ENGG1300
Introduction to Electrical Systems
Week 3

Lecturer:

Dr Philip Terrill

Recording of this Presentation

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For further information:

- PPL 3.20.06 Recording of Teaching at UQ
- UQ website: https://my.uq.edu.au/information-and-services/information-technology/software-and-web-apps/software-uq/zoom

Last Week

- Nodal Analysis
  - Solves first for node voltages, then currents (systematised application of KCL at each node of the circuit to derive simultaneous equations in terms of node voltages)
- Mesh Analysis
  - Solves first for mesh currents, then for branch currents and voltages (systematised application of KVL around each mesh of the circuit to derive simultaneous equations in terms of currents)
- Continued to use lab equipment or circuit simulations to test how well our models describe the circuit in the “real world”.

Limitations of Week 1 and 2 Analysis

- We clearly need to be able to model systems more complex than resistive networks with a single source
- For complex circuits, with multiple sources, we need more structured ways to solve a circuit:
  - **Node** and **Mesh (as per last week)**
  - BUT might there be easier ways to solve for some circuits?
  - Note that in most cases it is the effect of *multiple* sources that make analysis more complicated – Is there a way we can simplify this analysis somehow (rather than solving one complicated problem, solve a few simpler ones)?

i.e. Can we break a big problem down into multiple smaller, easier to solve problems?

This week ahead

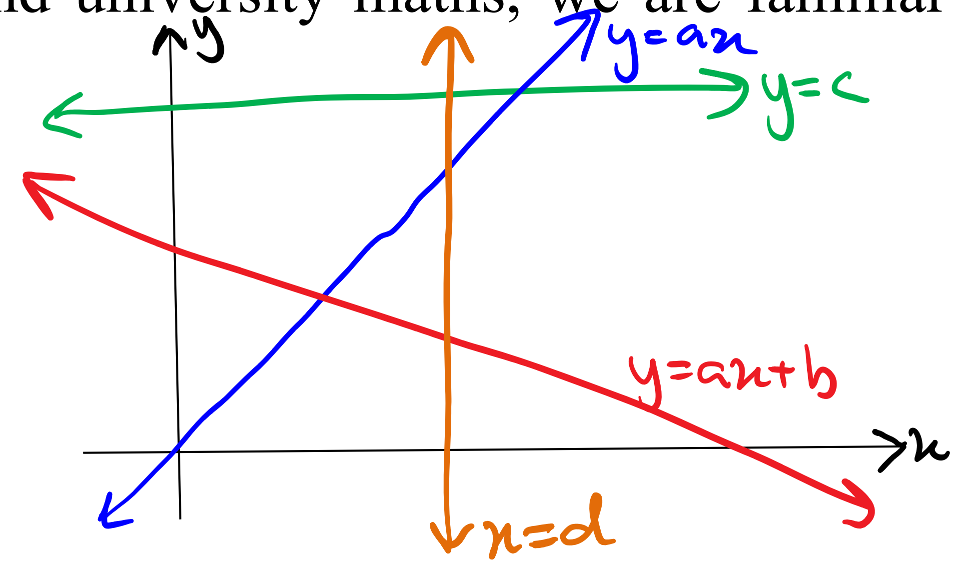
- Further methods to solve DC Circuits
  - Superposition
  - Equivalent Circuits (Norton + Thevenin)
  - Non-linear circuits
- Session 3A – Lab (V) (Superposition, Norton, Thevenin)
  - Labs will also continue to use lab equipment or circuit simulation.
- Session 3B – Laboratory (VI):
  - The LED, continuation of equivalent circuits and Load Lines (last DC lab)

- On-line Quiz 2 due 4pm Today (Monday 8/3/21).
- On-line Quiz 3 due 4pm next Monday 15/3/21.

- Mid-Sem Test: Monday of Week 6: Full details in next weeks lecture. See Course profile for basic details.

Linear Circuits and Systems

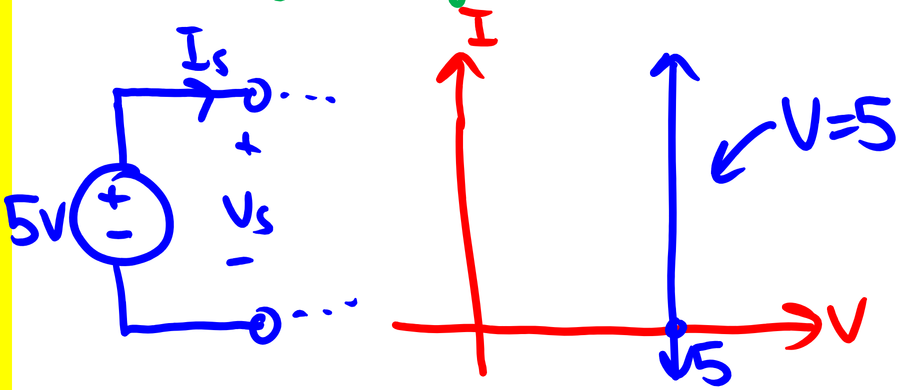
- From high school and university maths, we are familiar with linear equations:
  - $y = c$
  - $y = ax$
  - $y = ax + b$
  - $x = d$



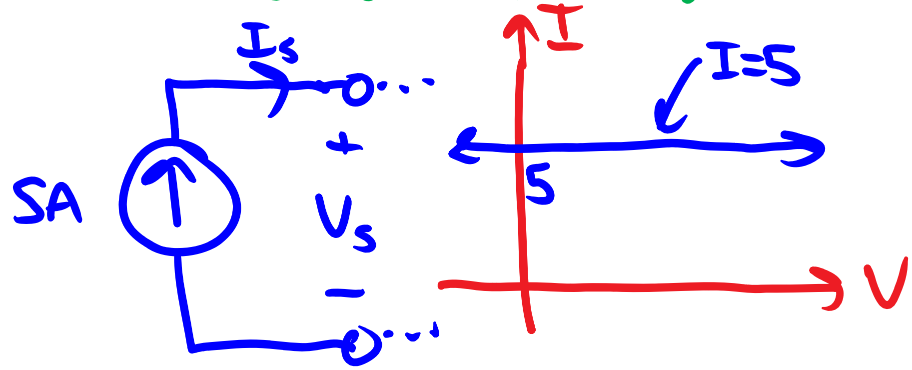
- A *linear system* is a system that we can *model* with linear equations.
  - Mathematically, linear systems have well known solutions and are there for easy to solve.
  - And, they have key properties that allow us to break down a complex problem into multiple smaller problems (superposition)

Review of Components so Far:

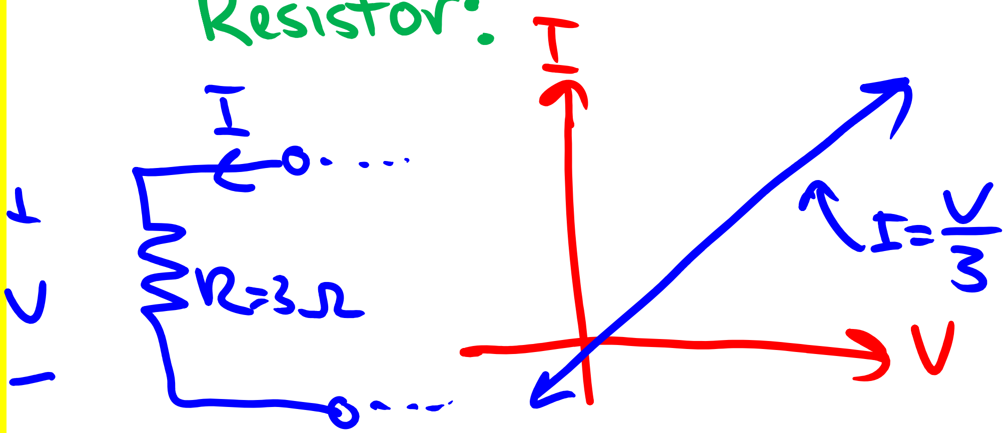
Voltage Source:



Current Source:



Resistor:



- All the components we have used so far are linear.
- Thus, any circuit comprised of resistors, voltage sources and current sources is also linear!
- i.e. The relationship between voltage and current at any component, branch or port of the circuit is linear

Superposition

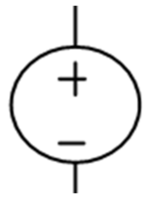
- If the circuit has multiple sources, then we can set all except one to zero, and find the response (branch currents, voltages) for that one source.
  - A zeroed voltage source (0V) is a short circuit,
  - A zeroed current source (0A) is an open circuit.
- "Response to all the sources is the sum of responses to each individual source"
  - This is an important property of linear systems that is exploited in mathematics, physics and across all engineering disciplines.

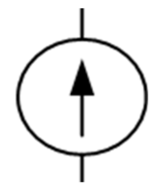
Steps for applying superposition

1. Identify each independent source in the circuit (i.e. ideal voltage sources; ideal current sources).
2. Identify the first source, and set all other sources to zero
   a) Re-draw the circuit so that you can solve it in an intuitive way!
   b) Calculate each node/branch voltage
   c) Calculate each branch current
3. Repeat Step 2 for each source in the circuit (i.e. solve the circuit for each source with all other sources set to zero).
4. Calculate complete branch currents and node/branch voltages by adding the components (calculated in septs 2-3) together
5. Once voltages and currents have been calculated, power dissipated by each component can be calculated
   - Because power is NOT linear, you cannot calculate power from each source and add together

Setting Sources to Zero

- What happens when we "set sources to zero"?
  - Lets think about it intuitively





Voltage Source?

