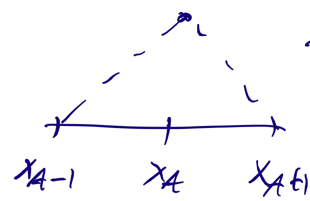
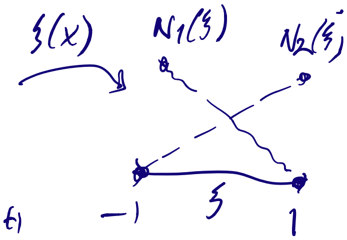
Finite-element method

Basis functions :

1) tent function
linear functions





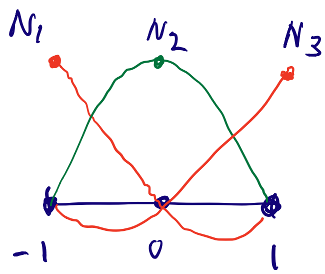
for reference element $u(\xi) = \sum_{a=1}^{2} u_a N_a(\xi)$

with $\begin{cases} N_1(\xi) = \frac{1}{2}(1-\xi) \\ N_2(\xi) = \frac{1}{2}(1+\xi) \end{cases}$

polynomial $P(\xi) = a_0 + a_1 \xi$

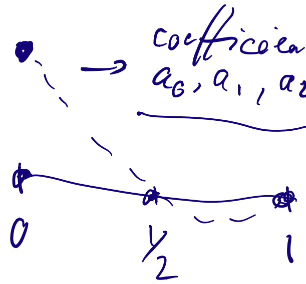
quadratic basis function : polynomials

$P(\xi) = a_0 + a_1\xi + \underline{a_2 \xi^2}$



$u(\xi) = \sum_{a=1}^{3} u_a N_a(\xi)$

with $\begin{cases} N_1(\xi) = \frac{1}{2}(\xi-1)\xi \\ N_2(\xi) = 1-\xi^2 \\ N_3(\xi) = \frac{1}{2}(\xi+1)\xi \end{cases}$



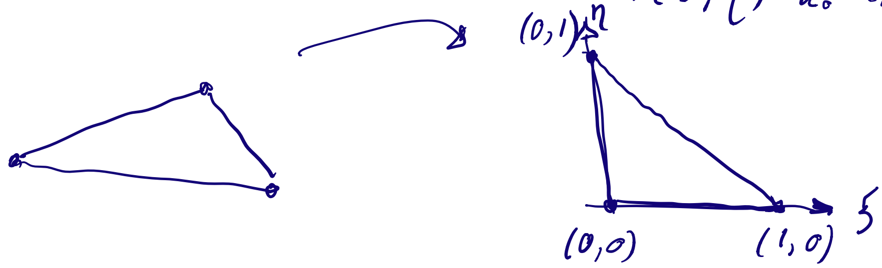
$\rightarrow$ coefficient a_0, a_1, a_2

or range $[0,1]$:

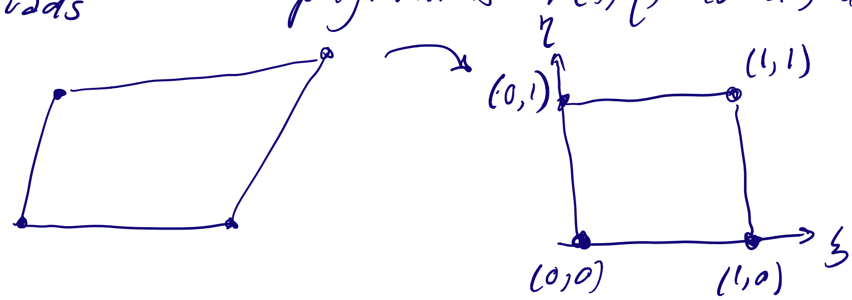
$\rightarrow \begin{cases} N_1(\xi) = 1 - 3\xi + 2\xi^2 \\ N_2(\xi) = 4\xi - 4\xi^2 \\ N_3(\xi) = -\xi + 2\xi^2 \end{cases}$

2D triangles linear polynomials

$$P(\xi,\eta)=a_0+a_1\xi+a_2\eta$$



quads polynomials $P(\xi,\eta)=a_0+a_1\xi+a_2\eta+a_3\xi\eta$

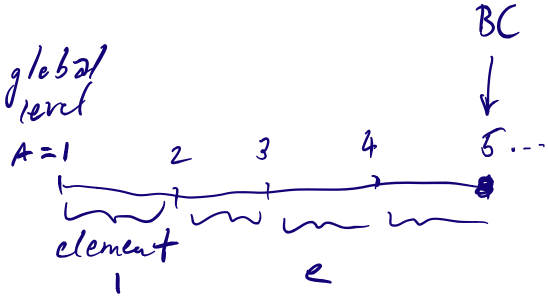


etcetera...

