



INFS4205/7205 Advanced Techniques for High Dimensional Data
Spatial Data Organization I

Semester 1, 2021

University of Queensland

+ Assignment 1

- Due 21 Apr 21, 4pm
- Blackboard submission
- Contents
 1. Database Design
 2. Spatial Query
 3. Query Execution
- No implementation, No actual code, No standard solution...
 - Design your own from scratch
 - Plain English description / Pseudo-code
 - Clear and concise

+ Advanced Techniques for High Dimensional Data

- ❑ Course Introduction
- ❑ Introduction to Spatial Databases
- ❑ Spatial Data Organization
- ❑ Spatial Query Processing
- ❑ Managing Spatiotemporal Data
- ❑ Managing High-dimensional Data
- ❑ Other High-dimensional Data Applications
- ❑ When Spatial Temporal Data Meets AI
- ❑ Route Planning
- ❑ Trends and Course Review

+ Learning Objectives

■ What we will cover

- Basic principles of managing multidimensional data
 - 2-D as example
 - How to construct index
- Representative data access methods for point, line and polygon data
 - How to use index
- Processing of some spatial operations using data access methods
 - Point query, Range / Window query, ...

■ Goals

- Understand major types of multidimensional data access methods, and their strengths and limits
- Understand how point query, window query and join query are processed using various data access methods

+ Readings

- R. Güting, An Introduction to Spatial Database Systems, *The VLDB Journal*, 3:4, 1994
- Oracle Business and Technical White Papers
- V. Gaede and O Günther, Multidimensional Access Methods, *ACM Computing Surveys*, 30:2, 1998
- J. Orenstein and F. Manola, PROBE Spatial Data Modeling and Query Processing in an Image Database Application, *IEEE Transactions on Software Engineering*, 14:5, 1988
- T. Brinkhoff, H.-P. Kriegel and B. Seeger, Efficient Processing of Spatial Joins Using R-Trees, SIGMOD'93

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+ Index

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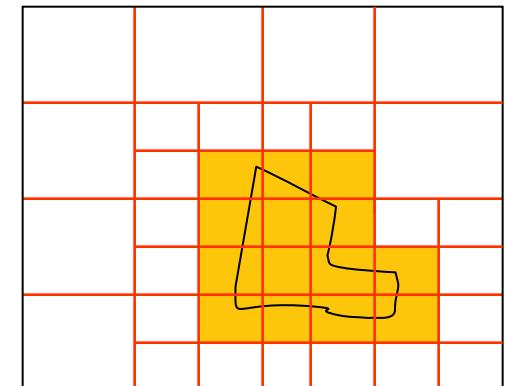
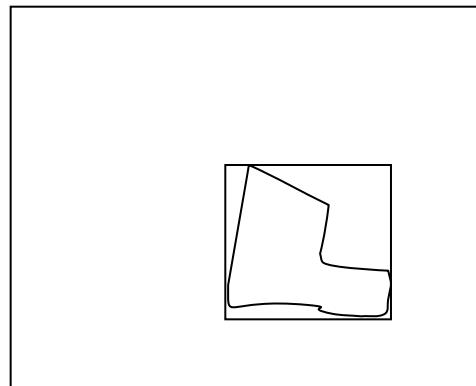
- A database index is similar to a book index
- Book index: lists important terms in alphabetical order with a list of page number(s) where the term appears
- Searching in a book:
 1. Search the book index for a term to find a list of addresses (i.e., page numbers)
 2. Use these addresses to retrieve the specified pages
 3. Search for the term in each page
- Otherwise, search through the entire book word by word (\approx linear search)

+ Spatial Indexing

- Purpose:
 - Efficiency in processing spatial selection, join and other spatial operations
- Two strategies to organize space and objects
 - Map spatial objects into 1D space and use a standard index structure (B-tree)
 - Dedicated external data structures
- Basic ideas
 - Approximation
 - Bounding box, Grids
 - Hierarchical Data Organization

+ Object Approximation

- A fundamental idea of spatial indexing is the use of approximation
- Continues Approximation
 - Object centric
 - Example:
 - Use of MBRs (Minimum Bounding Rectangles)
 - R-Tree
- Grid Approximation
 - Space centric
 - Faster mapping
 - Uniform / Non-uniform
 - High-D?
 - Example:
 - Quad-Tree



+ What We Need to Know

For each multidimensional access method:

- Motivation
 - Why it is proposed
 - Good for what type of data
 - Points? Line? Polygons?
- Operations for creating and maintaining an index
 - Insert, delete data items
- Query operations using an index
 - Point query and window query
 - Spatial join queries and other queries

+ Background Knowledge

- Disks and files
- Basic indexing methods in relational DBMS
 - B-Tree
 - Hashing
 - Bitmap
- Query processing using indexes
 - What to achieve, and how?

+ Secondary Storage Device

■ Magnetic Tape

■ Still in use???

- Cheap per GB: 2 cent
- Better durability: 50 years or longer
- Suit for backup of PB historical data
 - Telecom
 - Energy Efficient

■ Fast sequential read/write

- LTO 8: 12TB 360MB/s
- LTO 9: 25TB 708MB/s
- LTO 10: 48TB 1100MB/s



不过我们可以首先看看《SQUI》Commodore 64版为何口碑会好



IBM TS4500 Tape Library

Next-generation cloud storage library.

- Maximum capacity with LTO-8 / TS1160: 278 PB / 351 PB
- Drive Type(s): LTO and/or TS1100
- Maximum number of drives: 128



+ Secondary Storage Device

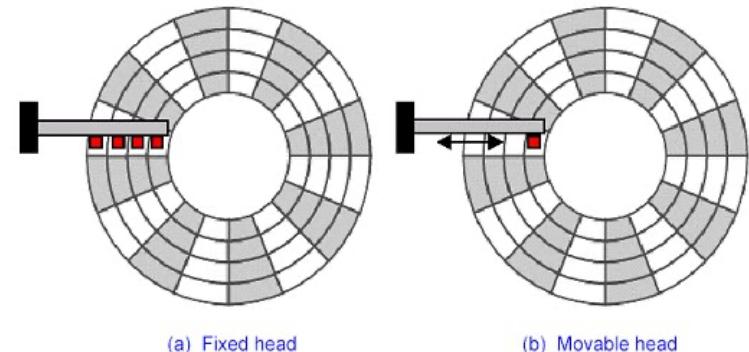
- (S)DBMS stores information on **hard disks**
- This has major implications for DBMS design:
 - **READ**: transfer data from disk to main memory
 - **WRITE**: transfer data from RAM to disk
 - In the unit of block
 - 512, 1K, 4K, 64M,...
 - Both are high-cost operations (I/O), so must be planned carefully!
 - External Memory Complexity: The number of I/Os that need to be performed
- Why not store everything in memory?
 1. Costs
 2. Main memory is volatile
 - PCM: Phase-Change Memory

+ Magnetic Disks

- Main advantage over tapes
 - Random access
- Data is stored and retrieved in units called disk blocks or pages or buckets
- Unlike RAM, time to retrieve a page varies depending upon location on disk

+ Accessing a Disk Block

- Time to access (read/write) a disk block:
 1. **Seek Time:** Moving arms to position disk head on track
 2. **Rotational Delay:** Waiting for block to rotate under head
 - 5400 / 7200 rpm
 3. **Transfer Time:** Actually moving data to/from disk surface
- Seek Time and Rotational Delay dominate
- Key to lower I/O cost: reduce seek/rotation delays!
Hardware vs. software solutions?



+ Index Structure

- An index is an **auxiliary** file that makes it more efficient to search for a record in the data file
- An index is usually specified on one field of the file (although it could be specified on several fields)
 - For another field? Build a new index for it
- One form of an index is a file of entries:

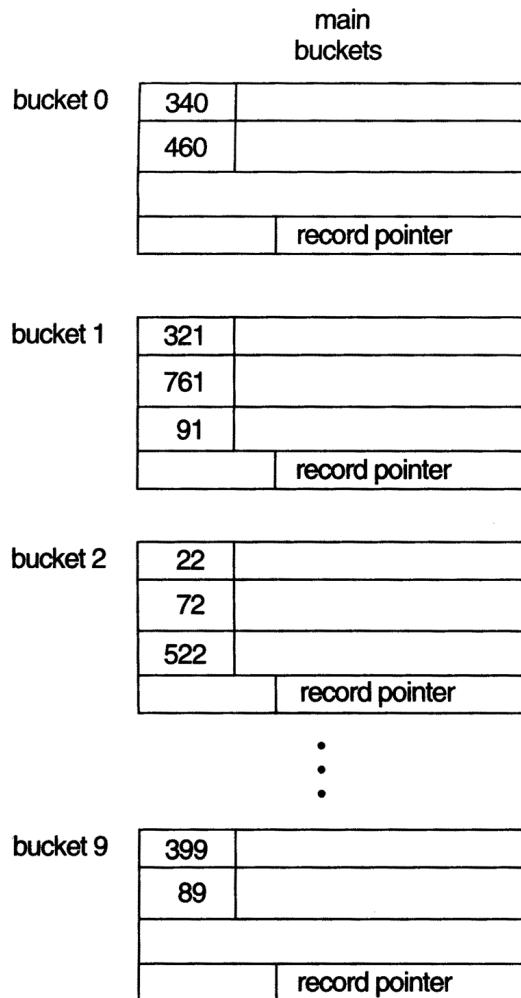
<field value, pointer to record>

 - ordered by field value
- The index file occupies **less disk blocks** than the data file
 - Because its **entries** are much **smaller**
- A binary search on the index yields a **pointer** to the file record
 - Extra I/O access is needed to **fetch** data

+ Data Access Methods

- One dimensional
 - Hashing and B-Trees
- Line Data
 - Segment Tree, Interval Tree
- Point data
 - Hashing: GRID and EXCELL
 - Hierarchical
 - Quadtree: point and region quadtrees
 - kd-Tree
 - Z-values and B-tree
- Polygon data
 - Transformation: End point mapping and z-values
 - Overlapping: R-tree and R*-tree
 - Clipping: R⁺-tree

+ Static Hashing



- **Hashing** converts the key of a record into an address in which the record is stored
 - Like a dictionary
 - Why not an array with direct addressing?
 - $O(1)$ worst case
 - Key number $k \ll N$ possible keys, waste space

+ Static Hashing

- Hashing converts the key of a record into an address in which the record is stored
- A **hash function** is used to map the key to the *relative* address of a bucket in which the record is stored.
 - If a file is allocated m buckets, the hash function must convert a key k into the relative address of the block:
 - $h(k) \in \{0, \dots, m - 1\}$
- A **bucket** is either one disk block or a cluster of contiguous blocks
- A **table** stored in the header of the file maps relative bucket numbers to disk block addresses

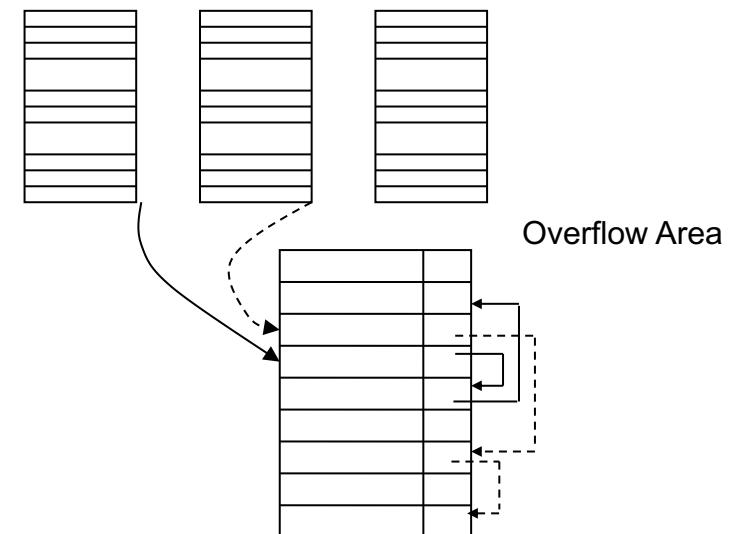
+ Collision

- Insertion of a new record may lead to collision

- No space in $b = h(k)$
 - Data Skewness

- Chaining: Use overflow buckets

- Example: Common overflow area for all blocks in a file
 - Each block has a pointer to its first record in the overflow area
 - Records belonging to the same block are linked by pointers
 - Hash could decay into linked list
 - Distribution does not meet expectation



+ Hashing Functions

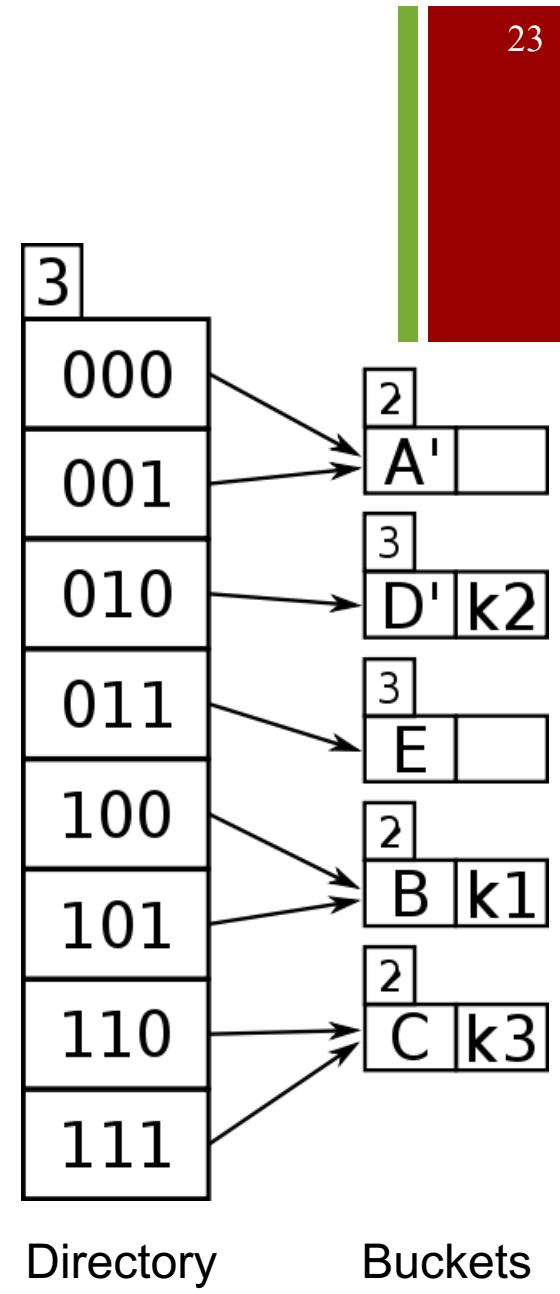
- A good hash functions must
 - Be computed efficiently
 - Minimize the number of collisions by spreading keys around the file as evenly and uniform as possible
- Example of hash functions
 - Truncation
 - Ignore part of the key and use the remaining part directly as the index
 - e.g. key is an 8-digit number, hash table has 1000 entries
 - Use the first, fourth, and eighth digit to make the hash function
 - 12345678 maps to 148
 - Not uniformly
 - Folding: Break up the key in parts and combine them in some way
 - 12345678 maps to $123+456+78=657$
 - Division: $h(k) = k \bmod m$

