CMP 405

CALCULATIONS + * / - 1ST SEM

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NOTE :

This note was derived from 2 seperate youtube videos, feel free to check them out yourself

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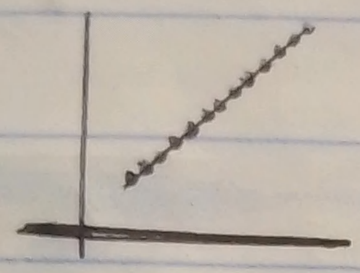
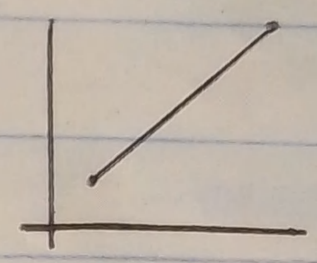
DDA Line Drawing Algorithm #1

DDA (stands for Digital Differential Analyzer) algorithm is used for drawing a line.

A Line is a collection of points over a path as shown below



But for simplicity we only draw a straight line without the points in between



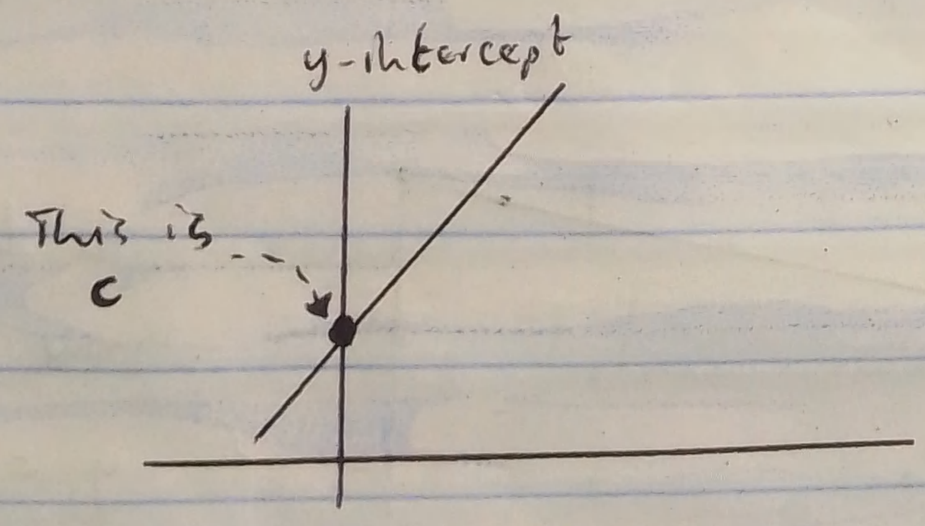
Now a line is defined by using the below Line Equation

$$\boxed{y = mx + c}$$

Some equations would have b instead of c. It doesn't change anything, feel free to use whichever letter you want

- m is the slope of the Line
- c is the intersection point of the Line and the y-intercept

e.g

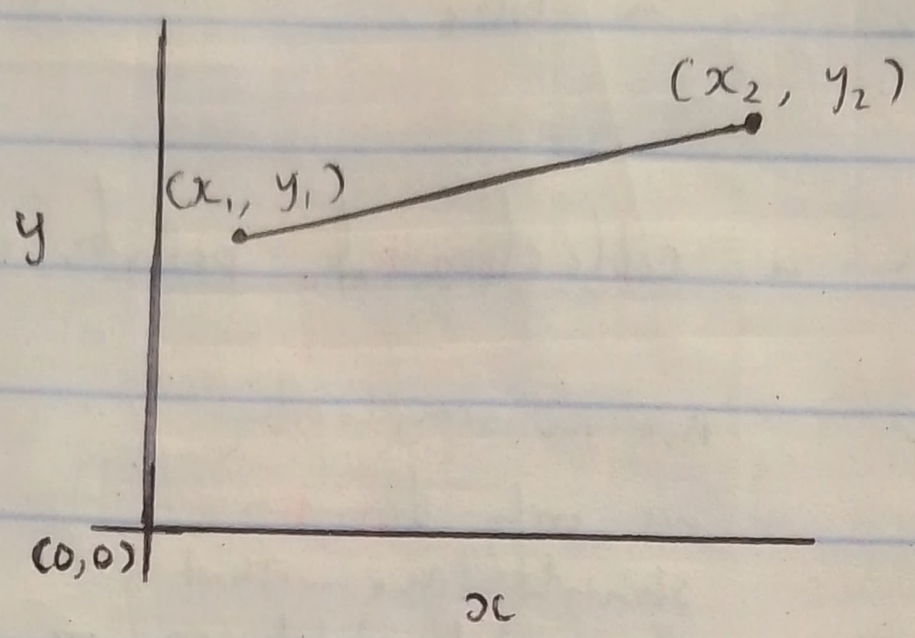


Fun fact:

- if you know the slope (m) and the y-intercept (c), then you can know any point on the line

Slope of a Line

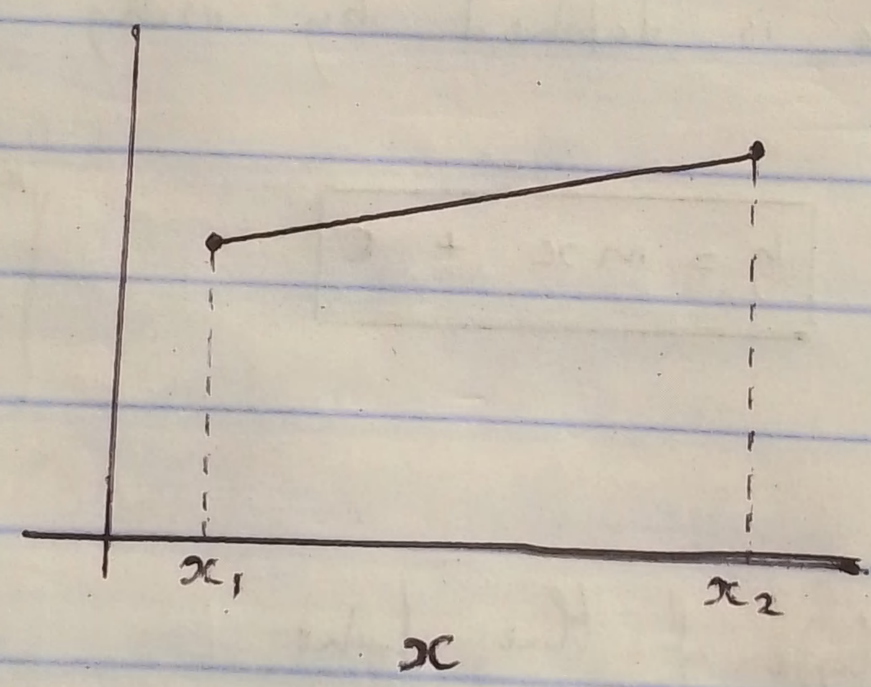
If we had a line segment as shown below



point A = (x_1, y_1)

point B = (x_2, y_2)

To get the x coordinates (x_1 and x_2) we would



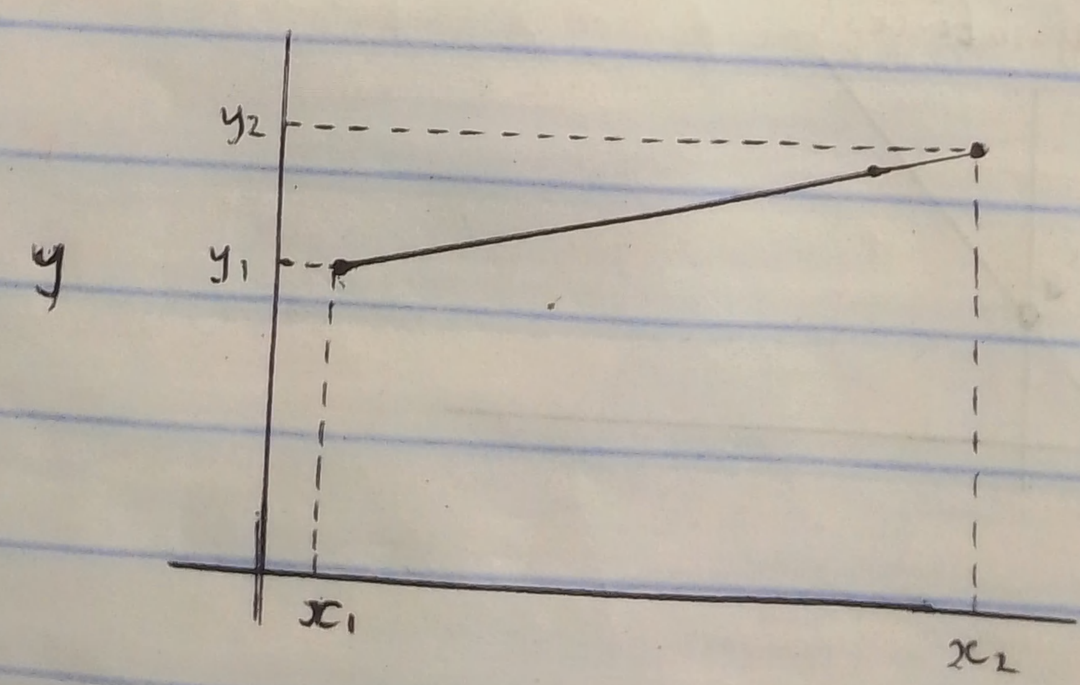
with this we would know x_1 and x_2

$x = (x_1, x_2)$

e.g

$x = (1, 8)$

To get the y coordinates (y_1 and y_2) we would



with this --- we would know y_1 and y_2

$y = (y_1, y_2)$

e.g

$y = (5, 7)$

Now to get the slope of our line m, we use the formula

$$m = \frac{\Delta y}{\Delta x} \quad \text{or} \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Δy or dy basically means "Difference in y" which means

$$y_2 - y_1$$

Δx or dy basically means "Difference in x" which means

$$x_2 - x_1$$

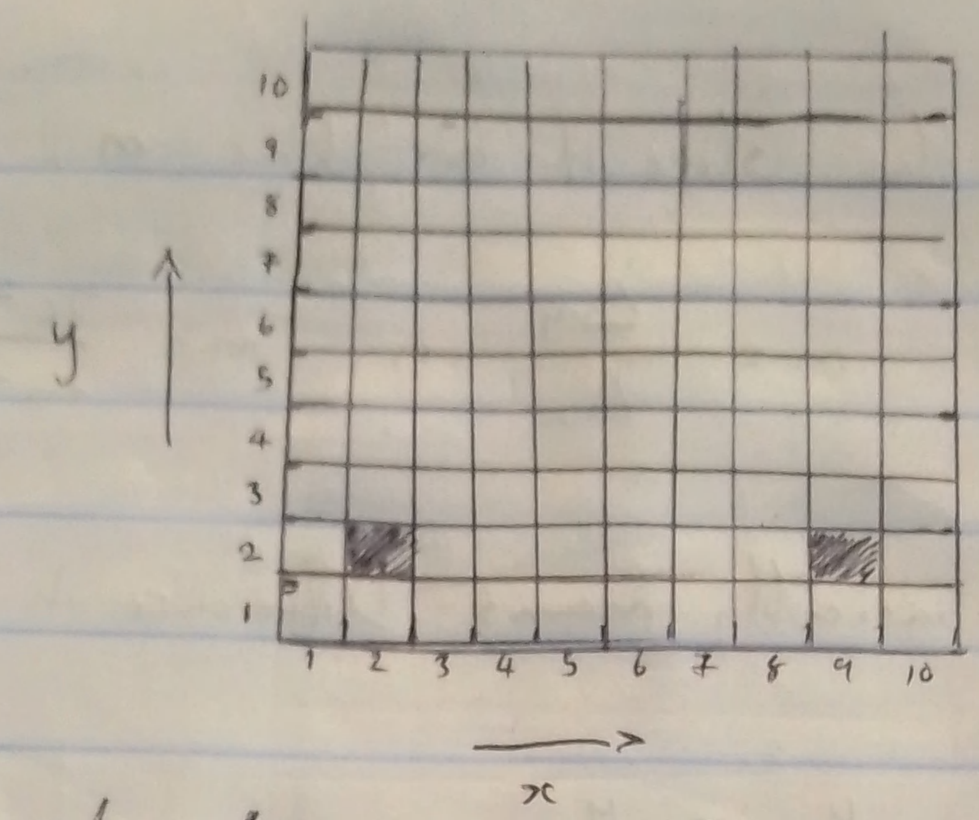
With this knowledge we can now calculate the slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$$

Practice Exercise 1

Now given the below screen with Point A (2, 2) and Point B (9, 2) where Point A is (x_1, y_1) and Point B is (x_2, y_2)

Screen



Point A

$(x_1, y_1) = (2, 2)$

Point B

$(x_2, y_2) = (9, 2)$

STEP 1: To get slope:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$$

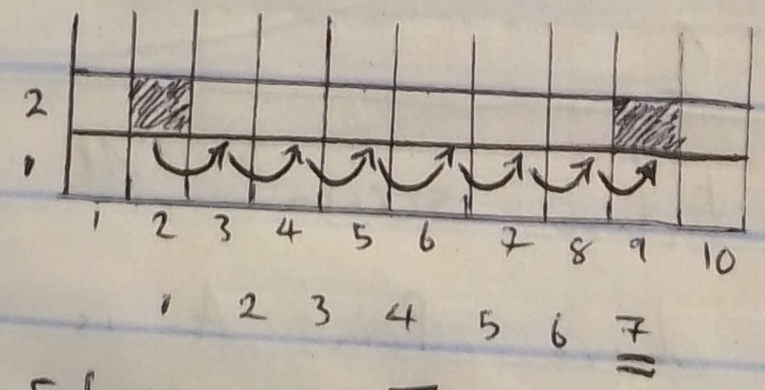
$$\boxed{\begin{aligned} \Delta y &= 0 \\ \Delta x &= 7 \end{aligned}}$$

$x_1 = 2, \quad x_2 = 9, \quad y_1 = 2, \quad y_2 \; 2$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - 2}{9 - 2} = \frac{0}{7} = 0$$

