20 Holdout Classes Among Skelet's 43 Hardly Non-Regular Turing Machines Daniel Briggs

Abstract

github.com/danbriggs/Turing/blob/master/doc/record7-19-20.txt is where the reader is encouraged to go to get the state diagrams.

Introduction

In 2003, Skelet (Georgi Georgiev) released a 6218-line uncommented Pascal program "bbfind" that goes through the approximately 150 million (by the program's count) essentially distinct 5-state Turing machines and for each one attempts to either prove that it never halts when run on a blank tape or run it until it does. After a run of about two weeks, only 164 machines had stumped the program. Of these, 54 began with writing 0 to the tape, and so their runs were equivalent with an offset in number of steps to machines among the other 110. These 110 machines included 67 which were found to be "shift recursive" and so easily provable never to halt by hand.

Skelet dubbed the 43 remaining machines "Hardly non-regular," and although his work has never been independently verified, his list has remained the starting point for anyone interested in finishing the Busy Beaver of 5 ever since.

The 27th of these 43 machines was classified as "BL_2" by Skelet, and the other 42 were not classified. Note that for this reason, the machines I refer to as #s 28–43 are often referred to as 27–42 by other authors—including myself, previously.

From this list of 43, #2, #5, and #6 are equivalent with an offset in step number, #13 is equivalent to #12, #29 is equivalent to #23, #s 21, 28, and 39 are equivalent with an offset, and #41 is equivalent with an offset to #30; thus we are left with 36 essentially distinct Turing machines to study.

Of these 36, #s 16, 24, and 38 reach a phase (of at least 1 trillion steps) of equivalent action with an offset, but with several bits beyond what seems to be the working tape that are different; thus they have not been proved equivalent, but deserve to be talked about together. The situation is the same with machines #19 & #42. So instead of 36 machines, we talk of 33 different studies to be made.

Of these, I (and others independently) have discovered that machines 2 (and so 5 and 6), 14, 18, and 25 are trivially proved never to halt. I also proved machines #8, #9, #10, #11, and #12 (and so #13) never to halt. Machines #21 (and so #28 and #39), #31 and #32 were proved by univerz (Pavel Kropitz). univerz in fact showed the non-halting of twelve machines, but we need only reference these three for our reduction in number.

Thus there are 21 studies that need to be performed; thus this document is organized into 21 sections. Machine #4 was seen to be trivial in the course of producing this document, hence the title "20 Holdout Classes."

Contents

1	3
3	4
4	9
7	9
15	11
$16{\sim}24{\sim}38$	12
17	12
$19{\sim}42$	12
20	12
22	12
23=29	12
26	12
BL_2	12
30=41	12
33	12
34	12
35	12
36	12
37	12
40	12
43	12

The cleanest way to study HNR#3 is probably to inspect moments when the tape head is on the right; at these moments, the whole tape tends to be comprised of long runs of 1s with just a very few isolated 0s—often just one 0—in between. Ignoring step numbers when the tape head is on the right after it had also recently—within the past 10 steps—been at the right, we find that it achieves this configuration in state A with bit 1, state C with bit 1, state D with bit 1, or state C with bit 0—these results are for when the machine has reached the rightmost bit of the swath ever accessed, not just the current nonzero swath.

When it is in state A and there is only one 0 in between, it seems that both swaths of 1s, excluding the 1 at the tape head, may always be of even length. In order to get a handle on what it does by the time it reaches the right again afterwards, after leaving the right for at least 10 steps, I simulated runs starting from this type of configuration with two swaths of 1s of all possible even lengths up to 118, and recorded how many steps it took to get back to the right, how many swaths of 1s there were afterwards, how long the leftmost and rightmost of these were, and what state it ended up in—this time, "on the right" meaning at the rightmost 1. Tables 1, 2, 3, 4, and 5 give this data for swaths of 1s of even length \leq 28; Table 6 shows moments when the tape head is on the right when started on a blank tape.