- What happens when we set a voltage source to zero?
- Imagine a voltage source that supplies zero volts (i.e. no potential different across that branch of the circuit)
- This is equivalent to a short circuit

- So when we "zero" a voltage source, we can replace it in the circuit with a short circuit

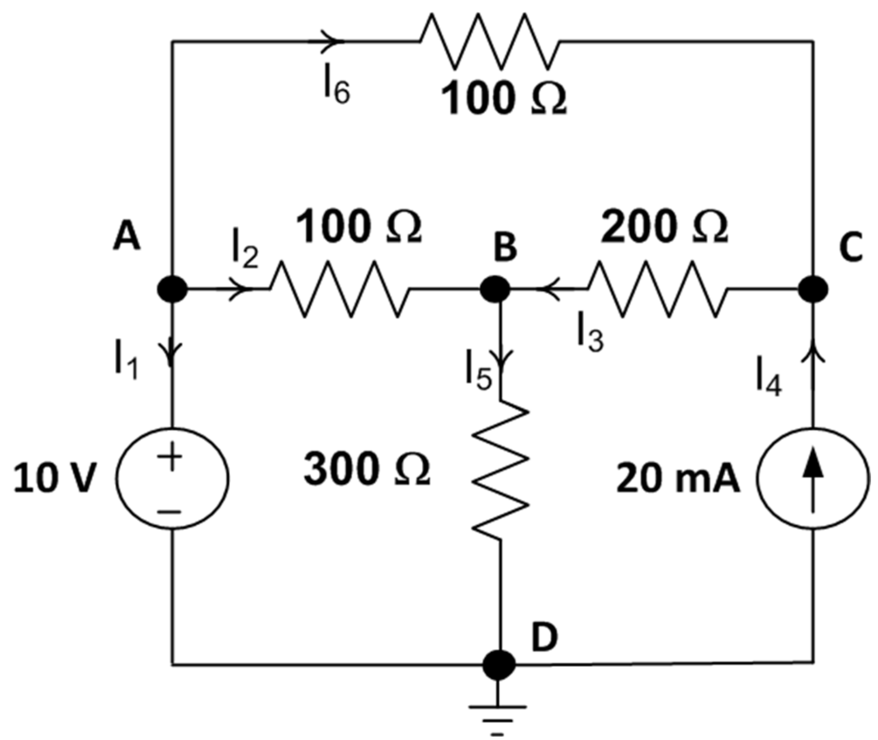
Current Source?

- What happens when we set a current source to zero?
- Imagine a current source that supplies zero amperes (i.e. no current through that branch of the circuit)
- This is equivalent to an open circuit

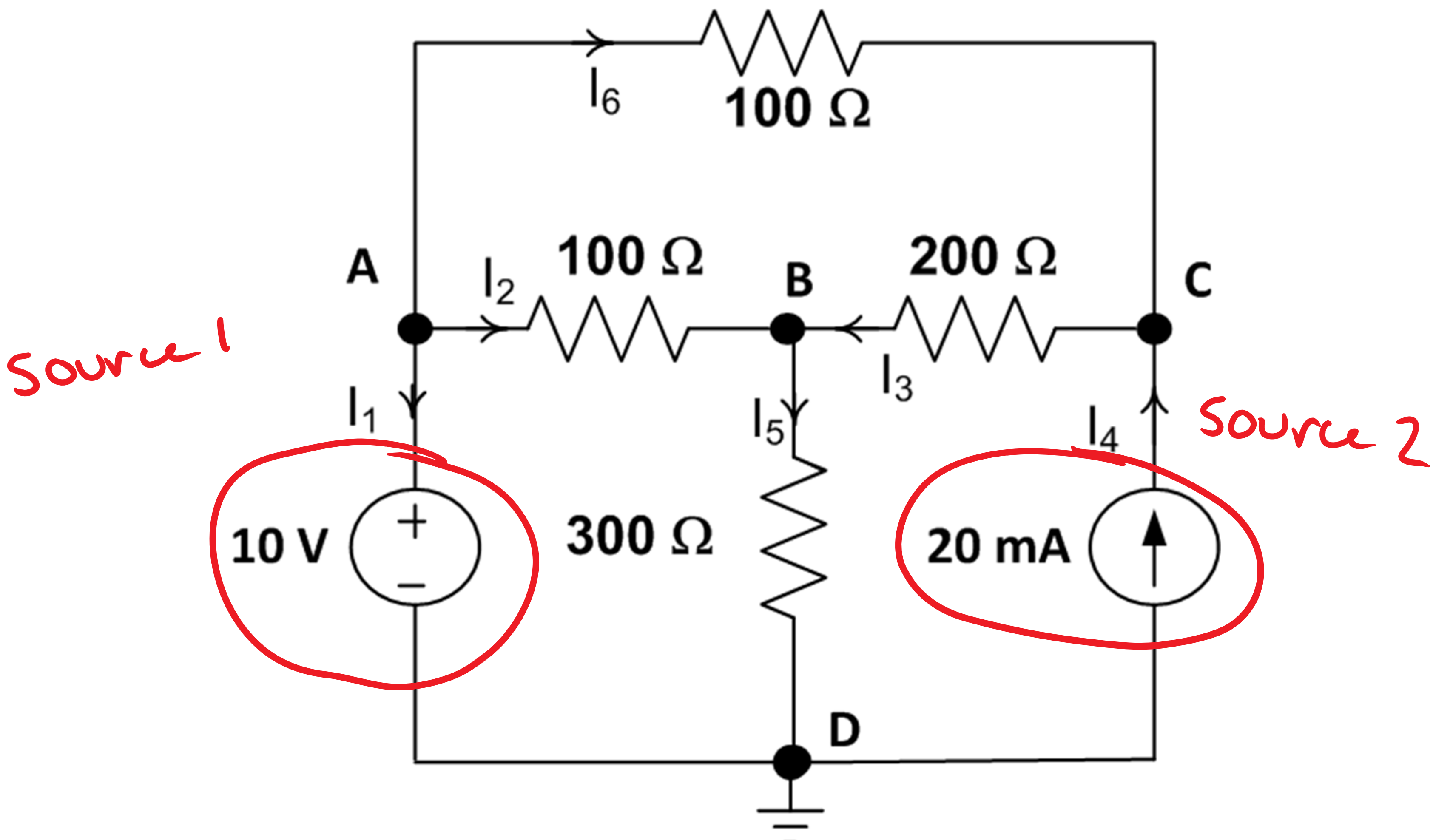
- So when we "zero" a current source, we can replace it in the circuit with an open circuit

Superposition - Example

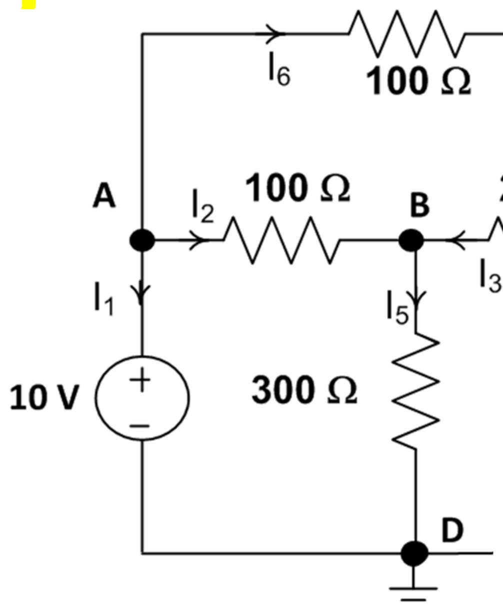
1. Identify each source in the circuit
2. Set current source to zero (open circuit).
   – Calculate: V_{A_1}, V_{B_1}, V_{C_1}, V_{D_1}, I_{1_1}, I_{2_1}, I_{3_1}, I_{4_1}, I_{5_1}, I_{6_1}
3. Set voltage source to zero (short circuit).
   – Calculate: V_{A_2}, V_{B_2}, V_{C_2}, V_{D_2}, I_{1_2}, I_{2_2}, I_{3_2}, I_{4_2}, I_{5_2}, I_{6_2}
4. Calculate actual branch currents and voltages + currents
   – Adding the components together: V_A, V_B, V_C, V_D, I_1, I_2, I_3, I_4, I_5, I_6
5. Once voltages and currents have been calculated, power dissipated by each component can be calculated

