How To Write Fast Numerical Code

Optimal Binary Search Trees

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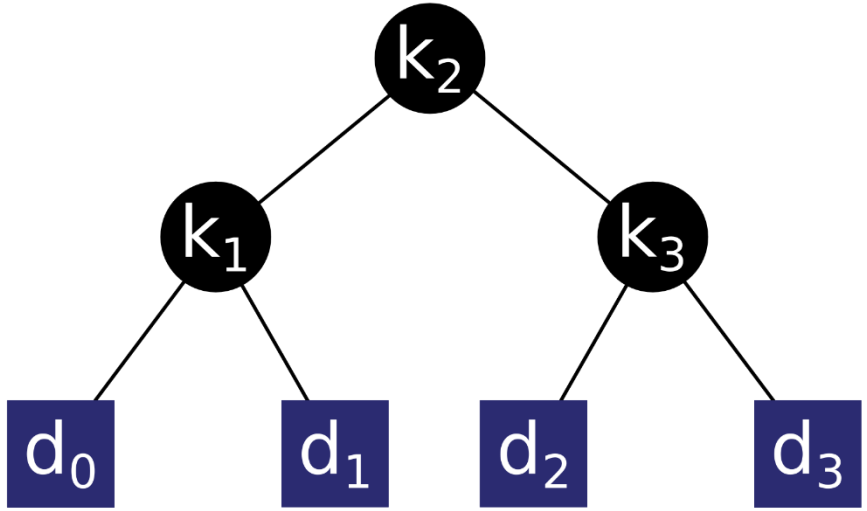
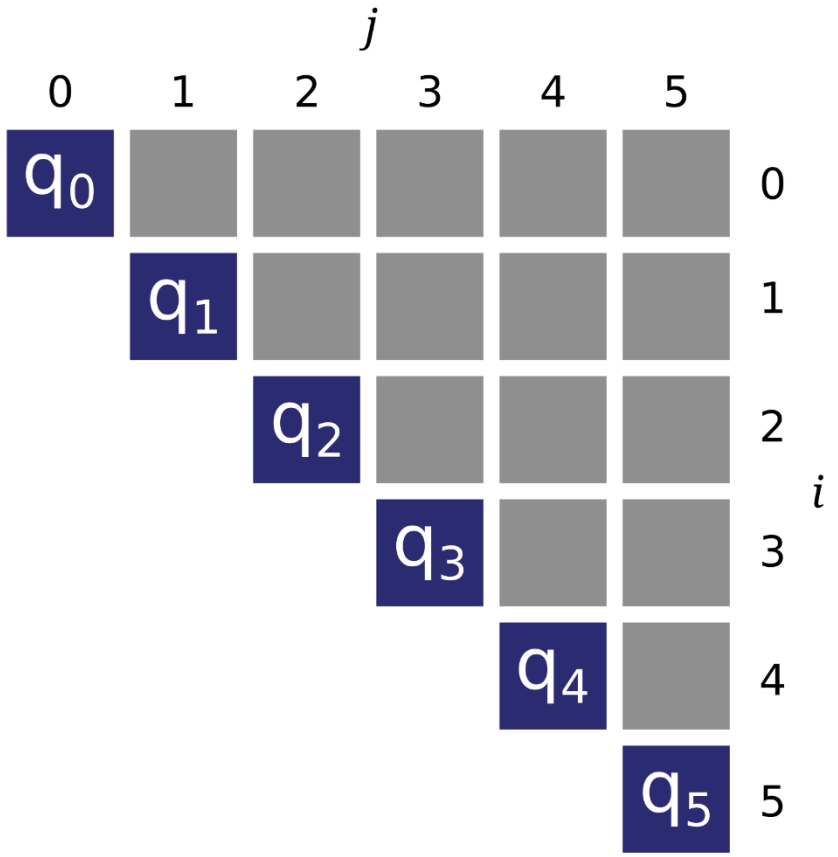
Problem Description

- Given:
  - Key Weights $p_1, \dots, p_n$
  - Dummy Weights $q_0, \dots, q_n$
- Objective:
  - Minimize Expected Lookup Cost
  - $\sum d(k_i) \cdot p_i + \sum d(d_j) \cdot q_j$



Algorithm: Dynamic Programming



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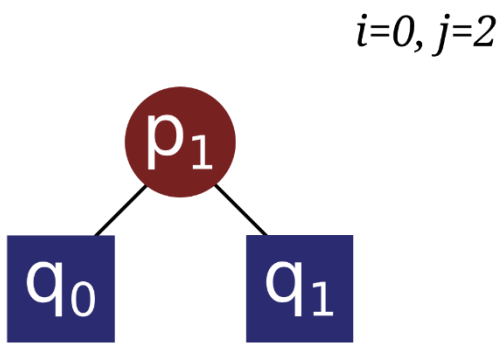

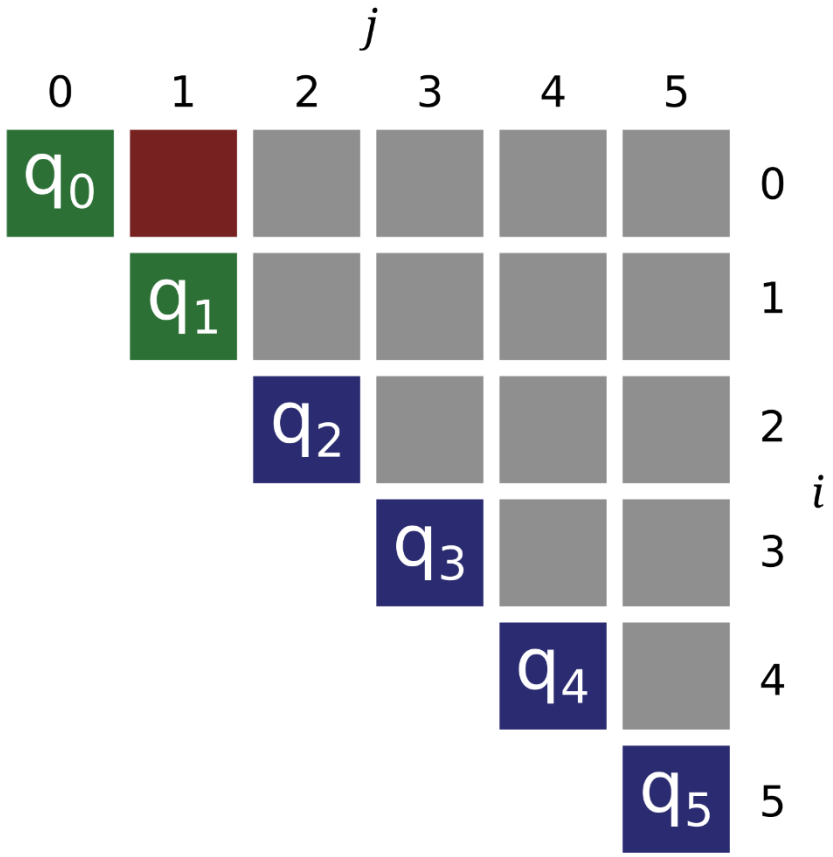

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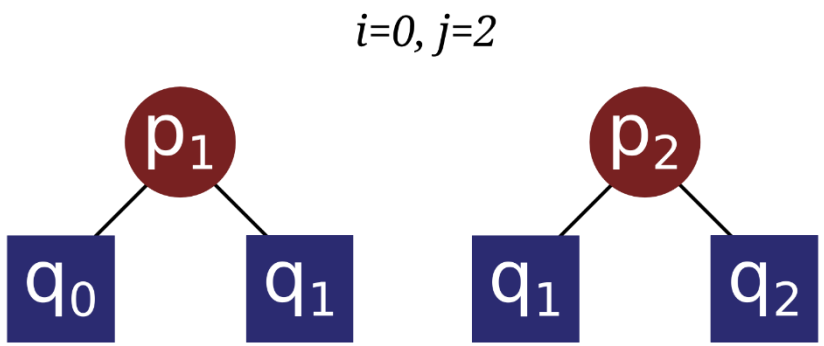
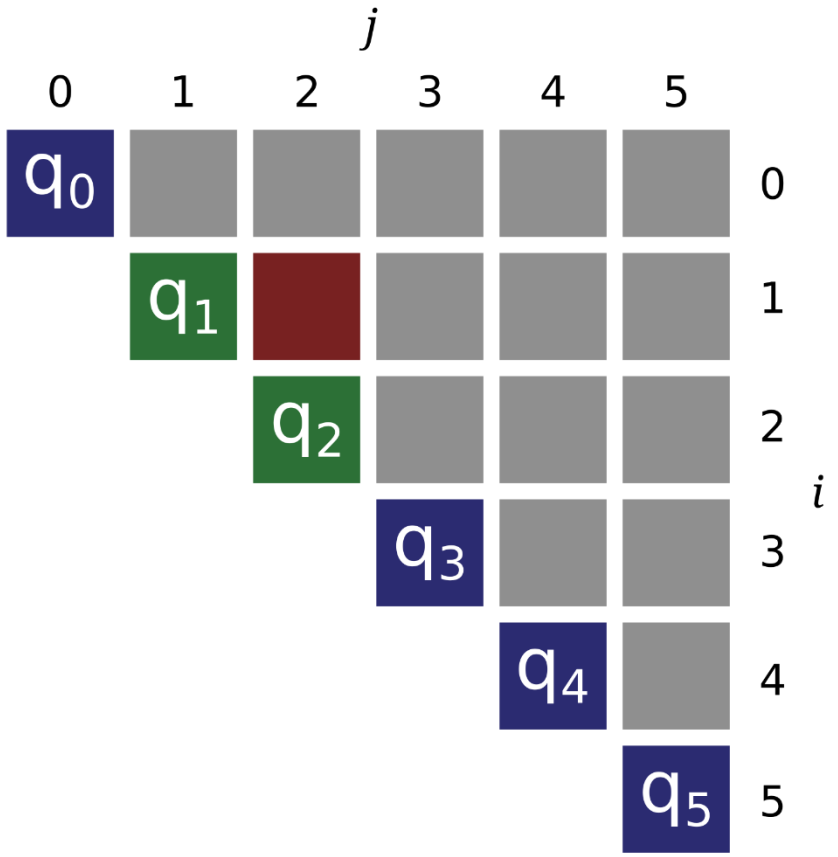
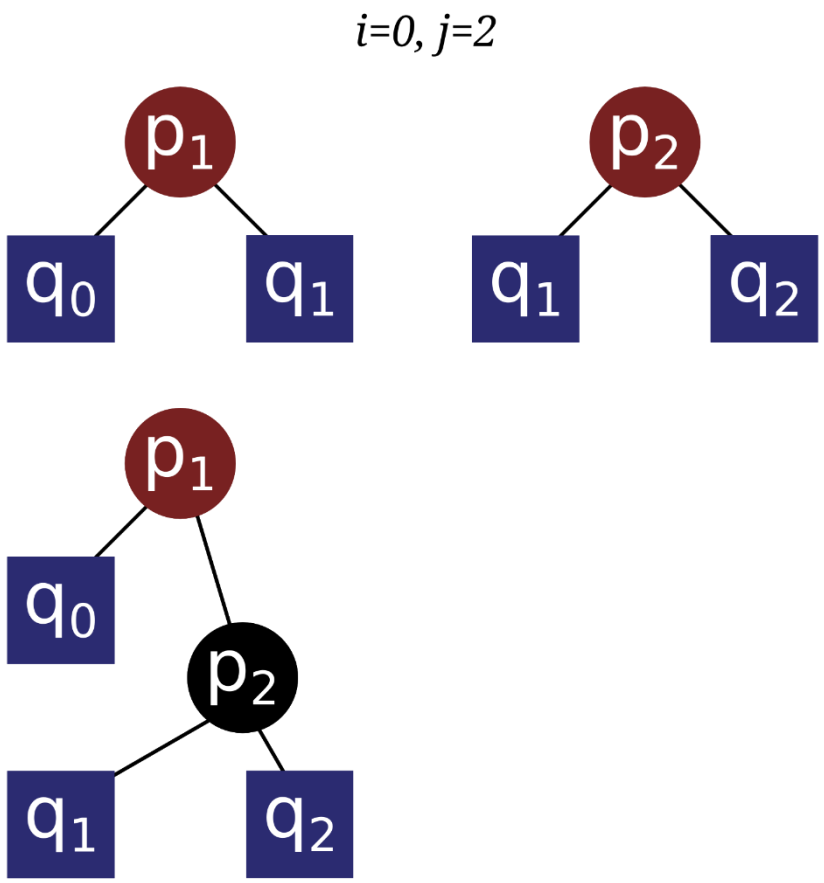
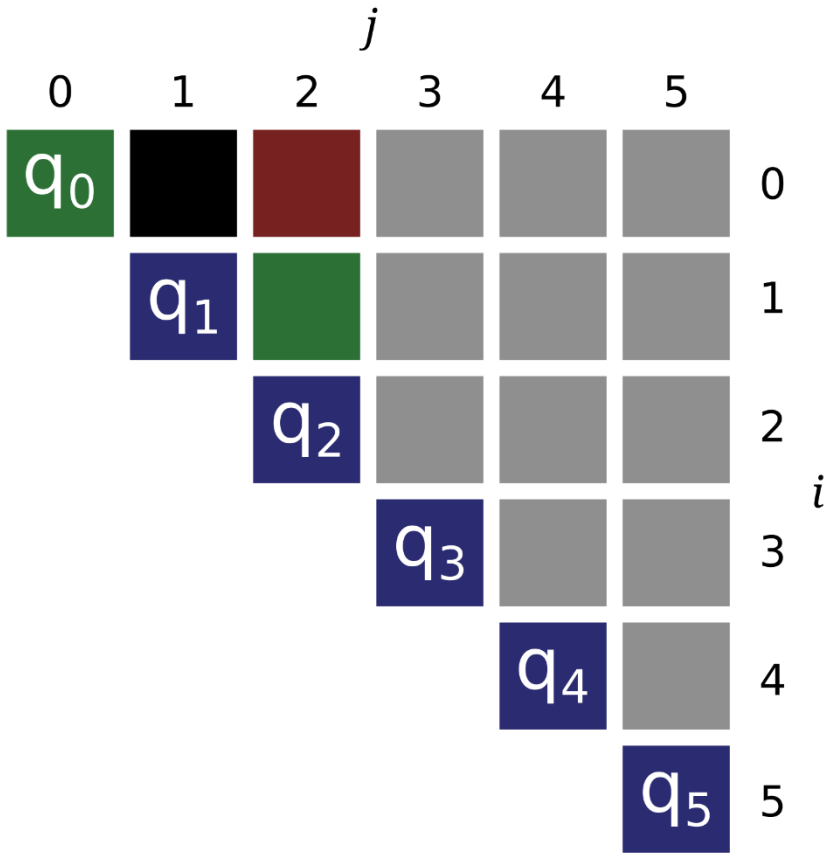


