The 21—or only 10—Holdout Classes Among Skelet's 43 Hardly Non-Regular Turing Machines

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Introduction

In 2003, Skelet (Georgi Georgiev) released a 6218-line uncommented Pascal program "bbfind" that goes through the approximately 150 million (by the program's count) essentially distinct 5-state Turing machines and for each one attempts to either prove that it never halts when run on a blank tape or run it until it does. After a run of about two weeks, only 164 machines had stumped the program. Of these, 54 began with writing 0 to the tape, and so their runs were equivalent with an offset in number of steps to machines among the other 110. These 110 machines included 67 which were found to be "shift recursive" and so easily provable never to halt by hand.

Skelet dubbed the 43 remaining machines "Hardly non-regular," and although his work has never been independently verified, his list has remained the starting point for anyone interested in finishing the Busy Beaver of 5 ever since.

The 27th of these 43 machines was classified as "BL_2" by Skelet, and the other 42 were not classified. Note that for this reason, the machines I refer to as #s 28–43 are often referred to as 27–42 by other authors—including myself, previously.

From this list of 43, #2, #5, and #6 are equivalent with an offset in step number, #13 is equivalent to #12, #29 is equivalent to #23, #s 21, 28, and 39 are equivalent with an offset, and #41 is equivalent with an offset to #30; thus we are left with 36 essentially distinct Turing machines to study.

Of these 36, #s 16, 24, and 38 reach a phase (of at least 1 trillion steps) of equivalent action with an offset, but with several bits beyond what seems to be the working tape that are different; thus they have not been proved equivalent, but deserve to be talked about together. The situation is the same with machines #19 & #42. So instead of 36 machines, we talk of 33 different studies to be made.

Of these, I (and others independently) have discovered that machines 2 (and so 5 and 6), 14, 18, and 25 are trivially proved never to halt. I also proved machines #8, #9, #10, #11, and #12 (and so #13) never to halt. Machines #21 (and so #28 and #39), #31 and #32 were proved by univerz (Pavel Kropitz). univerz in fact showed the non-halting of twelve machines, but we need only reference these three for our reduction in number.

Thus there are 21 studies that need to be performed; thus this document is organized into 21 sections. Machines #4, 16∼24∼38, 19∼42, 20, 23=29, 30=41, 36, 37, 40, and 43 showed that they will be of various degrees of easy but arduous in the course of producing this document, and in fact a proof for #36 is included herein. #s 34 and #35 showed that they should be considered equivalent to each other with proof. Of the 10 remaining classes, #BL_2 stands out as likely very easy, and #15 may well be easy as well.

Contents

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86	D	$(110)^3(10)^1 1i$
418	D	$(110)^3(10)^2(110)^6(10)^1 1i$
1046	D	$(110)^1(10)^1(110)^7(10)^2(110)^2(10)^1(110)^3(10)^1 1i$
3536	D	$(110)^{11}(10)^2(110)^4(10)^1(110)^3(10)^3(110)^2(10)^1(110)^{12}(10)^1 1i$
15546	D	$(110)^{39}(10)^2(110)^{12}(10)^2(110)^8(10)^1(110)^{16}(10)^1(110)^3(10)^1 1i$
54098	D	$(110)^{91}(10)^1(110)^1(10)^1(110)^{11}(10)^2(110)^8(10)^1(110)^3(10)^2(110)^6(10)^1 1i$
79572	D	$(110)^{89}(10)^1(110)^{11}(10)^1(110)^{28}(10)^1(110)^7(10)^2(110)^2(10)^1(110)^3(10)^1 1i$
2422302	D	$(110)^{139}(10)^2(110)^{634}(10)^2(110)^{76}(10)^2(110)^6(10)^1(110)^{11}(10)^1(110)^{12}(10)^1 1i$
3027880	D	$(110)^{331}(10)^2(110)^{586}(10)^2(110)^{64}(10)^3(110)^{11}(10)^1(110)^{11}(10)^1(110)^3(10)^1 1i$

The above shows steps when HNR#1 is on the right edge of the working tape. There are no other such steps among the first billion steps, other than nearly adjacent steps. HNR#1 has been perhaps the most intensely studied of all of Skelet's 43 machines over the years. It turns out that it is certainly among the hardest nine to solve, and could well be the hardest of all. My notes regarding HNR#1 are scattered among old computers, dead forums, and looseleaf paper, so it would not be appropriate to try to do the study of HNR#1 justice here before first compiling these notes.

Although steps when it is at the right are displayed above, it is more common to study HNR#1 using steps when it is at the left (see figure on the following page). Suffice it to say that it at first appears to be counting up in quaternary in its own representations of 0, 1, 2, and 3, but it is constantly truncating the amount of working space it gives itself to count up, and when it runs out of space to increment a number, it can end up in a variety of different "phases."

So new representations of what can appear on stretches of tape are defined, and replacement rules for strings of those symbols are striven for. E.g., it leaves the left side and comes back between the following steps:

3284 A $o10(110)^{10}1010(110)^4 10(110)^3 101010(110)^3 1010(110)^{10}1111$

3822 A $o10(110)^8 10110100(110)^3 10(110)^2 11110101101001(110)^2 10(110)^{11}1011010011$,

So one begins by letting say $X = 110$ and some other alphabet assignments and attempts to discover and prove the replacement rules among strings of them.

(HNR#1 uses a lot of 110 and 10 when it is at the left as well as the right, but for HNRs in general it is of course not the case that the same pieces will be found when the tape head is to either side.)

This sound daunting, but it is always possible that a very general lemma that does not need to take into account the particulars of the string can be got; in other words, "whenever HNR#1 has tape head at the left end of the tape and the tape is composed of such-and-such strings in any quantity in any order, it always reaches another such configuration in the future." See the proof for HNR#36 herein for comparison.

On the other hand, if its halting depends on some fine detail, one could say that its proof is still likely quite a ways off.

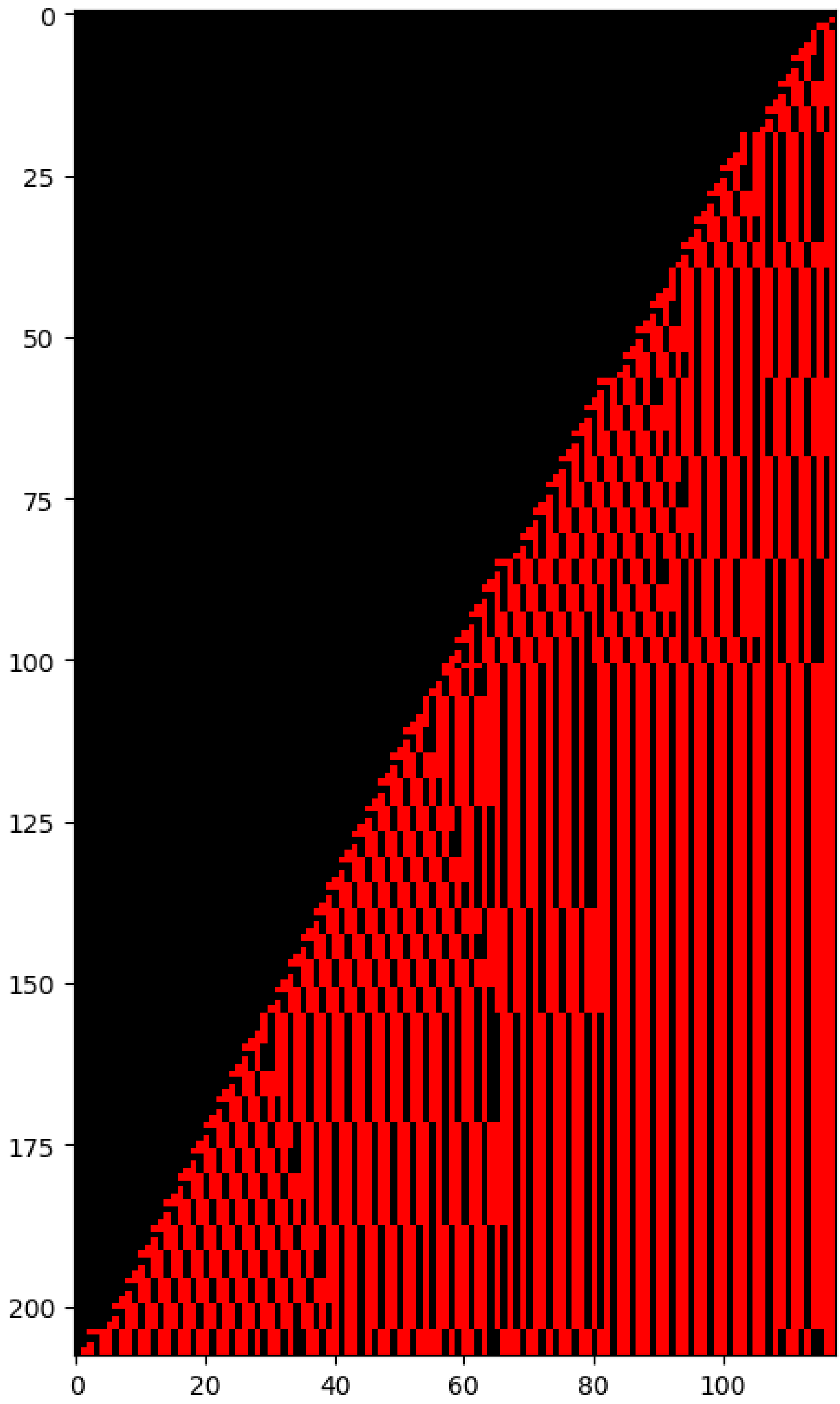

Figure 1: Steps through 3824 when HNR#1 is on the left edge of the working tape. Red means 1 and black means 0, and all such figures in this document are right-justified or left-justified, so it should not be assumed that bits are in the exact same location from one such step to the next.

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The cleanest way to study HNR#3 is probably to inspect moments when the tape head is on the right; at these moments, the whole tape tends to be comprised of long runs of 1s with just a very few isolated 0s—often just one 0—in between. Ignoring step numbers when the tape head is on the right after it had also recently—within the past 10 steps—been at the right, we find that it achieves this configuration in state A with bit 1, state C with bit 1, state D with bit 1, or state C with bit 0—these results are for when the machine has reached the rightmost bit of the swath ever accessed, not just the current nonzero swath.

When it is in state A and there is only one 0 in between, it seems that both swaths of 1s, excluding the 1 at the tape head, may always be of even length. In order to get a handle on what it does by the time it reaches the right again afterwards, after leaving the right for at least 10 steps, I simulated runs starting from this type of configuration with two swaths of 1s of all possible even lengths up to 118, and recorded how many steps it took to get back to the right, how many swaths of 1s there were afterwards, how long the leftmost and rightmost of these were, and what state it ended up in—this time, "on the right" meaning at the rightmost 1. Tables 1, 2, 3, 4, and 5 give this data for swaths of 1s of even length ≤ 28; Table 6 shows moments when the tape head is on the right when started on a blank tape.

43	51	85	93	133	141	187	195	247	255	313	321	385	393	463
27	33	39	45	51	57	63	69	75	81	87	93	99	105	111
119	171	179	237	245	309	317	387	395	471	479	561	569	657	665
63	69	75	81	87	93	99	105	111	117	123	129	135	141	147
273	279	285	291	297	303	309	315	321	327	333	339	345	351	357
105	111	117	123	129	135	141	147	153	159	165	171	177	183	189
275	345	353	429	437	519	527	615	623	717	725	825	833	939	947
153	159	165	171	177	183	189	195	201	207	213	219	225	231	237
527	579	587	645	653	717	725	795	803	879	887	969	977	1065	1073
207	213	219	225	231	237	243	249	255	261	267	273	279	285	291
461	549	557	651	659	759	767	873	881	993	1001	1119	1127	1251	1259
267	273	279	285	291	297	303	309	315	321	327	333	339	345	351
849	855	861	867	873	879	885	891	897	903	909	915	921	927	933
333	339	345	351	357	363	369	375	381	387	393	399	405	411	417
677	783	791	903	911	1029	1037	1161	1169	1299	1307	1443	1451	1593	1601

Table 1: Number of steps taken to once again arrive at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i\ A$ for $0 \leq m, n \leq 14$. Here m is the row number and n is the column number, both indexed from 0.

2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
3	3	3	3	3	3	3	3	3	3	3	3	3	3	3
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2

Table 2: Number of swaths of 1s produced upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i\ A$ for $0 \le m, n \le 14$. Here m is the row number and n is the column number; both are indexed from 0. Note there seems to be no dependence on the number of 1s to the right of the 0.

A	C	A	C	A	C	A	C	A	C	A	C	A	C	A
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
C	A	C	A	C	A	C	A	C	A	C	A	C	A	C
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
D	D	D	D	D	D	D	D	D	D	D	D	D	D	D
C	A	C	A	C	A	C	A	C	A	C	A	C	A	C

Table 3: State of the machine upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i\ A$ for $0 \le m, n \le 14$. Here m is the row number and n is the column number, both indexed from 0.

