ENGG1300
Introduction to Electrical Systems
Week 2

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For further information:

- PPL 3.20.06 Recording of Teaching at UQ
- UQ website: https://my.uq.edu.au/information-and-services/information-technology/software-and-web-apps/software-uq/zoom

Last Week

- Electric Quantities
  - Voltage, Current, Resistance, Conductance, Power
- Components and their Laws:
  - Ohm's law (Resistors) , Voltage & Current Sources, Open & Closed Circuits
- Circuit Laws: KCL, KVL, Power
- Series & Parallel Resistors
- Voltage & Current Dividers
- Solving Simple Circuits

- Introduction to the electronics lab and equipment

The Wider Picture

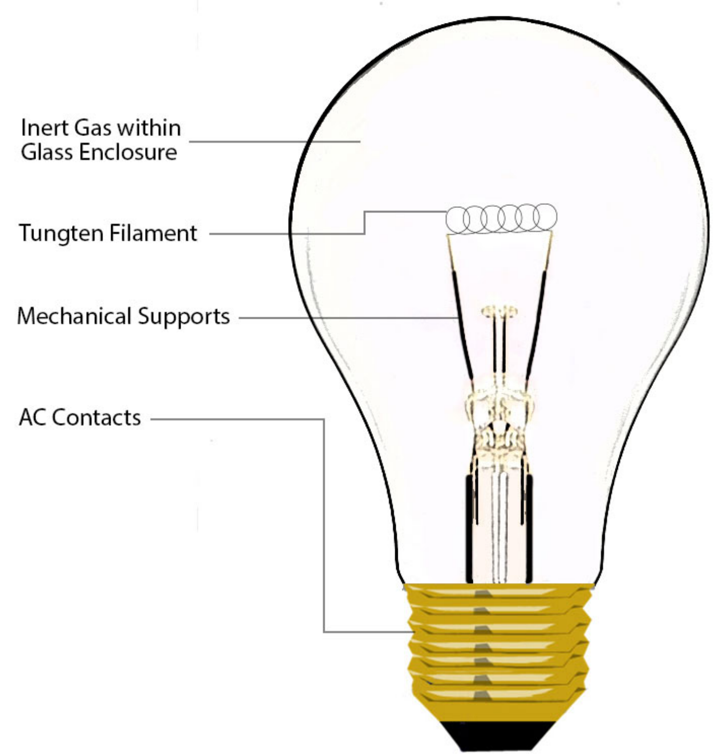
- **WHY?? What use are resistor networks?**

- Resistors are important circuit elements in their own right (which are essential in the design of electronic circuitry) but only one of many elements we'll learn about.
- Resistors are the simplest elements to analyse:
  - We can begin to understand voltage, current and power relationships in circuits.
  - No matter how complex a resistor network is, we can reduce it to a one-port resistor model (successive series and parallel simplifications).
  - With voltage and current dividers we can then find voltage and current at each branch of circuit.
- Resistors are equally important as *models* of the electrical behaviour of other circuit elements, *e.g.*, the light bulb.