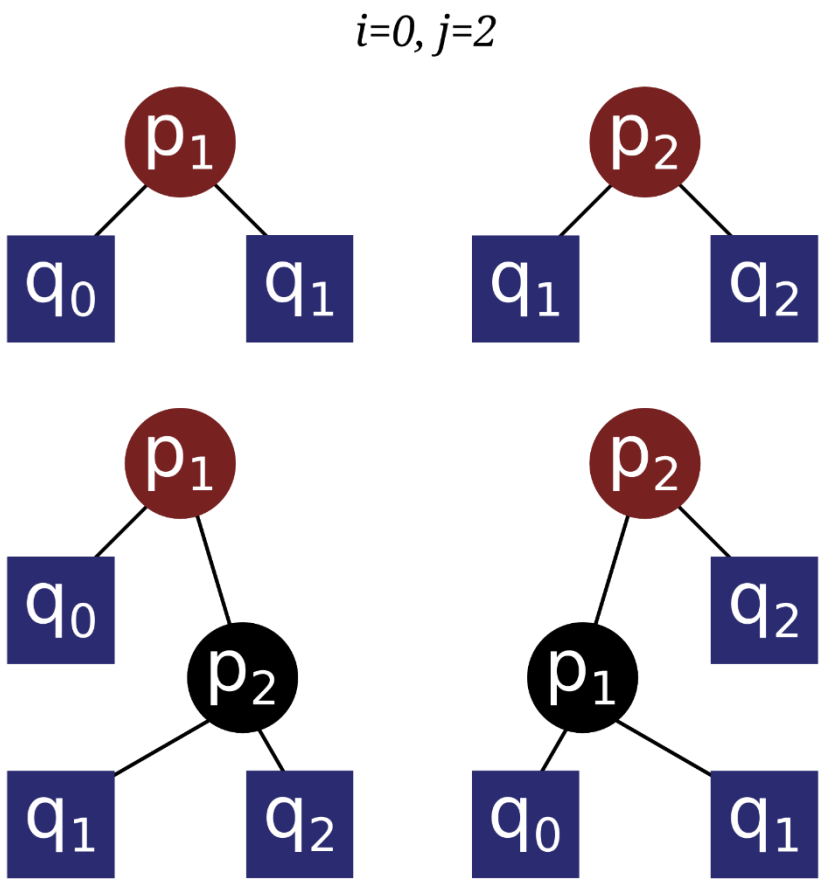


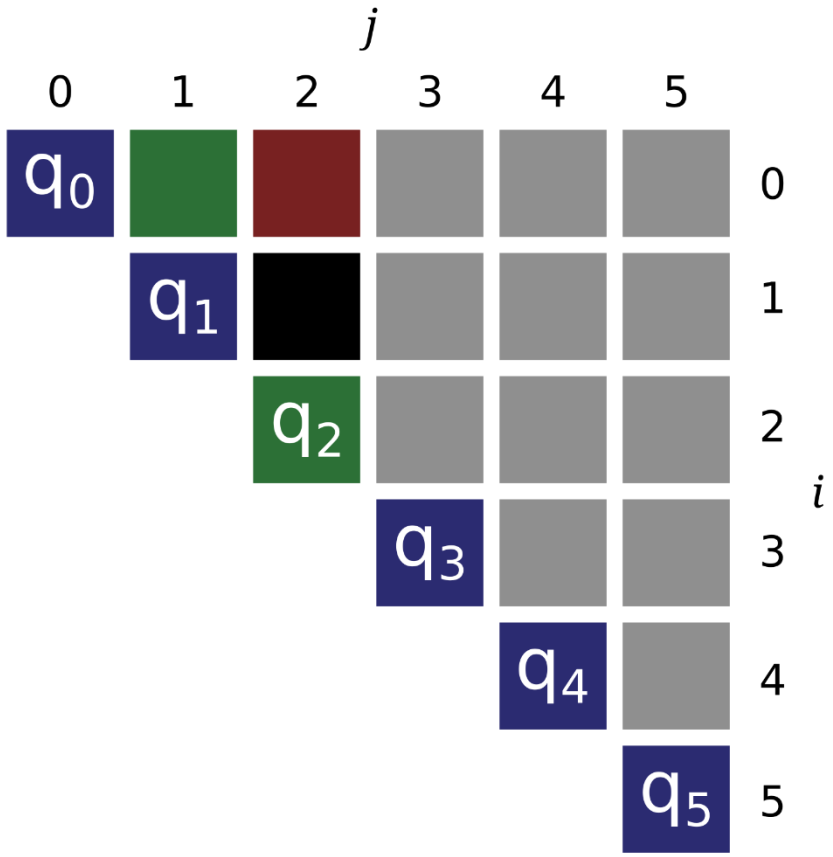
Algorithm: Dynamic Programming



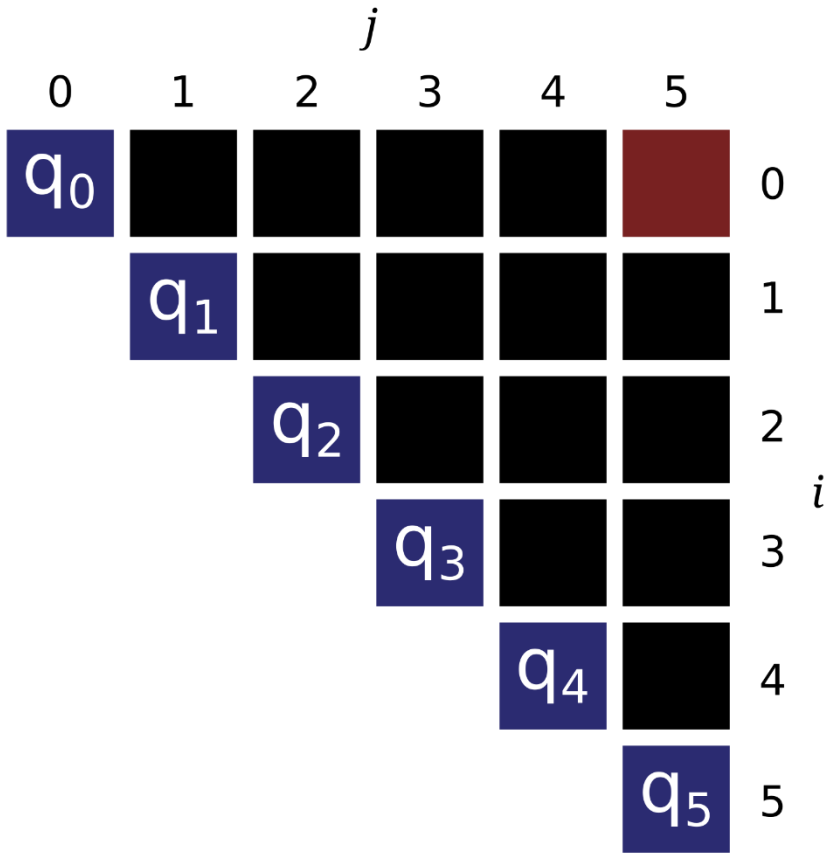


Algorithm: Dynamic Programming



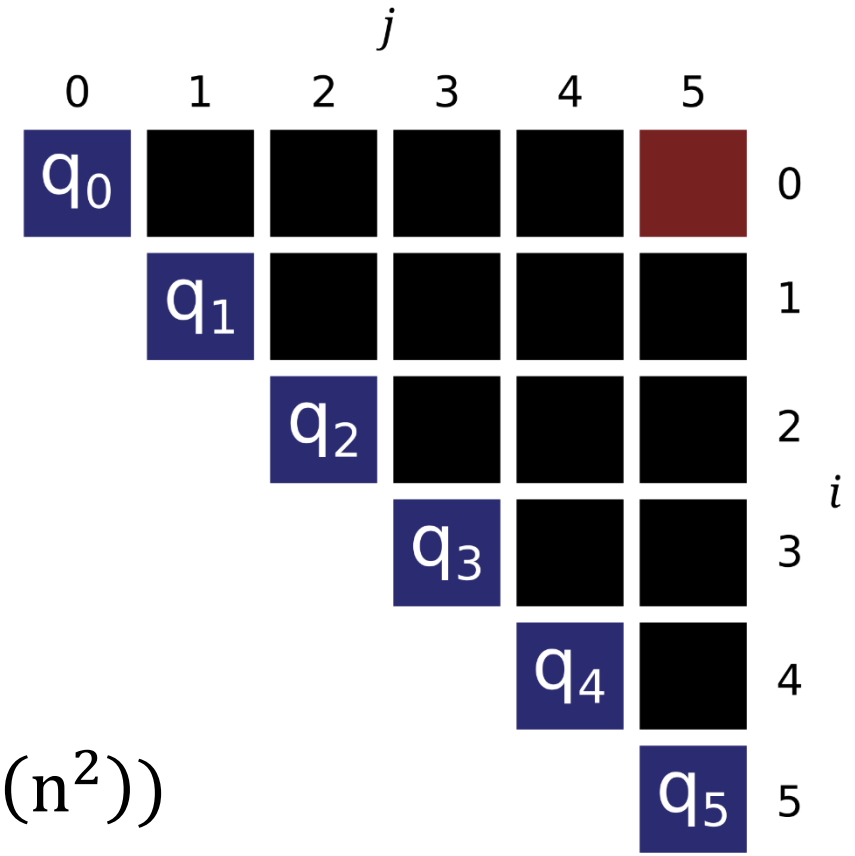


Algorithm: Dynamic Programming



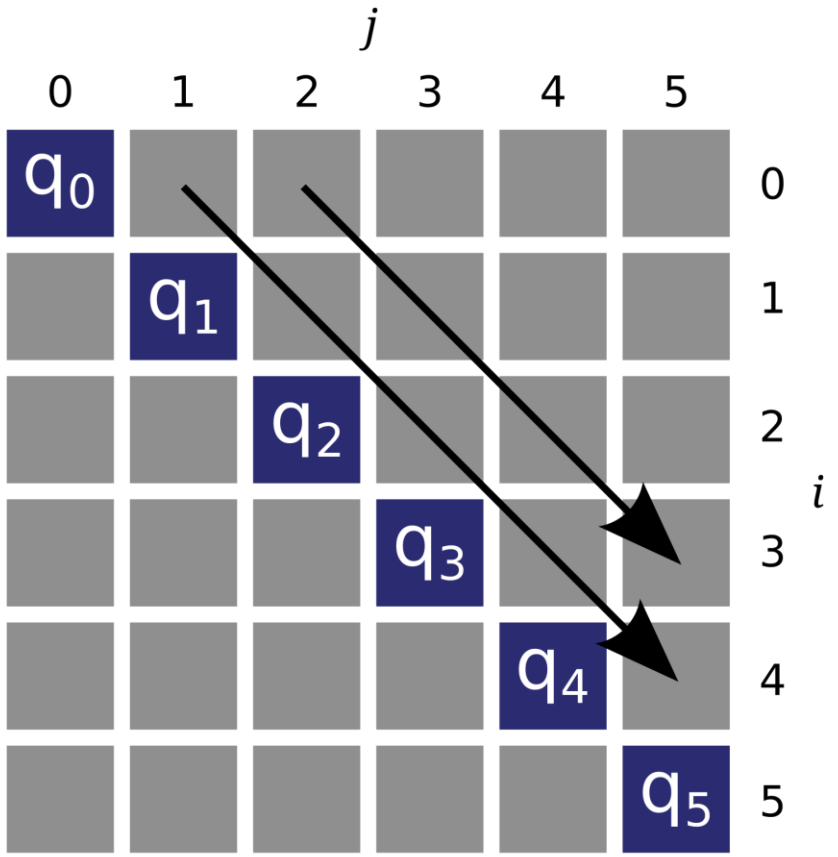
Algorithm: Dynamic Programming

- per entry
  - h additions
  - h comparisions
- $\mathrm{O}(n^2)$ entries



- $\mathrm{C}(\mathrm{add}, \mathrm{cmp}) = \mathrm{C}(\frac{1}{3}n^3 + \mathrm{O}(\mathrm{n}^2), \frac{1}{6}n^3 + \mathrm{O}(\mathrm{n}^2))$

Reference Implementation



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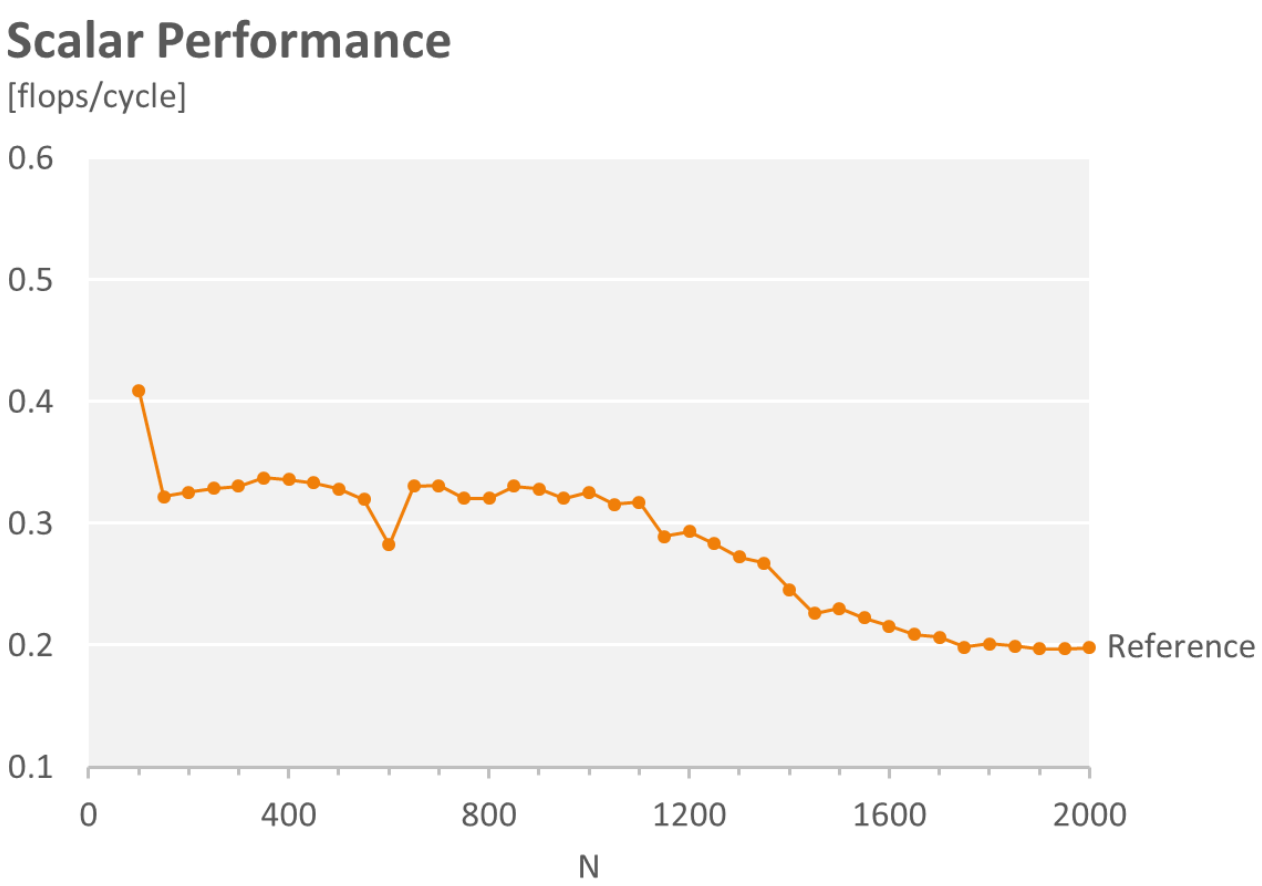