2	2	4	4	6	6	8	8	10	10	12	12	14	14	16
10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
2	4	4	6	6	8	8	10	10	12	12	14	14	16	16
2	2	2	2	2	2	2	2	2	2	2	2	2	2	2
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
4	6	6	8	8	10	10	12	12	14	14	16	16	18	18
6	6	6	6	6	6	6	6	6	6	6	6	6	6	6
4	4	4	4	4	4	4	4	4	4	4	4	4	4	4
8	8	8	8	8	8	8	8	8	8	8	8	8	8	8
6	8	8	10	10	12	12	14	14	16	16	18	18	20	20
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
10	10	10	10	10	10	10	10	10	10	10	10	10	10	10
12	12	12	12	12	12	12	12	12	12	12	12	12	12	12
8	10	10	12	12	14	14	16	16	18	18	20	20	22	22

Table 4: Length of the leftmost swath of 1s produced upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i\ A$ for $0 \leq m, n \leq 14$. Here m is the row number and n is the column number, both indexed from 0.

5	7	7	9	9	11	11	13	13	15	15	17	17	19	19
10	12	14	16	18	20	22	24	26	28	30	32	34	36	38
13	13	15	15	17	17	19	19	21	21	23	23	25	25	27
12	14	16	18	20	22	24	26	28	30	32	34	36	38	40
4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
14	16	18	20	22	24	26	28	30	32	34	36	38	40	42
19	19	21	21	23	23	25	25	27	27	29	29	31	31	33
16	18	20	22	24	26	28	30	32	34	36	38	40	42	44
13	13	15	15	17	17	19	19	21	21	23	23	25	25	27
18	20	22	24	26	28	30	32	34	36	38	40	42	44	46
25	25	27	27	29	29	31	31	33	33	35	35	37	37	39
20	22	24	26	28	30	32	34	36	38	40	42	44	46	48
4	6	8	10	12	14	16	18	20	22	24	26	28	30	32
22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
31	31	33	33	35	35	37	37	39	39	41	41	43	43	45

Table 5: Length of the rightmost swath of 1s produced upon arriving at the rightmost 1 bit after leaving for at least 10 steps starting with configuration $1^{2m}01^{2n}i\ A$ for $0 \leq m, n \leq 14$. Here m is the row number and n is the column number, both indexed from 0.

43	A	1	6	$1^{2}01^{4}i$
83	D	0	13	$1^{13}i$
268	A	1	18	$1^{8}01^{10}i$
572	D	2	30	$1^{4}01^{13}01^{13}i$
923	A	1	34	$1^{14}01^{20}i$
1137	D	1	41	$1^{6}01^{35}i$
1300	D	1	47	$1^{2}01^{45}i$
1457	D	0	53	$1^{53}i$
2512	A	1	58	$1^{28}01^{30}i$
4262	A	1	68	$1^{24}01^{44}i$
5244	D	2	80	$1^{10}01^{23}01^{47}i$
6881	C	1	84	$1^{36}01^{48}i$
7035	C	1	84	$1^{38}01^{46}o$
7731	D	1	89	$1^{18}01^{71}i$
8146	D	1	95	$1^{8}01^{87}i$
8675	D	2	106	$1^{4}01^{13}01^{89}i$
11762	A	1	110	$1^{52}01^{58}i$
16410	A	1	120	$1^{44}01^{76}i$
21486	C	1	130	$1^{50}01^{80}i$
21736	C	1	130	$1^{52}01^{78}o$
27456	C	1	138	$1^{52}01^{86}i$
27724	C	1	138	$1^{54}01^{84}o$
28930	D	1	143	$1^{26}01^{117}i$
29609	D	1	149	$1^{12}01^{137}i$
35900	C	1	158	$1^{72}01^{86}i$
36168	C	1	158	$1^{74}01^{84}o$
38004	D	1	163	$1^{36}01^{127}i$
46117	A	1	172	$1^{74}01^{98}i$
48005	D	1	179	$1^{36}01^{143}i$
57450	A	1	188	$1^{82}01^{106}i$
59656	D	1	195	$1^{40}01^{155}i$
61763	D	2	206	$1^{16}01^{33}01^{157}i$
71594	A	1	210	$1^{98}01^{112}i$
74478	D	1	217	$1^{48}01^{169}i$
86025	C	2	231	$1^{10}01^{109}01^{112}i$
86371	C	2	231	$1^{10}01^{111}01^{110}o$
99063	A	1	234	$1^{68}01^{166}i$
114997	A	1	244	$1^{102}01^{142}i$
118151	D	1	251	$1^{50}01^{201}i$
119604	D	1	257	$1^{24}01^{233}i$
121147	D	2	268	$1^{10}01^{23}01^{235}i$
137166	C	1	272	$1^{130}01^{142}i$
137602	C	1	272	$1^{132}01^{140}o$
159556	A	1	280	$1^{104}01^{176}i$
167144	D	2	292	$1^{40}01^{73}01^{179}i$
184393	C	1	296	$1^{132}01^{164}i$
184895	C	1	296	$1^{134}01^{162}o$
189755	D	1	301	$1^{66}01^{235}i$
191778	D	1	307	$1^{32}01^{275}i$
193873	D	3	323	$1^{4}01^{23}01^{19}01^{277}i$
214312	A	2	327	$1^{4}01^{165}01^{158}i$
240390	C	1	332	$1^{86}01^{246}i$
241138	C	1	332	$1^{88}01^{244}o$
247234	D	2	342	$1^{34}01^{63}01^{245}i$
272173	A	1	346	$1^{160}01^{186}i$
286901	D	3	363	$1^{16}01^{75}01^{83}01^{189}i$
307178	A	2	367	$1^{16}01^{173}01^{178}i$
338230	C	1	372	$1^{108}01^{264}i$
339032	C	1	372	$1^{110}01^{262}o$
342914	D	1	377	$1^{54}01^{323}i$
344841	D	1	383	$1^{26}01^{357}i$
346240	D	1	389	$1^{12}01^{377}i$
380071	C	1	398	$1^{192}01^{206}i$
380699	C	1	398	$1^{194}01^{204}o$
389825	D	1	403	$1^{96}01^{307}i$
396816	D	3	419	$1^{10}01^{49}01^{51}01^{309}i$
429039	A	2	423	$1^{10}01^{207}01^{206}i$
471335	C	1	428	$1^{116}01^{312}i$
472281	C	1	428	$1^{118}01^{310}o$
476709	D	1	433	$1^{58}01^{375}i$
478906	D	1	439	$1^{28}01^{411}i$
522833	A	1	448	$1^{214}01^{234}i$
533739	D	1	455	$1^{106}01^{349}i$
537694	D	1	461	$1^{52}01^{409}i$

Table 6: The step numbers when the tape head was at the rightmost bit of the swath ever accessed after leaving the right for at least 10 steps; what state the machine was in; how many intermediate zeros there were; how many 1s there were other than at the tape head; what the tape looked like. Run started on a blank tape.

Repeating the analysis done in Tables 1, 2, 3, 4, and 5, but with the machine starting in state C instead of A gives much simpler data: there are always two swaths; the number of steps and lengths of the left and right swaths are linear functions of the lengths of the original swaths; the machine ends up in state D as long as the right swath has at least six 1s. The machine starting in state D is exactly analogous to starting in state A, but the tape head is one bit further to the right and the step number is advanced by three. C with tape head at a 0 bit always comes from D with tape head at a 1 bit but otherwise of the same form.

Thus, it would suffice to understand the patterns shown in Tables 1, 2, 3, 4, and 5 to predict the action of HNR#3 from situations consisting of at most two swaths of 1s. However, situations with more than two swaths as shown in Table 6 may have to be understood separately, and could increase the difficulty of showing that HNR#3 never halts or accelerating it until it does.

Out of the first billion steps, the maximum number of swaths of 1s when the tape head is on the right is 6, and this first occurs at step 6333029, when the machine is in state D, and the tape is $1^4\ 0\ 1^{23}\ 0\ 1^{53}\ 0\ 1^{197}\ 0\ 1^{259}\ 0\ 1^{550}$.

The table also shows quite a few pairs of steps where the first of two 1's exponents is halved and one subtracted; the criteria under which this does or does not occur are not clear. For this reason, HNR#3 could easily prove to be as difficult as say the Collatz conjecture.

An argument backtracking from a halt state may be capable of sidestepping all this detailed analysis, but this has so far proven not to be the case with any of the HNRs (although see HNR#36 where halting would be achieved iff an even number of 0s appeared between the tape head and the first 1 in state D while it always has an odd number of 0s in that position.).