43	51	85	93	133	141	187	195	247	255	313	321	385	393	463
27	33	39	45	51	57	63	69	75	81	87	93	99	105	111
119	171	179	237	245	309	317	387	395	471	479	561	569	657	665
63	69	75	81	87	93	99	105	111	117	123	129	135	141	147
273	279	285	291	297	303	309	315	321	327	333	339	345	351	357
105	111	117	123	129	135	141	147	153	159	165	171	177	183	189
275	345	353	429	437	519	527	615	623	717	725	825	833	939	947
153	159	165	171	177	183	189	195	201	207	213	219	225	231	237
527	579	587	645	653	717	725	795	803	879	887	969	977	1065	1073
207	213	219	225	231	237	243	249	255	261	267	273	279	285	291
461	549	557	651	659	759	767	873	881	993	1001	1119	1127	1251	1259
267	273	279	285	291	297	303	309	315	321	327	333	339	345	351
849	855	861	867	873	879	885	891	897	903	909	915	921	927	933
333	339	345	351	357	363	369	375	381	387	393	399	405	411	417
677	783	791	903	911	1029	1037	1161	1169	1299	1307	1443	1451	1593	1601

Table 1: Number of steps taken to once again arrive at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i$ A for $0 \le m, n \le 14$. Here m is the row number and n is the column number, both indexed from 0.

```
2
    2
        2
            2
                2
                     2
                         2
                             2
                                 2
                                      2
                                                   2
                                                       2
                                                           2
1
    1
        1
            1
                1
                     1
                         1
                             1
                                 1
                                      1
                                          1
                                              1
                                                   1
                                                       1
                                                           1
2
    2
        2
            2
                2
                     2
                         2
                             2
                                 2
                                      2
                                          2
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                                                  2
                                                       2
                                                           2
    2
        2
            2
                2
                     2
                         2
                             2
                                 2
                                      2
                                          2
                                              2
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                                                       2
2
                                                           2
                                      3
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3
    3
        3
            3
                3
                     3
                         3
                             3
                                 3
                                              3
                                                  3
                                                       3
                                                           3
2
    2
        2
            2
                2
                     2
                         2
                             2
                                 2
                                      2
                                          2
                                              2
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    2
            2
                2
                     2
                                 2
                                      2
                                          2
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                                                       2
2
        2
                         2
                             2
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                                                           2
                2
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                                          2
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2
    2
        2
            2
                     2
                         2
                             2
                                 2
                                              2
                                                   2
                                                           2
    3
        3
            3
                3
                     3
                             3
                                 3
                                      3
                                          3
                                                   3
                                                       3
3
                         3
                                              3
                                                           3
2
    2
        2
            2
                2
                     2
                         2
                             2
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2
    2
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            2
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                         2
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    2
        2
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                                                       2
                                                           2
3
    3
        3
            3
                3
                     3
                         3
                             3
                                 3
                                      3
                                          3
                                              3
                                                   3
                                                       3
                                                           3
    2
2
        2
            2
                2
                     2
                         2
                             2
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                                                           2
    2
            2
                2
                     2
                         2
                                 2
                                      2
                                          2
                                                       2
                                                           2
2
        2
                             2
                                              2
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```

Table 2: Number of swaths of 1s produced upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i$ A for $0 \le m, n \le 14$. Here m is the row number and n is the column number; both are indexed from 0. Note there seems to be no dependence on the number of 1s to the right of the 0.

Α C Α C Α C Α С Α С Α C Α C Α D D D D D D D D D D D D D D D С С Α С C C С Α С A Α C Α Α Α D С Α C Α С Α \mathbb{C} Α С Α С Α C Α C D D D D D D D D D D D D D D D С С C С Α C Α С Α С A Α Α Α C D D D D D D D D D D D D D D D С С С С C C С Α Α Α С Α Α Α Α D С С C C C Α C Α C Α Α C Α Α Α

Table 3: State of the machine upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i$ A for $0 \le m, n \le 14$. Here m is the row number and n is the column number, both indexed from 0.

