



# A modular framework for stabilizing deep reinforcement learning control

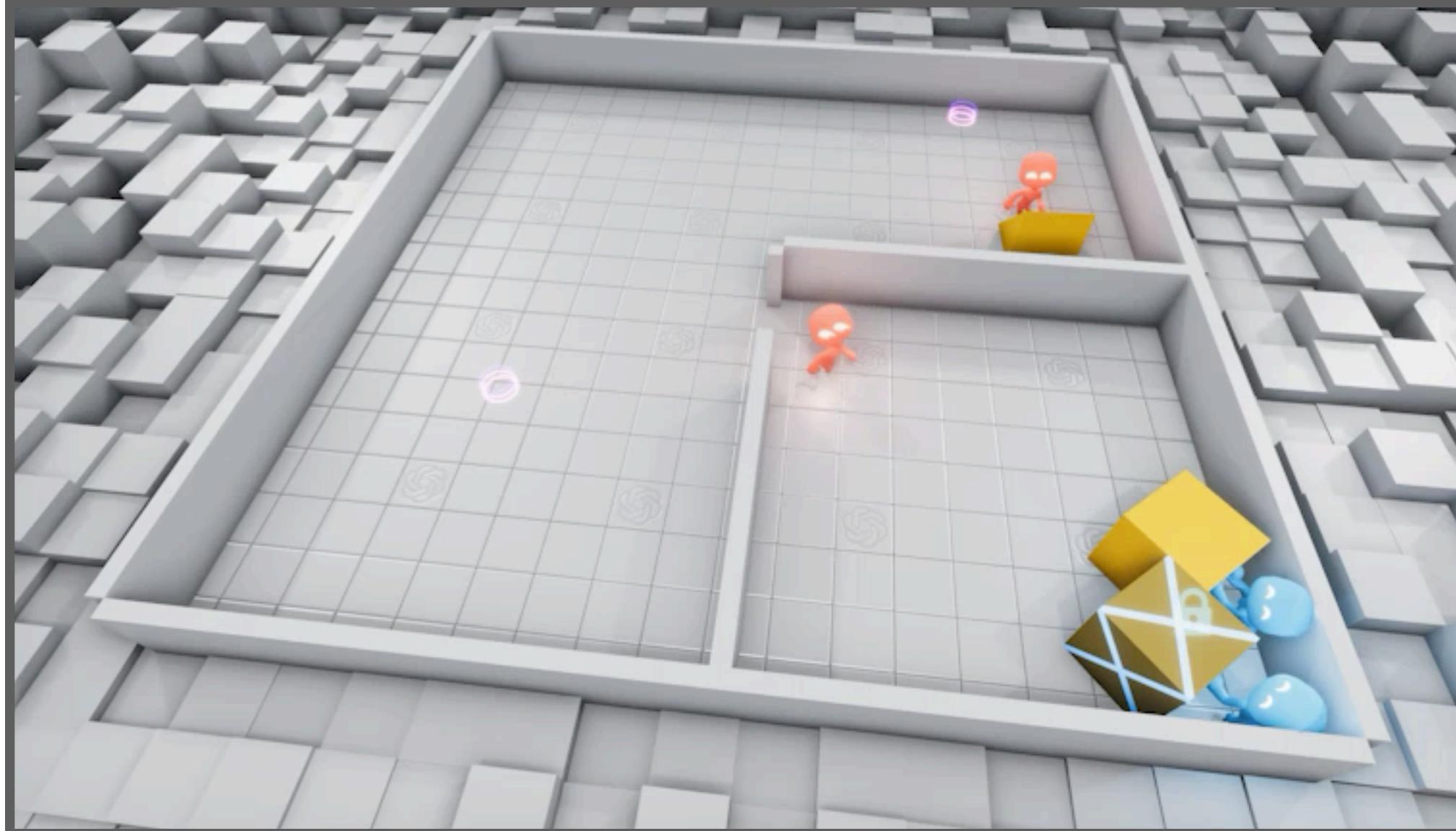
IFAC World Congress 2023

Nathan Lawrence ~ University of British Columbia ~ [lawrence@math.ubc.ca](mailto:lawrence@math.ubc.ca)

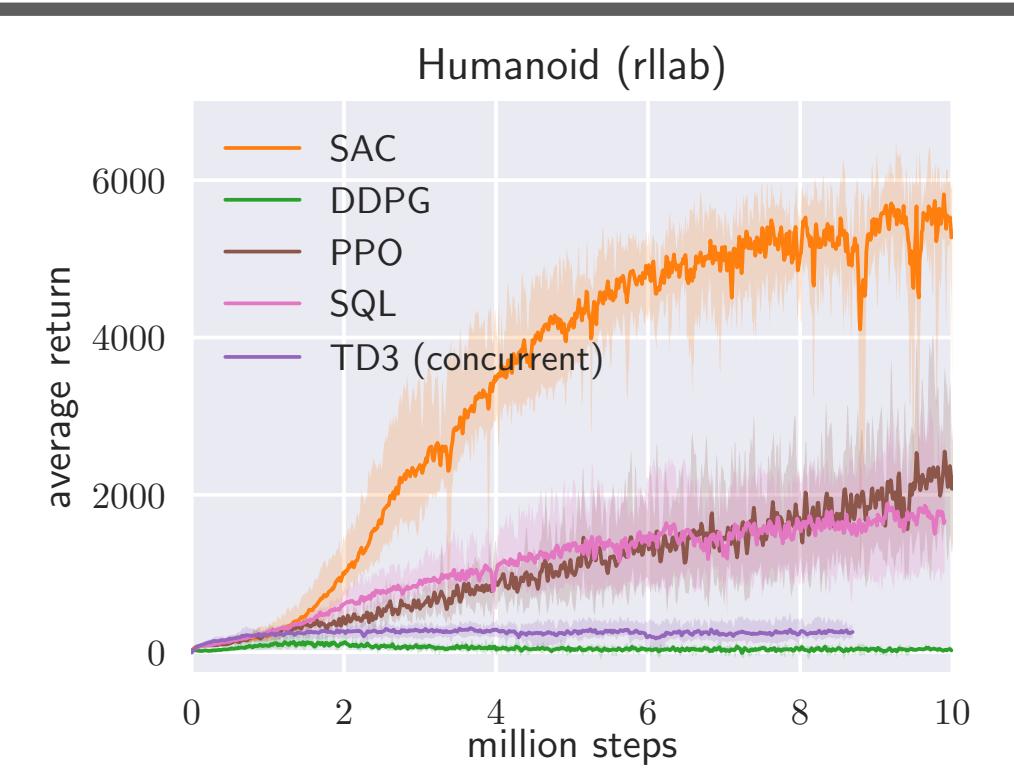
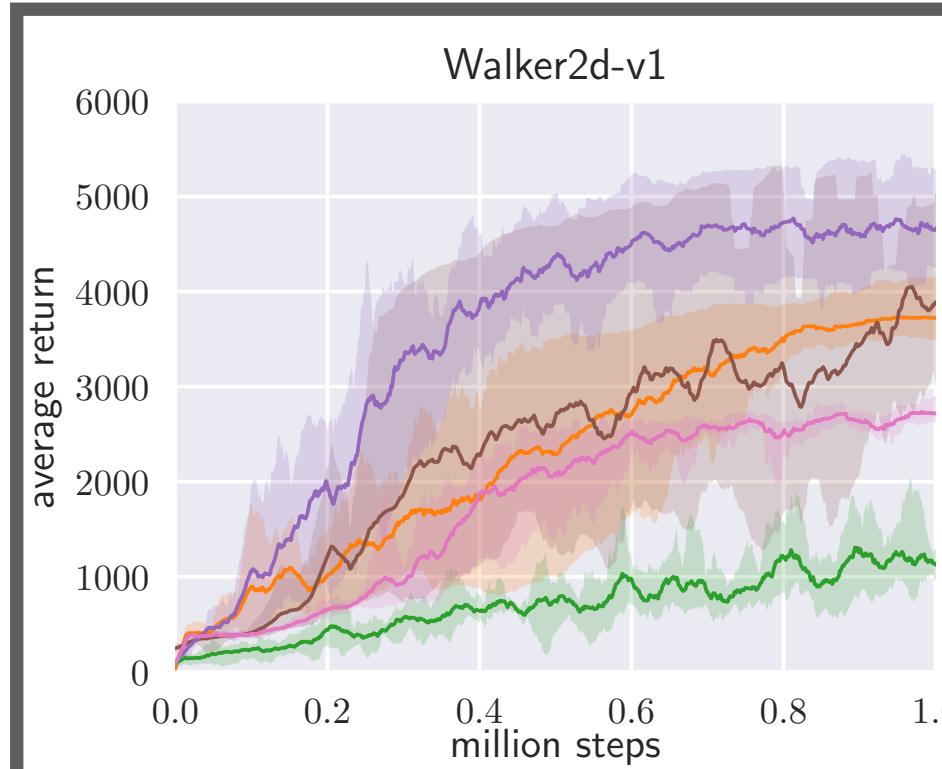
Philip Loewen, Shuyuan Wang, Michael Forbes, Bhushan Gopaluni