4

Machine 4 seems to be a trivial one, the proof just needs to be carried out. It counts up in binary using 11 as 0 and 10 as 1, with less significant digits to the left, on the left side of the tape, with an additional o11 at the left end in C. Then after a number of steps increasing by a constant difference of 12 modified by four times the location in the number of the 0 to be turned into a 1 (increasing when that bit is *closer*), it produces the next number, written in the same binary scheme, with an extra number 0 (again, written 11) as padding on the right before the ending 011. For example,

```
 882  C  o1110101110111111111111111011           11.  Then after 145,
1027  C  o111111101011111111111111111011         12.  Then after 165 (+20 3/1),
1192  C  o11101110101111111111111111111011       13.  Then after 173 (+8 1/2),
1365  C  o1111101010111111111111111111111011     14.  Then after 189 (+16 2/1),
1554  C  o111010101011111111111111111111111011   15.  Then after 185 (-4 1/5),
1739  C  o11111111111011111111111111111111111011 16.  Then after 213 (+28).
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Table 7: Machine 4. The numbers before and after the slash portray which bit in is to be turned from a 0 to a 1 previously/currently. The difference between the two numbers determines the multiple of 4 offset from 12 more the step difference will be above the previous step difference.

Thus, as it is creating more space for itself to count up in far faster than it is counting up, it will never halt.

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33	A	$1^{2}0^{1}1^{2}$	105757	D	$1^{155}0^{1}1^{81}0^{1}1^{4}$
57	C	1^{10}	120999	A	$1^{161}0^{1}1^{84}$
188	D	$1^{9}0^{1}1^{6}$	121489	A	$1^{158}0^{1}1^{86}$
276	C	$1^{20}0^{1}1^{2}$	123997	C	$1^{208}0^{1}1^{42}$
355	C	1^{28}	125270	C	$1^{236}0^{1}1^{20}$
746	A	$1^{19}0^{1}1^{14}$	141651	D	$1^{141}0^{1}1^{124}$
810	A	$1^{16}0^{1}1^{16}$	162381	A	$1^{173}0^{1}1^{102}$
1602	D	$1^{19}0^{1}1^{23}0^{1}1^{4}$	162907	A	$1^{170}0^{1}1^{104}$
2234	A	$1^{35}0^{1}1^{16}$	170467	C	$1^{172}0^{1}1^{73}0^{1}1^{40}$
2346	A	$1^{32}0^{1}1^{18}$	186702	A	$1^{161}0^{1}1^{128}$
2640	C	$1^{48}0^{1}1^{8}$	187192	A	$1^{158}0^{1}1^{130}$
3049	C	$1^{50}0^{1}1^{13}0^{1}1^{4}$	191812	C	$1^{230}0^{1}1^{64}$
4426	D	$1^{39}0^{1}1^{32}$	211699	A	$1^{151}0^{1}1^{135}0^{1}1^{23}0^{1}1^{4}$
5816	C	$1^{42}0^{1}1^{19}0^{1}1^{23}0^{1}1^{4}$	212159	A	$1^{148}0^{1}1^{137}0^{1}1^{23}0^{1}1^{4}$
7139	D	$1^{41}0^{1}1^{47}0^{1}1^{4}$	232791	A	$1^{213}0^{1}1^{99}0^{1}1^{4}$
9473	D	$1^{69}0^{1}1^{28}$	233437	A	$1^{210}0^{1}1^{101}0^{1}1^{4}$
12905	A	$1^{65}0^{1}1^{42}$	259509	D	$1^{207}0^{1}1^{112}$
13107	A	$1^{62}0^{1}1^{44}$	284371	D	$1^{163}0^{1}1^{149}0^{1}1^{22}$
17183	A	$1^{73}0^{1}1^{42}$	309195	A	$1^{233}0^{1}1^{106}$
17409	A	$1^{70}0^{1}1^{44}$	309901	A	$1^{230}0^{1}1^{108}$
21971	A	$1^{77}0^{1}1^{46}$	340719	A	$1^{205}0^{1}1^{142}$
22209	A	$1^{74}0^{1}1^{48}$	341341	A	$1^{202}0^{1}1^{144}$
26915	A	$1^{65}0^{1}1^{61}0^{1}1^{10}$	370731	A	$1^{177}0^{1}1^{155}0^{1}1^{28}$
27117	A	$1^{62}0^{1}1^{63}0^{1}1^{10}$	371269	A	$1^{174}0^{1}1^{157}0^{1}1^{28}$
31493	D	$1^{95}0^{1}1^{44}$	399339	D	$1^{245}0^{1}1^{118}$
37941	D	$1^{89}0^{1}1^{60}$	403579	C	$1^{312}0^{1}1^{58}$
45261	A	$1^{99}0^{1}1^{60}$	405584	C	$1^{348}0^{1}1^{28}$
45565	A	$1^{96}0^{1}1^{62}$	438615	D	$1^{203}0^{1}1^{182}$
47041	C	$1^{134}0^{1}1^{30}$	446809	C	$1^{302}0^{1}1^{90}$
47840	C	$1^{156}0^{1}1^{14}$	449912	C	$1^{354}0^{1}1^{44}$
48453	C	$1^{170}0^{1}1^{6}$	487277	A	$1^{219}0^{1}1^{188}$
49018	C	$1^{180}0^{1}1^{2}$	487941	A	$1^{216}0^{1}1^{190}$
49577	C	1^{188}	496785	C	$1^{318}0^{1}1^{94}$
57768	A	$1^{99}0^{1}1^{94}$	500104	C	$1^{372}0^{1}1^{46}$
58072	A	$1^{96}0^{1}1^{96}$	501965	C	$1^{402}0^{1}1^{22}$
64426	C	$1^{98}0^{1}1^{51}0^{1}1^{49}0^{1}1^{10}$	503430	C	$1^{420}0^{1}1^{10}$
70891	D	$1^{101}0^{1}1^{101}0^{1}1^{10}$	504787	C	$1^{432}0^{1}1^{4}$
81739	D	$1^{153}0^{1}1^{64}$	546044	D	$1^{227}0^{1}1^{218}$
93763	A	$1^{113}0^{1}1^{97}0^{1}1^{23}0^{1}1^{4}$	557280	C	$1^{344}0^{1}1^{108}$
94109	A	$1^{110}0^{1}1^{99}0^{1}1^{23}0^{1}1^{4}$	608971	D	$1^{261}0^{1}1^{200}$

Table 8: HNR#7, tapehead at leftmost bit. See similarities of HNR#3.

15

31167	C	$i1^{0}$01111111010111111101010111110101110101010111011111110101010101011	diff	off
31203	C	$i1^{4}$0111010111011111110101011111010111010101011101111111010101010101011	+36	0
31267	C	$i1^{8}$0111011111011111110101011111010111010101011101111111010101010101011	+64	-8
31407	C	$i1^{12}$01110111011111111010101111101011101010101110111111101010101010101011	+140	-4
31703	C	$i1^{16}$01110111010101010110101111101011101010101110111111101010101010101011	+296	8
32271	C	$i1^{20}$01110111010111010111010111110101110101010111011111110101010101010101011	+568	-8
33415	C	$i1^{24}$0111011101011111011101011111010111010101011101111111010101010101011	+1144	-8
35711	C	$i1^{28}$0111011101011111111101011111010111010101011101111111010101010101011	+2296	-8
40315	C	$i1^{32}$0111011101011111110111011111010111010101011101111111010101010101011	+4604	-4
49527	C	$i1^{36}$0111011101011111110101111111010111010101011101111111010101010101011	+9212	-4
67963	C	$i1^{40}$0111011101011111110101010101110111010101011101111111010101010101011	+18436	4
104819	C	$i1^{44}$0111011101011111110101011101110111010101011101111111010101010101011	+36856	-8
178539	C	$i1^{48}$0111011101011111110101011111110111010101011101111111010101010101011	+73720	-8
325991	C	$i1^{52}$0111011101011111110101011111011111010101011101111111010101010101011	+147452	-4
620903	C	$i1^{56}$0111011101011111110101011111010101110101011101111111010101010101011	+294912	0
1210719	C	$i1^{60}$0111011101011111110101011111010111110101011101111111010101010101011	+589816	-8
2390363	C	$i1^{64}$0111011101011111110101011111010111011101011101111111010101010101011	+1179644	-4
4749655	C	$i1^{68}$0111011101011111110101011111010111010111011101111111010101010101011	+2359292	-4
9468243	C	$i1^{72}$0111011101011111110101011111010111010101111101111111010101010101011	+4718588	-4
18905427	C	$i1^{76}$0111011101011111110101011111010111010101010111111111010101010101011	+9437184	0
37779787	C	$i1^{80}$0111011101011111110101011111010111010101011111111111010101010101011	+18874360	-8
75528531	C	$i1^{84}$0111011101011111110101011111010111010101011101010101110101010101011	+37748744	8
151025995	C	$i1^{88}$0111011101011111110101011111010111010101011101110101110101010101011	+75497464	-8
302020931	C	$i1^{92}$0111011101011111110101011111010111010101011101111101110101010101011	+150994936	-8
604010811	C	$i1^{96}$0111011101011111110101011111010111010101011101111111110101010101011	+301989880	-8
1207990583	C	$i1^{100}$0111011101011111110101011111010111010101011101111111011101010101011	+603979772	-4
2415950131	C	$i1^{104}$0111011101011111110101011111010111010101011101111111010111010101011	+1207959548	-4
4831869231	C	$i1^{108}$0111011101011111110101011111010111010101011101111111010101110101011	+2415919100	-4
38654736675	C	$i1^{120}$0111011101011111110101011111010111010101011101111111010101010101111	+19327352828	-4

Table 9: Machine 15. Let i be the row in the table above, indexed from -1; let $x_i = 36 * 2^i$. Let d_i be the difference in step number between the ith row and the $i-1$st row, and let k_i be the place of the first 1 bit that was a 0 bit in the previous line, indexing from 0 just after the compressed portion. It is observed that $d_i - x_i$ relates to $k_i - 2i$ in the following linear fashion: $d_i - x_i = -16 + 2(k_i - 2i)$. Also, $k_i - 2i$ always seems to be 2, 4, 6, 8, or 10. Also, the positions of the new 1s, highlighted in red, seem to be injective and surjective among every other position throughout the swath other than at the very end. Furthermore, once all this work is done, only the second 0 and the last 0 have been changed, and everything else in its "workspace" is as at the beginning.

			diff
76761382320	C	$i1^{0}$01111111111111111111111111110101010101011111111101011101010101010111011111101011101010111110111010111010101010111110111011111011111101111$(01)^{30}1$	
76761382396	C	$i1^{4}$0111010101010101010101010101110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+76
76761382460	C	$i1^{8}$011101110101010101010101010101110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+64
76761382596	C	$i1^{12}$01110111110101010101010101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+136
76761382876	C	$i1^{16}$01110111111101010101010101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+280