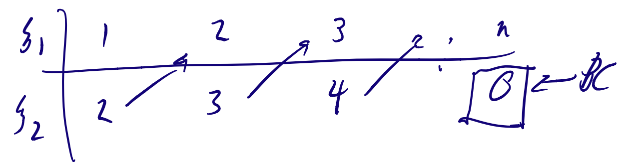
Unstructured mesh:

global node $A = LM(a, e)$

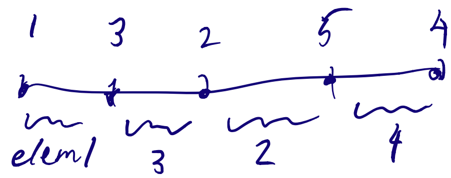
indexing between local and global level



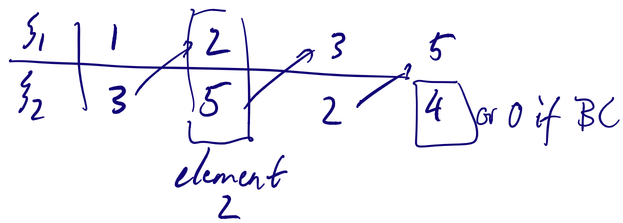
$LM(a, e)$



$iglob = LM(a_i, e_l)$



$\longrightarrow LM(a, e)$ unstructured meshes



compared to FD:

$$\frac{f_{i-1} + f_{i+1}}{2\Delta x}$$ based on regular meshes

$$x_j = j\Delta x$$

Heat transfer

Based on the conservation of energy and assuming Fourier's law, we can write the 1D heat equation as following (in absence of convection)

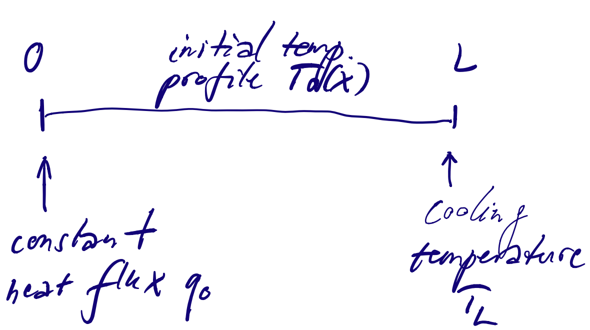
$$\rho c_V \, \partial_t T = \partial_x (K \, \partial_x T) + f$$

with T: temperature $T = T(x,t)$

c_V: specific heat

K: conductivity

f: heat source/sink



The boundary conditions are

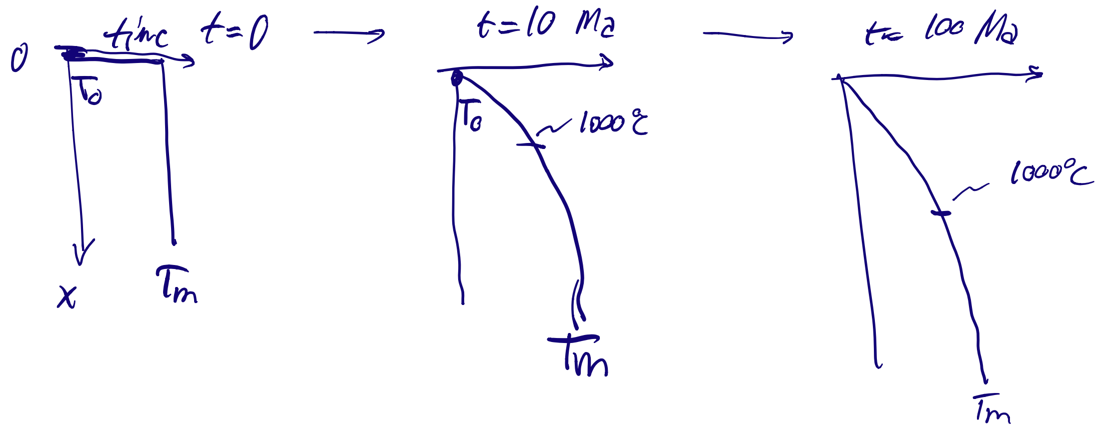
$$\begin{cases} T(L,t) = T_L \\ \partial_x T(0,t) = -q_0 \\ T(x,0) = T_0(x) \end{cases}$$

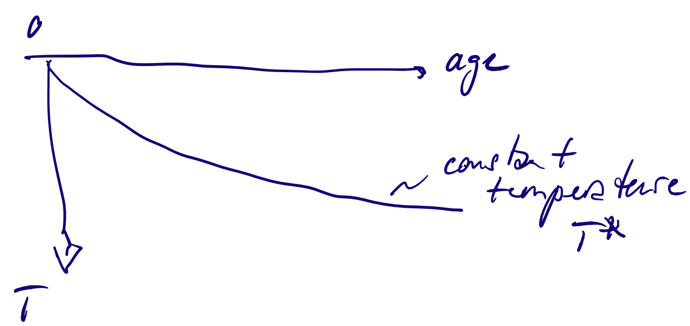
Finite-element approach:

1.) weak form
2.) define local element contributions
3.) assembly
4.) time scheme

Half-space cooling model

For an oceanic lithosphere, assume a uniformly hot plate (half-space) slowly cooling down.





Diffusion equation simplified for seafloor spreading (assumes lateral diffusion much smaller than vertical)

$$\boxed{\rho c_v \partial_t \hat{T} = \partial_x (k \partial_x \hat{T})}$$

with $\hat{T}(x,t) = \dfrac{T(x,t) - T_m}{T_0 - T_m}$ non-dimensional temperature

T_0: surface temperature (surface ~ 0°C)
T_m: initial/mantle temperature (mantle ~ 1300°C)

and boundary conditions

$$\begin{cases} T(0,t) = T_0 & \text{cooling down} \\ T(x\to\infty, t) = T_m & \\ T(x,0) = T_m & \text{assumes initially "hot" plate} \end{cases}$$