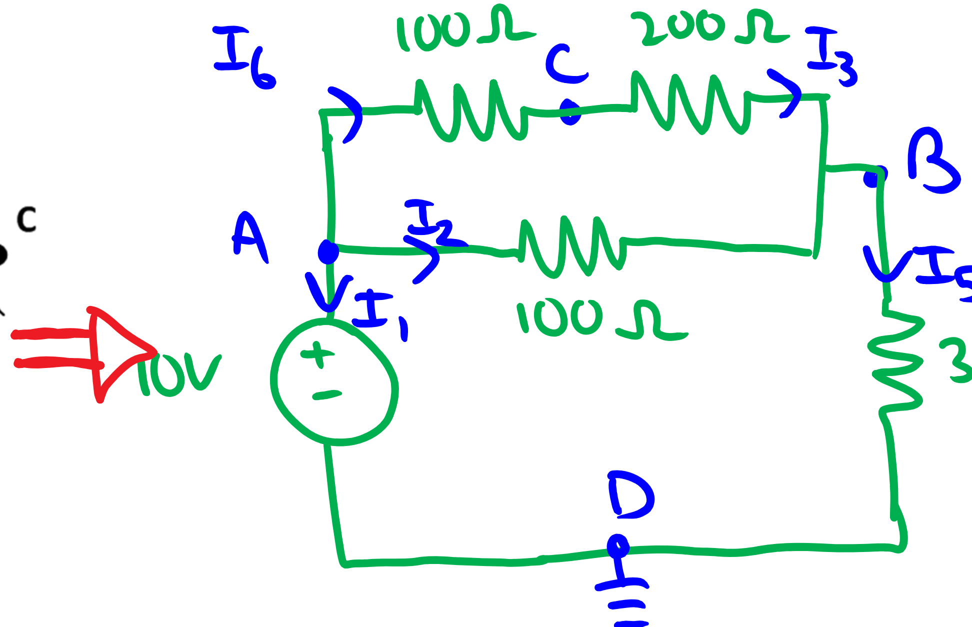


Step 1: Identify Sources



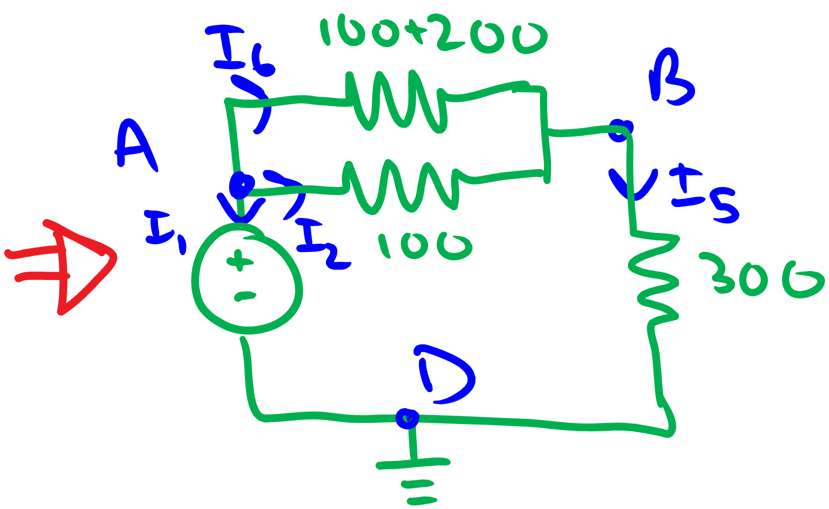
Step 2: Voltage source (zero other sources)

