Computational geophysics

Equations: 1D heat equation

$$\rho c_v \partial_t T = \partial_x (K \partial_x T) + f$$

ρ: density $\rho = \rho(x)$
c_v: specific heat $c_v = c_v(x)$
K: conductivity $K = K(x)$

homogeneous materials $\rho = \text{const}$, $c_v = \text{const}$, $K = \text{const}$:

$$\partial_t T = \underbrace{\frac{K}{\rho c_v}}_{D \equiv \text{const}} \partial_x^2 T$$

D: diffusivity

$$\boxed{\partial_t T = D \partial_x^2 T}$$ diffusion equation

1D wave equation

$$\rho \partial_t^2 s = \partial_x (\mu \partial_x s) + f$$

μ: (shear) modulus $\mu = \mu(x)$
ρ: density $\rho = \rho(x)$
s: displacement $s(x,t)$

homogeneous material $\rho = \text{const}$, $\mu = \text{const}$

$$\partial_t^2 s = \underbrace{\frac{\mu}{\rho}}_{c^2 = \text{const}} \partial_x^2 s$$

$$\boxed{\partial_t^2 s = c^2 \partial_x^2 s}$$

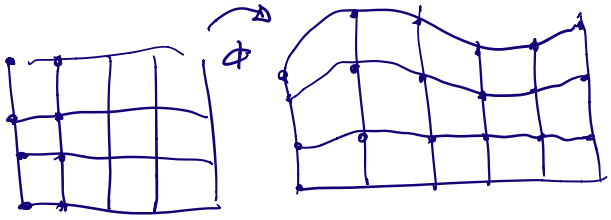
methods:
- finite-difference, pseudo-spectral } strong form < low-order, infinite order
- finite-element, spectral-element } weak form < low-order, high-order

time schemes: implicit

explicit — conditionally stable (CFL condition)

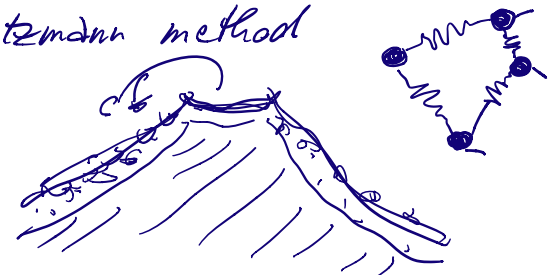
missing points / details:

- finite-difference:
  - staggered-grids
  - rotated
  - curvilinear



- absorbing boundary conditions:
  - PML
  - Somerfeld
  - Clayton-Enquist

- other methods:
  - finite-volume method ← low-order
  - discontinuous Galerkin method ← high-order
  - Lattice-Boltzmann method



  - particle-in-cell ← non-linear equations Lagrangian
  - ...