Problem Set 3

Tsunami

The open ocean propagation of tsunami waves is approximated by the long-wavelength, shallow-water equation:

$$\begin{aligned} \partial_t^2 P &= \nabla \cdot (v^2 \, \nabla P) && (1) \\ &= \partial_x v^2 \, \partial_x P + \partial_y v^2 \, \partial_y P + v^2 (\partial_x^2 P + \partial_y^2 P) && (2) \end{aligned}$$

where $P = P(x, y, t)$ is the height of tsunami waves above sea level, $v = \sqrt{gH(x, y)}$ the wave speed, g the gravity acceleration, H the ocean depth.

When the term v^2 is not changing rapidly, an approximation form of the governing equation that will accurately predict the speed, dispersion and focusing of the waves is:

$$\partial_t^2 P = v^2 \, \nabla^2 P \tag{3}$$

Equation (3) is called a '*homogeneous*' differential equation. There is no source term acting (and it assumes that v^2 is constant, i.e., describes waves in a homogeneous medium). The extra terms in equation (2) arise due to '*inhomogeneous*' material properties.

Problem:

An example code is provided to you, which can solve the homogeneous equation to 2nd-order in space and time.

(a) modify the code to (keep 2nd-order scheme in time) make it 4th-order in space

(b) derive formula for the inhomogeneous terms, both in 2nd-order and 4th-order

Additional information:

1. The C-code can be compiled with:

   ```
   cc -o class_tsunami class_tsunami.c -lm    (use ‘gcc’ instead of ‘cc’ on linux)
   ```

2. Then the code is executed by simply typing:

   ```
   ./class_tsunami
   ```

3. The output is in the file 'slice.out', which can be viewed in Matlab with the script ‘class_movie.m’