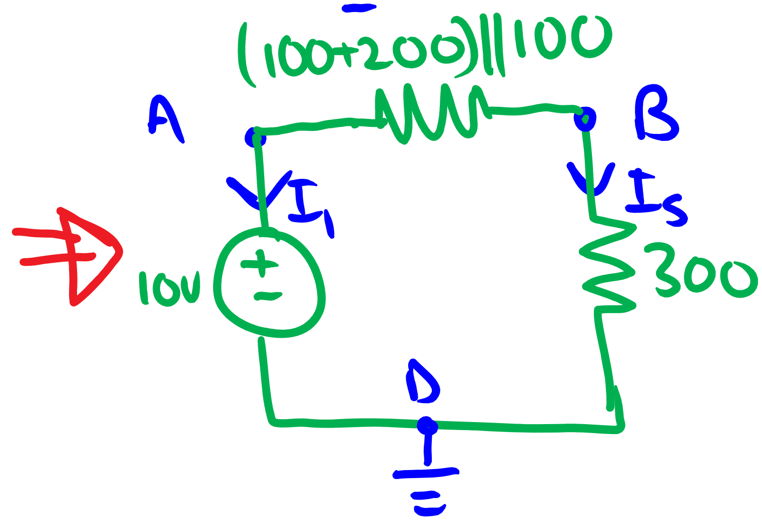


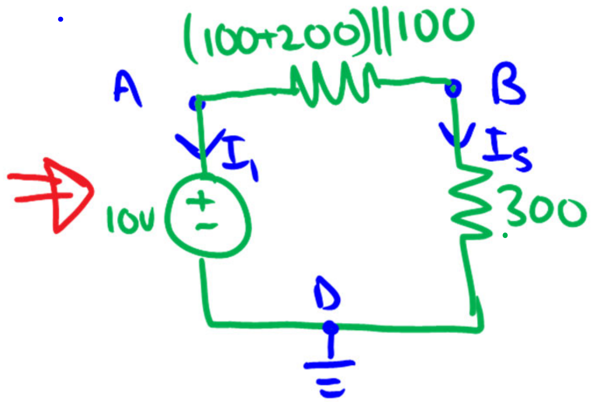


$I_4 = 0$

$I_6 = I_3$



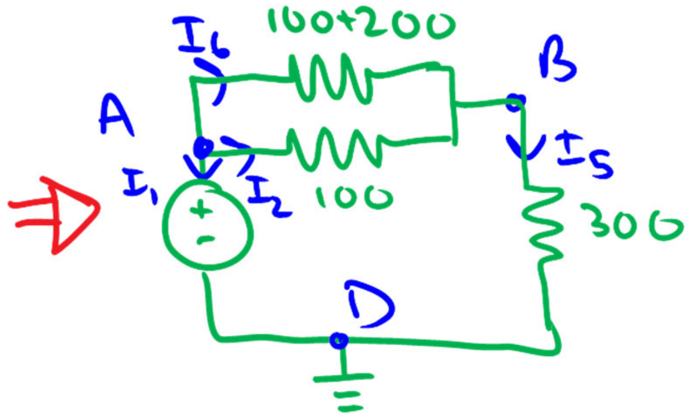




$$V = I \times R \Rightarrow 10 = I_S(300+75)$$

$$\therefore I_S = 0.027A \qquad I_1 = -0.027A$$

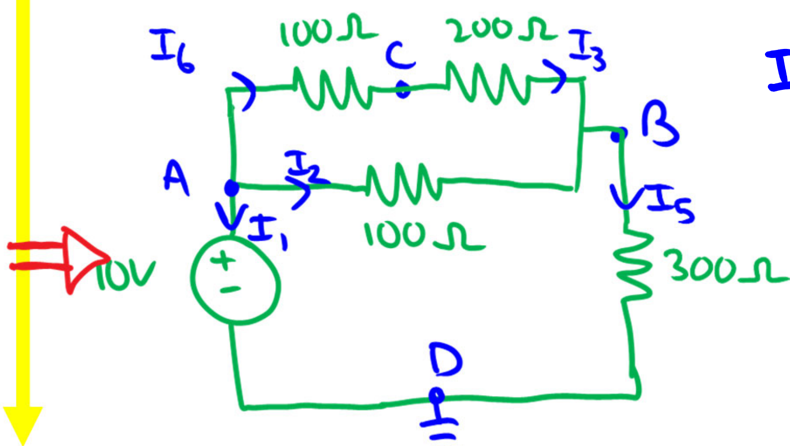
$$V_B = I_S \times 300 = 8V$$



$I_2 + I_6 = I_S \Rightarrow$ Current divider

$$I_2 = I_S\left(\frac{300}{300+100}\right) = 0.02A$$

$$I_6 = I_S\left(\frac{100}{100+300}\right) = 0.0067A$$



$$I_3 = I_6$$

$$V_C = 10 - (100 \times I_6)$$
$$= 9.33V$$

$V_A = 10V$

$V_B = 8V$

$V_C = 9.33V$

$V_D = 0V$

$I_1 = -27mA$

$I_2 = 20mA$

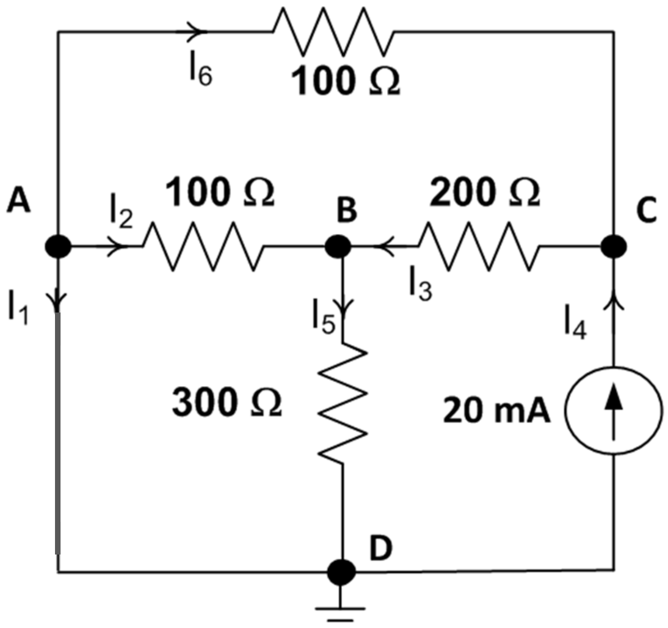
$I_3 = 6.7mA$

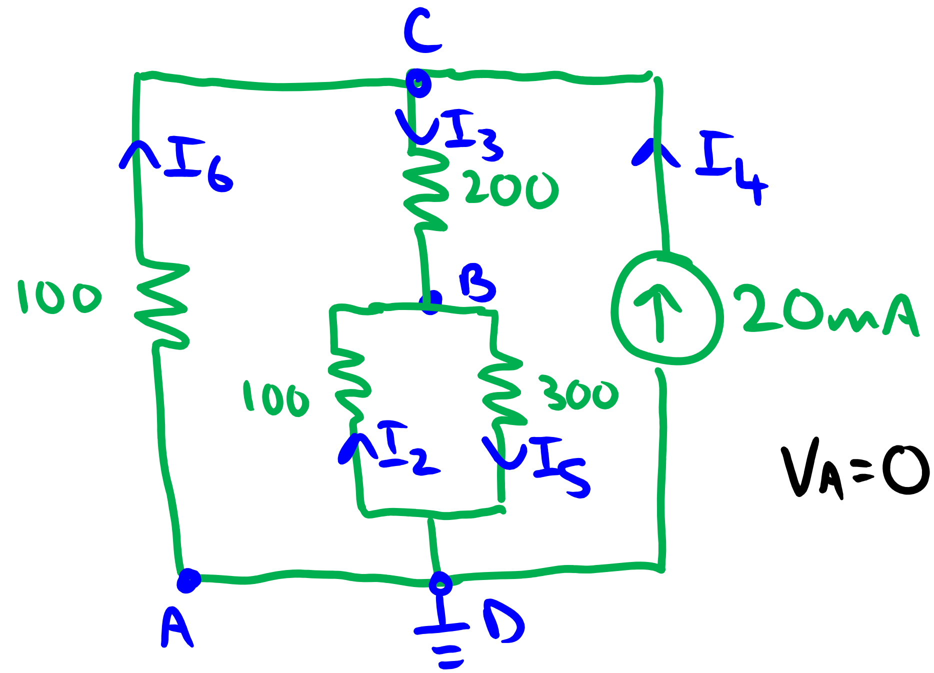
$I_4 = 0A$

$I_5 = 27mA$

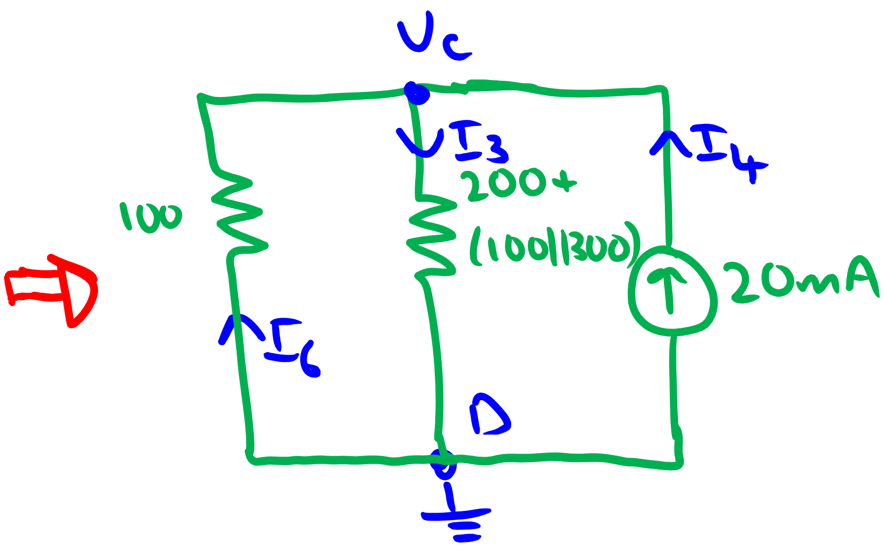
$I_6 = 6.7mA$

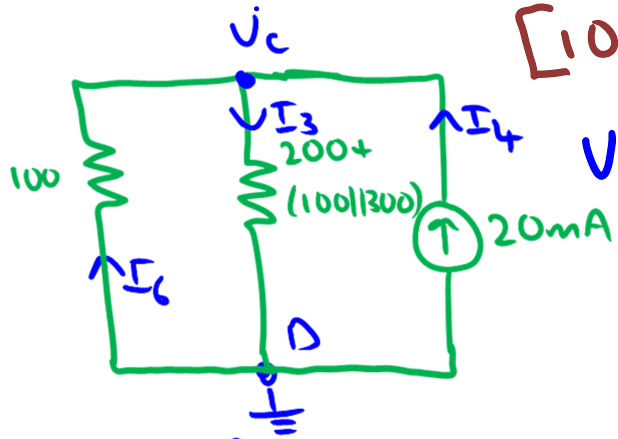
Step 3: Current source (zero other sources)

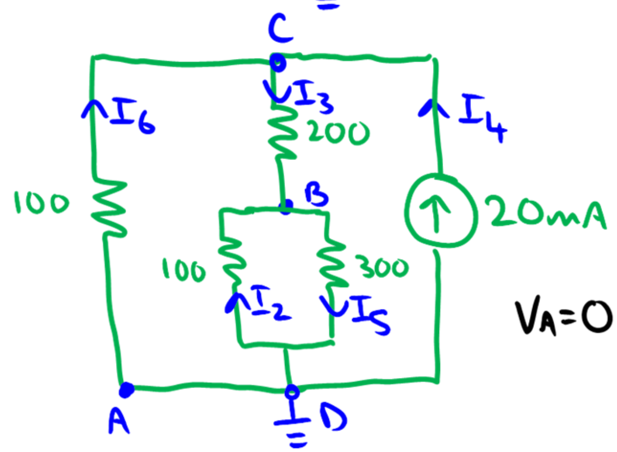


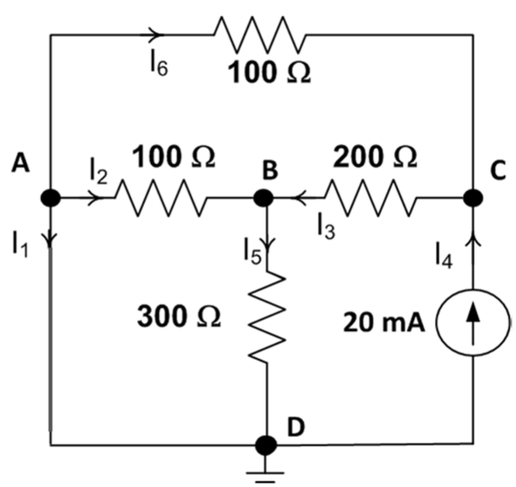


$V_A = 0V$









$$[100||300 = 75,\ 100||275 = 73.3]$$

$$V_C = I_4\left(100||(200+100||300)\right)$$

$$= 0.02 \times 73.3 = 1.47V$$

$$I_6 = -\frac{V_C}{100} = -14.7\,mA$$

$$I_3 = \frac{V_C}{275} = 5.33\,mA$$

$V_A = 0V$
$$V_B = V_C\left(\frac{75}{200+75}\right) = 0.4V$$

$$I_5 = \frac{V_B}{300} = 1.33\,mA$$

$$I_2 = \frac{-V_B}{100} = -4\,mA$$

$$I_1 = -(I_2 + I_6) = 18.7\,mA$$

$V_A = 0V$

$V_B = 0.4V$

$V_C = 1.47V$

$V_D = 0V$

$I_1 = 18.7mA$

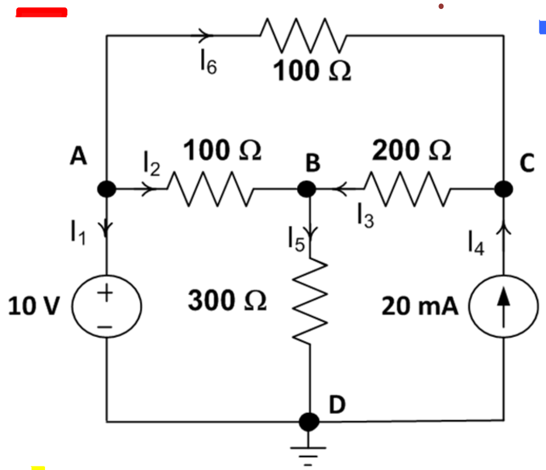
$I_2 = -4mA$

$I_3 = 5.3mA$

$I_4 = 20mA$

$I_5 = 1.33mA$

$I_6 = -14.7mA$



Step 4: Complete Solution

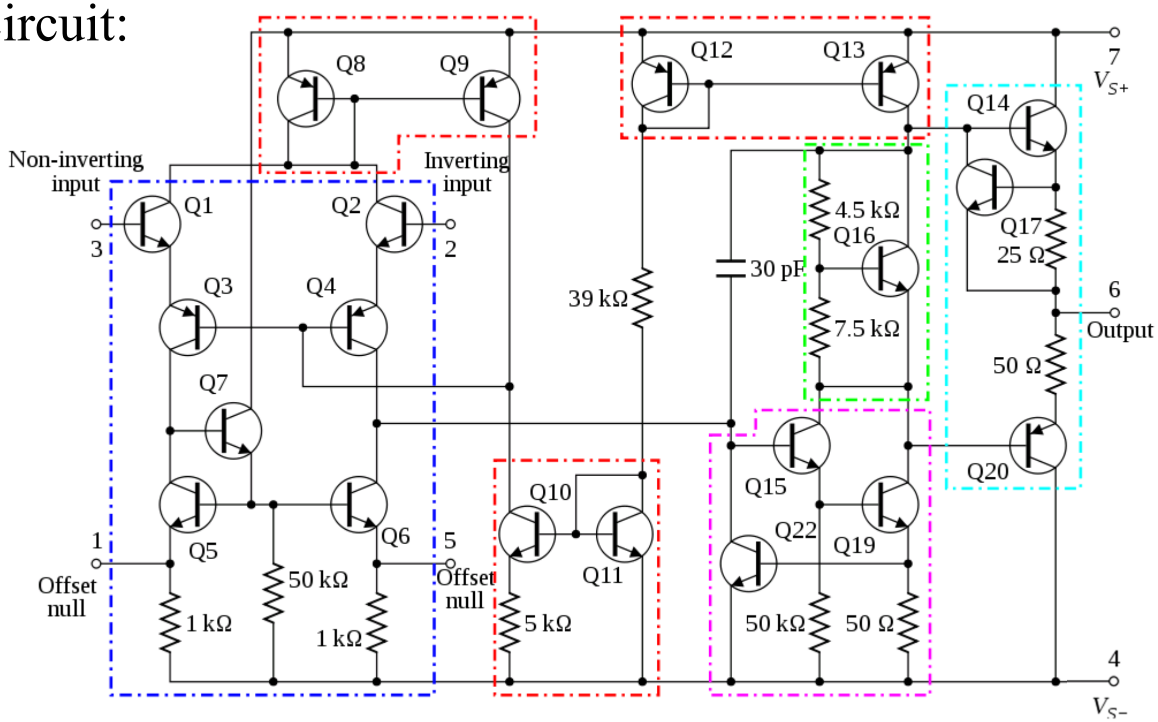
Voltage Source:	Current Source	Complete Solution [Sum of components]
$V_A = 10V$	$V_A = 0V$	$V_A = 0+10 = 10V$
$V_B = 8V$	$V_B = 0.4V$	$V_B = 8+0.4 = 8.4V$
$V_C = 9.33V$	$V_C = 1.47V$	$V_C = 9.33+1.47 = 10.8V$
$V_D = 0V$	$V_D = 0V$	$V_D = 0+0 = 0V$
$I_1 = -27mA$	$I_1 = 18.7mA$	$I_1 = 18.7-27 = -8mA$
$I_2 = 20mA$	$I_2 = -4mA$	$I_2 = 20-4 = 16mA$
$I_3 = 6.7mA$	$I_3 = 5.3mA$	$I_3 = 6.7+5.3 = 12mA$
$I_4 = 0A$	$I_4 = 20mA$	$I_4 = 20mA$
$I_5 = 27mA$	$I_5 = 1.33mA$	$I_5 = 26.7+1.33 = 28mA$
$I_6 = 6.7mA$	$I_6 = -14.7mA$	$I_6 = 6.7-14.7 = -8mA$