+ Pros and Cons of Hashing

- Excellent performance for searching on equality on the key used for hashing
- Records are not ordered
 - Any search other than on equality is very expensive
 - e.g. Range Query
- Prediction of total number of buckets is difficult
 - Allocate a large space
 - Estimate a “reasonable” size and periodically reorganize

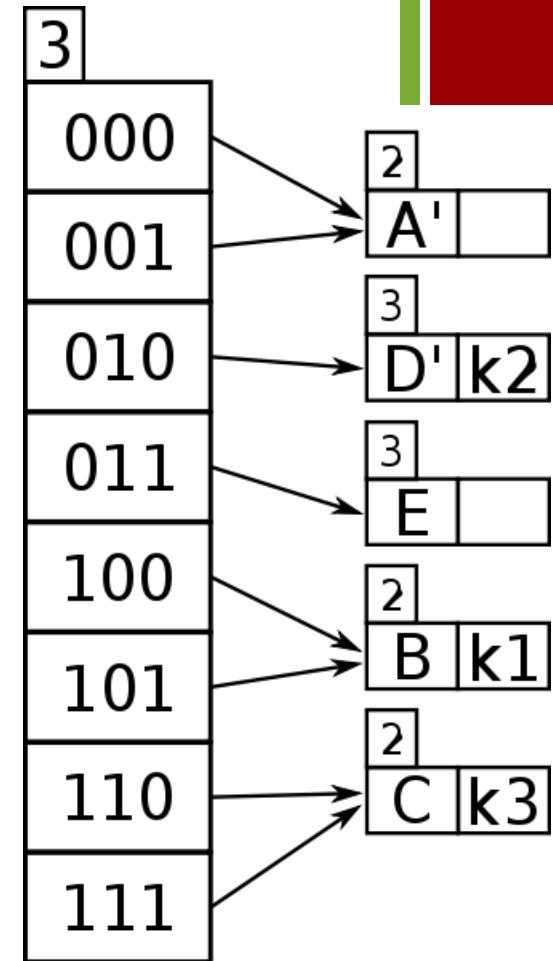
+ Extendible Hashing

- The file is structured into two levels:
 - Directory and Buckets
 - The directory has 2^d entries (d : global depth)
 - Each entry points to a bucket
 - Use some hashing function that generates a string of bits
 - The first/last d digits used as index into the directory
 - Prefix: Only the first d' bits are used
 - Suffix: *mod* operation, the last d' bits are used
 - Several entries can point to the same bucket
 - Local depth d' ($d' \leq d$)



+ Extendible Hashing Adjustment

- The directory size can be doubled or halved
 - Double: $d = d + 1$
 - When a bucket with $d' = d$ overflows
 - Half: $d = d - 1$
 - When $d > d'$ for all buckets
 - No directory size change
 - $d' = d - 1$
 - Split the bucket when overflow
- At most two-level search, highly efficient
 - No *just-in-case* space waste, no overflow page, no chain search
 - No space is allocated in extendible hashing for future growth, but additional buckets can be allocated dynamically as needed



+ Linear Hashing

■ Dynamic hashing scheme

- The file size grows linearly, bucket by bucket
- No directory
 - The file starts with m main buckets
 - Buckets are numbered from 0 to $m - 1$
- A family of hash functions h_0, h_1, h_2, \dots
 - $h_i(k) = k \bmod (2^i \times m)$
 - Initial hashing function $h_0(k) = k \bmod m$
 - h_{i+1} doubles the range of h_i
- Still has overflow
 - Split only when the *load factor* = $\frac{|item|}{|bucket\ capacity|}$ is met
 - Otherwise, use overflow page
 - Split the buckets with round-robin
 - Round ends when all initial buckets are split
 - Only two neighboring hash functions exist at the time

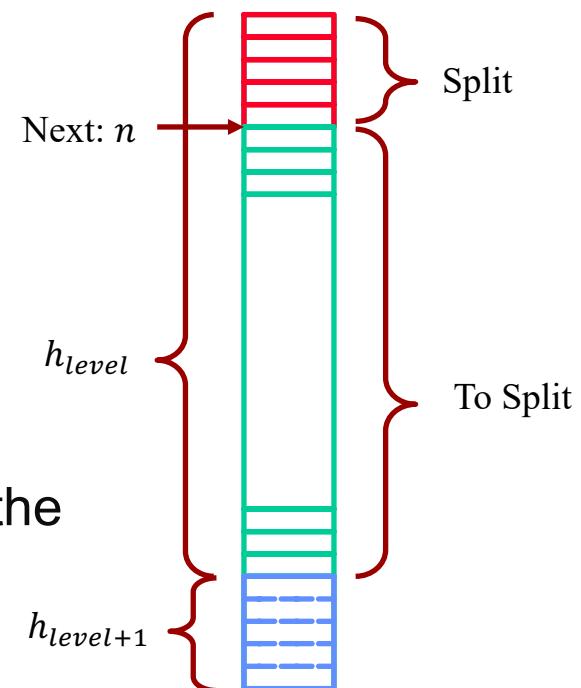
+ Linear Hashing

- We keep the following info:

- n : a pointer to the bucket that should be split next
- Initially, $n = 0$
 - 0 to n have been split
 - n to $2^i \times m$ to be split

- Search

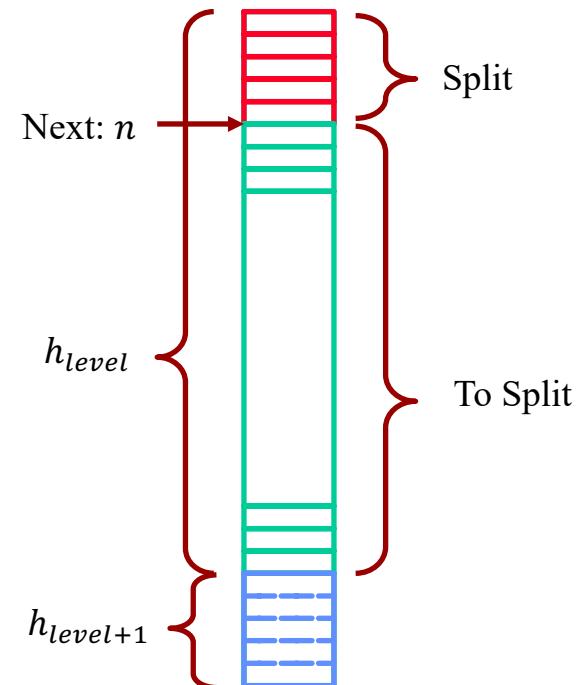
- $h_{level}(k)$ is larger than n
 - Search inside the bucket
- Otherwise, also apply $h_{level+1}(k)$ to find out the exact location



+ Insertion of a Record

If there is a collision, do:

- Push record to an overflow bucket
- (Grow) A new bucket is appended at the end of the hash table
- (Split) Records in bucket number n (including those in the overflow space) are hashed again using
 - $h_1(k) = k \bmod (2m)$ (will return either n or $m + n$)
 - These records will either remain in bucket n or in bucket $m + n$
- If $n = m$ (all original buckets have been split)
 - Set $n = 0$
 - $h_1(k) = k \bmod (2m)$
 - $h_2(k) = k \bmod (4m)$



+ Linear Hashing

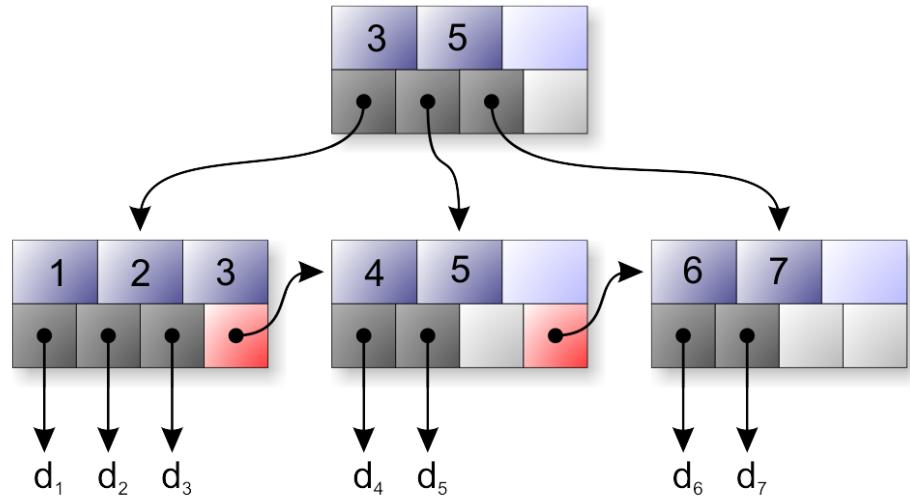
- Advantage
 - The file size grows linearly, bucket by bucket
- Disadvantage
 - No guarantee that a split relieves the overloaded bucket
 - Still requires overflow buckets and chaining

+ Hashing Summary

- Best for equality search, bad for range search
- Static Hashing can lead to long overflow chain
- Extendible Hashing avoids overflow by splitting a full bucket when a new data entry is added
 - Directory to keep track of buckets, doubles periodically
 - Can be large with skewed data
- Linear Hashing avoids directory by splitting buckets round-robin, and using overflow pages
 - Overflow pages not likely to be long
 - Space utilization could be lower than Extendible Hashing

+ About B-Tree

- What does “B” stands for?
- How to perform a search for a given key value?
- How to insert a value?
- Any overhead?
- When to create a B-tree index, and on what attributes?



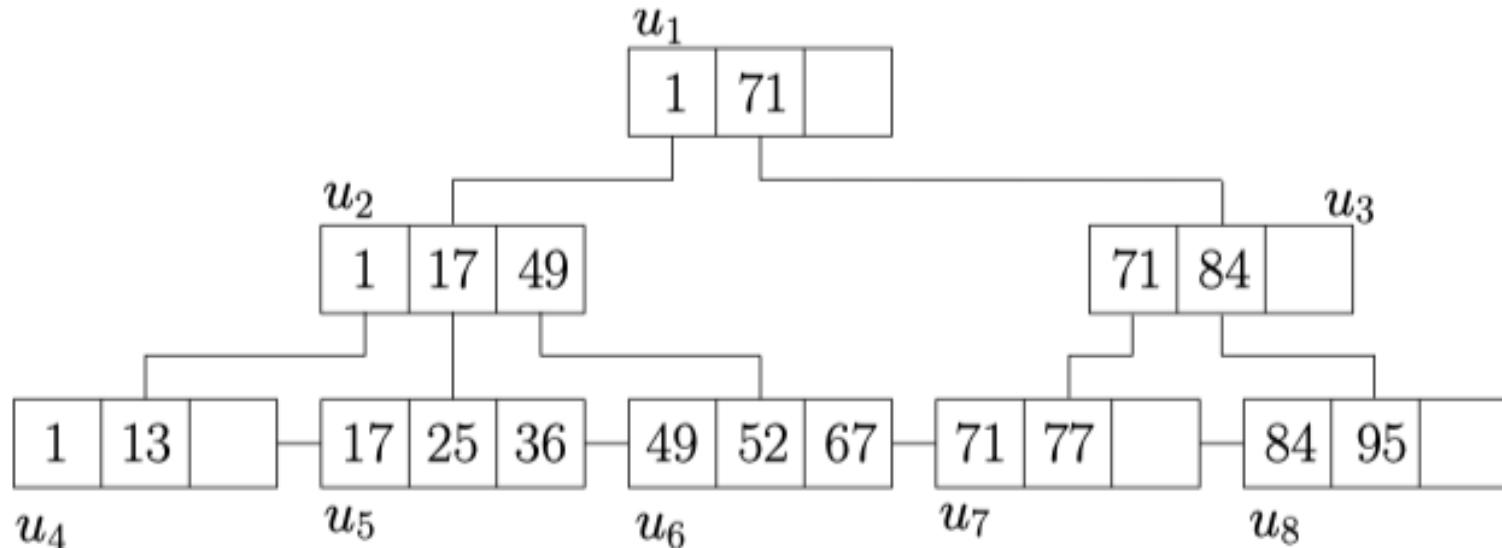
...B-tree and B⁺-tree: what's the difference?