Our slope $m = 0$ //

STEP 2: Next: <u>How many steps</u> do we have to take from Point A to Point B (Count it above in the screen)



Steps = 7

STEP 3: How many times should we increment in x

$$xinc = \frac{\Delta x}{steps} = \frac{7}{7} = 1$$

STEP 4: How many times should we increment in y

$$yinc = \frac{\Delta y}{steps} = \frac{0}{7} = 0$$

0 means we won't be incrementing in

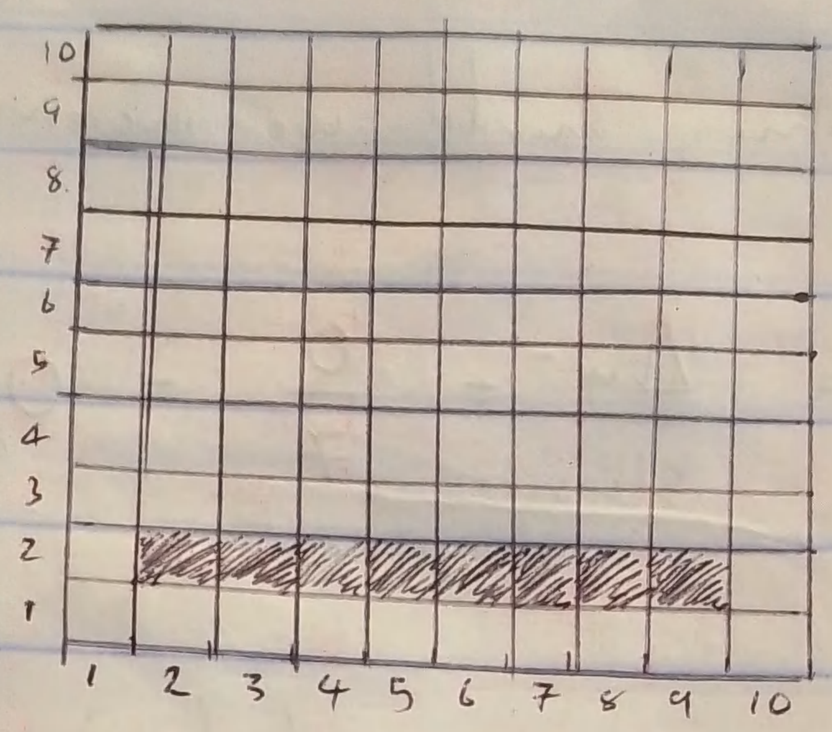
STEP 5: We make the Points (Starting from Point A (2, 2))

$\boxed{xinc = 1}$ $\boxed{yinc = 0}$

x	y	
2	2	# x_1 and y_1 Point A
3	2	# $2 + xinc = 3$ | $2 + yinc = 2$
4	2	# $3 + xinc = 4$ | $2 + yinc = 2$
5	2	# $4 + xinc = 5$ | $2 + yinc = 2$
6	2	# $5 + xinc = 6$ | $2 + yinc = 2$
7	2	# $6 + xinc = 7$ | $2 + yinc = 2$
8	2	# $7 + xinc = 8$ | $2 + yinc = 2$
9	2	# $8 + xinc = 9$ | $2 + yinc = 2$

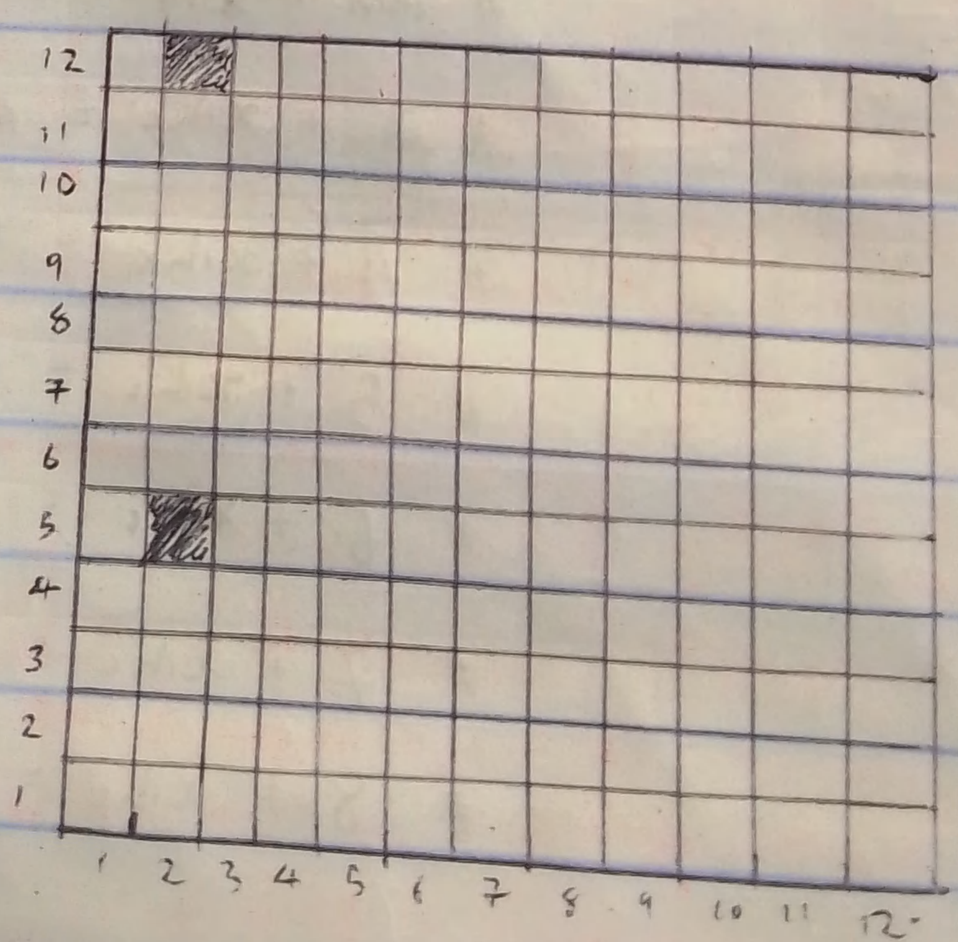
STEP 6 : We want to paint the pixel points we got from Step 5

(2, 2) (3, 2) (4, 2) (5, 2) (6, 2) (7, 2) (8, 2) (9, 2)



Practice Exercise 2

Given Point A and Point B, where Point A is (x_1, y_1) (2, 5) and Point B is (x_2, y_2) (2, 12)



STEP 1: Calculate slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$$

$$\Delta y = y_2 - y_1 = 12 - 5 = 7$$
$$\Delta x = x_2 - x_1 = 2 - 2 = 0$$

$x_1 = 2, \quad x_2 = 2, \quad y_1 = 5, \quad y_2 = 12$

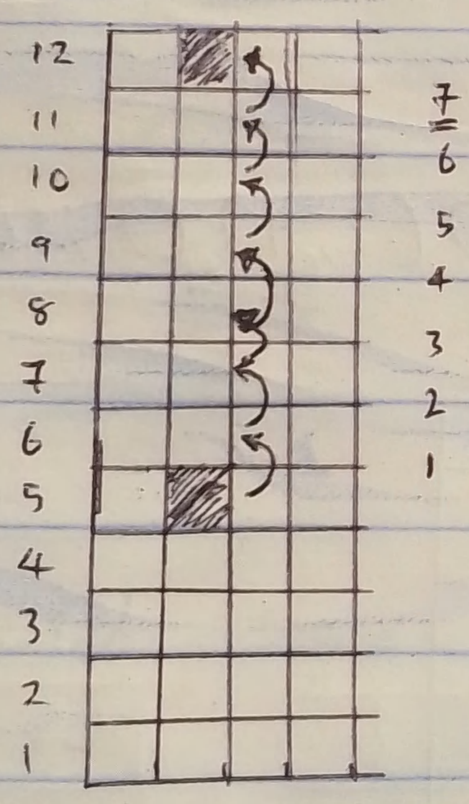
$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{12 - 5}{2 - 2} = \frac{7}{0} = \infty$$

Infinity

Because slope is vertical and greater than 1

Our slope $m = \infty$

STEP 2: Calculate steps



steps = 7

STEP 3: Calculate xinc

$$x_{inc} = \frac{\Delta x}{7} = \frac{0}{7} = 0$$

0 means we won't be incrementing x

STEP 4: Calculate yinc

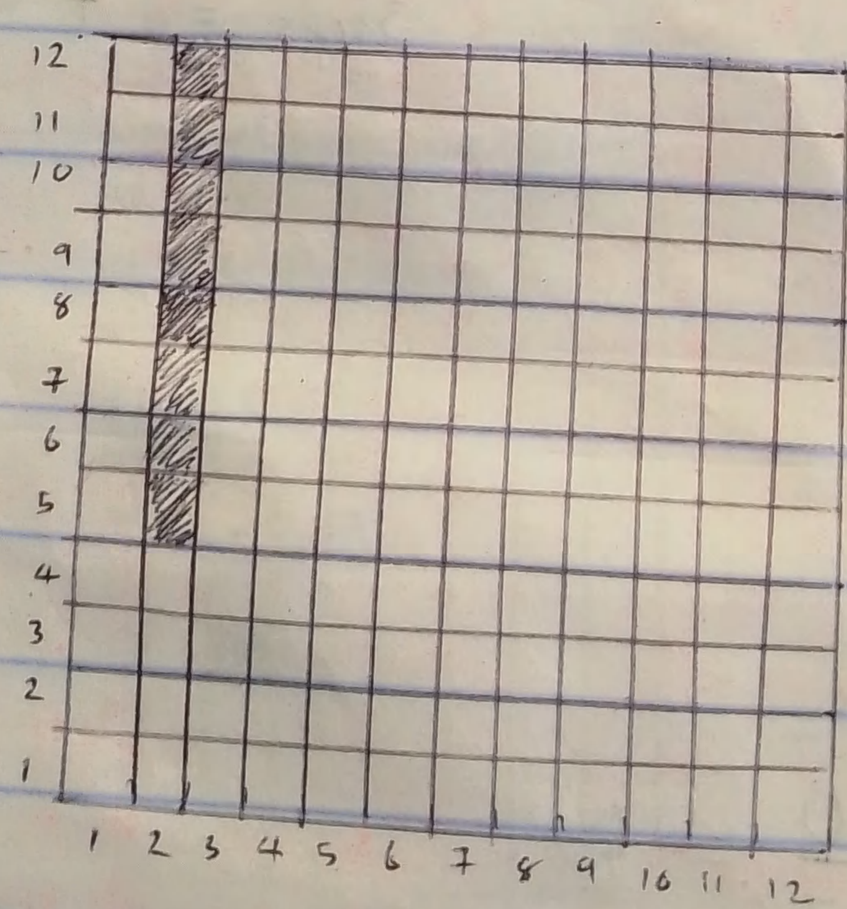
$$y_{inc} = \frac{\Delta y}{7} = \frac{7}{7} = 1$$

STEP 5: Take Points (Point A (x_1, y_1) or $(2, 5)$)

x	y	$x_{inc} = 0$	$y_{inc} = 1$
2	5	# $2 + x_{inc} = 2$	$5 + y_{inc} = 6$
2	6	# $2 + x_{inc} = 2$	$6 + y_{inc} = 7$
2	7	# $2 + x_{inc} = 2$	$7 + y_{inc} = 8$
2	8	# $2 + x_{inc} = 2$	$8 + y_{inc} = 9$
2	9	# $2 + x_{inc} = 2$	$9 + y_{inc} = 10$
2	10	# $2 + x_{inc} = 2$	$10 + y_{inc} = 11$
2	11	# $2 + x_{inc} = 2$	$11 + y_{inc} = 12$
2	12		

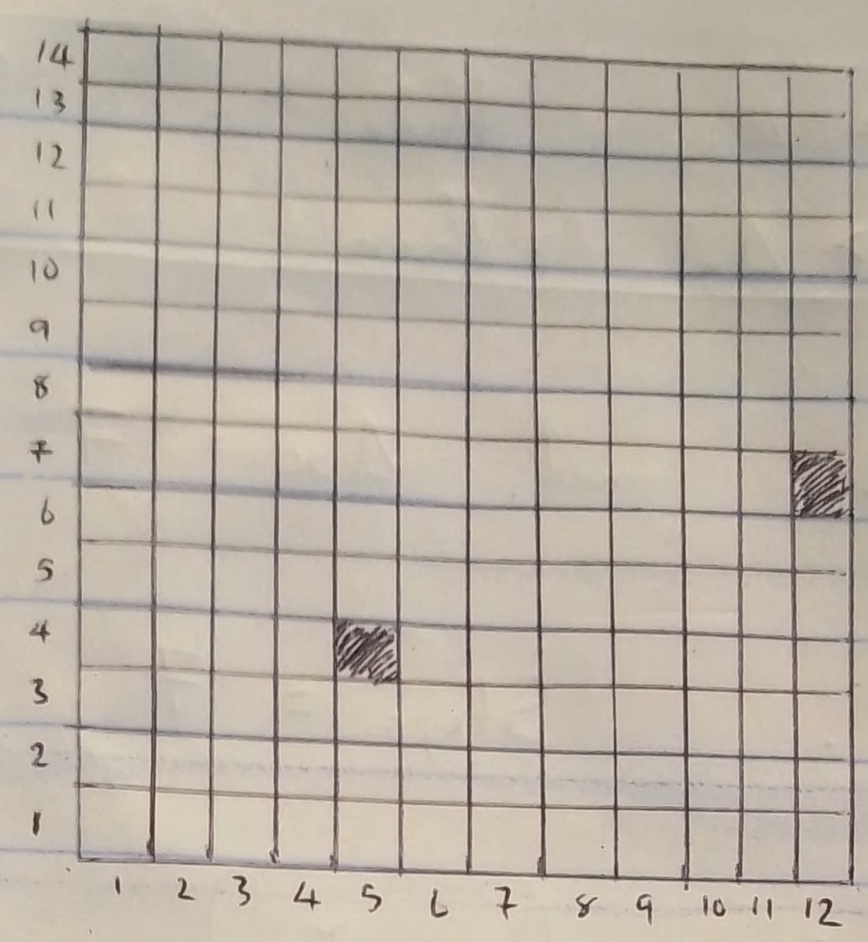
STEP 6: Paint Pixel Points

(2, 5) (2, 6) (2, 7) (2, 8) (2, 9) (2, 10) (2, 11) (2, 12)