Some discussion on engineering modelling

- One of the challenges in modern engineering is the reliable modelling of increasingly complicated systems:
  - A modern computer may contain literally millions of discrete electronic components (resistors, capacitors, transistors etc)
  - The same is true with regard to the total number of individual elements making up cars, aircraft, automation systems in mine, buildings, oil refinaries etc. etc.
- In order to analyse the behaviour or design such systems *it is essential we have predictable, intuitive models to describe them.* Clearly, a model which consists of a million variables is:
  a. Complicated, or impossible to solve; and
  b. Very difficult to interpret or explain how each individual element relates to each other element in the whole system
- All engineering disciplines use the method of dividing the complete system into subsystems (and maybe sub-sub-systems):
  - At a basic level, we can directly and intuitively *relate each individual element to the behaviour of the subsystem* (and derive simpler, more interpretable models for the subsystem)
  - At the higher level, we can directly intuitively *relate each sub-system to the behaviour of the system as a whole.* At this level, we do not have to care about the individual elements of the sub-systems (if we have done a good job above!)

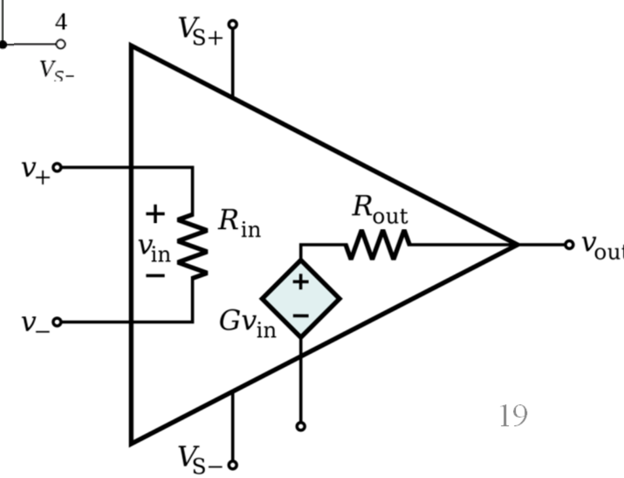
Equivalent Circuits in Electronics

- We apply this to increasingly complicated systems, like this amplifier circuit:



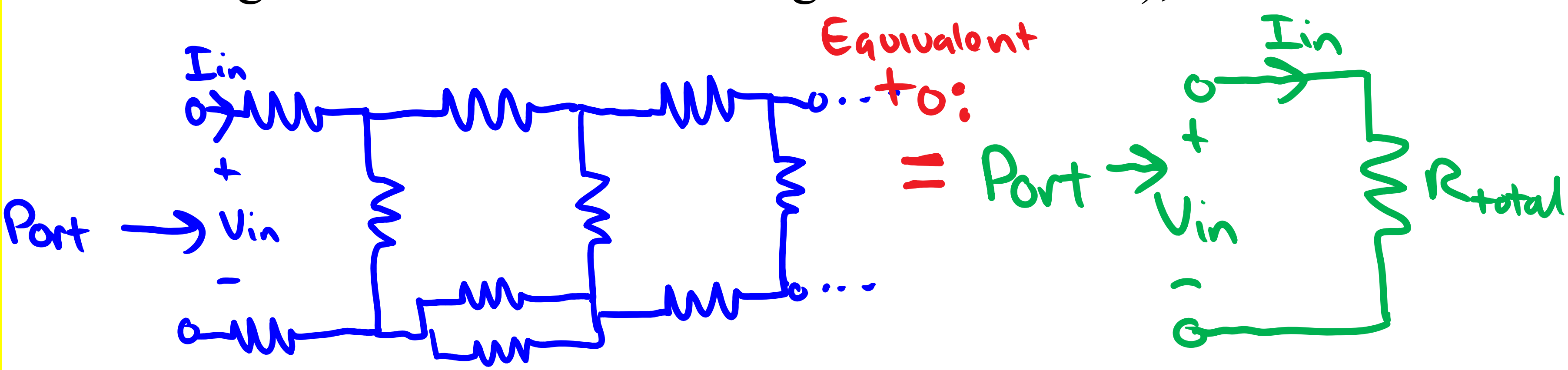
- Which we design to be modelled more simply with a single circuit element (the op-amp):

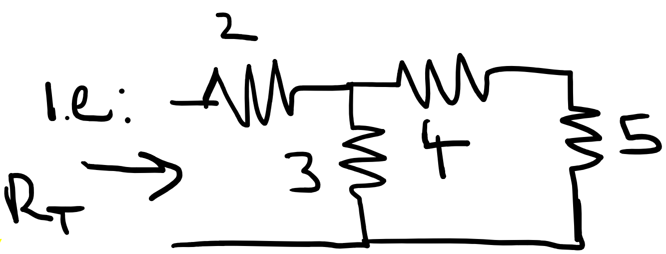
More details about op-amps in Week 11/12!



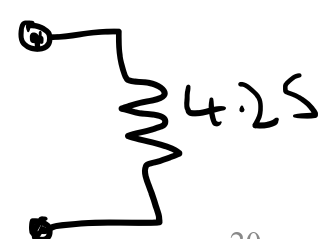
Equivalent Circuits in Electronics

- In electronics, we model subsystems using a general technique *called equivalent circuits*
- We already saw this in week 1: "one port resistor model" – any network of resistors – as viewed at a "port" - can be modelled by a single resistor (even though this does not tell us about the voltages/currents across/through each resistor), i.e.:





$\Rightarrow$ $R_{Total} = 2 + 3 \| (4+5)$
$= 4.25$

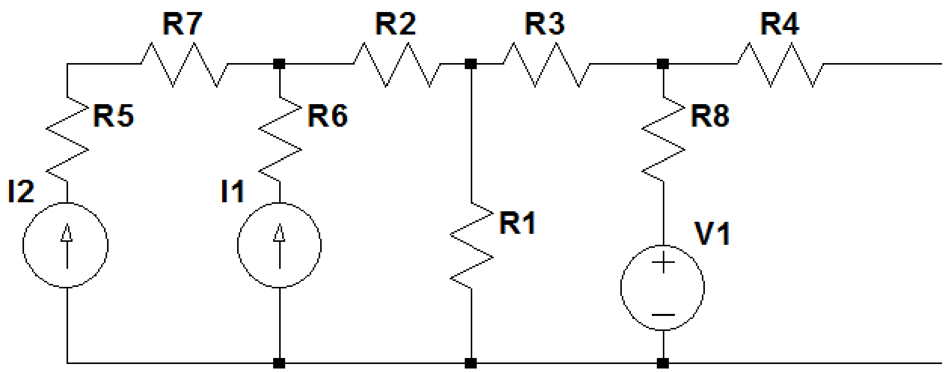


Example: Why Equivalent Circuits?

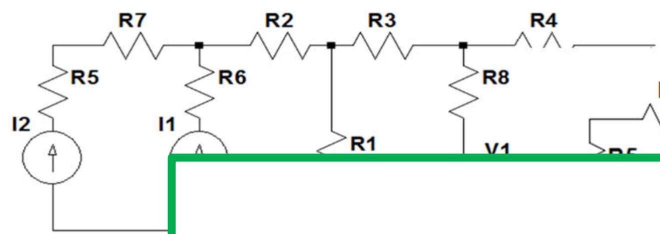
- You are an power electronics engineer, and you have developed a system which uses solar panels to charge a lead acid 12V battery.
- This system is then used to power a number of scientific devices for doing fieldwork in remote parts of Australia. It is essential for calibration purposes you know the specific voltages supplied to equipment at any time (i.e. to meet power specifications for each instrument….)
- However, the specific devices you are connecting to your power system vary throughout the day……

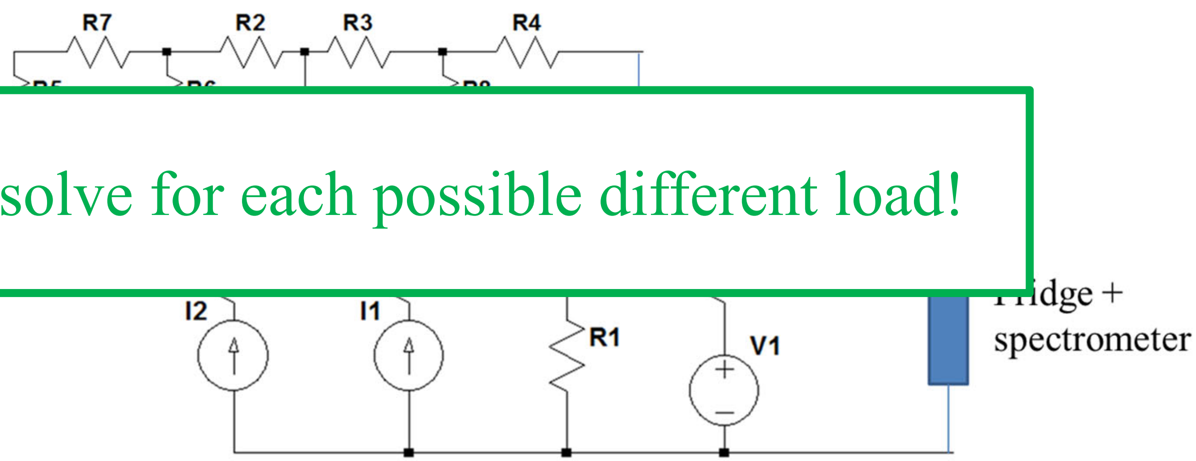
Why - Example?