```
2
     2
          4
                4
                     6
                          6
                               8
                                    8
                                         10
                                              10
                                                    12
                                                         12
                                                              14
                                                                   14
                                                                        16
10
     12
          14
               16
                    18
                         20
                               22
                                    24
                                         26
                                              28
                                                    30
                                                         32
                                                              34
                                                                   36
                                                                        38
2
     4
          4
                6
                     6
                          8
                               8
                                    10
                                         10
                                              12
                                                    12
                                                         14
                                                              14
                                                                   16
                                                                        16
     2
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2
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                               2
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                                               2
                                                    2
                                                         2
                                                              2
                                                                    2
                                                                         2
     4
          4
                4
                     4
                          4
                               4
                                    4
                                          4
                                               4
                                                    4
                                                              4
                                                                    4
                                                                         4
4
                                                         4
4
     4
          4
                4
                     4
                          4
                               4
                                    4
                                          4
                                               4
                                                    4
                                                         4
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                                                                    4
                                                                         4
4
     6
          6
                8
                     8
                         10
                               10
                                    12
                                         12
                                              14
                                                    14
                                                         16
                                                              16
                                                                   18
                                                                        18
6
     6
          6
                6
                     6
                          6
                               6
                                    6
                                          6
                                               6
                                                    6
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                                                              6
                                                                    6
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                                                                         4
4
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                4
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                                    4
                                          4
                                               4
                                                    4
                                                         4
                                                              4
                                                                    4
8
     8
          8
                8
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                               8
                                    8
                                          8
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                                                    8
                                                              8
                                                                    8
                                                                         8
                                                         8
6
     8
          8
               10
                    10
                         12
                               12
                                    14
                                         14
                                              16
                                                    16
                                                         18
                                                              18
                                                                   20
                                                                        20
                               10
                                                                        10
10
     10
          10
               10
                    10
                         10
                                    10
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                                              10
                                                    10
                                                         10
                                                              10
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10
     10
          10
               10
                    10
                         10
                               10
                                    10
                                         10
                                              10
                                                    10
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                                                              10
                                                                   10
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12
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          12
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                    12
                         12
                               12
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                                              12
                                                    12
                                                         12
                                                              12
                                                                   12
                                                                        12
8
     10
          10
               12
                    12
                         14
                               14
                                    16
                                         16
                                              18
                                                    18
                                                         20
                                                              20
                                                                   22
                                                                        22
```

Table 4: Length of the leftmost swath of 1s produced upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i$ A for $0 \le m, n \le 14$. Here m is the row number and n is the column number, both indexed from 0.

5	7	7	9	9	11	11	13	13	15	15	17	17	19	19
10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
13	13	15	15	17	17	19	19	21	21	23	23	25	25	27
12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
14	16	18	20	22	24	26	28	30	32	34	36	38	40	42
19	19	21	21	23	23	25	25	27	27	29	29	31	31	33
16	18	20	22	24	26	28	30	32	34	36	38	40	42	44
13	13	15	15	17	17	19	19	21	21	23	23	25	25	27
18	20	22	24	26	28	30	32	34	36	38	40	42	44	46
25	25	27	27	29	29	31	31	33	33	35	35	37	37	39
20	22	24	26	28	30	32	34	36	38	40	42	44	46	48
4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
31	31	33	33	35	35	37	37	39	39	41	41	43	43	45

Table 5: Length of the rightmost swath of 1s produced upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i$ A for $0 \le m, n \le 14$. Here m is the row number and n is the column number, both indexed from 0.

