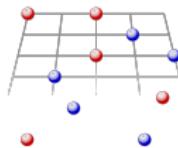


# Topologically non-trivial electronic and magnetic states in doped copper kagome lattices

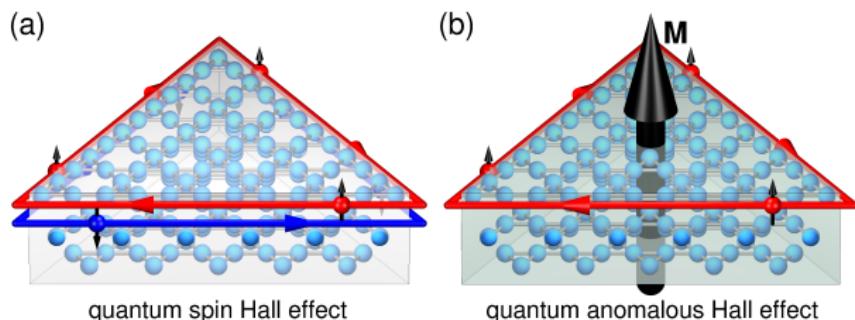
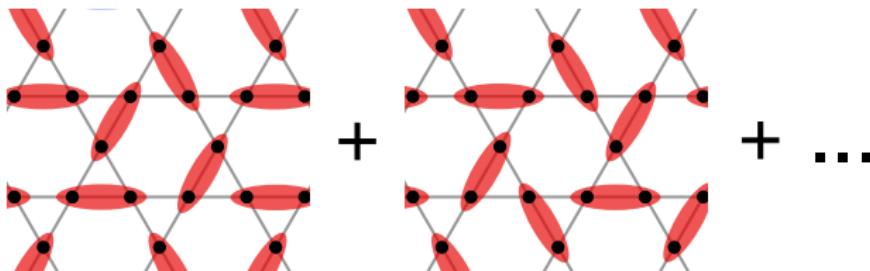
Daniel Guterding, Harald O. Jeschke, Roser Valentí  
Institut für Theoretische Physik

March 18, 2016



# Non-trivial topology on the kagome lattice: quantum spin liquid and quantum Hall effects

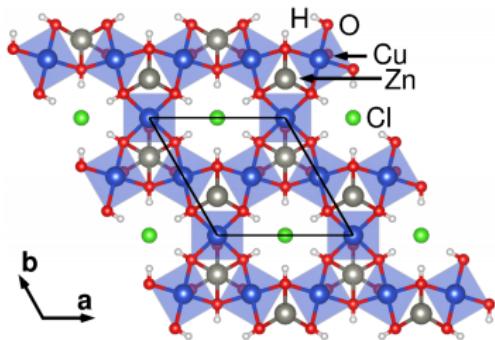
- highly frustrated lattice
- QSL in the AFM Heisenberg model
- topological electronic edge states (QSHE, QAHE)
- spintronics
- non-abelian anyons (e.g. TI-SC interface)
- topological quantum computer



# Copper kagome materials: Herbertsmithite and Barlowite

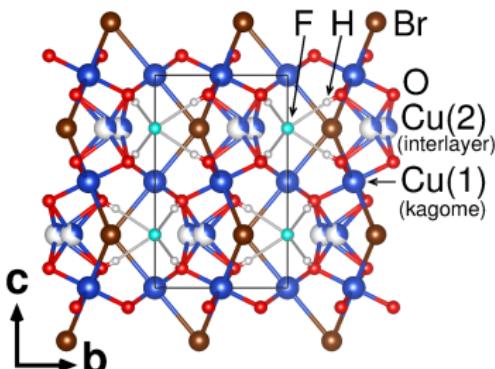
## Herbertsmithite $\text{ZnCu}_3(\text{OH})_6\text{Cl}_2$

- kagome lattice of Cu  $3d^9$  ( $\text{Cu}^{2+}$ ) ions
  - AFM NN  $J_1 = 182.4$  K, other  $J_i < 7$  K
  - no magnetic order down to 50 mK
- [Helton et al., PRL 98, 107204 (2007)]
- Cu-Zn antisite-disorder of a few percent



## Barlowite $\text{Cu}_4(\text{OH})_6\text{FBr}$

- $\text{Cu}^{2+}$  between layers, ordered AFM
- AFM kagome exchange  $J_3 = 178$  K
- FM interlayer exchange  $J_1 = -205$  K

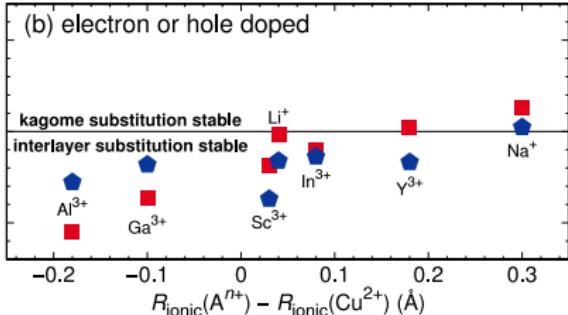
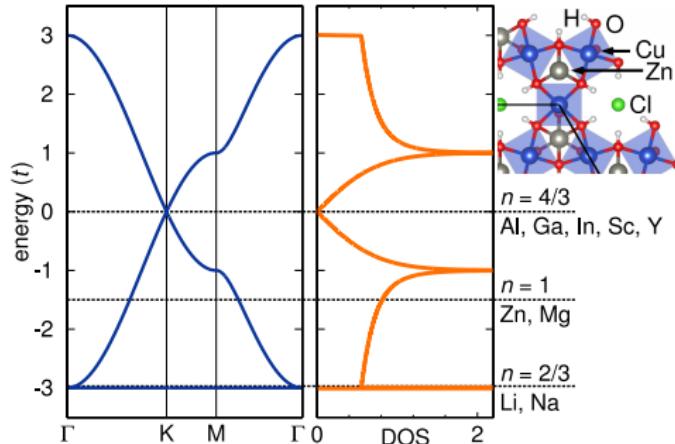
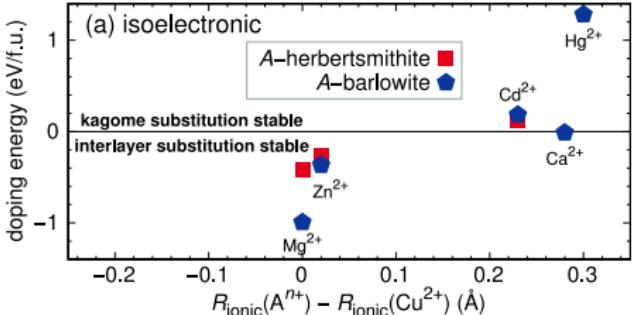


Jeschke, Salvat-Pujol, Valentí, PRB 88, 075106 (2013)

Jeschke, Salvat-Pujol, Gati, Hoang, Wolf, Lang, Schlueter, Valentí, PRB 92, 094417 (2015)

# Stability analysis for substituted Herbertsmithite $ACu_3(OH)_6Cl_2$ and Barlowite $ACu_3(OH)_6FBr$

- $A = Li^+, Na^+$ : FM
- $A = Mg^{2+}, Zn^{2+}$ : QSL
- $A = Ga^{3+}, Sc^{3+}$ : Dirac metal
- do dopants occupy interlayer or kagome site? → DFT study
- many materials in principle stable!
- small dopants favored