- You worked out that the electrical model for your power system is:



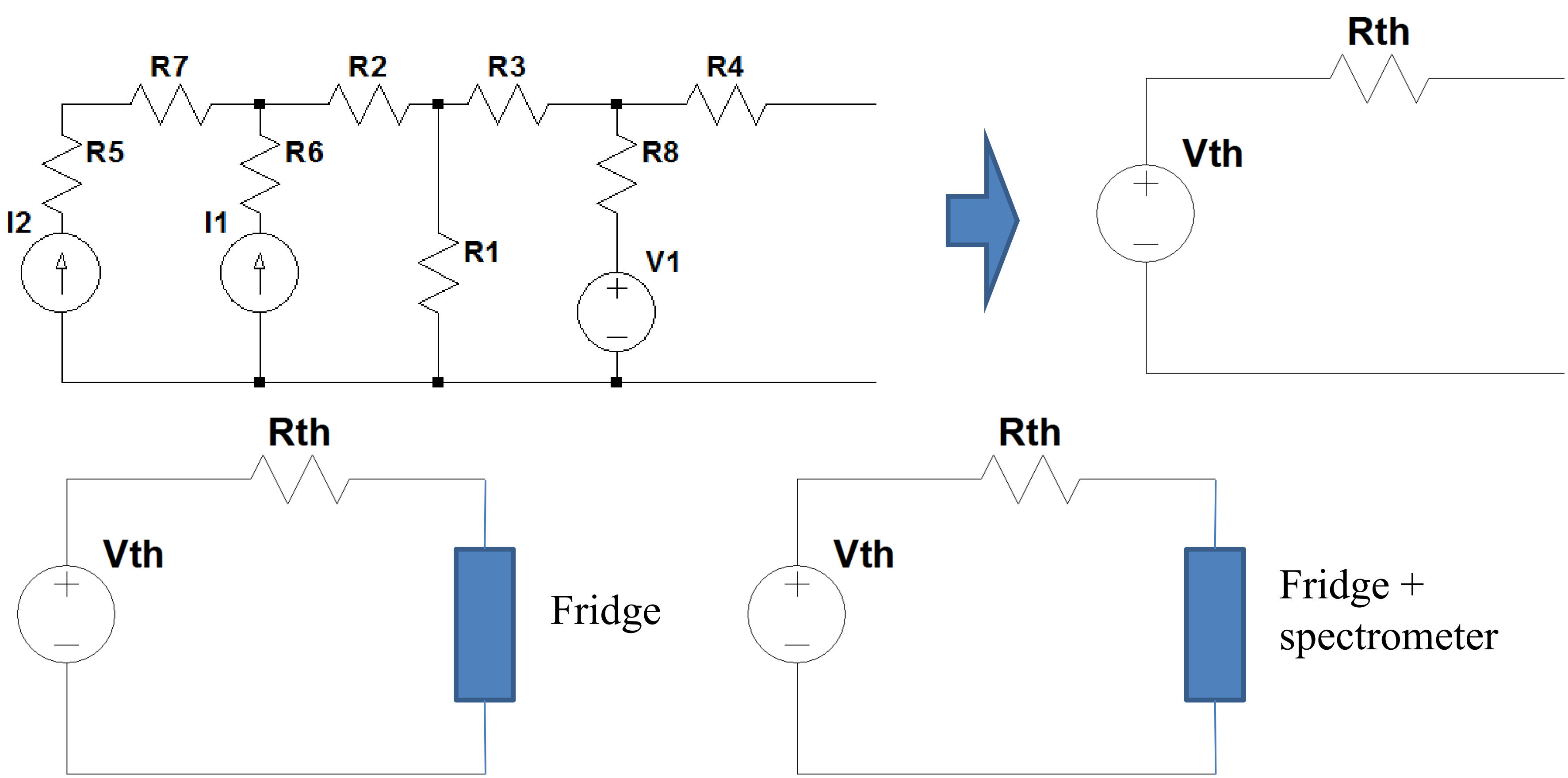
- Which could be connected to:
  - Spectrum analyser
  - RF signal strength meter
  - Digital scales
  - 12V fridge
  - Lighting……



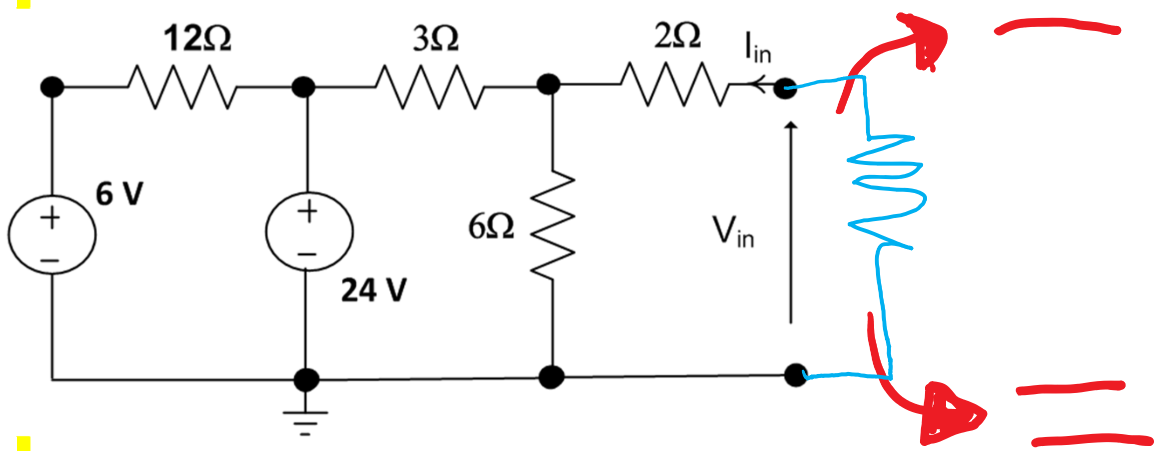


Annoying to solve for each possible different load!

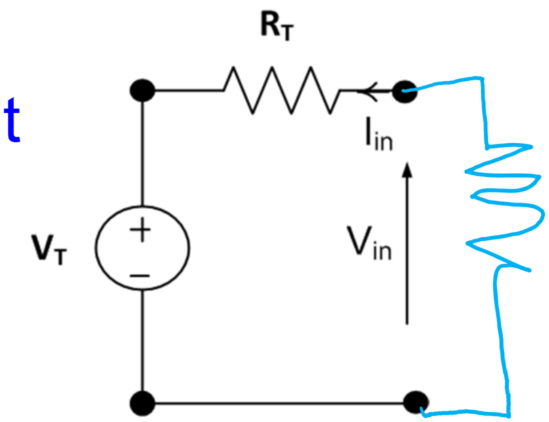
Wouldn't it be great to be able to find a simpler model for the circuit to allow us to simply solve for each different load?

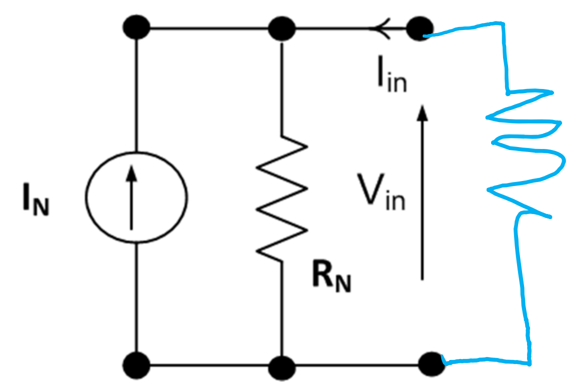


One port networks: We can replace any one-port network of voltage sources, current sources and resistors with a circuit consisting of a voltage source and single resistor; or current source and resistor.



Thevenin Equivalent Circuit



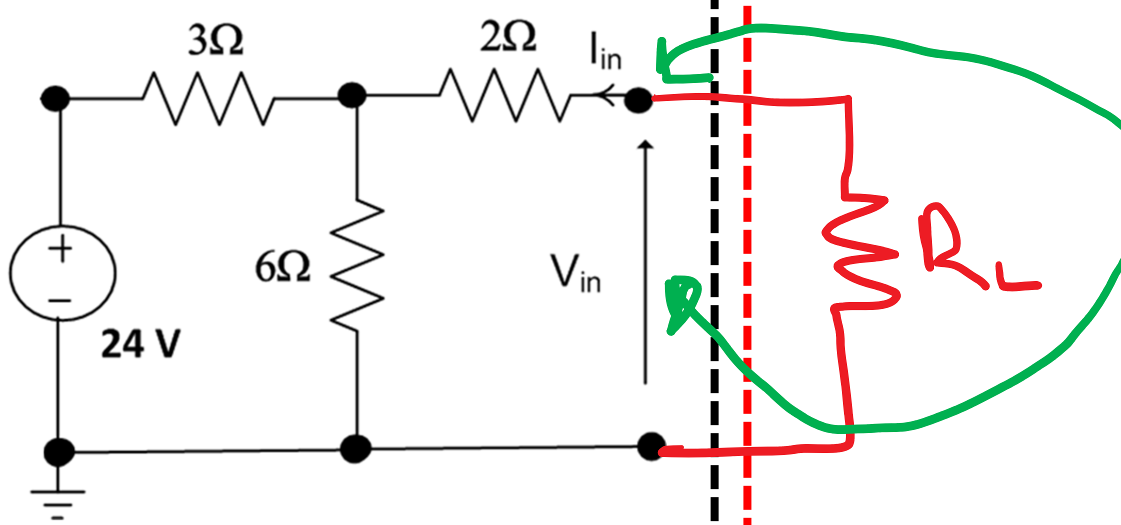


Norton Equivalent Circuit

How does this work?