```
1^201^4i
                                                                                        1^{102}01^{142}i
    43
          Α
               1
                      6
                                                          114997
                                                                      Α
                                                                           1
                                                                                244
                                                                                        1^{50}01^{201}i
                            1^{13}i
    83
          D
               0
                     13
                                                          118151
                                                                      D
                                                                           1
                                                                                251
                            1^{8}01^{10}i
                                                                                        1^{24}01^{233}i
                                                                                257
   268
          Α
               1
                     18
                                                          119604
                                                                      D
                                                                           1
                            1^401^{13}01^{13}i
                                                                                        1^{10}01^{23}01^{235}i
                                                                           2
                                                                                268
   572
          D
               2
                     30
                                                          121147
                                                                      D
                                                                                        1^{130}01^{142}i
                            1^{14}01^{20}i
   923
                     34
                                                                                272
          Α
               1
                                                          137166
                                                                      С
                                                                           1
                                                                                        1^{132}01^{140}o
                            1^{6}01^{35}i
                                                                                272
 1137
          D
               1
                     41
                                                          137602
                                                                      C
                                                                           1
                                                                                        1^{104}01^{176}i
                            1^201^{45}i
 1300
          D
               1
                     47
                                                          159556
                                                                      Α
                                                                           1
                                                                                280
                                                                                        1^{40}01^{73}01^{179}i
                            1^{53}i
 1457
          D
               0
                     53
                                                          167144
                                                                      D
                                                                           2
                                                                                292
                            1^{28}01^{30}i
                                                                                        1^{132}01^{164}i
 2512
                     58
                                                          184393
                                                                      С
                                                                           1
                                                                                296
          Α
               1
                                                                                        1^{134}01^{162}o
                            1^{24}01^{44}i
               1
                     68
                                                                           1
                                                                                296
 4262
          Α
                                                          184895
                                                                      С
                            1^{10}01^{23}01^{47}i
                                                                                        1^{66}01^{235}i
                                                                                301
 5244
          D
               2
                     80
                                                          189755
                                                                           1
                                                                      D
           C
                            1^{36}01^{48}i
                                                                                307
                                                                                        1^{32}01^{275}i
 6881
               1
                     84
                                                          191778
                                                                           1
                                                                      D
 7035
                            1^{38}01^{46}o
                                                          193873
                                                                           3
                                                                                323
                                                                                        1^{4}01^{23}01^{19}01^{277}i
          С
               1
                     84
                                                                      D
                                                                                        1^{4}01^{165}01^{158}i
                            1^{18}01^{71}i
 7731
          D
               1
                     89
                                                          214312
                                                                      Α
                                                                           2
                                                                                327
                            1^801^{87}i
                                                                                        1^{86}01^{246}i
 8146
          D
                     95
                                                          240390
                                                                      C
                                                                                332
               1
                                                                           1
                            1^{4}01^{13}01^{89}i
                                                                                        1^{88}01^{244}o
               2
                                                                                332
 8675
          D
                    106
                                                          241138
                                                                      C
                                                                           1
                            1^{52}01^{58}i
                                                                                        1^{34}01^{63}01^{245}i
11762
                    110
                                                          247234
                                                                      D
                                                                           2
                                                                                342
          Α
               1
                            1^{44}01^{76}i
                                                                                        1^{160}01^{186}i
16410
          Α
               1
                    120
                                                          272173
                                                                           1
                                                                                346
                                                                      Α
                            1^{50}01^{80}i
                                                                                        1^{16}01^{75}01^{83}01^{189}i
                    130
21486
          \mathbb{C}
                                                          286901
                                                                           3
                                                                                363
               1
                                                                      D
                                                                                        1^{16}01^{173}01^{178}i
                            1^{52}01^{78}o
21736
          C
               1
                    130
                                                          307178
                                                                           2
                                                                                367
                                                                      Α
                                                                                        1^{108}01^{264}i
                            1^{52}01^{86}i