76761383444	C	$i1^{20}$01110111111111010101010101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+568
76761384588	C	$i1^{24}$01110111111111110101010101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+1144
76761386884	C	$i1^{28}$01110111111111111101010101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+2296
76761391484	C	$i1^{32}$01110111111111111111010101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+4600
76761400692	C	$i1^{36}$01110111111111111111110101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+9208
76761419116	C	$i1^{40}$01110111111111111111111101011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+18424
76761455972	C	$i1^{44}$01110111111111111111111111011110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+36856
76761529692	C	$i1^{48}$01110111111111111111111111111110101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+73720
76761677144	C	$i1^{52}$0111011111111111111111111111011101010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+147452
76761972052	C	$i1^{56}$0111011111111111111111111111010111010101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+294908
76762561872	C	$i1^{60}$0111011111111111111111111111010101110101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+589820
76763741516	C	$i1^{64}$0111011111111111111111111111010101011101111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+1179644
76766100808	C	$i1^{68}$0111011111111111111111111111010101010111111111110101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+2359292
76770819412	C	$i1^{72}$0111011111111111111111111111010101010101010101011101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+4718604
76780256588	C	$i1^{76}$0111011111111111111111111111010101010101110101011101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+9437176
76799130948	C	$i1^{80}$0111011111111111111111111111010101010101111101011101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+18874360
76836879676	C	$i1^{84}$0111011111111111111111111111010101010101111111011101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+37748728
76912377140	C	$i1^{88}$0111011111111111111111111111010101010101111111111101110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+75497464
77063372080	C	$i1^{92}$0111011111111111111111111111010101010101111111110111110101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+150994940
77365361968	C	$i1^{96}$0111011111111111111111111111010101010101111111110101011101010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+301989888
77969341736	C	$i1^{100}$0111011111111111111111111111010101010101111111110101111010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+603979768
79177301284	C	$i1^{104}$0111011111111111111111111111010101010101111111110101110111010101011101111110101110101011111011101011101010101011111011101111101111110111$(01)^{30}1$	+1207959548

Table 10: The machine's behavior after returning to the left at step 76761382305 B. It has set up a new task for itself in that it has written new stuff further to the right of where it had stuff previously, and it has gone back to having no 1s to the left of the workspace. It repeats its prior behavior, again taking nearly exactly $36 * 2^{n/4}$ steps to install a new 1 and return to the left, where n is the number of 1s after the i on the left (the first difference is significantly greater than 36 since the first new 1 to install is so far to the right), again injectively and bijectively ostensibly until $n/4$ is at or near 93, and then when $n = 94$ it would likely either write a new workspace for itself or do something new entirely. So in order to continue analyzing this machine effectively, an acceleration should be written that is able to automate away these $36 * 2^{93}$ or so steps. After all this work is done, seemingly only the second 0 and the last 0 would be altered, all the rest of the bits in the workspace becoming as they were before the work started.

Ostensibly the pack of 1s on the left is telling the machine to go to at least the $\frac{n}{4} + 2$nd even bit after the pack (indexing from 0 at the first bit after the pack) and turn the first 0 bit encountered from that point on into a 1, turning alternate 1 bits that it encounters between that index and that bit back into 0s. The most primitive way that a Turing machine could carry out a task like this of course takes exponential time in the number of 1 bits on the left, matching the pattern in step differences that we see, and this would also explain why the bits of the working tape end up nearly exactly as they were before the work began once the machine has accomplished this for each bit.

Conjecture. *For any string* $x = a_0 \ldots a_m$ *with each of the* a_j *being* 10 *or* 11, *and any nonnegative integer* $n \leq m$, *let* j *be the least index* $j \geq n$ *such that* $a_j = 10$*; suppose such index exists. Then*

$$0^4 i 1^{4n} 011x \; \mathtt{C} \vdash \; i1^{4n+4} 011\overline{x} \; \mathtt{C} \qquad 36 \cdot 2^n - 8 + 4(j-n),$$

where $\overline{x} = a_0 a_1 \ldots a_{n-1} \overline{a_n}\, \overline{a_{n+1}} \ldots \overline{a_j} a_{j+1} \ldots a_m$, *for* $a = 10$, $\overline{a} = 11$, *and vice versa.*

The conjecture has been verified to produce each subsequent line of either of the two tables above from any given current line running the Python script found at github.com/danbriggs/Turing/blob/master/paper/accelerate_15_old.py, which uses stepfiguration.py from the same directory. Assuming the conjecture is true, one obtains

356526731314189519247709158424 $\mathtt{C}$
$i1^{372}$0111011111111111111111111111010101010101111111110101110101010101110/
11111010111010101111101110101110101010101111101110111110111110111010101/
0101010101010101010101010101010101010101010101010101111/

i.e., the machine runs for at least 356 octillion or $3.56 \cdot 10^{29}$ steps.

Pushing beyond this 356 octillion boundary will require applying the specific lemmas that would combine to provide the conjecture above in a manner very similar to that that would prove the conjecture. Scratch notes can be found at noteson15.txt.

16∼24∼38

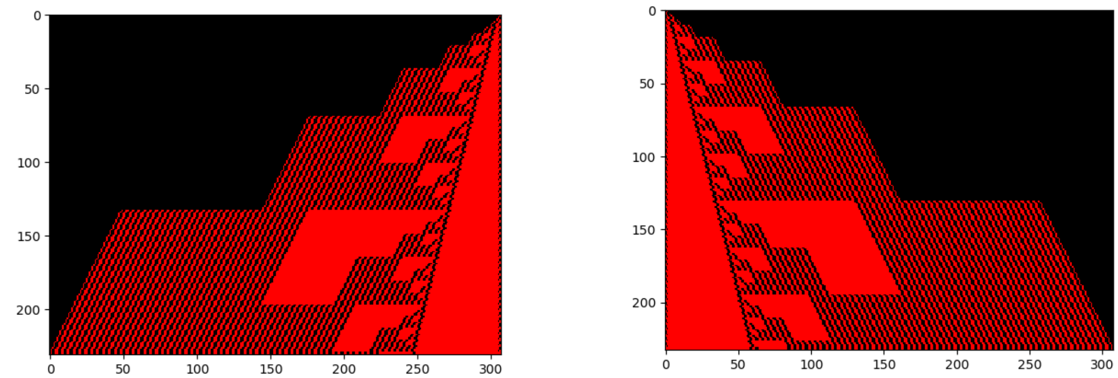


Figure 2: HNRs #16 & 24, tape head at extreme for up to 30000 steps.

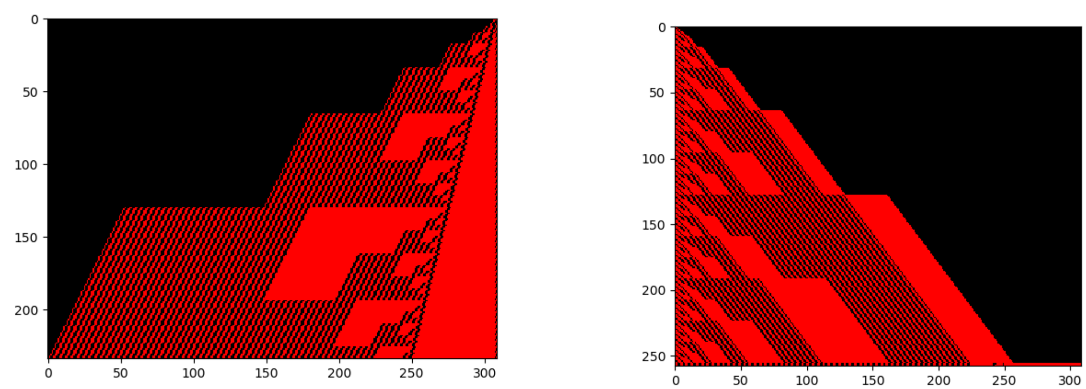


Figure 3: HNR #38, head at right, left resp. These three machines require but a routine proof.

17

0	A	o
2	C	i
18	B	$(10)^{1}1i$
34	C	$(10)^{2}i$
66	B	$(10)^{1}1(10)^{1}1i$
88	D	$(10)^{2}1(10)^{1}i$
89	B	$(10)^{2}1(10)^{1}1o$
90	C	$(10)^{2}1(10)^{2}o$
91	D	$(10)^{2}1(10)^{2}0o$
95	C	$(10)^{2}1(10)^{2}1i$
219	C	$(10)^{4}1(10)^{2}1i$
335	B	$(10)^{4}1(10)^{2}1(10)^{1}1i$
451	C	$(10)^{4}1(10)^{2}1(10)^{2}i$
583	B	$(10)^{4}1(10)^{4}1(10)^{1}1i$
723	C	$(10)^{6}1(10)^{2}1(10)^{2}i$
895	B	$(10)^{1}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1111	C	$(10)^{8}1(10)^{2}1(10)^{2}i$
1323	B	$(10)^{3}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1675	C	$(10)^{8}1(10)^{4}1(10)^{2}i$
1911	B	$(10)^{3}1(10)^{6}1(10)^{4}1(10)^{1}1i$
2335	C	$(10)^{1}1(10)^{8}1(10)^{4}1(10)^{2}i$
2679	B	$(10)^{3}1(10)^{8}1(10)^{4}1(10)^{1}1i$
3175	C	$(10)^{3}1(10)^{8}1(10)^{4}1(10)^{2}i$
3687	B	$(10)^{3}1(10)^{8}1(10)^{6}1(10)^{1}1i$
4223	C	$(10)^{3}1(10)^{10}1(10)^{4}1(10)^{2}i$
4807	B	$(10)^{5}1(10)^{8}1(10)^{6}1(10)^{1}1i$
5507	C	$(10)^{3}1(10)^{12}1(10)^{4}1(10)^{2}i$
6163	B	$(10)^{7}1(10)^{8}1(10)^{6}1(10)^{1}1i$
6545	D	$(10)^{12}1(10)^{8}1(10)^{2}1(10)^{1}i$
6546	B	$(10)^{12}1(10)^{8}1(10)^{2}1(10)^{1}1o$
6547	C	$(10)^{12}1(10)^{8}1(10)^{2}1(10)^{2}o$
6548	D	$(10)^{12}1(10)^{8}1(10)^{2}1(10)^{2}0o$
6552	C	$(10)^{12}1(10)^{8}1(10)^{2}1(10)^{2}1i$
8200	C	$(10)^{12}1(10)^{8}1(10)^{4}1(10)^{2}1i$
9944	C	$(10)^{14}1(10)^{8}1(10)^{4}1(10)^{2}1i$
11988	C	$(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1i$
13212	B	$(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1(10)^{1}1i$
14436	C	$(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1(10)^{2}i$
15676	B	$(10)^{16}1(10)^{8}1(10)^{4}1(10)^{4}1(10)^{1}1i$
16924	C	$(10)^{16}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
18204	B	$(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
19524	C	$(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
20940	B	$(10)^{1}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
22480	C	$(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
24032	B	$(10)^{3}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
25876	C	$(10)^{20}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
27500	B	$(10)^{3}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
29512	C	$(10)^{1}1(10)^{20}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