# Magnetic exchange couplings

- map DFT+U to Heisenberg model
- $U = 6 \text{ eV}$  and  $J_H = 1 \text{ eV}$  on Cu 3d

## Mg-Barlowite $\text{MgCu}_3(\text{OH})_6\text{FBr}$

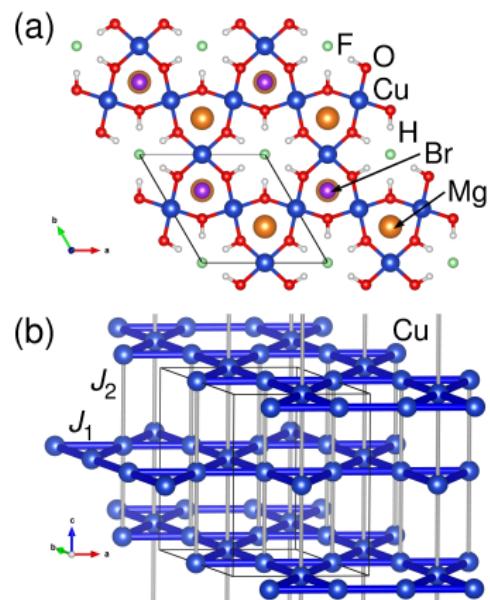
- AFM  $J_1 = 226 \text{ K}$ ,  $J_2/J_1 = 0.06$
- spin-liquid predicted for  $|J_2/J_1| < 0.15$

[Götze, Richter, arXiv:1510:04898]

- QSL with low anti-site disorder

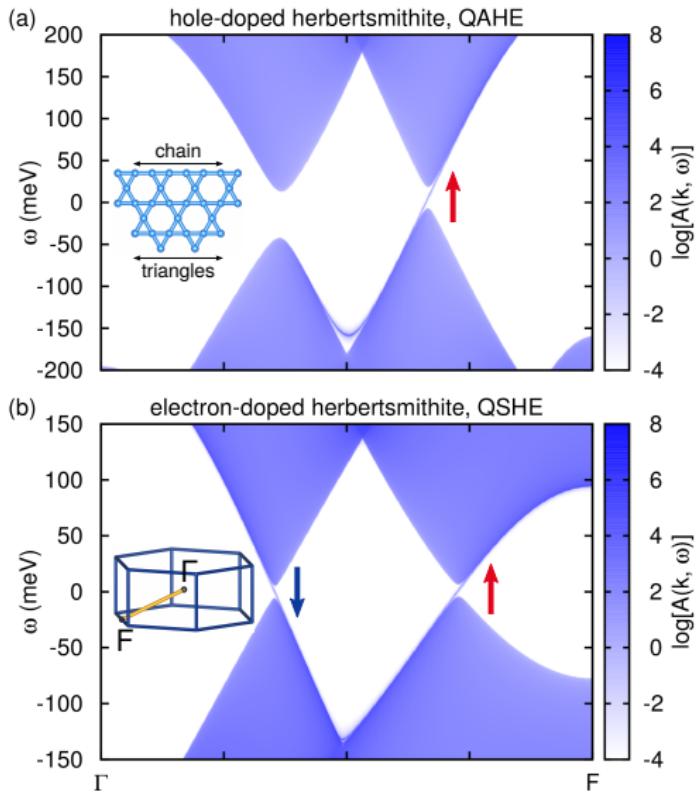
## Li-Herbertsmithite $\text{LiCu}_3(\text{OH})_6\text{Cl}_2$

- FM kagome coupling  $J_1 = -544 \text{ K}$
- MF Curie temperature  $T_C \approx 1160 \text{ K}$



# Topological surface states in doped Herbertsmithite

- Li-Herbertsmithite is 2/3 filled FM,  $\{\uparrow \frac{2}{3}, \downarrow 0\}$
- Ga-Herbertsmithite is 4/3 filled,  $\{\uparrow \frac{2}{3}, \downarrow \frac{2}{3}\}$
- FM Dirac metal, SOC gap at Dirac point
- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0; (111)$
- QAHE on (001) surface
- Barlowite is topologically trivial, additional band crossings



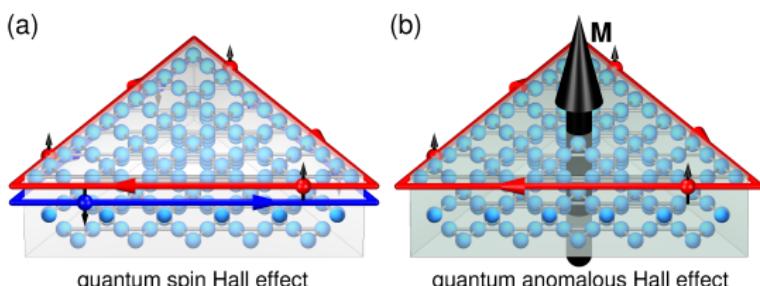
# Doped kagome lattice Mott insulators

## Summary

- investigated stability of new copper kagome materials
- Mg-Barlowite is a new spin-liquid candidate
- Li-Herbertsmithite shows the quantum anomalous Hall effect
- (Al, Ga, In, Sc)-Herbertsmithite show the quantum spin Hall effect
- preprint available: **arXiv:1511.05686** (version 1)

## Outlook

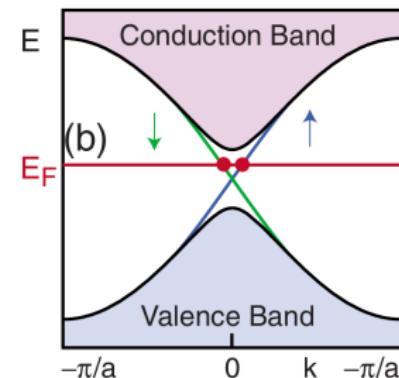
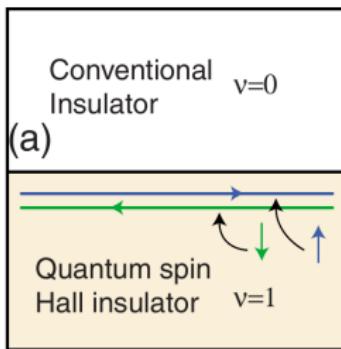
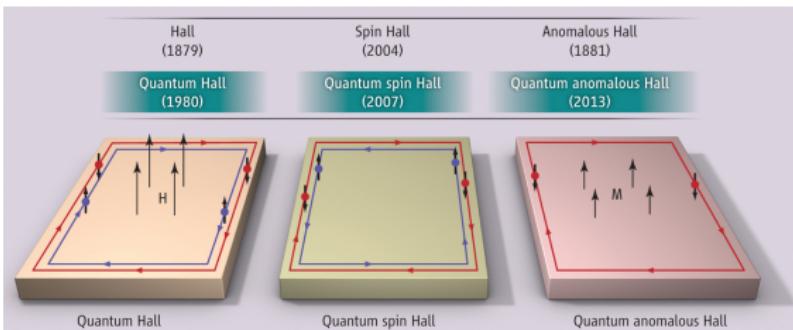
- material synthesis
- gating with ionic liquids
- deposition of alkalis
- intermediate doping