+ B-Tree

- Each leaf node has between $B/2$ and B data elements, where $B \geq 3$
- The only exception takes place when the leaf is the root, in which case it can have any number of elements
- All the leaf nodes are at the same level
 - Each internal node has between $B/2$ and B child nodes
 - Except that the root can have as few as 2 child nodes

+ B-Tree Structure

- For any node u , denote by S_u the set of elements in the subtree of u . Now let u be an internal node with child nodes v_1, \dots, v_f ($f \leq B$)
 - All the data elements in S_{v_i} must be strictly smaller than any data element in S_{v_j} for any $1 \leq i < j \leq f$
 - For each v_i ($i \leq f$), u stores the smallest element $r(v_i)$ in S_{v_i} , which is referred to as a routing element



+ B-Tree Successor and Predecessor Search

■ Successor and Predecessor Search

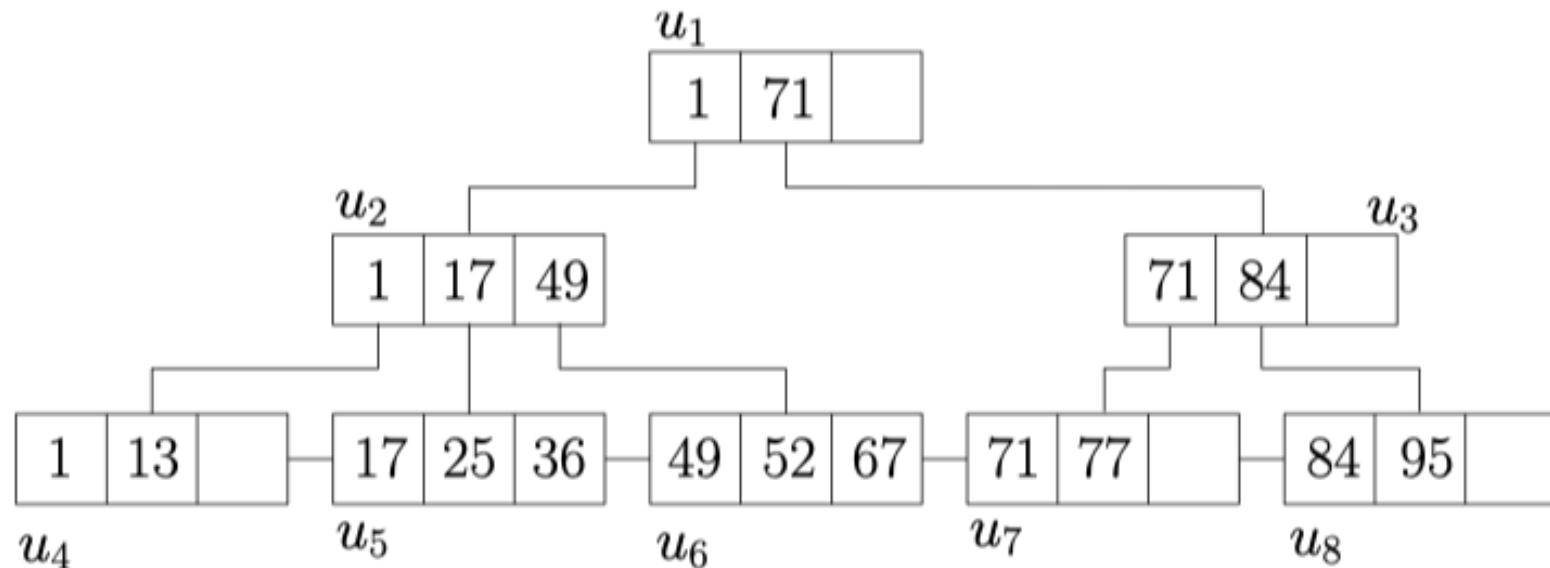
- Let S be a set of real values
- The successor / predecessor of a value x in S is the smallest/largest value in S that is at least / most x

- Example: $S = \{1, 13, 17, 25\}$
 - Successors of 10 and 13 are both 13
 - Predecessor of 12 is 1

- Algorithm
 1. Set u to the root of T
 2. If u is a leaf, return the successor of x among the elements in u
 3. Otherwise, find the predecessor of x among the elements in u .

+ B-Tree Successor and Predecessor Search

- u_1, u_2, u_5 are accessed to retrieve the successor of 20
 - Cost = 3 I/Os



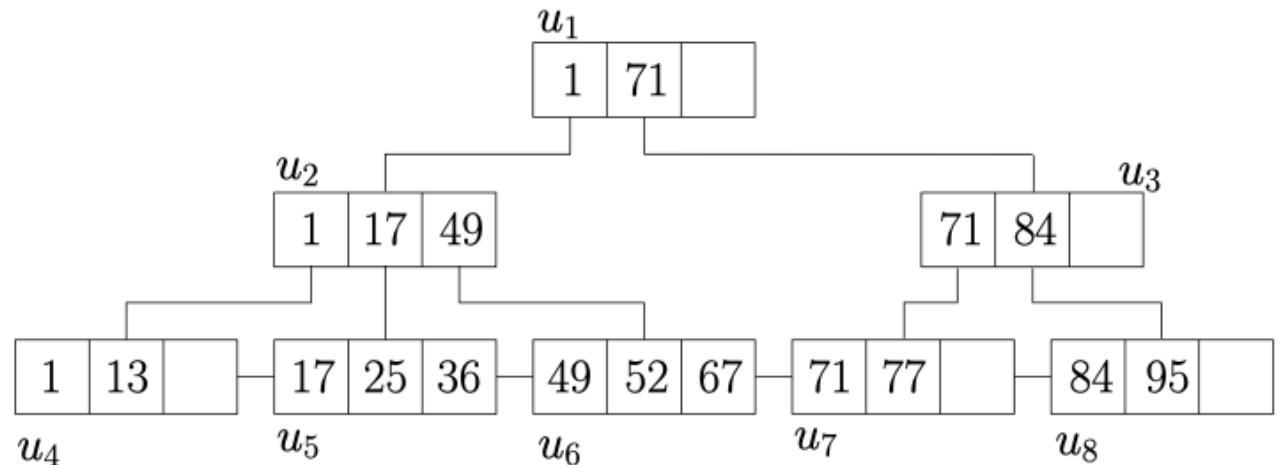
+ B-Tree Range Query

■ $q = [x, y]$

1. Locate the leaf u containing the successor of x
2. Report all elements in u that fall in q
3. If no element in u is greater than y
 - Set u to the succeeding leaf node, and go Step 2

■ $q = [20, 75]$

- $u_1, u_2, u_5, u_6, \text{ and } u_7$



+ B-Tree Insertion

■ To insert a value x into a B-Tree

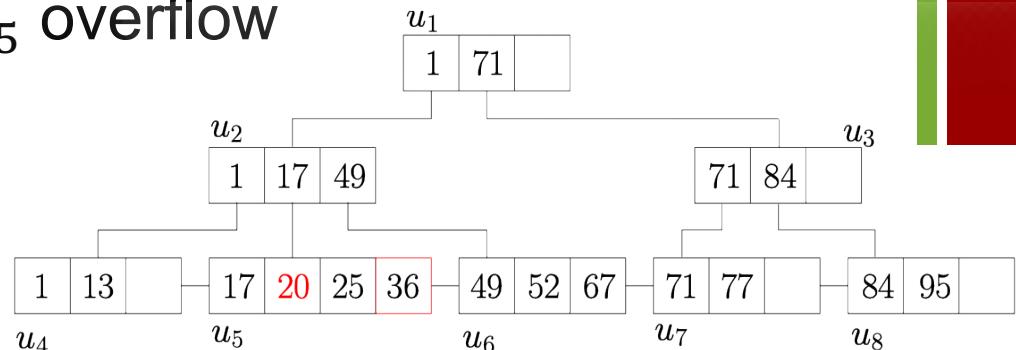
1. Find the leaf node u that should accommodate x without violating the B-Tree definition, add x to u
2. If u has no more than B elements, the insertion is complete
 - Otherwise, u overflows

■ Overflow Handling

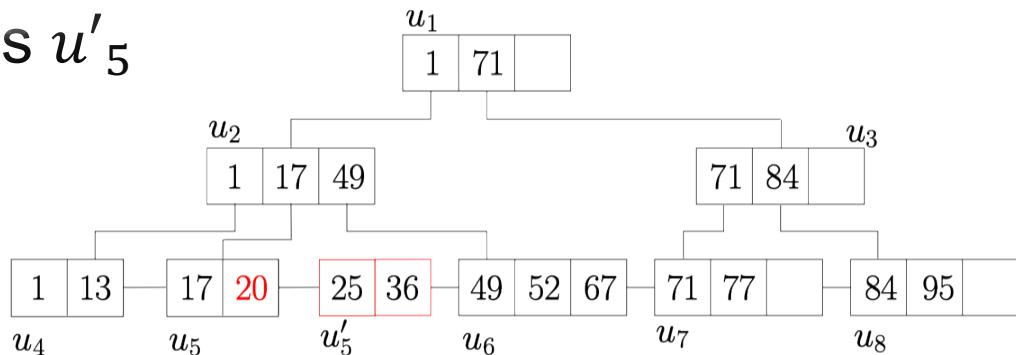
1. Create a new node u'
2. Split the element of u in two halves (u and u')
3. Insert $r(u')$ into the parent p of u
4. If p has no more than B elements, done
 - Otherwise, hand the overflow at p in the same way

+ B-Tree Insertion

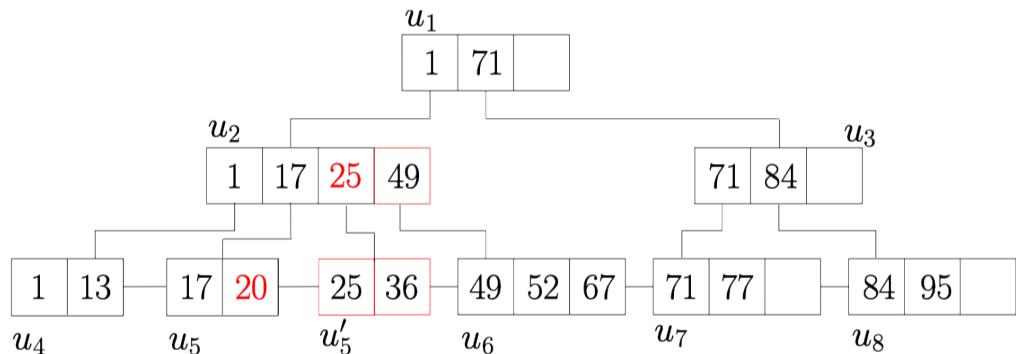
- Insertion of 20 make u_5 overflow



- u_5 splits, and generates u'_5

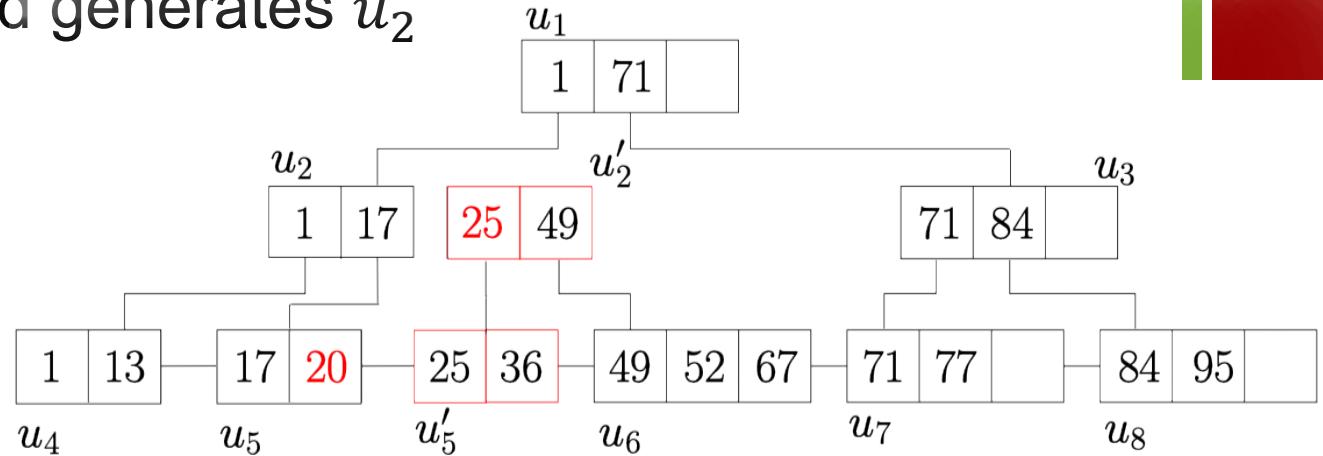


- $r(u'_5) = 25$ is added to u_2 , which overflows

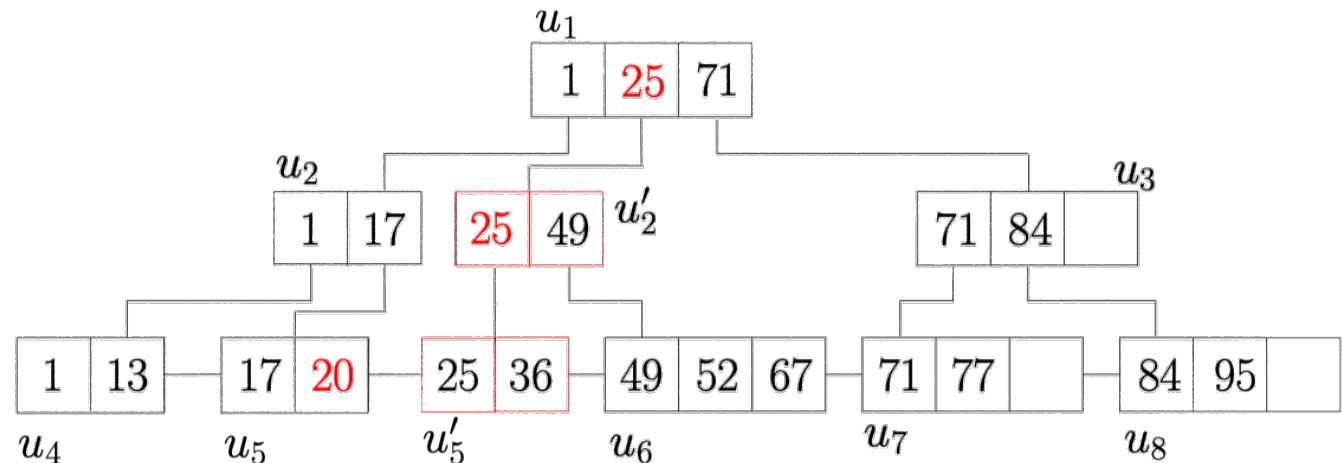


+ B-Tree Insertion

- u_2 splits, and generates u'_2



- $p(u'_2)$ is added to the root, done!