31388	B	$(10)^{3}1(10)^{20}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
33568	C	$(10)^{3}1(10)^{20}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
35780	B	$(10)^{3}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
38048	C	$(10)^{3}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
40428	B	$(10)^{5}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
43004	C	$(10)^{3}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
45552	B	$(10)^{7}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
48564	C	$(10)^{3}1(10)^{24}1(10)^{10}1(10)^{8}1(10)^{2}1(10)^{2}i$
51152	B	$(10)^{7}1(10)^{20}1(10)^{14}1(10)^{4}1(10)^{4}1(10)^{1}1i$
54268	C	$(10)^{3}1(10)^{26}1(10)^{10}1(10)^{8}1(10)^{2}1(10)^{2}i$
57024	B	$(10)^{9}1(10)^{20}1(10)^{14}1(10)^{4}1(10)^{4}1(10)^{1}1i$
60504	C	$(10)^{3}1(10)^{28}1(10)^{10}1(10)^{8}1(10)^{2}1(10)^{2}i$
63428	B	$(10)^{11}1(10)^{20}1(10)^{14}1(10)^{4}1(10)^{4}1(10)^{1}1i$
67396	C	$(10)^{3}1(10)^{28}1(10)^{12}1(10)^{8}1(10)^{2}1(10)^{2}i$
70408	B	$(10)^{11}1(10)^{22}1(10)^{14}1(10)^{4}1(10)^{4}1(10)^{1}1i$
74608	C	$(10)^{5}1(10)^{28}1(10)^{12}1(10)^{8}1(10)^{2}1(10)^{2}i$
77992	B	$(10)^{11}1(10)^{24}1(10)^{14}1(10)^{4}1(10)^{4}1(10)^{1}1i$
82424	C	$(10)^{7}1(10)^{28}1(10)^{12}1(10)^{8}1(10)^{2}1(10)^{2}i$
86340	B	$(10)^{11}1(10)^{24}1(10)^{14}1(10)^{6}1(10)^{4}1(10)^{1}1i$
90828	C	$(10)^{7}1(10)^{28}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}i$
94848	B	$(10)^{11}1(10)^{26}1(10)^{14}1(10)^{6}1(10)^{4}1(10)^{1}1i$
99568	C	$(10)^{9}1(10)^{28}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}i$
104016	B	$(10)^{11}1(10)^{28}1(10)^{14}1(10)^{6}1(10)^{4}1(10)^{1}1i$
108968	C	$(10)^{11}1(10)^{28}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}i$
113968	B	$(10)^{11}1(10)^{28}1(10)^{16}1(10)^{6}1(10)^{4}1(10)^{1}1i$
119040	C	$(10)^{11}1(10)^{30}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}i$
124272	B	$(10)^{13}1(10)^{28}1(10)^{16}1(10)^{6}1(10)^{4}1(10)^{1}1i$
129828	C	$(10)^{11}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}i$
135292	B	$(10)^{15}1(10)^{28}1(10)^{16}1(10)^{6}1(10)^{4}1(10)^{1}1i$
141552	C	$(10)^{11}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{4}1(10)^{2}i$
147040	B	$(10)^{15}1(10)^{28}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{1}1i$
153372	C	$(10)^{11}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}i$
158980	B	$(10)^{15}1(10)^{30}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{1}1i$
165576	C	$(10)^{13}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}i$
171700	B	$(10)^{15}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{1}1i$
178560	C	$(10)^{15}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}i$
185436	B	$(10)^{15}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{6}1(10)^{1}1i$
192336	C	$(10)^{15}1(10)^{32}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{2}i$
199284	B	$(10)^{15}1(10)^{32}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{1}1i$
206320	C	$(10)^{15}1(10)^{34}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{2}i$
213532	B	$(10)^{17}1(10)^{32}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{1}1i$

Table 11: Machine 17.

221140	C	$(10)^{15}1(10)^{36}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{2}i$
228616	B	$(10)^{19}1(10)^{32}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{1}1i$
236960	C	$(10)^{15}1(10)^{36}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{2}i$
244572	B	$(10)^{19}1(10)^{34}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{1}1i$
253212	C	$(10)^{17}1(10)^{36}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{2}i$
261428	B	$(10)^{19}1(10)^{36}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{1}1i$
270364	C	$(10)^{19}1(10)^{36}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{2}i$
279348	B	$(10)^{19}1(10)^{36}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{1}1i$
288436	C	$(10)^{19}1(10)^{38}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{2}i$
297716	B	$(10)^{21}1(10)^{36}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{1}1i$
307464	C	$(10)^{19}1(10)^{40}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{2}i$
317040	B	$(10)^{23}1(10)^{36}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{1}1i$
327696	C	$(10)^{19}1(10)^{40}1(10)^{18}1(10)^{12}1(10)^{4}1(10)^{2}i$
337344	B	$(10)^{23}1(10)^{36}1(10)^{22}1(10)^{8}1(10)^{6}1(10)^{1}1i$
348168	C	$(10)^{19}1(10)^{42}1(10)^{18}1(10)^{12}1(10)^{4}1(10)^{2}i$
358112	B	$(10)^{25}1(10)^{36}1(10)^{22}1(10)^{8}1(10)^{6}1(10)^{1}1i$
369652	C	$(10)^{19}1(10)^{44}1(10)^{18}1(10)^{12}1(10)^{4}1(10)^{2}i$
379892	B	$(10)^{27}1(10)^{36}1(10)^{22}1(10)^{8}1(10)^{6}1(10)^{1}1i$
392352	C	$(10)^{19}1(10)^{44}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{2}i$
402744	B	$(10)^{27}1(10)^{38}1(10)^{22}1(10)^{8}1(10)^{6}1(10)^{1}1i$
415564	C	$(10)^{21}1(10)^{44}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{2}i$
426680	B	$(10)^{27}1(10)^{40}1(10)^{22}1(10)^{8}1(10)^{6}1(10)^{1}1i$
439860	C	$(10)^{23}1(10)^{44}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{2}i$
451980	B	$(10)^{27}1(10)^{40}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{1}1i$
465248	C	$(10)^{23}1(10)^{44}1(10)^{22}1(10)^{12}1(10)^{4}1(10)^{2}i$
477536	B	$(10)^{27}1(10)^{42}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{1}1i$
491164	C	$(10)^{25}1(10)^{44}1(10)^{22}1(10)^{12}1(10)^{4}1(10)^{2}i$
504232	B	$(10)^{27}1(10)^{44}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{1}1i$
518220	C	$(10)^{27}1(10)^{44}1(10)^{22}1(10)^{12}1(10)^{4}1(10)^{2}i$
532272	B	$(10)^{27}1(10)^{44}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{1}1i$
546444	C	$(10)^{27}1(10)^{46}1(10)^{22}1(10)^{12}1(10)^{4}1(10)^{2}i$
560856	B	$(10)^{29}1(10)^{44}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{1}1i$
575864	C	$(10)^{27}1(10)^{48}1(10)^{22}1(10)^{12}1(10)^{4}1(10)^{2}i$
590636	B	$(10)^{31}1(10)^{44}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{1}1i$
597290	D	$(10)^{60}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{1}i$
597291	B	$(10)^{60}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{1}1o$
597292	C	$(10)^{60}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}o$
597293	D	$(10)^{60}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}0o$
597297	C	$(10)^{60}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{2}1(10)^{2}1i$
628321	C	$(10)^{60}1(10)^{32}1(10)^{14}1(10)^{8}1(10)^{4}1(10)^{2}1i$
659441	C	$(10)^{60}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1i$
690945	C	$(10)^{62}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1i$
723709	C	$(10)^{64}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1i$
740873	B	$(10)^{64}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1(10)^{1}1i$
758037	C	$(10)^{64}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{2}1(10)^{2}i$
775217	B	$(10)^{64}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{4}1(10)^{4}1(10)^{1}1i$
792405	C	$(10)^{64}1(10)^{32}1(10)^{16}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
809625	B	$(10)^{64}1(10)^{32}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
826885	C	$(10)^{64}1(10)^{32}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
844241	B	$(10)^{64}1(10)^{34}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
861765	C	$(10)^{66}1(10)^{32}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
879641	B	$(10)^{1}1(10)^{64}1(10)^{34}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
897961	C	$(10)^{68}1(10)^{32}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
916357	B	$(10)^{3}1(10)^{64}1(10)^{34}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
935653	C	$(10)^{68}1(10)^{34}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
954313	B	$(10)^{3}1(10)^{66}1(10)^{34}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
974161	C	$(10)^{1}1(10)^{68}1(10)^{34}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