# Reinforcement learning

Maximizing reward through experience

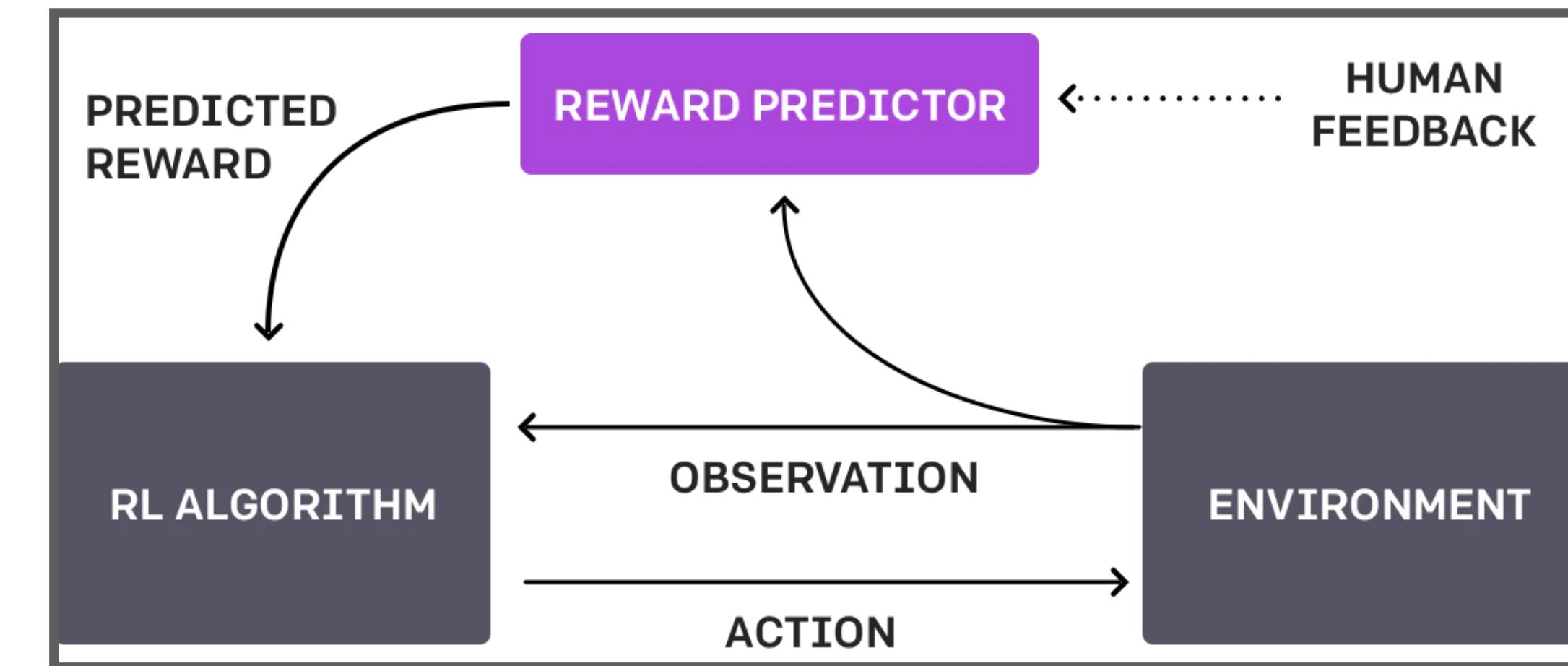


<https://openai.com/research/emergent-tool-use#surprisingbehaviors>

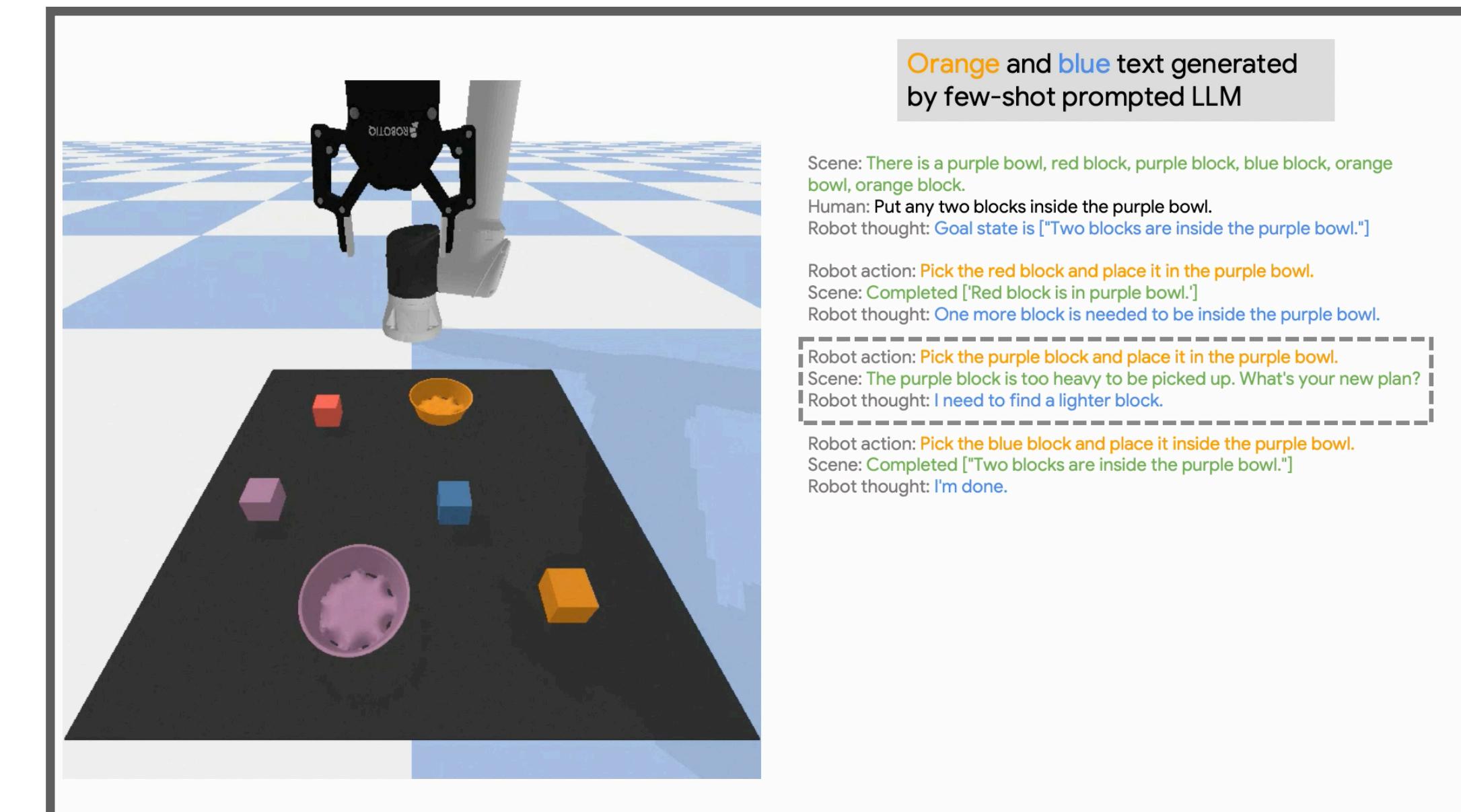


Haarnoja, Tuomas, et al. "Soft actor-critic: Off-policy maximum entropy deep reinforcement learning with a stochastic actor." 2018.

ChatGPT



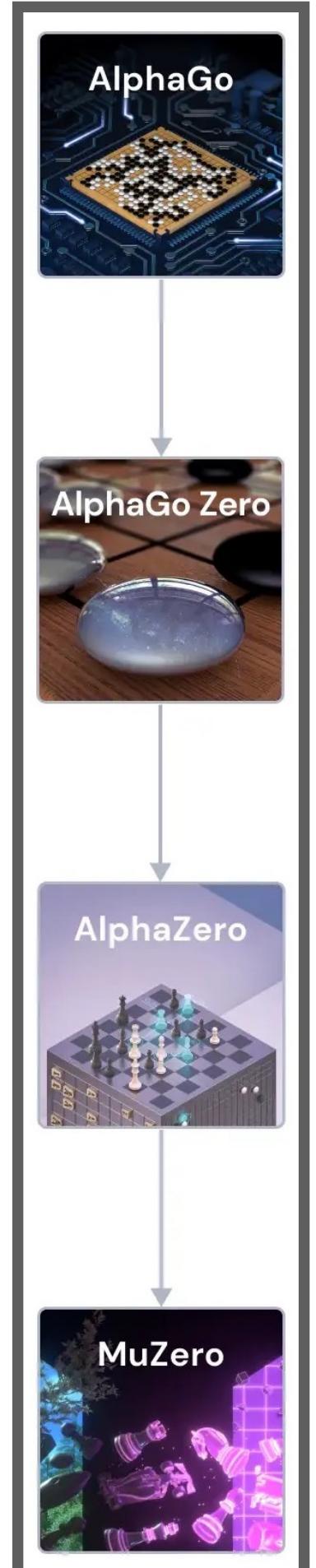
<https://openai.com/research/learning-from-human-preferences>



<https://innermonologue.github.io/>

<https://www.deepmind.com/blog/muzero-mastering-go-chess-shogi-and-atari-without-rules>

Chess, Go, Shogi



# RL in PSE?



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## Computers and Chemical Engineering

journal homepage: [www.elsevier.com/locate/compchemeng](http://www.elsevier.com/locate/compchemeng)

A review On reinforcement learning: Introduction and applications in industrial process control

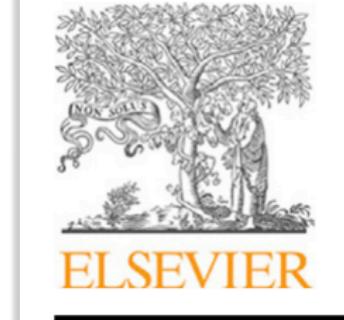
Rui Nian, Jinfeng Liu\*, Biao Huang

PROCESS SYSTEMS ENGINEERING

AIChE JOURNAL

Toward self-driving processes: A deep reinforcement learning approach to control

Steven Spielberg<sup>1</sup> | Aditya Tulsyan<sup>1</sup> | Nathan P. Lawrence<sup>2</sup> | Philip D. Loewen<sup>2</sup> | R. Bhushan Gopaluni<sup>1</sup>



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Review article

Reinforcement learning for batch process control: Review and perspectives  
Haeun Yoo<sup>1</sup>, Ha Eun Byun<sup>1</sup>, Dongho Han, Jay H. Lee\*



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## Computers and Chemical Engineering

journal homepage: [www.elsevier.com/locate/compchemeng](http://www.elsevier.com/locate/compchemeng)

Reinforcement Learning – Overview of recent progress and implications for process control

Joohyun Shin<sup>a</sup>, Thomas A. Badgwell<sup>b</sup>, Kuang-Hung Liu<sup>b</sup>, Jay H. Lee<sup>a,\*</sup>

# Can reinforcement learning help maintain control loops?

## It's complicated

In favor

- Leverage observed data to improve operations
- **Minimize prior domain knowledge**
- Automated maintenance on a variety of systems

Against

- Additional algorithmic complexity
- Auto-tuners exist already (but are often idle)
- **Stability during and after training**
- Sample efficiency

*Our goal is to balance the automation and scalability of reinforcement learning with control-theoretic tools to create efficient and safe improvements*

# Reinforcement learning over all stable behaviour

## Topics for today

1. Willems' lemma

Data-based characterization of dynamics

2. Youla-Kučera parameterization

Recipe for all stabilizing controllers

3. Learning algorithms

A module to shape system behavior

*Combining these elements gives a modular setup that decouples learning and stability*