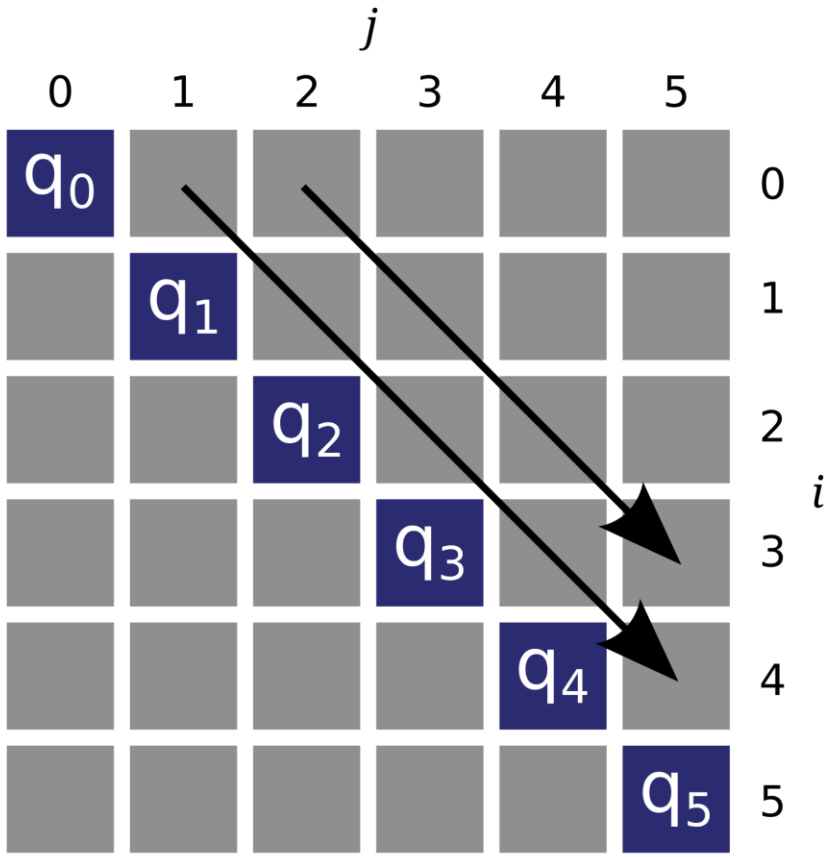

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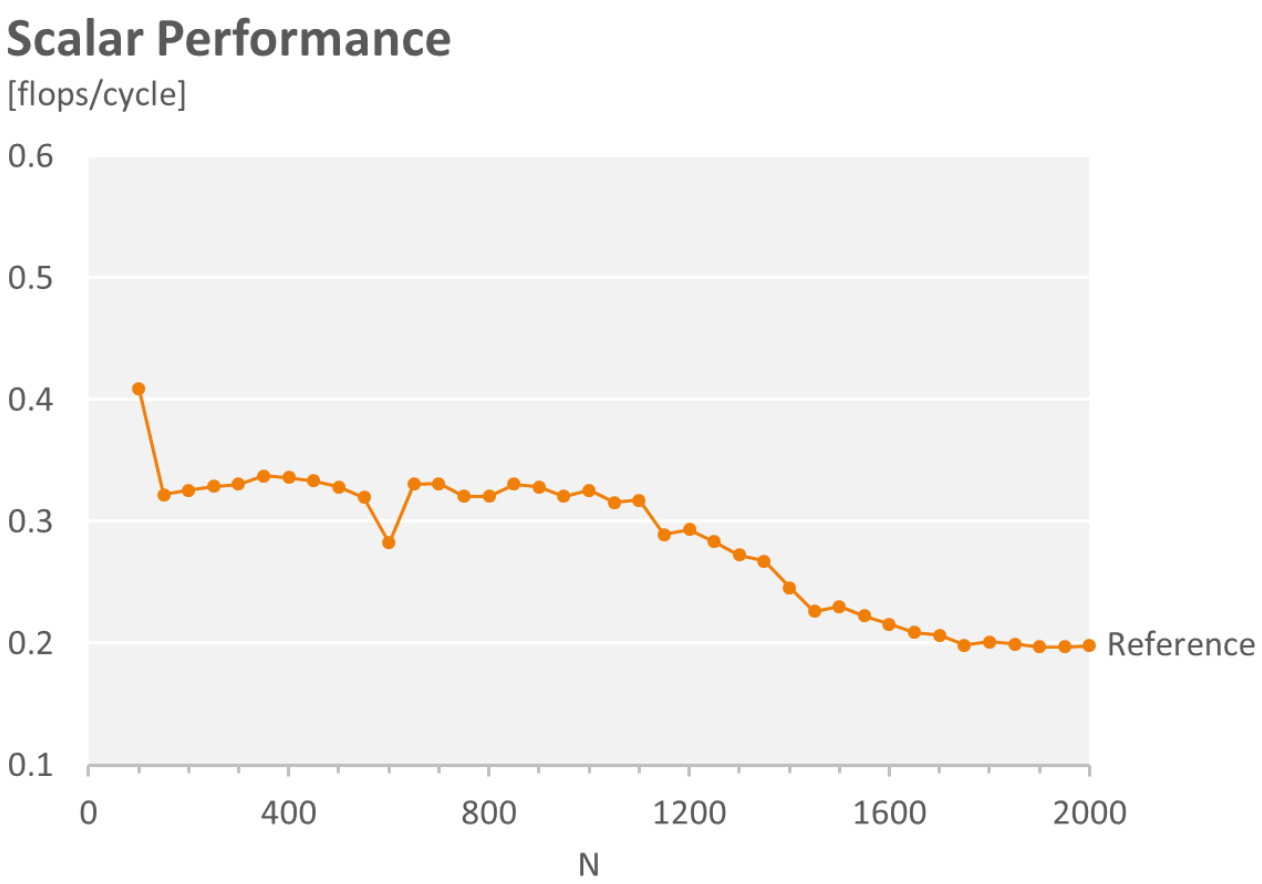

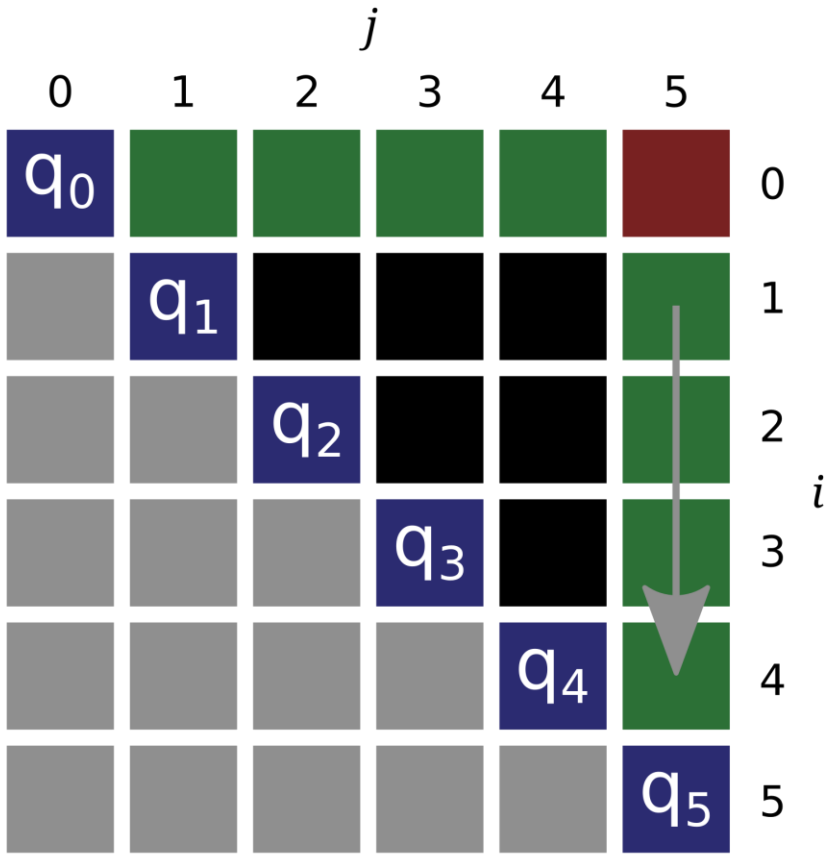

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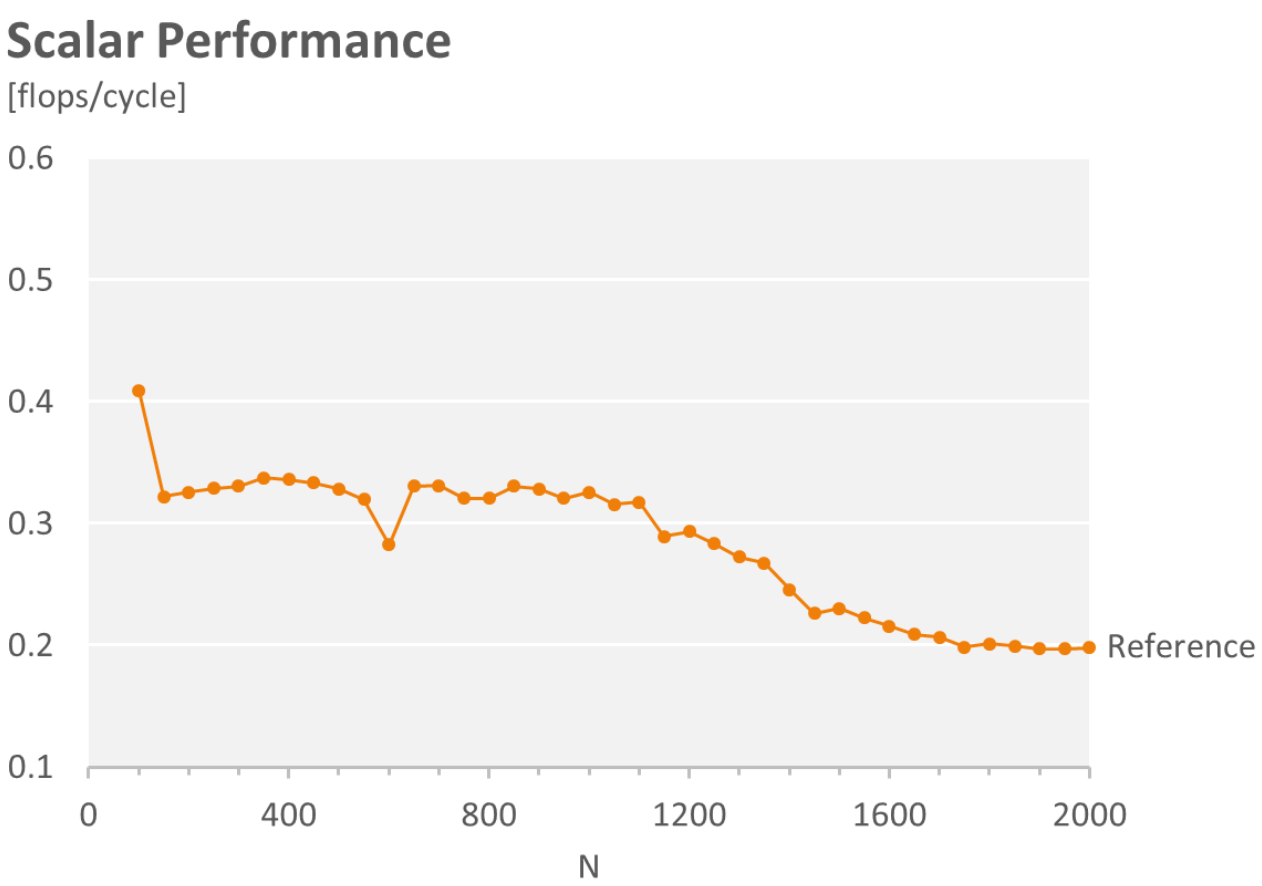
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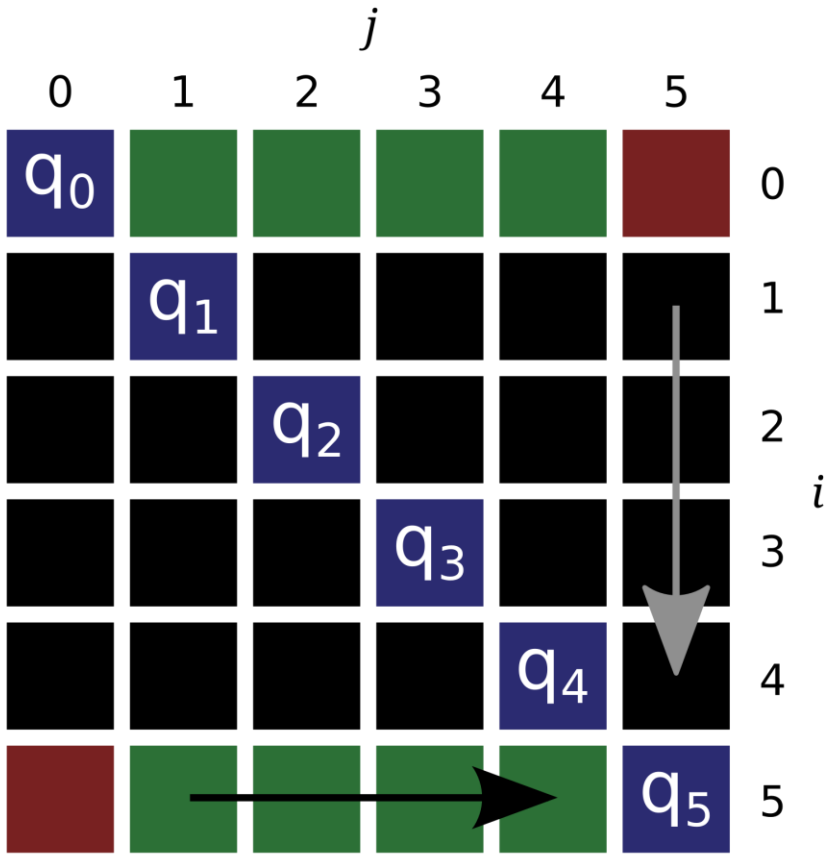
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Transposed


Scalar Performance
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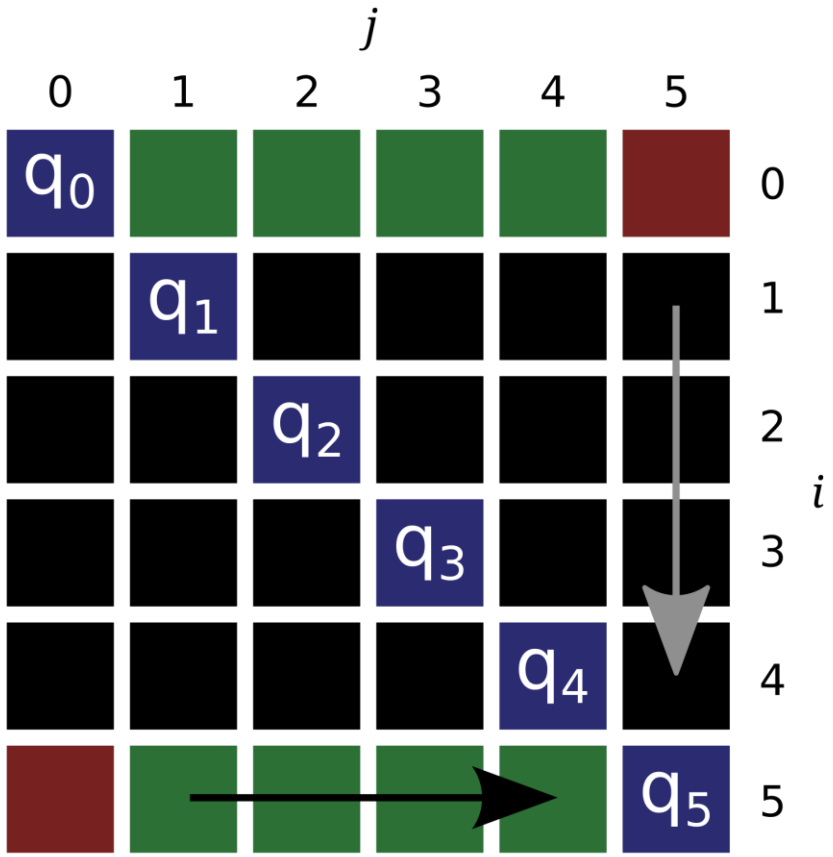
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Bottom Up