993649	B	$(10)^{3}1(10)^{68}1(10)^{34}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1014049	C	$(10)^{3}1(10)^{68}1(10)^{34}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1034545	B	$(10)^{3}1(10)^{68}1(10)^{36}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1055225	C	$(10)^{3}1(10)^{70}1(10)^{34}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1076273	B	$(10)^{5}1(10)^{68}1(10)^{36}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1097837	C	$(10)^{3}1(10)^{72}1(10)^{34}1(10)^{18}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1119437	B	$(10)^{7}1(10)^{68}1(10)^{36}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1142157	C	$(10)^{3}1(10)^{72}1(10)^{34}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1163893	B	$(10)^{7}1(10)^{68}1(10)^{38}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1186909	C	$(10)^{3}1(10)^{74}1(10)^{34}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1209197	B	$(10)^{9}1(10)^{68}1(10)^{38}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1233153	C	$(10)^{3}1(10)^{76}1(10)^{34}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1255993	B	$(10)^{11}1(10)^{68}1(10)^{38}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1281109	C	$(10)^{3}1(10)^{76}1(10)^{36}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1304229	B	$(10)^{11}1(10)^{70}1(10)^{38}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1329961	C	$(10)^{5}1(10)^{76}1(10)^{36}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1354029	B	$(10)^{11}1(10)^{72}1(10)^{38}1(10)^{16}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1380377	C	$(10)^{7}1(10)^{76}1(10)^{36}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1405697	B	$(10)^{11}1(10)^{72}1(10)^{38}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1432197	C	$(10)^{7}1(10)^{76}1(10)^{38}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1457813	B	$(10)^{11}1(10)^{74}1(10)^{38}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1484929	C	$(10)^{9}1(10)^{76}1(10)^{38}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1511549	B	$(10)^{11}1(10)^{76}1(10)^{38}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1539281	C	$(10)^{11}1(10)^{76}1(10)^{38}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1567125	B	$(10)^{11}1(10)^{76}1(10)^{40}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1595169	C	$(10)^{11}1(10)^{78}1(10)^{38}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1623629	B	$(10)^{13}1(10)^{76}1(10)^{40}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1652733	C	$(10)^{11}1(10)^{80}1(10)^{38}1(10)^{20}1(10)^{8}1(10)^{6}1(10)^{2}1(10)^{2}i$
1681809	B	$(10)^{15}1(10)^{76}1(10)^{40}1(10)^{18}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1712361	C	$(10)^{11}1(10)^{80}1(10)^{38}1(10)^{20}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
1741509	B	$(10)^{15}1(10)^{76}1(10)^{40}1(10)^{20}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1772229	C	$(10)^{11}1(10)^{80}1(10)^{40}1(10)^{20}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
1801689	B	$(10)^{15}1(10)^{78}1(10)^{40}1(10)^{20}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1833057	C	$(10)^{13}1(10)^{80}1(10)^{40}1(10)^{20}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
1863609	B	$(10)^{15}1(10)^{80}1(10)^{40}1(10)^{20}1(10)^{10}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1895625	C	$(10)^{15}1(10)^{80}1(10)^{40}1(10)^{20}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
1927673	B	$(10)^{15}1(10)^{80}1(10)^{40}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
1959777	C	$(10)^{15}1(10)^{80}1(10)^{40}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
1991993	B	$(10)^{15}1(10)^{80}1(10)^{42}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2024425	C	$(10)^{15}1(10)^{82}1(10)^{40}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2057289	B	$(10)^{17}1(10)^{80}1(10)^{42}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2090869	C	$(10)^{15}1(10)^{84}1(10)^{40}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2124381	B	$(10)^{19}1(10)^{80}1(10)^{42}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2159369	C	$(10)^{15}1(10)^{84}1(10)^{42}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2193209	B	$(10)^{19}1(10)^{82}1(10)^{42}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2228877	C	$(10)^{17}1(10)^{84}1(10)^{42}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2263897	B	$(10)^{19}1(10)^{84}1(10)^{42}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2300245	C	$(10)^{19}1(10)^{84}1(10)^{42}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2336705	B	$(10)^{19}1(10)^{84}1(10)^{44}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2373397	C	$(10)^{19}1(10)^{86}1(10)^{42}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2410537	B	$(10)^{21}1(10)^{84}1(10)^{44}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2448465	C	$(10)^{19}1(10)^{88}1(10)^{42}1(10)^{22}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2486285	B	$(10)^{23}1(10)^{84}1(10)^{44}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2525841	C	$(10)^{19}1(10)^{88}1(10)^{42}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2563829	B	$(10)^{23}1(10)^{84}1(10)^{46}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2603745	C	$(10)^{19}1(10)^{90}1(10)^{42}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2642413	B	$(10)^{25}1(10)^{84}1(10)^{46}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2683621	C	$(10)^{19}1(10)^{92}1(10)^{42}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2722969	B	$(10)^{27}1(10)^{84}1(10)^{46}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2765769	C	$(10)^{19}1(10)^{92}1(10)^{44}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2805461	B	$(10)^{27}1(10)^{86}1(10)^{46}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2849005	C	$(10)^{21}1(10)^{92}1(10)^{44}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2889997	B	$(10)^{27}1(10)^{88}1(10)^{46}1(10)^{20}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$
2934285	C	$(10)^{23}1(10)^{92}1(10)^{44}1(10)^{24}1(10)^{10}1(10)^{6}1(10)^{2}1(10)^{2}i$
2977001	B	$(10)^{27}1(10)^{88}1(10)^{46}1(10)^{22}1(10)^{12}1(10)^{4}1(10)^{4}1(10)^{1}1i$

Table 12: Machine 17 continued.

19∼42

Figure 4: HNR #19, tape head at left end for up to 300000 steps.

Figure 5: HNR #42 similarly. These two require but a routine proof.

20

Let

$$\begin{aligned} T &= 1001001000 \\ U &= 10010010010101001101 \\ V &= 1010011010 \\ W &= 10011001001001010. \end{aligned}$$

Then we observe the following:

`22613`	`E`	$T^{09}V^{1.5}WU^{03.5}00i$
`70779`	`E`	$T^{16}V^{2.5}WU^{07.5}00i$
`145633`	`E`	$T^{23}V^{3.5}WU^{11.5}00i$
`247175`	`E`	$T^{30}V^{4.5}WU^{15.5}00i$
`375405`	`E`	$T^{37}V^{5.5}WU^{19.5}00i$
`530323`	`E`	$T^{44}V^{6.5}WU^{23.5}00i$,

where the half-integer indicates an additional half a repetition.

So at intervals of $48166 + 26688k$, the machine just adds seven Ts, one V, and four Us.

So the proof for HNR#20 will be easy if possibly arduous.

22

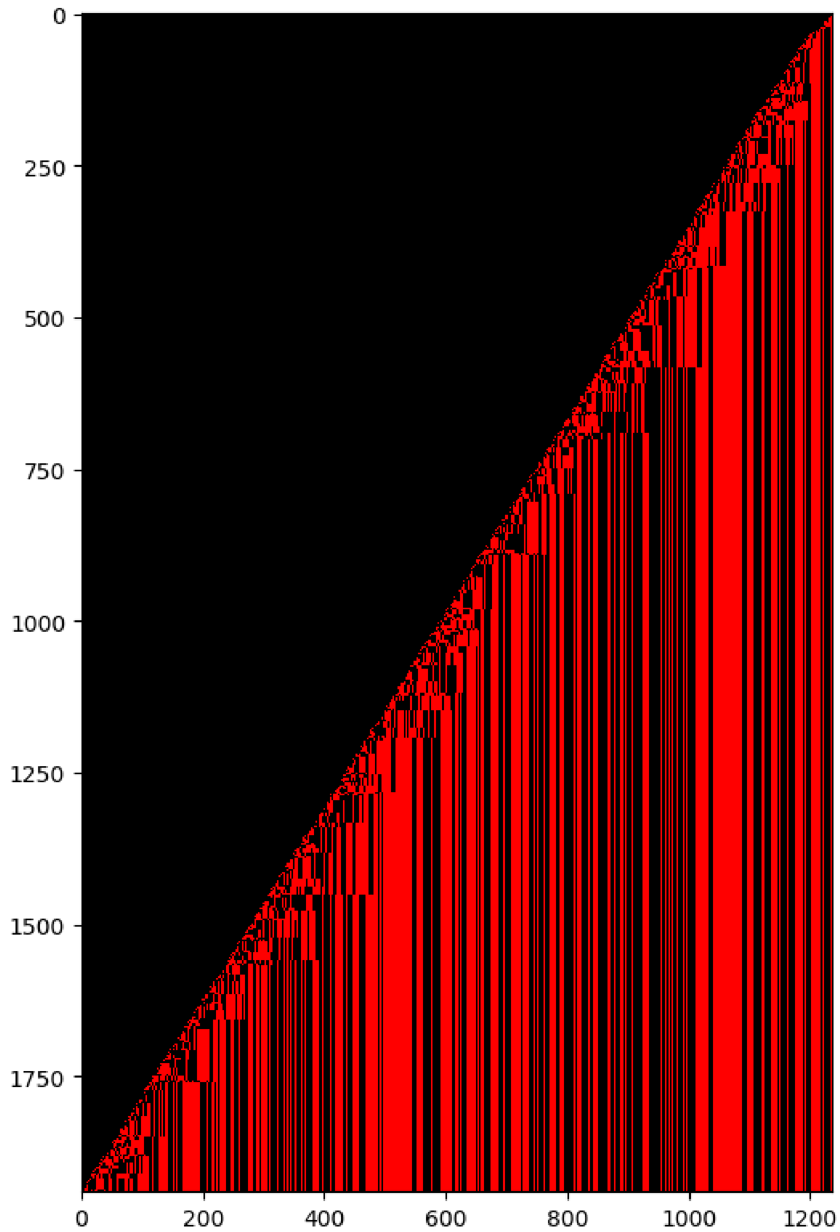


Figure 6: HNR #22, tape head at left end for 199576 steps.

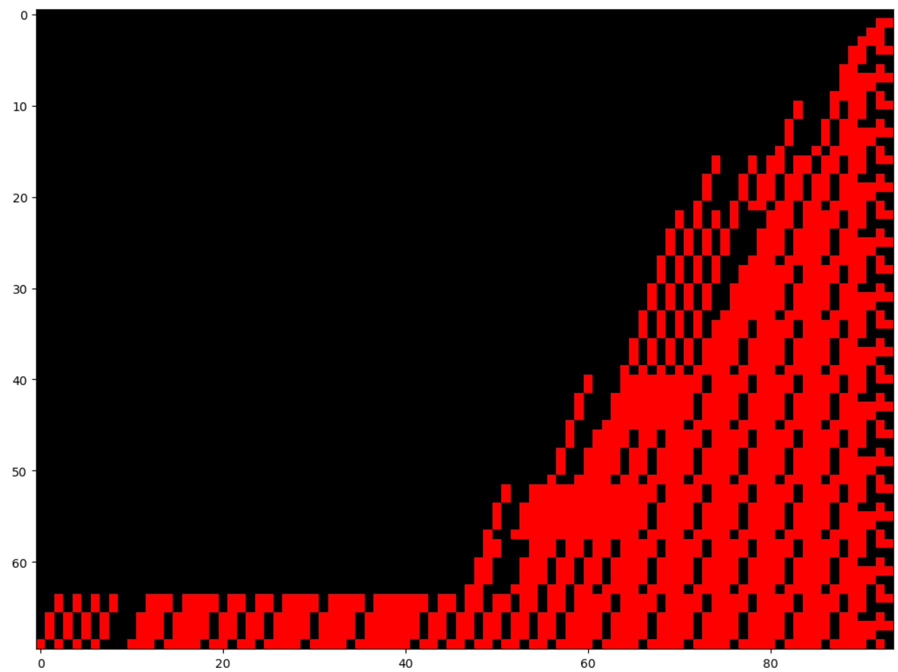


Figure 7: HNR #22, tape head at right end for 4606 steps.
It does not return to the right by step 3000000.

23=29

888	C	$(10)^2 0(100)^8(10)^7 i$
2820	C	$(10)^2 0(100)^{18}(10)^9 i$
7082	C	$(10)^2 0(100)^{32}(10)^{11} i$
15114	C	$(10)^2 0(100)^{50}(10)^{13} i$
28716	C	$(10)^2 0(100)^{72}(10)^{15} i$
50048	C	$(10)^2 0(100)^{98}(10)^{17} i$
81630	C	$(10)^2 0(100)^{128}(10)^{19} i$
126342	C	$(10)^2 0(100)^{162}(10)^{21} i$
187424	C	$(10)^2 0(100)^{200}(10)^{23} i$

The step differences are 1932, 4262, 8032, 13602, 21332, 31582, 44712, and 61082; thus the second differences are 2330, 3770, 5570, 7730, 10250, 13130, and 16370, the third differences are 1440, 1800, 2160, 2520, 2880, and 3240, and the fourth differences are all 360.

So this machine will be fairly easy to prove never halts.

26

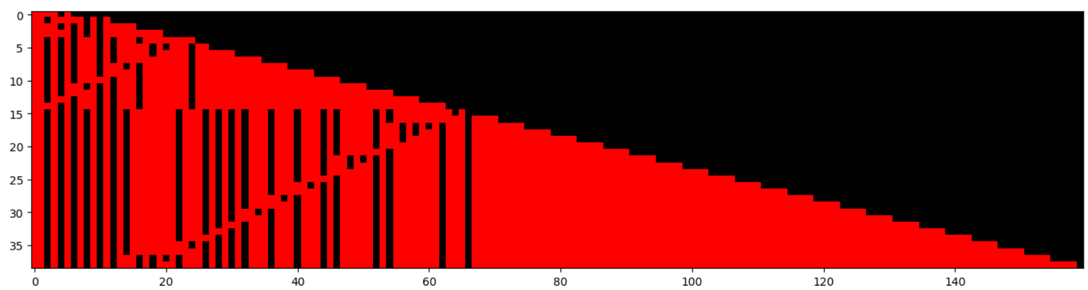


Figure 8: HNR #26, tape head at right end for 151029682 steps.
Each step number is roughly twice the previous.

The steps when HNR#26 has reached the right end are 20, 84, 120, 184, 318, 343, 371, 435, 579, 859, 1431, 2583, 4879, 9483, 18695, 34848, 34858, 34902, 34966, 35102, 35382, 35954, 37110, 39406, 44006, 53218, 71650, 108506, 182234, 329682, 624594, 1214410, 2394054, 4753346, 9471934, 18909118, 37783478, 75532222, 151029686 shown in this picture. Denote this sequence by a, and let Δs denote the sequence $s_{i+1} - s_i$ for any sequence s. Then it is observed that $\Delta a - \Delta^2 a$ is the sequence 92, 8, -6, 243, 22, -8, -16, 8, -12, -8, 8, -12, -4, 2271, 32296, -24, 24, -8, -8, -12, -12, 16, -8, -12, -8, 8, -16, 8, -16, 8, -12, -4, -4, -8, 8, -24, 24.

In other words, the intervals between steps fail to double by very little in most transitions, and where they fail greatly coordinates with the transition around the 15 mark in the picture.

BL_2

10	C	$i1011$
51	C	$i101(110)^{2}11$
214	C	$i101(110)^{1}1(110)^{5}11$
1063	C	$i101(110)^{1}1(110)^{2}10(110)^{12}11$
5834	C	$i101(110)^{1}1(110)^{2}10(110)^{2}1(110)^{32}11$
34397	C	$i101(110)^{1}1(110)^{2}10(110)^{2}1(110)^{6}1(110)^{80}11$