# Appendix

# Topological insulators: introduction

- topological insulator fundamentally different from trivial insulator
- insulating bulk
- gapless surface states protected by time-reversal symmetry
- robust against disorder, weak interactions, etc.
- dissipation-free transport through surface states



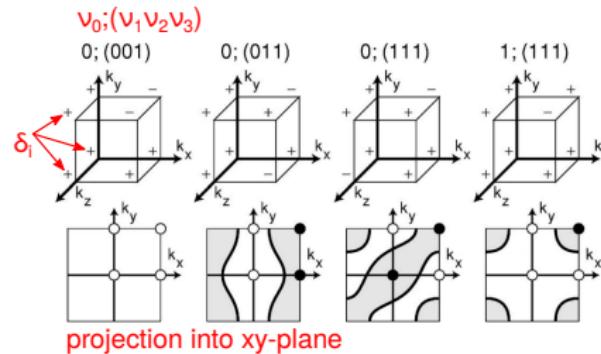
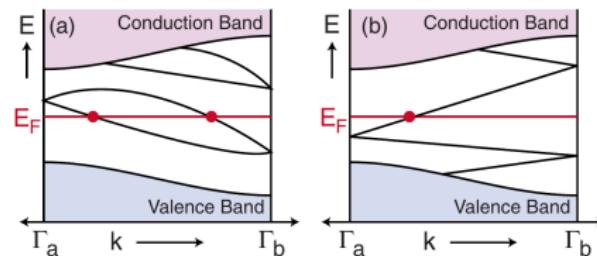
# Topological insulators: computational aspects

- time-reversal invariance  
 $E(\vec{k}) = E(-\vec{k})$
- Kramer's theorem: time-reversal symmetry  $\rightarrow$  twofold degeneracy
- connectivity between time-reversal invariant points  $\Gamma_i$  matters
- take product of parity eigenvalues for occupied bands

$$\delta_i = \prod_{m=1}^N \xi_{2m}(\Gamma_i)$$

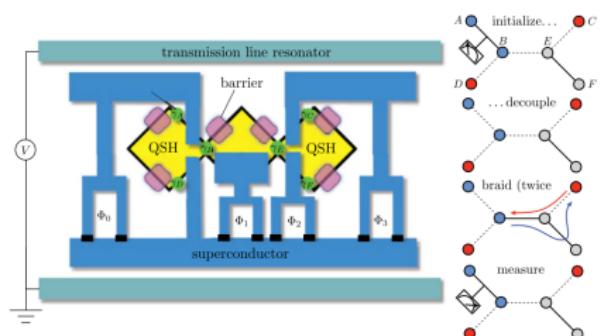
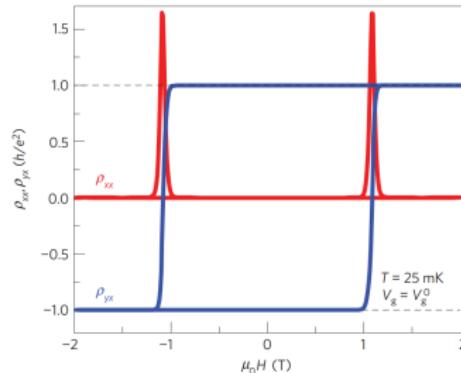
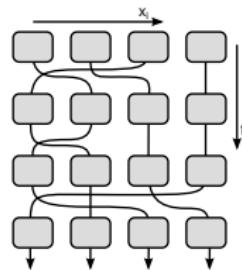
$$(-1)^{\nu_0} = \prod_{i=1}^8 \delta_i; \rightarrow \nu_0; (\nu_1, \nu_2, \nu_3)$$

$$(-1)^{\nu_k} = \prod_{n_k=1; n_j \neq k=0,1} \delta_{i=(n_1 n_2 n_3)}$$



# Topological insulators: application

- in QAH insulator all currents are spin-polarized → spintronics
- $\rho_{xx} = 0$  at zero field for QAHE
- Majorana zero modes (non-abelian anyons) at TI-SC interface
- braiding records history of the system
- fault-tolerant quantum computer

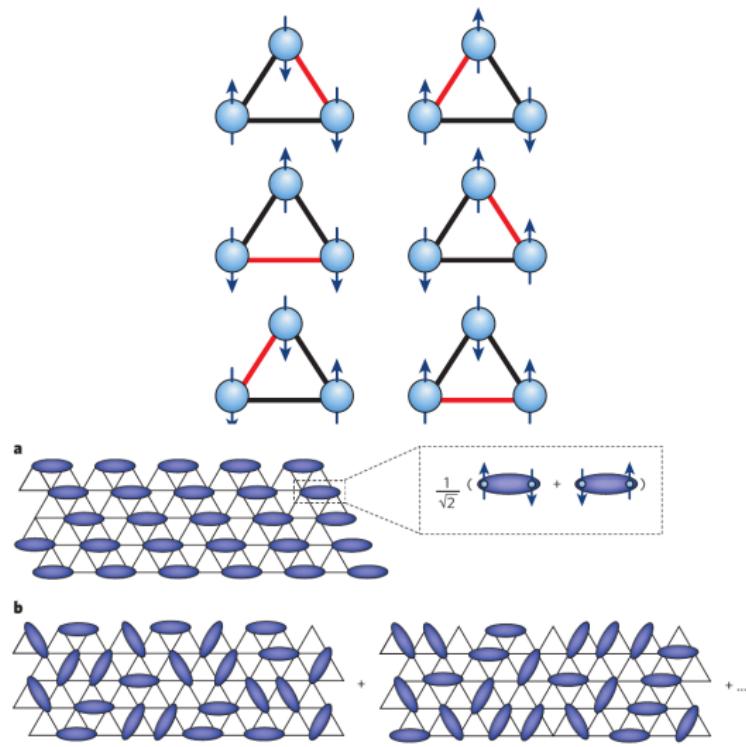


Wilczek, Nat. Phys. 5, 614 (2009); Chang et al., Nat. Mater. 14, 473 (2015)

Das Sarma et al., Quantum Information 1, 15001 (2015); van Heck et al., Phys. Scr. T164, 014007 (2015) 11 / 7

