Optimal control based feedback linearization for position control of DC motor

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Abstract— **This paper proposes the position control of DC motor. Two methods are used for position control, LQR method and feedback linearization. We show that these methods without load torque are stable, but, when load is added to the motor's shaft, LQR and feedback linearization can not make efficient input signal for reference tracking in output. To solve this problem, we combined these methods and will show by using combined method, the position of shaft tracks reference in presence of large torque. For validation of new controller, we compared response with LQR and feedback linearization. Simulation results show stable response of new method.**

Keywords-DC motor; optimal control; nonlinear control;

I. INTRODUCTION

Direct current (DC) motors have been widely used in many industrial applications such as electrical vehicles, steel rolling mills, electric cranes, and robotic manipulators due to precise, wide simple and continuous control characteristics. DC motors are characterized by: ability to produce full continuous torque, controlled braking is relatively simple and low cost as compared with similar AC drives at high powers. Traditionally, rheostatic armature control method was widely used for the speed control of flow power DC motors. However, the controllability, cheapness, higher efficiency, higher current carrying capabilities of static power converters brought a major change in the performance of electrical drives [1].

The proportional-integral-derivative (PID) controller is widely used in many control applications because of its simplicity and effectiveness. Though the use of PID control has been along history in the field of control engineering, the three controller gain parameters are usually fixed. The disadvantage of PID controller is poor capability of dealing with system uncertainty, i.e. parameter variations and external disturbance. [2-5]. To improve system response, optimal control are used to design controller for DC motor. In this method, by using state feedback and solve Riccati equation, feedback gain is computed. In simulation, we show, by using only LQR method, system's response can not track reference in presence of load torque. To solve this problem some researchers combined LQR with intelligent system [6]. An approach to stabilization and trajectory tracking control for systems is feedback linearization [7-9]. In input-output linearization method, system model converts to the linear canonical model, then, control signal are produced by using state feedback. To consider the efficiency of LQR and feedback linearization method, [10] used them and show the LQR method can make better response.

In this paper, we show that without load torque, LQR makes response better than feedback linearization, but, when load torque increases, both of them have problem to track reference. To overcome this problem, we combined two methods. At first, system is linearized and for designing input, we use linearized state and input matrices to solve Riccati equation, till produce optimal input. We show that this new controller can make stable response in presence of load torque and output of system can track reference with minimum error.

The paper is organized as follow: In Section 2, model of system is introduced. LQR, feedback linearization controller are designed in section 3. In section 4, simulations results are shown. Finally in section 5, conclusion is presented.

II. DYNAMIC OF SYSTEM

The block diagram of feedback control system for DC motor drives is shown in Fig. 1. The control objective is to make the motor speed and position follow the references. The dynamics of a DC motor is given:

$$\begin{cases} v_t = R_a I_a + L_a \dfrac{dI_a}{dt} + E_a \\ T = J\dfrac{d\omega}{dt} + B\omega + T_l \\ T = k_t I_a \\ E_a = k_a \omega \\ \omega = \dfrac{d\theta}{dt} \end{cases} \tag{1}$$

where, $v_t, E_a, R_a, L_a, I_a, k_t, k_a, \theta, \omega$ are supply voltage that tuned by controller (V), back emf (V), armature resistance

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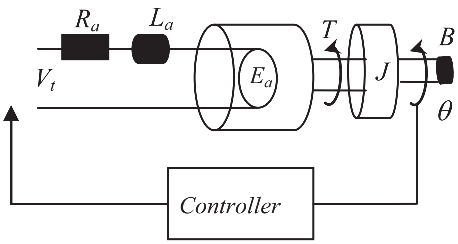

Figure 1. Description of closed loop system

(Ohm), armature inductance (H), armature current (A), torque factor constant (Nm/A), motor constant (Vs/rad), position and velocity, respectively.
Let's define state vector:

$$[x_1, x_2, x_3]^T = [\theta, \omega, I_a]^T \quad (2)$$

New parameters are defined as:

$$D_1 = \frac{k_T}{J}, D_2 = \frac{B}{J}, D_3 = \frac{R_a}{L_a}, D_4 = \frac{K_a}{L_a} \quad (3)$$