27456
          С
               1
                    138
                                                          338230
                                                                      C
                                                                           1
                                                                                372
                                                                                        1^{110}01^{262}o
27724
                            1^{54}01^{84}o
          C
                    138
                                                                                372
               1
                                                          339032
                                                                      С
                                                                           1
                            1^{26}01^{117}i
                                                                                377
                                                                                        1^{54}01^{323}i
                    143
28930
          D
               1
                                                          342914
                                                                      D
                                                                           1
                            1^{12}01^{137}i
                                                                                        1^{26}01^{357}i
                    149
                                                                                383
29609
          D
               1
                                                          344841
                                                                      D
                                                                           1
                            1^{72}01^{86}i
                                                                                        1^{12}01^{377}i
          C
                    158
                                                                           1
                                                                                389
35900
               1
                                                          346240
                                                                      D
          С
               1
                    158
                            1^{74}01^{84}o
                                                          380071
                                                                      C
                                                                           1
                                                                                398
                                                                                        1^{192}01^{206}i
36168
                                                                                        1^{194}01^{204}o
                            1^{36}01^{127}i
                    163
                                                                                398
38004
          D
               1
                                                          380699
                                                                      С
                                                                           1
                                                                                        1^{96}01^{307}i
                            1^{74}01^{98}i
                                                                                403
46117
           Α
               1
                    172
                                                          389825
                                                                      D
                                                                           1
                                                                                        1^{10}01^{49}01^{51}01^{309}i
                            1^{36}01^{143}i
48005
          D
               1
                    179
                                                          396816
                                                                      D
                                                                           3
                                                                                419
                                                                                        1^{10}01^{207}01^{206}i
                            1^{82}01^{106}i
                    188
                                                                                423
57450
                                                          429039
                                                                           2
          Α
               1
                                                                      Α
                            1^{40}01^{155}i
                                                                                428
                                                                                        1^{116}01^{312}i
                                                                      С
59656
               1
                    195
                                                          471335
                                                                           1
          D
                                                                                        1^{118}01^{310}o
                            1^{16}01^{33}01^{157}i
               2
                    206
                                                                                428
61763
          D
                                                          472281
                                                                      C
                                                                           1
                            1^{98}01^{112}i
                                                                                        1^{58}01^{375}i
                                                                                433
71594
          Α
               1
                    210
                                                          476709
                                                                      D
                                                                           1
                            1^{48}01^{169}i
                                                                                        1^{28}01^{411}i
74478
          D
               1
                    217
                                                          478906
                                                                      D
                                                                           1
                                                                                439
                            1^{10}01^{109}01^{112}i
                                                                                        1^{214}01^{234}i
86025
          C
               2
                    231
                                                          522833
                                                                           1
                                                                                448
                                                                      Α
                            1^{10}01^{111}01^{110}o
                                                                                        1^{106}01^{349}i
86371
           С
               2
                    231
                                                          533739
                                                                      D
                                                                                455
                                                                           1
                                                                                        1^{52}01^{409}i
                            1^{68}01^{166}i
                    234
                                                                                461
99063
          Α
               1
                                                          537694
                                                                      D
                                                                           1
```

Table 6: The step numbers when the tape head was at the rightmost bit of the swath ever accessed after leaving the right for at least 10 steps; what state the machine was in; how many intermediate zeros there were; how many 1s there were other than at the tape head; what the tape looked like. Run started on a blank tape.

Repeating the analysis done in Tables 1, 2, 3, 4, and 5, but with the machine starting in state C instead of A gives much simpler data: there are always two swaths; the number of steps and lengths of the left and right swaths are linear functions of the lengths of the original swaths; the machine ends up in state D as long as the right swath has at least six 1s. The machine starting in state D is exactly analogous to starting in state A, but the tape head is one bit further to the right and the step number is advanced by three. C with tape head at a 0 bit always comes from D with tape head at a 1 bit but otherwise of the same form.

Thus, it would suffice to understand the patterns shown in Tables 1, 2, 3, 4, and 5 to predict the action of HNR#3 from situations consisting of at most two swaths of 1s. However, situations with more than two swaths as shown in Table 6 may have to be understood separately, and could increase the difficulty of showing that HNR#3 never halts or accelerating it until it does.

Out of the first billion steps, the maximum number of swaths of 1s when the tape head is on the right is 6, and this first occurs at step 6333029, when the machine is in state D, and the tape is $1^4 \ 0 \ 1^{23} \ 0 \ 1^{53} \ 0 \ 1^{197} \ 0 \ 1^{259} \ 0 \ 1^{550}$.

An argument backtracking from a halt state may be capable of sidestepping all this detailed analysis; it is clear from the machine diagram