# Key ingredients

## State-space model

- Define the system equations

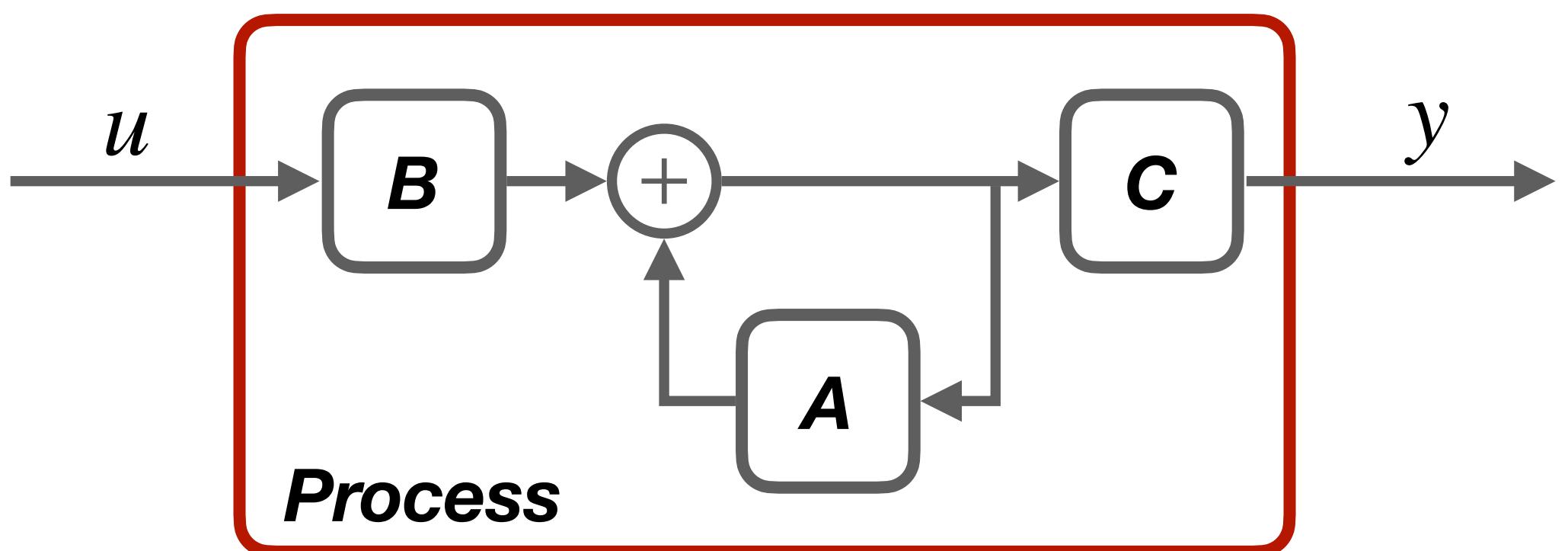
$$x_{t+1} = Ax_t + Bu_t$$

$$y_t = Cx_t$$

where

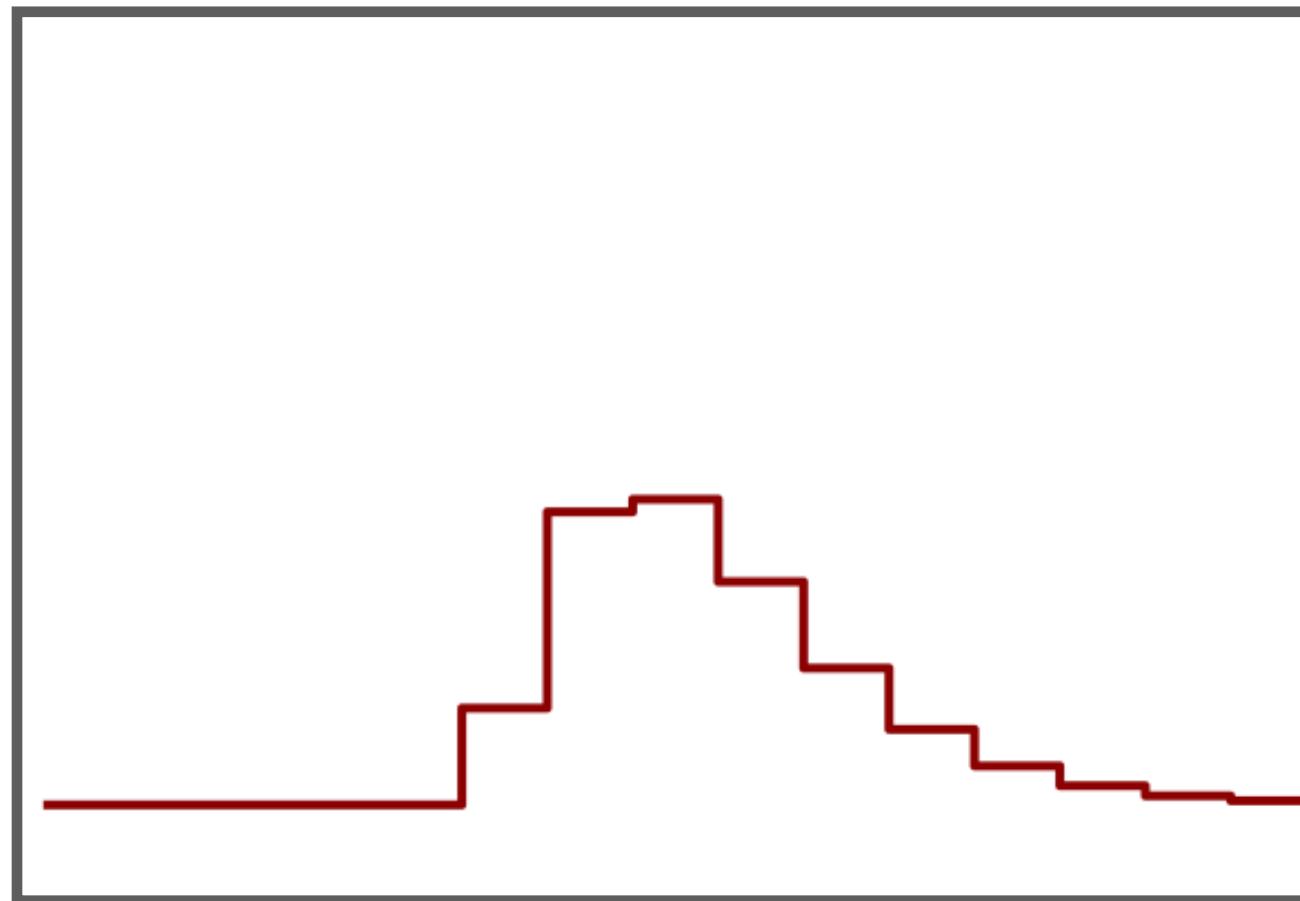
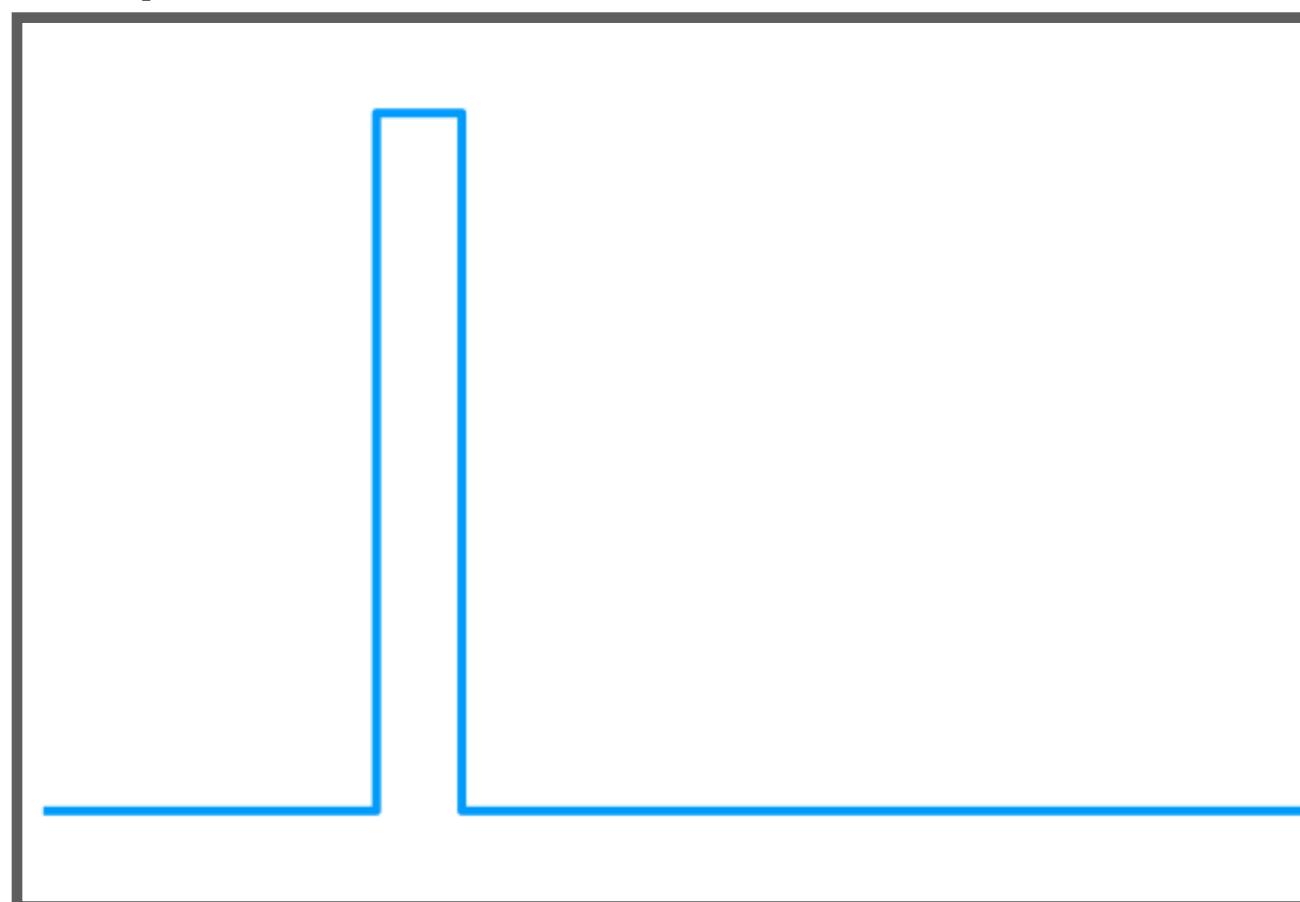
$$A \in \mathbb{R}^{n \times n}, B \in \mathbb{R}^{n \times 1}, C \in \mathbb{R}^{1 \times n}$$