Resistors in practice

Heater Element

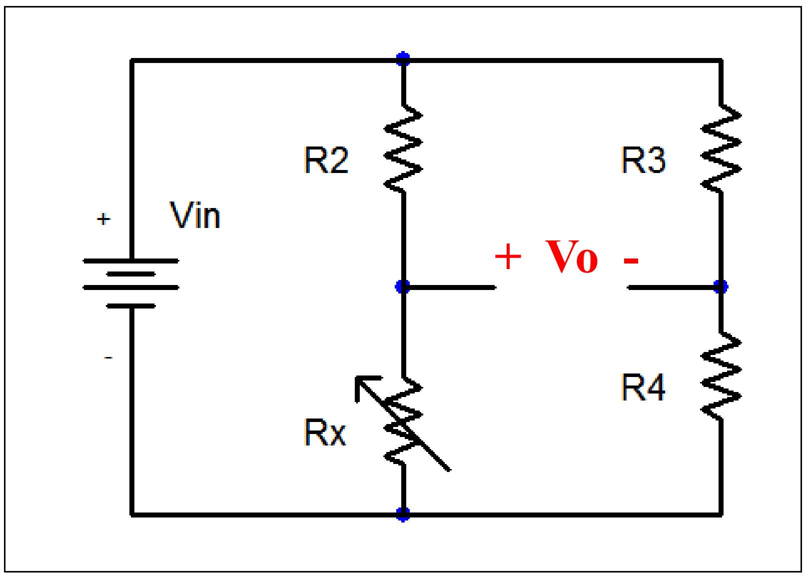
Incandescent Lamp

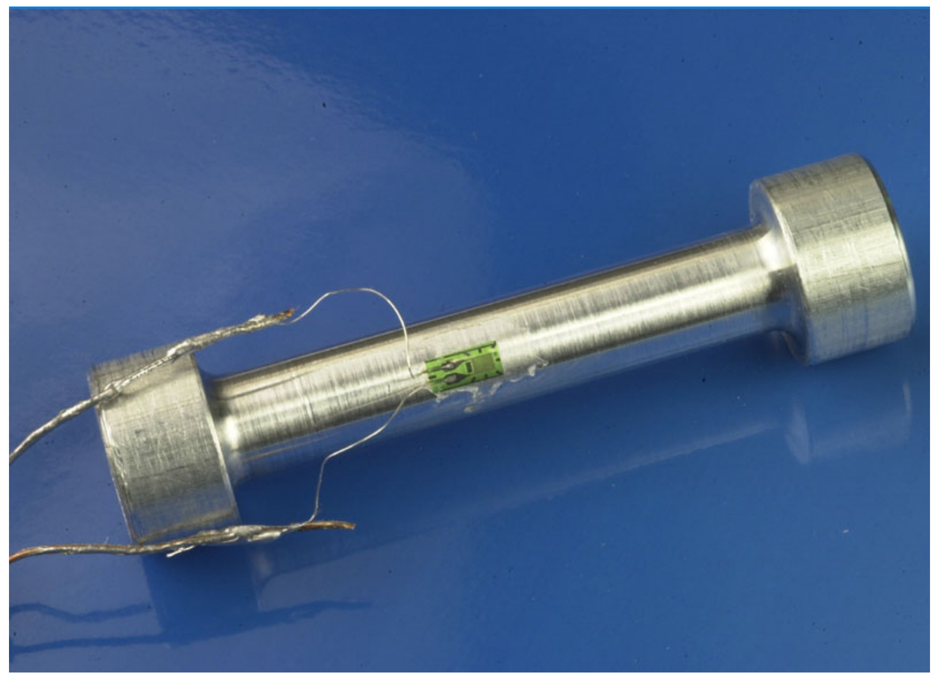


http://www.microchip.com/pagehandler/en-us/technology/intelligentLighting/technology/incandescent.html

Resistors in practice

- Resistive Sensors (*e.g.*, strain gauge, light-dependent resistor)

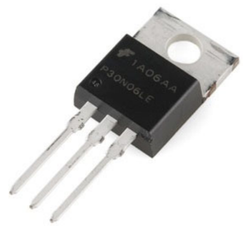


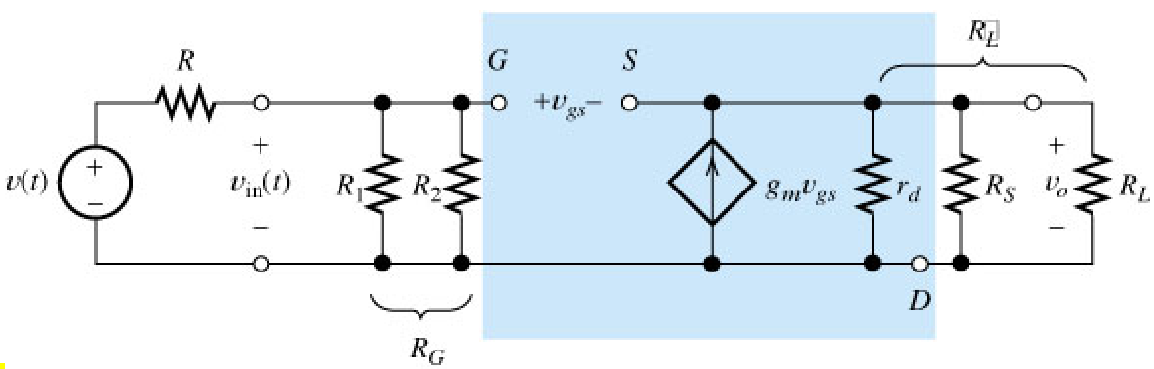


http://www.doitpoms.ac.uk/tlplib/BD3/images/strain-gauge-full.jpg

Limitations of Week 1 Analysis:

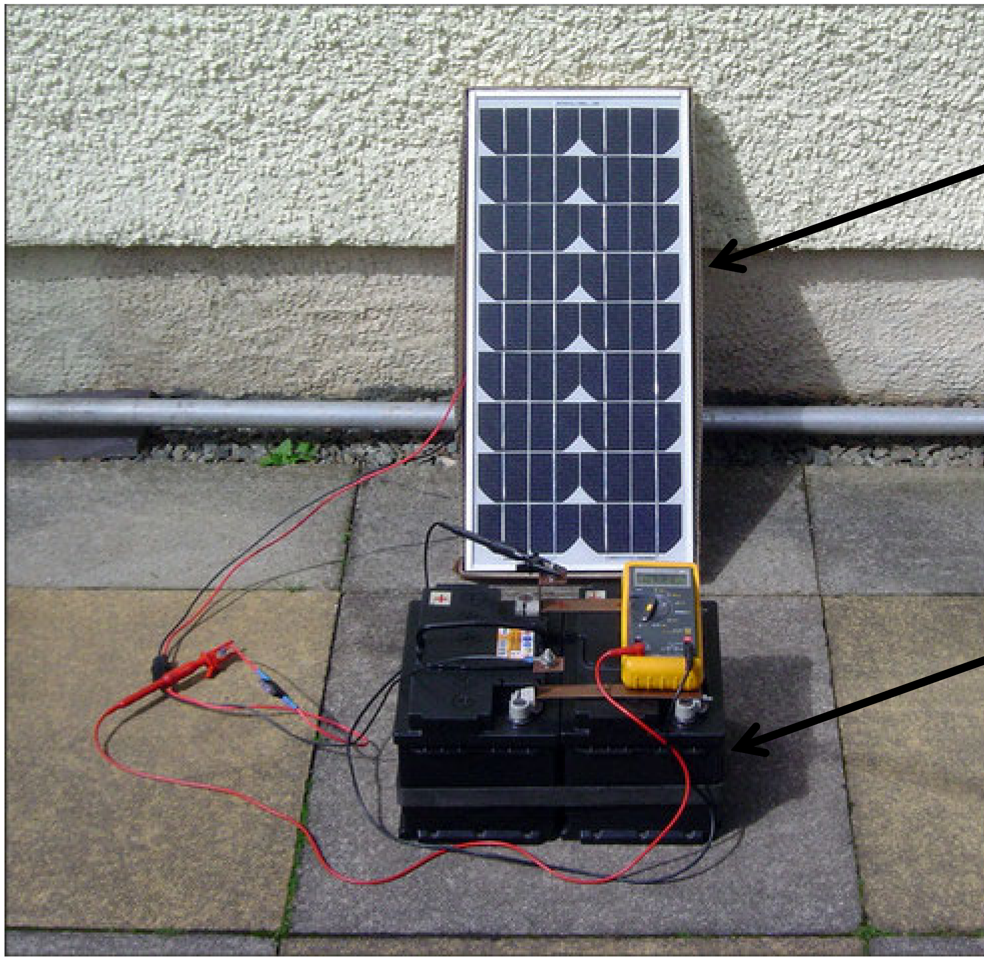
- Limited number of practical systems are modelled with resistors only
- Thus need Systematised method for predicting the behaviour of circuits with multiple sources
- E.g. Amplifiers





Charging 12 V Batteries with Solar Panel

Regulator: Modelled as an amplifier, which we model with sources + resistors



Modelled as a current source

Modelled as a voltage source

http://www.nowpublic.com/tech-biz/solar-panel-battery-trickle-charging-2w0daa

This Week

- For more complex circuits, with multiple sources, we need more structured ways to solve a circuit.
- If a circuit has 10 unknowns (currents and voltages), then we will need a structured way to get these 10 equations to find these 10 unknown variables.