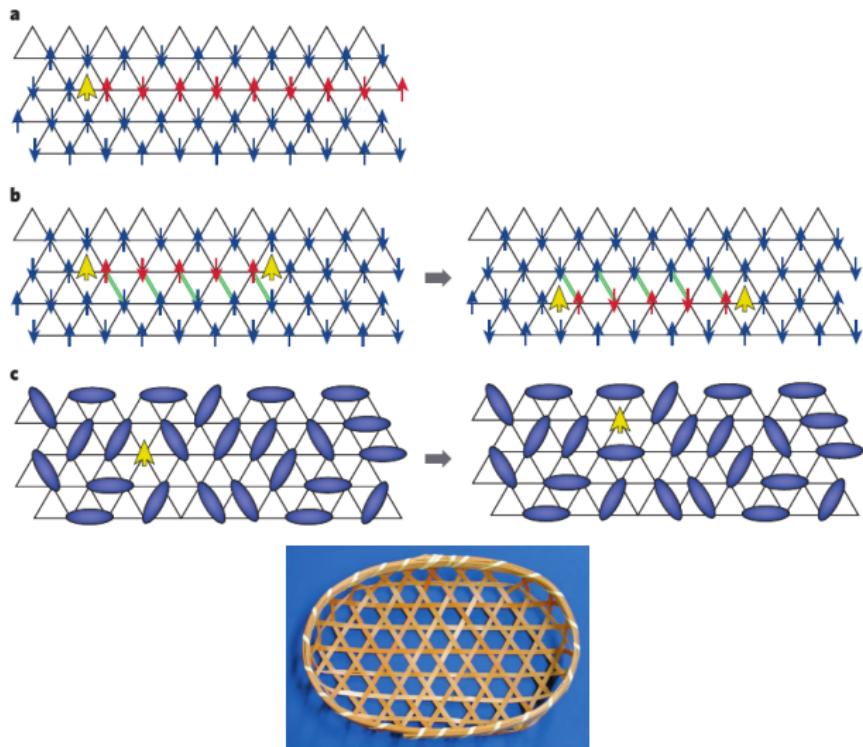
# Quantum spin-liquids: introduction

- frustration leads to degenerate ground state
- highly correlated well-formed local moments, but no static order (liquid-like)
- **no spontaneous symmetry-breaking**
- described by lattice covered with valence bonds
- static valence bonds give valence bond solid (VBS)
- **superposition gives resonating valence bond (RVB) state → QSL**



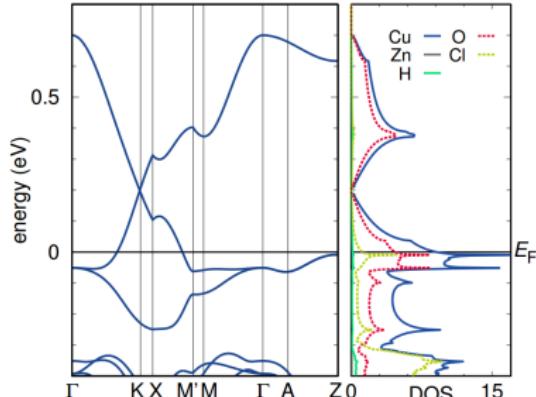
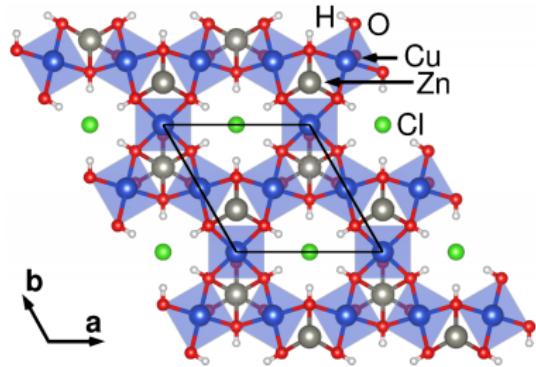
# Quantum spin-liquids: excitations

- usual excitations are **electron-like** ( $S = 1/2$ ,  $q = \pm e$ ) or **magnon-like** ( $S = 1$ ,  $q = 0$ )
- fractionalized excitations in QSL: **spinons** ( $S = 1/2$ ,  $q = 0$ )
- spinons can be gapped or gapless
- probe spinons with thermodynamics, neutrons, etc.



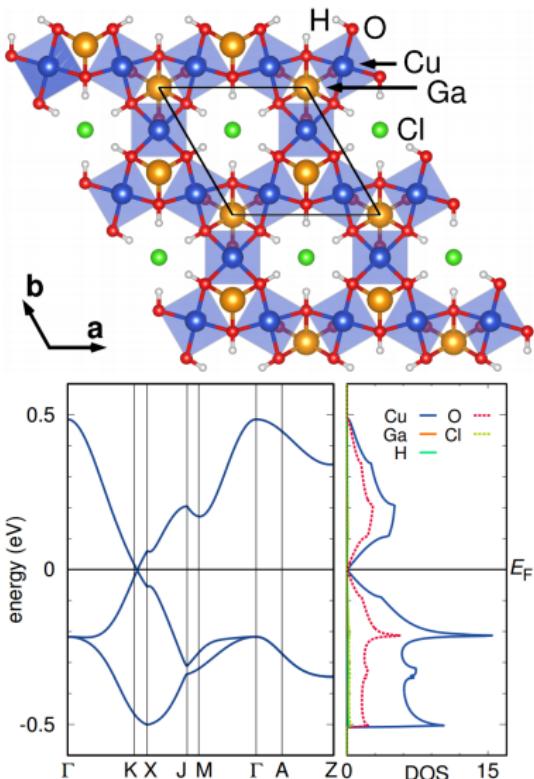
# Herbertsmithite $[ZnCu_3(OH)_6Cl_2]$ : spin-liquid candidate on the kagome lattice

- kagome lattice of Cu  $3d^9$  ( $Cu^{2+}$ ) ions
  - Mott insulator
  - antiferromagnetic NN exchange  
 $J_1 = 182.4$  K
  - various other couplings  $J_i < 7$  K
  - no magnetic order down to 50 mK
- [Helton et al., PRL 98, 107204 (2007)]
- neutron experiments see continuum of (fractionalized) excitations
- [Han et. al, Nature 492, 406 (2012)]
- Cu-Zn antisite-disorder of a few percent, some studies claim only Zn site disorder



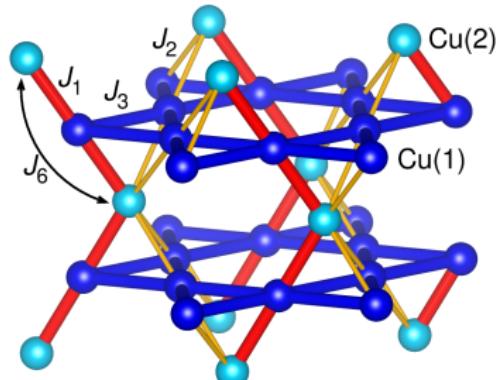
# Ga-Herbertsmithite $[GaCu_3(OH)_6Cl_2]$ : predicted exotic ferromagnet or superconductor

- substitute  $Zn^{2+}$  by  $Ga^{3+}$
- electron doping moves Fermi level to Dirac point
- Dirac metal robust in presence of correlations (DCA)
- vary Zn:Ga ratio to obtain intermediate electron doping
- competition between FM and f-wave SC predicted

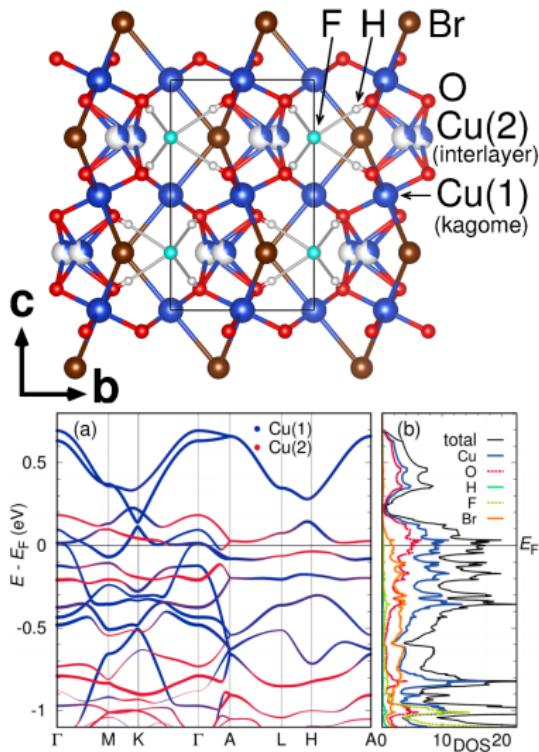


# Barlowite $[\text{Cu}_4(\text{OH})_6\text{FBr}]$ : kagome lattice antiferromagnet

- kagome lattice of  $\text{Cu}^{2+}$  ions
- $\text{Cu}^{2+}$  also between kagome layers
- **antiferromagnetic Mott insulator**
- AFM kagome exchange  $J_3 = 178 \text{ K}$
- FM interlayer exchange  $J_1 = -205 \text{ K}$
- remove interlayer coupling for QSL