- Inputs, outputs are scalars for simplicity

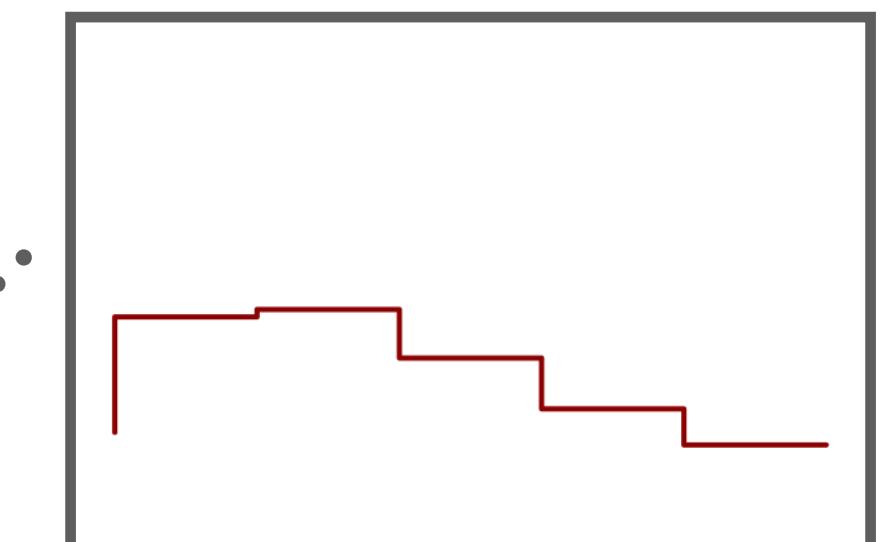
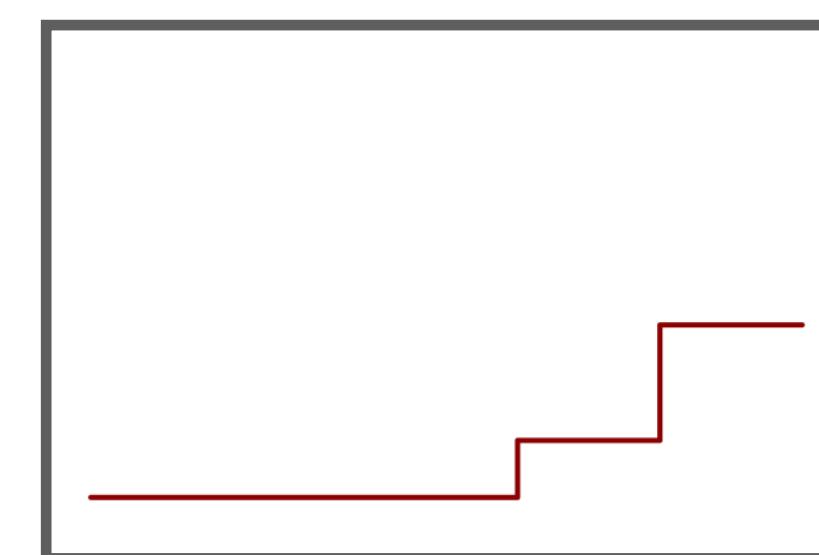
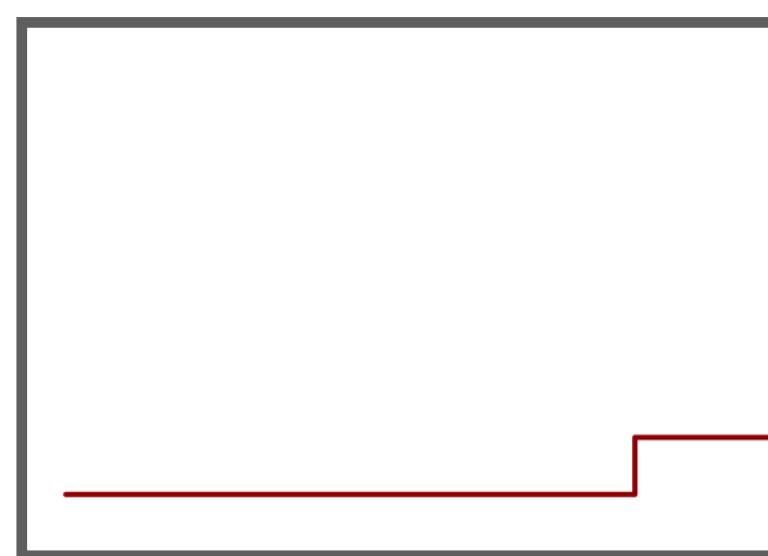
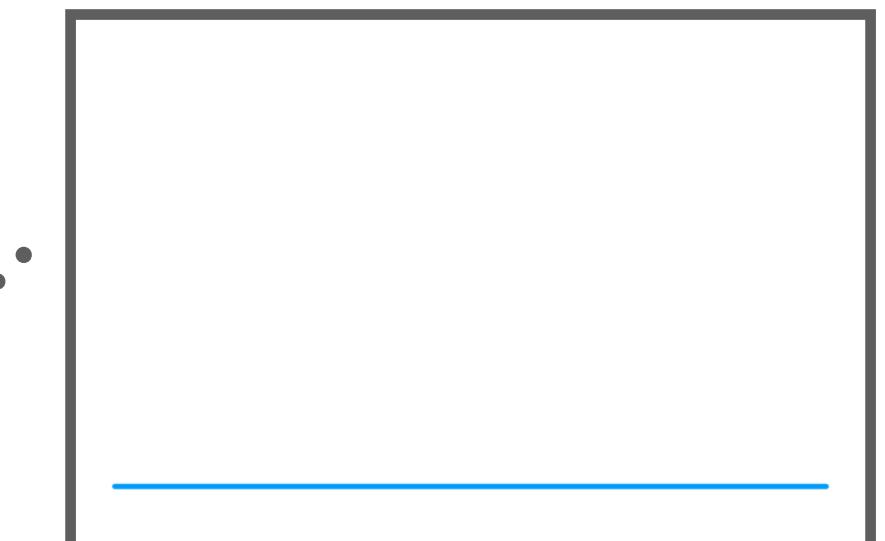
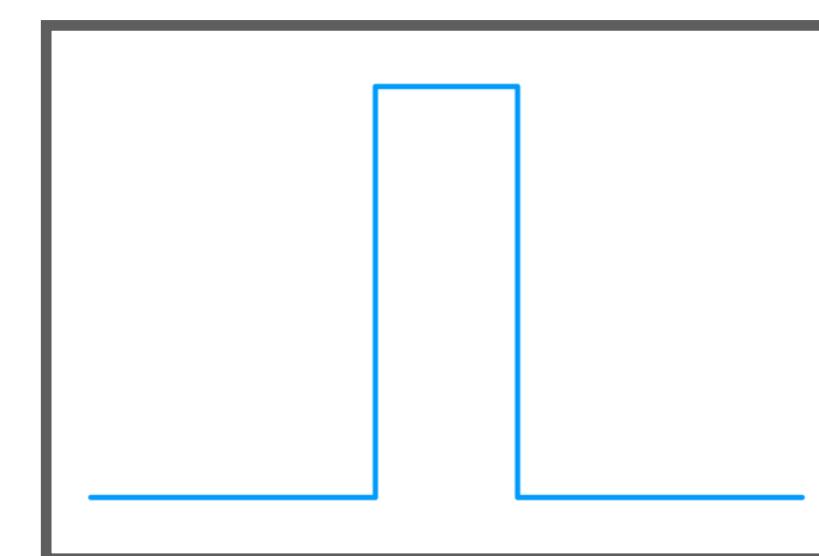
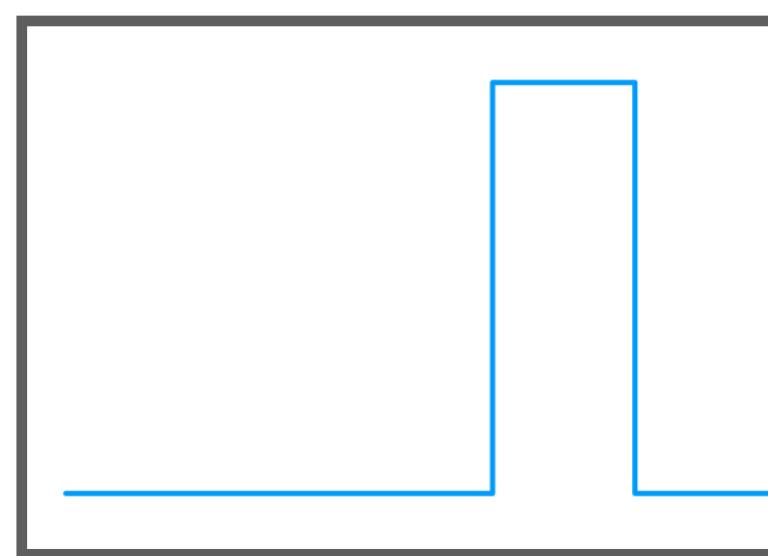