Practice Exercise 3 ($m < 1$ example)

Given Point A (x_1, y_1) (5, 4) and Point B (x_2, y_2) (12, 7)



STEP 1: Calculate slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$$

$\Delta y = y_2 - y_1 = 7 - 4 = 3$

$\Delta x = x_2 - x_1 = 12 - 5 = 7$

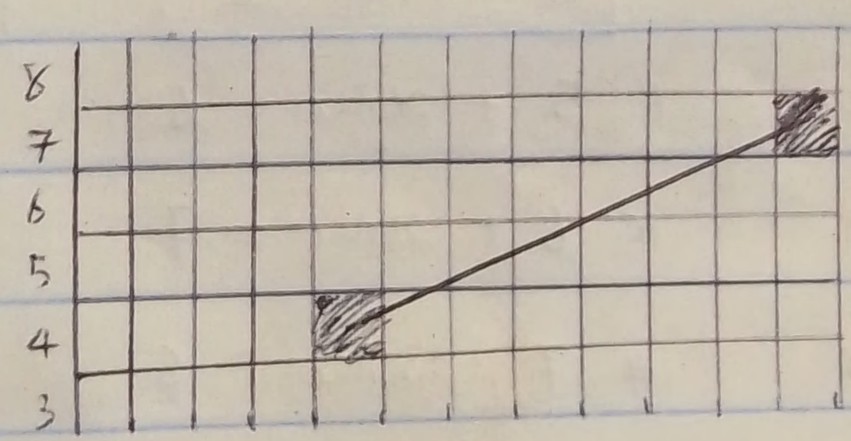
$x_1 = 5, \quad x_2 = 12, \quad y_1 = 4, \quad y_2 = 7$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7 - 4}{12 - 5} = \frac{3}{7} = 0.4$$

Notice:

m (or slope) < 1

STEP 2: Calculate steps



Steps = ?

To get the steps in such a scenario where we don't know which base to count from Point A to Point B.

We say - Steps will be the largest number between Δy and

if $\Delta_y > \Delta_x$ then Steps $= \Delta_y$
if $\Delta_x > \Delta_y$ then Steps $= \Delta_x$

$\Delta_y = 3$ and $\Delta_x = 7$

Therefore : Steps $= 7$ } the largest amongst Δ_y and Δ_x

STEP 3 : Calculate x_{inc} (or x_k)

$$x_{inc} = \frac{\Delta_x}{steps} = \frac{7}{7} = 1$$

STEP 4 : Calculate y_{inc} (or y_k)

$$y_{inc} = \frac{\Delta_y}{steps} = \frac{3}{7} = 0.4$$

STEP 5 : Take Points

x	y		
5	4	$\boxed{x_{inc} = 1}$	$\boxed{y_{inc} = 0.4}$
6	4.4	# $5 + x_{inc} = 6$	$4 + y_{inc} = 4.4$
7	4.8	# $6 + x_{inc} = 7$	$4.4 + y_{inc} = 4.8$
8	5.2	# $7 + x_{inc} = 8$	$4.8 + y_{inc} = 5.2$
9	5.6	# $8 + x_{inc} = 9$	$5.2 + y_{inc} = 5.6$
10	6	# $9 + x_{inc} = 10$	$5.6 + y_{inc} = 6$
11	6.4	# $10 + x_{inc} = 11$	$6 + y_{inc} = 6.4$
12	6.8	# $11 + x_{inc} = 12$	$6.4 + y_{inc} = 6.8$

Tip
- y_{inc} is the same as y_k
- x_{inc} is the same as x_k

STEP 5.5: Round decimal points

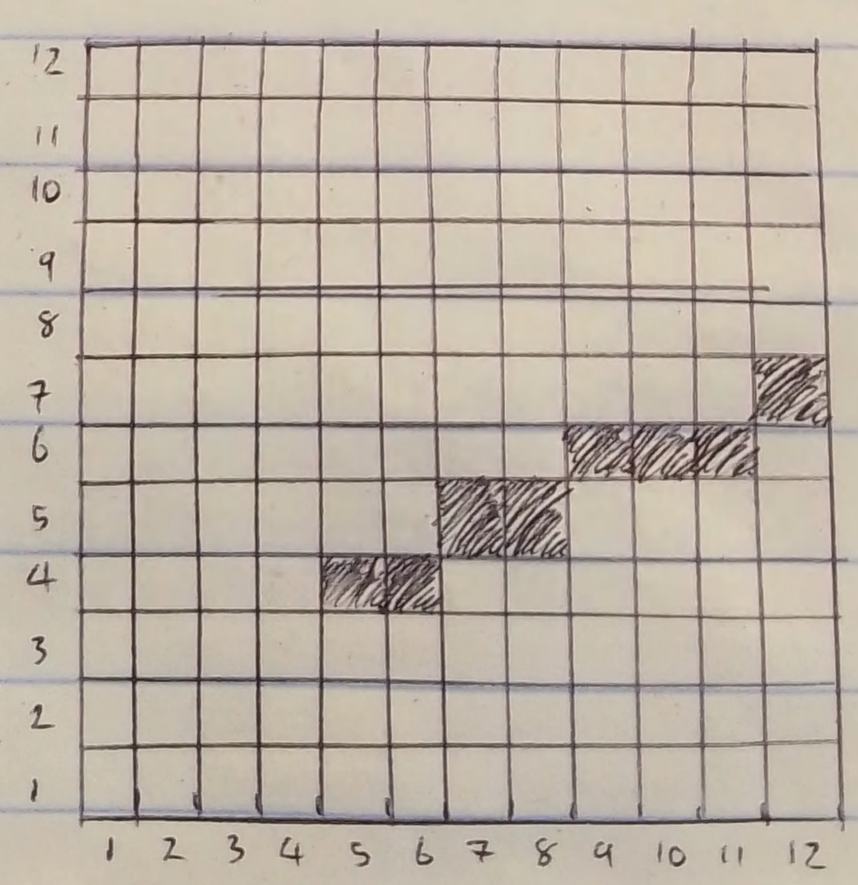
Rounding Rule
.4 below - is rounded down, .5 above - is rounded up

x	y	y (Rounded)	
5	4	4	# 4 becomes 4
6	4.4	4	# 4.4 becomes 4
7	4.8	5	# 4.8 becomes 5
8	5.2	5	# 5.2 becomes 5
9	5.6	6	# 5.6 becomes 6
10	6	6	# 6 becomes 6
11	6.4	6	# 6.4 becomes 6
12	6.8	7	# 6.8 becomes 7

Δx

STEP 6: Paint Pixel Points

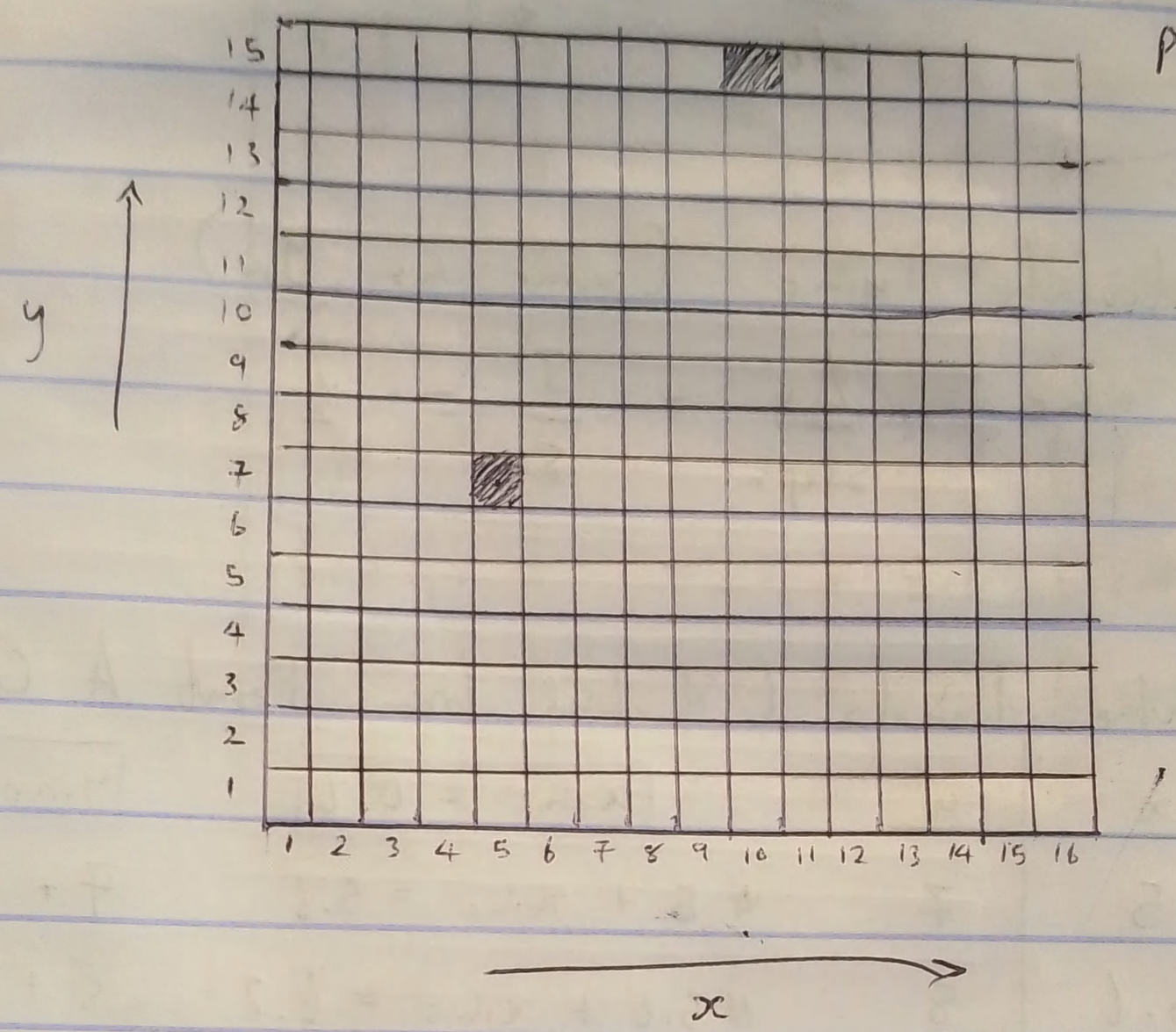
(5, 4) (6, 4) (7, 5) (8, 5) (9, 6) (10, 6) (11, 6) (12, 7



Practice Exercise 4 ($m > 1$)

Given Point A (x_1, y_1) $(5, 7)$ and Point B (x_2, y_2) $(10, 15)$

Plot the Pixels



STEP 1: Calculate Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$$

$$\boxed{\begin{aligned} \Delta y &= y_2 - y_1 = 15 - 7 = 8 \\ \Delta x &= x_2 - x_1 = 10 - 5 = 5 \end{aligned}}$$

$x_1 = 5, \quad x_2 = 10, \quad y_1 = 7, \quad y_2 = 15$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{15 - 7}{10 - 5} = \frac{8}{5}$$

STEP 2: Calculate steps

Steps = Larges number between Δy and Δx

Steps = Δy

Steps = 8

STEP 3: Calculate x_{inc} (same as x_k)

$$x_{inc} = \frac{\Delta x}{steps} = \frac{5}{8} = 0.6$$

STEP 4: Calculate y_{inc} (same as y_k)

$$y_{inc} = \frac{\Delta y}{steps} = \frac{8}{8} = 1$$

STEP 5: Take Points (Starting from Point A (x_1, y_1) (5, 7)