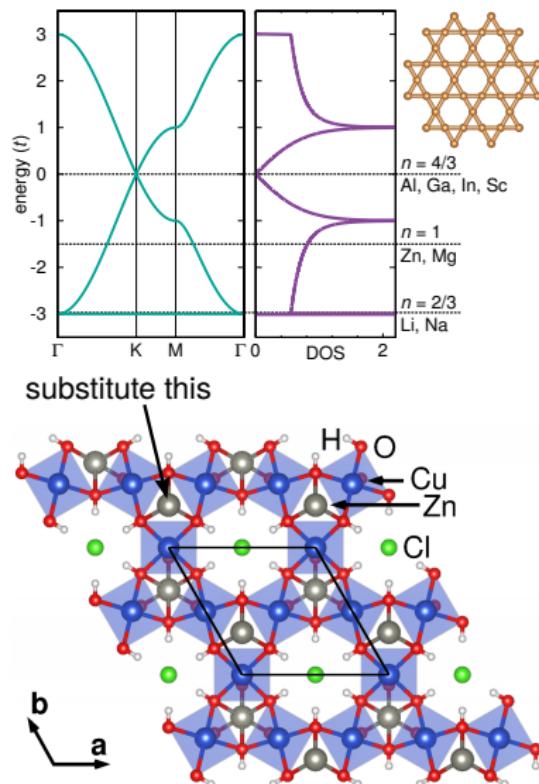


Jeschke, Salvat-Pujol, Gati, Hoang, Wolf, Lang, Schlueter, Valentí, PRB 92, 094417 (2015)



# Chemical modifications of Barlowite and Herbertsmithite

- we investigate  
A-Herbertsmithite [ACu<sub>3</sub>(OH)<sub>6</sub>Cl<sub>2</sub>] and  
A-Barlowite [ACu<sub>3</sub>(OH)<sub>6</sub>FBr]
- we propose dopants for realizing QAHE,  
QSHE and QSL
- hole-doping (QAHE): Li<sup>+</sup>, Na<sup>+</sup>
- electron-doping (QSHE):  
Al<sup>3+</sup>, Ga<sup>3+</sup>, In<sup>3+</sup>, Sc<sup>3+</sup>, Y<sup>3+</sup>
- isoelectronic doping (QSL):  
Mg<sup>2+</sup>, Ca<sup>2+</sup>, Zn<sup>2+</sup>, Cd<sup>2+</sup>, Hg<sup>2+</sup>
- for all proposed compounds we analyse  
the stability of crystal structures
- Is kagome lattice distorted upon doping?



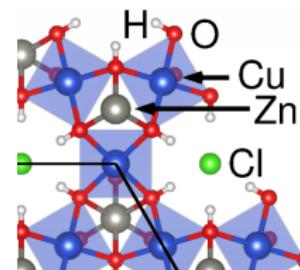
# Preparation and analysis of crystal structures

## structure preparation and relaxation

- replace interlayer site starting from Herbertsmithite/Barlowite
- density functional theory (DFT) calculations, GGA-PBE functional
- GPAW code, projector-augmented wave (PAW) basis,  
1000 eV plane-wave cutoff
- **unit cell shape and internal coordinates relaxed** until forces < 10 meV/Å

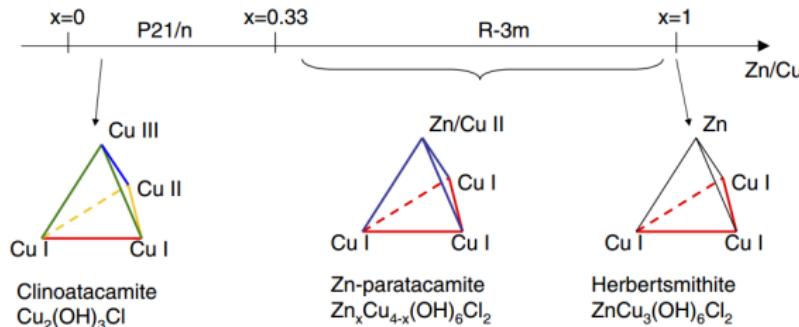
## stability tests for $\text{ACu}_3(\text{OH})_6\text{Cl}_2$ and $\text{ACu}_3(\text{OH})_6\text{FBr}$

- 1 does the dopant go in at all?
- 2 **does the dopant like the kagome environment better?**
- 3 what about vacancies instead of A?
- 4 what about Cu impurities instead of A?



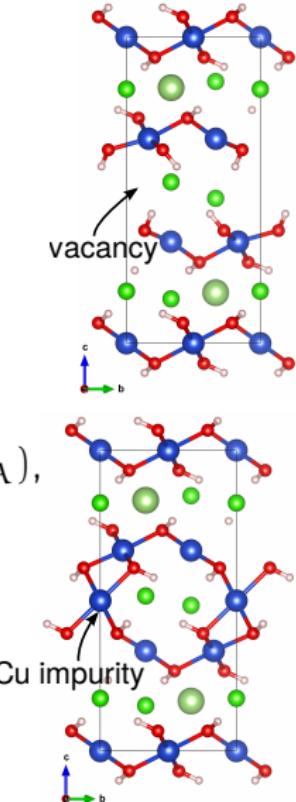
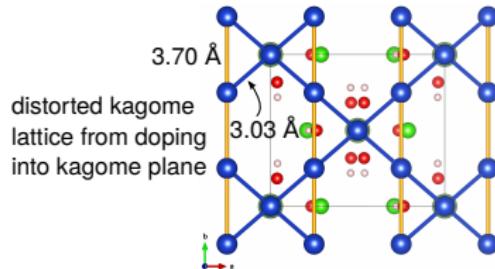
# Formation energies

- total energies from full-potential local orbital (FPLO) code
- model growth process by solid state reaction
- compare energies of  $\text{ACu}_3(\text{OH})_6\text{Cl}_2 + \text{Cu}$  to  $2 \times \text{Cu}_2(\text{OH})_3\text{Cl} + \text{A}$
- dopant metal energies from crystals, e.g. fcc-Cu
- formation energy is defined as  $E_{\text{form}} = E_{\text{AH}} + E_{\text{Cu}} - (E_{\text{C}} + E_{\text{A}})$
- negative formation energy  $\rightarrow \text{ACu}_3(\text{OH})_6\text{Cl}_2$  is formed
- positive formation energy  $\rightarrow \text{Cu}_2(\text{OH})_3\text{Cl}$  is formed