Scalar Performance
[flops/cycle]
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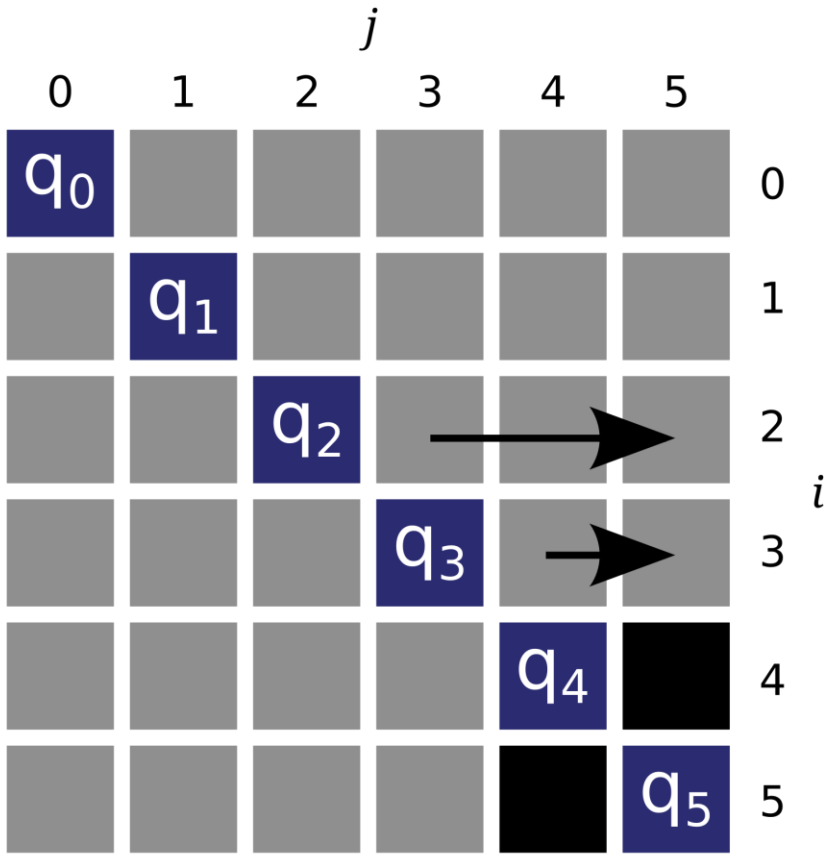
Reference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Bottom Up


Scalar Performance
[flops/cycle]
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Bottom Up
Transposed
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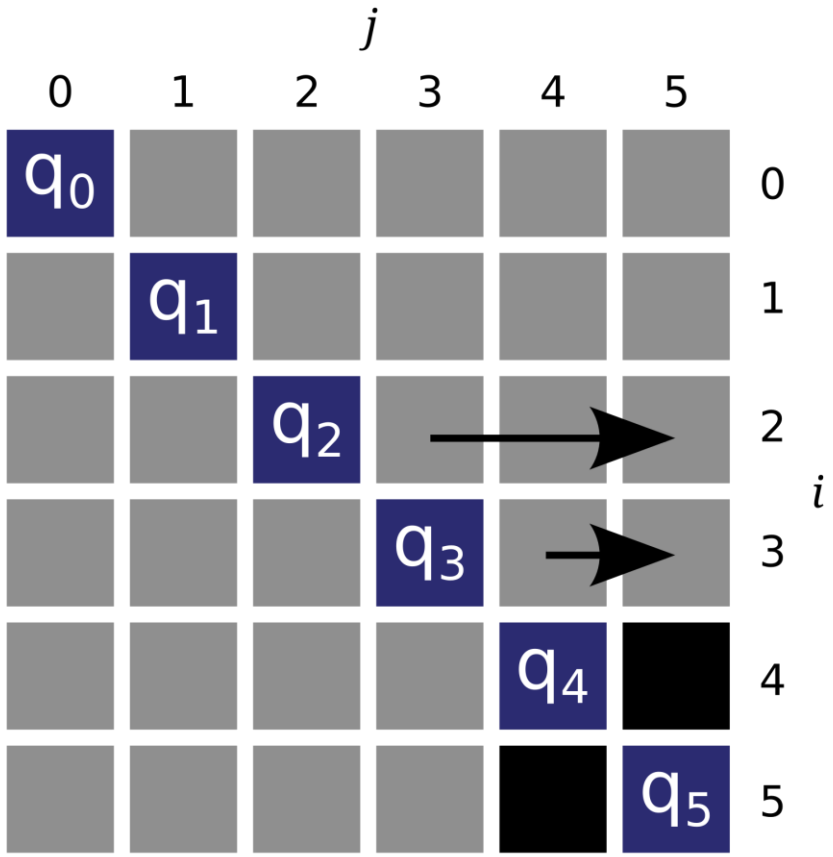
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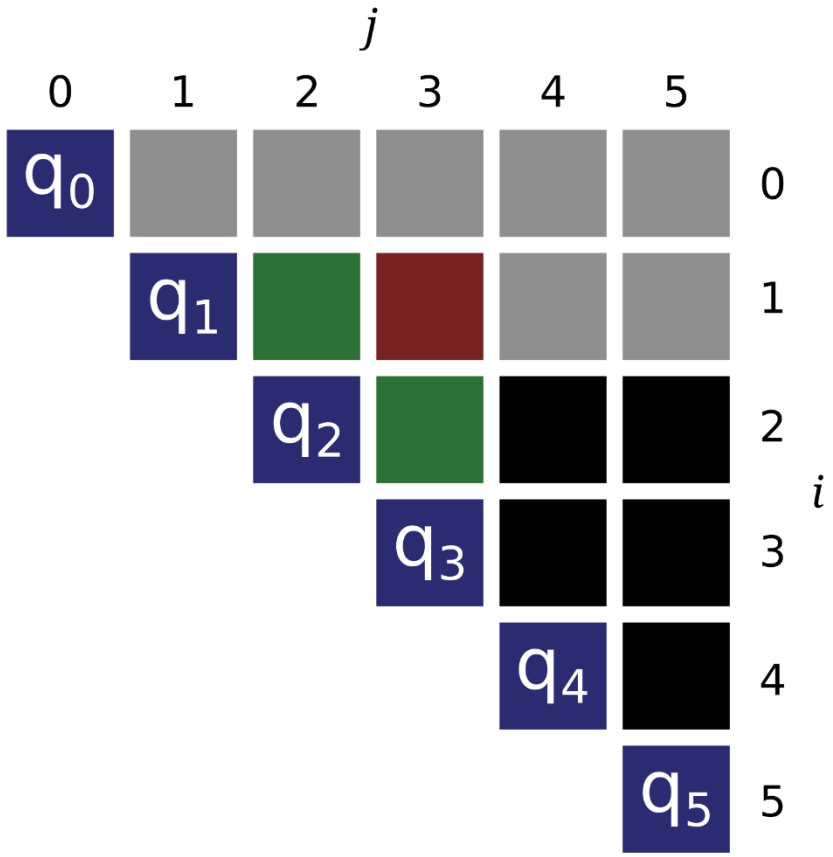
Swap Loops: Triangle



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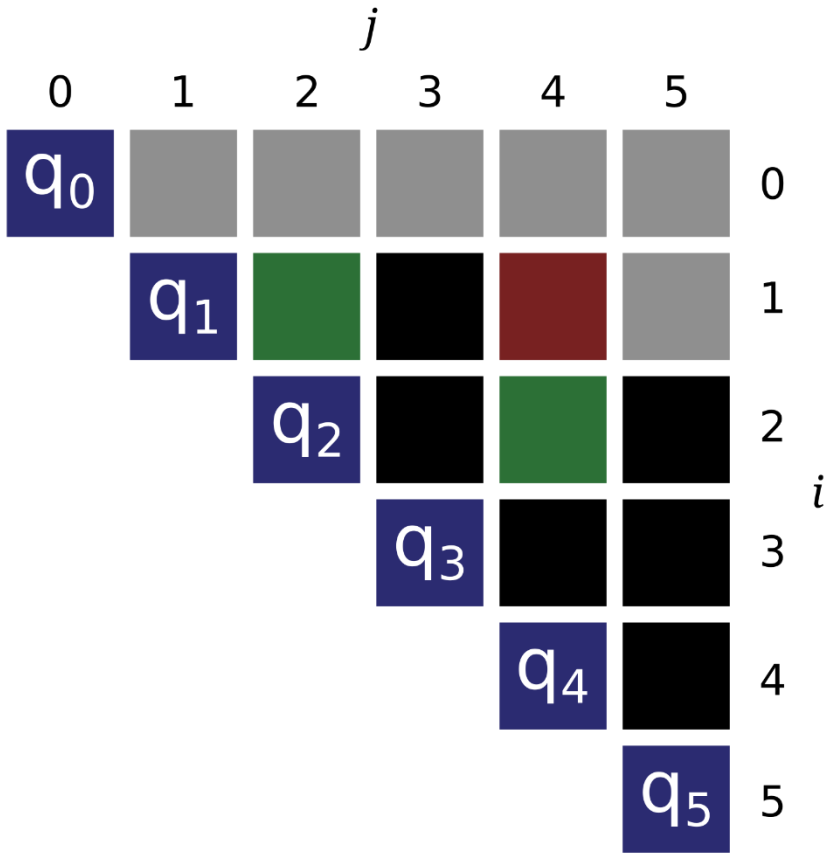

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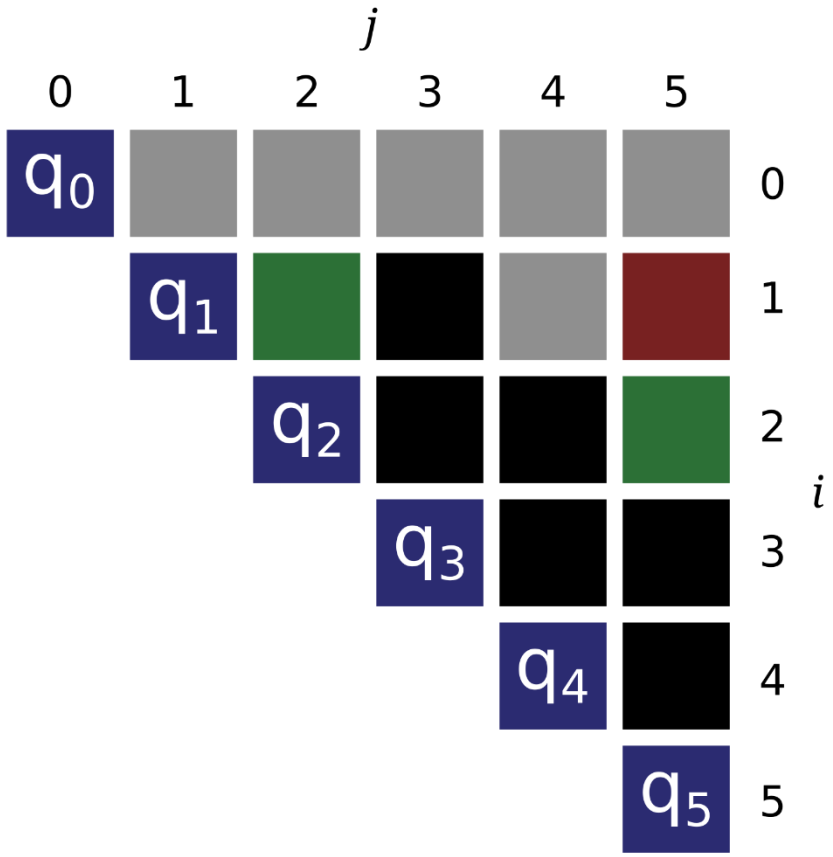
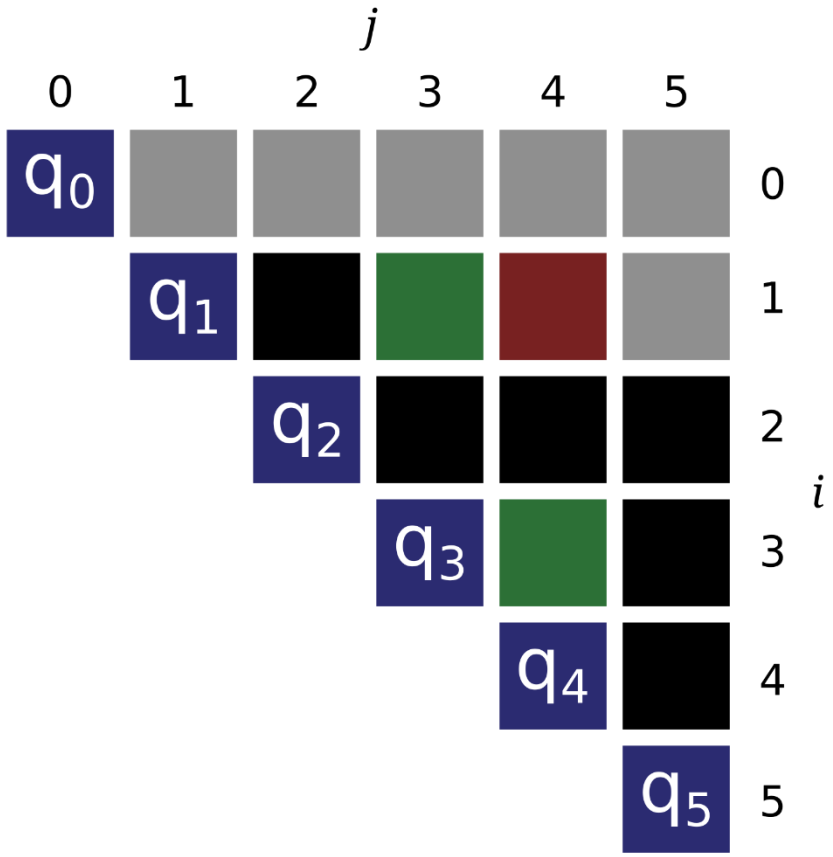


Swap Loops: Triangle



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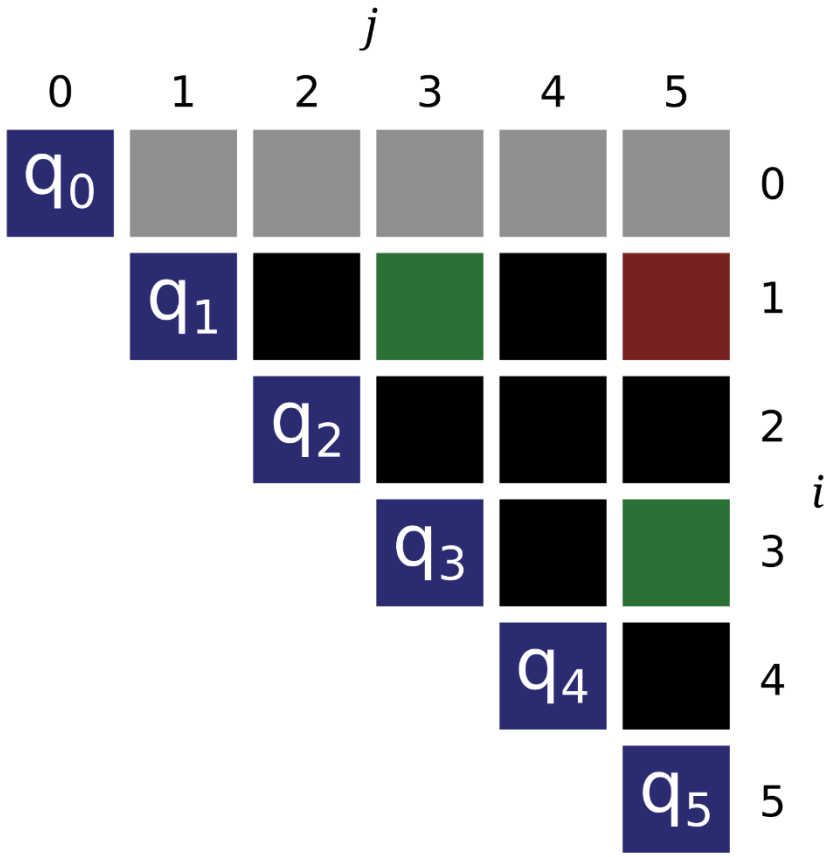