+ B-Tree Deletion

- To delete a value x from a B-Tree

1. Find the leaf node u that contains x , remove x from u
 2. If x is the smallest element in u , adjust the routing elements in the ancestors of u appropriately
 3. If u is the root or has at least $\frac{B}{2}$ elements, the deletion is complete
- Otherwise, u underflows

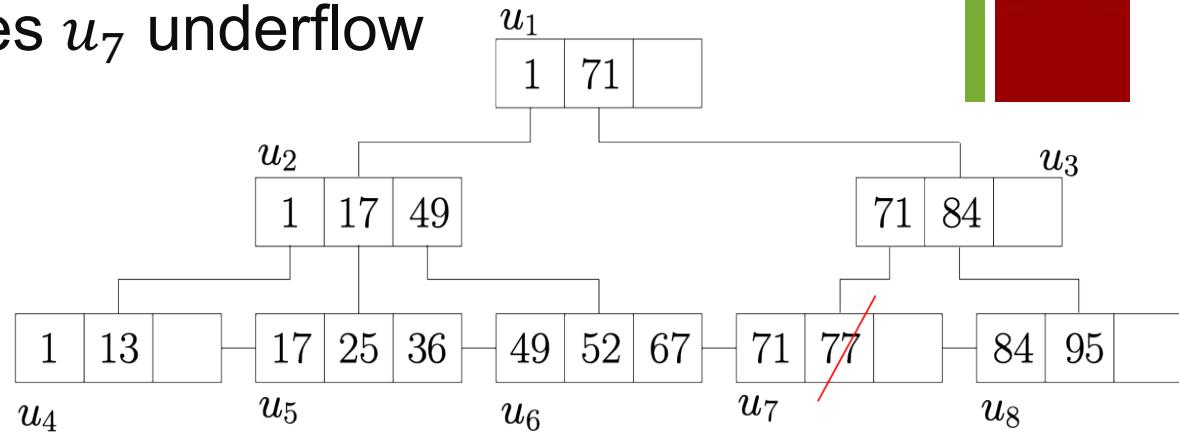
+ B-Tree Deletion

■ Underflow Handling

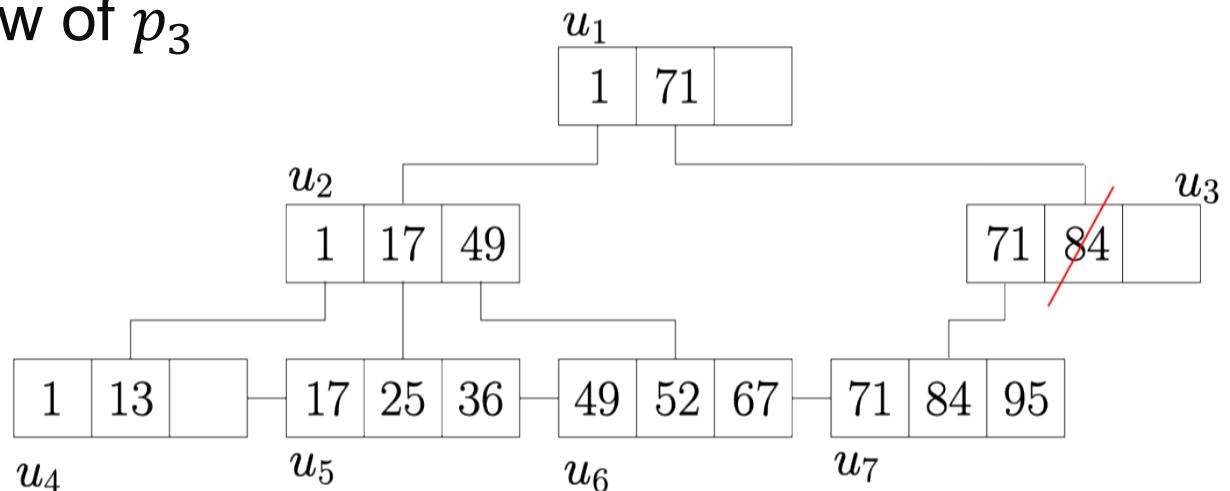
- Let u be the node that underflows, and u' be the right sibling of u (if u does not have a right sibling, set u' to its left sibling, and swap u and u' in the below):
 1. If u and u' contain no more than B elements in total, perform a **merge**
 - Put all the elements in u
 - Remove $r(u')$ from the parent p of u
 - If p underflows, handle it in the same way
 2. Otherwise, perform a **share**
 - Distribute the elements in u and u' equally between them
 - Modify $r(u')$ in p
 - If the root ends up having only one child, remove the root

+ B-Tree Deletion

- Deletion of 77 makes u_7 underflow

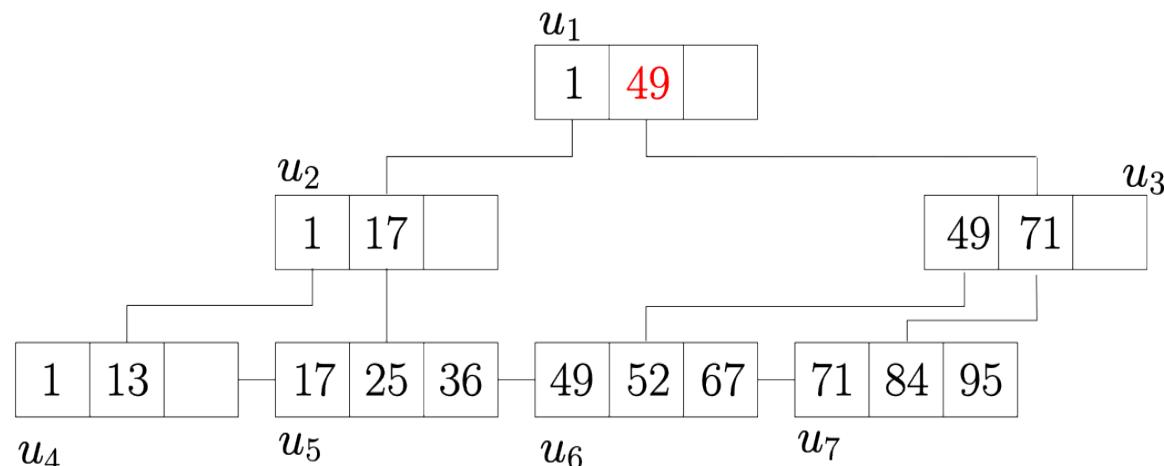
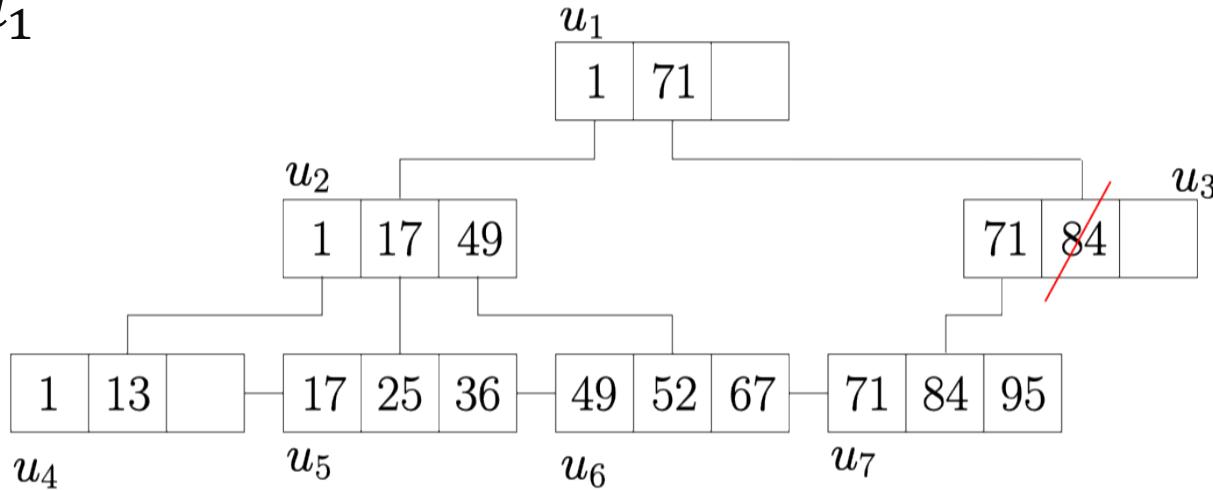


- Merge u_7 and u'_7 , and remove $r(u'_7) = 84$ from p_3 , causing underflow of p_3

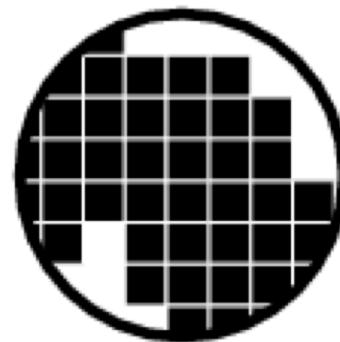
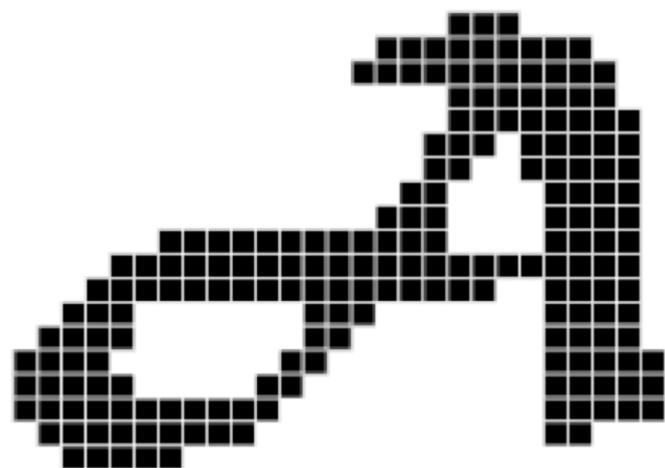


+ B-Tree Deletion

- Perform a share between u_2 and u_3 . Update $r(u_3) = 49$ in u_1



+ Bitmap



+ Bitmap

SID	Name	Gender
6007	Peter	M
6100	Ann	F
6107	Bob	M
6207	Jane	F
6240	Suzy	F
6350	Ben	M
6420	Peter	M
6500	Jenn	F

M	F
1	0
0	1
1	0
0	1
0	1
1	0
1	0
0	1

+ Bitmap

SID	Name	Gender	State	M	F	VIC	QLD	NSW
6007	Peter	M	VIC	1	0	1	0	0
6100	Ann	F	QLD	0	1	0	1	0
6107	Bob	M	NSW	1	0	0	0	1
6207	Jane	F	QLD	0	1	0	1	0
6240	Suzy	F	NSW	0	1	0	0	1
6350	Ben	M	NSW	1	0	0	0	1
6420	Peter	M	QLD	1	0	0	1	0
6500	Jenn	F	VIC	0	1	1	0	0

+ Bitmap

SID	Name	Gender	State
6007	Peter	M	VIC
6100	Ann	F	QLD
6107	Bob	M	NSW
6207	Jane	F	QLD
6240	Suzy	F	NSW
6350	Ben	M	NSW
6420	Peter	M	QLD
6500	Jenn	F	VIC

QLD
0
1
0
1
0
0
1
0

- **SELECT * FROM Students S WHERE S.state = “QLD”**

- Return the Row_ids of: all “1”s in the “QLD” bitmap
 - Rows: 2, 4, 7

+ Bitmap

SID	Name	Gender	State	QLD	F
6007	Peter	M	VIC	0	0
6100	Ann	F	QLD	1	1
6107	Bob	M	NSW	0	0
6207	Jane	F	QLD	1	1
6240	Suzy	F	NSW	0	1
6350	Ben	M	NSW	0	0
6420	Peter	M	QLD	1	0
6500	Jenn	F	VIC	0	1

- SELECT * FROM Students S WHERE S.state = “QLD” and S.Gender = “F”
- Return the Row_ids of: all “1”s in the intersection of “F” and “QLD” bitmaps
 - Rows: 2, 4

+ Bitmap Size

- Size of each bitmap (in bits) is equal to the number of rows in the relation
- Number of bitmaps for a field is equal to the number of distinct values of that field
- Total space needed to index one field (in bits)
 - = number of distinct values × number of rows
- Whereas, file size (in bits)
 - = record size in bits × number of rows
- In general, bitmap indexes are space-efficient

+ Bitmap Limitations

- Not good for data that is modified regularly
 - Updates will require modifying all the associated bitmap indexes
- Used in warehouse data sets which are large and are not updated frequently
 - Mainly for Online Analytical Processing (OLAP)
- How to use it to index the spatial data?