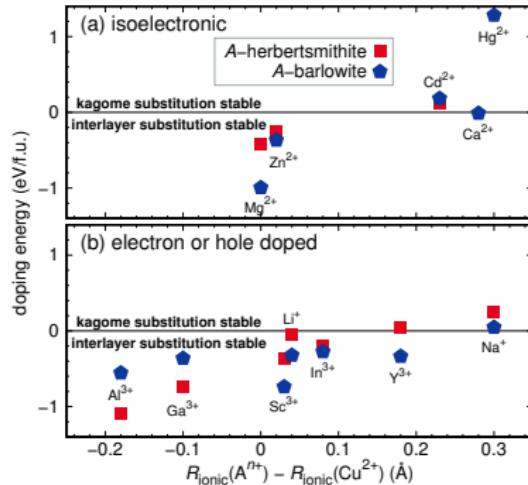
# Doping Energies

- evaluate energies of various defects, fully relaxed structures
- dopant A on kagome site and Cu on interlayer  
 $\text{Cu}(\text{ACu}_2)(\text{OH})_6\text{Cl}_2$
- compare energy directly to  $\text{ACu}_3(\text{OH})_6\text{Cl}_2$
- vacancy structure  $\text{A}_{0.66}\text{Cu}_3(\text{OH})_6\text{Cl}_2$
- impurity structure  $\text{A}_{0.66}\text{Cu}_{3.33}(\text{OH})_6\text{Cl}_2$
- doping energy:  $E_{\text{dop}} = E_{\text{AH}} + y \cdot E_{\text{Cu}} - (E_{\text{AHM}} + x \cdot E_A)$ ,  
where  $E_{\text{AHM}}$  belongs to  $\text{A}_{1-x}\text{Cu}_{3+y}(\text{OH})_6\text{Cl}_2$



# Summary of formation and doping energies

- variety of materials energetically possible
- vacancies and large fractions of Cu impurites irrelevant
- interlayer site favors small dopants
- hole-doped Li-Herbertsmithite stable
- various electron doped materials stable
- Mg-Barlowite strongly favors interlayer doping, probably reduced disorder

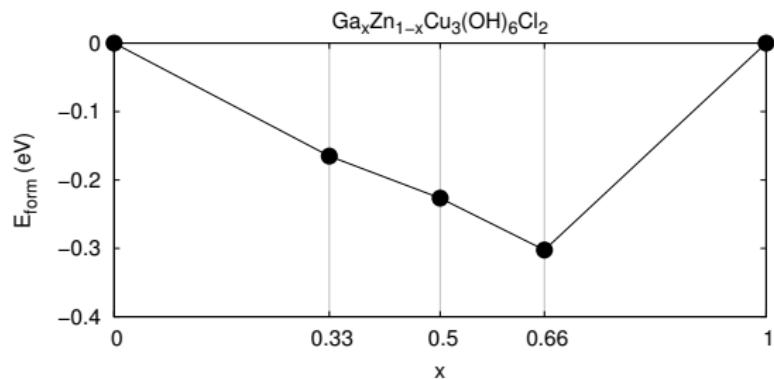


case	A-herbertsmithite, A =	Li <sup>1+</sup>	Na <sup>1+</sup>	Mg <sup>2+</sup>	Ca <sup>2+</sup>	Zn <sup>2+</sup>	Cd <sup>2+</sup>	Hg <sup>2+</sup>	Sc <sup>3+</sup>	Y <sup>3+</sup>	Al <sup>3+</sup>	Ga <sup>3+</sup>	In <sup>3+</sup>
1	ACu <sub>3</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-2.660	-2.517	-5.333	n/a	-2.082	-1.461	n/a	-7.300	-7.144	-6.257	-2.939	-3.080
2	ACu <sub>3</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-0.041	+0.257	-0.421	n/a	-0.261	+0.128	n/a	-0.372	+0.043	-1.101	-0.736	-0.203
3	A <sub>0.66</sub> Cu <sub>3</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-0.847	-0.735	-1.689	n/a	-0.694	n/a	n/a	-2.481	-2.398	-2.141	-1.176	-1.062
4	A <sub>0.66</sub> Cu <sub>3.33</sub> (OH) <sub>6</sub> Cl <sub>2</sub>	-1.497	-1.350	-2.308	n/a	-1.321	n/a	n/a	-3.019	-2.878	-2.659	-1.711	-1.585
	A-barlowite, A =												
1	ACu <sub>3</sub> (OH) <sub>6</sub> FBr	-2.860	-2.247	-5.089	-5.808	-2.074	-1.420	+1.317	-7.498	-7.400	-5.605	-2.895	-2.919
2	ACu <sub>3</sub> (OH) <sub>6</sub> FBr	-0.322	+0.047	-0.994	-0.012	-0.367	+0.183	+1.282	-0.738	-0.336	-0.555	-0.363	-0.276
	ionic radius in pm	76	102	72	100	74	95	102	75	90	54	62	80

# Doping energies for small concentrations

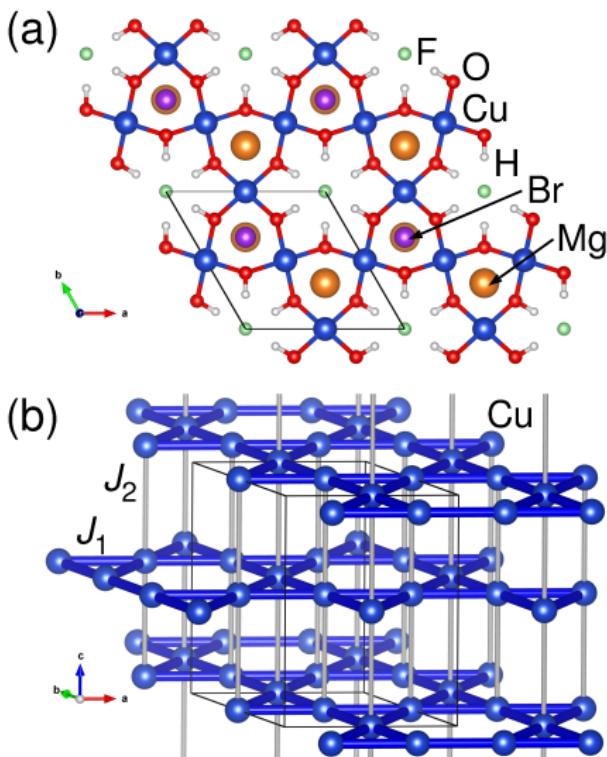
- Herbertsmithite very stable against excess copper on interlayer site
- Herbertsmithite susceptible to excess dopants in kagome plane
- more work needed to understand impurity concentration, arrangement, etc.
- solid solution  $\text{Ga}_x\text{Zn}_{1-x}\text{Cu}_3(\text{OH})_6\text{Cl}_2$  should exist

$A =$	$\text{Zn}^{2+}$	$\text{Mg}^{2+}$
$\text{A}_{0.83}\text{Cu}_{3.17}(\text{OH})_6\text{Cl}_2$	-0.464	-0.831
$\text{A}_{1.17}\text{Cu}_{2.83}(\text{OH})_6\text{Cl}_2$	+0.210	+0.803