$\boxed{x_{inc} = 0.6}$ $\boxed{y_{inc} = 1}$

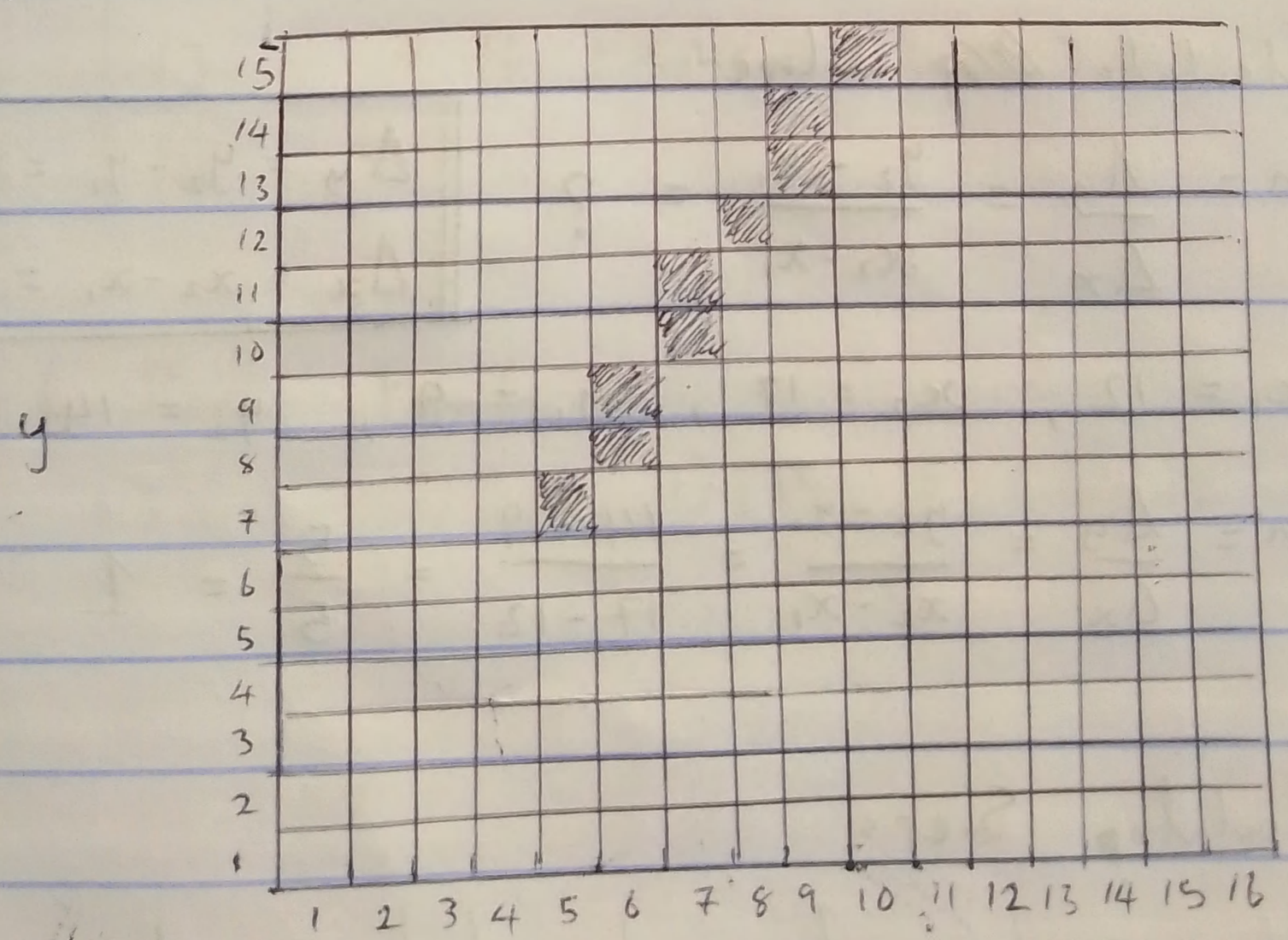
x	y		
5	7	# $5 + x_{inc} = 5.6$	$7 + y_{inc} = 8$
5.6	8	# $5.6 + x_{inc} = 6.2$	$8 + y_{inc} = 9$
6.2	9	# $6.2 + x_{inc} = 6.8$	$9 + y_{inc} = 10$
6.8	10	# $6.8 + x_{inc} = 7.4$	$10 + y_{inc} = 11$
7.4	11	# $7.4 + x_{inc} = 8$	$11 + y_{inc} = 12$
8	12	# $8 + x_{inc} = 8.6$	$12 + y_{inc} = 13$
8.6	13	# $8.6 + x_{inc} = 9.2$	$13 + y_{inc} = 14$
9.2	14	# $9.2 + x_{inc} = 9.8$	$14 + y_{inc} = 15$
9.8	15		

STEP 5.5: Take Points (Round decimal points)

x (Rounded)	x	y	
5	5	7	# 5 becomes 5
6	5.6	8	# 5.6 becomes 6
6	6.2	9	# 6.2 becomes 6
7	6.8	10	# 6.8 becomes 7
7	7.4	11	# 7.4 becomes 7
8	8	12	# 8 becomes 8
9	8.6	13	# 8.6 becomes 9
9	9.2	14	# 9.2 becomes 9
10	9.8	15	# 9.8 becomes 10

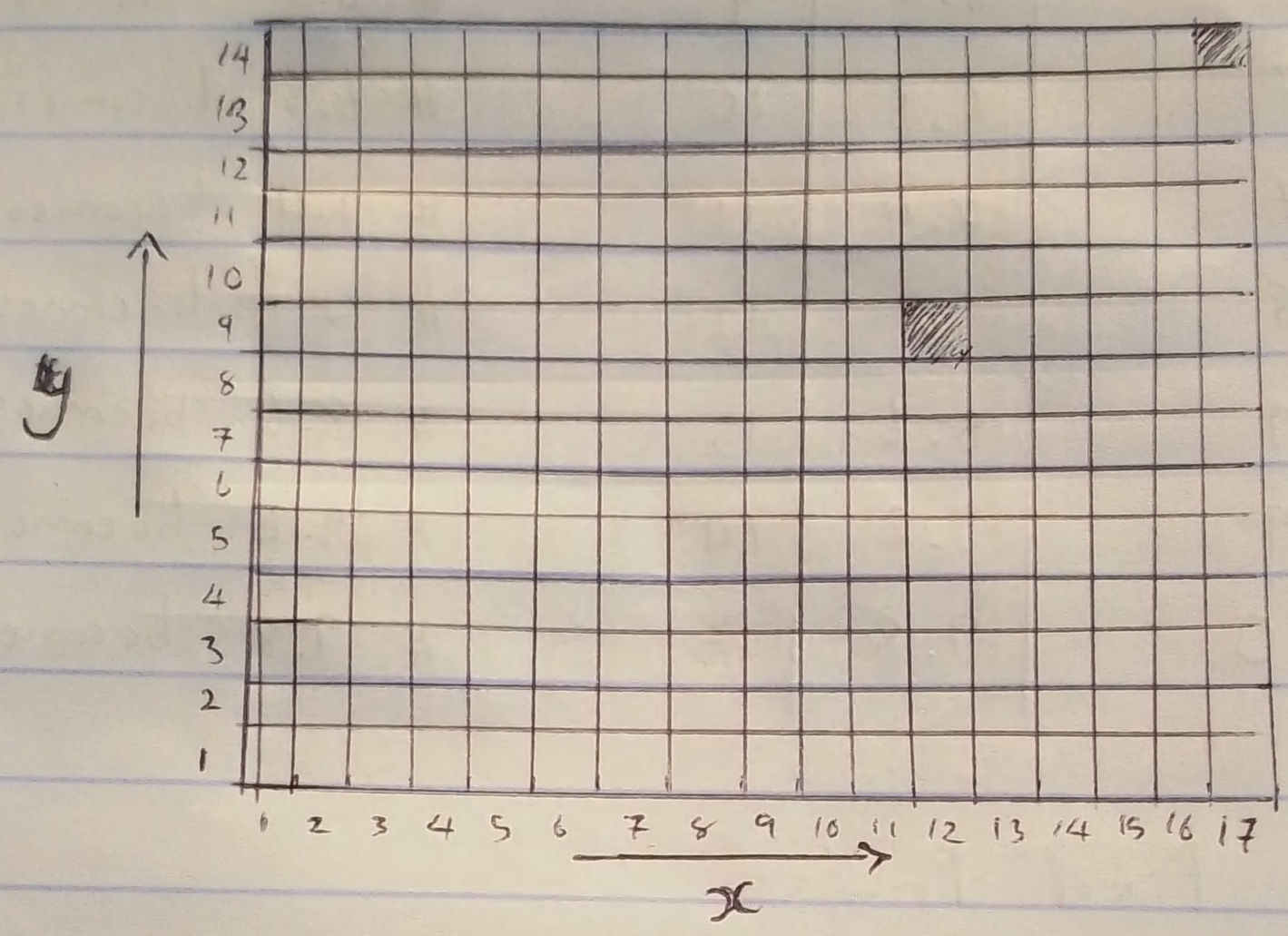
STEP 6: Paint Pixel Points

(5, 7) (6, 8) (6, 9) (7, 10) (7, 11) (8, 12) (9, 13) (9, 14)
(10, 15)



Practice Exercise 5 ($m = 1$)

Given Point A (x_1, y_1) $(12, 9)$ and Point B (x_2, y_2) $(17, 14)$



STEP 1 : Calculate ~~Step~~ Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$$

$$\Delta y = y_2 - y_1 = 14 - 9 = 5$$
$$\Delta x = x_2 - x_1 = 17 - 12 = 5$$

$x_1 = 12$, $x_2 = 17$, $y_1 = 9$, $y_2 = 14$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{14 - 9}{17 - 12} = \frac{5}{5} = 1$$

STEP 2 : Calculate Steps

Steps = Largest number between Δy and Δx

Both are the same so we can use any

$$\text{Steps} = \Delta y$$
$$\text{Steps} = 5$$

STEP 3: Calculate x_{inc}

$$x_{inc} = \frac{\Delta x}{\text{steps}} = \frac{5}{5} = 1$$

STEP 4: Calculate y_{inc}

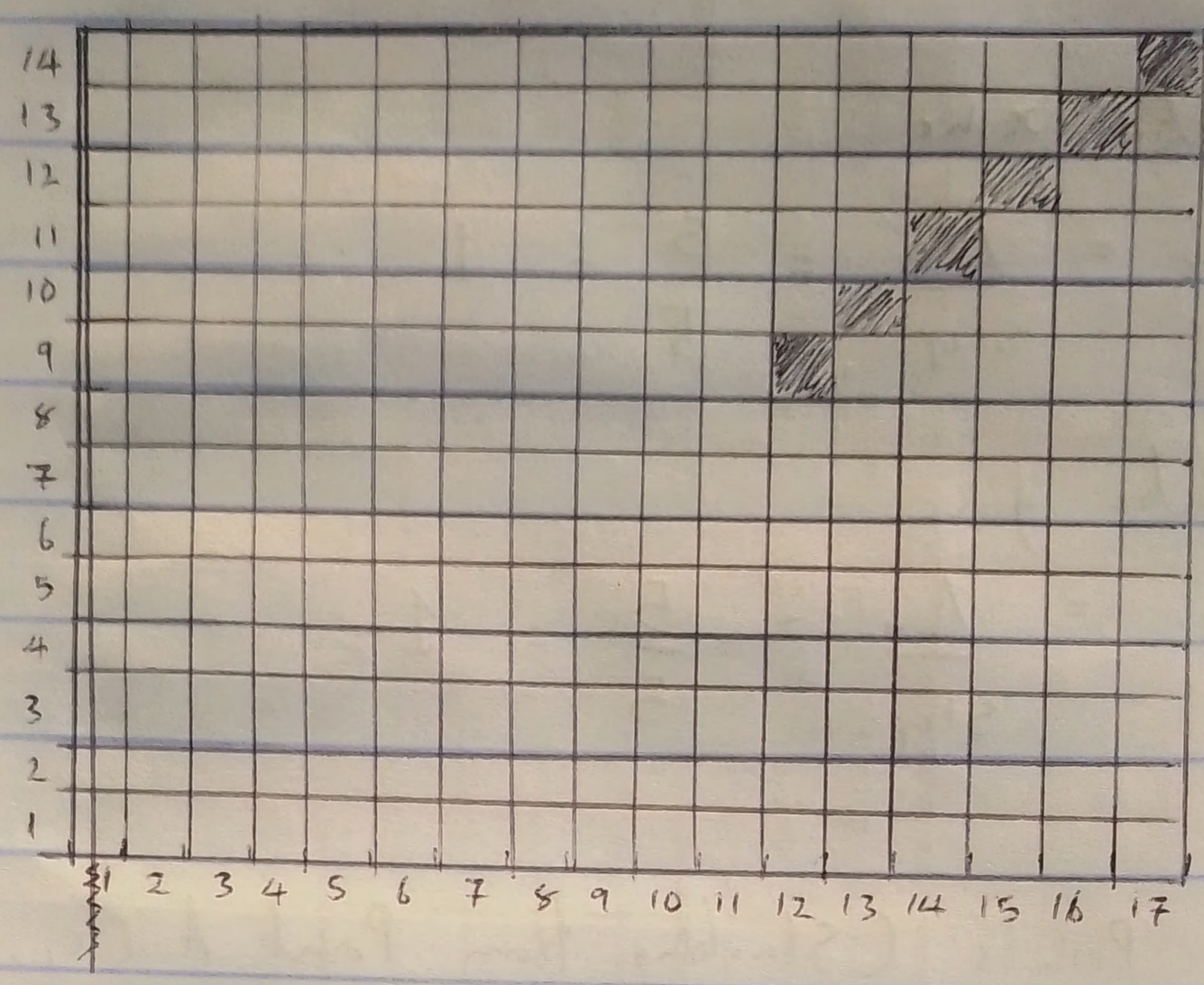
$$y_{inc} = \frac{\Delta y}{\text{steps}} = \frac{5}{5} = 1$$

STEP 5: Take Points (Starting from Point A (x_1, y_1) $(12, 9)$

x	y	$\boxed{x_{inc} = 1}$	$\boxed{y_{inc} = 1}$
12	9	# $12 + x_{inc} = 13$	$9 + y_{inc} = 10$
13	10	# $13 + x_{inc} = 14$	$10 + y_{inc} = 11$
14	11	# $14 + x_{inc} = 15$	$11 + y_{inc} = 12$
15	12	# $15 + x_{inc} = 16$	$12 + y_{inc} = 13$
16	13	# $16 + x_{inc} = 17$	$13 + y_{inc} = 14$
17	14		

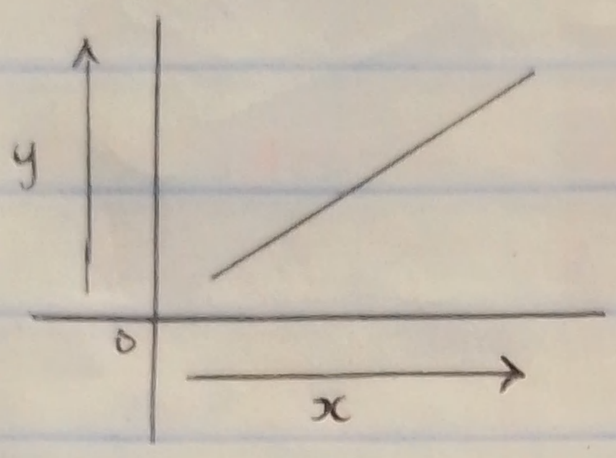
STEP 6: Paint Pixel Points

$(12, 9)$ $(13, 10)$ $(14, 11)$ $(15, 12)$ $(16, 13)$ $(17, 14)$

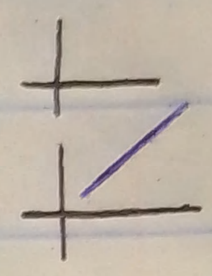

14
13
12
11
10
9
8
7
6
5
4
3
2
1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17

