# Computational geophysics

Equations: 1D heat equation

$$\rho\, c_v\, \partial_t T = \partial_x (K\, \partial_x T) + f$$

$\rho$ : density $\rho = \rho(x)$
$c_v$ : specific heat $c_v = c_v(x)$
$K$ : conductivity $K = K(x)$

homogeneous materials $\rho = \text{const}$, $c_v = \text{const}$, $K = \text{const}$ :

$$\partial_t T = \underbrace{\frac{K}{\rho\, c_v}}_{D \equiv \text{const}} \partial_x^2 T$$

$D$ : diffusivity

$$\boxed{\partial_t T = D\, \partial_x^2 T}$$ diffusion equation

1D wave equation

$$\rho\, \partial_t^2 s = \partial_x (\mu\, \partial_x s) + f$$

$\mu$ : (shear) modulus $\mu = \mu(x)$
$\rho$ : density $\rho = \rho(x)$
$s$ : displacement $s(x,t)$

homogeneous material $\rho = \text{const}$, $\mu = \text{const}$

$$\partial_t^2 s = \underbrace{\frac{\mu}{\rho}}_{c^2 = \text{const}} \partial_x^2 s$$

$$\boxed{\partial_t^2 s = c^2 \partial_x^2 s}$$

methods:

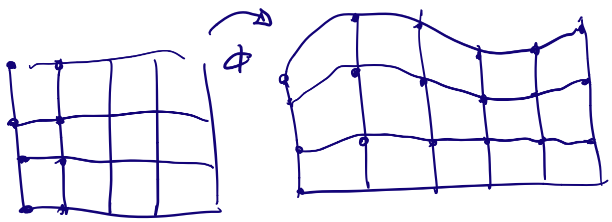
$$\left.\begin{array}{l}\text{finite-difference}\\ \text{pseudo-spectral}\end{array}\right\}\ \text{strong form} \begin{cases}\text{low-order}\\ \text{infinite order}\end{cases}$$

$$\left.\begin{array}{l}\text{finite-element}\\ \text{spectral-element}\end{array}\right\}\ \text{weak form} \begin{cases}\text{low-order}\\ \text{high-order}\end{cases}$$

time schemes:
- implicit
- explicit — conditionally stable (CFL condition)

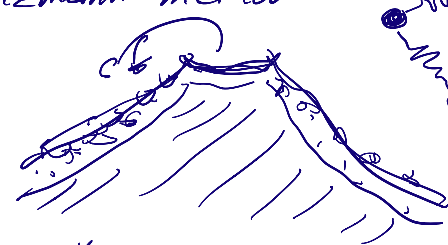
missing points / details:

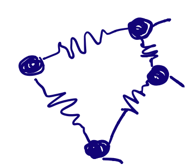
- finite-difference:
  - staggered-grids
  - rotated
  - curvilinear




- absorbing boundary conditions:
  - PML
  - Somerfeld
  - Clayton-Enquist

- other methods:
  - finite-volume method ← low-order
  - → discontinuous Galerkin method ← high-order
  - Lattice-Boltzmann method





  - particle-in-cell ← non-linear equations
  - ...