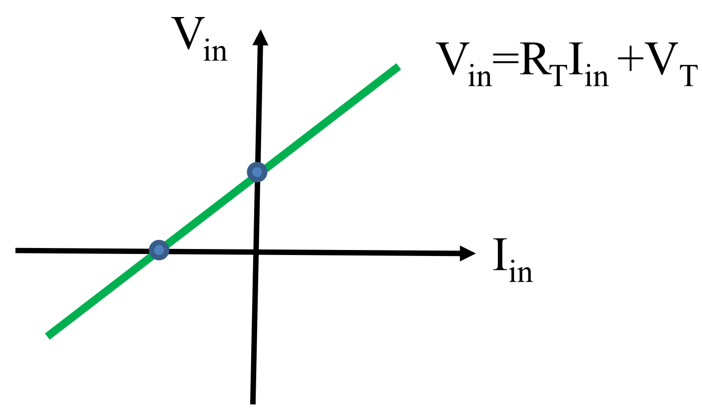
Consider the one-port network

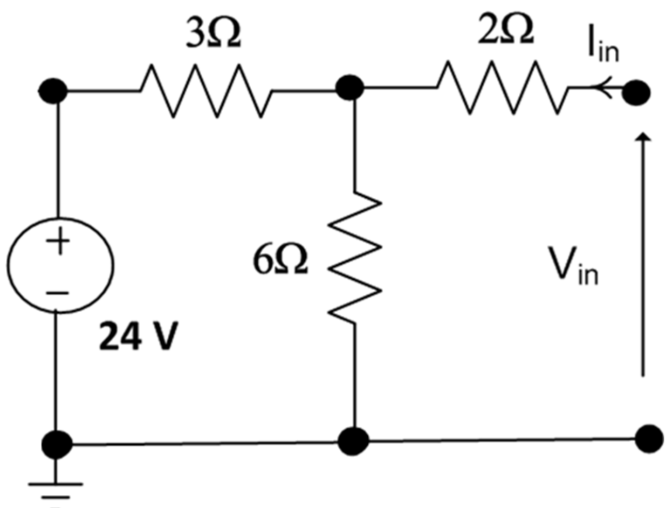
Connected to any load

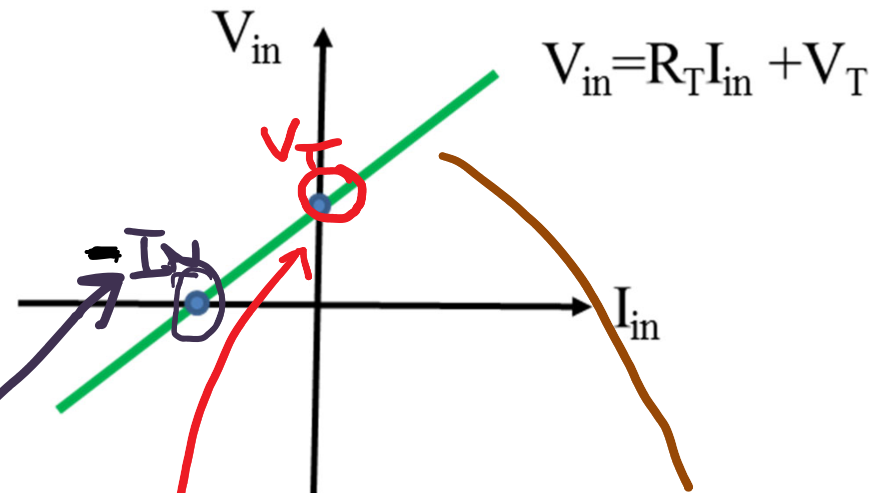


The relationship between I_{in} and V_{in} with different loads gives when plotted, I_{in} vs. V_{in} gives a straight-line (defined by two parameters)

Because the one port network contains independent voltages sources, current sources, and resistors is a linear circuit – It, can be explained by a linear model







V_T is V_{in} when $I_{in}=0$ (i.e: when port is open circuit)

R_T is the gradient and is the resistive part of the circuit

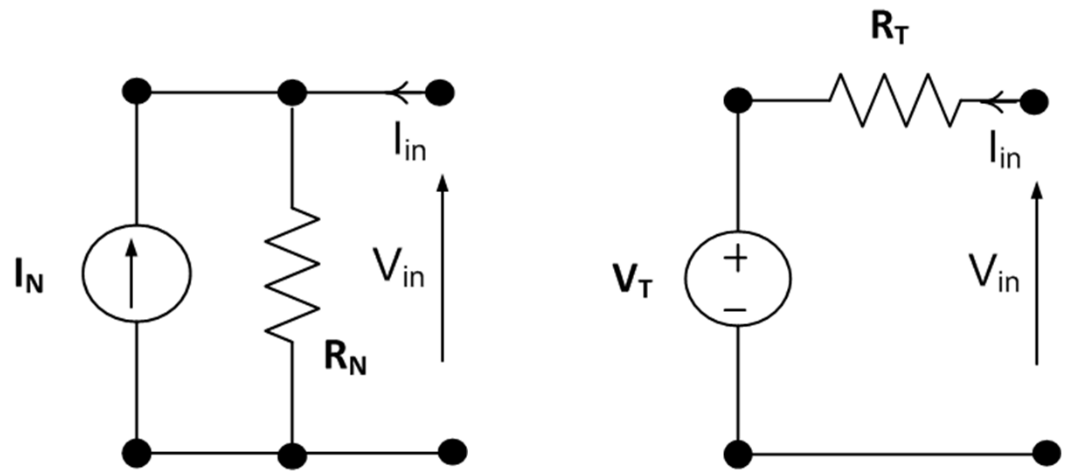
We can parameterise the line by any two points; or the gradient and a point!

I_N is $-I_{in}$ when $V_{in}=0$ (i.e: when port is short circuit)

$$I_{in} = V_{in}/R_T - V_T/R_T$$
$$I_{in} = V_{in}/R_T - I_N$$

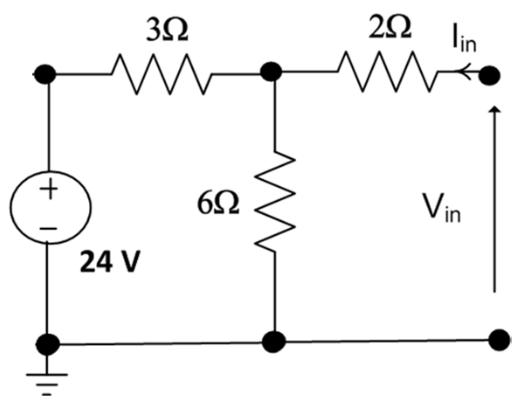
$$R_T = R_N \; ; \; I_N = \frac{V_T}{R_T}$$

These mathematical models as schematics:

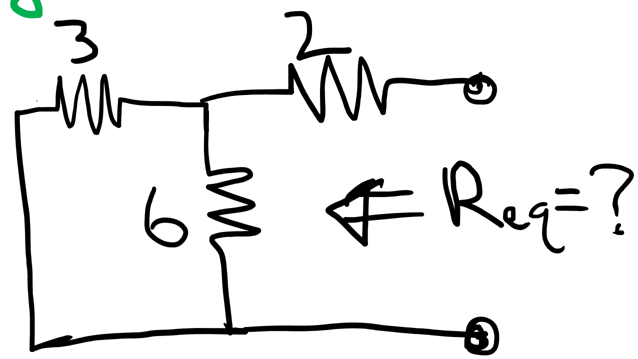


Thevenin (Internal) Resistance

- What is the resistive component of this circuit?
  1. Set the sources to zero (just considering resistive part and ignoring sources), and redraw the circuit
  2. Calculate equivalent resistance



1. Sources to zero and redraw:



2. Calculate equivalent resistance:

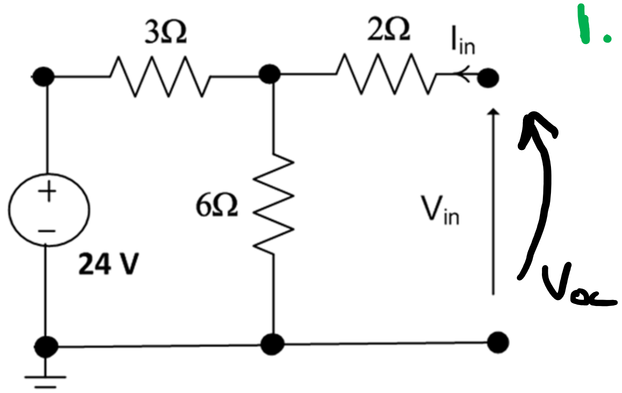
↳ note 6Ω resistor is in parallel with 3Ω

i.e.:

↳ $R_{eq} = 2 + 3 \| 6 = 2 + 2 = 4\,\Omega$

Thevenin (Open Circuit) Voltage

- What is the Open circuit voltage of the circuit?
  1. Consider an open circuit at the port
  2. What is the voltage across the port?