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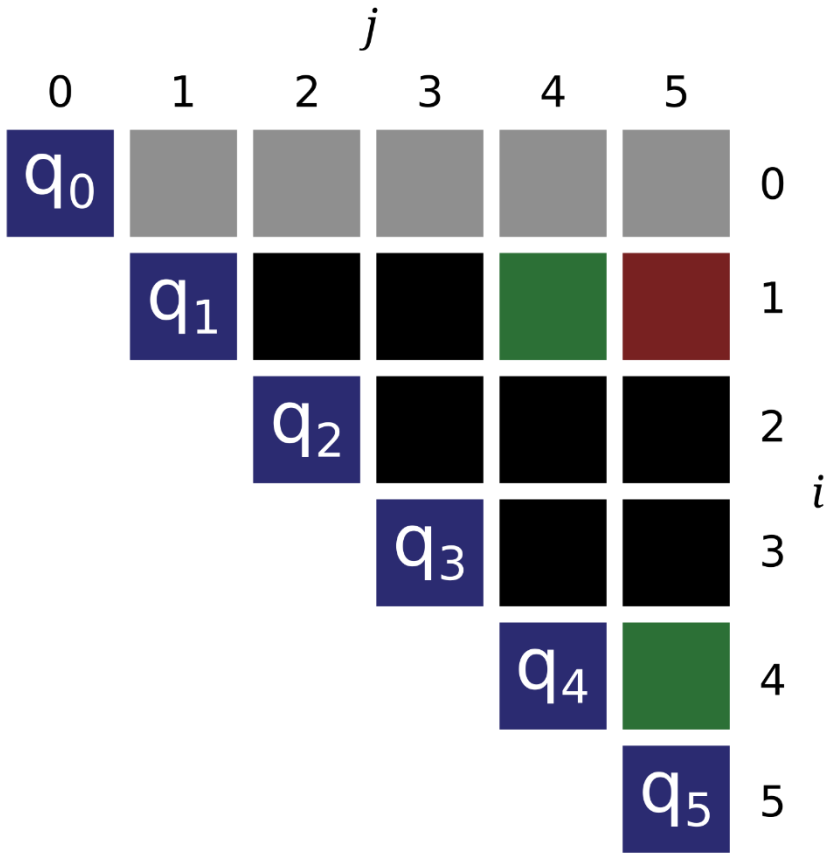

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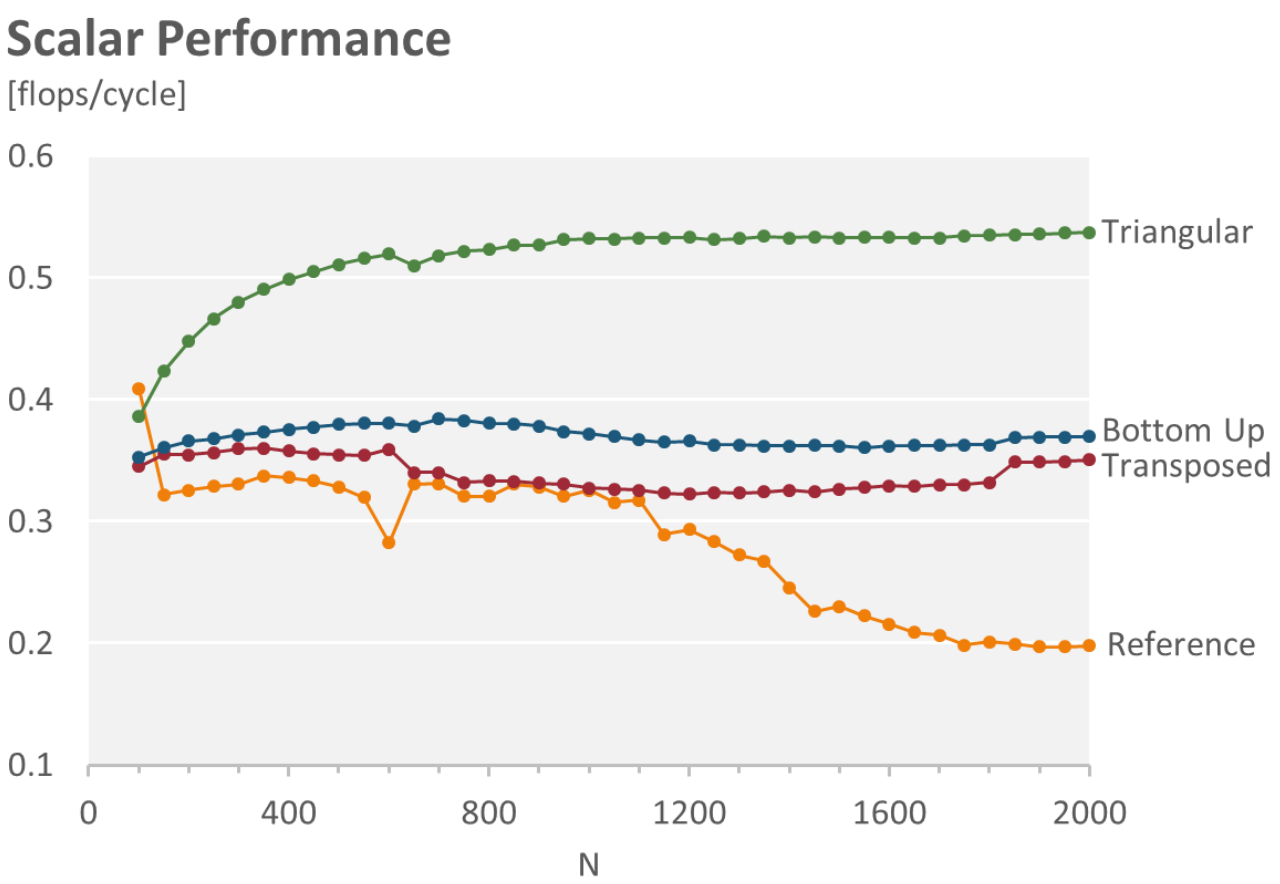

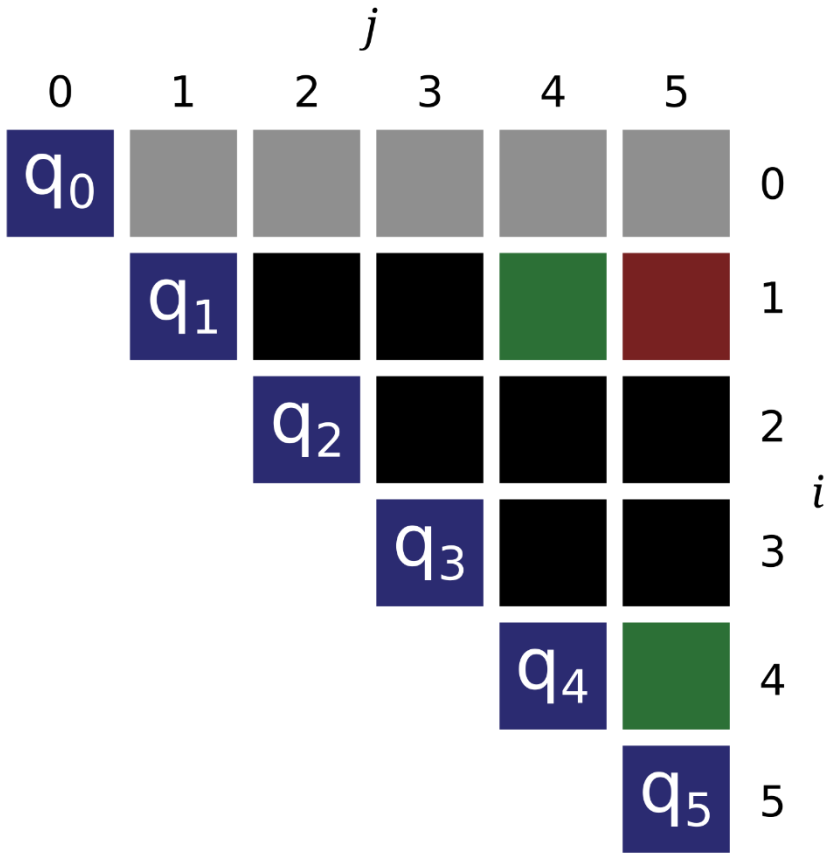

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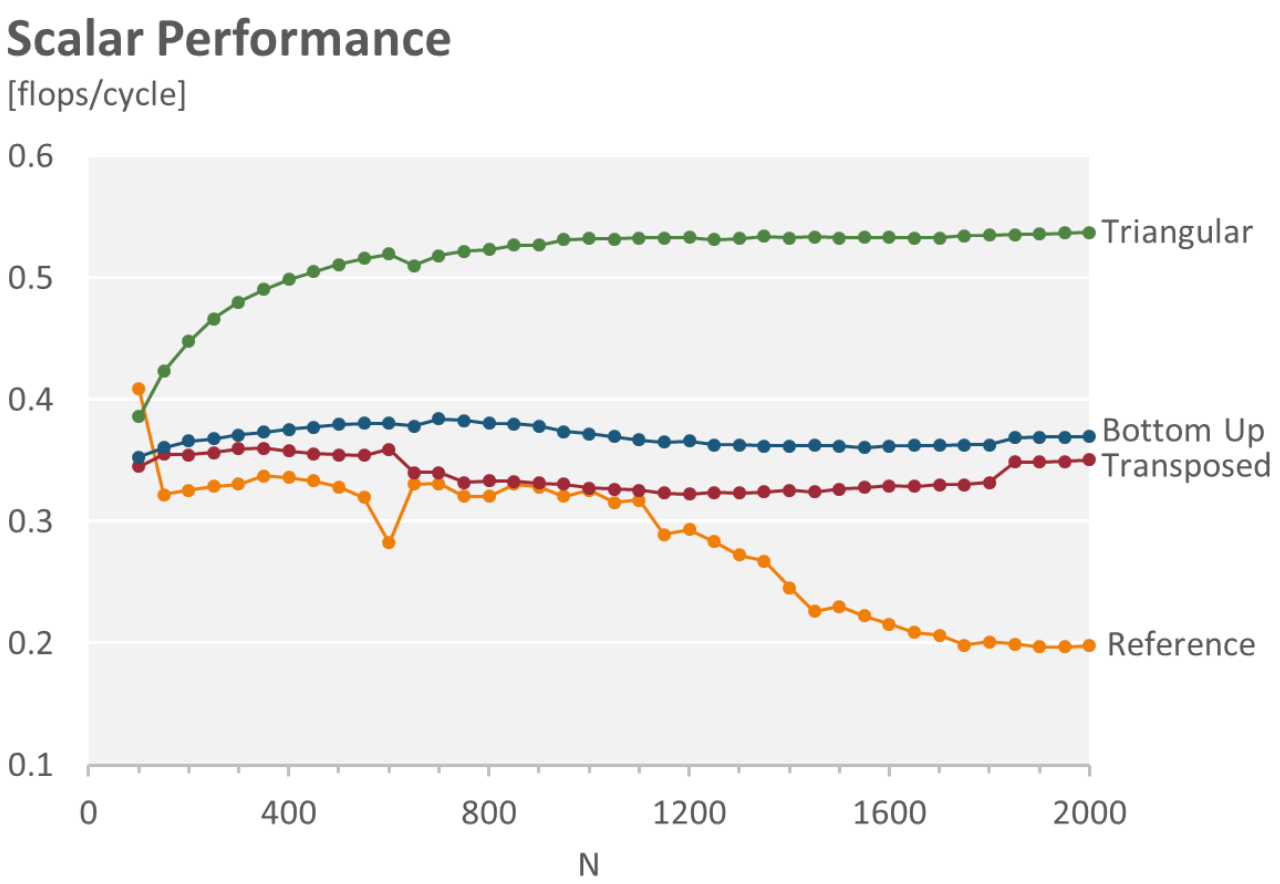
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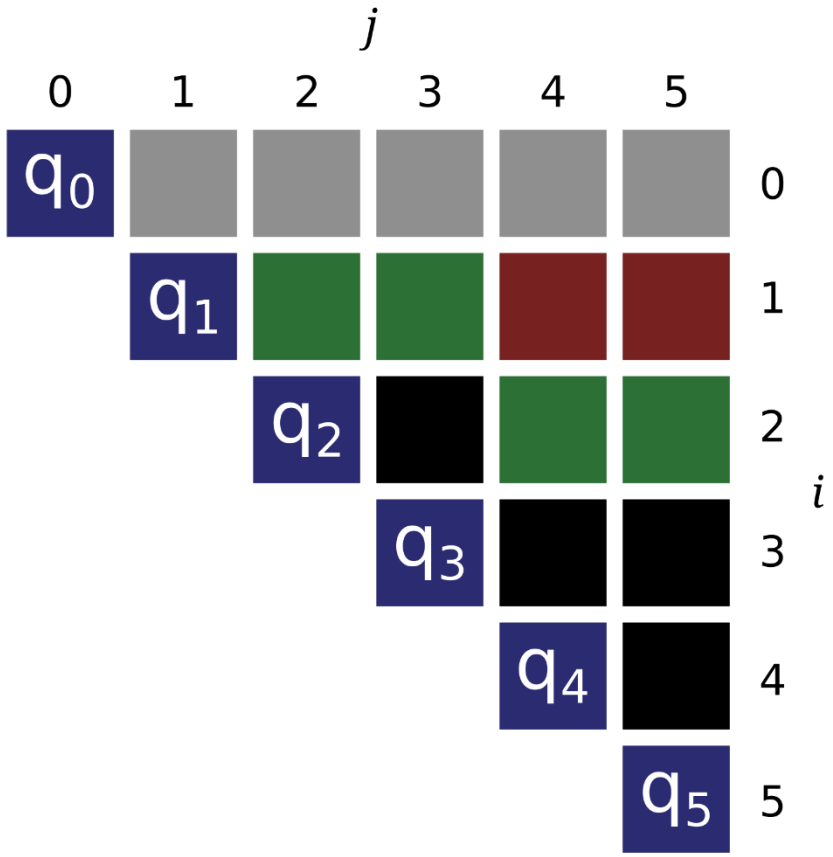
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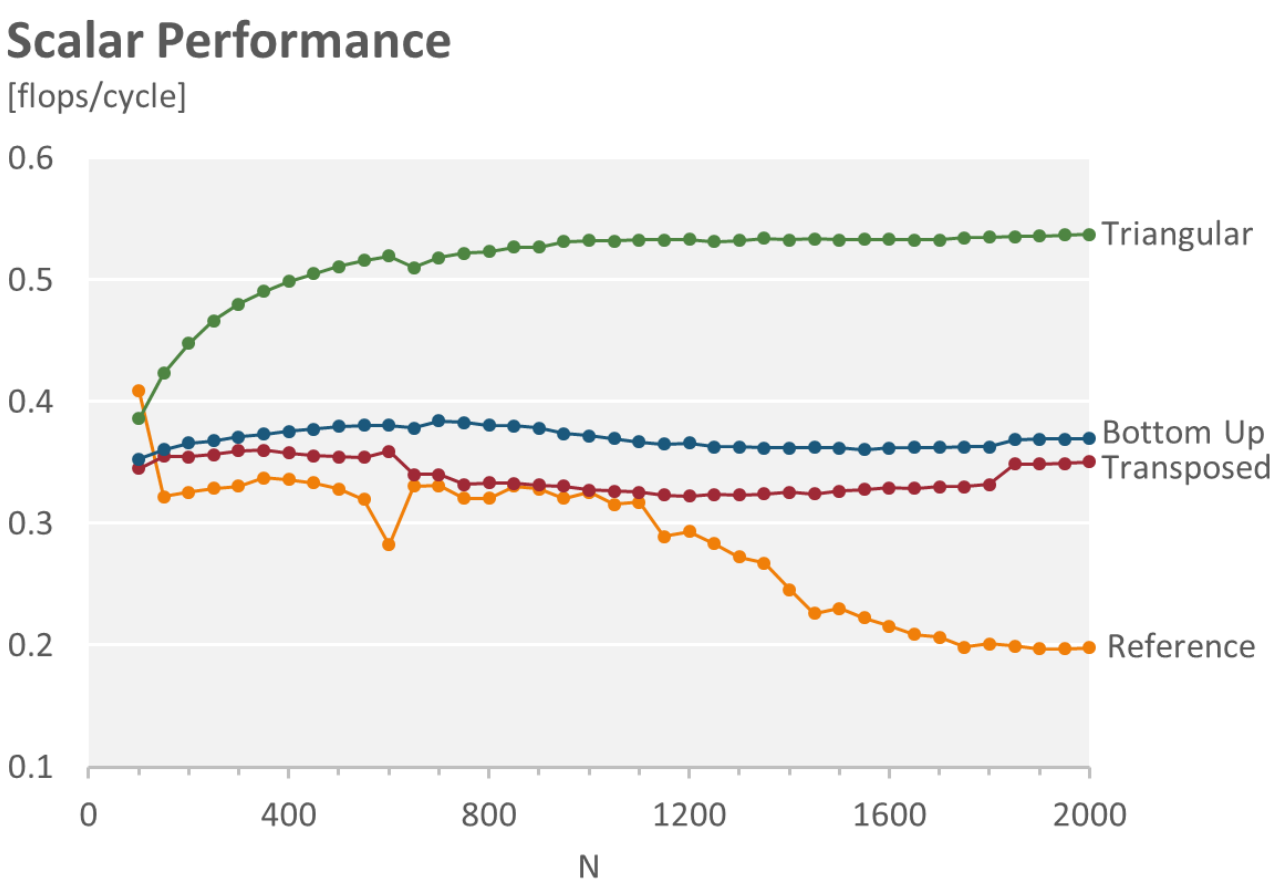
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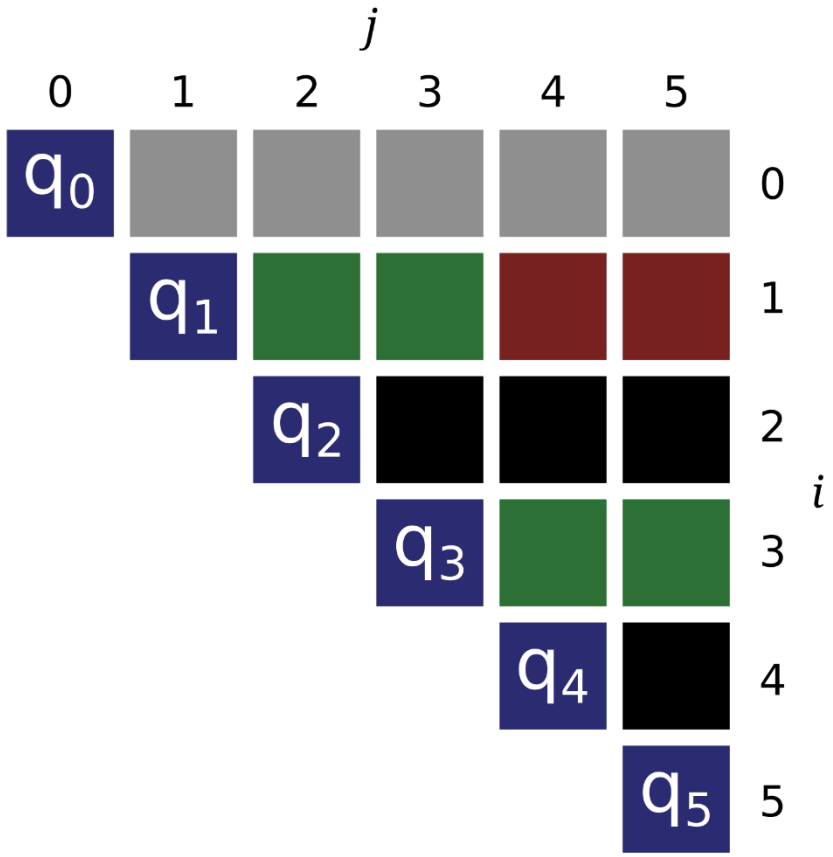
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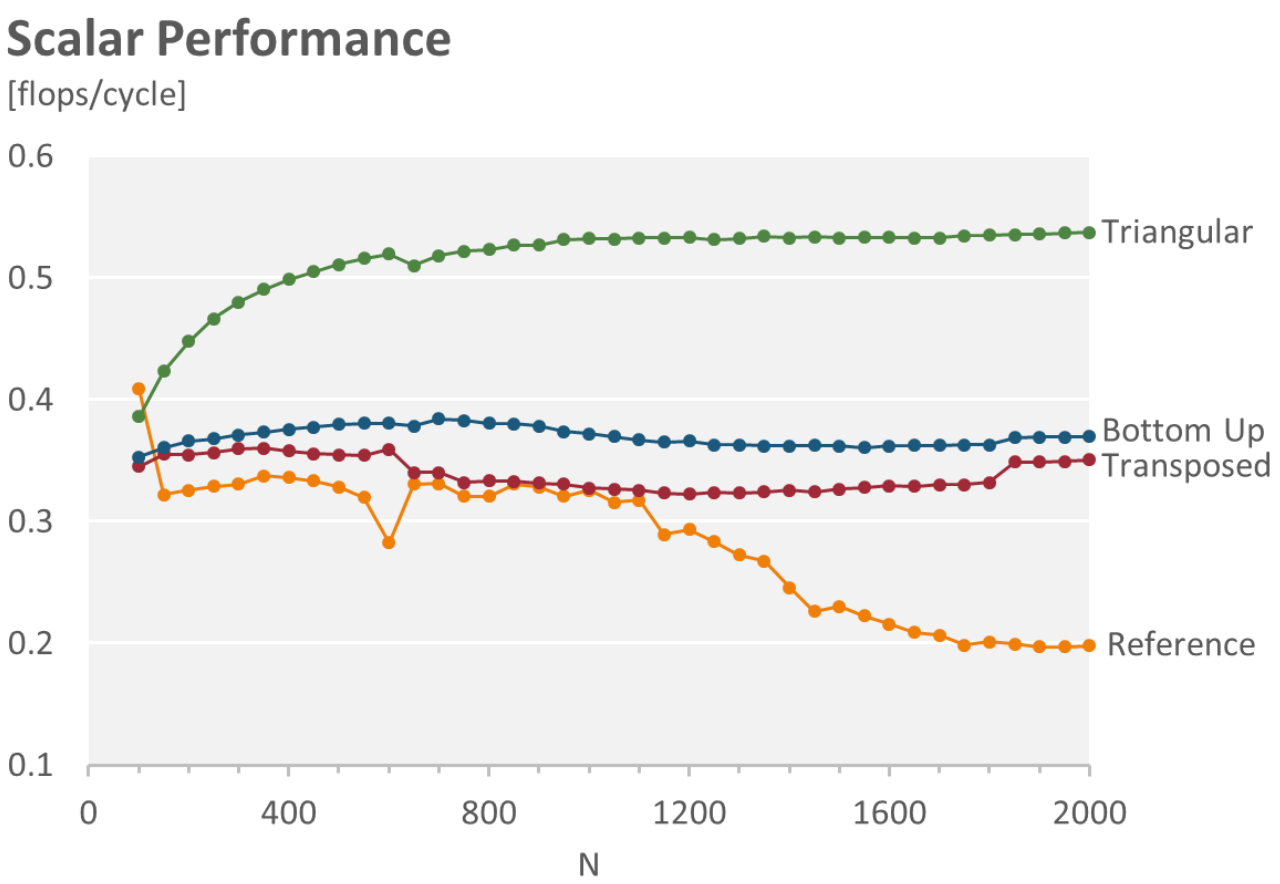

