ENGG1300
Introduction to Electrical Systems
Week 7 – Filters + Communication Systems

Lecturer:

Dr Philip Terrill

Plan For Rest of Semester:

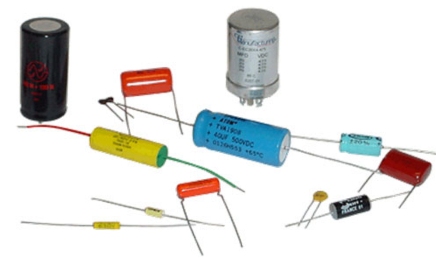
- Week 7: Filters + Communications 1
- Week 8: Filters + Communications 2
- Week 9: Monday Public Holiday
  - No lecture – Anzac day
  - No lab A
  - Lab B: Design project work
- Week 10: Monday Public holiday (but Monday timetable on Tuesday
  - Lecture: Power Systems 1
  - No lab A
  - Lab B: Design project work
- Week 11: Power Systems 2
- Week 12: Control & Op-amps 1
- Week 13: Control & Op-amps 2

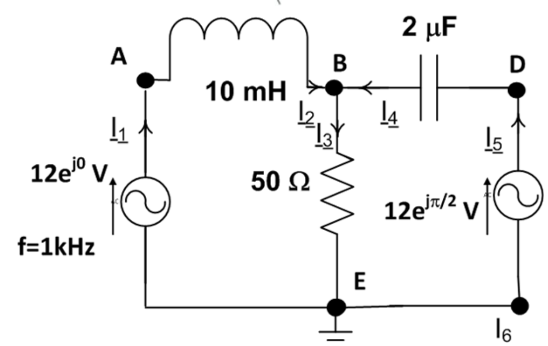
What did we do in Week 6?

- Phasor Diagrams – representing circuit solutions graphically
  - An intuitive way of visualising KCL and KVL with phasors (sum of currents to node = 0; or sum of voltages around mesh = 0)
- AC Nodal & Mesh Analysis
  - Extend our DC circuit analysis techniques to solve of sinusoidal time-varying circuits

So where are we up to?

1. We have seen that we can model inductors and capacitors using frequency dependant complex impedances
2. Circuits containing inductors and capacitors thus have a frequency dependence



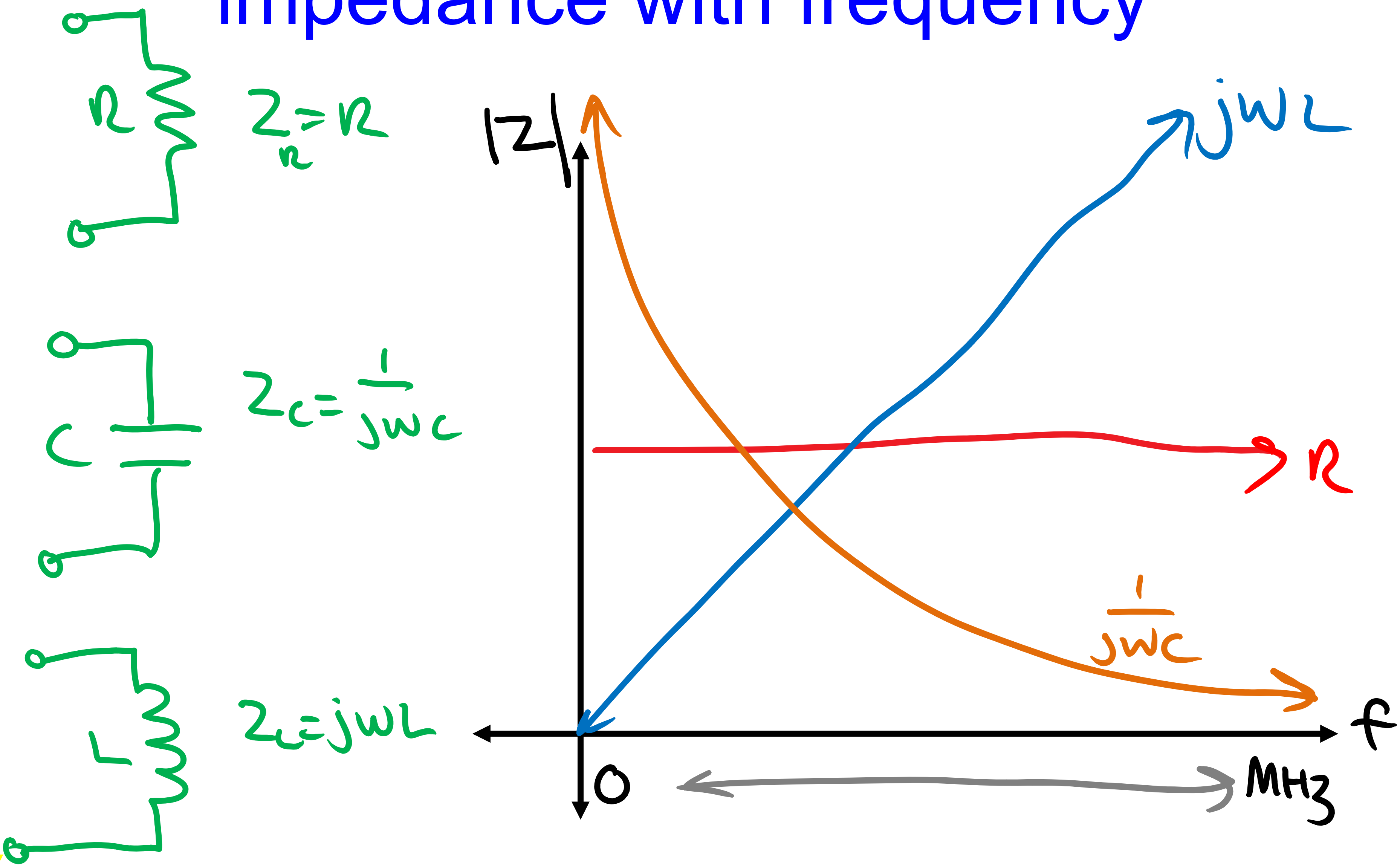


f-=1000; $\omega = 6283$;

$Z_2 = j\omega L = j\,62.83$;

$Z_3 = 50$, $Z_4 = -j79.6$

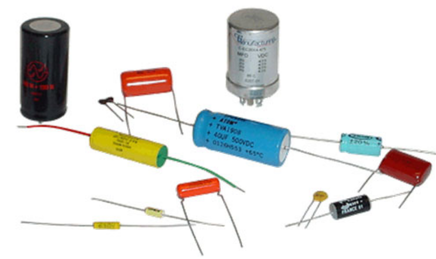
Magnitude of Component impedance with frequency

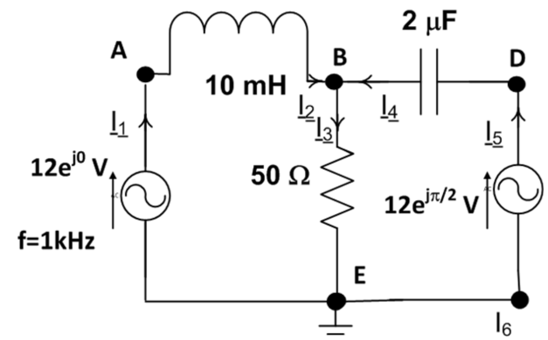


So where are we up to?

1. We have seen that we can model inductors and capacitors using frequency dependant complex impedances
2. Circuits containing inductors and capacitors thus have a frequency dependence
3. We have solved such circuits by substituting in for a particular source frequency (i.e. substituting in for f or w)
4. However, we can also make frequency the variable of interest to model how the circuit behaves at different frequencies

   i.e. model circuit as a function of frequency: $G(\omega)$ =?
5. We would like to plot this frequency response in an intuitive manner to help us understand the behaviour of the circuit.





f-=1000; $\omega = 6283$;
$Z_2 = j\omega L = j\ 62.83$;
$Z_3 = 50$, $Z_4 = -j79.6$

