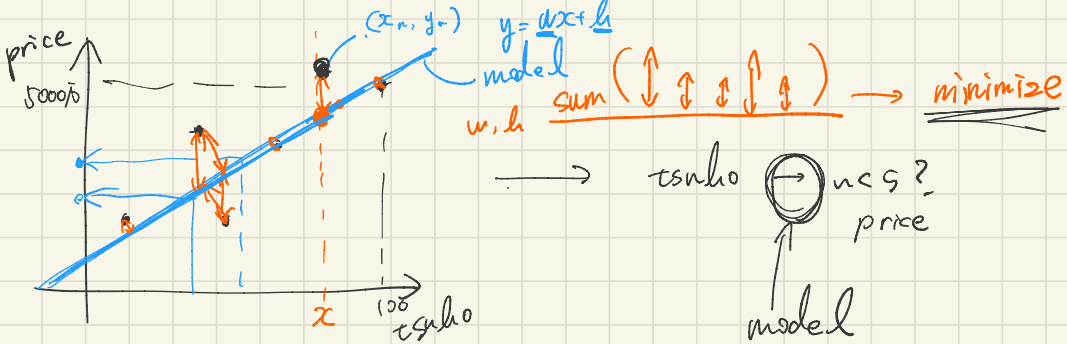


Data : $(1, 5), \cdots, (100, 3000)$

3 ⟶ 15 ?

Data : x : tsubo , y : price (x, y) ⟶ $(x_1, y_1), \cdots, (x_n, y_n)$ 个数: n

model : $y_p = wx + h$ (price = y_p, tsubo = x, w, h : parameter)

最小二乗法

Loss : $L = \sum_{i=1}^{n} (\dot{y}_{p_i} - \dot{y}_i)^2$

(予測 − 値)^2 ⟶ 0 に理想 ⟶ 最小

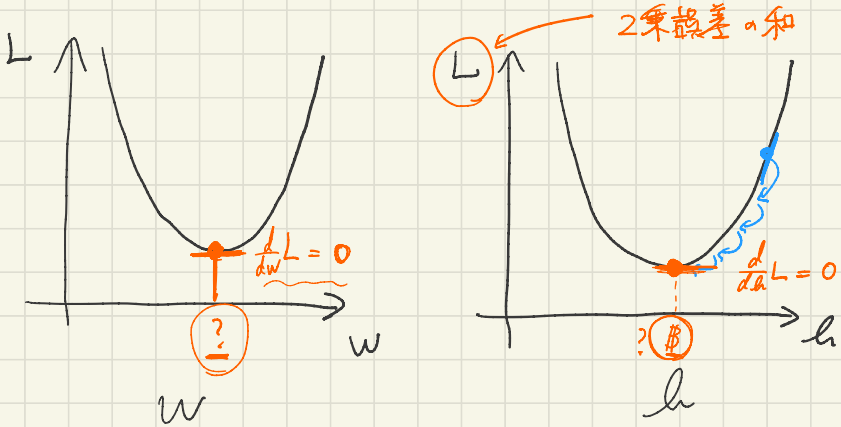
$$= \sum_{n} (wx_i + h - y_i)^2$$

$$= \sum_{n} (w^2 x_i^2 + h^2 + y_i^2 + whx_i + wx_i y_i - hy_i)$$

$$L(w) = \sum_{n} (x_i^2 w^2 + (hx_i + x_i y_i) w + h^2 + y_i^2 - hy_i) \quad 2次$$

$$L(h) = \sum_{n} (h^2 + (wx_i - y_i) h + w^2 x_i^2 + y_i + wx_i y_i) \quad 2次$$

$(y = ax^2 + bx + c)$



$$L = \sum_n \left(\underbrace{wx_i}_{x_i} + \underbrace{b}_{1} - \underbrace{y_i}_{y_i} \right)^2$$

$\left(\bigcirc^2 \to 2 \cdot \bigcirc \cdot \bigcirc$ の時の微次
$x^n \to n x^{n-1} x' \right)$

$$\frac{\partial L}{\partial w} = \sum_n 2 (wx_i + b - y_i) \cdot x_i$$

$$= 2 \sum_n x_i (wx_i + b - y_i) \quad (= 0)$$

$$\sum_n x_i (wx_i + b - y_i) = 0 \quad \cdots > \quad w = ? \quad \cdots (a)$$

$b' = 1$

$$\frac{\partial L}{\partial b} = \sum_n 2 \cdot (wx_i + b - y_i) \cdot 1$$

$$= 2 \sum_n (wx_i + b - y_i) \quad (= 0)$$

$$\sum_n (wx_i + b - y_i) = 0 \quad \cdots > \quad b = ? \quad \cdots (b)$$

(a), (b) より,

$$\begin{cases} \sum_n x_i (w x_i + b - y_i) = 0 \quad \cdots (c) \\ \sum_n (w x_i + b - y_i) = 0 \quad \cdots (d) \end{cases} \rightarrow \begin{cases} w = ? \\ b = ? \end{cases}$$