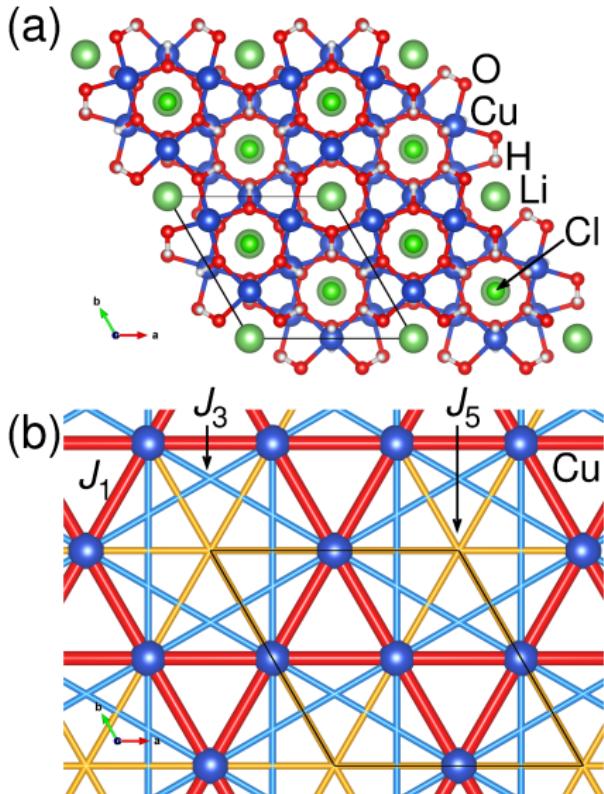
# Heisenberg model for Mg-Barlowite

- DFT+U calculation with  $U = 6$  eV and  $J_H = 1$  eV on Cu 3d
- map energies of DFT spin-configurations to Heisenberg model
- AFM kagome coupling  $J_1 = 226$  K
- interlayer coupling  $J_2 = 13.4$  K
- $J_2/J_1 = 0.06$
- spin-liquid predicted for  $|J_2/J_1| < 0.15$   
[Götze, Richter, arXiv:1510.04898]
- Mg-Barlowite is likely a quantum spin-liquid with low anti-site disorder



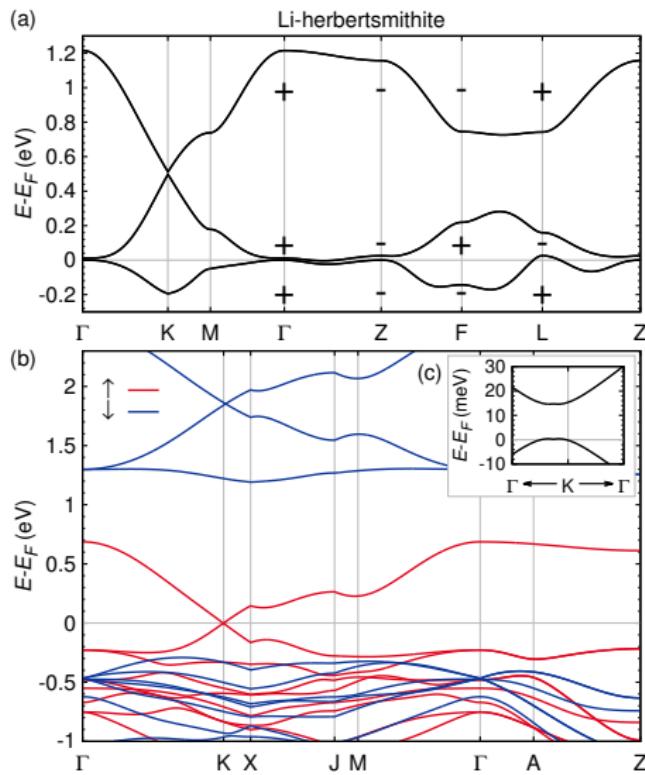
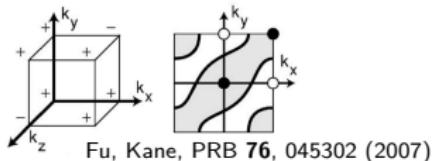
# Heisenberg model for Li-Herbertsmithite

- DFT+U calculation with  $U = 6$  eV and  $J_H = 1$  eV on Cu 3d
- large FM kagome coupling  $J_1 = -544$  K
- negligible interplane couplings
- mean-field Curie temperature  $T_C = -\frac{2}{3}S(S + 1) \sum_i z_i J_i$
- $J_i = \{-544, 39, -50\}$  K and  $z_i = \{4, 4, 6\} \rightarrow T_C \approx 1160$  K
- Li-Herbertsmithite is a very robust ferromagnet



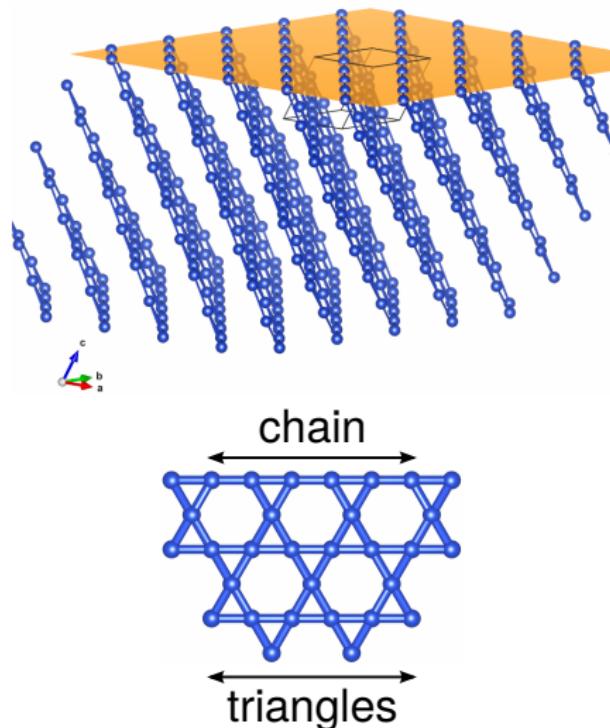
# Electronic properties of Li-Herbertsmithite

- Ga-Herbertsmithite is  $4/3$  filled,  $\{\uparrow 2/3, \downarrow 2/3\}$
- Li-Herbertsmithite is  $2/3$  filled FM,  $\{\uparrow 2/3, \downarrow 0\}$
- FM Dirac metal is suspicious of QAHE
- SOC opens gap at Dirac point
- check topological indices, parity eigenvalues from DFT
- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0; (111)$
- QAHE on (001) surface



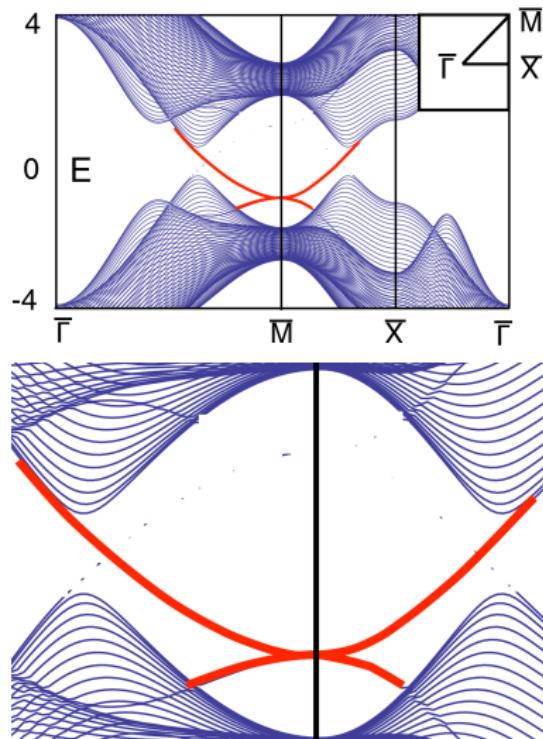
# Surface termination of Herberthsmithite