+ Data Access Methods

- One Dimensional
 - Hashing and B-Trees
- Line Data
 - Interval Tree, Segment Tree
- Point Data
 - Hashing: GRID and EXCELL
 - Hierarchical
 - Quadtree: Point and Region Quadtrees
 - kd-Tree
 - Z-values and B-tree
- Polygon Data
 - Transformation: End point mapping and z-values
 - Overlapping: R-tree and R*-tree
 - Clipping: R⁺-tree

+ Line Data

- Arbitrary direction and shape
 - Use MBR and treat like polygons
 - Treated as trajectory data
- Straight line segments with perpendicular directions
 - Align parallel the axis
 - Interval Tree
- Straight line segments with different directions
 - Use Point Index techniques to index the two end points separately
 - Segment Tree

+ 2D Range Tree

■ 1D Range Tree

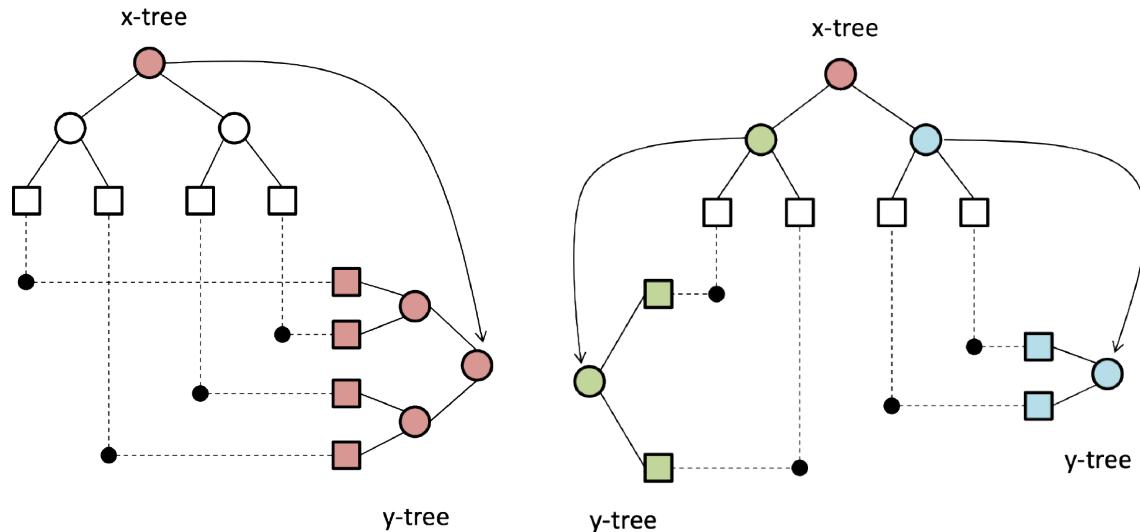
- Leaf nodes are linked in sorted order using a doubly linked list

■ Binary Tree of Binary Tree

- One Binary Tree in X (x-Tree)
- One Binary Tree in Y for each node in the X-Tree

■ Complexity

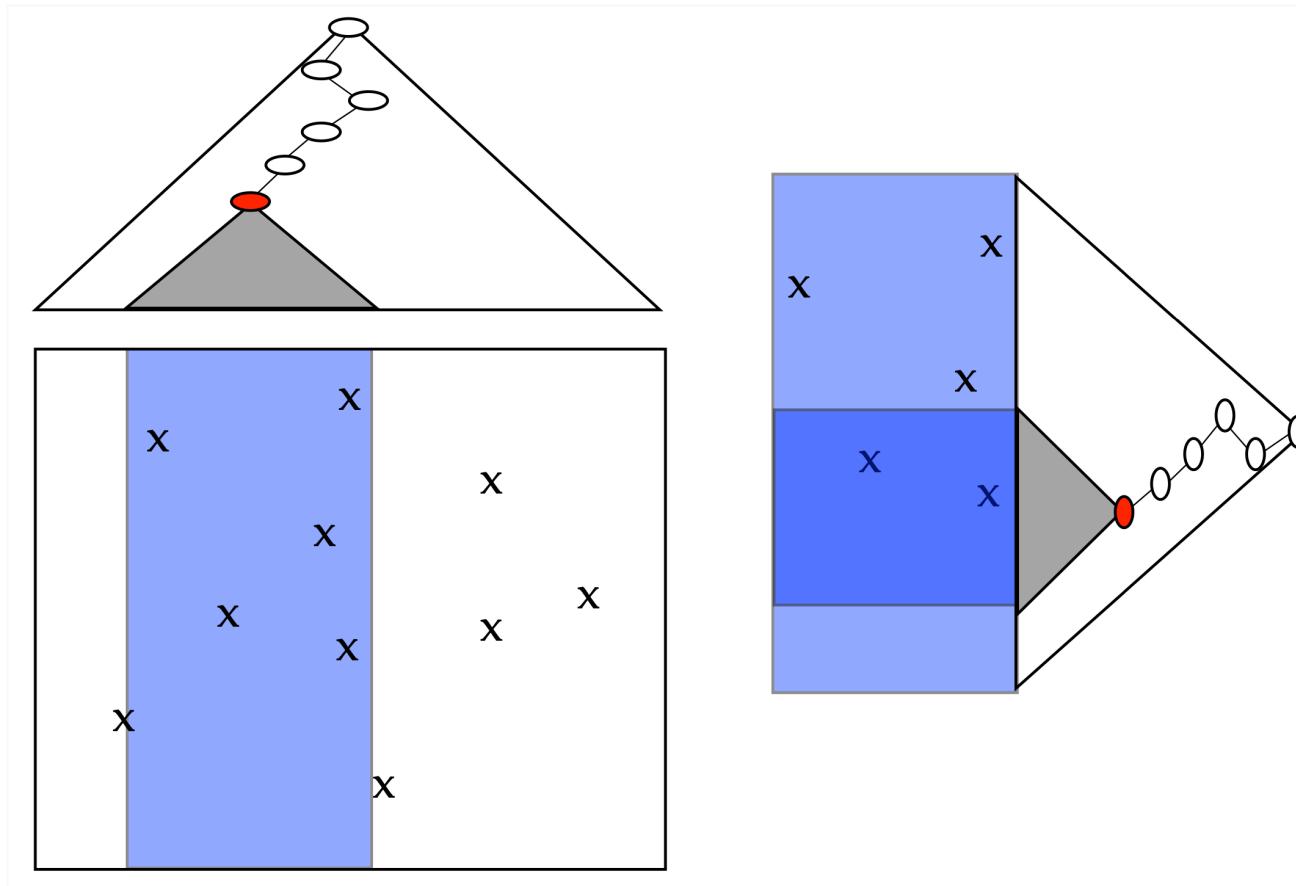
- Space: $O(n \log n)$
- Time: $O(n \log n)$



+ 2D Range Tree Search

- Range Query $[x_1, x_2] \times [y_1, y_2]$

- $O(\log^2 n + k)$ complexity

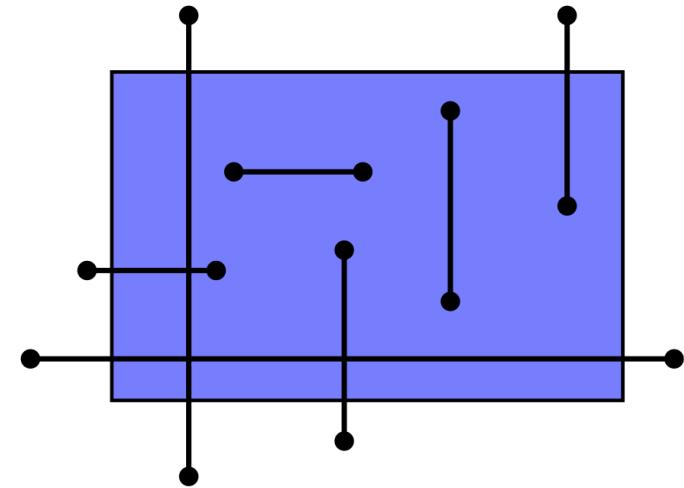
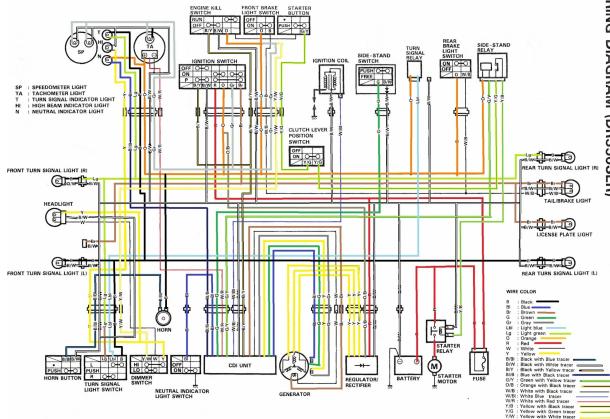


+ Interval Tree

■ Axis-Aligned segments

■ Range Query

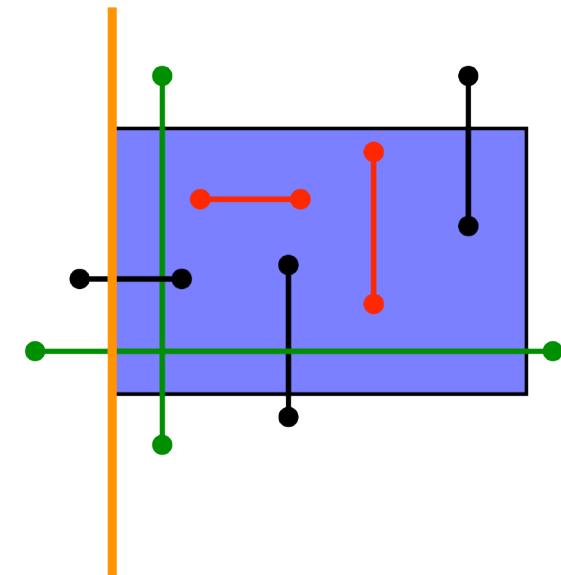
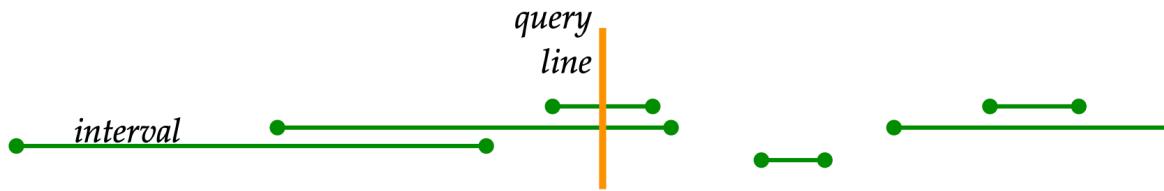
- Return all segments that have any part of them inside the rectangle
- Wiring Diagrams, Genes on genomes



+ Interval Tree

1. Segments with at least one endpoint in the rectangle
 - Black lines and Red lines
 - 2D Range Tree
 - Any point index structures

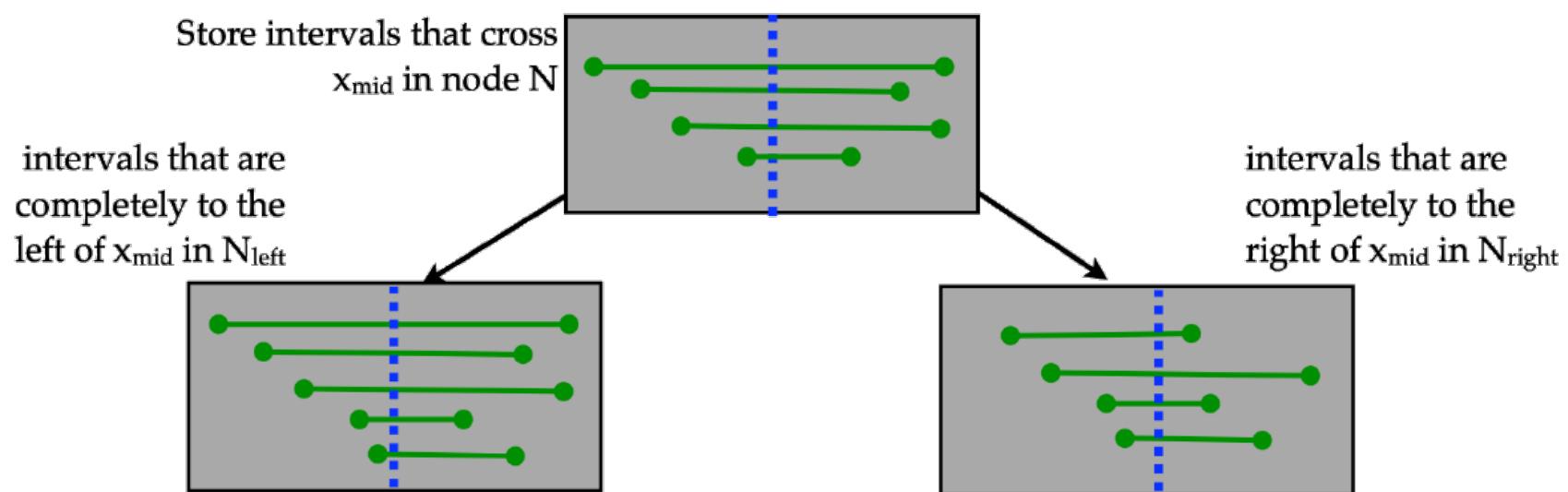
2. Segments with no endpoints in the range
 - Green lines
 - Consider just horizontal segments
 - They must cross a vertical side of the region
 - Subproblem
 - Given a vertical line, find segments that it crosses
 - Y is irrelevant for this subproblem



+ Interval Tree Construction

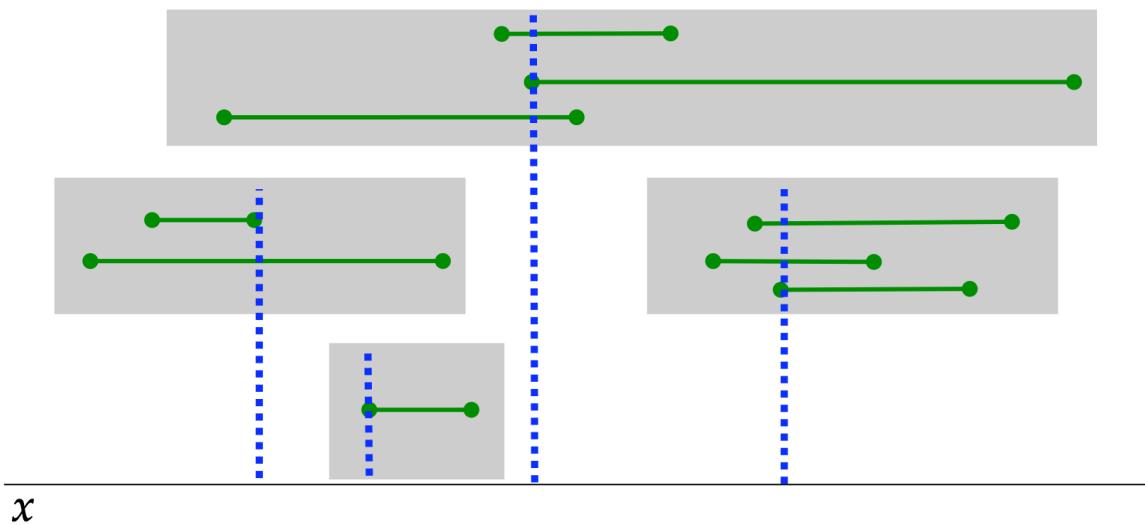
- Recursively build tree on intervals as follows:

- Sort $2n$ endpoints
- x_{mid} is the median point
 - Store the intervals crosses x_{mid}
 - Left child: intervals completely smaller than x_{mid}
 - Right end point smaller than x_{mid}
 - Right child: intervals completely larger than x_{mid}



+ Interval Tree Construction

- Store x_{mid} with each node
- Approximately balanced because by choosing the median, we split the set of end points up in half each time
 - Height is $O(\log n)$
- $O(n)$ Storage
 - Each interval stored once
 - Fewer than n nodes (each node contains at least one interval)
- $O(n \log n)$ Construction time

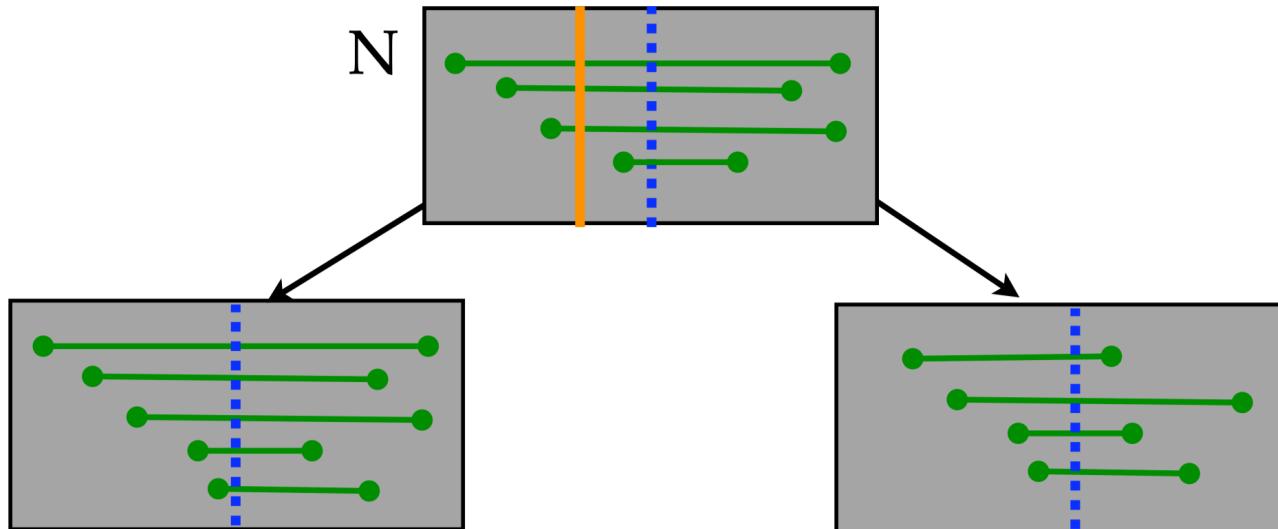


+ Interval Tree Search

■ Query x_q : a vertical line

- Suppose we are at node N

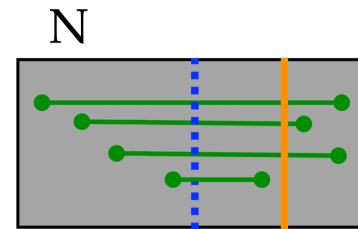
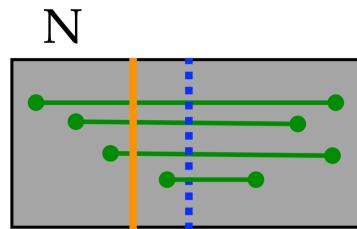
- If $x_q < x_{med}$, skip the right subtree
- If $x_q \geq x_{med}$, skip the left subtree
- Always have to search the intervals stored at current node
- $\log N$ nodes to check



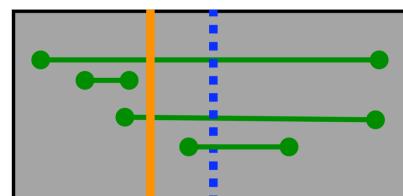
+ Interval Tree Search

■ Current Node Search

- Store each interval in two sorted list stored at node
 - List L sorted by increasing left endpoint
 - List R sorted by decreasing right endpoint
- Search List depending on which side of x_{med} the query is on
 - If $x_q < x_{med}$, search L , output all until a left endpoint $> x_q$
 - If $x_q \geq x_{med}$, search R , output all until a right endpoint $< x_q$



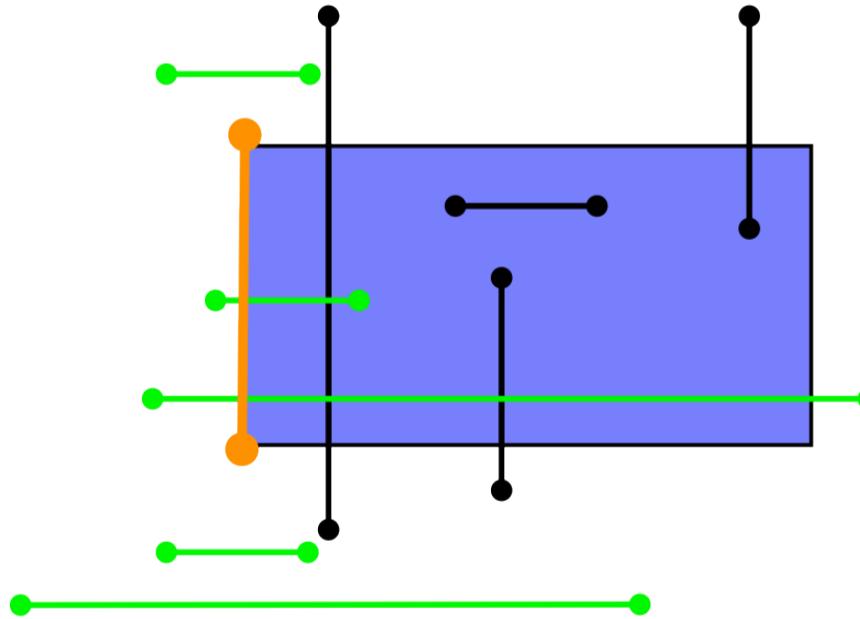
- It works because we know each segment intersects x_{med}



+ Interval Tree

■ Vertical Segment Search

- Instead of infinite vertical lines, we have finite segments as query
 - Somehow have to remove the ones that don't satisfy the y -constraints
 - Interval trees \rightarrow candidates
 - How to remove the ones that don't satisfy the y -constraints?

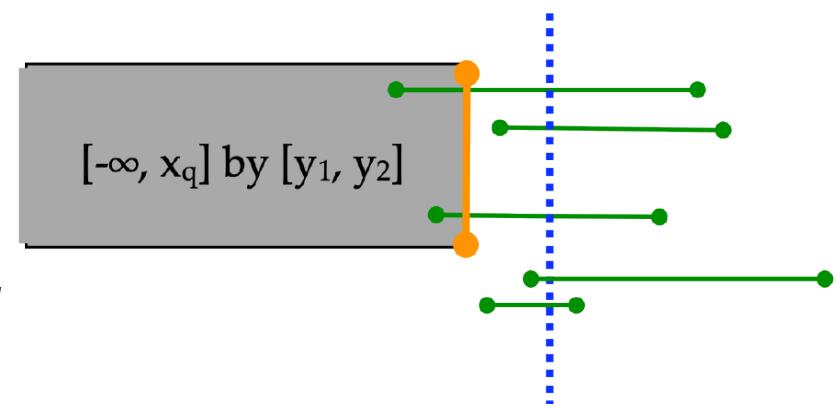


+ Interval Tree

■ Vertical Segment Search

- Use 2D Range Tree instead of sorted lists to hold segments at each node
 - Execute a range query on a semi-infinite range on the 2d-range tree on the end points stored at each node of the interval tree.
 - Two range trees R_{left} and R_{right} that store points to the left and to the right of x_{mid}

- Time complexity
 - $O(\log^3 n + k)$ with two trees
 - $O(\log^2 n + k)$ with *fractional cascading*
- Space complexity
 - Each interval stored at one node
 - Total space for the range trees is $O(n \log n)$



+ Priority Search Tree

- Space Complexity $O(n)$

- 1-Side Range Query

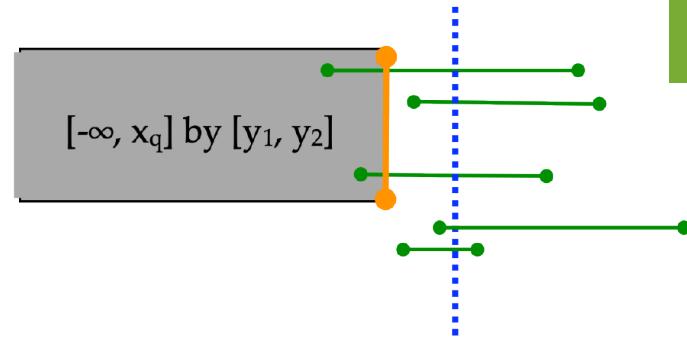
- $[-\infty, x], [y_1, y_2]$

- Easy in 1D

- Just walk the sorted list from left to right or right to left

- 2D: $x < 20$ and $25 < y < 70$

- Somehow integrate the the information about y -coordinate in the structure



+ Priority Search Tree

■ 1-Side Range Query

- $[-\infty, x], [y_1, y_2]$
- Find the highest/lowest value of x
 - Heap
 - Freedom in how to partition the set into 2 subsets

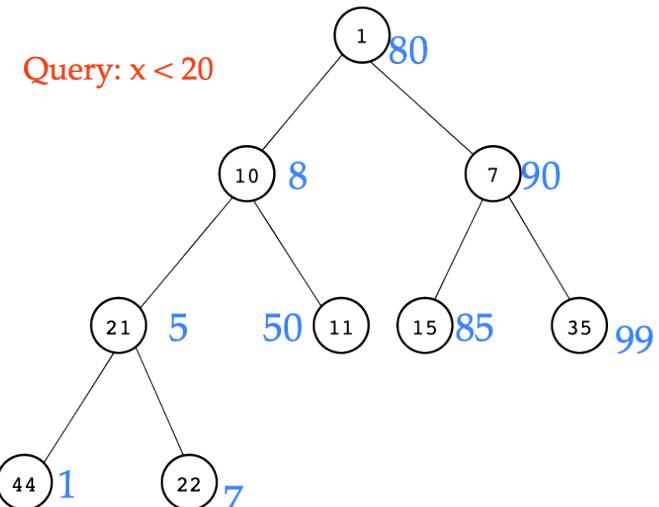
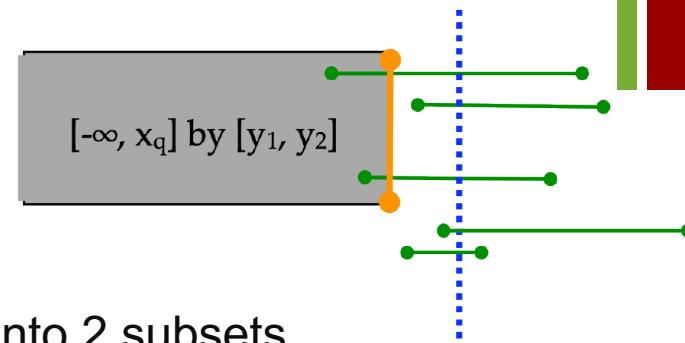
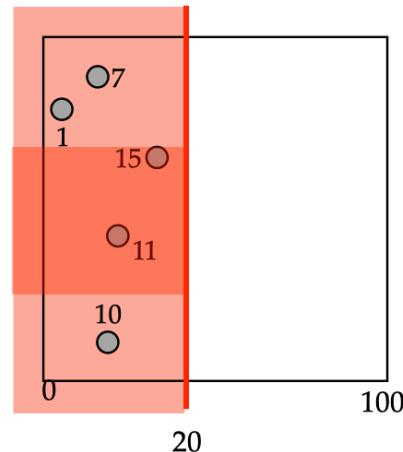
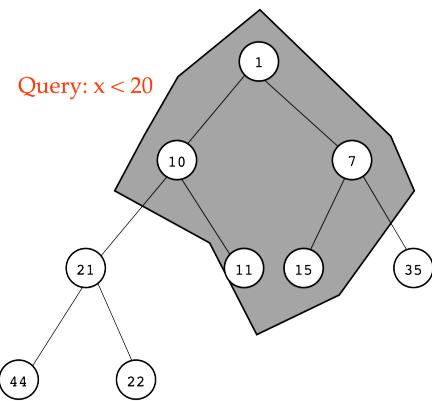
■ Find the range of y

- Binary Search Tree

■ Combine them!