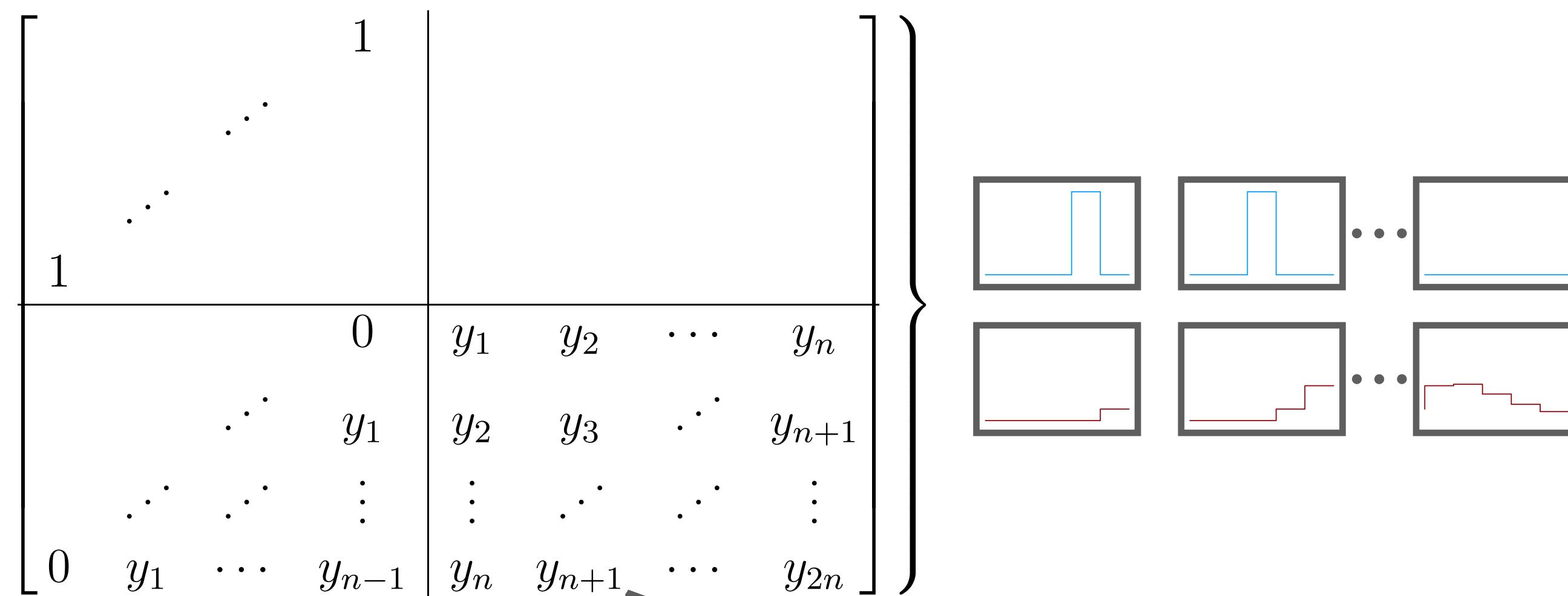
# Willems' fundamental lemma – a special case (Picture form)

Impulse



What is the span of these data vectors?





→ Full-rank data matrix!

$$\underbrace{\begin{bmatrix} C \\ CA \\ \vdots \\ CA^{n-1} \end{bmatrix}}_{\mathcal{O}} \underbrace{\begin{bmatrix} B & AB & \cdots & A^{n-1}B \end{bmatrix}}_{\mathcal{C}} = \begin{bmatrix} CB & CAB & \cdots & CA^{n-1}B \\ CAB & CA^2B & \cdots & CA^nB \\ \vdots & \vdots & \ddots & \vdots \\ CA^{n-1}B & CA^nB & \cdots & CA^{2n-1}B \end{bmatrix}$$

# Willems' fundamental lemma – general case

**Data  $\iff$  models**

- Given a signal  $z = \{z_t\}_{t=0}^{N-1}$ , define its Hankel matrix of order  $L$ :

“Persistently exciting” if full rank

$$H_L(z) = \begin{bmatrix} z_0 & z_1 & \cdots & z_{N-L} \\ z_1 & z_2 & \cdots & z_{N-L+1} \\ \vdots & \vdots & \ddots & \vdots \\ z_{L-1} & z_L & \cdots & z_{N-1} \end{bmatrix}$$

- Let  $\{u_t, y_t\}_{t=0}^{N-1}$  be a trajectory where  $u$  is persistently exciting of order  $L + n$ . Then  $\{\bar{u}_t, \bar{y}_t\}_{t=0}^{L-1}$  is a trajectory if and only if there exists  $\alpha$  such that

Static, collected data

$$\begin{bmatrix} H_L(u) \\ H_L(y) \end{bmatrix} \alpha = \begin{bmatrix} \bar{u} \\ \bar{y} \end{bmatrix}$$

All possible data

# A dynamic Willems' lemma

## Carrying a trajectory forward

- Start with Willems' lemma:

$$\begin{bmatrix} H_L(u) \\ H_L(y) \end{bmatrix} \alpha_0 = \begin{bmatrix} \bar{u} \\ \bar{y} \end{bmatrix}$$

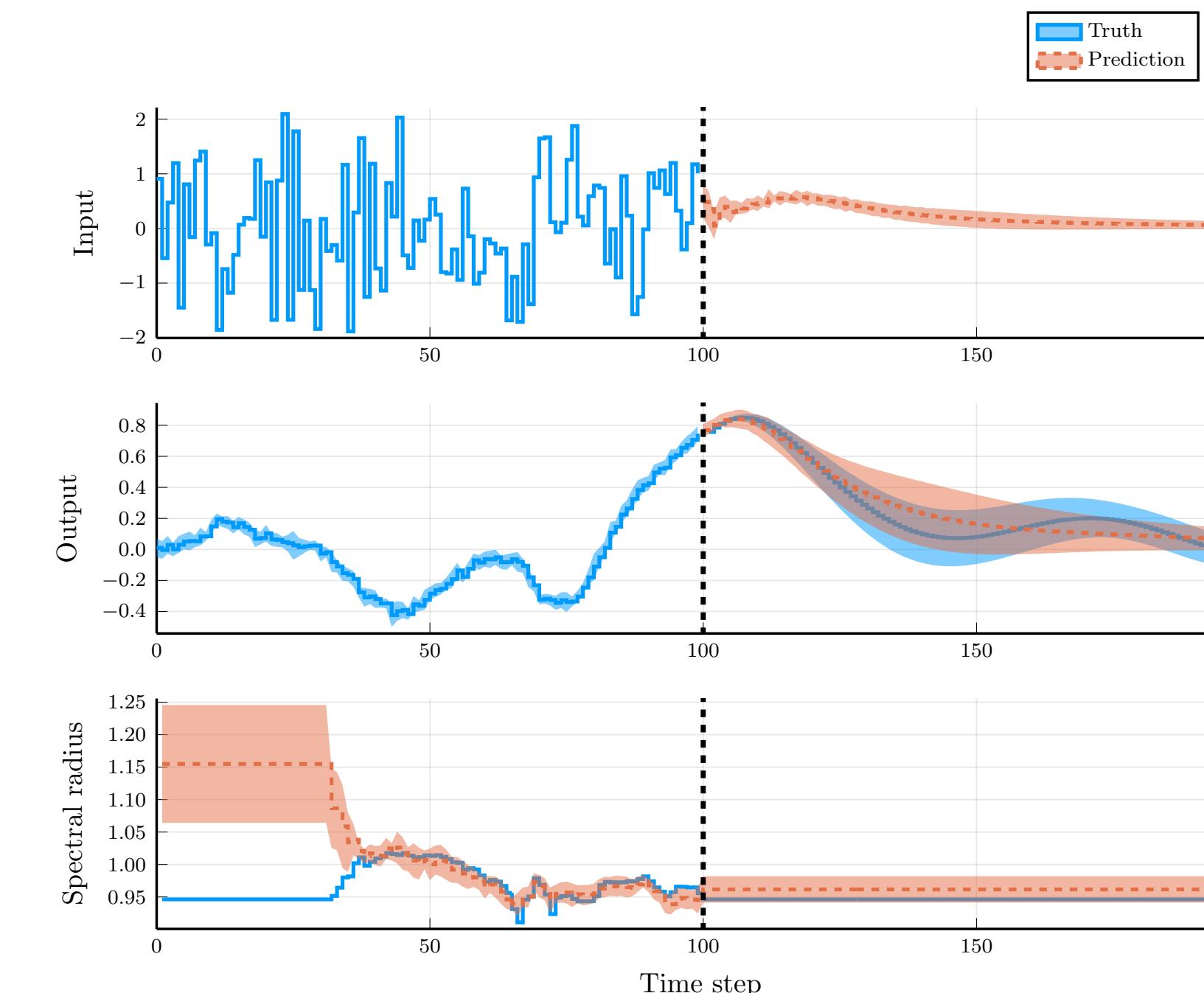
- How to advance to the next output? Consider nested Hankel matrices:

$$\underbrace{\begin{bmatrix} y_0 & y_1 & \dots & y_{N-L} & y_{N-L+1} \\ y_1 & y_2 & \dots & y_{N-L+1} & y_{N-L+2} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ y_{L-1} & y_L & \dots & y_{N-1} & y_N \end{bmatrix}}_{H_L(y)} \quad \underbrace{\begin{bmatrix} \bar{u}' \\ \bar{y}' \end{bmatrix}}_{H'_L(y)}$$

$$\begin{bmatrix} \bar{u}' \\ \bar{y}' \end{bmatrix} = \underbrace{\begin{bmatrix} H'_L(u) \\ H'_L(y) \end{bmatrix}}_{H'_L(y)} \alpha_0$$

Multiply previous solution by shifted Hankel matrix —  
Then repeat!