209800	C	$i101(110)^{1}1(110)^{2}10(110)^{2}1(110)^{6}1(110)^{14}1(110)^{200}11$
1298303	C	$i101(110)^{1}1(110)^{2}10(110)^{2}1(110)^{6}1(110)^{14}1(110)^{34}1(110)^{500}11$
8082056	C	$i101(110)^{1}1(110)^{2}10(110)^{2}1(110)^{6}1(110)^{14}1(110)^{34}1(110)^{84}1(110)^{1250}11$
50432059	C	$i101(110)^{1}1(110)^{2}10(110)^{2}1(110)^{6}1(110)^{14}1(110)^{34}1(110)^{84}1(110)^{209}1(110)^{3125}11$
315021908	C	$i101(110)^{1}1(110)^{2}10(110)^{2}1(110)^{6}1(110)^{14}1(110)^{34}1(110)^{84}1(110)^{209}1(110)^{522}10(110)^{7812}11$

Let a_i and b_i denote the exponents of the last and second to last swath of 110s during a given step when the tape head has reached the left edge of the tape anew, as written above.

Then it appears to be the case that

$$\begin{aligned} b_{i+1} &= \lceil a_i/6 \rceil + t \\ a_{i+1} &= \lfloor 5a_i/2 \rfloor \end{aligned}$$

where t is 1 if a_i is $5 \pmod 6$ and 0 otherwise.

Seems the step number tends to multiplying by 25/4 from each step to the next.

The sequence of differences in step number seems to tend to multiplying by 25/4 as well, with the error incurred by assuming this being about $11\%, 4.4\%, 1.8\%, .71\%$, and $.23\%$ of the previous difference by the end. Also, the ratio between each of these errors and the next seems to be very close to 5/2 half the time.

Thus denote by $\{x_i\}_{i=0}^{10}$ the sequence of step numbers mentioned above, and let

$$\begin{aligned} y_i &= x_{i+1} - x_i, \\ z_i &= \frac{25}{4} y_i - y_{i+1}, \text{ and} \\ w_i &= \frac{5}{2} z_i - z_{i+1}. \end{aligned}$$

Let us align the values we get for $\{w_i\}_{i=0}^{7}$ against $a_i \pmod 6$:

63.375	-110.875	82.375	23.625	23.625	23.625	23.625	23463.375		
0	2	5	0	2	2	2	2	2	5

So it seems that w_i is $23{}^5/{}_8$ whenever all of a_i, a_{i+1}, and a_{i+2} are not $5 \pmod 6$.

Although it is too early to tell, it seems likely that HNR#BL_2 will not be very difficult.

30=41

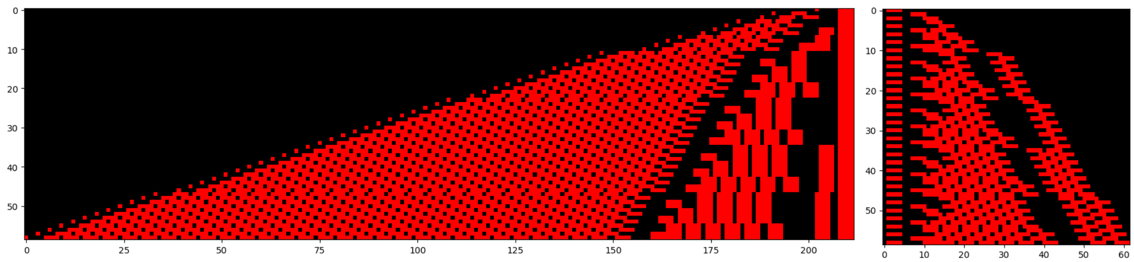


Figure 9: HNR #30, tape head at left end for 30000 steps.
The second image shows it left-justified with the beginning repetitive portion deleted.

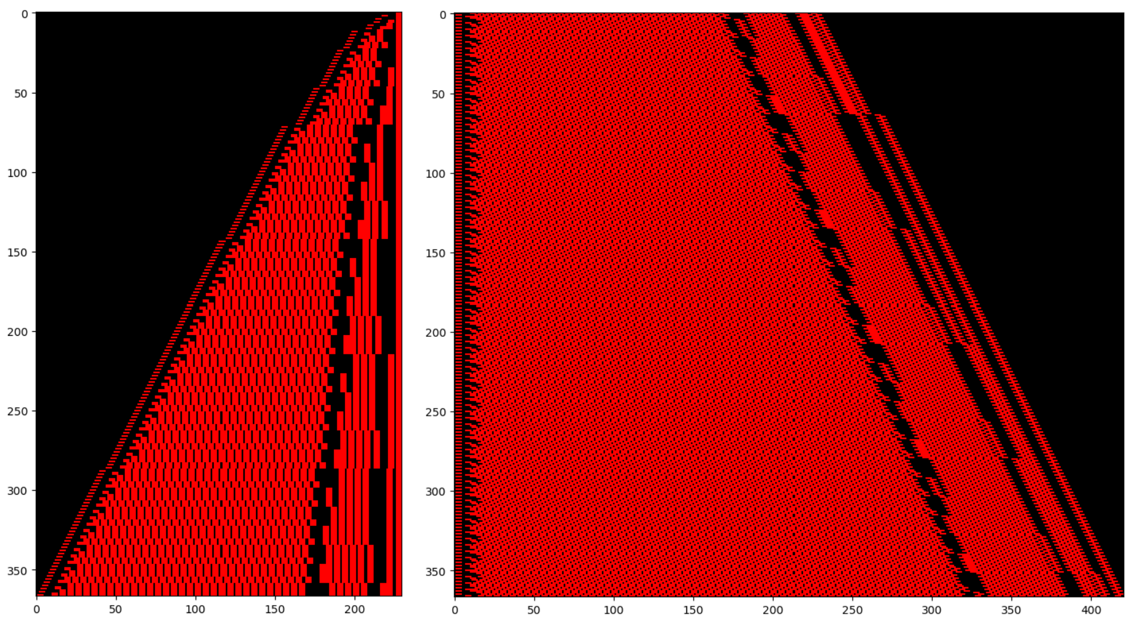


Figure 10: Tape head at left, repetitive portion deleted, right-justified, 1000000 steps.
The second image shows the same left-justified from step 1000000 to 3939415.

The action of this machine appears to be of a fractal nature, but it is more difficult to describe than that of HNR#16 or 19.

On the next page, we will see that looking at moments when the tape head is on the right side helps us figure out much more easily what to do to prove that this machine never halts.

Looking at moments when the tape head is on the right gives us a much clearer picture of what to do. If we list all steps from 31 to 300000000 when the tape head is on the right, it follows a 5/2/5/5/5 pattern where the 5 is E-B-C-D-A and the 2 is C-D. Taking only those C moments from the 2, hence, rows 5 (mod 22) indexing from 0, we obtain the following steps:

92	C	$(11110)^2 11i$
1842	C	$(11110)^6 1^{14}(11110)^2 11i$
42652	C	$(11110)^{36} 1^{14}(11110)^6 1^{14}(11110)^2 11i$
1376222	C	$(11110)^{216} 1^{14}(11110)^{36} 1^{14}(11110)^6 1^{14}(11110)^2 11i$
48568752	C	$(11110)^{1296} 1^{14}(11110)^{216} 1^{14}(11110)^{36} 1^{14}(11110)^6 1^{14}(11110)^2 11i$

Studying the step numbers, it appears that their ratio is trending towards 36, which is not surprising given the lengths of strings involved. Thus let the sequence of five steps above be $\{x_i\}_{i=0}^4$, and define $y_i = 36x_i - x_{i+1}$. We obtain

$$1470, 23660, 159250, 975240.$$

The descending tens places suggest taking the difference, so let $z = \Delta y$. We get

$$22190, 135590, 815990.$$

The ratios of subsequent terms are very close to 6, so let $w_i = z_{i+1} - 6z_i$.

We find that w is the constant sequence 2450 of length 2 so far. Substituting in terms of the original sequence, this is to say

$$2450 = -x_{i+3} + 43x_{i+2} - 258x_{i+1} + 216x_i, \text{ or} \tag{1}$$
$$x_{i+3} = 43x_{i+2} - 258x_{i+1} + 216x_i - 2450. \tag{2}$$

Since $43 \cdot 48568752 - 258 \cdot 1376222 + 216 \cdot 42652 - 2450 = 1742601442$, let us inspect step 1742601442 to confirm that our result was not just an artifact of having too many parameters available to adjust. The output of the Java program Turing, when written with exponents using blit30end.py, gives:

1742601442 C $(11110)^{7776} 1^{14}(11110)^{1296} 1^{14}(11110)^{216} 1^{14}(11110)^{36} 1^{14}(11110)^6 1^{14}(11110)^2 11i$

Thus the following is overwhelmingly likely to be true:

Conjecture. *At steps $\{x_i\}_{i=1}^{\infty}$ where $x_1 = 92$, $x_2 = 1842$, and $x_3 = 42652$, and (2), HNR#30 is in state C with tape head at its rightmost 1 bit and tape*

$$\left(\prod_{j=1}^{i} (11110)^{6^{i+1-j}} 1^{14}\right) (11110)^2 111.$$

The proof will likely be not too difficult, but it might be very arduous.

33

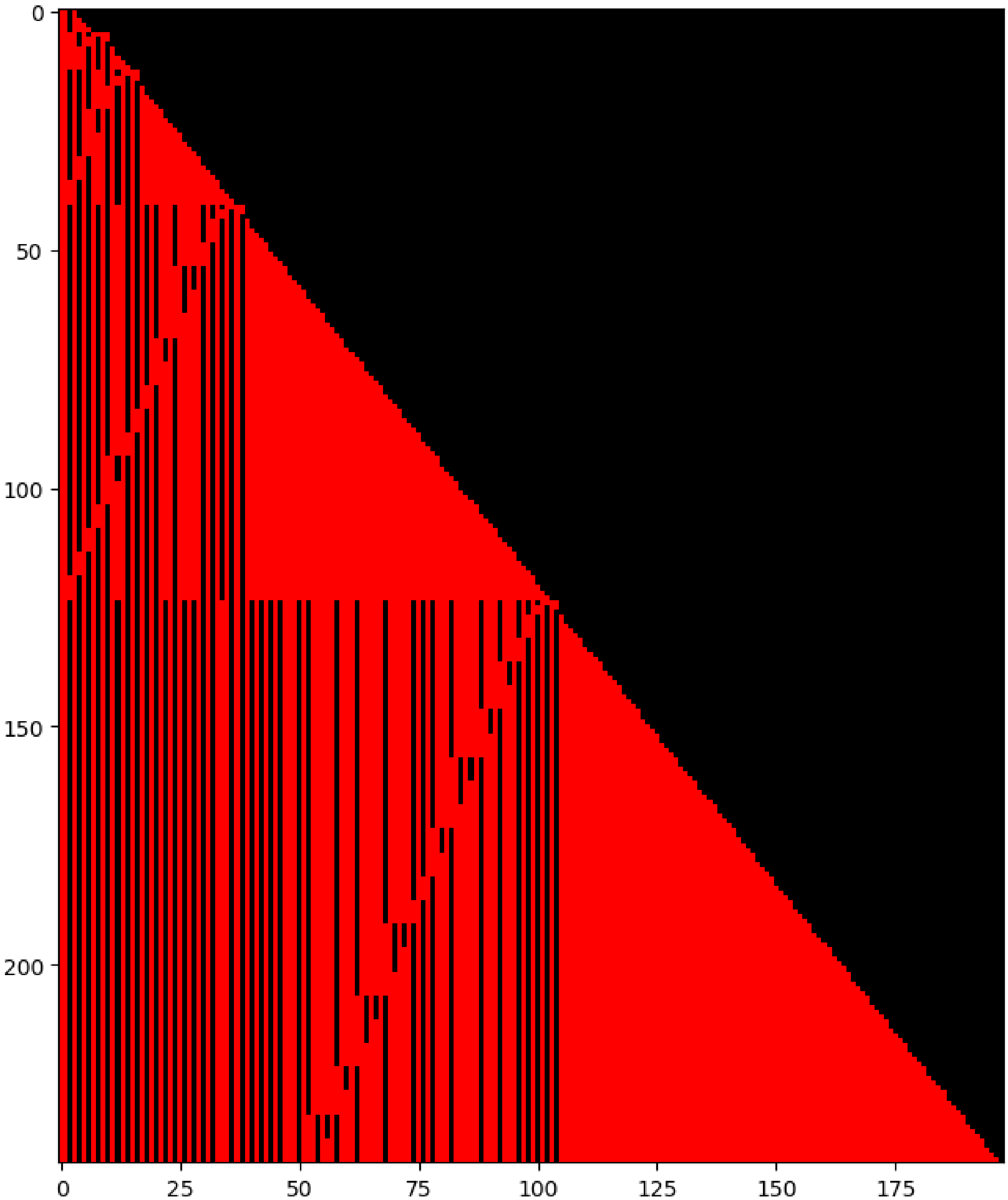


Figure 11: Tape head at right from step 10 to step 238493660.
There is not another moment in the first 300 million steps.

It almost always hits the right in A and stays on the right for five steps in A-D-B-E-A, except for moments during the four clear transitional periods shown where it does E and then D-A eight steps later; these are at steps 74, 177, 1980 and 3612743.

If we instead try to determine moments when it has reached the left, we get an extraordinary sparse set of steps:

47	E	$1^6 I$
144	E	$1^{12} I$
1785	E	$1^{18} IOIII$
3612042	E	$1^{20} IIOOIOIIIIOIIIII$

Tape head at leftmost 1. $I = 0001, O = 0000$.

The 47, 144, 1785 and 3612042 correspond to a mere 27, 33, 205 steps and 701 steps, respectively, behind the four clear transitional periods shown in the figure where it was on the left. So it is safe to assume that it never gets back to the left until it's done "doing its computation" on the previous string, whatever that may mean to it.

74	E	1111010111i
177	E	1101010111010111i
1980	E	$1(10)^5 11(10)^4 11101111101010111i$
3612743	E	$1(10)^{16} 11(10)^6 111010111110111011111011111010101110111110111011101010111i$

Tape head at rightmost bit for transitional period.

It does not reach the left again within the first 3 billion steps.

Probably a significant acceleration would be required to find even the next moment when it reaches the left. It also might be a good idea to run experiments like $1^{6n}0001\ E$ with the tape head on the left and take a look at what the tape looks like the first moment the head hits the right.

Maybe it's doing something like Ackermann's function.

34

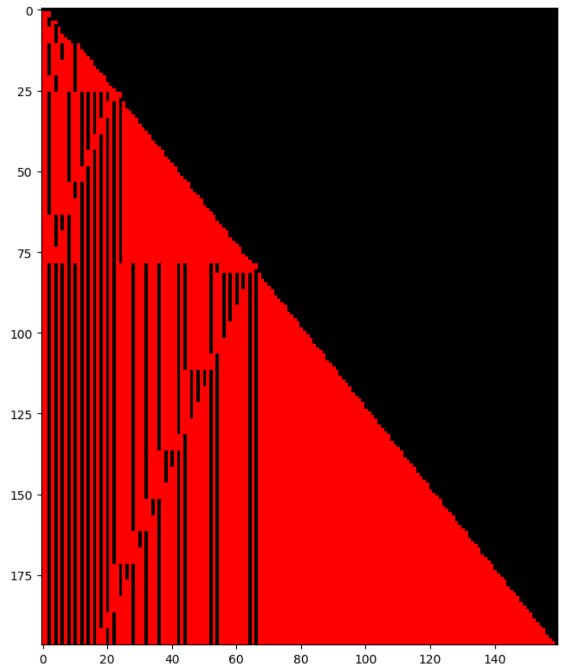


Figure 12: Tape head at right until step 218136190.
There is not another moment in the first 300 million steps.

This machine seems to be very similar to machine #33; it even seems to have the same orientation and with Skelet's program's choice use the same states A-D-B-E-A for usual rightmost bit moments and E-D-A for transitional moments, only with two instead of eight steps from the E to the D.

12	D	$i011$
51	D	$i01^{3}0I$
90	D	$i01^{4}I1$
391	D	$i01^{9}0OII$
32112	D	$i01^{24}IIIIIOIOOI$

Tape head at leftmost 1. $I = 0001, O = 0000$.
There are no other steps under 3 billion (other than adjacent steps) where the tape head is on the left.

35

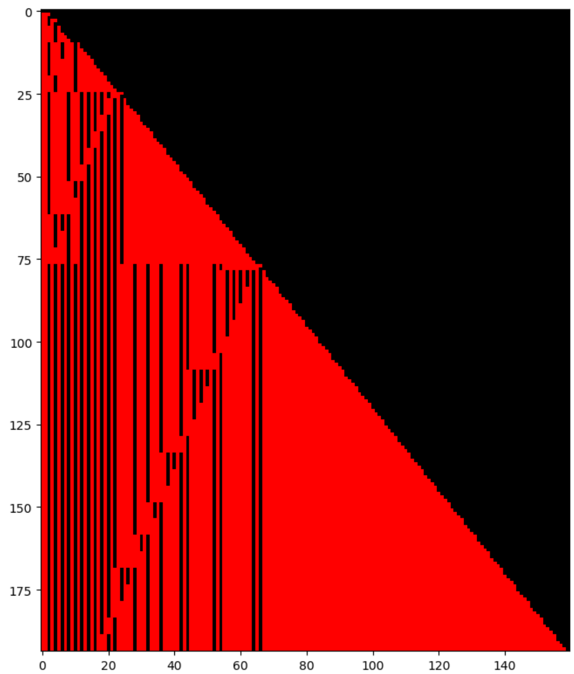


Figure 13: Tape head at right until step 201356448.
There is not another moment in the first 300 million steps.

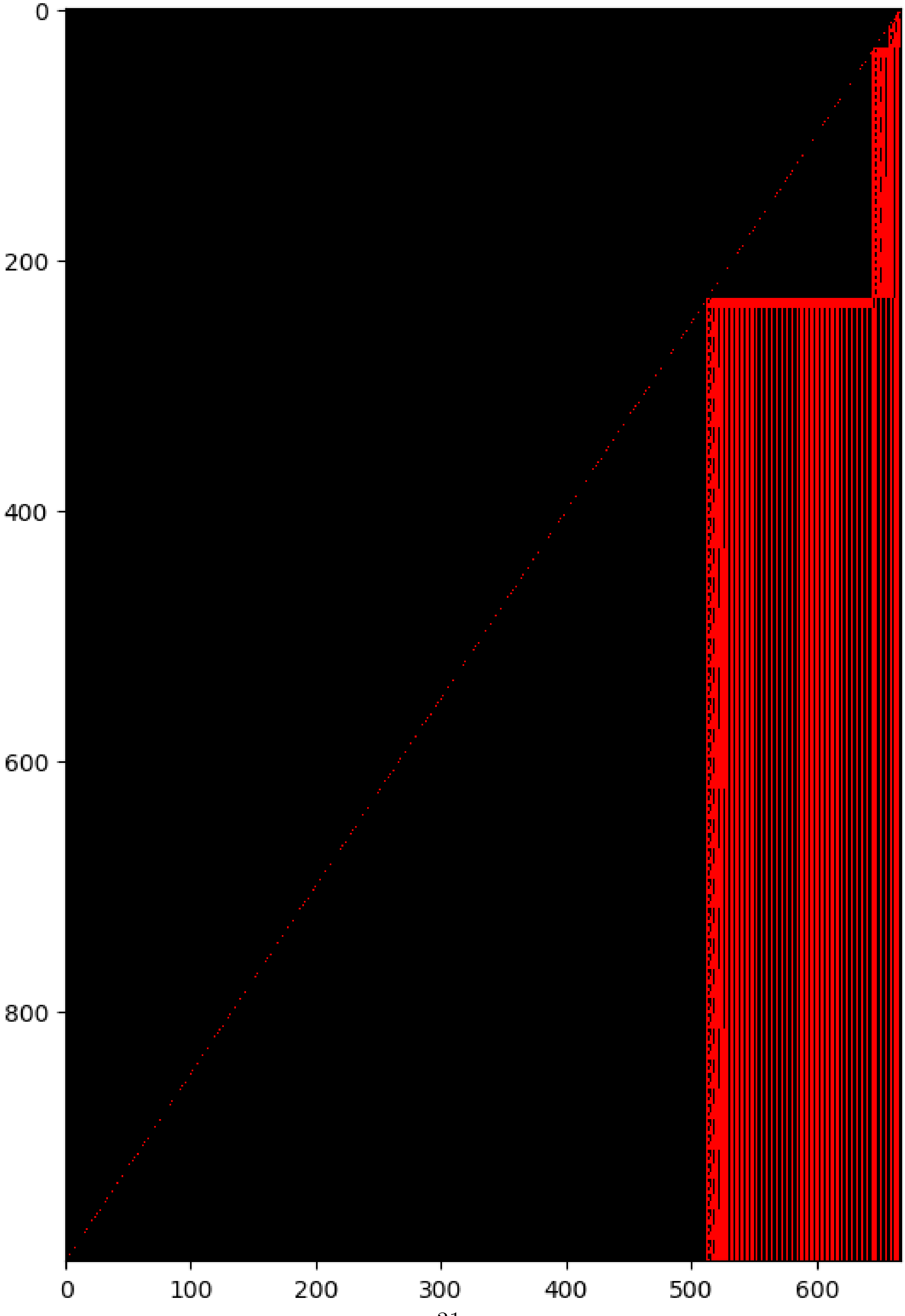
This machine seems to be very similar to machine #s 33 and 34; it even seems to have the same orientation and with Skelet's program's choice use the same states A-D-B-E-A for usual rightmost bit moments. It uses E-A, adjacent steps, for transitional moments.

10	D	$i011$
45	D	$i01^3 0I$
80	D	$i01^4 I1$
355	D	$i01^9 0OII$
29630	D	$i01^{24} IIIIIOIOOI$

Tape head at leftmost 1. $I = 0001, O = 0000$.
There are no other steps under 3 billion (other than adjacent steps) where the tape head is on the left.

Notice that these configurations are the exact same, even with the same orientation and state, as those of HNR#34. The differences in step number between a leftmost configuration of HNR#34 and the equivalent configuration of #35 are 2, 6, 10, 36, 2482.

36



The figure above shows the steps where the tape head is at left until step 144081. It never seems to make it to the right after step 40.

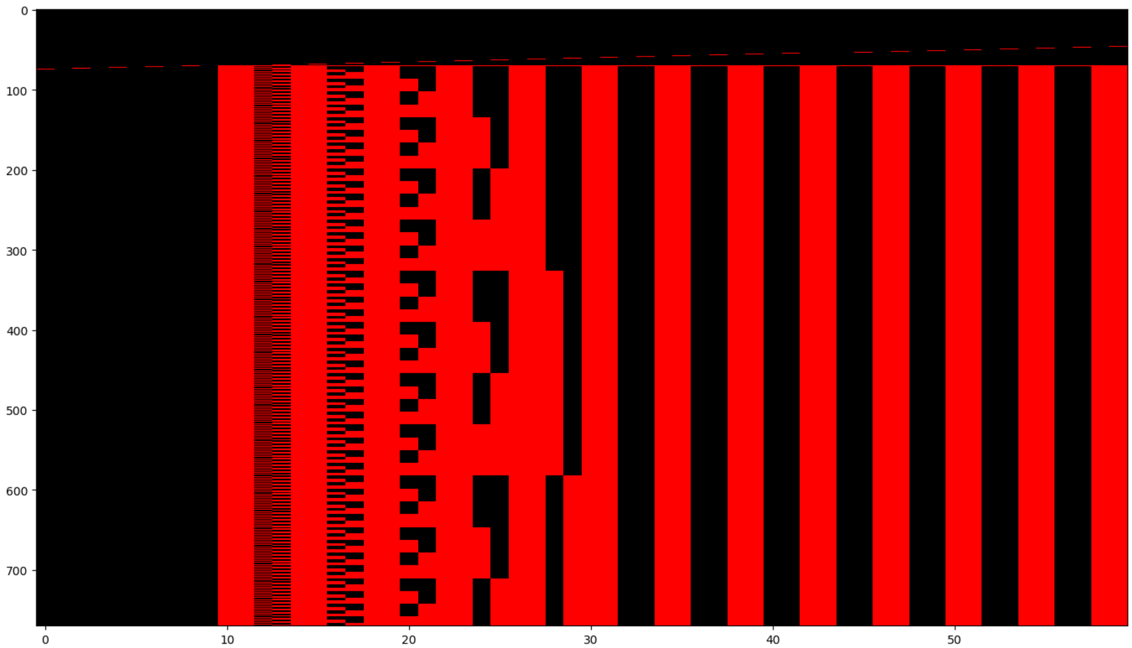


Figure 14: Zooming in on a particular part of the image and stretching the aspect ratio reveals that it is definitely binary counting going on.

