Mathematics and Methods in Machine Learning

Home Exercises 1

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1. $$A = \begin{bmatrix} 2 & 5 & -1 & 3 & 6 \\ 1 & 0 & 0 & -2 & 0 \\ 4 & 1 & -2 & 0 & 7 \\ 0 & 3 & 5 & 1 & -1 \end{bmatrix}$$

Matrix A is rectangular. It's 4×5.

$A(2,3) = 0$, $A(1,2) = 5$, $A(3,4) = 0$, $A(4,5) = -1$

2. $$A(i,j) = \begin{cases} i - j, & i < j \\ 0, & i = j \\ i + 2j, & i > j \end{cases}$$

$$A = \begin{bmatrix} 0 & -1 & -2 \\ 4 & 0 & -1 \\ 5 & 7 & 0 \\ 6 & 8 & 10 \end{bmatrix}$$

3. a) $\sum_{n=1}^{100} n$

b) $\sum_{n=2}^{10} (-1)^n \cdot \frac{1}{n}$

c) $\sum_{n=1}^{n} a_{nn}$

d) $\sum_{n=1}^{n} a_{kn} x_n$

e) $\sum_{n=1}^{n} a_{kn} b_{np}$

4. $A = \begin{bmatrix} 2 & 5 & -1 & 3 & 6 \\ 1 & 0 & 0 & -2 & 0 \\ 4 & 1 & -2 & 0 & 7 \\ 0 & 3 & 5 & 1 & -1 \end{bmatrix}$

a) $\sum_{j=1}^{5} A(3,j) = A(3,1) + A(3,2) + A(3,3) + A(3,4) + A(3,5) =$

$= 4 + 1 - 2 + 0 + 7 = 10$

b) $\sum_{k=1}^{4} A(k,k) = A(1,1) + A(2,2) + A(3,3) + A(4,4) =$

$= 2 + 0 - 2 + 1 = 1$

c) $\sum_{i=1}^{4} A(i,1)\,A(i,3) = A(1,1) \cdot A(1,3) + A(2,1) \cdot A(2,3) + A(3,1) \cdot A(3,3)$

$+ A(4,1) \cdot A(4,3) = 2 \cdot (-1) + 1 \cdot 0 + 4 \cdot (-2) + 0 \cdot 5 =$

$= -2 + 0 - 8 + 0 = -10$

5. $A = \begin{bmatrix} 1 & -3 \\ 2 & 4 \\ 0 & -1 \end{bmatrix}$, $B = \begin{bmatrix} a & 3 \\ 1 & -2 \\ x & 2 \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

$D = \begin{bmatrix} 1, & -5, & x \end{bmatrix}$

a) $A + B = \begin{bmatrix} 1+a, & 0 \\ 3, & 2 \\ x, & 1 \end{bmatrix}$

b) $5A = \begin{bmatrix} 5 & -15 \\ 10 & 20 \\ 0 & -5 \end{bmatrix}$

c) $C + D \Longrightarrow$ not defined

d) $A + C \Longrightarrow$ not defined

e) $-B = \begin{bmatrix} -a & -3 \\ -1 & 2 \\ -x & -2 \end{bmatrix}$

f) $B - A = \begin{bmatrix} a-1 & 6 \\ -1 & -6 \\ x & 3 \end{bmatrix}$

6. $A = \begin{bmatrix} 1 & 3 & 5 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -1 \\ 3 \\ 2 \\ 4 \end{bmatrix}$

$AB = -1 + 9 + 10 + 4 = 22$

$$BA = \begin{bmatrix} (-1\cdot1) & (-1\cdot3) & (-1\cdot5) & (-1\cdot1) \\ (3\cdot1) & (3\cdot3) & (3\cdot5) & (3\cdot1) \\ (2\cdot1) & (2\cdot3) & (2\cdot5) & (2\cdot1) \\ (4\cdot1) & (4\cdot3) & (4\cdot5) & (4\cdot1) \end{bmatrix} = \begin{bmatrix} -1 & -3 & -5 & -1 \\ 3 & 9 & 15 & 3 \\ 2 & 6 & 10 & 2 \\ 4 & 12 & 20 & 4 \end{bmatrix}$$

7. $A = \begin{bmatrix} 1 & -1 & 1 \\ -3 & 2 & -1 \\ -2 & 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 1 & 2 & 3 \end{bmatrix}$

$$AB = \begin{bmatrix} 1-2+1, & 2-4+2, & 3-6+3 \\ -3+4-1, & -6+8-2, & -9+12-3 \\ -2+2+0, & -4+4+0, & -6+6+0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$BA = \begin{bmatrix} 1-6-6, & -1+4+3, & 1-2+0 \\ 2-12-12, & -2+8+6, & 2-4+0 \\ 1-6-6, & -1+4+3, & 1-2+0 \end{bmatrix} = \begin{bmatrix} -11 & 6 & -1 \\ -22 & 12 & -2 \\ -11 & 6 & -1 \end{bmatrix}$$