BRESENHAM'S LINE DRAWING ALGORITHM #2

Line



Supposing the left has our graphic system we draw and a line in it



Any line in our graphic system is represented by the below formula

$$y = mx + c$$

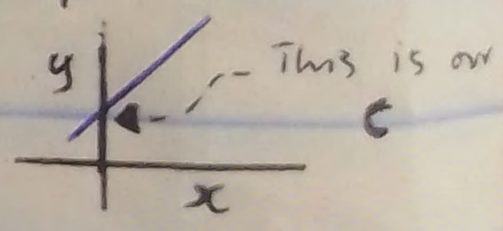
~~Explanation~~ Explanation:

m (is the slope of a line) $= \frac{\Delta y}{\Delta x}$ $\frac{\text{\#Difference in } y}{\text{\#Difference in } x}$

x (is simply the x coordinate of the graphic system)

y (is the y co-ordinate of the graphic system)

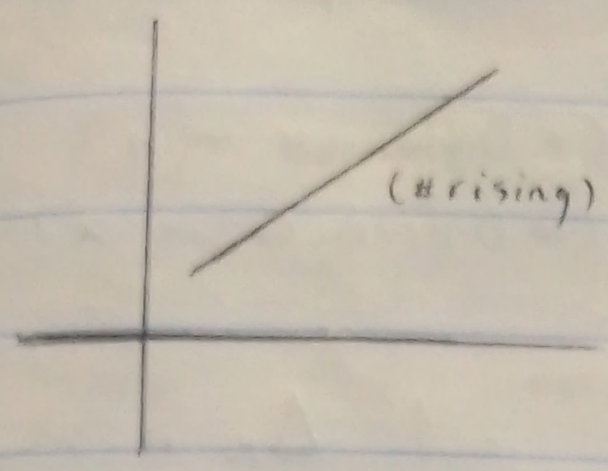
c (is the y-intercept of the line, which is the point where the line intersects the the y-axis)



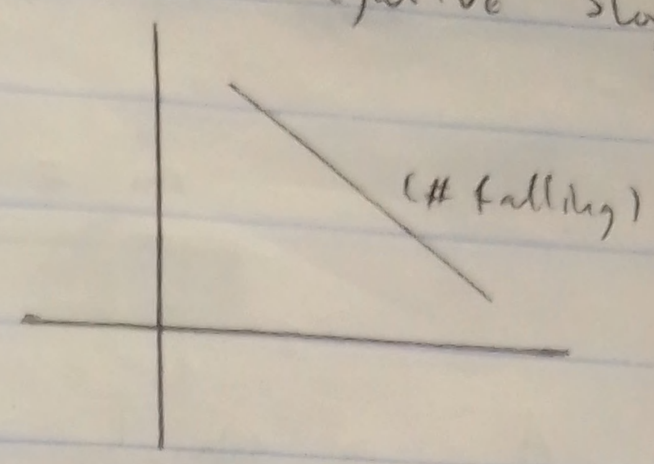
m is the slope of a line, which means m determines the angle at which the line rises or falls. A Positive slope indicates a rising line, while a Negative slope indicates a falling line

2

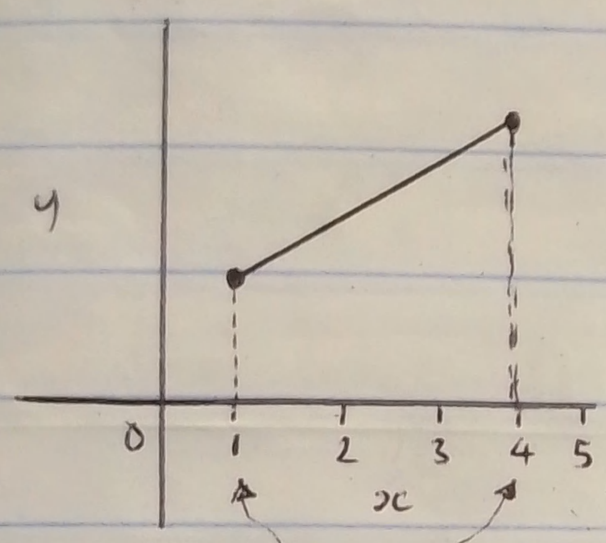
(+) Positive Slope (m)



(−) Negative slope



x is simply the x-coordinates. e.g

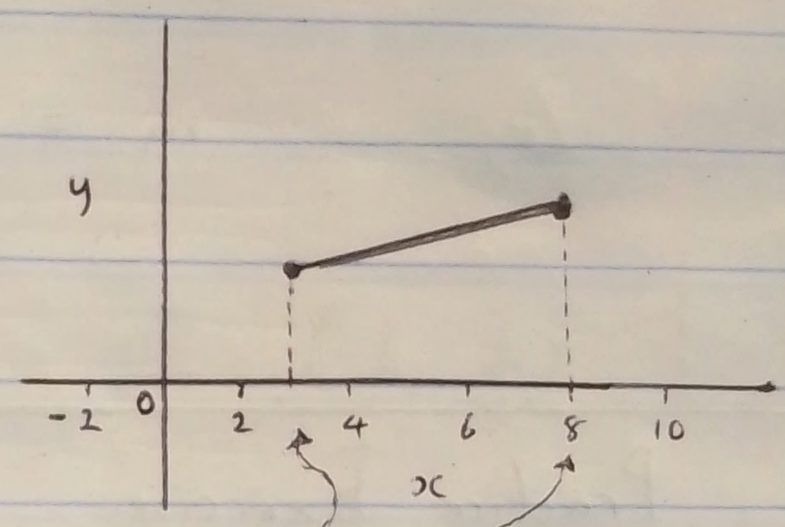


$x = (\underline{1}, 4)$

Coordinates of x

Note: $x_1 = 1$

$x_2 = 4$



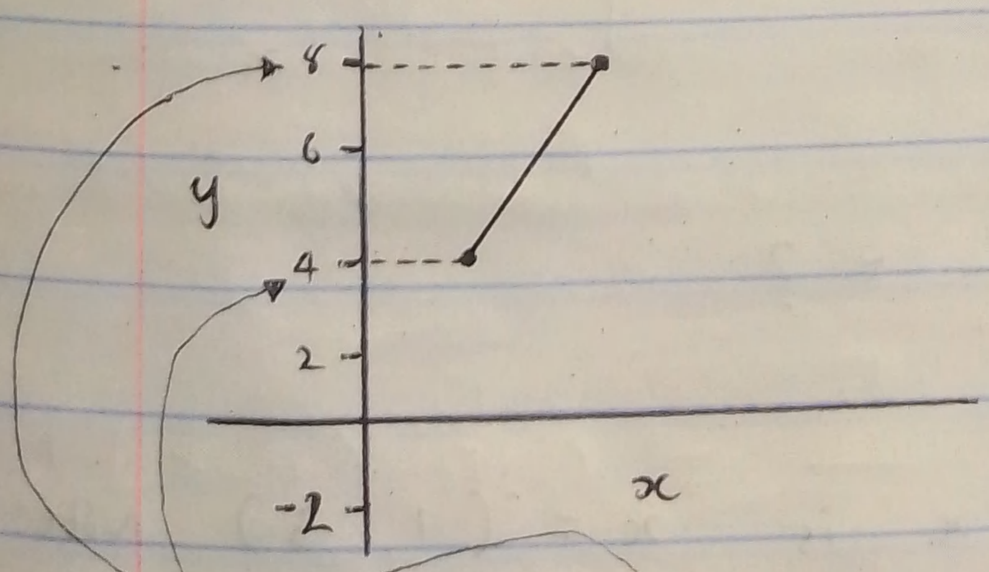
$x = (2.5, 8)$

Coordinates of x

Note: $x_1 = 2.5$

$x_2 = 8$

y is simply the y-coordinate

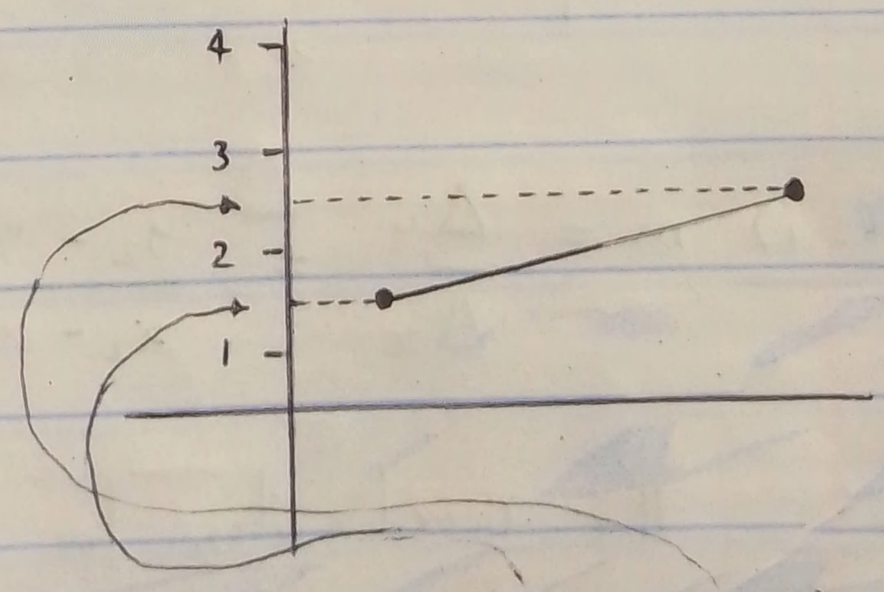


$y = (4, 8)$

Coordinates of y

Note: $y_1 = 4$

$y_2 = 8$



$y = (1.5, 2.5)$

Coordinates of y

Note: $y_1 = 1.5$

$y_2 = 2.5$

A slope is m. And formula for m is below

$$m = \frac{\Delta y}{\Delta x} \quad \begin{matrix}(\text{\# Difference in } y) \\ (\text{\# Difference in } x)\end{matrix}$$

Δy is simply $y_2 - y_1$ and Δx is simply $x_2 - x_1$

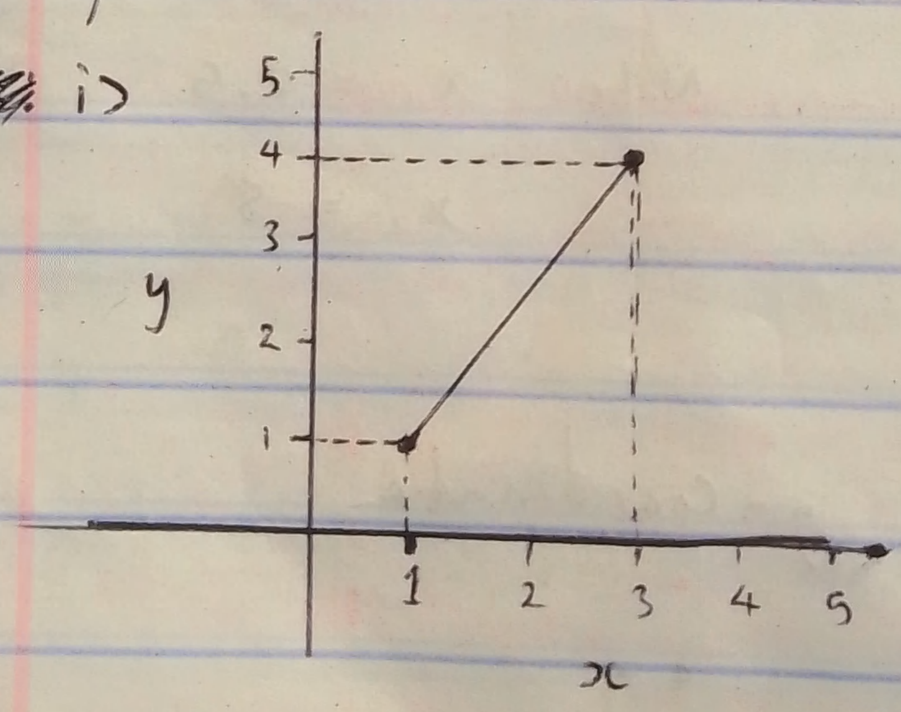
So we can say the formula for slope of a line m is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

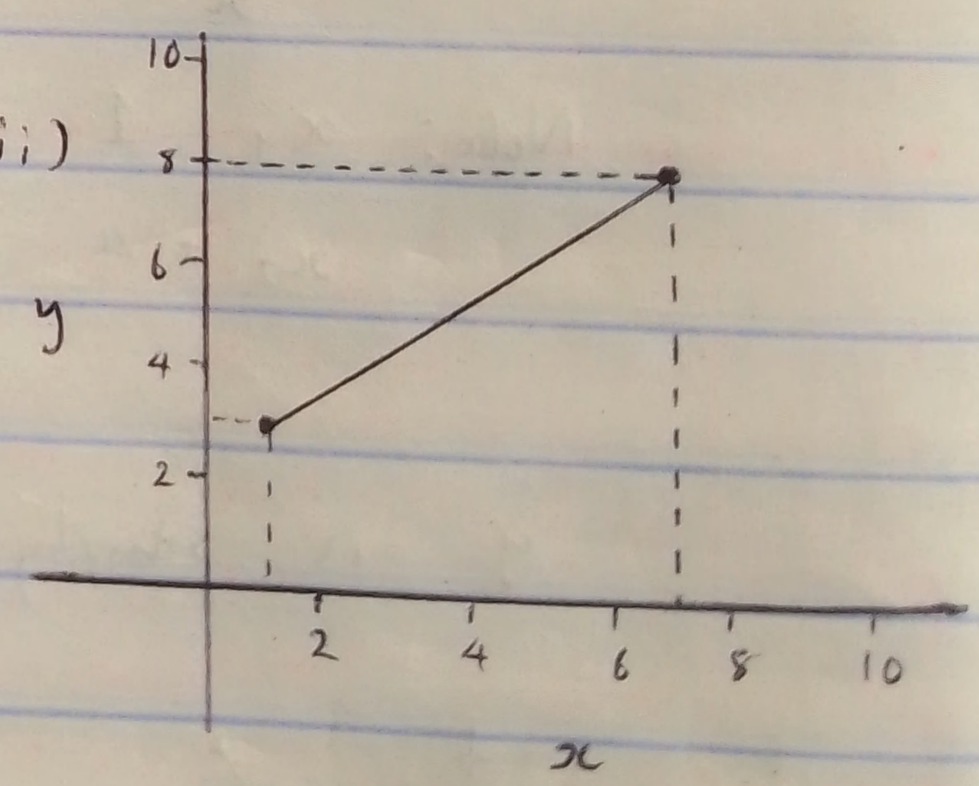
Practice Exercise

1 Calculate the slope of the line in the below Graph systems

i)



ii)



ans: i) $m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$

The coordinates for x is $x = (1, 3)$ Note: $x_1 = 1$

$x_2 = 3$

The coordinates for y is $y = (1, 4)$ Note: $y_1 = 1$

$y_2 = 4$

Back to calculation:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{4 - 1}{3 - 1} = \frac{3}{2} = 1.5$$