$$H_L(y)\alpha' = H'_L(y)\alpha$$

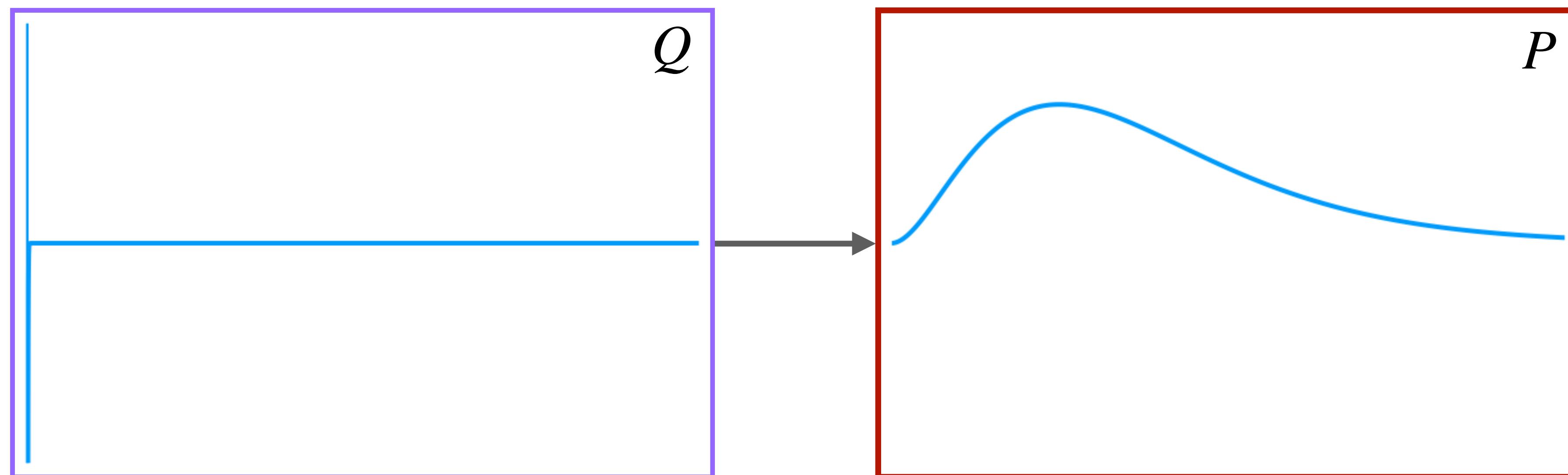


**So far we have characterized a system in terms of data ... how do we drive its behaviour?**

# Youla-Kučera parameterization

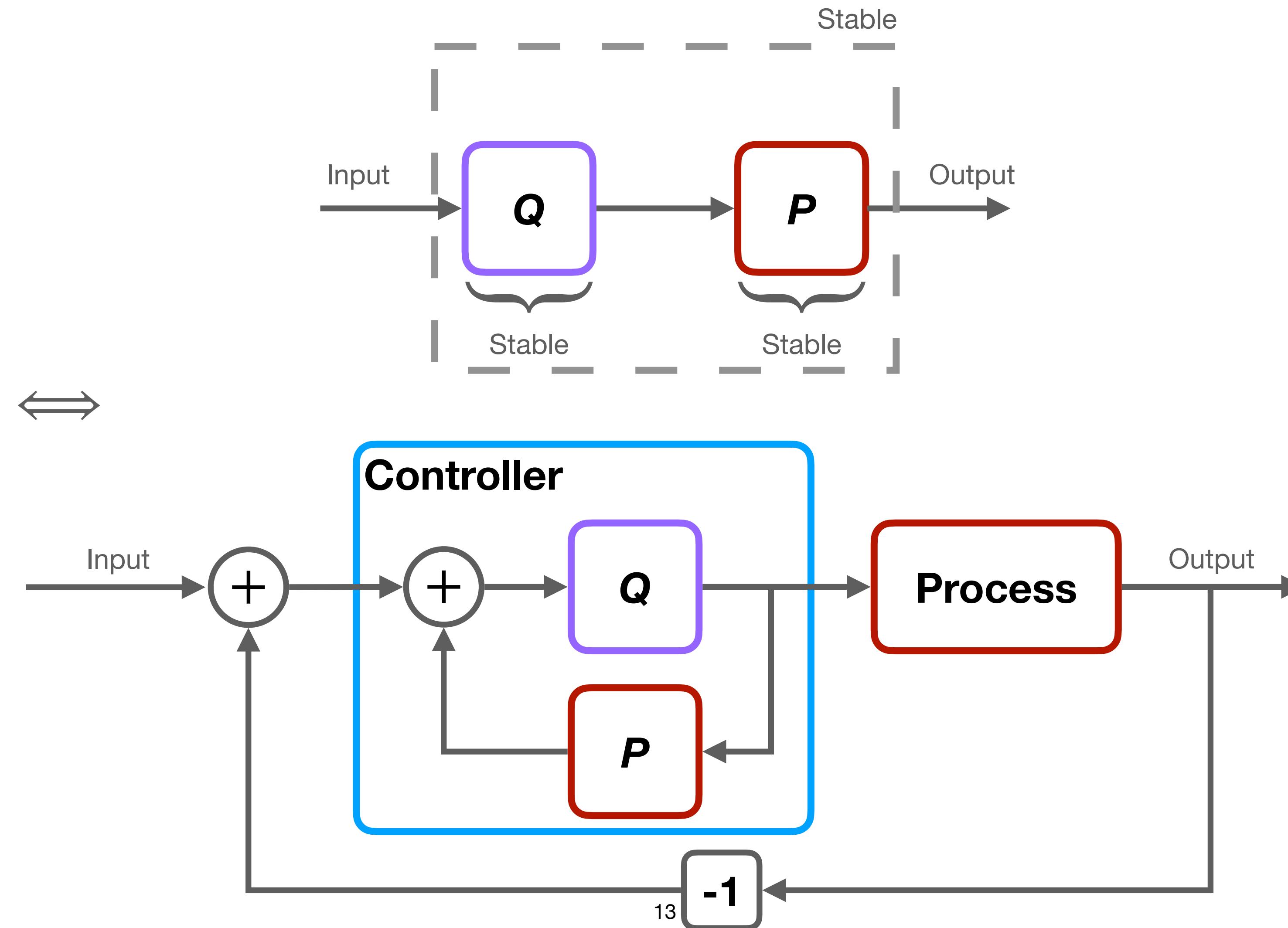
## All stabilizing controllers

- Hard: Given a controller  $K$ , is it stabilizing? What is the set of all stabilizing controllers?
- Easier: What happens when you probe  $P$  with stable dynamics  $Q$ ?



# $Q$ “parameter” characterizes stable behaviour

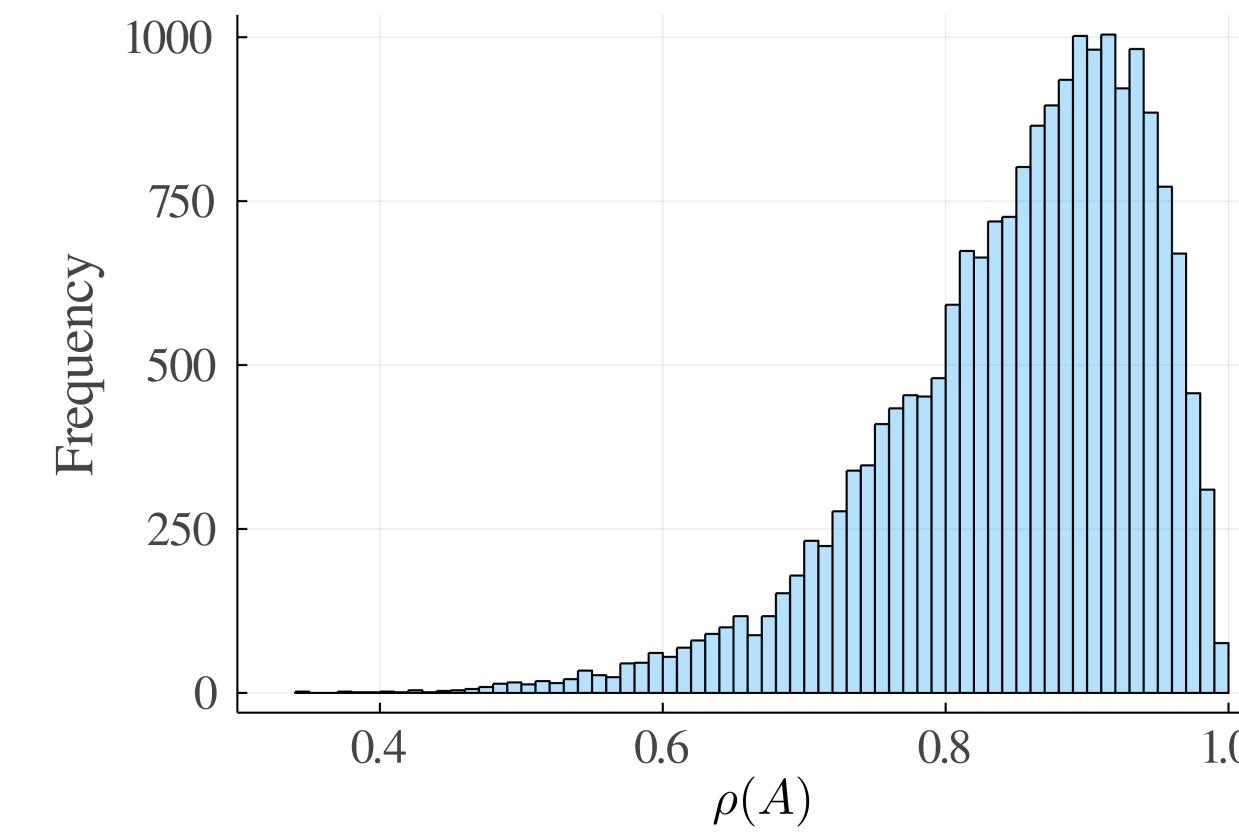
## How do we turn it into a controller?



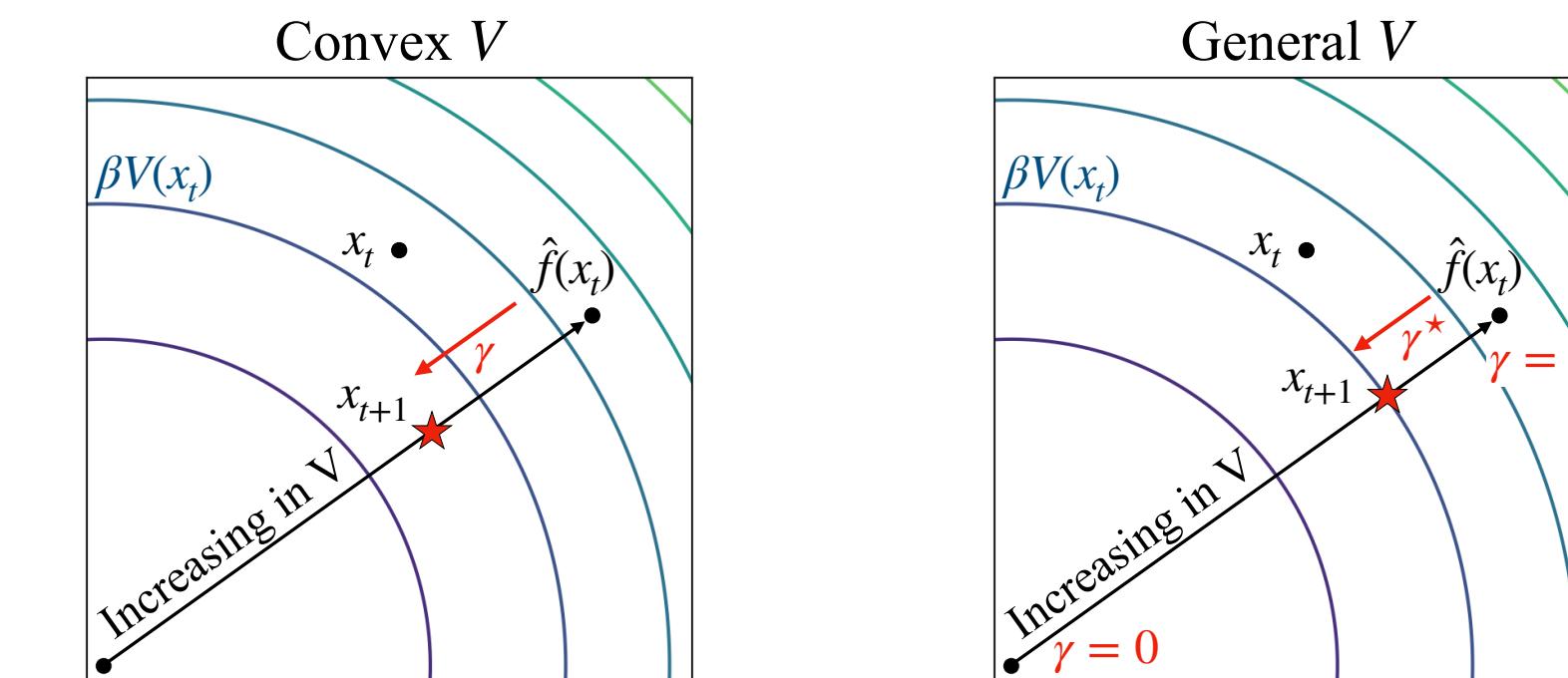
# Learning stable systems ( $Q$ in Youla-Kučera)