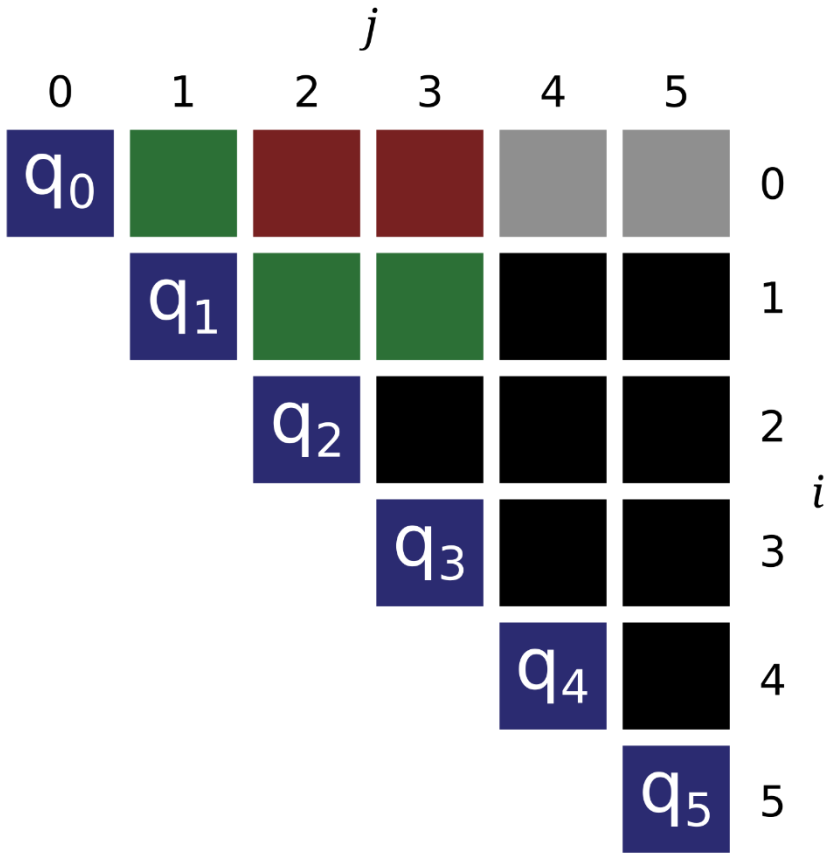

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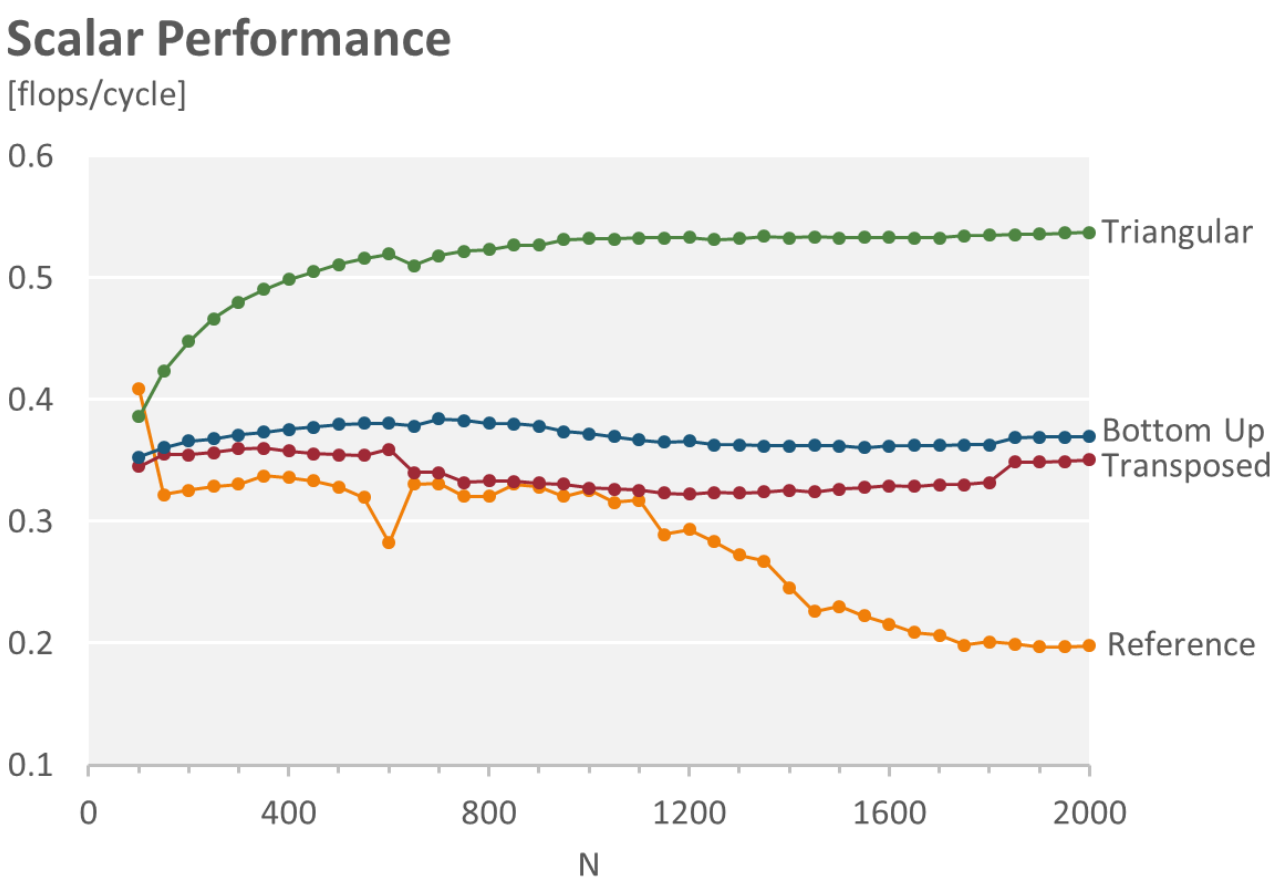