- $\nu_0; (\nu_1, \nu_2, \nu_3) = 0$ ; (111) is stack of 2D systems
- (001) termination has edge of kagome layer at surface
- termination influences details of surface bands, not qualitative physics
- surface states are generic in Herberthsmithite system
- doping only controls Fermi level
- electron doped-materials show QSHE, two spins at Fermi level
- Barlowite bands are topologically trivial, additional band crossings

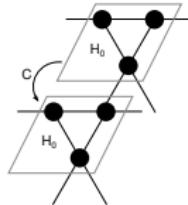


# How to calculate surface states

- full-relativistic DFT calculations, projective Wannier functions
- tight binding model for Cu states  $(n, j, m_j) = (3, 5/2, \pm 5/2)$
- minimum number of layers to allow bulk-surface distinction  $N \sim 20$
- diagonalization effort grows with  $N^3$
- always two surfaces in TB slab, experiment observes one at a time
- some authors just erase bands from the dual surface (justified, but ugly)
- proper way to calculate surface states: Green's functions



# State on the surface of Herberthsmithite



- iterative Green's function method

[Sancho et al., J. Phys. F: Met. Phys. **14**, 1205 (1984)]

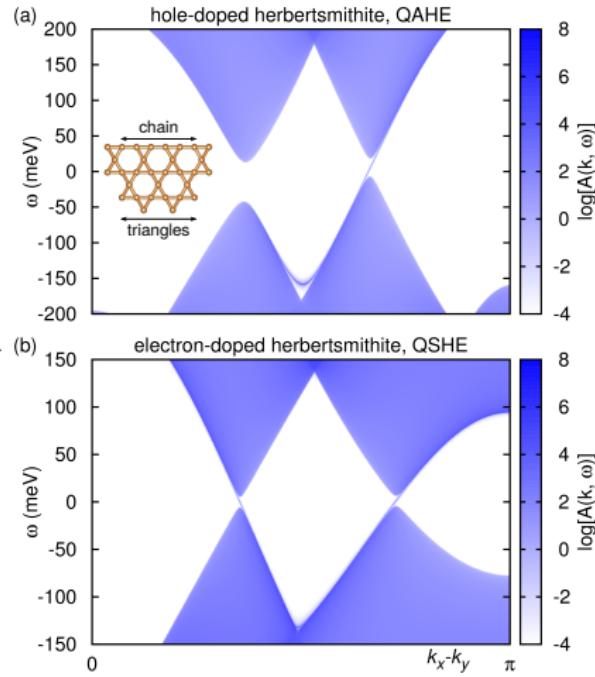
- initial condition  $G_{ij}^{(1)}(\omega) = (\omega - H_0)_{ij}^{-1}$

- surface  $G_{ij}^{(N)}(\omega) = \left[ (\omega - H_0 - CG^{(N-1)}C^\dagger)^{-1} \right]_{ij}$

- dual surface  $G_{ij}^{(N)}(\omega) = \left[ (\omega - H_0 - C^\dagger G^{(N-1)} C)^{-1} \right]_{ij}$

- spectral function

$$A(k, \omega) = -\text{Im} \text{ Tr} [G(k, \omega)] / \pi$$

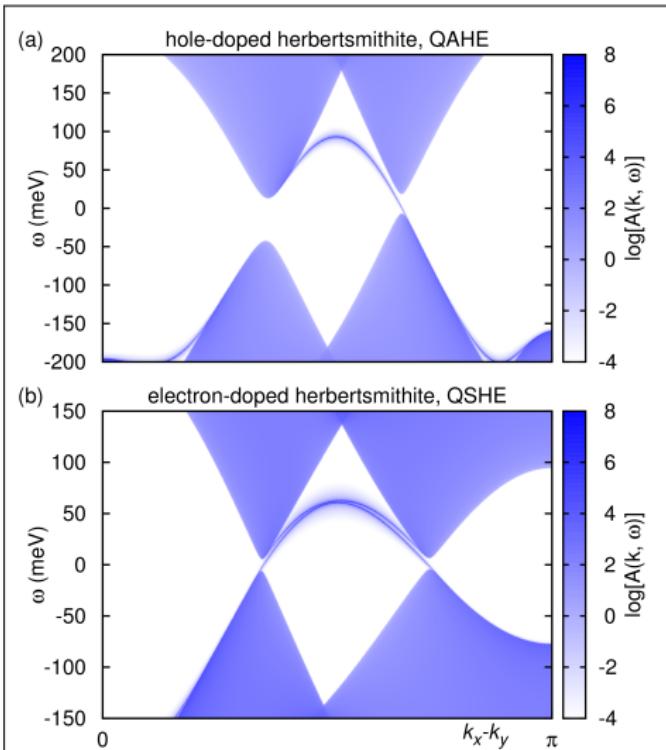
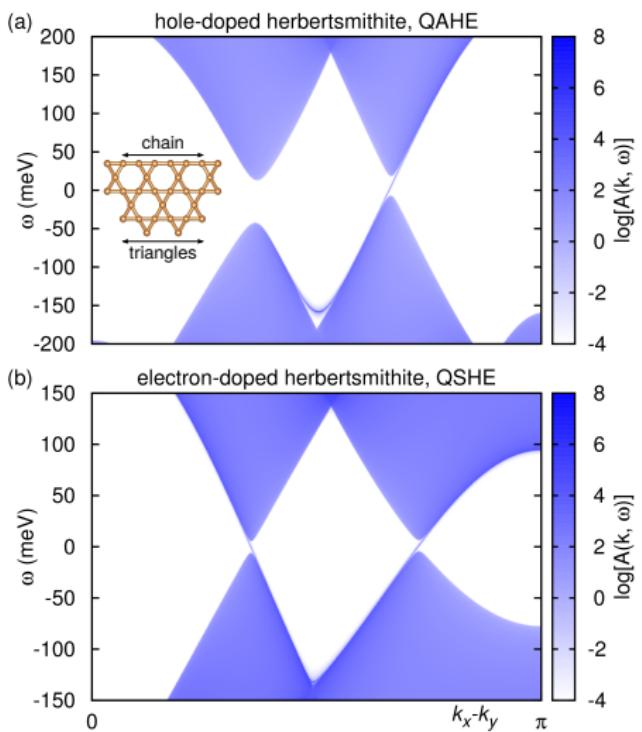


Guterding, Jeschke, Valentí, arXiv:1511.05686

- $N = 10^5$

- $\omega \rightarrow \omega + i \cdot 10^{-5} \text{ eV}$

# State on the dual surface of Herberthsmithite



# Test for Green's function code: crystalline topological insulator