Communications + Electronics

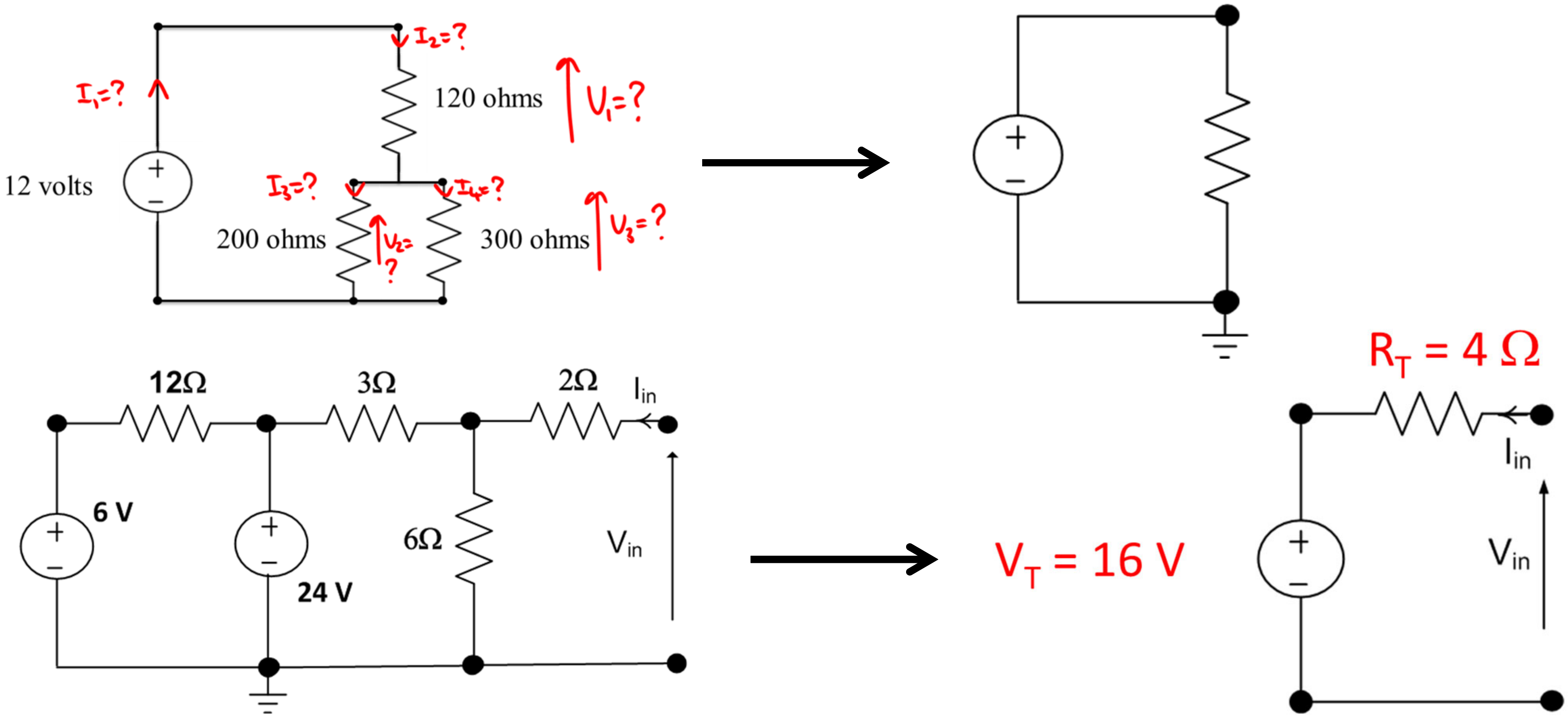
- Electronic communication systems rely on frequency dependant behaviour:
  - To remove “noise” from a signal of interest
  - To modulate information
  - To transmit information
  - To de-modulate information
- In weeks 7-8 we will be discussing techniques to model and intuitively present frequency dependant behaviour

- As Engineers, what we want to know is:
  How can we exploit frequency dependant behaviour in real world Communications + Electronics Engineering?

This Week

- Two port networks
  [We have previously seen one port networks as a way of simplifying the model behaviour of a circuit with **ONE** terminal]
- Transfer Functions
  [The function we use to model a two-port network]
- Frequency Response
  [The frequency dependant behaviour of the transfer function]
- Bode Plots
  [visualising this frequency dependant behaviour]
- Filters
  [practical application for this frequency dependant behaviour]

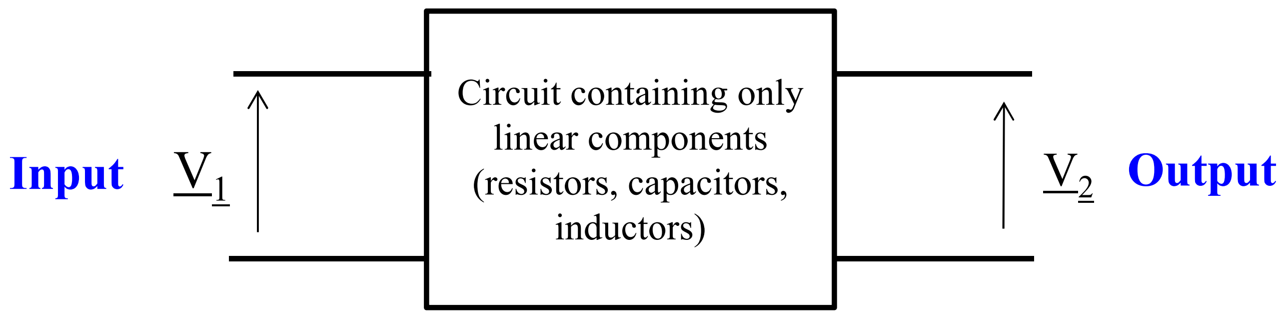
One-Port Networks



- Thevenin equivalent circuits and one port resistor models: Allow us solve complex circuit once. This is now a “module” to which you can connect any arbitrary load.

- However, we often work with circuits that have two ports –an input and an output

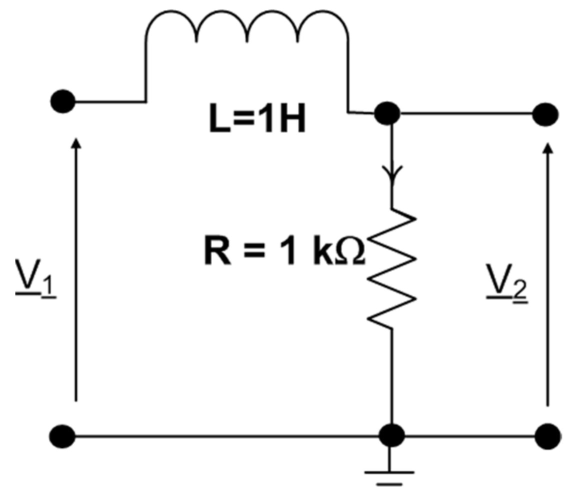
Two Port Networks with Sinusoidal Inputs



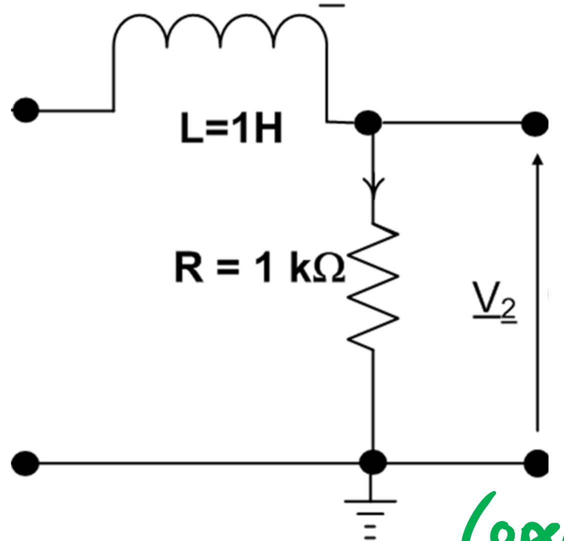
- We are only considering a special cases of two port networks:
  - V_1 is a voltage on the "input" port and we will connect a time varying voltage source here.
  - V_2 is the open-circuit voltage on the "output" port (i.e. we assume $I_2 = 0$)
- We can simplify the network further by assuming that all signals (i.e. input voltages) are sinusoidal, and therefore can be represented by phasors.

A Simple Example

Consider the following two port network:



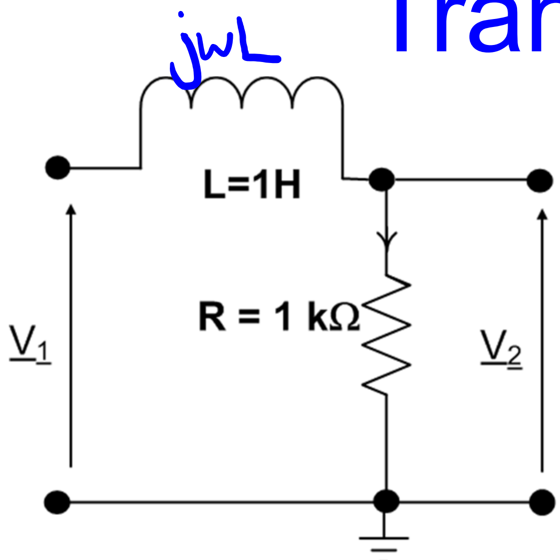
We might investigate in the lab:

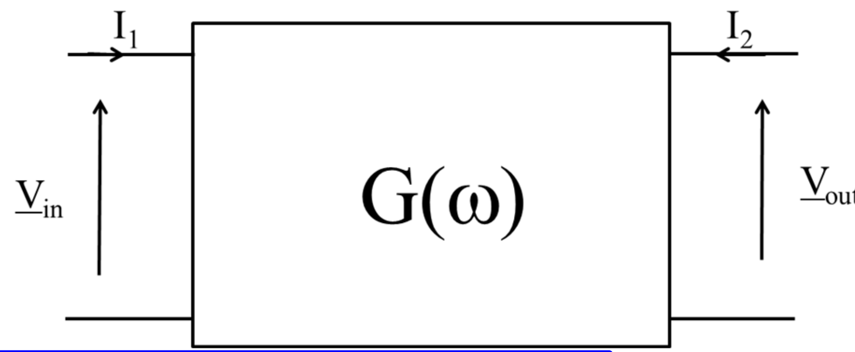


Might be our function generator

Our oscilloscope (open circuit)

Transfer Functions





Where: $V_{out} = V_{in} \times G(\omega)$

Or: $G(\omega) = \frac{V_{out}}{V_{in}}$

* $G(\omega) = \frac{V_{out}}{V_{in}} = \frac{V_2}{V_1}$
* For V_2 apply voltage divider:

$$V_2 = V_1\left(\frac{R}{R+j\omega L}\right) = V_1\left(\frac{1000}{1000+j\omega}\right)$$

$$\therefore G(\omega) = \frac{V_2}{V_1} = \underbrace{\frac{1000}{1000+j\omega}}_{\text{complex number}} = \overset{|G(\omega)|}{A}\, e^{j\overset{\angle G(\omega)}{\phi}}$$

$$G(\omega) = \frac{1000}{1000+j\omega} = \frac{\overset{1e^{j0}}{1}}{\underbrace{1+\frac{j\omega}{1000}}_{\left(\sqrt{1+\frac{\omega^2}{10^6}}\right)e^{j\left(\tan^{-1}\frac{\omega}{1000}\right)}}}$$

$$G(\omega) = \frac{1e^{j0}}{\left(\sqrt{1+\frac{\omega^2}{10^6}}\right)e^{j\left(\tan^{-1}\left(\frac{\omega}{1000}\right)\right)}}$$

$$|G(\omega)| = \left|\frac{1e^{j0}}{\left(\sqrt{1+\frac{\omega^2}{10^6}}\right)e^{j\left(\tan^{-1}\left(\frac{\omega}{1000}\right)\right)}}\right| = \frac{1}{\sqrt{1+\frac{\omega^2}{10^6}}}$$

$$\angle G(\omega) = \angle\frac{1e^{j0}}{\sqrt{1+\frac{\omega^2}{10^6}}\,e^{j\left(\tan^{-1}\left(\frac{\omega}{1000}\right)\right)}} = -\tan^{-1}\left(\frac{\omega}{1000}\right)$$

Transfer Function

The transfer function, $G(\omega)$ has the following characteristics:

- $G(\omega)$ is a complex number, and we typically write it in polar form, i.e., as an amplitude and an angle.
- We usually call the amplitude, $|G(\omega)|$, the gain of the transfer function. [More correctly we should call it the voltage gain. Sometimes it is called magnitude].
- We usually call the angle, $\angle(G(\omega))$, the phase of the transfer function.
- $G(\omega)$ is a function of angular frequency, and typically it can vary considerably for different frequencies.
- Note that $G(\omega)$ does not depend on the amplitude or phase of the input voltage:

  $G(\omega)$ is the ratio between output voltage and input voltage phasors.