By using (2), (3), (1) in state space model can be written:

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= D_1 x_3 - D_2 x_2 - \frac{T_l}{J} \quad (4) \\ \dot{x}_3 &= -D_3 x_3 - D_4 x_2 + \frac{v_t}{L_a} \\ &\Rightarrow \dot{\boldsymbol{x}} = A\boldsymbol{x} + Bu + H \end{aligned}$$

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -D_2 & D_1 \\ 0 & -D_4 & -D_3 \end{bmatrix}, B = \begin{bmatrix} 0 \\ 0 \\ 1/La \end{bmatrix}, H = \begin{bmatrix} 0 \\ -\frac{T_l}{J} \\ 0 \end{bmatrix}, u = v_t$$

Where A is state matrix, B is input matrix, H is matrix of load torque that makes undesired effects on system's response. Controller must omit effectiveness of load torque on output of system. We use this state space model to design LQR and feedback linearization.

III. DESIGN CONTROLLER

A. LQR controller

In this section, we use optimal control method to design controller. The controller design is based on trajectory error with respect to three states. At first, we consider some conditions to defined state errors. In system (4), we have:

$$\dot{x}_2 = D_1 x_3 - D_2 x_2 - \frac{T_l}{J} \quad (5)$$

$$k_T >> B \Rightarrow \frac{k_T}{J} >> \frac{B}{J} \Rightarrow D_1 >> D_2 \quad (6)$$

From (6), we can write:

$$\dot{x}_2 = D_1 x_3 - \frac{T_l}{J} \quad (7)$$

State errors are defined as:

$$e_1 = x_1 - r, e_2 = x_2 - \dot{r}, e_3 = \frac{\dot{e}_2}{D_1} = x_3 - \frac{\ddot{r}}{D_1} - \frac{T_l}{D_1 J} \quad (8)$$

By using LQR method, the control signal is computed:

$$u = -F\boldsymbol{e}, \ \boldsymbol{e} = [e_1, e_2, e_3]^T \quad (9)$$

where the feedback gain F are obtained by minimizing an infinite-horizon quadratic performance criterion:

$$\lim_{t \to \infty} \int_0^t (\boldsymbol{e}^T Q \boldsymbol{e} + u^T R u) dt \quad (10)$$

where $Q = diag([q_1, q_2, q_3])$, $q_i > 0, i = 1,2,3$. This problem is commonly referred to as linear quadratic regulator optimization. The feedback gain is given by:

$$F = -R^{-1} BP \quad (11)$$

The constant matrix P is the solution of the algebraic Riccati equation:

$$Q - PBR^{-1} B^T P + PA + A^T P = 0 \quad (12)$$

For simulation coefficients are chosen as:

$$Q = diag([5,5,1]), R = 0.5$$

B. Feedback linearization

In this section, Controller scheme is based on input output feedback linearization methodology. The controller design requires an explicit relationship between the output and input signals. This relationship is obtained by differentiating output function repeatedly until input appears. In this paper, the control objective is position of shaft tracks reference, therefore, shaft's position is chosen as main output. Output of system $y = x_1$ is indirectly linked to input v_t. To obtain relationship between output and input, it is necessary to differentiate the output y.

After differentiating the output three times, the following relationship is obtained:

$$\dddot{y} = -D_1 D_3 x_3 - D_1 D_4 x_2 - D_1 D_2 x_3 + D_2^2 x_2 + \frac{D_2 T_l}{J} + \frac{D_1}{L_a} v_t \quad (13)$$

We must notice that order of (13) and system (4) are same. In this case, no part of system has not been rendered unobservable by input output linearization. New state vector and matrices are gained:

$$\dot{\boldsymbol{z}} = A_f \boldsymbol{z} + B_f u$$

$$A_f = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, B_f = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad (14)$$