- $Q$  is a global parameter, but explicitly writing it down is difficult
- We represent  $Q$  using an unconstrained set of trainable parameters
- Yields stable models suitable for RL or supervised learning

Linear case – matrix factorization



Nonlinear case – stable DNN



Lawrence, Nathan, et al. "Almost surely stable deep dynamics." 2020.

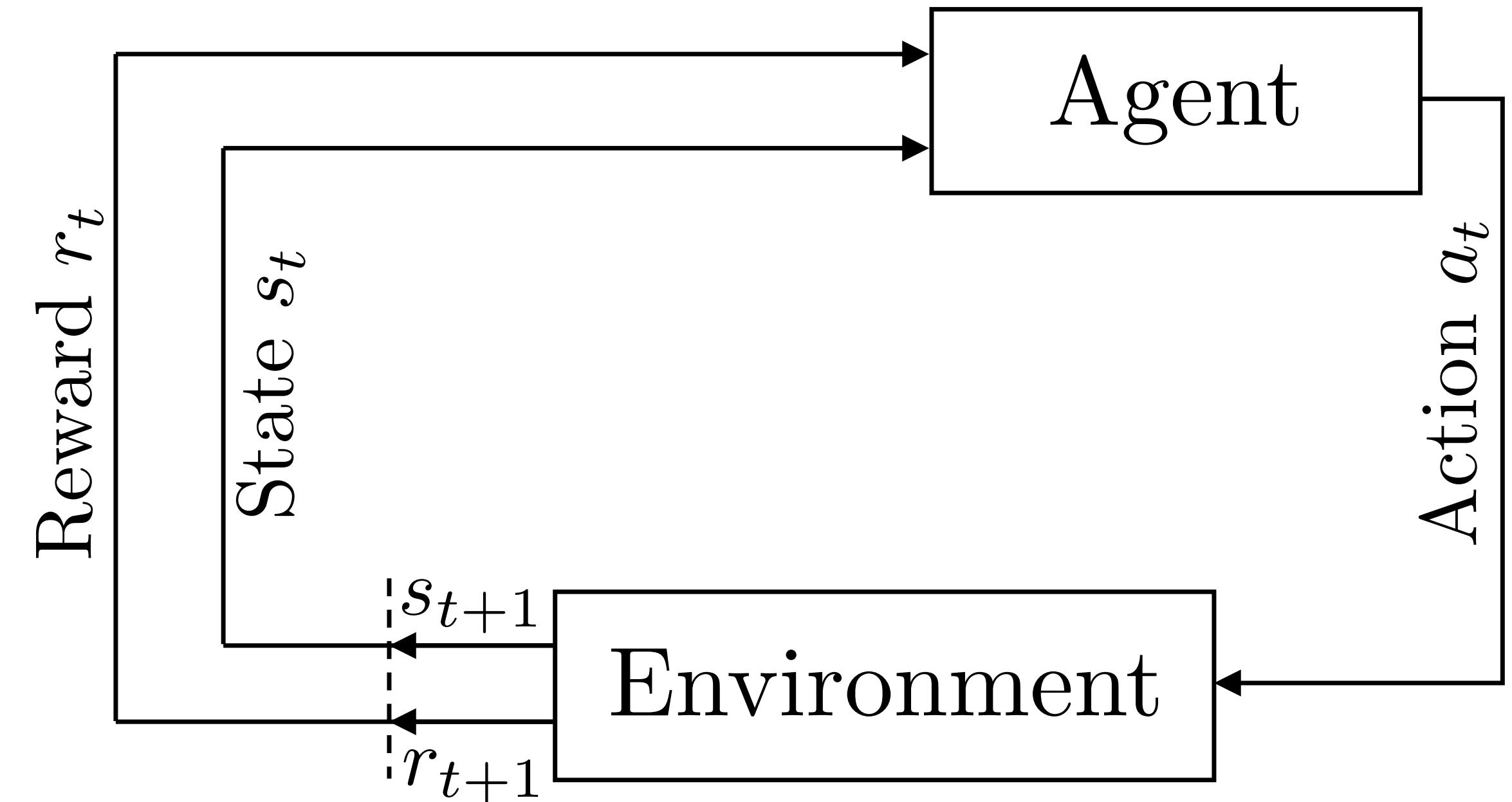
# Final ingredient: learning algorithms

# Reinforcement learning

## Business as usual

- A “policy”  $\pi$  interacts with an “environment”, generating a trajectory  $s_0, a_0, r_0, s_1, a_1, r_1, \dots$
- A “return” is accrued and averaged:  

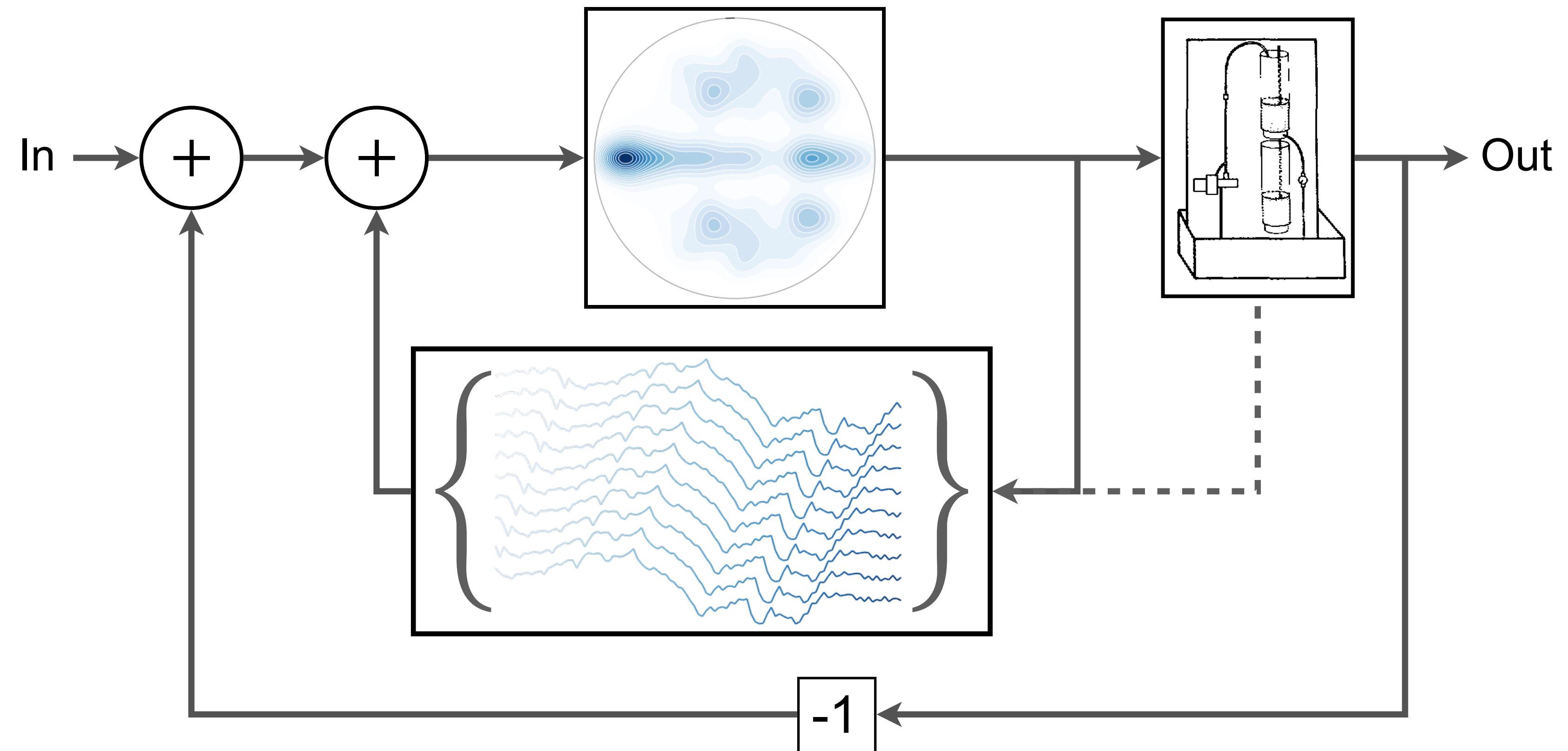
$$V(s) = \mathbb{E} \left[ \sum_{t=0}^{\infty} \gamma^t r(s_t, a_t) \right], \text{ where } s = s_0$$
- An “agent” tries to find the “best” policy