- $G(\omega)$ does depend on the impedance connected to the output port: We will only consider an open-circuit or very high impedance load on the output (e.g. oscilloscope probe)

  [In future courses, you will consider input and output impedances that allow you generalise – See ELEC3400]

Plotting Frequency Response

- Tabulating ω (2π*f*), Gain and Phase:

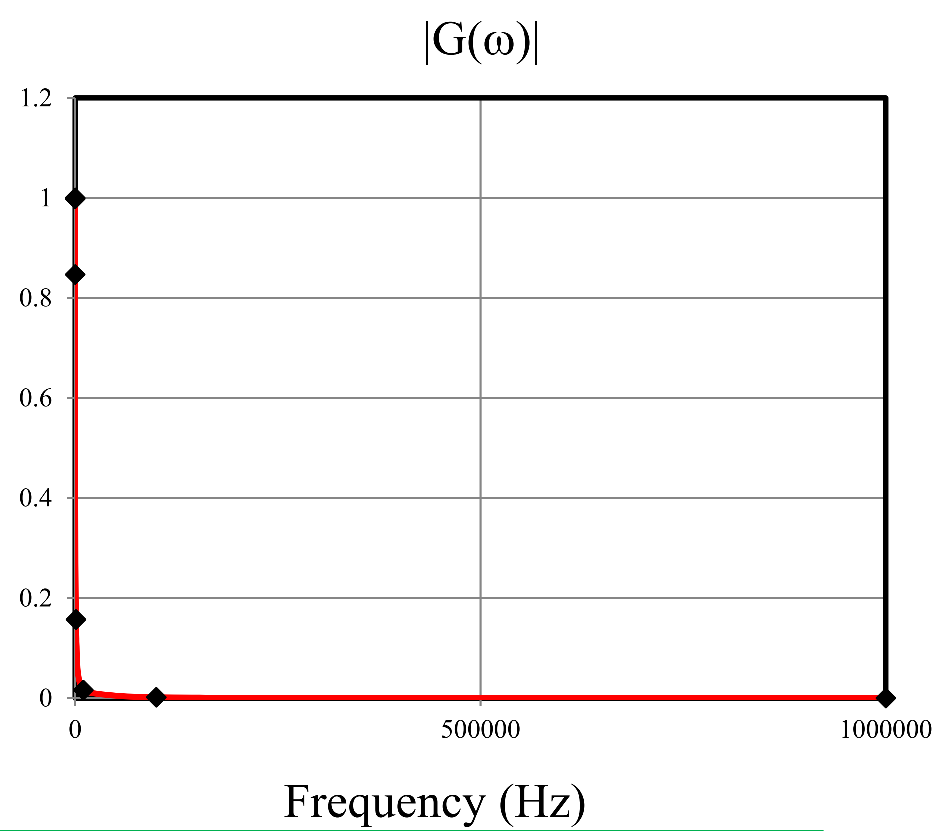
Gain: $|G(\omega)| = \frac{1}{\sqrt{(1 + \omega^2/10^6}}$; and phase: $\angle G(\omega) = -\tan^{-1}\frac{\omega}{1000}$

FREQUENCY
1 Hz
10 Hz
100 Hz
1000 Hz
10 kHz
100 kHz
1 MHz

Plotting Frequency Response

- We can plot this on a graph:

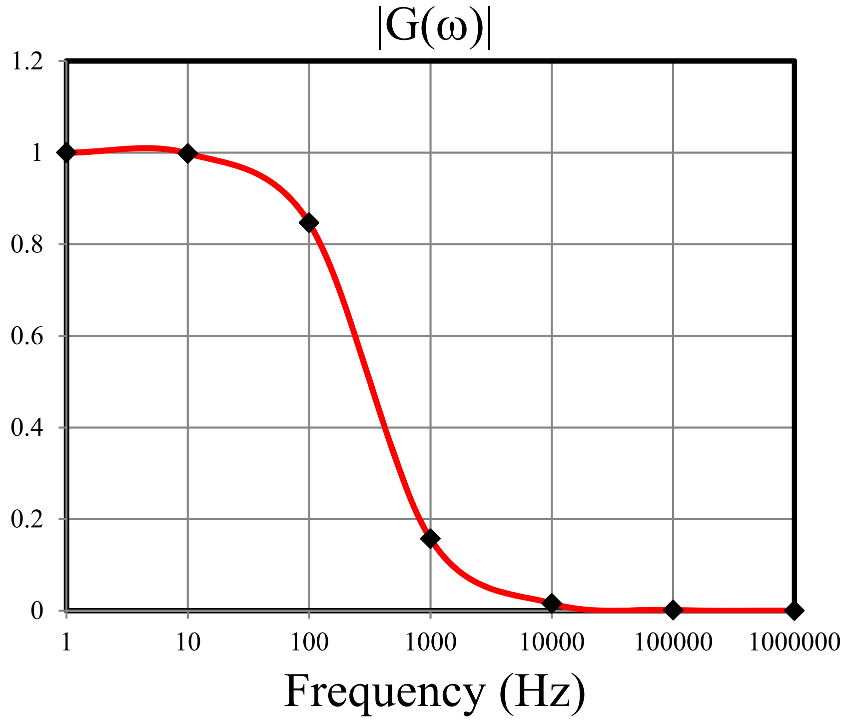
FREQUENCY	ω	GAIN , |G(ω)|
1 Hz	6.283 rad s^{-1}	0.99998
10 Hz	62.83 rad s^{-1}	0.998
100 Hz	628.3 rad s^{-1}	0.847
1000 Hz	6283 rad s^{-1}	0.157
10 kHz	62832 rad s^{-1}	0.0159
100 kHz	628319 rad s^{-1}	0.00159
1 MHz	6,283,185 rad s^{-1}	0.000159



Notice the first problem – we can't see the detail at low frequencies, because the values are all crowded at the left.

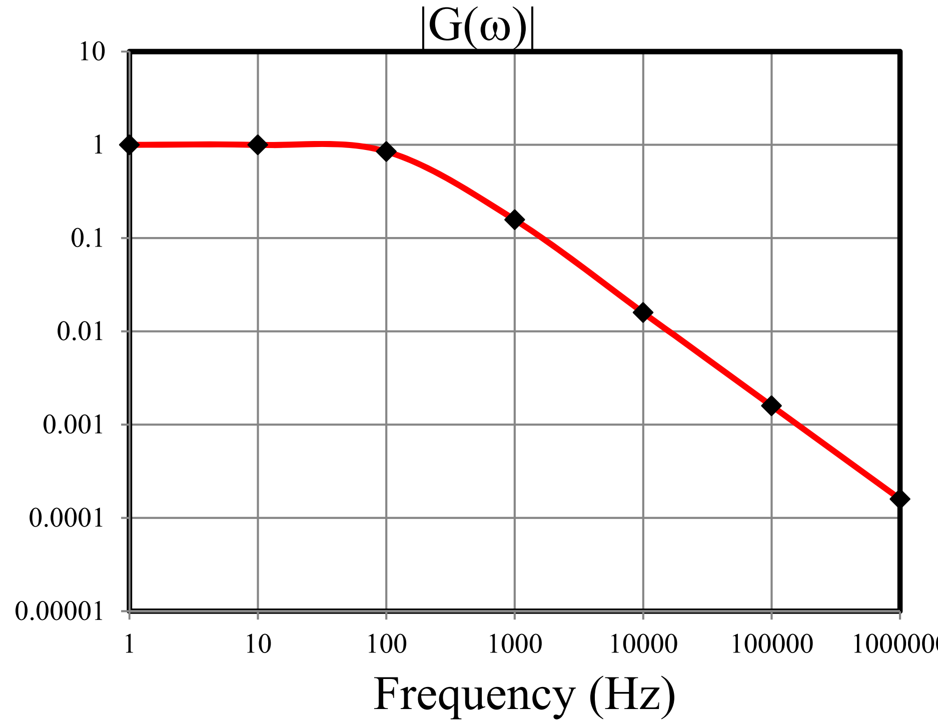
Plotting Frequency Response

- We can fix this by plotting frequency on a logarithmic scale



PROBLEM: We can't easily see the differences between small values on the gain axis.

- We can fix Y axis by also plotting gain on a logarithmic scale.



On this plot, gain is "approximately" two straight lines:

- 1-100Hz and
- 1kHz – 1 MHz,

With a transition between them

Think or Stretch

Can you think of some other measurements or scales which are logarithmic?