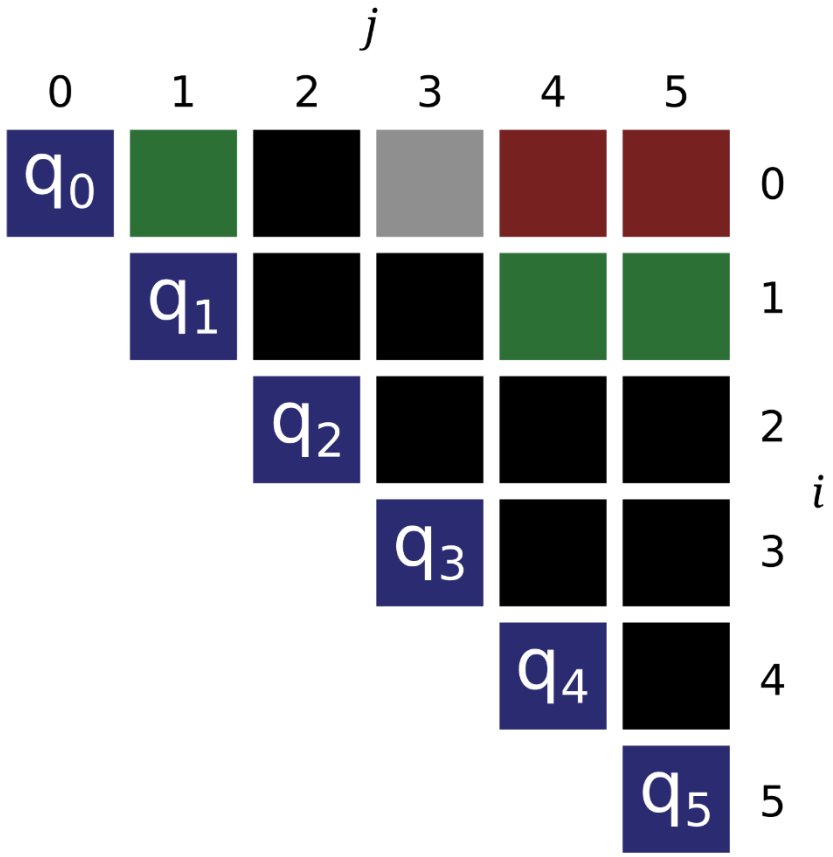

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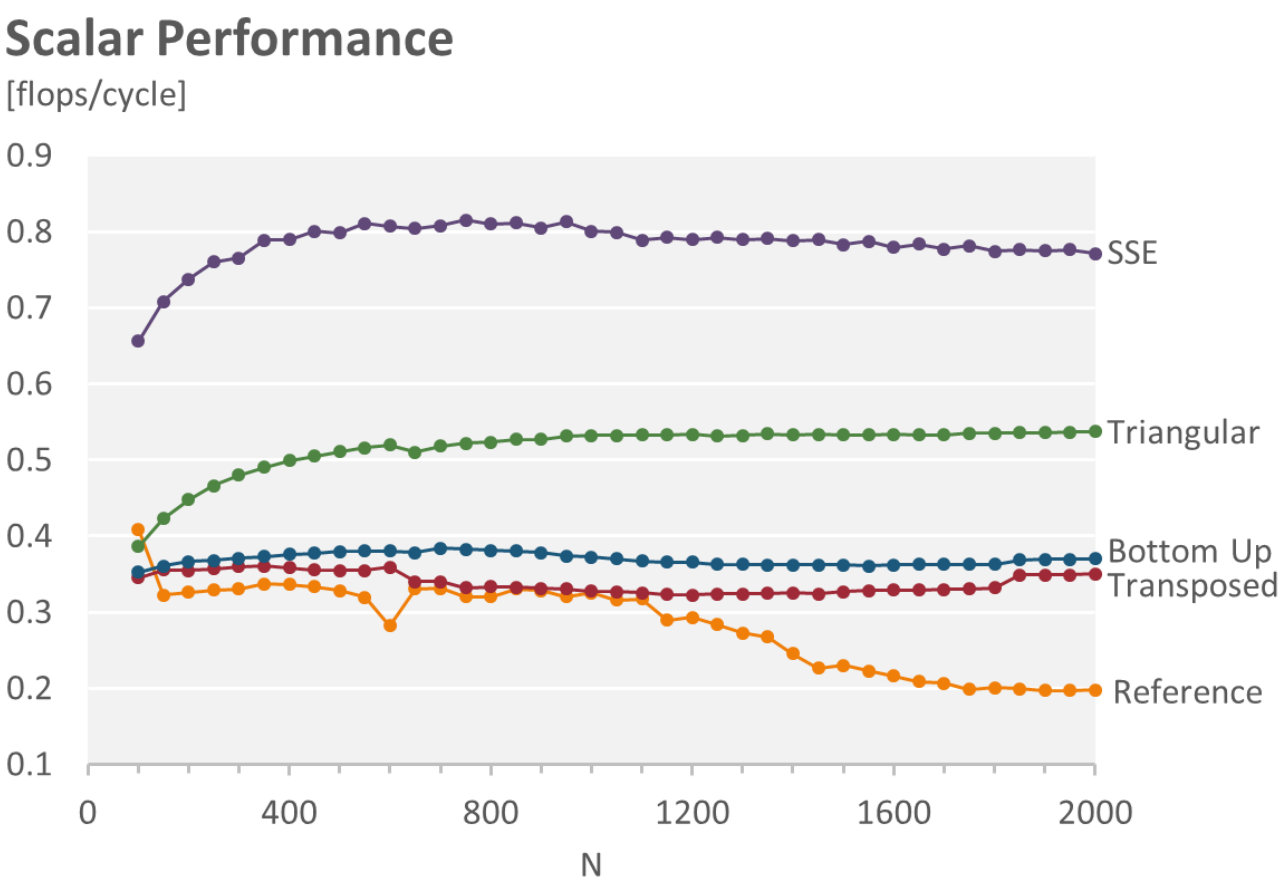

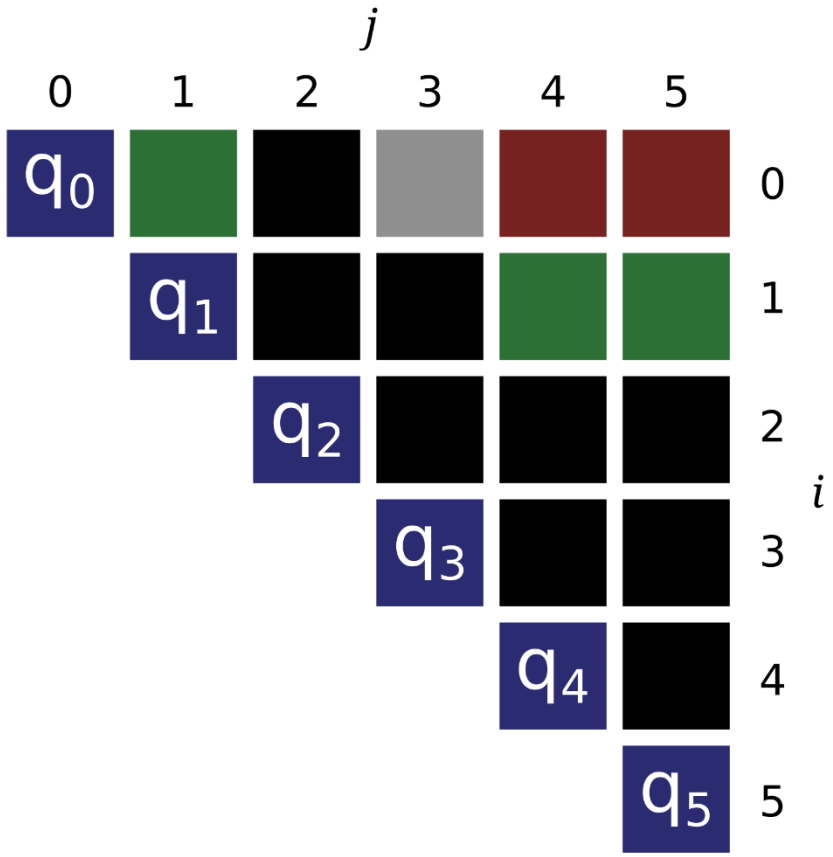

Vectorization Issues: Scheduling

Standard

```
a = load()
b = load()
c = load()
d = load()

e = add(a,b)
f = add(c,d)
g = add(e,f)

store(g)
```

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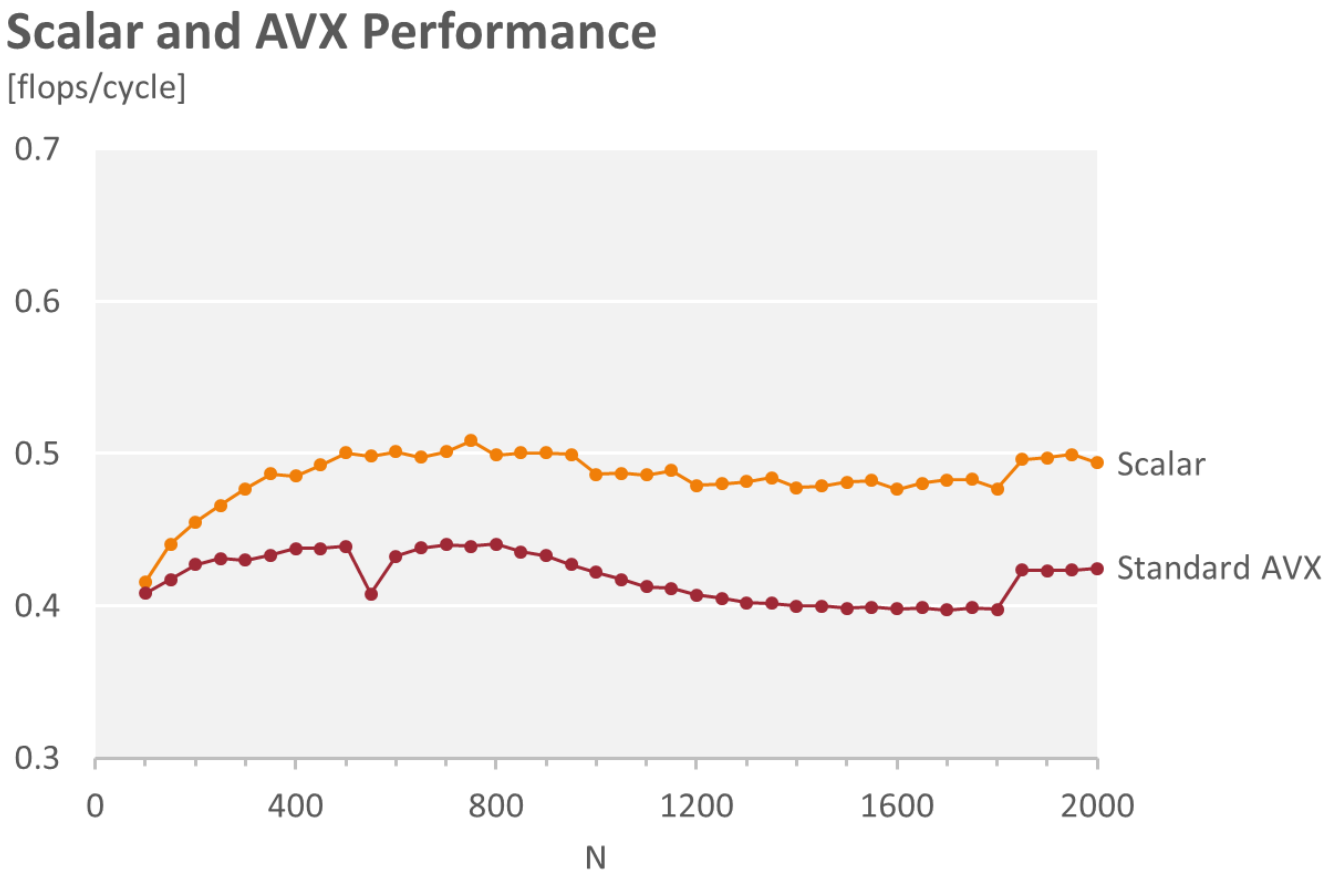

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a = load()
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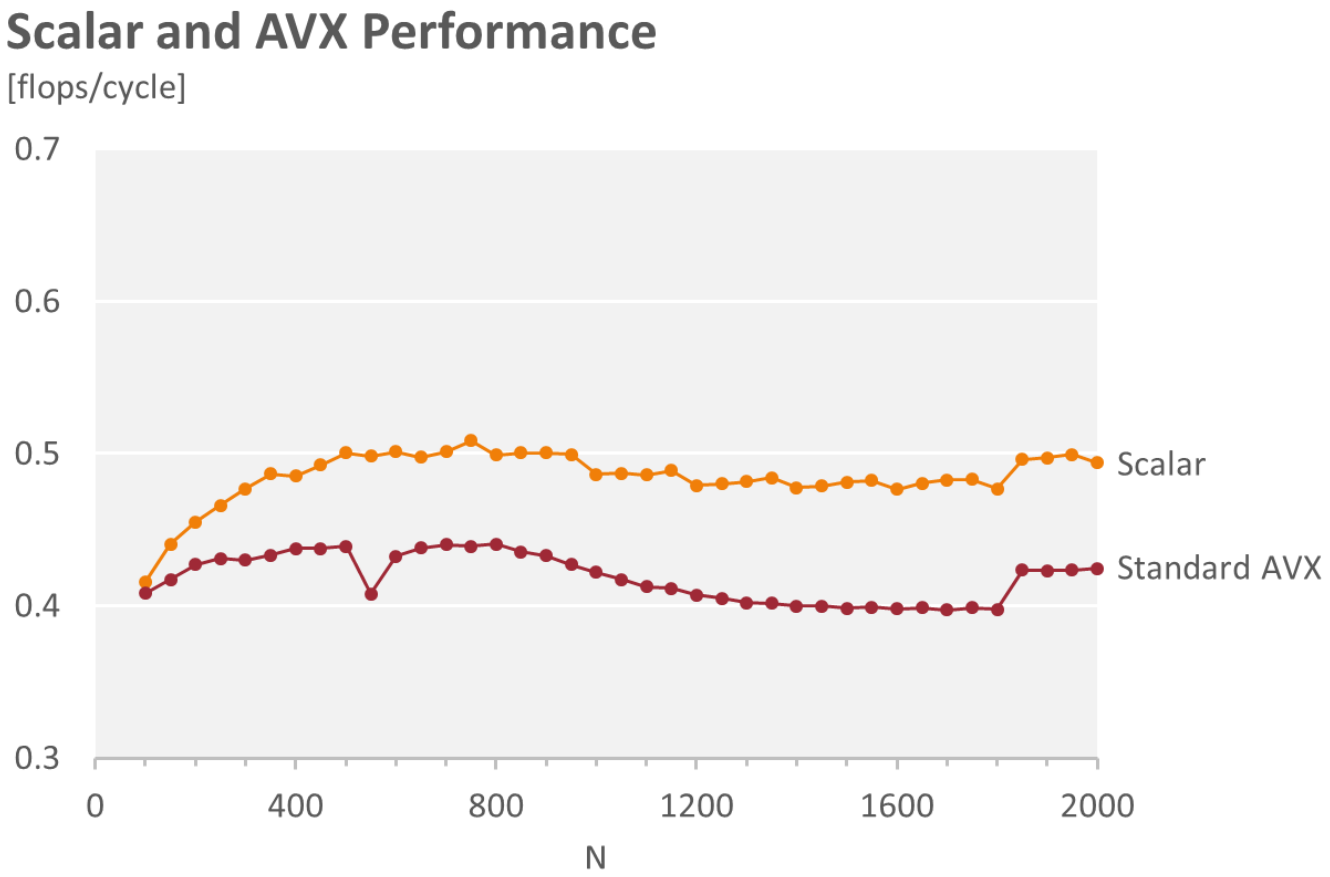
```

Rescheduled

```
a = load()
b = load()
e = add(a,b)

c = load()
d = load()
f = add(c,d)