1. consider open cct:
   - $I_{in} = 0$
   - Voltage across 2Ω resistor $= 0$

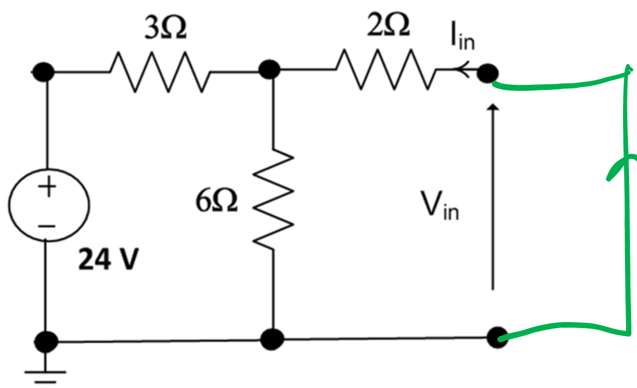
2. Calculat V_{oc}

↳ Apply potential divider (V_{oc} = Voltage across 6Ω resistor)

$$V_{oc} = 24\left(\frac{6}{6+3}\right) = 16V$$

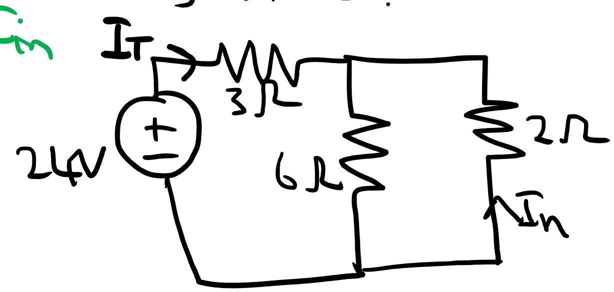
Norton (Short Circuit) Current

- What is the Short Circuit current of the circuit?
  1. Consider an <u>short circuit</u> at the port
  2. What is the current through this short circuit?



1. Consider short circuit:

↳ $V_{in} = 0$.



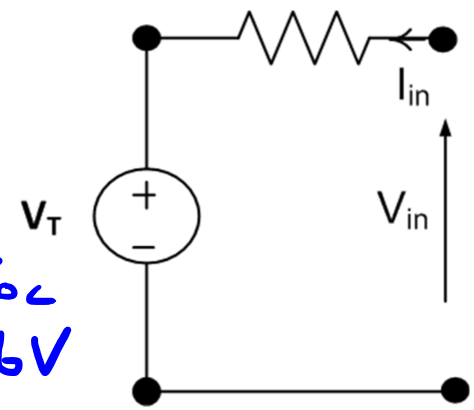
2. Calculate I_{in}:

- $I_T = \dfrac{24}{3 + 6||2} = 5\tfrac{1}{3}\,A$.

- Current divider for I_N: $I_{in} = \dfrac{-(6)(5\tfrac{1}{3})}{2+6} = -4A$

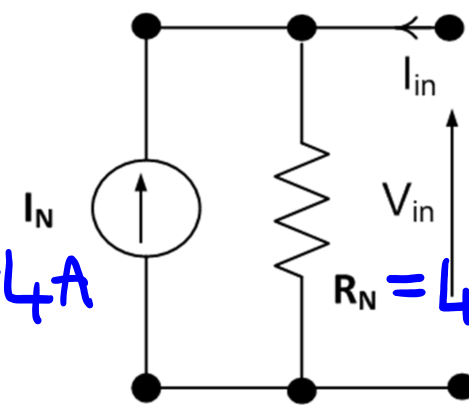
Thevenin Equivalent

$R_T = 4\,\Omega$



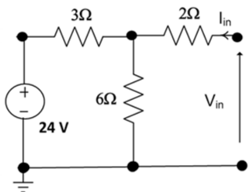
$= V_{oc}$
$= 16V$

Norton Equivalent

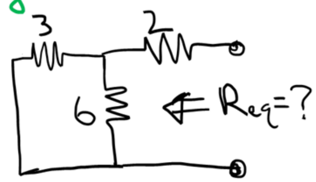


$= I_{sc} = -4A$

$R_N = 4\,\Omega$



1. Sources to zero and redraw:

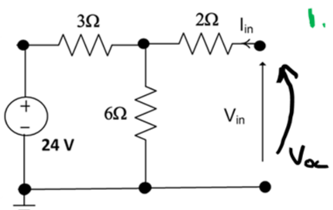


2. Calculate equivalent resistance:

↳ note 12Ω resistor is in parallel with 3Ω

i.e.:

↳ $R_{eq} = 2 + 3 \| 6 = \underline{\underline{2 + 2 = 4\,\Omega}}$



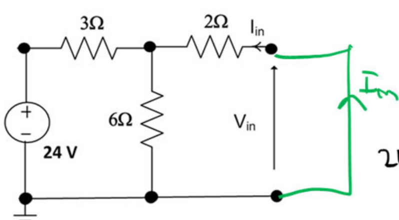
1. consider open cct:

- $I_{in} = 0$
- Voltage across 2Ω resistor = 0

2. Calculat V_{oc}

↳ Apply potential divider (V_{oc} = Voltage across 6Ω resistor)

$$V_{oc} = 24\left(\frac{6}{6+3}\right) = 16V$$



1. Consider short circuit:

↳ $V_{in} = 0$.

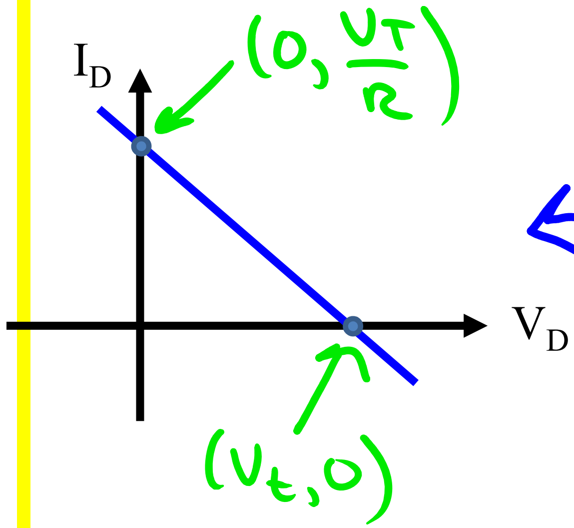
2. Calculate I_{IN}:

- $I_T = \dfrac{24}{3 + 6\|2} = 5\tfrac{1}{3}A$.
- Current divider for I_{IN}: $I_{in} = \dfrac{-(6)(5\frac{1}{3})}{2+6} = -4A$

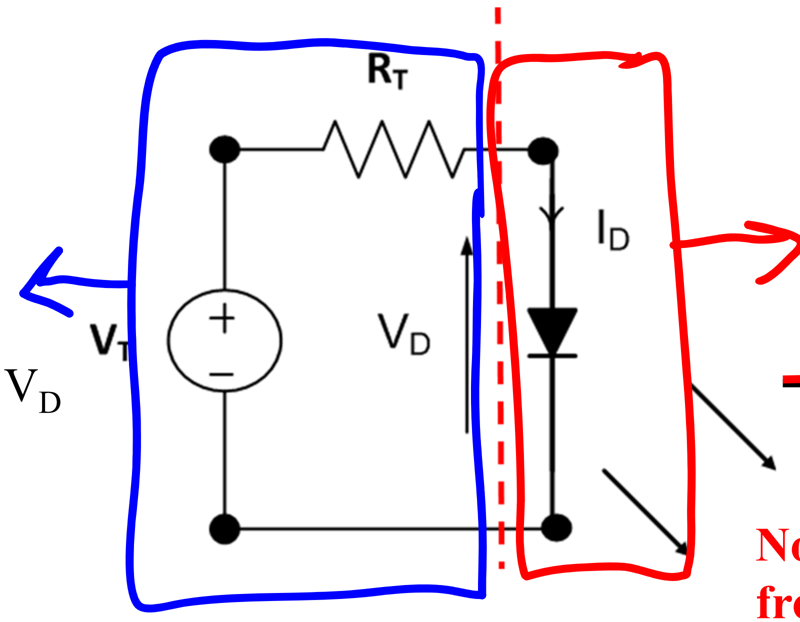
Why Equivalent Circuits?

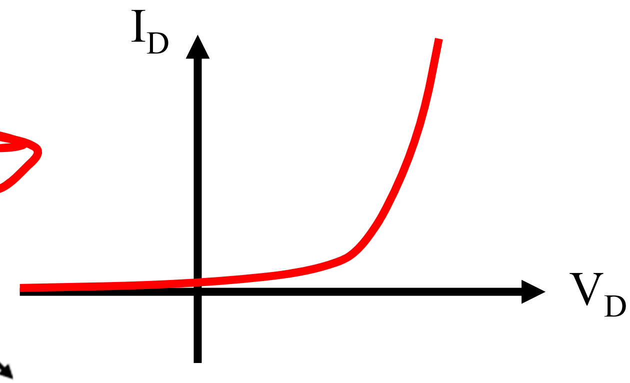
- As an intermediate step to solve more complicated circuits – simplify part of a circuit containing sources and resistors.
- Allow us to analyse the complex part of a circuit once – and then apply it to multiple examples….
- Solve complex circuit once. This is now a "module" to which you can connect any arbitrary load.

Load-line for non-linear components



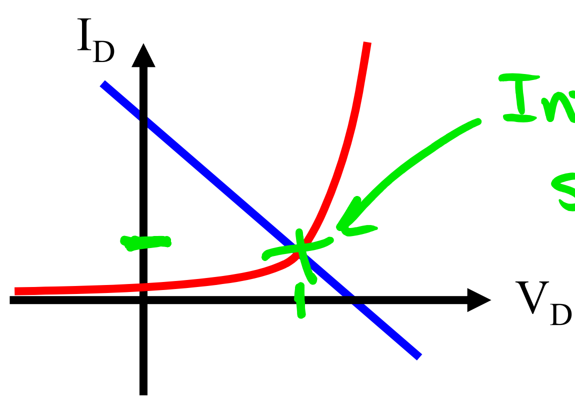
$I_D = (V_t - V_D)/R_T$





Note: We can get this curve from the datasheet, or we can measure it

See Lab (VI) for further details



Intersection is the solution to circuit

Summary

- Superposition is one more tool in our circuit solution toolbox –
  - Reduce circuit to multiple, smaller problems
- Thevenin and Norton equivalent circuits allow us to model complex linear one-port networks with a simpler circuit.
  - And then connect to any other circuit…

- Next Week:
  - AC Circuits
  - Be prepared for complex numbers – have a calculator and manual.