ANSWER:
Audio: sound pressure level
(washing machine = 50 dB, jet engine = 120 dB)
Optical fibre loss (0.4dB/km) = 10% loss each km
Richter Scale for Earthquakes

Decibels

- The decibel is a logarithmic measure of gain.
- It is measures the relative power of two signals (i.e. the ratio of two powers - .
- The bel, B, named after Alexander Graham Bell, is defined as:

  Gain in Bels = $log_{10}(\frac{P_2}{P_1})$

- Instead, we use decibels, dB. [10dB = 1 B]

  Gain in decibels =$10log_{10}(\frac{P_2}{P_1})$

- Power in a resistor is related to voltage by

  $P_R = \frac{V^2}{R}$, [P is proportional to V^2]

- So we can write:

  Gain in decibels =$10log_{10}\left(\frac{P_2}{P_1}\right) = 10log_{10}\left(\frac{{V_2}^2}{{V_1}^2}\right) = \boxed{20log_{10}\left(\frac{V_2}{V_1}\right)}$

It doesn't matter whether the voltages are peak, peak-to-peak or RMS, the ratio (gain) is the same!

This version for two port network voltage gains!

Bode Plot

- So, for: $|G(\omega)| = \frac{1}{\sqrt{(1+\omega^2/10^6}}$
- We can write: Gain in dB= $20log_{10}\left(\frac{1}{\sqrt{(1+\omega^2/10^6}}\right)$

Linear Y-axis

Log X-axis

FREQUENCY	ω	GAIN , |G(ω)|
1 Hz	6.283 rad s^{-1}	0.99998
10 Hz	62.83 rad s^{-1}	0.998
100 Hz	628.3 rad s^{-1}	0.847
1000 Hz	6283 rad s^{-1}	0.157
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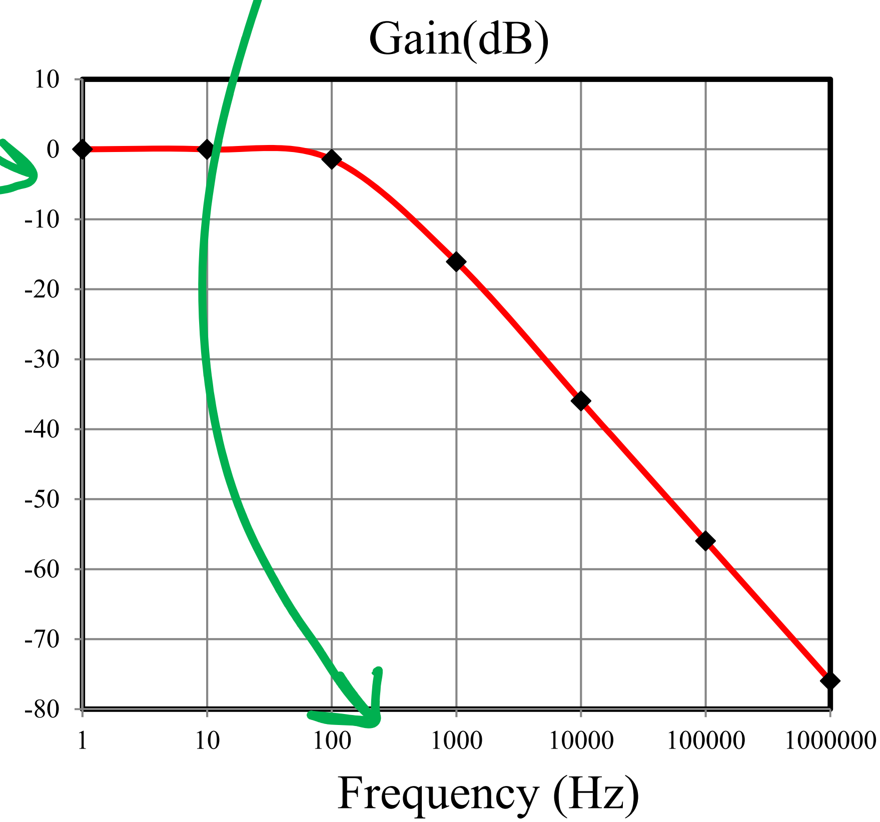
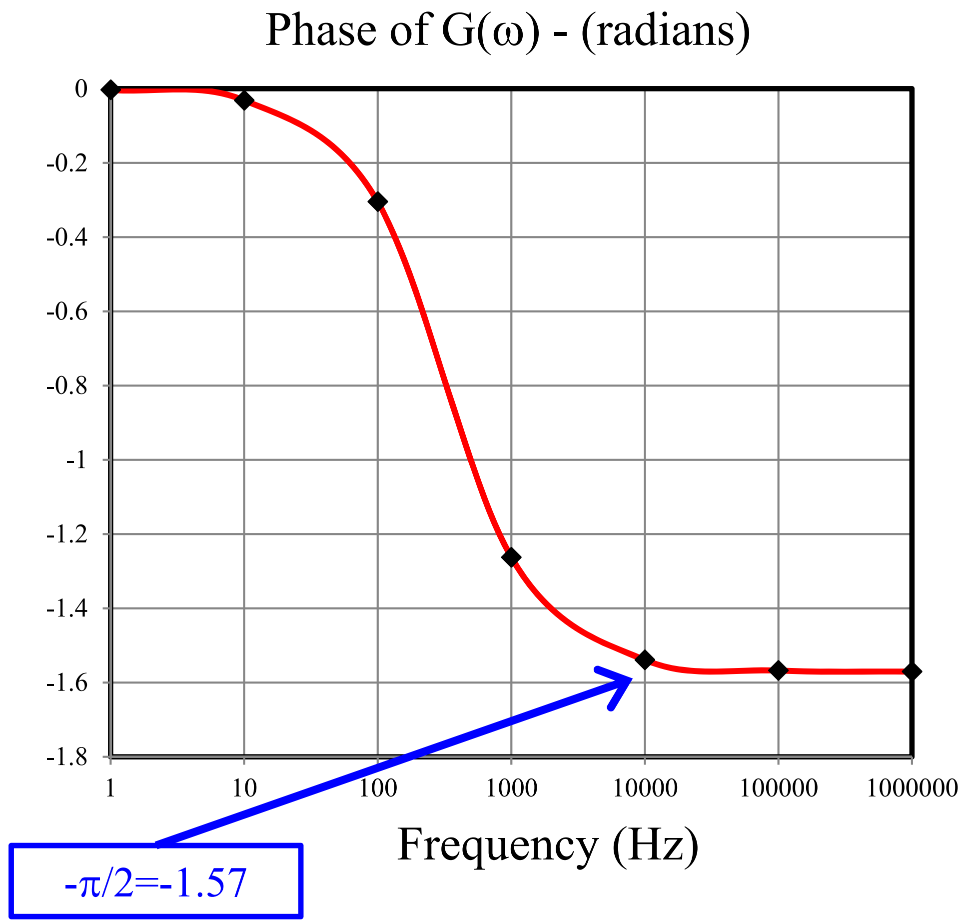


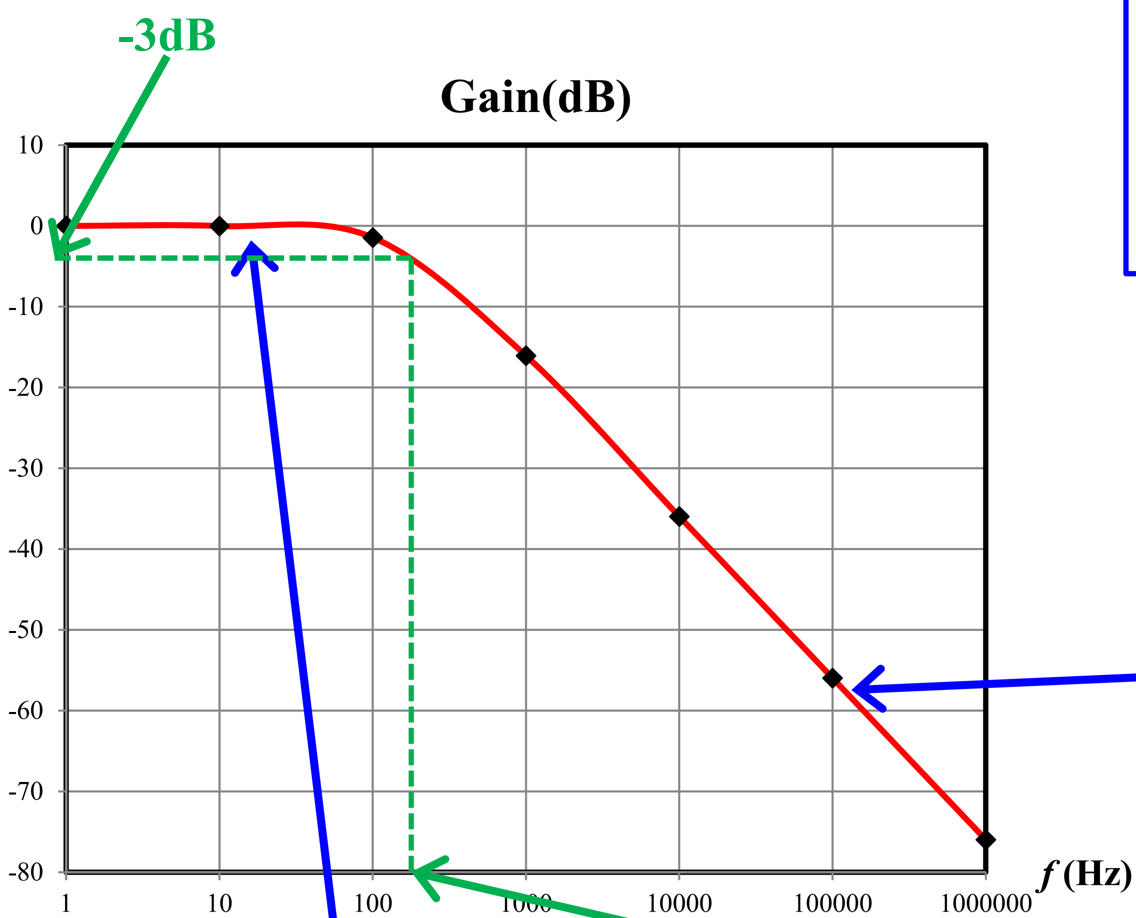
Bode Phase Plot

- We plot phase on a linear y axis, we still use a logarithmic frequency axis:

FREQUENCY	ω	PHASE of G(ω) radians
1 Hz	6.283 rad s^{-1}	-0.00314158
10 Hz	62.83 rad s^{-1}	-0.0314056
100 Hz	628.3 rad s^{-1}	-0.3043958
1000 Hz	6283 rad s^{-1}	-1.26262726
10 kHz	62832 rad s^{-1}	-1.53897608
100 kHz	628319 rad s^{-1}	-1.56761324
1 MHz	6,283,185 rad s^{-1}	-1.57047802



The Filter



We define the cut-off frequency as the "half power point":

i.e.: $\left|\frac{P_2}{P_1}\right| = \left|\frac{{V_2}^2}{{V_1}^2}\right| = \frac{1}{2}$

Therefore: $\left|\frac{V_2}{V_1}\right| = \frac{1}{\sqrt{2}}$

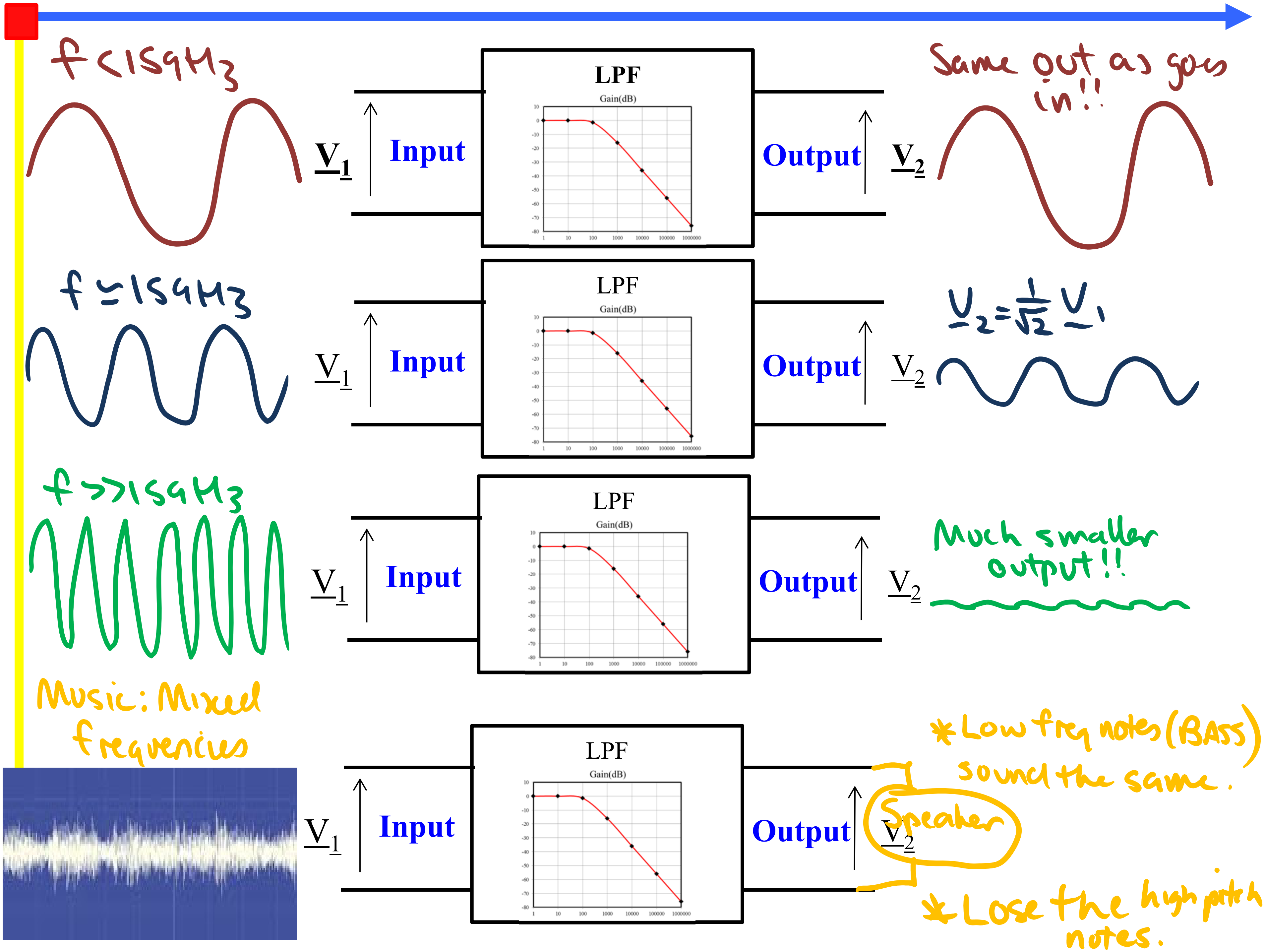
in dB: $20log_{10}\left(\frac{1}{\sqrt{2}}\right) = -3$

Higher frequencies are "blocked" (attenuated)

Lower frequencies pass through unaltered

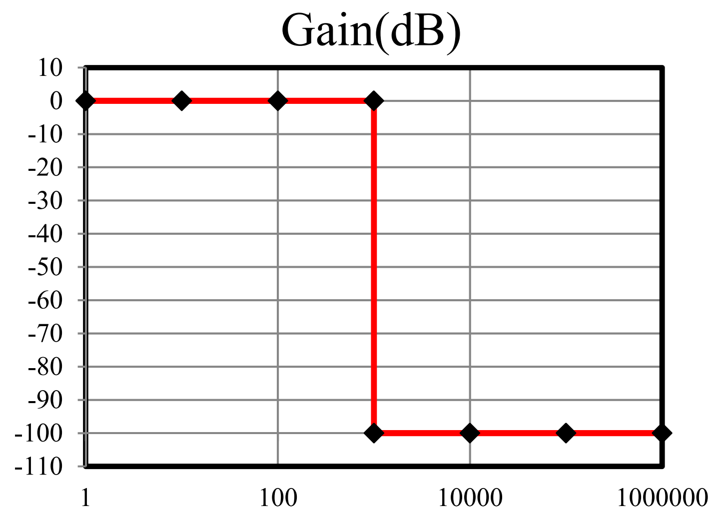
Cut-off Frequency = 159 Hz

We call this a low-pass filter

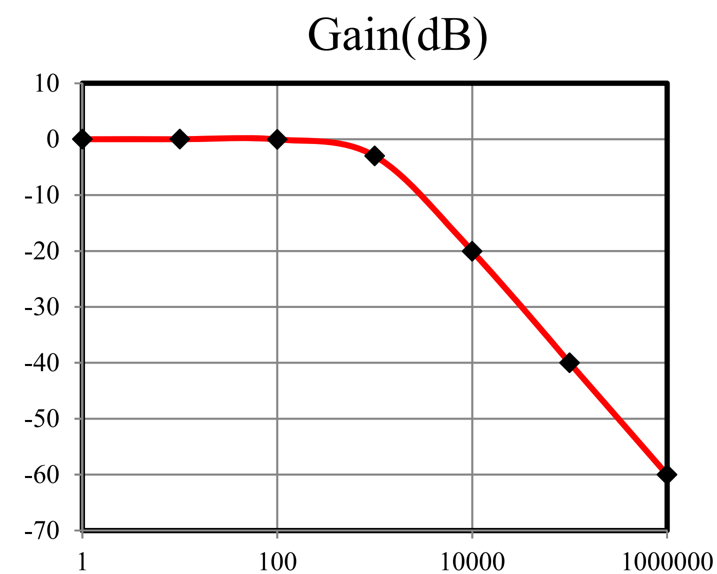

f < 159Hz
LPF
Gain(dB)
V1
Input
Output
V2
Same out as goes in!!
f ≈ 159Hz
V2 = 1/√2 V1
f >> 159Hz
Much smaller output!!
Music: Mixed frequencies
* Low freq notes (BASS) sound the same.
Speaker
* Lose the high pitch notes.

Low Pass Filter – Frequency Response

- "Ideal" Filter with a cut-off Frequency at 1 kHz:

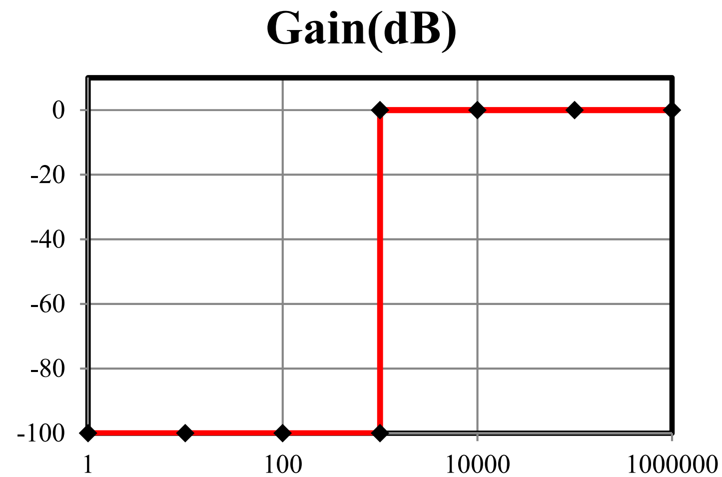


- Simple RL filter with a cut-off at 1KHz:

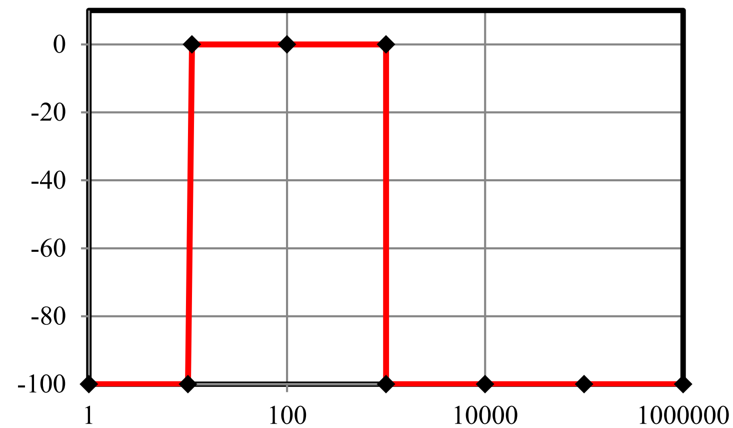


Other Ideal Filters:

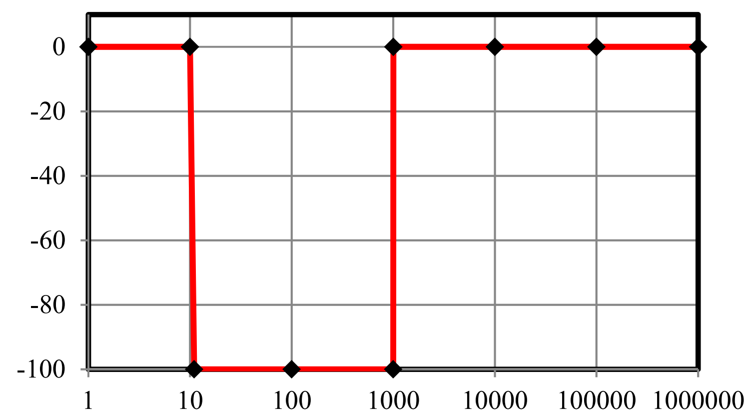
- High Pass:



- Band Pass:



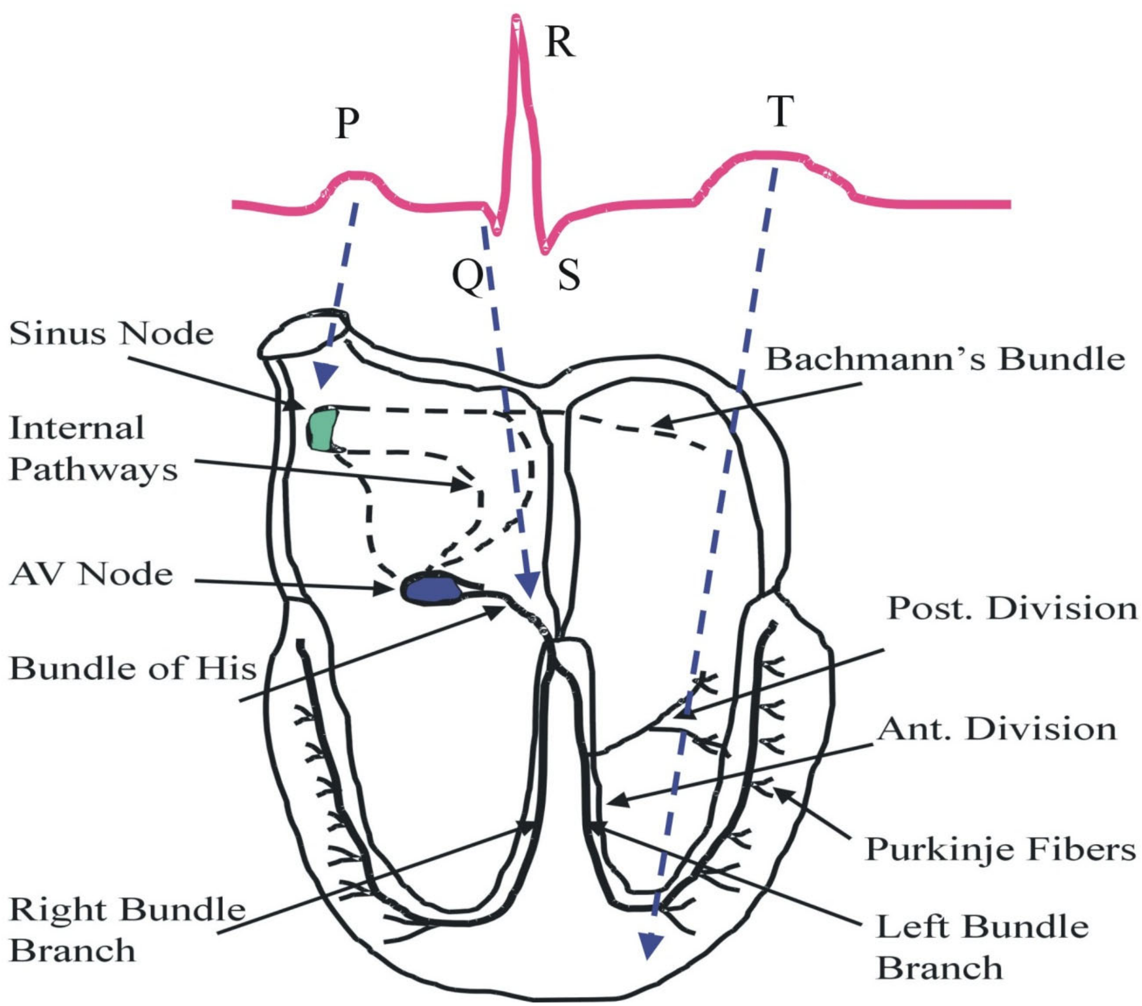
- Band Stop:



Can you think any practical applications of filters?

- In other words, why would we want to remove certain frequencies from a voltage signal?

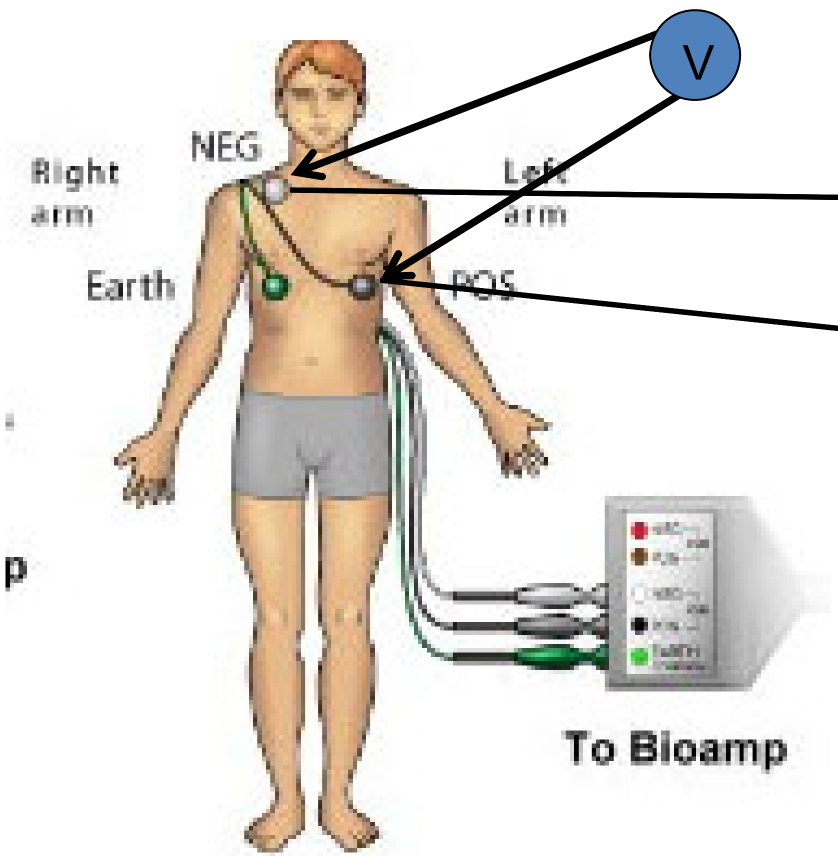
Filtering Example – ECG Example

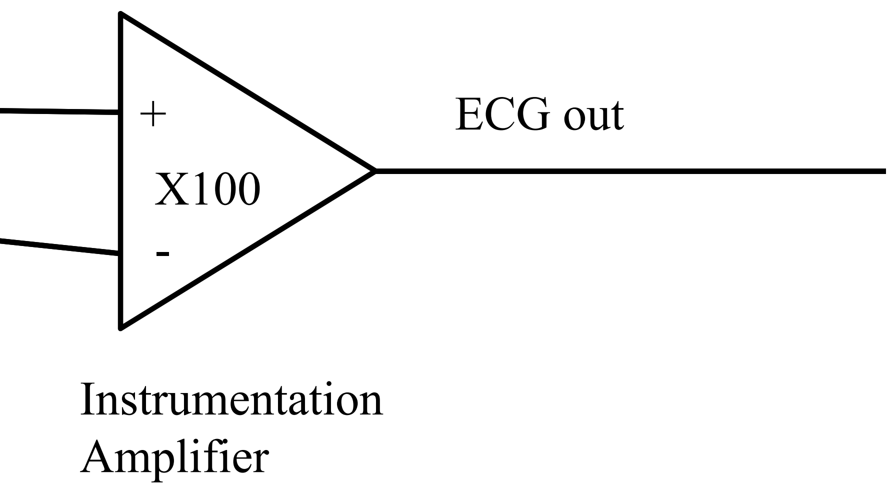


http://www.meditech.cn/meditech-edu/ecg-2.asp

Electrocardiogram (ECG)