If the input voltage is designed in the form:

$$v_t = \frac{L_a}{D_1}(u + D_1 D_3 x_3 + D_1 D_4 x_2 + D_1 D_2 x_3 - D_2^2 x_2 - D_2 \frac{T_l}{J}) \quad (15)$$

we will have linear system that can make stable response and good tracking by designing u with efficient control method. New state variables are chosen as:

$$z_1 = x_1, z_2 = x_2, z_3 = D_1 x_3 - D_2 x_2 - \frac{T_l}{J} \quad (16)$$

The state errors are gained:

$$e_{1f} = z_1 - r, e_{2f} = z_2 - \dot{r}, e_{3f} = z_3 - \ddot{r} \quad (17)$$

Control signal u is designed:

$$u = \dddot{r} - k_3 \ddot{e}_f - k_2 \dot{e}_f - k_1 e_f \quad (18)$$

where k_1, k_2, k_3 are positive constant. For simulation, these coefficients are chosen as:

$$k_1 = 5, k_2 = 12, k_3 = 7 \quad (19)$$

In simulation section, we will show system output can not chase reference by using this old method to design control signal, therefore, to overcome this problem we imply optimal control to design control signal by using new state error and matrix.

C. *LQR feedback linearization method*

In this section, we use LQR method to design control signal u for input voltage (15). When we use feedback linearization, we convert undesired model of system to canonical model with new state variables and matrices. To design LQR controller, we used new state and input matrices to solve Riccati equation to compute feedback gain. New state errors, state matrix and input matrix are chosen:

$$\boldsymbol{e}_f = [e_{1f}, e_{2f}, e_{3f}], A_f, B_f \quad (20)$$

The control signal is computed:

$$u_f = -F_f \boldsymbol{e}_f \quad (21)$$

where the feedback gain F is computed by following equation:

$$F_f = -R_f^{-1} B_f P_f \quad (22)$$

where P_f is gained by solving following Riccati equation:

$$Q - P_f B_f R_f^{-1} B_f^T P_f + P_f A_f + A_f^T P_f = 0 \quad (23)$$

By using new design, input voltage is gained:

$$v_t = \frac{L_a}{D_1}\left(u_f + D_1 D_3 x_3 + D_1 D_4 x_2 + D_1 D_2 x_3 - D_2^2 x_2 - D_2 \frac{T_l}{J}\right) \quad (24)$$

For simulation we use $R_f = 0.0005$.

IV. SIMULATION RESULTS

In this section simulation results are presented applying the proposed control scheme to control the position of DC motor. In order to validate the effectiveness of the controller, response of proposed FLQR (feedback linearization LQR) controller is compared with LQR and FL control methods. Equations (1), (9), (15), (24) are used for simulation. System properties are chosen as follow:

$$R_a = 2.06, L_a = 0.0238, k_t = 0.0235, k_a = 0.02352,$$

$$B = 12 \times 10^{-4}, J = 1.07 \times 10^{-3}, T_l = 0, T_l = 0.03$$

To simulate the different conditions experienced by DC motor, different loads are attached to the shaft of the motor and simulations are considered for different references. At first, we simulate system without load torque. Fig. 2 shows system position responses for three controllers. Without load torque, rise time of LQR is smaller than FL and FLQR.