The slope $m = 1.5$ //

ii) $$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = ?$$

The coordinates for x is $x = (1, 7)$ Note: $x_1 = 1$

$x_2 = 7$

The coordinates for y is $y = (3, 8)$ Note: $y_1 = 3$

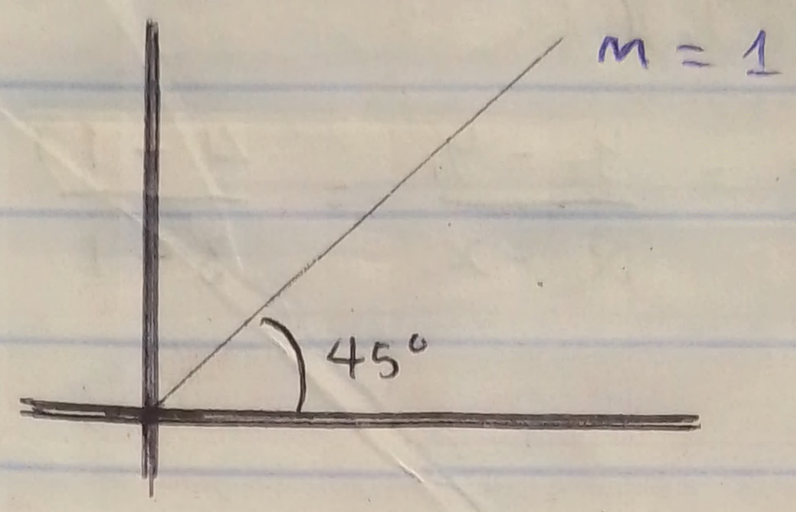
$y_2 = 8$

Back to calculation:

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{8 - 3}{7 - 1} = \frac{5}{6} = 0.8333$$ //

The slope $m = 0.8333$

- When a line has 45° angle as shown below

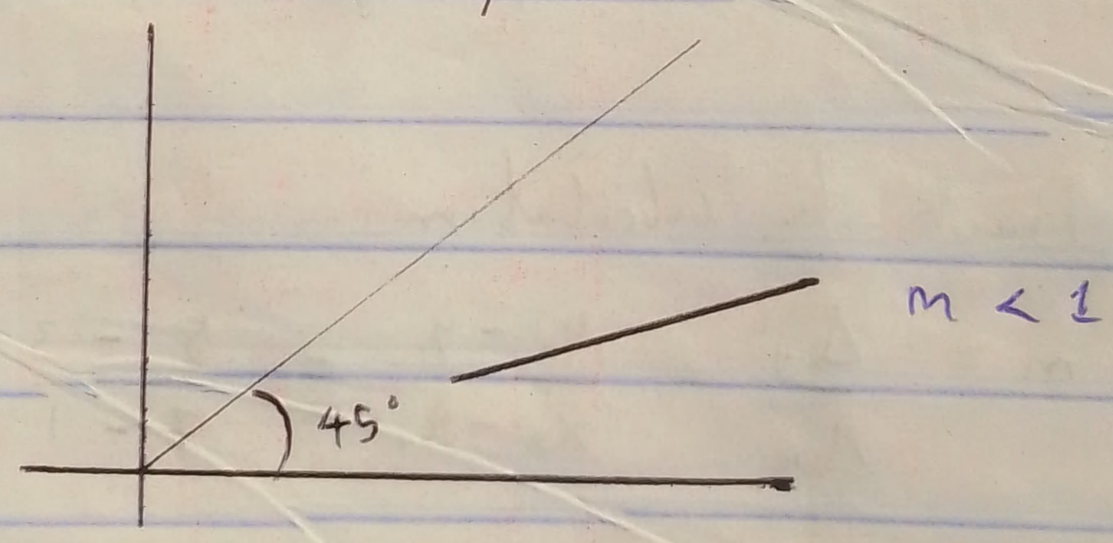


it means that the slope is $m = 1$. Because Δx and Δy would be the same

ex.

$$m = \frac{\Delta y}{\Delta x} = \frac{5}{5} = 1$$

- When a line has degree less than 45° as shown below



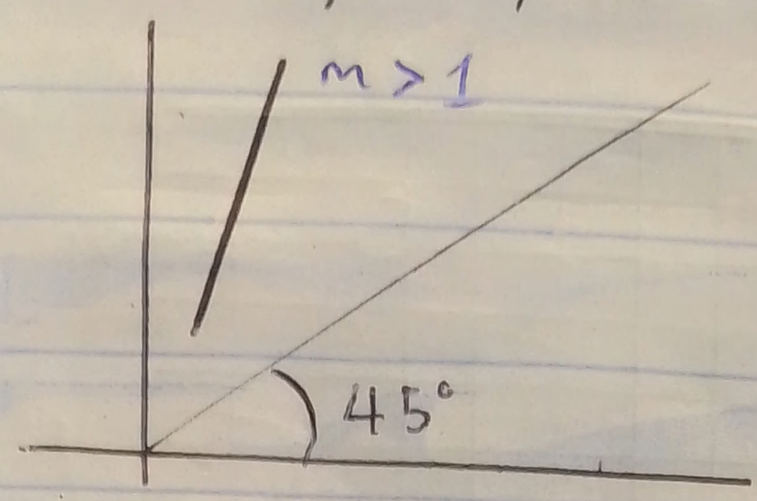
it means that the slope is $m < 1$ (less than 1). Because Δx would have a ~~lesser~~ greater number than Δy

ex.

$$m = \frac{\Delta y}{\Delta x} = \frac{5}{10} = 0.5 \ (m < 1)$$

- Difference in y is smaller $\Delta y = 5$
- Difference in x is larger $\Delta x = 10$

- When a line has degree greater than 45° as shown below



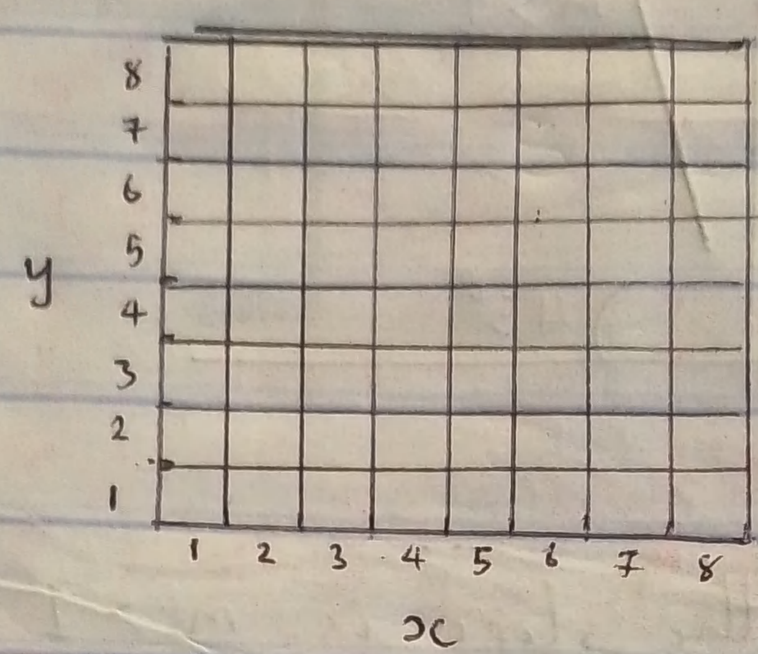
it means that the slope is $m > 1$ (greater than 1).
Because Δx would have a lesser number than Δy

ex.

$$m = \frac{\Delta y}{\Delta x} = \frac{7}{3} = 2.333 \quad (m > 1)$$

- Difference in y is larger $\Delta y = 7$
- Difference in x is smaller $\Delta x = 3$

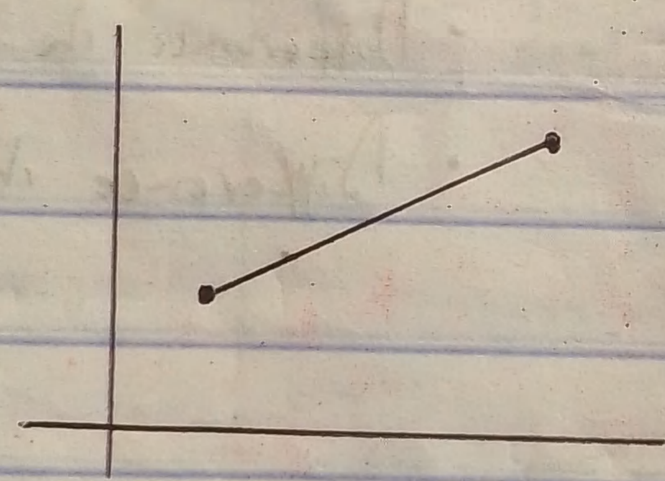
Now the below represents a screen (or a monitor)



Raster Graphics

- each box basically represents a pixel □ and pixels make up screens

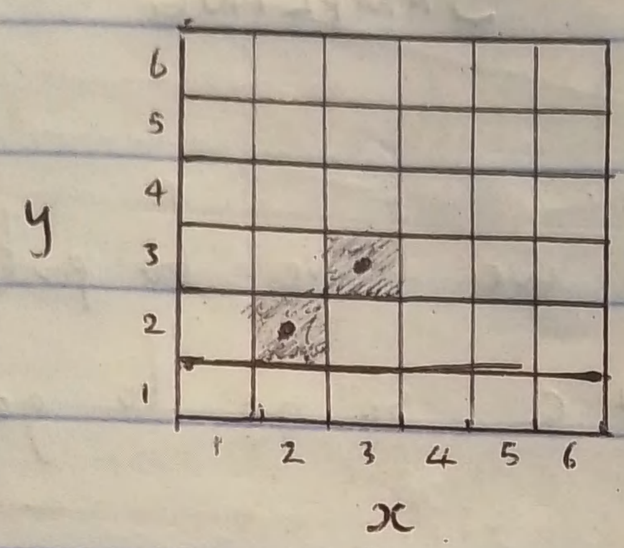
So far though, we've been using a <u>coordinate systems</u> like the one shown below



Where the formula for a line is $\boxed{y = mx + c}$, Vector
Now any line on our coordinate system can be defined by this formula which is a vector

☹ Sadly this formula $y = mx + c$ is not useful on screens (or monitors), <u>because</u> a screen is a collection of pixels

So If we want to draw a line (as we've been doing on coordinate systems) on a screen or monitor.
We must define it (the line) as a collection of pixels.



Raster Graphics

like this

(2, 2) (3, 3) # (x-value, y-value)
x y x y

Now converting a vector $y = mx + c$ into Raster (pixels) is called Rasterization.

In Summary

We would still use our formula $\boxed{y = mx + c}$ (Vector), but it would have to define our line formula to a collection of pixels as shown above i.e (2, 2) (3, 3)

Sampling

Using our current line equation $y = mx + c$ is how we get the different pixel points and to do so we need to do some SAMPLING

Sampling means we have to put 1 value into our equation $y = mx + c$ so as to get another value

But Why do we need Sampling?

Now for our equation $y = mx + c$, we want to know the value of y and x. To do that we need Sampling, we sample either y to get x OR sample x to get y

NOTE:

If m (slope) is greater than 1

- We sample y values (to get x values)

If m (slope) is less than 1

- We sample x values (to get y values)

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If m (slope) is greater than 1

- We sample y values (to get x values)

If m (slope) is less than 1

- We sample x values (to get y values)

Drawbacks of DDA

- Simple Summary

$\Delta x = x_2 - x_1$ # PERFECT

$\Delta y = y_2 - y_1$

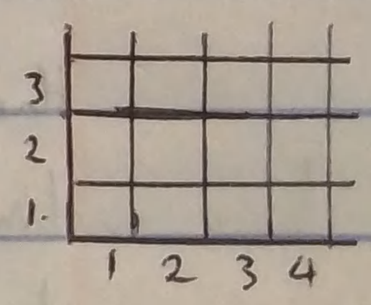
$Steps = \max(\Delta x, \Delta y)$ # PERFECT

$x_{inc} = \frac{\Delta x}{Steps}$ # This is where the PROBLEM

$y_{inc} = \frac{\Delta y}{Steps}$ Lies

Now because DDA does "division operations" to find y_{inc} and x_{inc} it means in some cases we would use floats.

We don't need floats when drawing lines on the screen (a reason being that screen only uses integers e.g



and doesn't use floats), we only need integers.