```
 29  D                o00011111
 64  D             o011011101101
100  D          o0000011001111101
120  D        o000000011101111101
146  D      o00000000011011111101
174  D    o0000000000011111111101
265  D o0110111111111110110011011
277  D o000111111111111110110011011
```

Tape head at left from 29 to 265. Recall that at step 40 it is on the right.
If there were an even number of intervening 0s, the machine would halt.

Any journey "there and back again" from a configuration as above, in state D on the left at a 0 with an odd number of intervening 0s before a string beginning with 1, consists of the following phases:

1. B-D alternation going to the right. By parity this necessarily culminates in a configuration like `186 D 11111111111i1111111101`.

2. A-E alternation going to the right (after one left step). This will yield a step like `160 E 111111111101o11111101` or `197 A 1111111111110101010o1`.

3. C-D-A-C going to the left in the latter case; the same beginning with the D in the former case. How this ends could in theory depend on how many 1s there were (mod 4). This phase in the first string of 1s after the first string of 0s at the starting step, such as 174, where there were 9.

4. Once it has gotten left of the "tie" bit (as in step 186), the latter case goes into an A-E-A-C-D phase, moving 1 to the left every 5 steps, followed by one final A-C-D at extreme left. The former case goes into straight Cs, going left.

This accounts for the vastly different results of ending at step 174 and 265 on the left. In fact, it seems to be the case that we get this "transition-producing" behavior only when the number of 1s in that swath is 1 (mod 4). In all cases, the whole string should probably be interpreted as a usual binary number, with least significant digit on the left, but with an extra pair of 1s between each pair of digits. One can see how the machine is thus counting up:

5717	D	$o0^{099}111111111101111110011011$
5941	D	$o0^{101}110011001111111110011011$
6153	D	$o0^{103}111011001111111110011011$
6371	D	$o0^{105}110111001111111110011011$
6591	D	$o0^{107}111111001111111110011011$
6823	D	$o0^{109}110011101111111110011011$
7051	D	$o0^{111}111011101111111110011011$
7285	D	$o0^{113}110111101111111110011011$
7521	D	$o0^{115}111111101111111110011011$
7771	D	$o0^{117}110011011111111110011011$
8015	D	$o0^{119}111011011111111110011011$
8265	D	$o0^{121}110111011111111110011011$
8517	D	$o0^{123}111111011111111110011011$
8781	D	$o0^{125}110011111111111110011011$
9041	D	$o0^{127}111011111111111110011011$
9307	D	$o0^{129}110111111111111110011011$
9575	D	$o0^{131}111111111111111110011011$
10414	D	$o0^{3}1^{134}0110011001101011011$
10694	D	$o0^{5}(1100)^{33}1111100110011001101011011$

Since there are 17 1s in a row in the second-to-last row, the machine is no longer able to interpret everything it meets in the "binary number in pairs with pairs of 1s ahead of each pair of bits" format.

So the machine has set up a whole new task for itself on step 10414, so to speak; or, it has set up the same old task for itself with a whole new number. It *is* able to view this new format as a binary number through the 136th tape bit (68th number bit), but the 138th violates the format, and so after 2^{66} passes after step 10694 it will increment this old new problem bit-pair and create a whole new number for itself, $2 * 2^{66} + 5 + 1$ tape bits longer than the old number, reinterpreting this as a binary number of $2^{66} + 3$ numerical bits to count all the way up. It will continue turning the counting number (of 0s) it has created for itself linearly with each pass into a space into which to count up in binary, extending by pairs of 0s to the left all the while (see also Figure). The problem bits (shown in red) recede to the left at each of these transitions.

If this analysis is correct—and it clearly needs to be made more precise—then HNR#36 never halts.

We make it more precise here.

Lemma. *Let HNR#36 be in state D with tape $o0^m1^n0x$, where m is odd, n is positive, and x is any string. Then it will return to another configuration with the same property later.*

#36	A	B	C	D	E
0	C1L	D1R	D0L	B1R	D1L
1	E1R	H1L	C0L	A1L	A0R

Proof. Refer to the phases mentioned above.

$$
\begin{array}{llll}
 & 0 & D & o0^m1^n0 \\
\vdash & m+1 & D & 1^{m+1}i1^{n-1}0 \\
\vdash & m+3 & E & 1^{m+1}i1^{n-1}0 \\
\vdash & m+n+3 & E & 1^{m+1}(01)^{n/2}o, \quad n \text{ even, or} \\
 & m+n+3 & A & 1^{m+1}(01)^{(n-1)/2}0o, \quad n \text{ odd.}
\end{array}
$$

If n is even, one obtains

$$
\begin{array}{llll}
 & m+2n+4 & D & 1^m i(0011)^{n/4}1 \\
\vdash & 6m+2n+5 & A & o1^{m+2}011(0011)^{\frac{n}{4}-1}1 \\
\vdash & 6m+2n+7 & D & o01^{m+3}011(0011)^{\frac{n}{4}-1}1, \quad n \equiv 0 \pmod 4\text{, or} \\
 & m+2n+4 & C & 1^m i11(0011)^{(n-2)/4}1 \\
\vdash & 2m+2n+5 & C & o0^{m+1}11(0011)^{(n-2)/4}1 \\
\vdash & 2m+2n+6 & D & o0^{m+2}11(0011)^{(n-2)/4}1, \quad n \equiv 2 \pmod 4.
\end{array}
$$

The $n \equiv 0 \pmod 4$ case is from the A-E-A-C-D phase mentioned above. So in the first case, we end up with one 0 after the D, whereas in the second case, we end up with $m + 2$ 0s after the D.

If n is odd, one obtains

$$
\begin{array}{llll}
 & m+n+5 & D & 1^{m+1}(01)^{(n-1)/2} < 01 \\
\vdash & m+2n+6 & D & 1^m i(0011)^{(n-1)/4}01, \quad n \equiv 1 \pmod 4\text{, or} \\
 & m+2n+6 & C & 1^m i11(0011)^{(n-3)/4}01, \quad n \equiv 3 \pmod 4.
\end{array}
$$

by A-C-C-D. Thus the $1 \pmod 4$ case finishes out like the $0 \pmod 4$ case, and the $3 \pmod 4$ case finishes out like the $2 \pmod 4$ case. □

Thus HNR#36 never halts.

37

12	A	$(10)^3$
71	A	$(10)^3 1 (10)^3$
400	A	$(10)^9 1 (10)^3 1 (10)^3$
2835	A	$(10)^{27} 1 (10)^9 1 (10)^3 1 (10)^3$
23252	A	$(10)^{81} 1 (10)^{27} 1 (10)^9 1 (10)^3 1 (10)^3$
202591	A	$(10)^{243} 1 (10)^{81} 1 (10)^{27} 1 (10)^9 1 (10)^3 1 (10)^3$
1803480	A	$(10)^{729} 1 (10)^{243} 1 (10)^{81} 1 (10)^{27} 1 (10)^9 1 (10)^3 1 (10)^3$
16172075	A	$(10)^{2187} 1 (10)^{729} 1 (10)^{243} 1 (10)^{81} 1 (10)^{27} 1 (10)^9 1 (10)^3 1 (10)^3$
145371292	A	$(10)^{6561} 1 (10)^{2187} 1 (10)^{729} 1 (10)^{243} 1 (10)^{81} 1 (10)^{27} 1 (10)^9 1 (10)^3 1 (10)^3$

Moments when the tape head is on the right.

HNR#37 will thus clearly be a simple one to prove never halts.

40

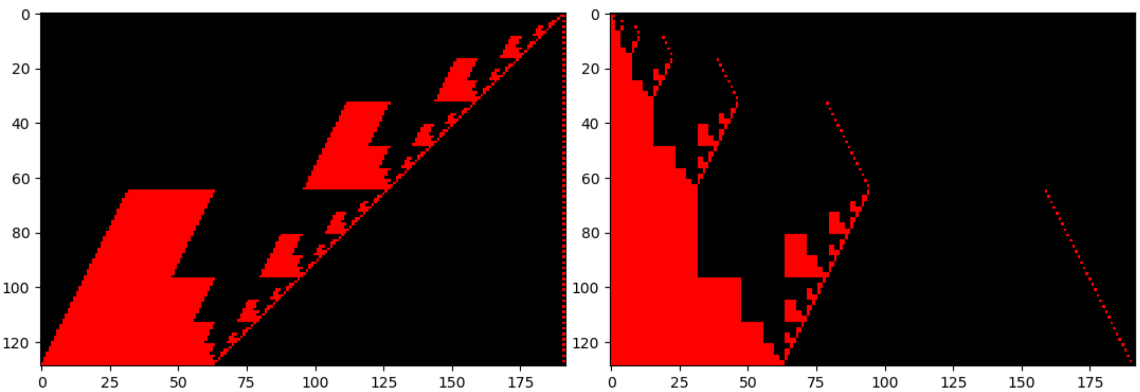


Figure 15: Tape head on the right, right-justified and left-justified respectively.

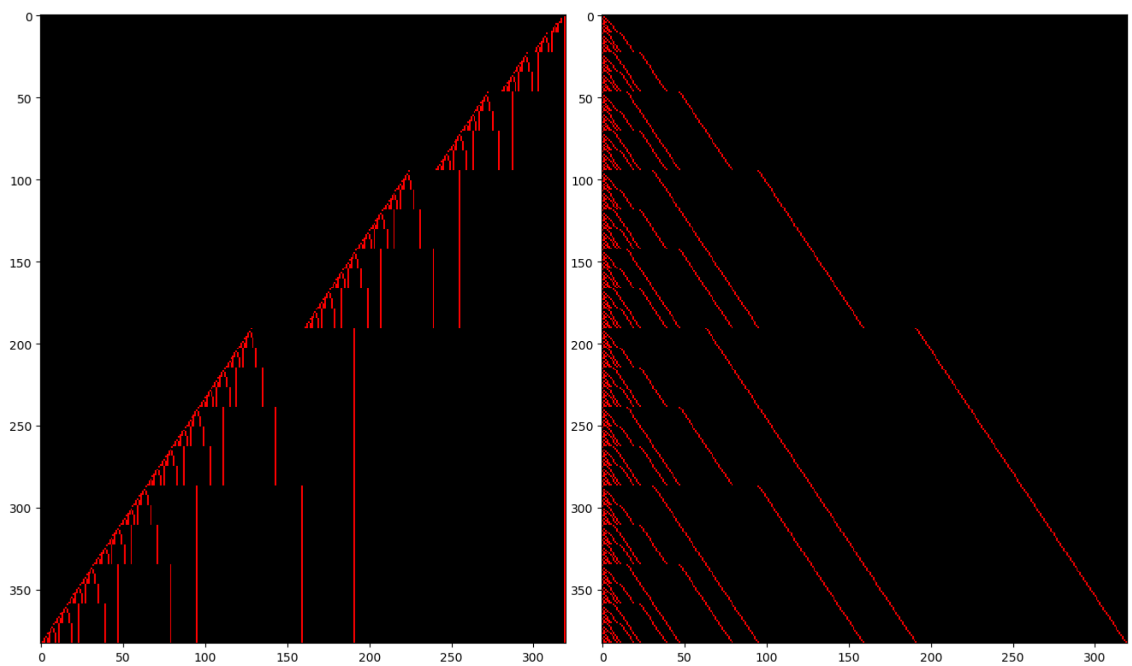


Figure 16: Tape head on the left, right-justified and left-justified respectively.

HNR#40 seems to just want to draw a fractal, so I'm lumping it in with the "easy but arduous to prove."

43

87	C	$o001101111111$
219	C	$o001101(1110110)^{1}1111111$
569	C	$o00110111111(1110110)^{2}1111111$
1536	C	$o001101^{11}(1110110)^{4}1111111$
4472	C	$o001101(1110110)^{1}1^{11}(1110110)^{8}1111111$
14462	C	$o00110111111(1110110)^{2}1^{11}(1110110)^{16}1111111$
50837	C	$o001101^{11}(1110110)^{4}1^{11}(1110110)^{32}1111111$
189101	C	$o001101(1110110)^{1}1^{11}(1110110)^{8}1^{11}(1110110)^{64}1111111$
727795	C	$o00110111111(1110110)^{2}1^{11}(1110110)^{16}1^{11}(1110110)^{128}1111111$
2853770	C	$o001101^{11}(1110110)^{4}1^{11}(1110110)^{32}1^{11}(1110110)^{256}1111111$

Moments when the tape head is on the left.

Non-halting of HNR#43 will thus clearly be an easy but possibly arduous proof.