```
Α
          В
                C
                      D
                            Ε
0
   C1L
         H1L
               D1R
                     A1R
                           COL
1
   AOR
         E1L
               BOL
                     C1R
                           D1L
```

that 010 A and 010 E are the only ways to achieve halting; these must come from 000 D, 010i B, respectively; the former is a dead-end, whereas the latter comes from 0101i C; unfortunately, this branches into three possibilities: 010110 A, 01001 D, and 010110 E. These respectively come from 010100 D, 0100i E, and 010110i B; these, from 010i00 C, 01001i B, and 0101101i C;

4

Machine 4 seems to be a trivial one, the proof just needs to be carried out. It counts up in binary using 11 as 0 and 10 as 1, with less significant digits to the left, on the left side of the tape, with an additional oll at the left end in C. Then after a number of steps increasing by a constant difference of 12 modified by four times the location in the number of the 0 to be turned into a 1 (increasing when that bit is *closer*), it produces the next number, written in the same binary scheme, with an extra 0 as padding on the right before its bookend 011. For example,

```
882
   C
             o11101011101111111111111111111111
                                   11.
                                      Then after 145,
1027
           o111111101011111111111111111111111
    C
                                   12.
                                      Then after 165 (+20 3/1),
1192
    C
          13.
                                      Then after 173 (+8 \ 1/2),
1365
    C
         14.
                                      Then after 189 (+16 2/1),
1554
    C
       15.
                                      Then after 185 (-4 1/5),
1739
                                      Then after 213 (+28).
      16.
```

Table 7: Machine 4. The numbers before and after the slash portray which bit in is to be turned from a 0 to a 1 previously/currently. The difference between the two numbers determines the multiple of 4 offset from 12 more the step difference will be above the previous step difference.

33	Α	$1^20^11^2$	105757	D	$1^{155}0^11^{81}0^11^4$
57	С	1^{10}	120999	Α	$1^{161}0^11^{84}$
188	D	$1^90^11^6$	121489	Α	$1^{158}0^11^{86}$
276	С	$1^{20}0^11^2$	123997	С	$1^{208}0^{1}1^{42}$
355	С	1^{28}	125270	С	$1^{236}0^11^{20}$
746	Α	$1^{19}0^11^{14}$	141651	D	$1^{141}0^11^{124}$
810	Α	$1^{16}0^11^{16}$	162381	Α	$1^{173}0^11^{102}$
1602	D	$1^{19}0^11^{23}0^11^4$	162907	Α	$1^{170}0^11^{104}$
2234	Α	$1^{35}0^11^{16}$	170467	С	$1^{172}0^11^{73}0^11^{40}$
2346	Α	$1^{32}0^11^{18}$	186702	Α	$1^{161}0^11^{128}$
2640	С	$1^{48}0^11^8$	187192	Α	$1^{158}0^11^{130}$
3049	С	$1^{50}0^11^{13}0^11^4$	191812	C	$1^{230}0^11^{64}$