- Measures electric field, on skin surface, generated by heart as it contracts

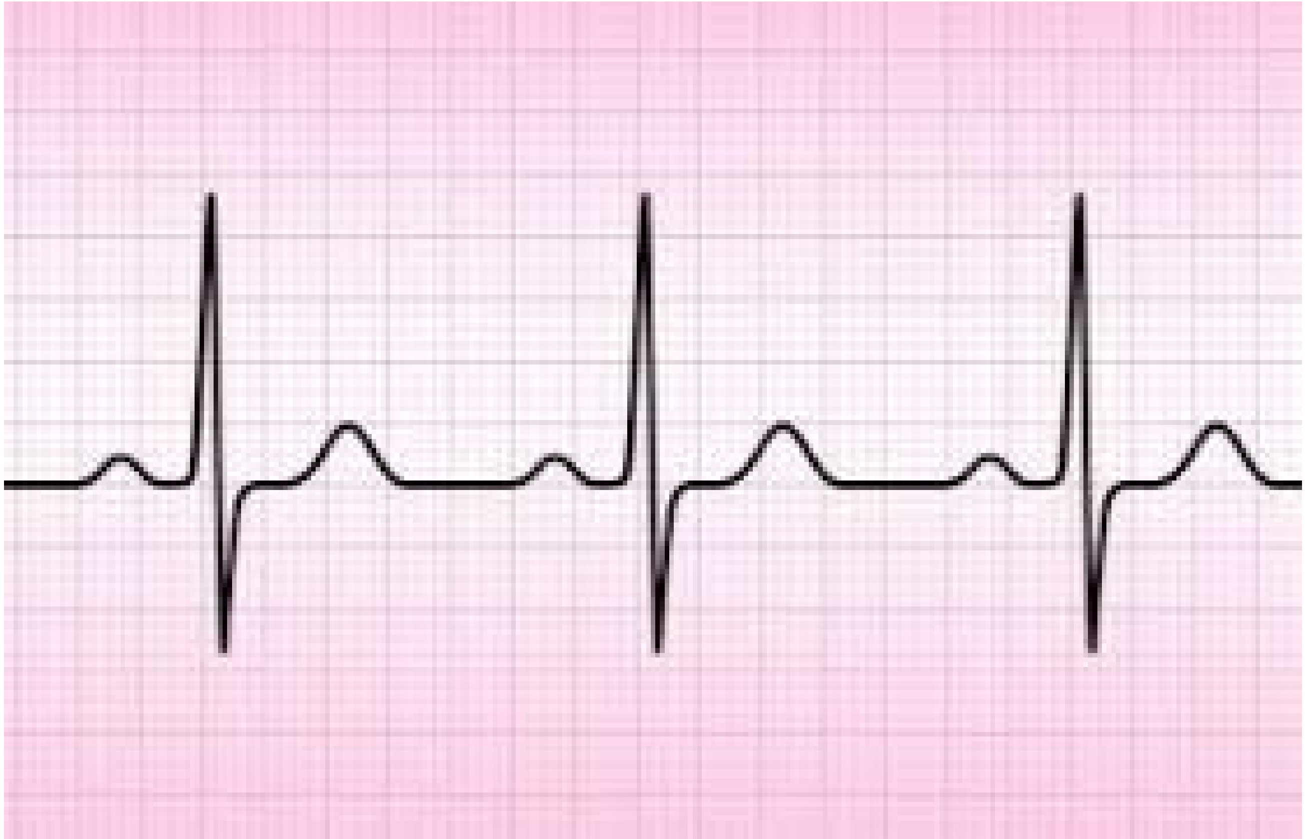




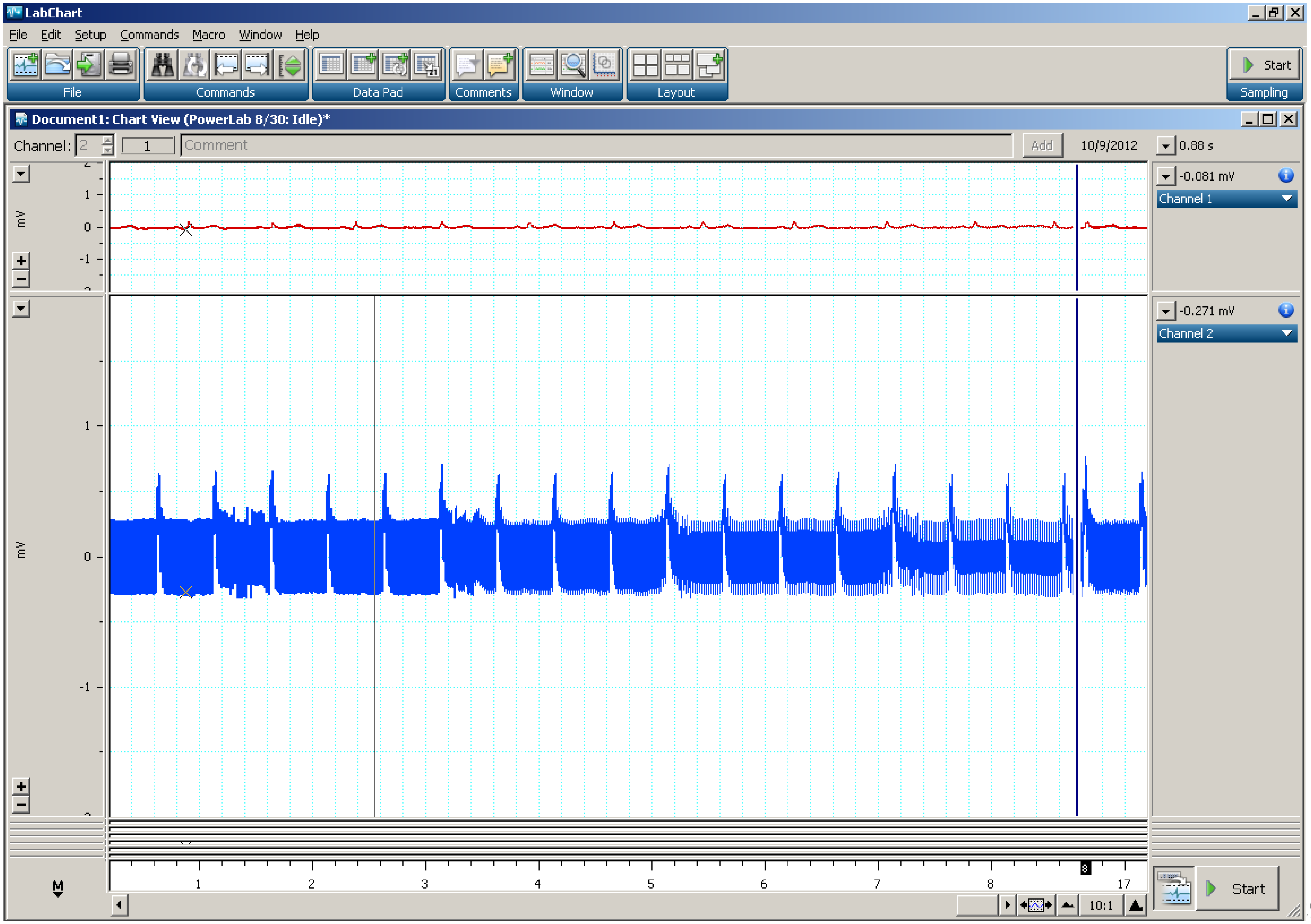
http://www.adinstruments.com/solutions/images_new/ecg_connect.jpg

Can think about this as an "amplified" oscilloscope for measuring the small voltages of the heart.
See ELEC2400, BIOE6403

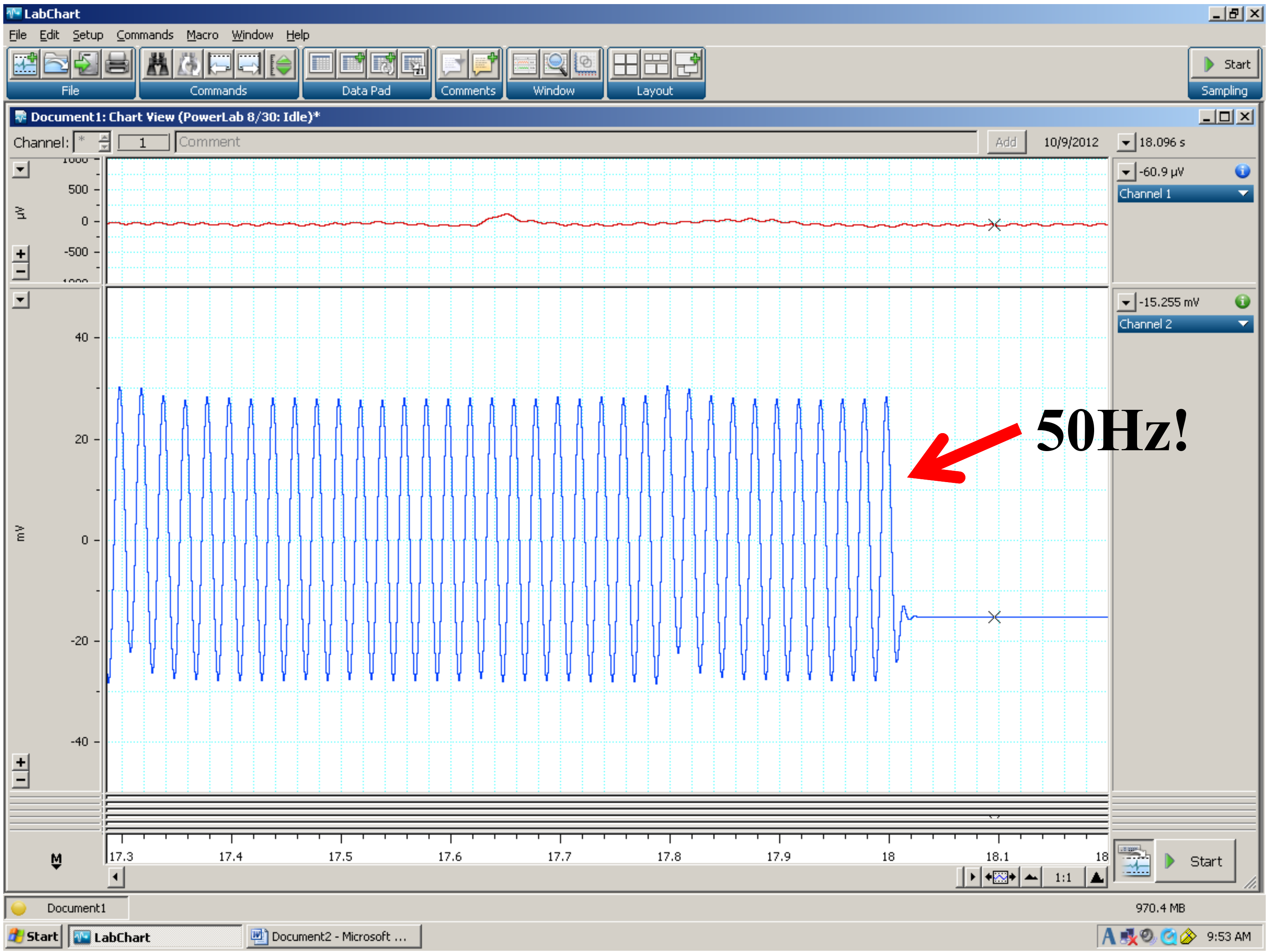
Text book ECG Trace:



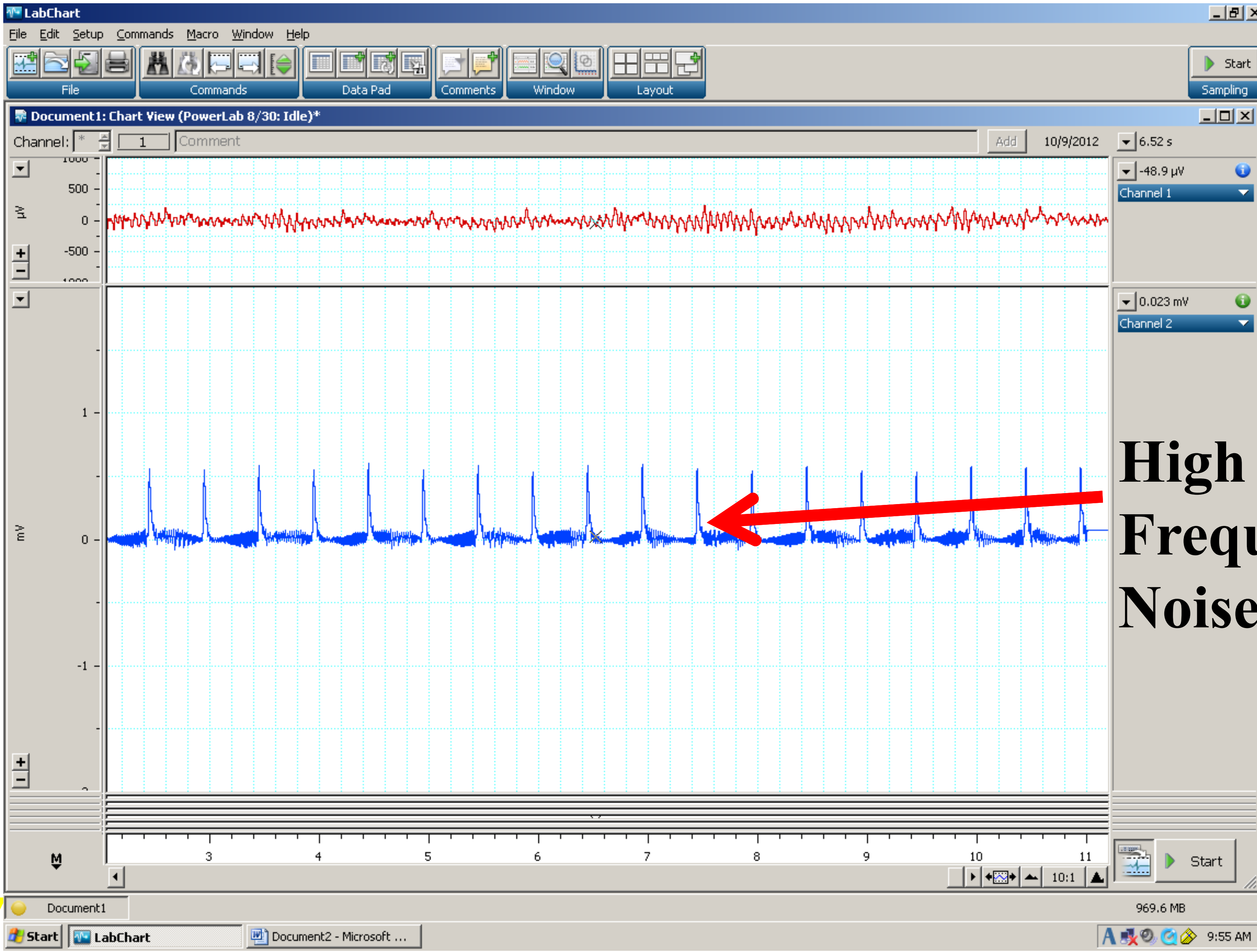
ECG Example: In the real world



ECG Example: Zoomed in:

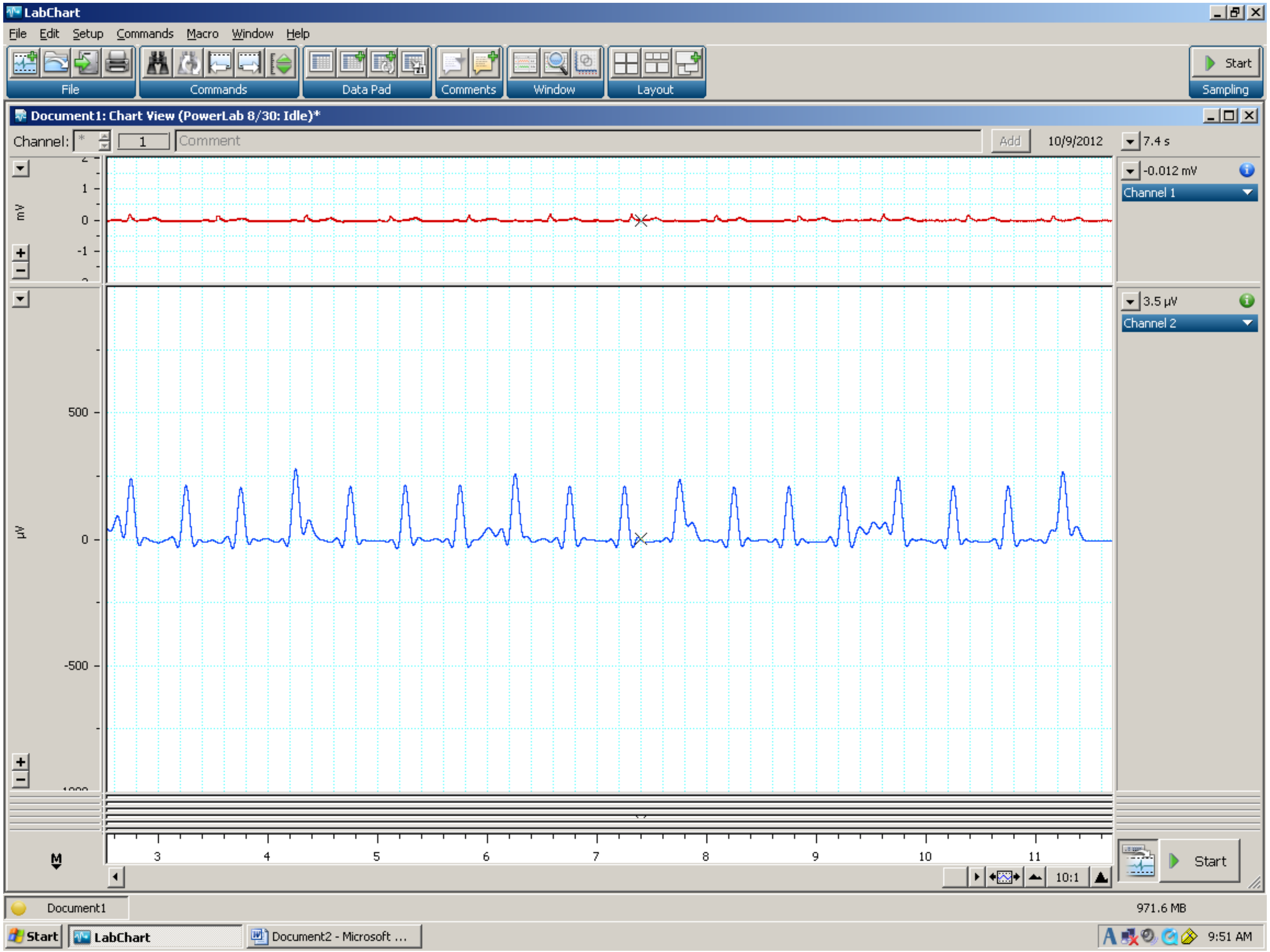


ECG Example: 50Hz Band Stop Filter



High Frequency Noise

ECG Example: 10Hz LPF:



This week ahead

- Prac 7A:
  - Transfer functions and frequency response of filters
- Prac 7B:
  - Transfer functions and frequency response of filter cont. and applications of filters to audio signals
- Flexible Students: Classes are back on-campus, see: https://about.uq.edu.au/coronavirus
- Until 12-noon Thursday 15th of April:
  - You need to bring your masks. Should be worn when social distancing is not possible.
  - Exemptions from mask wearing include where you need to be able to easily communicate such as teaching
  - Please "check-in" to practical rooms with QR codes
  - Continue to wash or sanitise your hands on entry to, and exit from our ENGG1300 practical room