g = add(e,f)
store(g)
```



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Rescheduled

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a = load()
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e = add(a,b)

c = load()
d = load()
f = add(c,d)

g = add(e,f)
store(g)
```
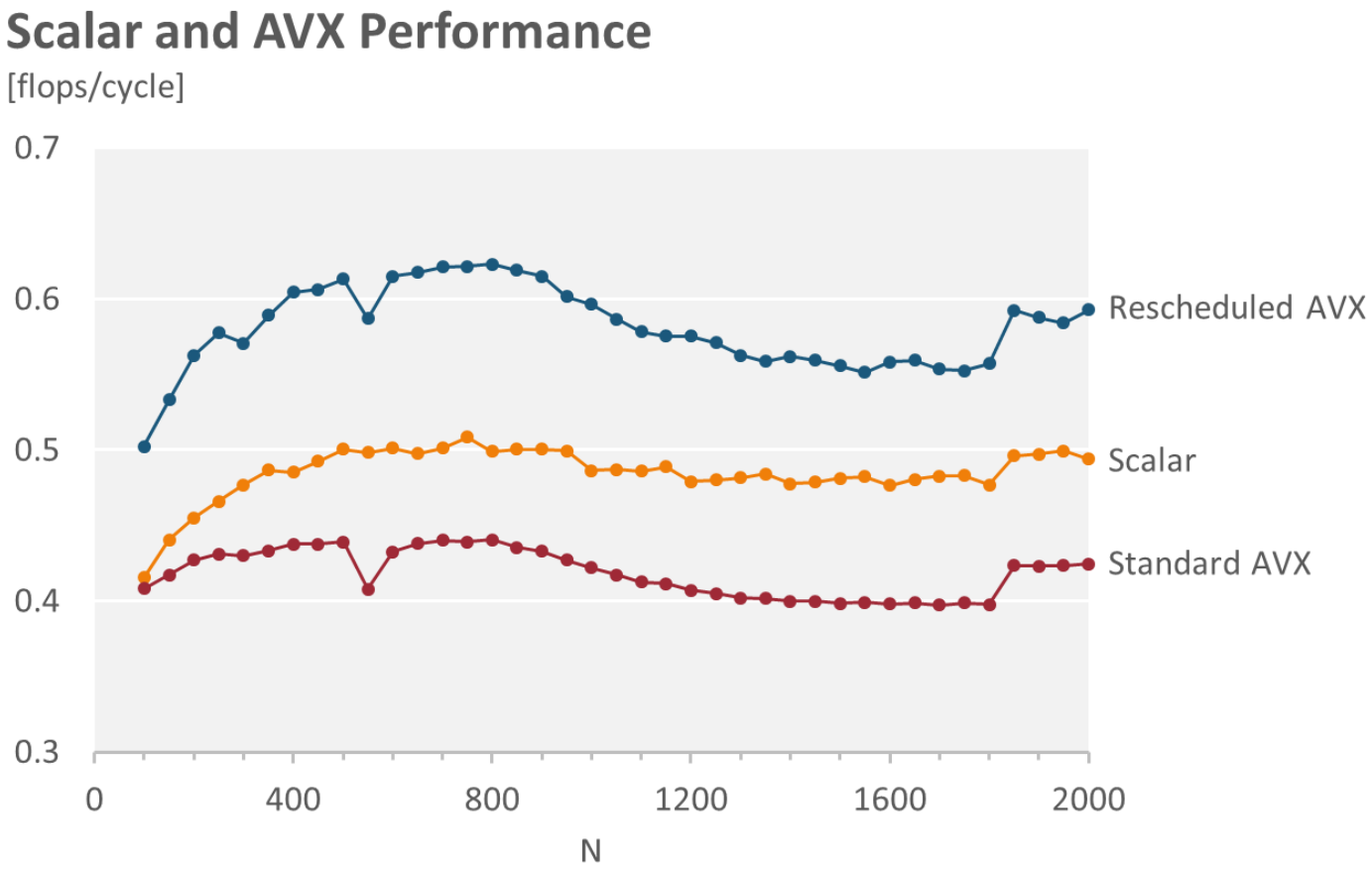
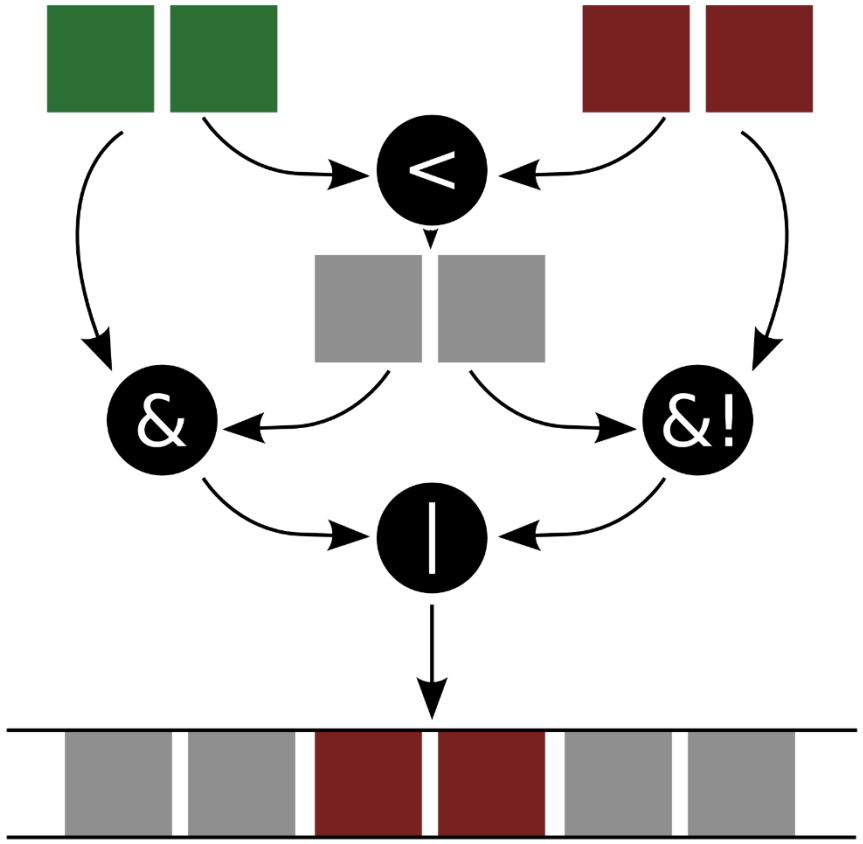
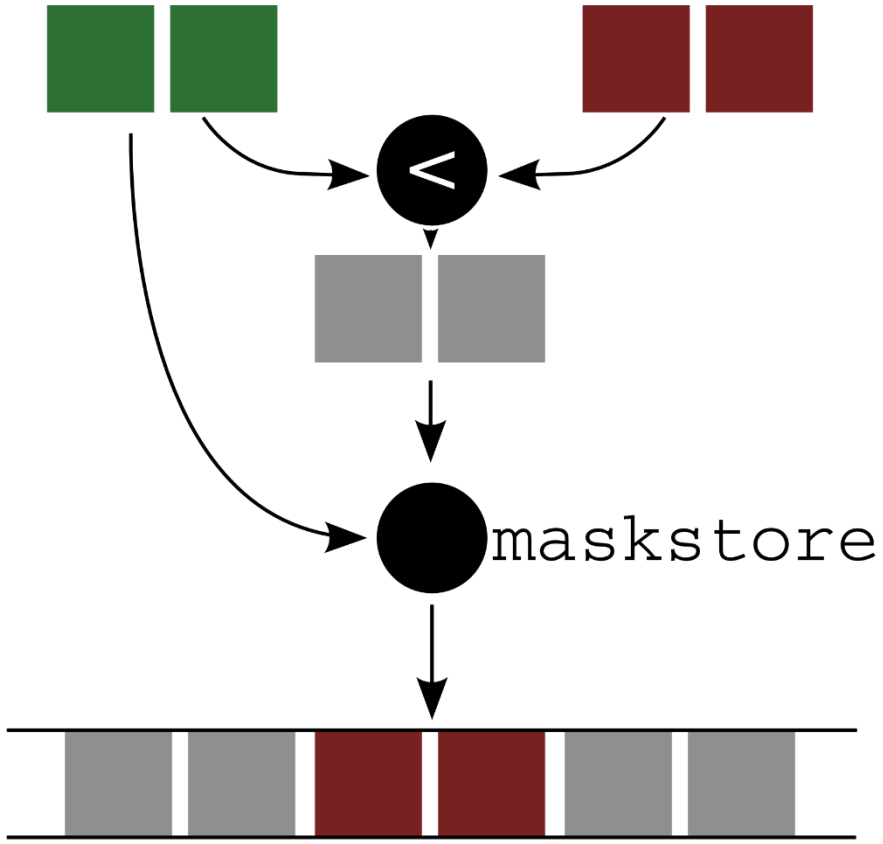



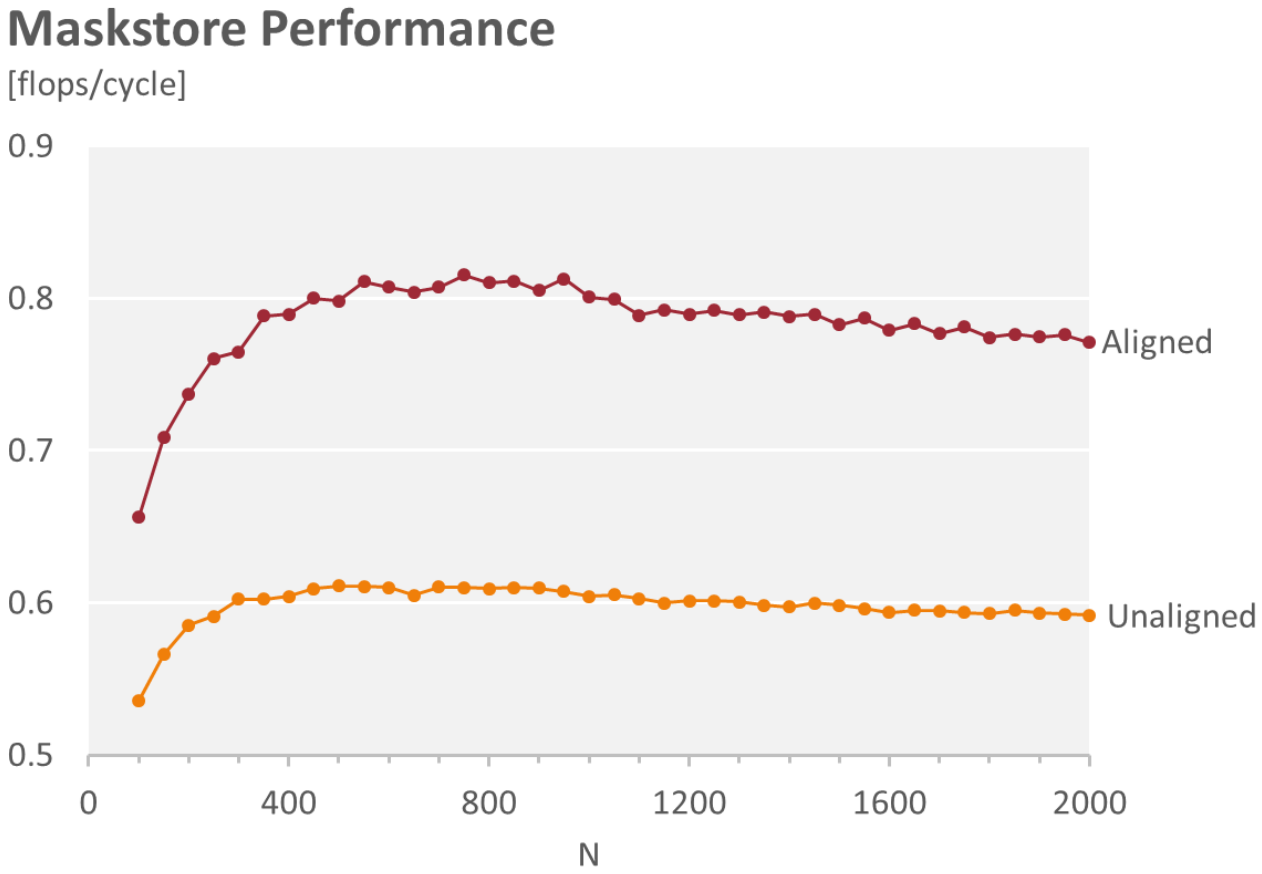
Vectorization Issues: Maskstore

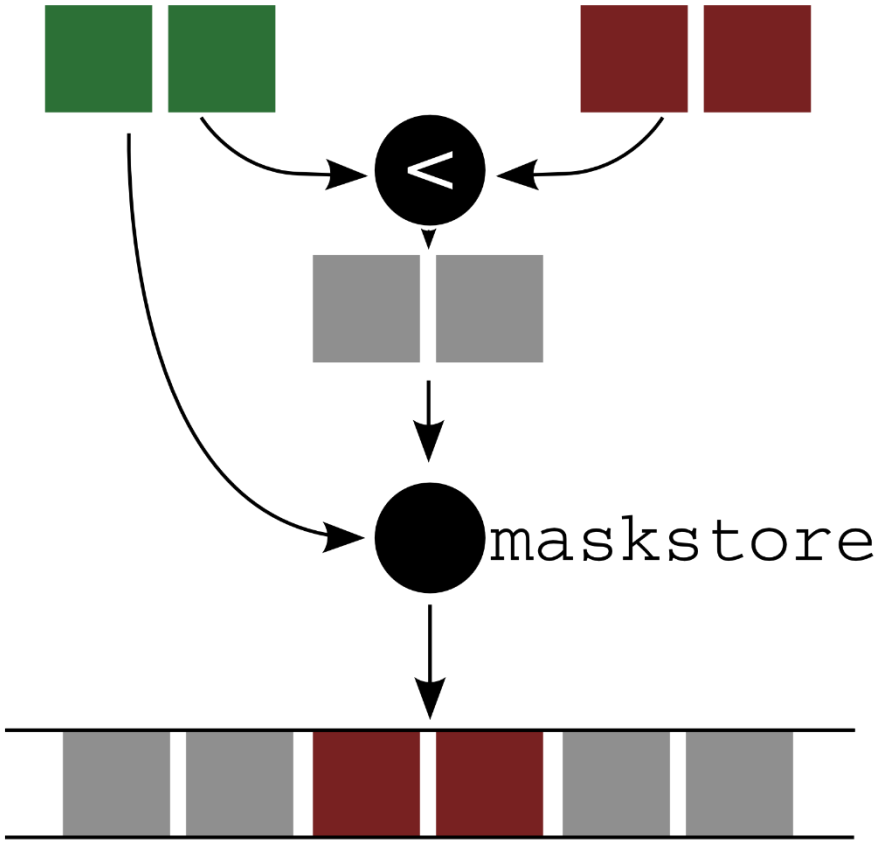


Vectorization Issues: Maskstore

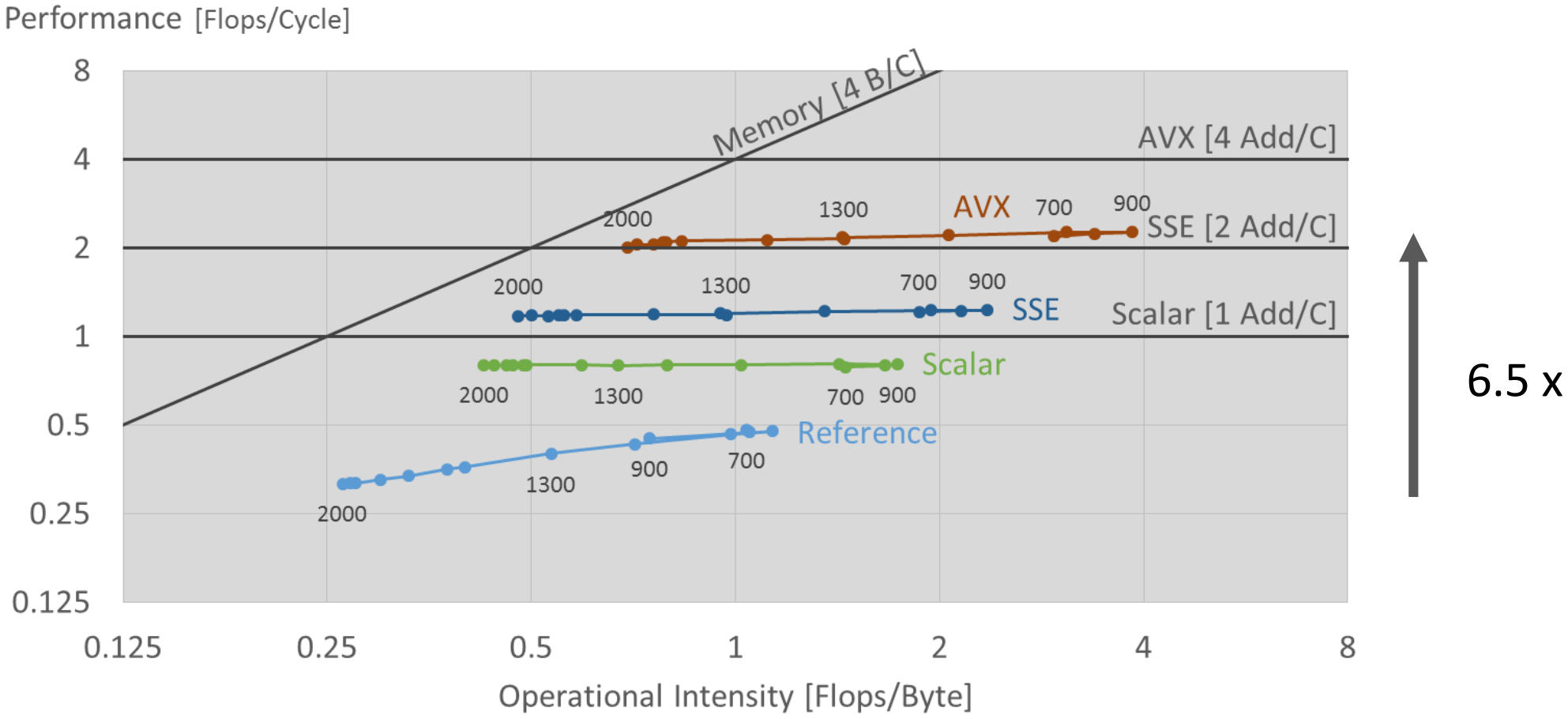


Vectorization Issues: Maskstore





Roofline Plot



6.5 x

Questions…?