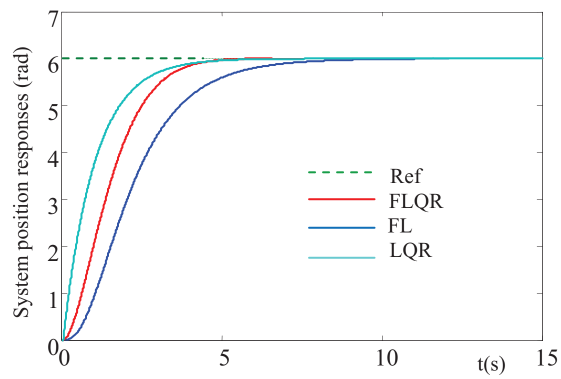

Figure. 2. Position of shaft without load torque for regulation problem

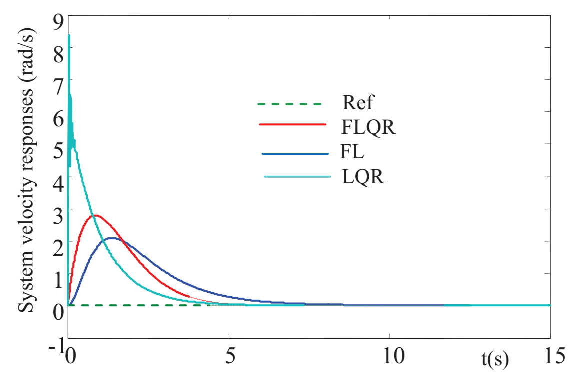

Figure. 3. Velocity of shaft without load torque for regulation problem

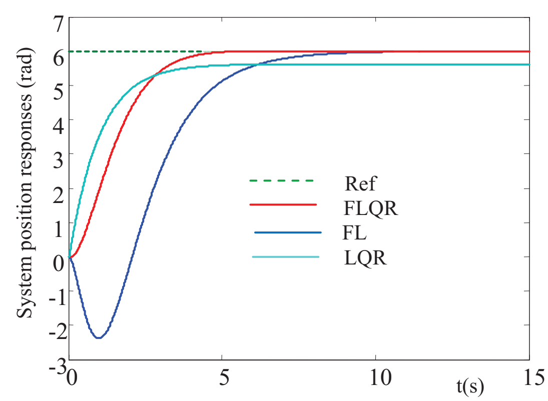

Figure 4. . Position of shaft in presence of load torque for regulation problem

In Fig. 3, simulation shows that optimal control makes intense changing in velocity and at first of simulation, initial velocity variation of shaft is fast and has large domain, this variation is not good in application. FL makes smooth variation in velocity, but its settling time is large. The velocity response of FLQR changes smooth and has settling time as same as LQR.

Again, we simulated system with previous properties, but increase load torque. We consider the effectiveness of

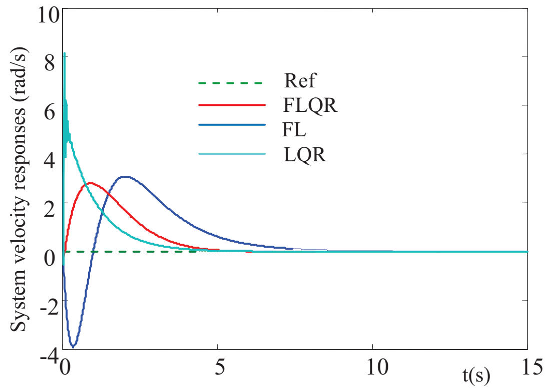

Figure. 5. Velocity of shaft in presence load torque for regulation problem

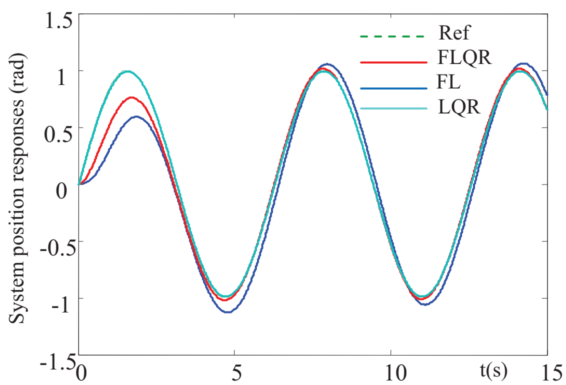

Figure. 6. Position of shaft without load torque for sinusoidal reference

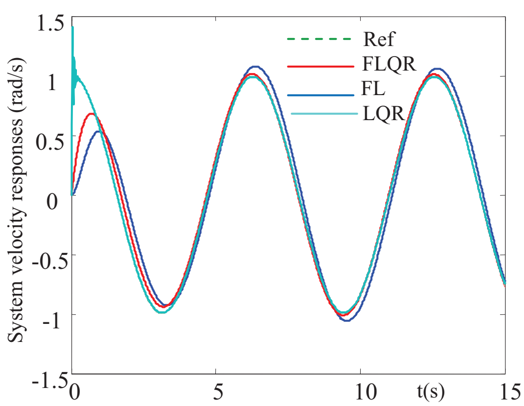

Figure. 7. Velocity of shaft without load torque for sinusoidal reference

load torque on system response. In Fig. 4, the position responses of system in presence of load torque are demonstrated. LQR response has large constant error that shows it can not track reference in presence of load torque. After 10s, FL response closes to the reference, but at initial times of simulation, it has undershoot that shows inability of controller to produce efficient control signal. In application,