■ Priority Search Tree

Heap on x -values:



+ Priority Search Tree

■ Construction

- Given a set of points P
 - p_{minx} is the one with the smallest x
 - y_{mid} : median of the y -coordinates of $P \setminus \{p_{minx}\}$
- Store point p_{minx} and y_{mid} in a node N
 - y_{mid} does not need to correspond to point p_{minx}

■ Split the points by y -coordinate

- $P_{left} = \{p \in P \setminus \{p_{minx}\} \text{ and } p.y \leq y_{mid}\}$
- $P_{right} = \{p \in P \setminus \{p_{minx}\} \text{ and } p.y > y_{mid}\}$

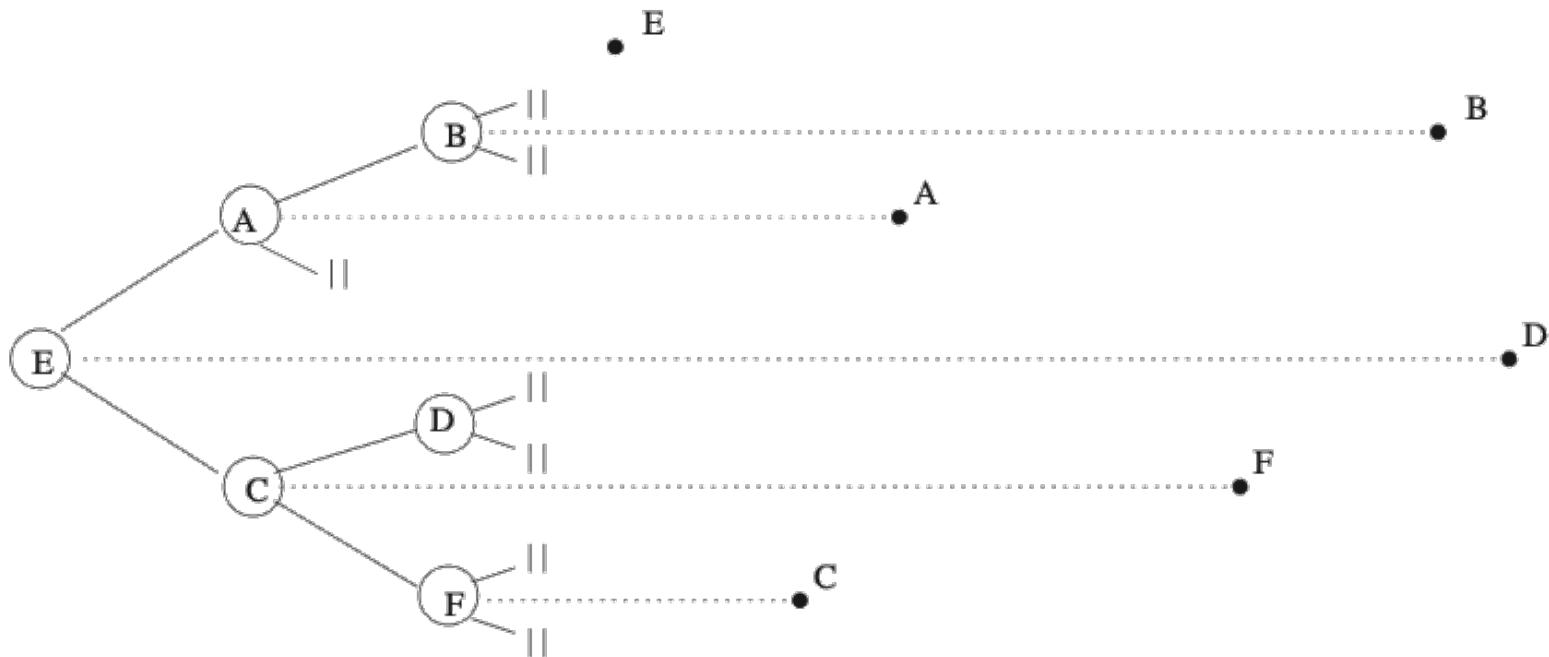
■ Recursively build left/right subtrees

- $O(n \log n)$

+ Priority Search Tree

■ Construction

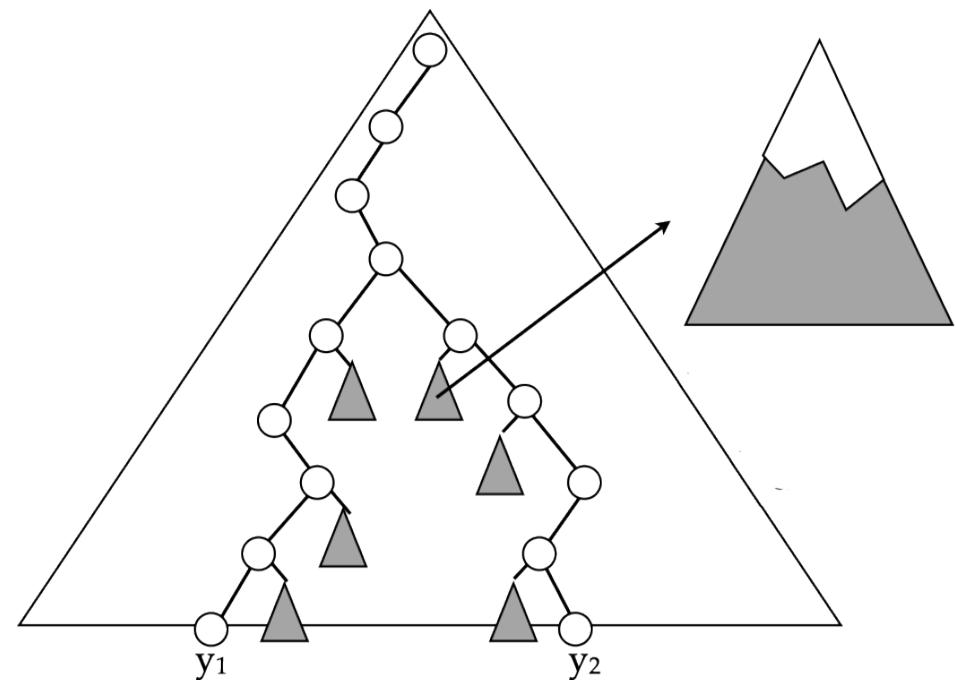
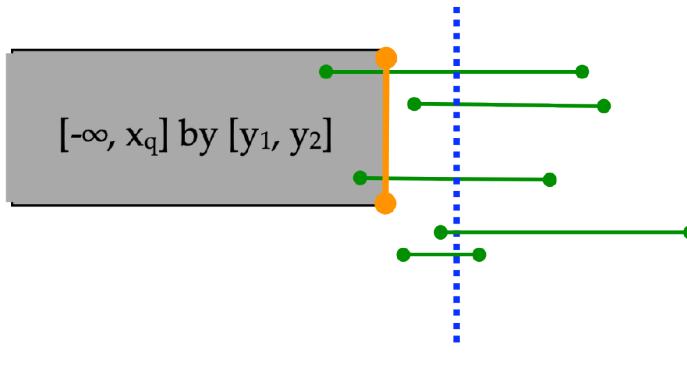
- Binary tree on y
- Each sub-tree is a heap on x



+ Priority Search Tree

■ Search

- Range search for y can be done in a 1-d range tree
 - Points in the grey sub-trees all satisfy the y -constraints
 - Points along the search paths may or may not
 - Must check them
- Each result sub-tree is a heap
 - Output the tops of the heap
- $O(\log n + k)$



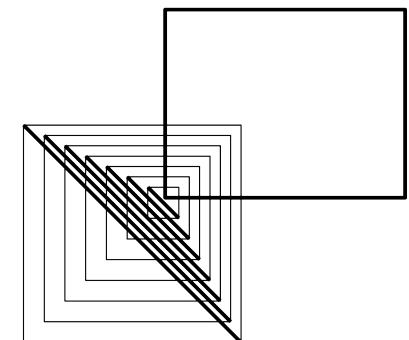
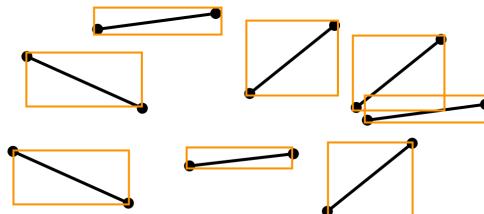
+ Segment Tree

■ Arbitrarily Oriented Segments

- No longer assume they are parallel to the axes
- Store the bounding boxes of each segment as a collection of 4 axis-parallel segments?
 - How to handle range queries on these kinds of segments
 - If a vertical line crosses a segment, it crosses its MBR
 - It may be that a vertical segment crosses a bounding box but doesn't cross the segment

■ Vertical Segment Stabbing Query

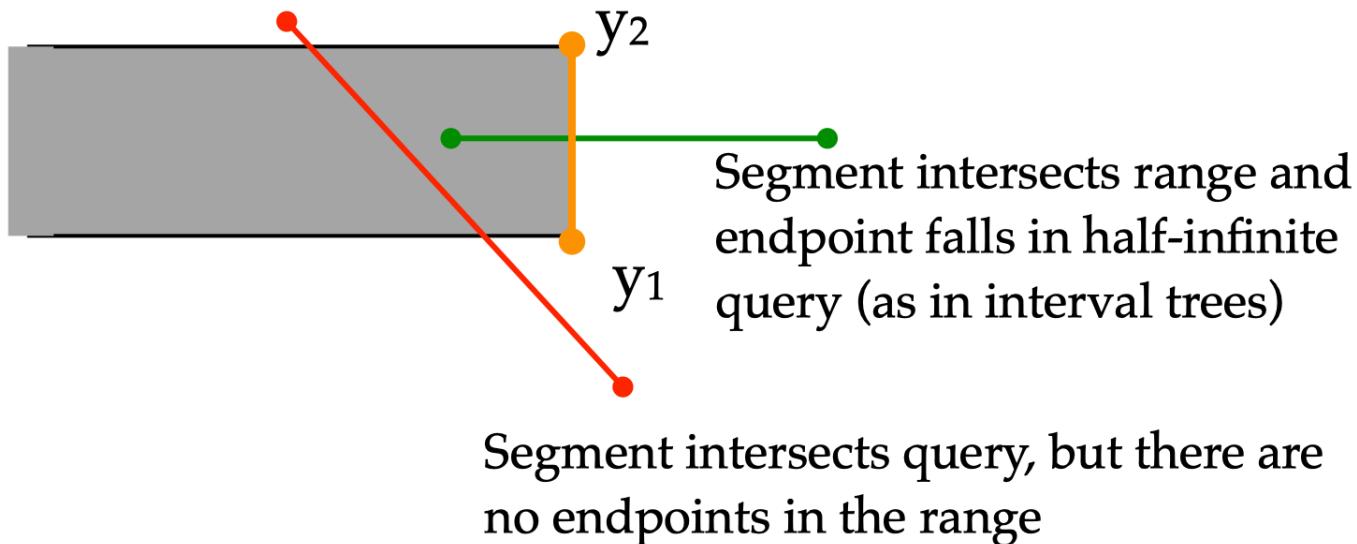
- Return all segments that intersect a vertical query segment
- Assume segments don't cross



+ Segment Tree

■ Interval Tree?

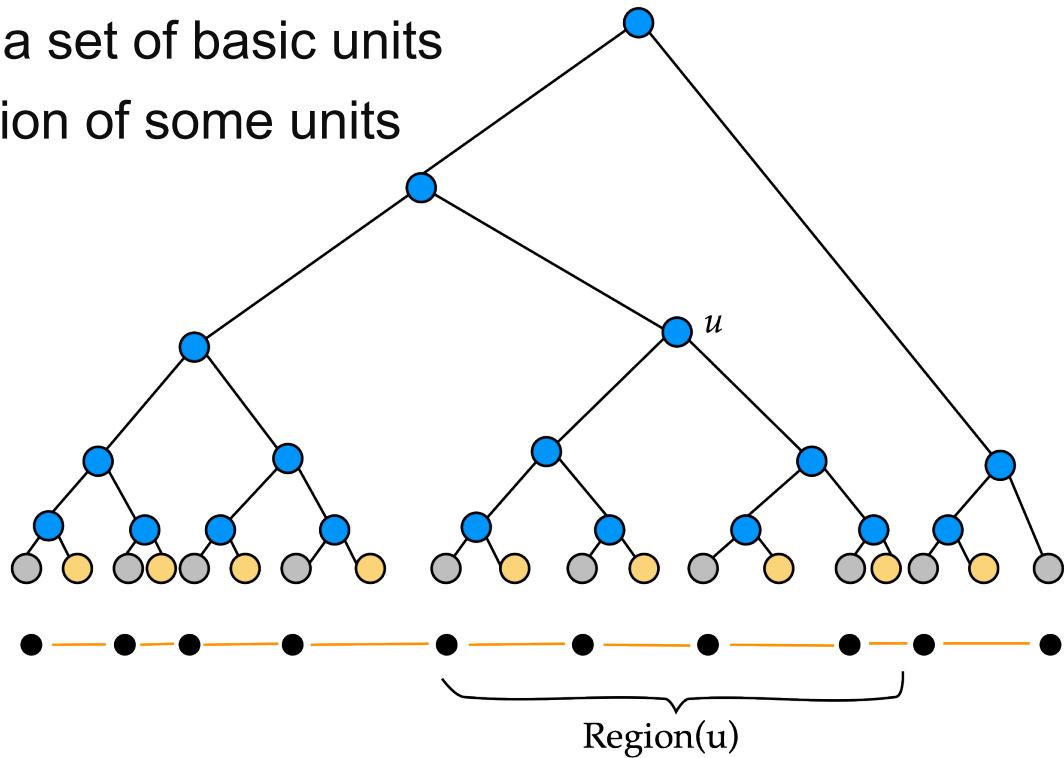
- No longer true that a query like $[-\infty, x]$ by $[y_1, y_2]$ will find the endpoints of satisfying segments



+ Segment Tree

■ Binary Search Tree

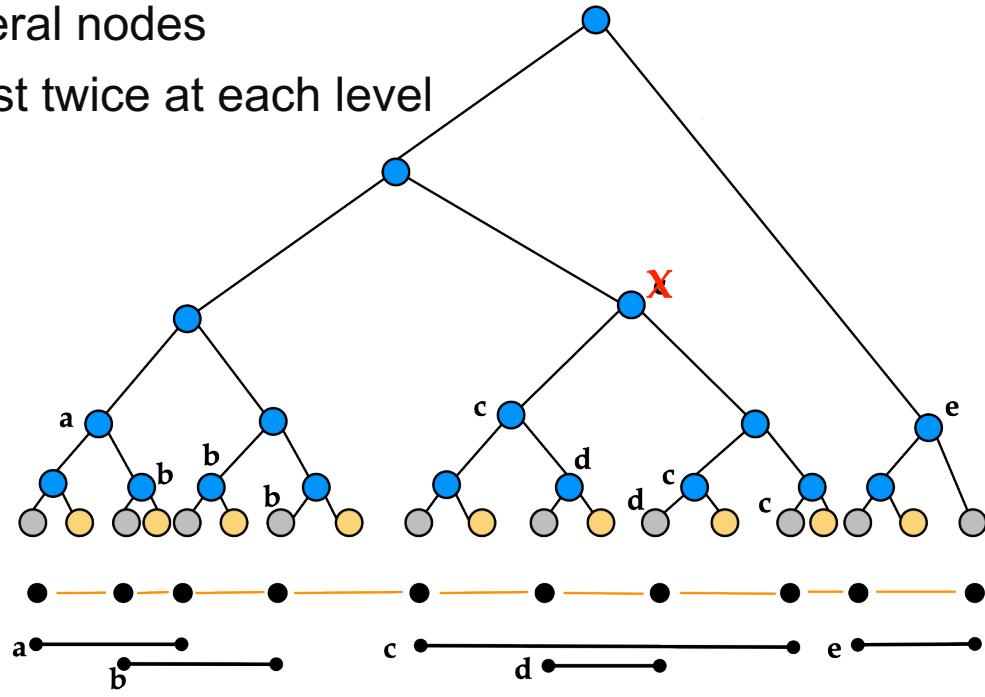
- Recursively partitions 1D space
- Each leaf stores an elementary region
- For internal node u , $\text{Region}(u)$ is the union of elementary regions in the subtree rooted at u
- The space is divided into a set of basic units
- Every segments is the union of some units