That's why we rounded the values in DDA Algorithm, DDA Algorithm in the real world automatically rounds the float values it gets into integers

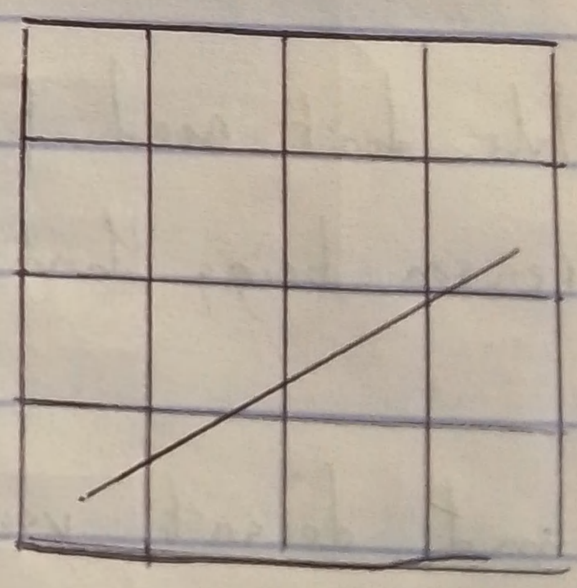
floats : numbers with decimals eg 1.5, 6.8 etc
integers : whole numbers e.g 1, 2, 3, 4 etc

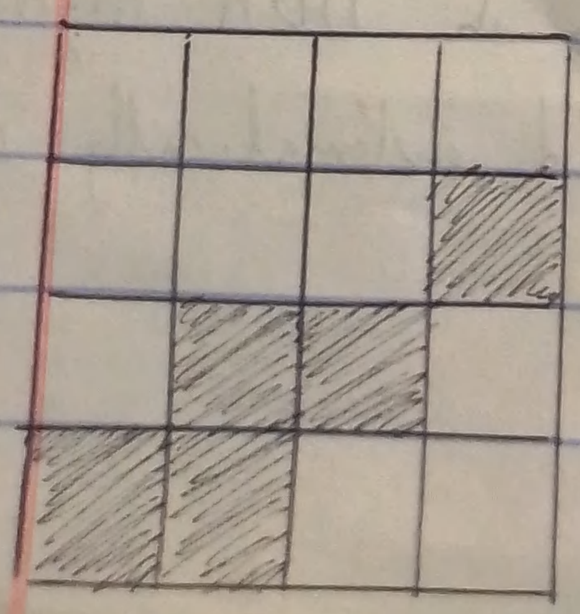
Now the problem is DDA uses floats in it's algorithm when it is not needed, and as we all know floats take more processing time than integers, it is easier and faster for the Computer to process integers than it is for them to process floats. So DDA uses floats and then performs additional operations to round the float values to integers. This is UNNECESSARY

This is where Bresenham's Line Drawing Algorithm Comes in

BRESENHAM'S ALGORITIM

Take a look at our screen on the right and notice how our line we want to ~~doesn't~~ follow a straight path and overlaps multiple pixels



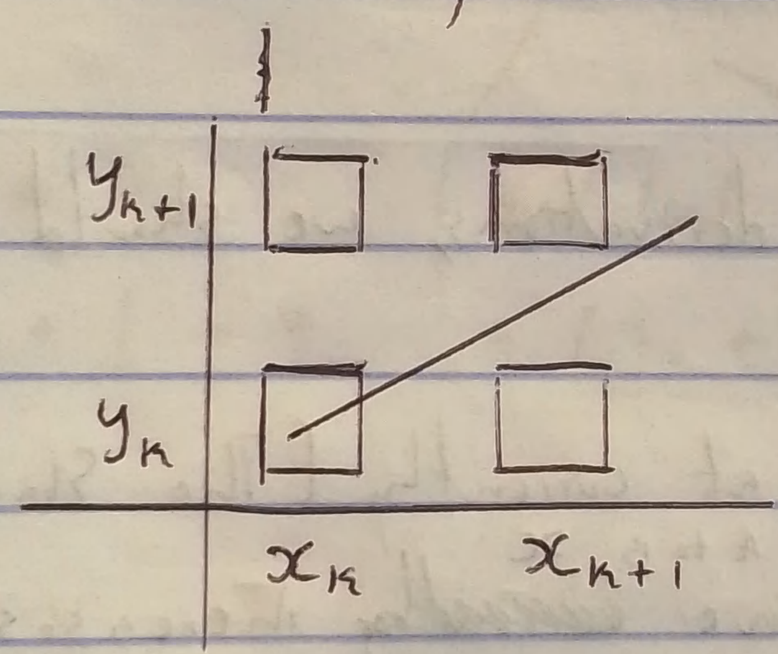


Trying to find out which pixel points we would take to draw this line would leave us with a crooked result as shown on the left. (Using DDA Algorithm that is)

But instead of using DDA and getting a crooked line, we want to use BRESENHAM'S ALGORITHM and get a <u>smooth line</u>

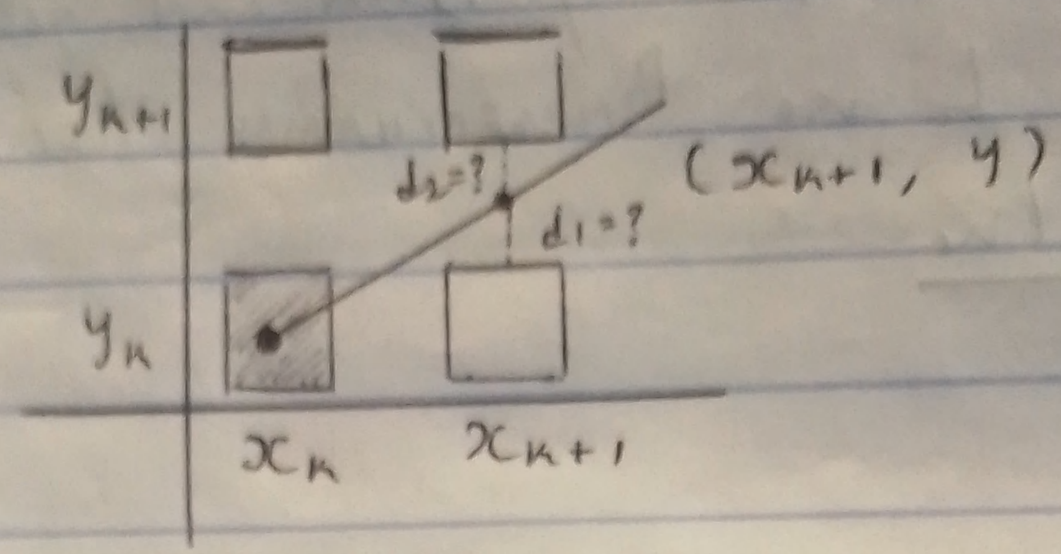
Bresenham's Algorithm only selects 1 pixel as it progresses and by doing so the result is a smooth line as supposed to DDA with crooked lines

How Bresenham's Algorithm works



Bresenham's Algorithm selects a pixel at a time and in a case where a line goes through 2 pixels at a time, Bresenham's Algorithm checks which of the pixels are closer to the line, whichever is closer gets selected over the other

To properly visualize this we use the below diagram that splits the pixels



Wherever the line goes through 2 pixels, we want to know which pixel is closer to the line in order to know which to select

Derivation

Before starting derivations, we should keep few things in mind

- Where we are at currently (The starting point, the pixel)
- Everytime we move from Point A to B, x increases by 1, and y increases sometimes and it doesn't some other time

$$x_{next} = x_k + 1$$

$$y_{next} = \begin{cases} y_k \\ y_{k+1} \end{cases}$$

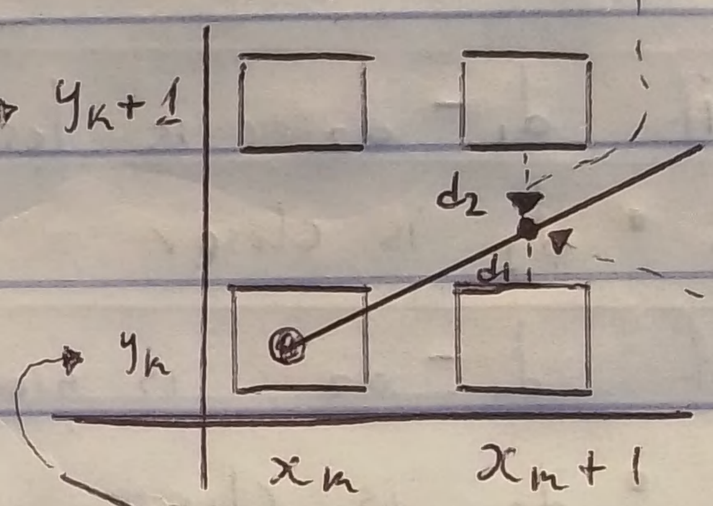
With the above we see that x always increases by 1, but y sometimes does and sometimes stays the same. So the tough part is finding out whether

y increases or not as the line from Point A to B progresses

◎ Starting Derivation

$y = mx + c$

$y = m(x_k + 1) + c$ # This is the line equation that gives us this point --



$d_1 = y - y_k$ # Actual $y - y_k$

$d_2 = y_k + 1 - y$ # $(y_k + 1)$ - Actual y

Replacing y with our line equation $y = m(x_k + 1) + c$

$d_1 = y - y_k = (m(x_k + 1) + c) - y_k$

$d_2 = y_k + 1 - y = y_k + 1 - (m(x_k + 1) + c)$

Open brackets

$d_1 = y - y_k = m(x_k + 1) + c - y_k$

$d_2 = y_k + 1 - y = y_k + 1 - m(x_k + 1) - c$

We want to now know between d_1 (distance 1) and d_2 (distance 2) which one is closer (or has a lesser distance) to <u>Actual y</u> ($\frac{t}{s}$), the one which is closer, we select its pixel

But how do we actually know which pixel is closer?

if $d_1 - d_2$ is less than 0
- d_1 is closer # we select y_k pixel

if $d_1 - d_2$ is greater than 0
- d_2 is closer # we select $y_k + 1$ pixel

Next we subtract

$$d_1 - d_2 = [m(x_k + 1) + c - y_k] - [y_k + 1 - m(x_k + 1) - c]$$

Open brackets

$$d_1 - d_2 = m(x_k + 1) + c - y_k - y_k - 1 + m(x_k + 1) + c$$

NOTICE THAT:

$$d_1 - d_2 = \underbrace{m(x_k + 1)}_{a} + \underbrace{c}_{b} - \underbrace{y_k}_{c} - \underbrace{y_k}_{c} - 1 + \underbrace{m(x_k + 1)}_{a} + \underbrace{c}_{b}$$

- Multiple values repeated. This means we'll add the ones that repeated:

a) $m(x_k + 1) + m(x_k + 1) = 2m(x_k + 1)$

b) $\underset{b}{+c} \; \underset{b}{+c} = 2c$

c) $\underset{c}{-y_m} - \underset{c}{y_m} = -2y_m$

Which means we'll now have:

RESULT

$$d_1 - d_2 = \underset{a}{2m(x_m + 1)} - \underset{c}{2y_m} + \underset{b}{2c}$$

$$d_1 - d_2 = 2m(x_m + 1) - 2y_m + 2c$$

BUT THERE IS A PROBLEM WITH OUR ABOVE

We have $\underline{m}$ existing in the formula, m is slope which means it is $\frac{\Delta y}{\Delta x}$ (This gives float results ☹)

We want to now remove m (which is $\frac{\Delta y}{\Delta x}$)

To do this (remove m) we multiply both sides of our formula by Δx

$$\Delta x(d_1 - d_2) = \Delta x\left[2m(x_m + 1) - 2y_m + 2c - 1\right]$$

Replace $\underline{m}$ with $\frac{\Delta y}{\Delta x}$

$$\Delta x(d_1 - d_2) = \Delta x\left[2 \cdot \frac{\Delta y}{\Delta x}(x_m + 1) - 2y_m + 2c - 1\right]$$

$$\Delta x\,(d_1 - d_2) = \underbrace{2\Delta y\,(x_m + 1)}_{\text{\# open bracket}} - 2\Delta x\, y_m + 2\Delta x c - \Delta x$$

Open Bracket

$$\Delta x\,(d_1 - d_2) = 2\Delta y\, x_m + 2\Delta y - 2\Delta x\, y_m + 2\Delta x c - \Delta x$$

So far
- We've successfully removed $\underline{m}$
- And Opened all the brackets

We want to now label our current Equation above as P_m (Decision Variable)

$$P_m = \Delta x\,(d_1 - d_2) = 2\Delta y\, x_m + \underbrace{2\Delta y}_{\text{constant}} - 2\Delta x\, y_m + \underbrace{2\Delta x c}_{\text{constant}} - \underbrace{\Delta x}_{\text{constant}}$$

Initial P_K FORMULA

But certain parts of our equation (or terms) are constant (They don't have x_m or y_m), So lets organise our Constants together

$$P_m = \Delta x\,(d_1 - d_2) = 2\Delta y\, x_m - 2\Delta x\, y_m + \underbrace{2\Delta y + 2\Delta x c - \Delta x}_{\text{Constants}}$$

Note: Constants don't have y_m or x_m

We want to now reduce our formula to terms that contain x_k or y_k

$$P_k = \Delta_x (d_1 - d_2) = 2\Delta_y x_k - 2\Delta_x y_k + \underbrace{2\Delta_y + 2\Delta_x c - \Delta_x}_{\text{Remove this terms}}$$

Reduced

$$P_k = \Delta_x (d_1 - d_2) = 2\Delta_y x_k - 2\Delta_x y_k$$

OR

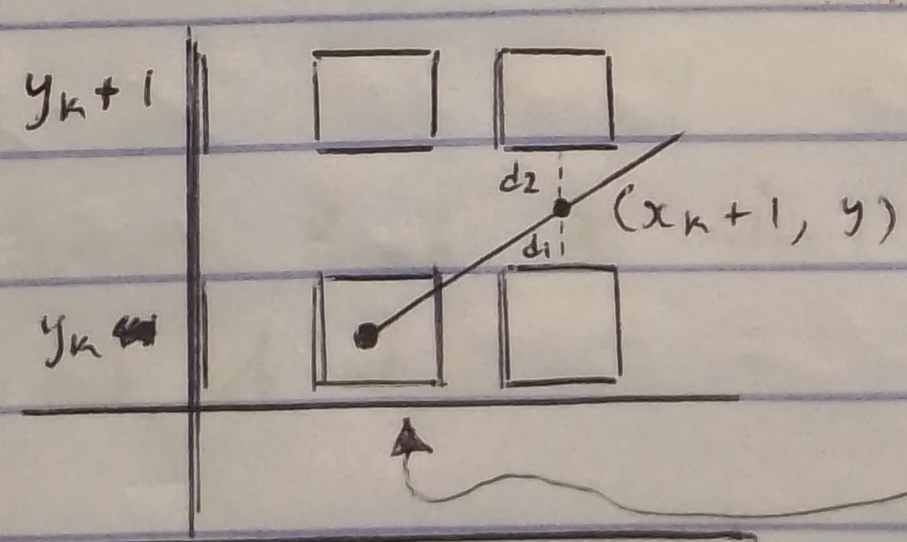
$$P_k = 2\Delta_y x_k - 2\Delta_x y_k$$

CONGRATS

We now have our P_k (Decision variable or Decision parameter), but note that as we move from one pixel to another, our P_k will be changing

