$$

$$\left. \frac{\partial}{\partial y} f(x, y) = \frac{1}{\frac{x}{y}} \cdot (-1) x y^{-2} = -\frac{y}{x} \cdot \frac{x}{y^2} \right]$$

$$= -\frac{1}{y}$$

$$c) f(x, y) = \frac{\sin x}{\cos y}$$

$$\frac{\partial}{\partial x} f(x, y) = \frac{1}{\cos y} \cdot \cos x = \frac{\cos x}{\cos y}$$

$$\frac{\partial}{\partial y} f(x, y) = \sin x \cdot \frac{\sin y}{\cos^2 y} \cdot \frac{1}{\cos y}$$

2) a)

$$f(x, y, z) = x^2 z + \ln(yz)$$

$$\left\{ \begin{array}{l} \frac{\partial}{\partial x} f(x, y, z) = 2xz + 0 = 2xz \\ \frac{\partial}{\partial y} f(x, y, z) = 0 + \frac{1}{yz} \cdot z = \frac{1}{y} \\ \frac{\partial}{\partial z} f(x, y, z) = x^2 + \frac{1}{yz} \cdot y = x^2 + \frac{1}{z} \end{array} \right.$$

$$\nabla f(x, y, z) = \begin{bmatrix} 2xz \\ \frac{1}{y} \\ x^2 + \frac{1}{z} \end{bmatrix}$$

2 b)

$$f(x, y) = \sqrt{x^2 + y^2}$$

$$\frac{\partial}{\partial x} f(x, y) = \frac{1}{2\sqrt{x^2 + y^2}} = \frac{\frac{\partial}{\partial x} (x^2 + y^2)}{2\sqrt{x^2 + y^2}}$$

$$= \frac{2x}{2\sqrt{x^2 + y^2}} = \frac{x}{\sqrt{x^2 + y^2}}$$

$$\frac{\partial}{\partial y} f(x, y) = \frac{2y}{2\sqrt{x^2 + y^2}} = \frac{y}{\sqrt{x^2 + y^2}}$$

3.

$$f(x) = x^4 + 5x^3 - 7x$$

$$f'(x) = 4x^3 + 15x^2 - 7$$

$$x_{i+1} = x_i - \alpha f'(x_i)$$

$$\alpha = 0,2$$

$$x_0 = 0$$

$$f'(x) = 4x^3 + 15x^2 - 7$$

$$x_1 = 0 - 0,2(0 + 0 - 7) = 1,4$$

$$x_2 = 1,4 - 0,2(10,976 + 29,4 - 7)$$

$$x_2 = -3,4752$$

~~$$x_3 = -3,4752 - 0,2(-41,97)$$~~

$$x_3 = -3,4752 \cdot 0,2 (-167,88 - 181,155226 + 7)$$

$$x_3 = -3,1$$

4) $f(x, y) = 3x^2 + 3xy + 2y^2 + 3x - 5y$
~~0/0/0~~

$$\frac{\partial}{\partial x} f(x, y) = 6x + 3y + 3$$

$$\frac{\partial}{\partial y} f(x, y) = 3x - 4y - 5$$

$$\Delta f(x, y) = \begin{bmatrix} 6x + 3y + 3 \\ 3x - 4y - 5 \end{bmatrix}$$

$$x_0 = \begin{bmatrix} 0 \\ 0 \end{bmatrix}, \quad \alpha = 0,3$$

$$x_1 = \begin{bmatrix} 0 \\ 0 \end{bmatrix} - 0,3 \begin{bmatrix} 3 \\ -5 \end{bmatrix} = \begin{bmatrix} 0,9 \\ -0,15 \end{bmatrix}$$

$$x_2 = \begin{bmatrix} 0,9 \\ -0,15 \end{bmatrix} - 0,3 \begin{bmatrix} 6(0,9) + 3(-0,15) + 3 \\ 3(0,9) - 4(-0,15) - 5 \end{bmatrix}$$

$$= \begin{bmatrix} -1,485 \\ 1,95 \end{bmatrix}$$

$$x_3 = \begin{bmatrix} -1,467 \\ 2,4965 \end{bmatrix}$$