(d) -

$$\sum_n w x_i + \sum_n b - \sum_n y_i = 0$$

$$\Leftrightarrow w \sum_n x_i + nb - \sum_n y_i = 0$$

$$\Leftrightarrow nb = \sum_n y_i - w \sum_n x_i$$

$$\Leftrightarrow b = \frac{1}{n} \left(\sum_n y_i - w \sum_n x_i \right)$$

$$= \frac{1}{n} \sum_n (y_i - w x_i) \quad \cdots (e)$$

$\left(\sum_n = \sum_{i=1}^{n} \right)$

$$\sum_{i=1}^{n} a_i = a_1 + a_2 + \cdots + a_n$$

$$\sum_{i=1}^{n} c = \underbrace{c + c + \cdots + c}_{n\text{개}} = nc$$

(e) 를 (c) 에 대입하면,

$$\sum_n x_i (w x_i + b - y_i) = 0$$

$$\Leftrightarrow \sum_n w x_i^2 + \sum_n b x_i - \sum_n x_i y_i = 0$$

$$\Leftrightarrow \sum_n w x_i^2 + \sum_n x_i \cdot \left\{ \frac{1}{n} \sum_n (y_i - w x_i) \right\} - \sum_n x_i y_i = 0 \rightarrow w = ?$$

(f)

$$(f) = \frac{1}{n} \sum_n x_i \left(\sum_n y_i - \sum_n w x_i \right)$$

$$= \frac{1}{n} \sum_n x_i \sum_n y_i - \frac{1}{n} w \sum_n x_i \sum_n x_i$$

$$= \frac{1}{n}\sum_n x_i \sum_n y_i - \frac{1}{n} w \left(\sum_n x_i\right)^2$$

(c) $\Leftrightarrow$ $\sum_n w x_i^2 + \frac{1}{n}\sum_n x_i \cdot \sum_n y_i - \frac{1}{n} w \left(\sum_n x_i\right)^2 - \sum_n x_i y_i = 0$

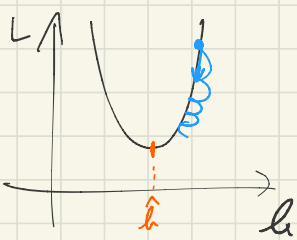
$\Leftrightarrow$ $w\left(\sum_n x_i^2 - \frac{1}{n}\left(\sum_n x_i\right)^2\right) + \frac{1}{n}\sum_n x_i \cdot \sum_n y_i - \sum_n x_i y_i = 0$

$\Leftrightarrow$

$$\hat{w} = \frac{\sum_n x_i y_i - \frac{1}{n}\sum_n x_i \cdot \sum_n y_i}{\sum_n x_i^2 - \frac{1}{n}\left(\sum_n x_i\right)^2}$$

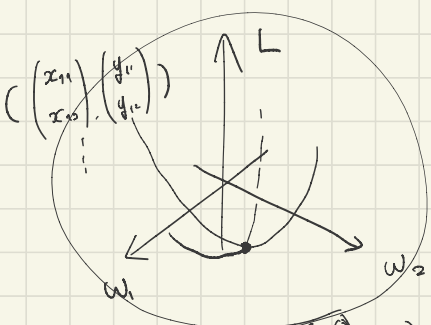
全部定数 解ける.

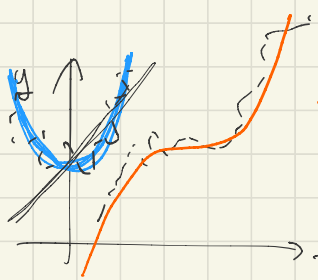
$$\hat{b} = \frac{1}{n}\sum_n (y_i - w x_i)$$



$y = \hat{w}x + \hat{b}$

最適なモデル.

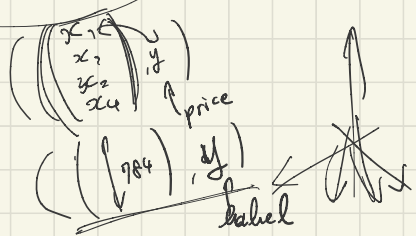




$y = w_1 x^3 + w_2 x^2 + w_3 x + b$

model

$y = w_1 x^2 + w_2 x + b$



$y = wx + b$