OR in other words

As we move from 1 pixel to another we need to take desicisions (hence our P_k (descision parameters) keeps changing), x is always increasing as we move but we need to descide whether y changes or stays the same



Our current $\boxed{P_k = 2\Delta_y x_k - 2\Delta_x y_k}$ refers to our initial pixel

In Summary

We need to find the descision parameter (P_m) for the next pixel as we move from the initial pixel to the next

$$\boxed{P_k = 2\Delta_y x_k - 2\Delta_x y_k}$$

Initial pixel

$$\boxed{\begin{array}{c} P_{k+1} = 2\Delta_y x_{next} - 2\Delta_x y_{next} \\ \text{OR more accurately} \\ P_{next} = 2\Delta_y x_{next} - 2\Delta_x y_{next} \end{array}}$$

Next Pixel

for every next pixel this is the formula

Now that we have the descision parameter (P_k) for the current pixel and the next pixel, we've to keep on changing the descision parameter and to do so we subtract

$P_{next} - P_k$ # Subtract current P_k as we move from the next P_k (or P_{next})

$$P_{next} - P_k = [2\Delta_y x_{next} - 2\Delta_x y_{next}] - [2\Delta_y x_k - 2\Delta_x y_k]$$

Open Brackets

$$P_{next} - P_k = \underbrace{2\Delta_y}_{a} x_{next} - \underbrace{2\Delta_x}_{b} y_{next} - \underbrace{2\Delta_y}_{a} x_k + \underbrace{2\Delta_x}_{b} y_k$$

Group Common terms

$$P_{next} - P_k = 2\Delta_y (x_{next} - x_k) - 2\Delta_x (y_{next} - y_k)$$

This is our FORMULA Now

- In our formula now we want to know what is x_{next} and y_{next}
- We already know what x_{next} will actually be - it will always be increased by 1 as we move, As for y_{next} we are not sure, sometimes y_{next} stays the same as we move and sometimes it is increased by 1

$$x_{next} = x_k + 1$$

$$y_{next} = \begin{cases} y_k \\ y_k + 1 \end{cases}$$

So we know now that x always increases by 1 and y does too but not always.

Our Current Formula :

$$P_{next} - P_k = 2\Delta_y (x_{next} - x_k) - 2\Delta_x (y_{next} - y_k)$$

NOTE :

1) if $P_{next} - P_k$ is less than 0
   - x_{next} increases by 1 $(x_k + 1)$
   - y_{next} stays the same (y_k)

2) if $P_{next} - P_k$ is greater than 0
   - x_{next} increases by 1 $(x_k + 1)$
   - y_{next} increases by 1 $(y_k + 1)$

With the above , we can therefore say

1) if $P_{next} - P_k$ is less than 0

$$P_{next} - P_k = 2\Delta_y (x_{next} - x_k) - 2\Delta_x (y_{next} - y_k)$$

Move $-P_k$ across

$$P_{next} = P_k + 2\Delta_y (x_{next} - x_k) - 2\Delta_x (y_{next} - y_k)$$

Substitute x_{next} and y_{next}

$$P_{next} = P_k + 2\Delta_y (\underbrace{x_k + 1}_{x_{next}} - x_k) - 2\Delta_x (\underbrace{y_k}_{y_{next}} - y_k)$$

Solve

$$P_{next} = P_k + 2\Delta_y (\cancel{x_k} - \cancel{x_k} + 1) - \cancel{2\Delta_x y_k} + \cancel{2\Delta_x y_k}$$

RESULT

$$P_{next} = P_k + 2\Delta_y$$

Conclussion

- So if $P_{next} - P_k$ is less than 0, we can use this formula to ~~[illegible]~~ get the next decision variable

2) If $P_{next} - P_k$ is greater than 0

$$P_{next} - P_k = 2\Delta_y (x_{next} - x_k) - 2\Delta_x (y_{next} - y_k)$$

move $-P_k$ across

$$P_{next} = P_k + 2\Delta_y (x_{next} - x_k) - 2\Delta_x (y_{next} - y_k)$$

substitute x_{next} and y_{next}

$$P_{next} = P_k + 2\Delta_y (\underbrace{x_k + 1}_{x_{next}} - x_k) - 2\Delta_x (\underbrace{y_k + 1}_{y_{next}} - y_k)$$

solve

$$P_{next} = P_k + 2\Delta_y (\cancel{x_k} - \cancel{x_k} + 1) - 2\Delta_x (\cancel{y_k} - \cancel{y_k} + 1)$$

RESULT

$$P_{next} = P_k + 2\Delta_y - 2\Delta_x$$

Conclussion

- Now if $P_{next} - P_k$ is greater than 0, we can use this formula to get the next descision variable

In most textbooks, Instead of using P_{next}, they use P_{k+1} -> They are both the same thing and are both referring to the next descision variable / parameter

FORMULA SUMMARY

If $P_k < 0$	$P_{next} = P_k + 2\Delta_y$
if $P_k > 0$	$P_{next} = P_k + 2\Delta_y - 2\Delta_x$

P_k here means $\boxed{P_{next} - P_k}$

Pheeww 😫.

"Keep it up if you've gotten this far, we almost done"

- We currently have the formulas for our P_{next} (next decision variable) when $P_k < 0$ and when $P_k > 0$.

Next

- Next : ?

We want to get the <u>Initial value</u> of P_k (Decision Variable)

Derivation of INITAL VALUE

To find the Initial value of P_k, we use the below formula

$$P_k = \Delta_x(d_1 - d_2) = 2\Delta_y x_k - 2\Delta_x y_k + \underbrace{2\Delta_y + 2\Delta_x C - \Delta_x}_{\text{constant}}$$

same as

$$P_k = 2\Delta_y x_k - 2\Delta_x y_k + 2\Delta_y + 2\Delta_x C - \Delta_x$$

Notice we got this formar (5 pages back) some time back and back then we discarded the constant section and used the rest in $2\Delta_y x_k - 2\Delta_x y_k$ to solve and get P_k for the next decision variable when $P_k < 0$ and when $P_k > 0$

BUT

To solve for initial value of P_k, we will use the entire formula including the constants

(x_1, y_1) $\quad y_1 = mx_1 + c$

$$P_1 = 2\Delta_y x_1 - 2\Delta_x y_1 + 2\Delta_y + 2\Delta_x c - \Delta_x$$

Firstly, we want to remove the constant c

(x_1, y_1) $\quad y_1 = mx_1 + c$ $\quad$ # Solve for c

$y_1 = \frac{\Delta y}{\Delta x} x_1 + c$ $\quad$ # replace $m = \frac{\Delta y}{\Delta x}$

$y_1 - \frac{\Delta y}{\Delta x} x_1 = c$ $\quad$ # move $\frac{\Delta y}{\Delta x} x_1$ across

organize

$$c = y_1 - \frac{\Delta y}{\Delta x} x_1$$

Substitute $c = y_1 - \frac{\Delta y}{\Delta x} x_1$ into equation

$$P_1 = 2\Delta_y x_1 - 2\Delta_x y_1 + 2\Delta_y + 2\Delta_x \left[y_1 - \frac{\Delta y}{\Delta x} x_1 \right] - \Delta_x$$

Open Brackets

open bracke

$$P_1 = 2\Delta_y x_1 - 2\Delta_x y_1 + 2\Delta_y + 2\Delta_x y_1 - 2\Delta_y x y_1 - \Delta_x$$

Calculate

$$P_1 = \cancel{2\Delta_y x_1} - \cancel{2\Delta_x y_1} + 2\Delta_y + \cancel{2\Delta_x y_1} - \cancel{2\Delta_x y_1} - \Delta_x$$

RESULT

$$P_1 = 2\Delta_y - \Delta_x$$

- We now have initial value for descision variable and
- Formula for incremental value also when $P_k < 0$ and when $P_k > 0$

Our Formulas

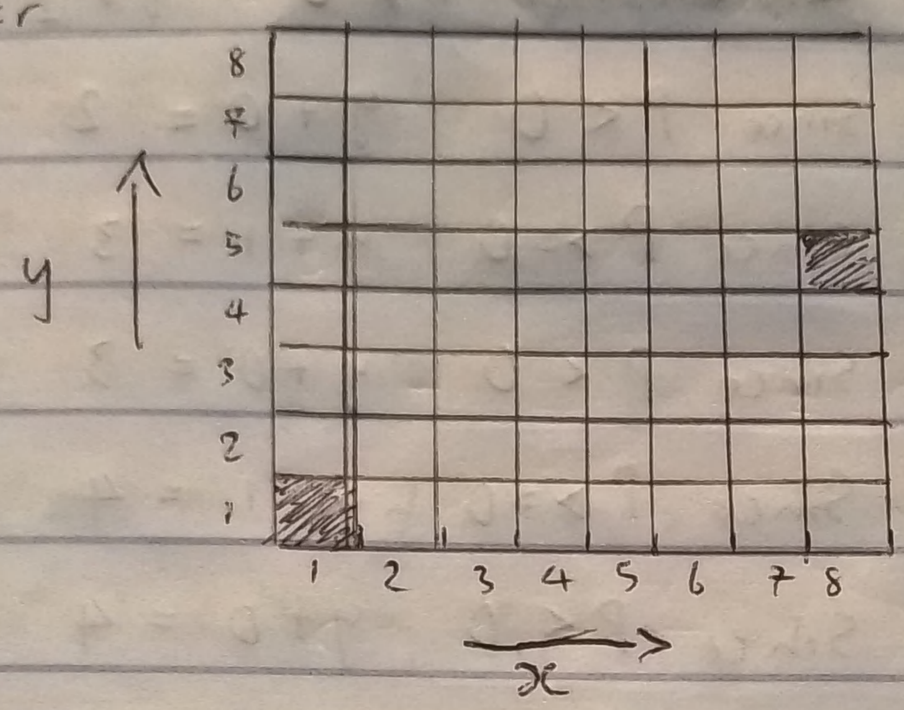
- $P_1 = 2\Delta y - \Delta x$
- if $P_k < 0$ $\quad P_{k+1} = P_k + 2\Delta y$
- if $P_k >= 0$ $\quad P_{k+1} = P_k + 2\Delta y - 2\Delta x$

Note: Δy is same as dy (difference in y)
Δx is same as dx (difference in x)

Practice Exercise

Given Point A (x_1, y_1) $(1, 1)$ and Point B (x_2, y_2) $(8, 5)$.
Plot the Pixel Points.

Answer



$m < 1$ or $\tan\theta < 45°$

STEP 1:

Formulas

- $P_1 = 2\Delta_y - \Delta_x$
- if $P_k < 0$ $\quad P_{k+1} = P_k + 2\Delta_y$
- if $P_k >= 0$ $\quad P_{k+1} = P_k + 2\Delta_y - 2\Delta_x$

What is our Δ_y, Δ_x and P_1 though

$\Delta_y = y_2 - y_1 = 5 - 1 = 4$

$\Delta_x = x_2 - x_1 = 8 - 1 = 7$

$P_1 = 2\Delta_y - \Delta_x$
$= 2(4) - (7)$
$= 8 - 7$
$= 1$

/ Generate Points

STEP 2: Take Points (starting from Point A (x_1, y_1) (1, 1))

$x_{inc} = 1$ $y_{inc} = ?$ $\Delta y = 4$ $\Delta x = 7$

x	y	P
1	1	1
2	2	-5
3	2	3
4	3	-3
5	3	5
6	4	-1
7	4	7
8	5	1

since $P >= 0$ $y + 1 = 2$ $P_{next} = P_k + 2\Delta_y - 2\Delta_x = 1 + 2(4) - 2(7) = 1 + 8 - 14 = -5$

since $P < 0$ $y + 0 = 2$ $P_{next} = P_k + 2\Delta_y = -5 + 2(4) = -5 + 8 = 3$

since $P >= 0$ $y + 1 = 3$ $P_{next} = P_k + 2\Delta_y - 2\Delta_x = 3 + 2(4) - 2(7) = 3 + 8 - 14 = -3$

since $P < 0$ $y + 0 = 3$ $P_{next} = P_k + 2\Delta_y = -3 + 2(4) = -3 + 8 = 5$

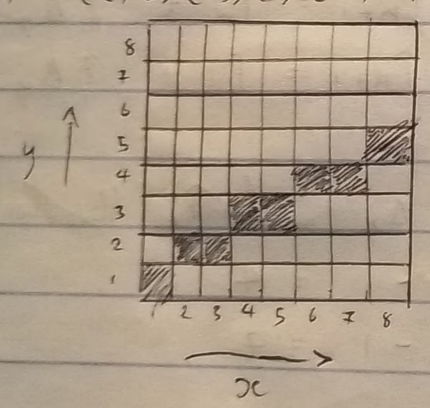
since $P >= 0$ $y + 1 = 4$ $P_{next} = P_k + 2\Delta_y - 2\Delta_x = 5 + 2(4) - 2(7) = 5 + 8 - 14 = -1$

since $P < 0$ $y + 0 = 4$ $P_{next} = P_k + 2\Delta_y = -1 + 2(4) = -1 + 8 = 7$

since $P >= 0$ $y + 1 = 5$ $P_{next} = P_k + 2\Delta_y - 2\Delta_x = 7 + 2(4) - 2(7) = 7 + 8 - 14 = 1$

STEP 3: Paint Pixel Points

(1, 1) (2, 2) (3, 2) (4, 3) (5, 3) (6, 4) (7, 4) (8, 5)



GOODLUCK !