+ Segment Tree

■ Store the segments in the Binary Search Tree

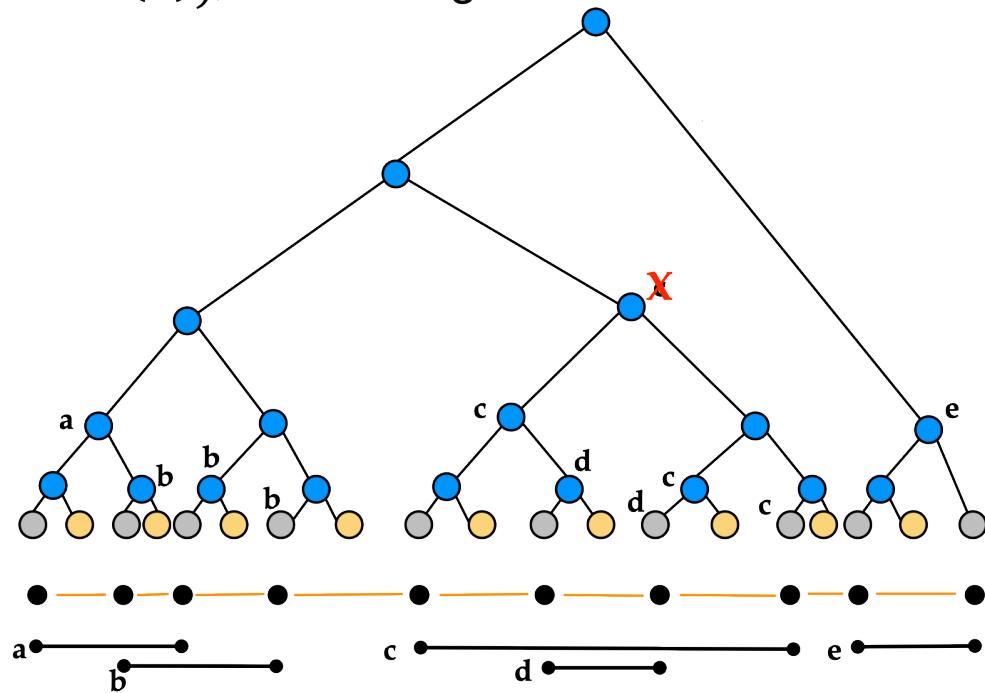
- Store the segment s at any node u that
 - Segment covers the entire $\text{Region}(u)$
 - It doesn't cover the entire $\text{Region}(\text{Parent}(u))$
 - Propagate segments up until we reach a node whose Region is not a subset of the segment
- Segments may be stored at several nodes
 - Each segment is stored at most twice at each level



+ Segment Tree

■ Vertical Line Query

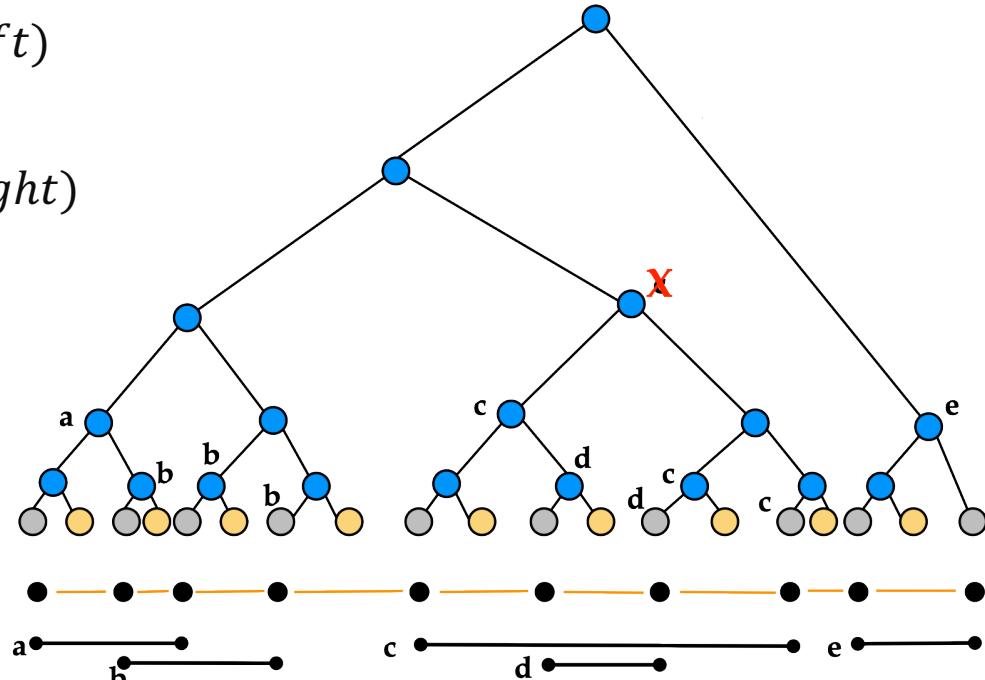
- Find the segment that intersect a given x
 - Binary search
 - Output every segment stored at the current node
 - If x falls into $\text{Region}(\text{leftchild}(u))$, take the left branch
 - Else if x falls into $\text{Region}(\text{rightchild}(u))$, take the right branch



+ Segment Tree

■ Construction

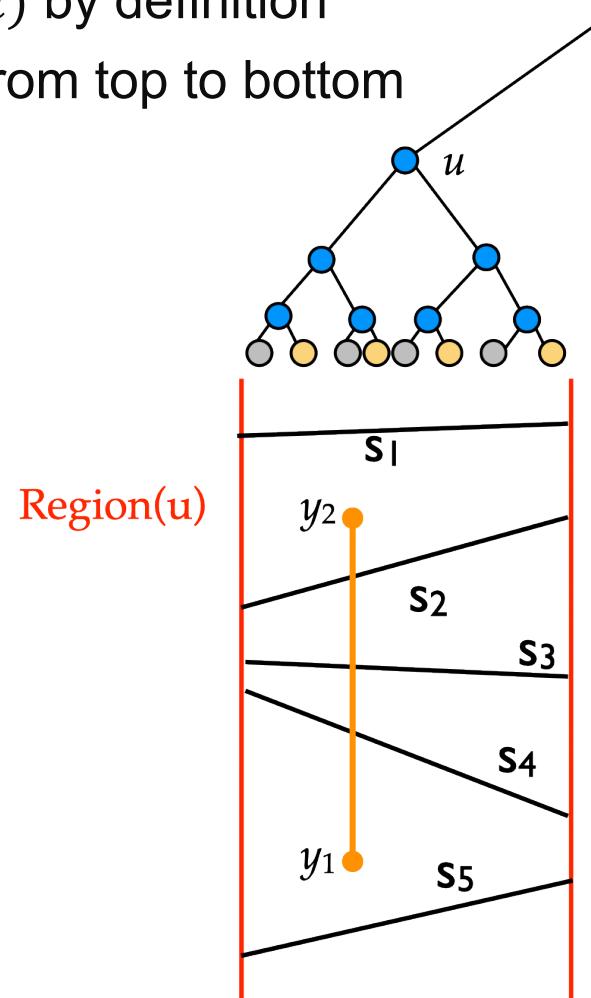
- Sort the segments
- Break into units
- Build balanced BST on them
- Insert the segment (x_1, x_2, u)
 - If $\text{Region}(u) \subset [x_1, x_2]$, $u.insert(x_1, x_2)$
 - If $[x_1, x_2]$ intersects $\text{Region}(u.left)$
 - Insert($x_1, x_2, u.left$)
 - If $[x_1, x_2]$ intersects $\text{Region}(u.right)$
 - Insert($x_1, x_2, u.right$)



+ Segment Tree

■ Vertical Segment Stabbing Query

- Segments stored at u all span $\text{Region}(u)$ by definition
- The segments can be linearly ordered from top to bottom
- Index them in binary search tree
- Range query between y_1 and y_2
- $O(\log^2 n + k)$



+ Data Access Methods

- One Dimensional
 - Hashing and B-Trees
- Line Data
 - Interval Tree, Segment Tree
- Point Data
 - Hashing: GRID and EXCELL
 - Hierarchical
 - Quadtree: Point and Region Quadtrees
 - kd-Tree
 - Z-values and B-tree
- Polygon Data
 - Transformation: End point mapping and z-values
 - Overlapping: R-tree and R*-tree
 - Clipping: R⁺-tree

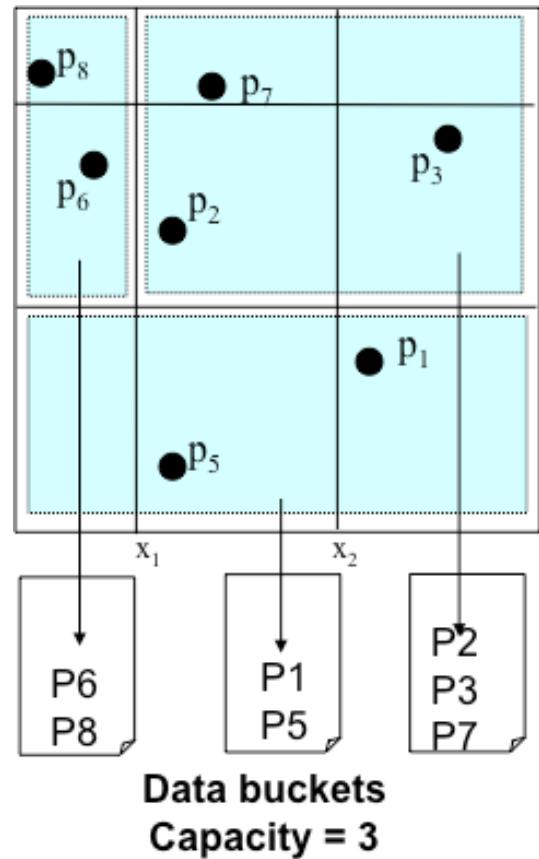
+ Grid File

■ Basic idea

- Superimpose a k -dimensional grid on the space
 - Only scan the data in a grid
 - Essentially, a 2D hash function
- Cells can be of varying sizes
- Cell-to-bucket mapping: many-to-1
 - *What about 1-to-many?*
- The grid definition (scales) is kept in memory
- The grid directory is kept on disk

■ Motivation

- Fixed-grid not suitable for non-uniformly distributed data



+ Grid File

- To answer a point query:
 - Use the scales to locate the cell
 - Read the cell from disk
 - The loaded cell contains a reference (pointer) to data bucket
 - Read data bucket
 - On average two disk accesses
- To answer a range query:
 - Examine all cells that overlap the search region
 - Read the corresponding data buckets(s)

+ Grid File

- To insert a point:

- Search (point query) the matching cell and data bucket
- If there is sufficient space, insert into data bucket
- Else: add a vertical or horizontal line to split, if necessary, and move data accordingly

- To delete a point:

- Search ...
- Merge if necessary

- Problems:

- Have to remember all the definitions of the grids
- Why not uniformly?

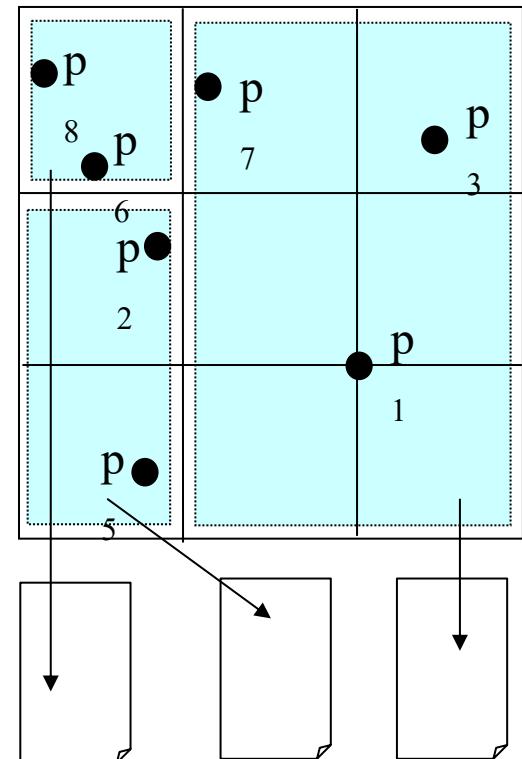
■ EXtendible CELL

■ Motivation

- Fixed grid is easier to manage and more efficient to use

■ Basic Ideas

- All cells are of the same size
 - No need to keep grid definition (scales) in memory
 - But it's still necessary to remember how to map grid cells to buckets



Data buckets
Capacity = 3

- Search:

- Efficient for point and range queries

- To insert a point:

- Search (point query) the matching cell and data bucket
 - If there is sufficient space, insert into data bucket
 - Else: add a vertical or horizontal line to split, if necessary, and move data accordingly
 - When split, the number of cells is doubled

+ Data Access Methods

- One Dimensional
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