4426	D	$1^{39}0^11^{32}$	211699	Α	$1^{151}0^11^{135}0^11^{23}0^11^4$
5816	С	$1^{42}0^11^{19}0^11^{23}0^11^4$	212159	Α	$1^{148}0^11^{137}0^11^{23}0^11^4$
7139	D	$1^{41}0^11^{47}0^11^4$	232791	Α	$1^{213}0^11^{99}0^11^4$
9473	D	$1^{69}0^{1}1^{28}$	233437	Α	$1^{210}0^{1}1^{101}0^{1}1^{4}$
12905	Α	$1^{65}0^{1}1^{42}$	259509	D	$1^{207}0^11^{112}$
13107	Α	$1^{62}0^11^{44}$	284371	D	$1^{163}0^11^{149}0^11^{22}$
17183	Α	$1^{73}0^{1}1^{42}$	309195	Α	$1^{233}0^11^{106}$
17409	Α	$1^{70}0^{1}1^{44}$	309901	Α	$1^{230}0^11^{108}$
21971	Α	$1^{77}0^11^{46}$	340719	Α	$1^{205}0^11^{142}$
22209	Α	$1^{74}0^11^{48}$	341341	Α	$1^{202}0^11^{144}$
26915	Α	$1^{65}0^11^{61}0^11^{10}$	370731	Α	$1^{177}0^11^{155}0^11^{28}$
27117	Α	$1^{62}0^{1}1^{63}0^{1}1^{10}$	371269	Α	$1^{174}0^11^{157}0^11^{28}$
31493	D	$1^{95}0^11^{44}$	399339	D	$1^{245}0^{1}1^{118}$
37941	D	$1^{89}0^11^{60}$	403579	С	$1^{312}0^11^{58}$
45261	Α	$1^{99}0^11^{60}$	405584	С	$1^{348}0^11^{28}$
45565	Α	$1^{96}0^11^{62}$	438615	D	$1^{203}0^11^{182}$
47041	С	$1^{134}0^11^{30}$	446809	С	$1^{302}0^11^{90}$
47840	С	$1^{156}0^11^{14}$	449912	С	$1^{354}0^11^{44}$
48453	С	$1^{170}0^11^6$	487277	Α	$1^{219}0^11^{188}$
49018	С	$1^{180}0^11^2$	487941	Α	$1^{216}0^{1}1^{190}$
49577	С	1^{188}	496785	С	$1^{318}0^11^{94}$
57768	Α	$1^{99}0^11^{94}$	500104	С	$1^{372}0^11^{46}$
58072	Α	$1^{96}0^11^{96}$	501965	С	$1^{402}0^11^{22}$
64426	С	$1^{98}0^{1}1^{51}0^{1}1^{49}0^{1}1^{10}$	503430	С	$1^{420}0^11^{10}$
70891	D	$1^{101}0^11^{101}0^11^{10}$	504787	С	$1^{432}0^11^4$
81739	D	$1^{153}0^11^{64}$	546044	D	$1^{227}0^11^{218}$
93763	A	$1^{113}0^{1}1^{97}0^{1}1^{23}0^{1}1^{4}$	557280	C	$1^{344}0^{1}1^{108}$
94109	Α	$1^{110}0^{1}1^{99}0^{1}1^{23}0^{1}1^{4}$	608971	D	$1^{261}0^{1}1^{200}$
			· -		-

15

```
31167
  +36
  31203
                   +64
                     =*2+-8
  31267
                   +140
                     =*2+12
  31407
                   +296
                     =*2+16
  31703
                   +568
                     =*2+-24
  32271
                   +1144
                     =*2+8
33415
  +2296
                     =*2+8
  35711
                   +4604
                     =*2+12
  40315
                   +9212
                     =*2+4
49527
  +18436
                     =*2+12
  67963
                   +36856
                    =*2+-16
  104819
                   +73720
                     =*2+8
  178539
                  +147452
                     =*2+12
325991
  +294912
                     =*2+8
620903
  +589816
                     =*2+-8
  1210719
                  +1179644
                     =*2+12
  +2359292
2390363
                     =*2+4
4749655
  +4718588
                     =*2+4
  9468243
                  +9437184
                    =*2+8
```

Table 9: Machine 15.

 $16{\sim}24{\sim}38$

 $19{\sim}42$

23=29

 BL_{-2}

30=41