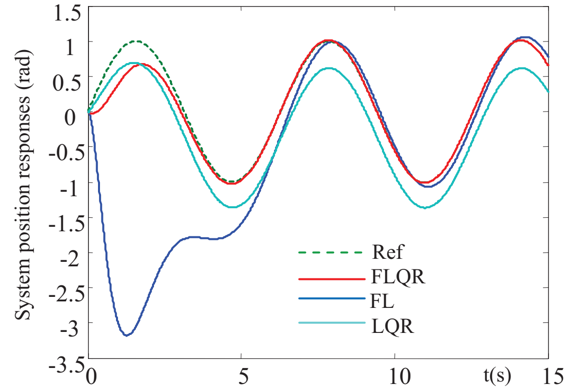

Figure. 8. Position of shaft in presence of load torque for sinusoidal reference

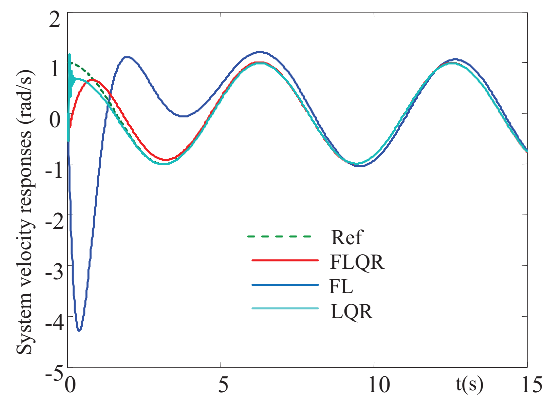

Figure. 9. Velocity of shaft in presence of load torque for sinusoidal reference

existence of undershoot means that shaft rotates in undesired direction. FLQR response closes to reference with smooth variation.

In Fig. 5, velocity responses are considered. At initial times, LQR response oscillates and changes fast. FL response changes smooth but sign of velocity changes and the settling time of response is large. Domain and settling time of FLQR response are small that shows FLQR can makes efficient control signal that results stable tracking in output.

To consider the ability of controllers for tracking of time varying reference, we use sinusoidal reference and simulate system without load torque and in presence of load torque. Fig. 6 shows system position response without load torque. LQR response tracks reference with minimum output error. At initial times, FL output has large error, this error becomes small during time, but yet, there is noticeable error. For FLQR response, at initial time, error is large, but position response closes to reference after 5 seconds. In Fig. 7, velocity responses show that without load torque, velocity of shaft has good response in three methods.

Again, we increase load torque and simulate system by using previous coefficients. Fig 8 shows that LQR response can not track reference and it has large constant error. This response shows LQR method is not good for tracking control. FL response has large undershoot. Undershoot makes undesired effects on system. FLQR response at initial time has error, but after 4s, response closes to reference with minimum error, also, response has not undershoot and varies smoothly. In Fig. 9 LQR and FLQR responses closes to reference smoothly but FL method, again, makes large undershoot in velocity response.

All these simulations show that FL and LQR have special properties that make different responses in different conditions. But, FLQR is stable method that has both FL and LQR properties. Combination of optimal control and nonlinear method makes new controller produces optimal input in presence of load torque.

V. CONCLUSION

In this paper, optimal and nonlinear methods for controlling DC motor are considered. We introduce LQR and feedback linearization methods separately, then, we combined these methods. At first, we use FL method to gain new canonical system with new state variables. Then, we use new state matrix and input matrix to solve Riccati equation and design optimal feedback gain. Responses of three methods are simulated and drawn together. In all simulation, FLQR has stable response and tracks reference in different condition. We showed that this controller is better than other methods that considered in this paper.

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