Advanced Topics in Control 2014: Robust Control and Convex Optimization

Exercise 11: Nominal $\mathcal{H}_2$ control design

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May 15, 2014

Please submit your solutions as a **single PDF** with filename *ATICXX-Surname-StudentNumber.pdf* (XX is the exercise number) to *ifaatic@ee.ethz.ch* until May 23th, 09.59 (next Friday). The subject of the email should be *[ATICXX] Surname-StudentNumber*.

Material from this exercise may (but doesn't have to) appear in your final report.

a) Design a nominal $\mathcal{H}_2$ controller for your project system by following the steps below.

1. Take the LTI representation of your system from Exercise 9 and remove uncertainties to give a nominal plant with which you can work (in the same way you did for the nominal $\mathcal{H}_\infty$ control design).
2. Determine the matrix components of Slide 10:15 with respect to your particular system. These will be used to pose the output feedback $\mathcal{H}_2$ design problem (minimize $\|\mathcal{F}_l(P(s), K(s))\|_{\mathcal{H}_2}$).
3. With the matrices defined, formulate the LMI conditions of Slide 10:18 for your system and solve the resulting SDP. Keep in mind that the given description is only valid for the special case where $D_k = 0$.
4. Use the process described on Slide 10:20 to retrieve the requested controller. How does this controller compare to the one given by **h2syn**? Compute the $\mathcal{H}_2$ gain of the two closed loop systems to support your claim.

b) Plug the designed controller (from SDP) to your system and check if the $\mathcal{H}_2$ gain respects the computed bound.

c) Compute the $\mathcal{H}_\infty$ gain of your designed closed loop system and compare with the bound achieved by the controller designed in Exercise 10. Give a short intuitive explanation of your results.