# Reinforcement learning over all stable behaviour

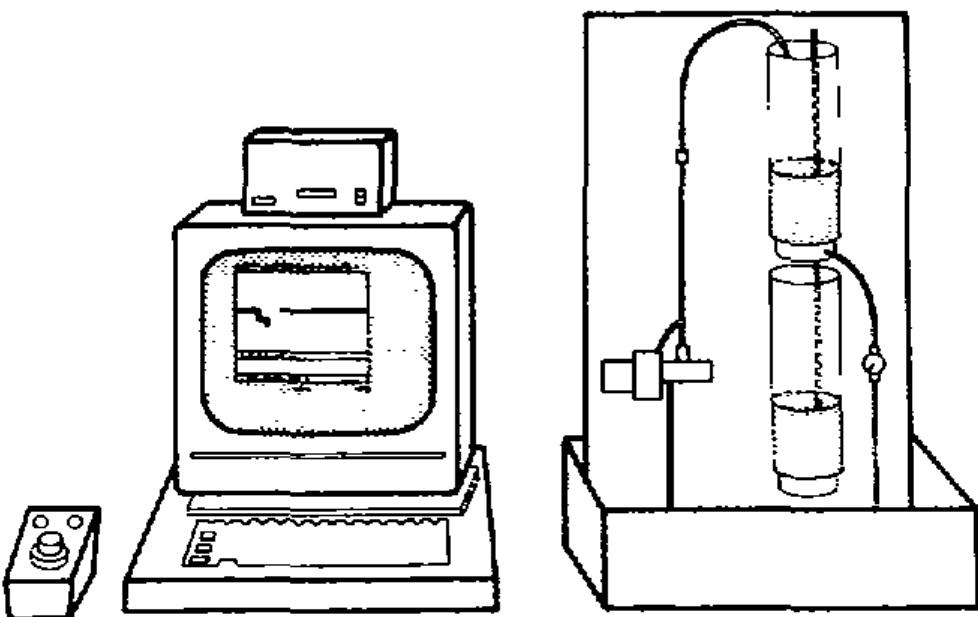
## A modular setup

1. Willems' lemma
2. Youla-Kučera
3. Learning algorithm



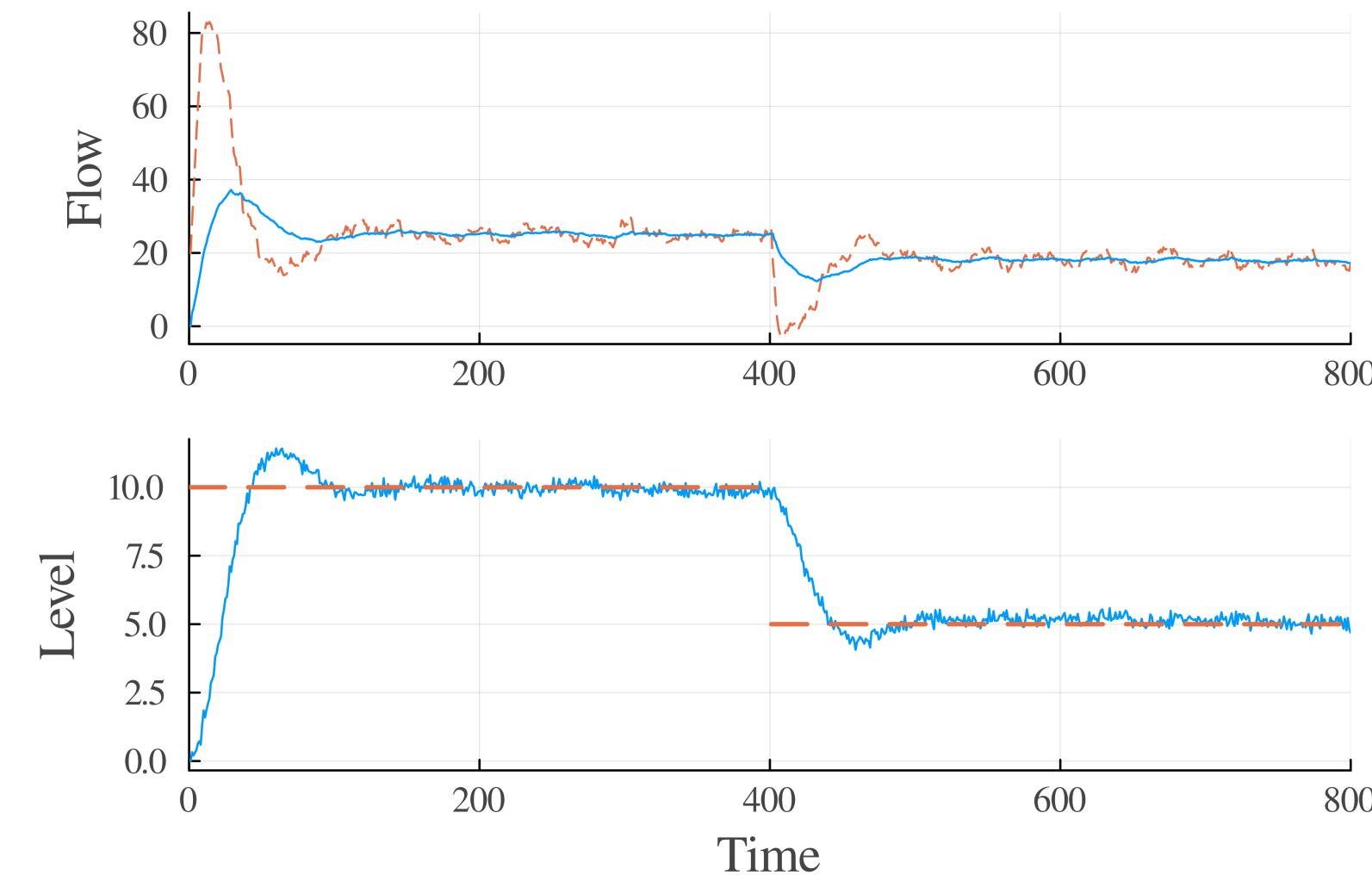
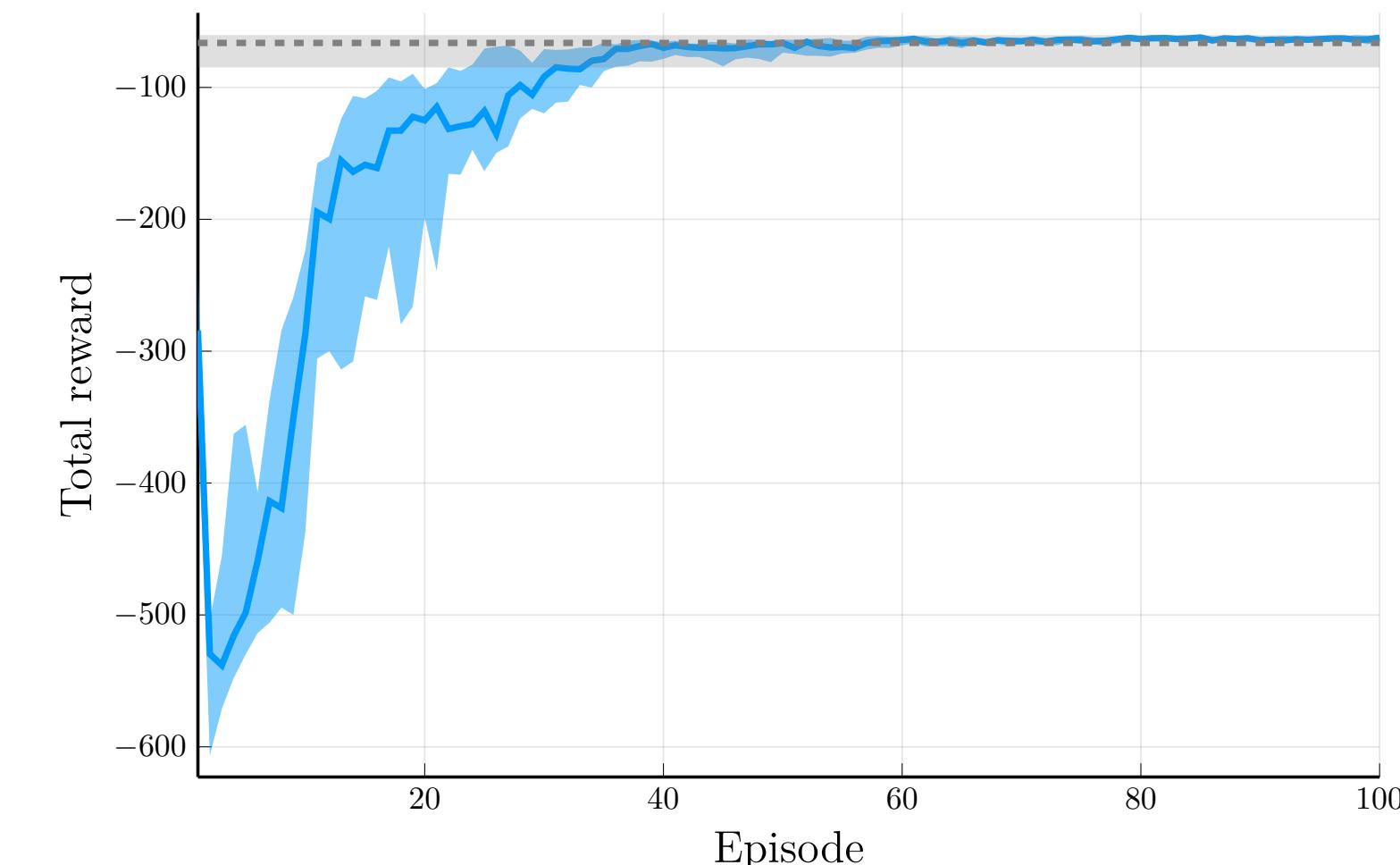
*Decouples learning and stability*

# Industrial example



Astrom, K., and A-B. Ostberg. "A teaching laboratory for process control.", 1986

- RL agent manipulates  $Q$  parameter
- End-to-end stable learning with DNN based control
  - ▶ Stable during and after training without loss in performance



# Conclusions

- Constant advances in deep RL push the boundaries of what is possible
- This success is often misaligned with industrial priorities
  - ▶ Performance is not the only metric
- We aim to preserve flexibility of general learning algorithms & maintain key system features



<https://process.honeywell.com/us/en/industries/sheet-manufacturing/pulp-and-paper>

# References

- Willems, Jan C., et al. “A note on persistency of excitation.” 2005.
- Anderson, Brian DO. “From Youla–Kucera to identification, adaptive and nonlinear control.” 1998.
- See also: Lawrence, Nathan P. “Deep reinforcement learning agents for industrial control system design.” Electronic Theses and Dissertations, University of British Columbia. 2023.

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