- It is also useful to have a structured way to solve these equations quickly by hand.
- Ultimately these structured methods also help us to *computationally simulate* and *solve* circuits (i.e. a structured *algorithm* that codes the method to solve any arbitrary circuit).

This Week

- We will introduce and apply two important methods for solving circuits – these will be used throughout the course:
  - Nodal analysis: first solves for node voltages [*exploits systematised KCL at each node over the circuit*] and then calculates branch currents.
  - Mesh analysis: first solves for a specific type of current called mesh currents [*exploits systematised KVL at each loop of a circuit*], then solves for branch currents and voltages.

This Week

- Lab 2A (nodal analysis)
- Lab 2B (mesh analysis)
- Labs will also continue use of lab equipment: please make sure you are having a go a this!
- Pre-reading: videos, examples, prep-quiz’s
- On-line Quiz 1: Due 4pm Monday 1st March (today).
- On-line Quiz 2: Due 4pm next Monday 8th March.

Node vs. Mesh Analysis

- If we know all the node voltages in a circuit, we can easily calculate the branch voltage differences, and the branch currents:

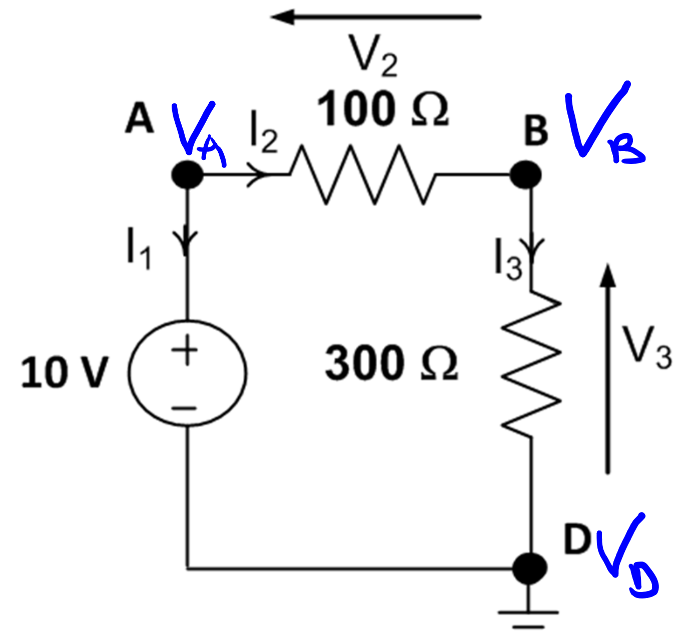
$$V_2 = V_A - V_B, \quad I_2 = \frac{V_2}{100}$$

$$V_3 = V_B - V_D, \quad I_3 = \frac{V_3}{300}$$

- Similarly, if we know the currents, we can subsequently calculate the branch and node voltages in a circuit:

$$V_2 = I_2 \times 100, \quad V_3 = I_3 \times 300$$

$$V_B = V_3; \quad V_A = V_2 + V_3$$



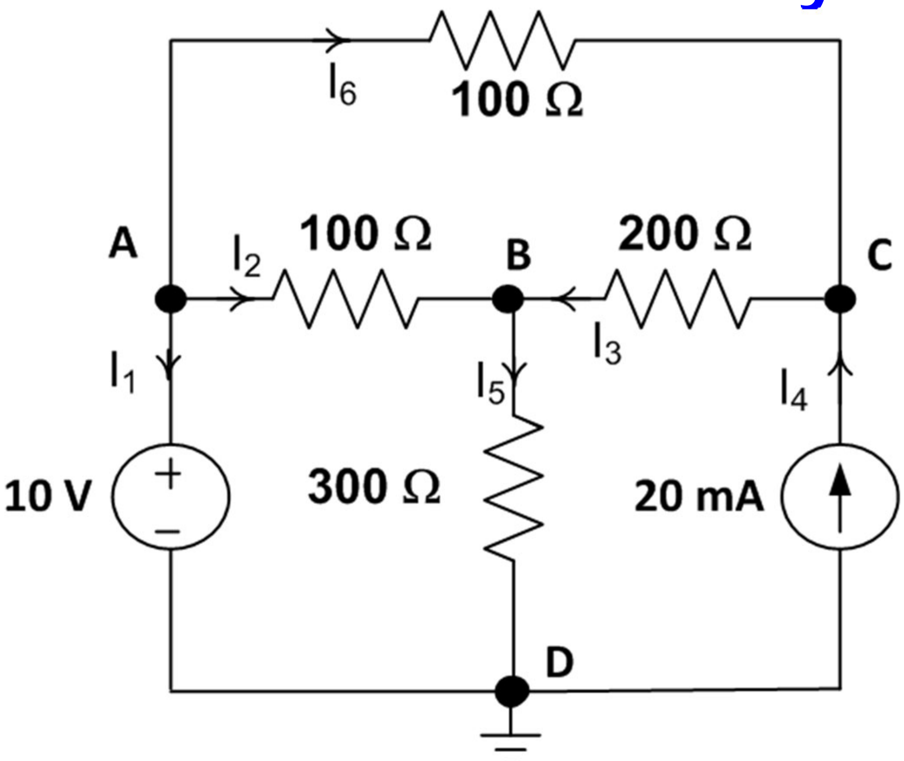
- So we can first solve for node voltages by systematically applying Kirchoff's current law at each node (Nodal Analysis);
- Or, we can first solve for branch currents by systematically applying Kirchoff's voltage law to each loop of the circuit (Mesh analysis).

Nodal Analysis

We will use a method with 7 specific steps to follow to get the necessary equations & solve them:

1. Voltage at ground node = 0V
2. Equations for voltage & current sources
3. Equations for Resistors
4. Apply KCL at all nodes
5. Eliminate currents from equations, and solve for node voltages
6. Solve for branch currents
7. If needed, solve for branch voltages and powers

Nodal Analysis - Example



List Unknowns

$V_A = 10V$ $I_1 =$

$V_B =$ $I_2 =$

$V_C =$ $I_3 =$

$V_D = 0V$ $I_4 = 0.02A$

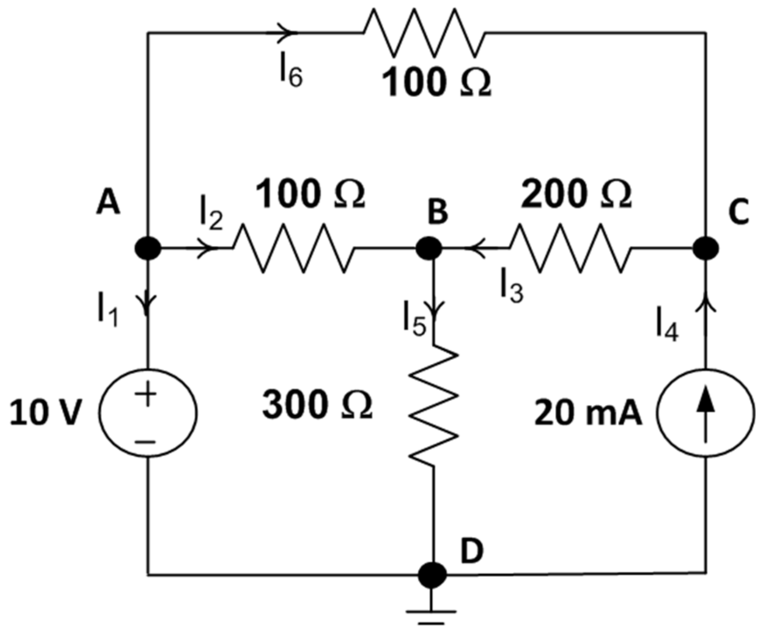
$I_5 =$

$I_6 =$

* Step 1: Voltage at ground node = 0
* Step 2: Component equations for voltage + current sources

↳ $V_A - V_D = 10V \Rightarrow V_D = 0, V_A = 10V.$

↳ $I_4 = 20mA = 0.02A$



$V_A = 10V$ $I_1 =$

$V_B =$ $I_2 =$

$V_C =$ $I_3 =$

$V_D = 0V$ $I_4 = 0.02A$

$I_5 =$

$I_6 =$

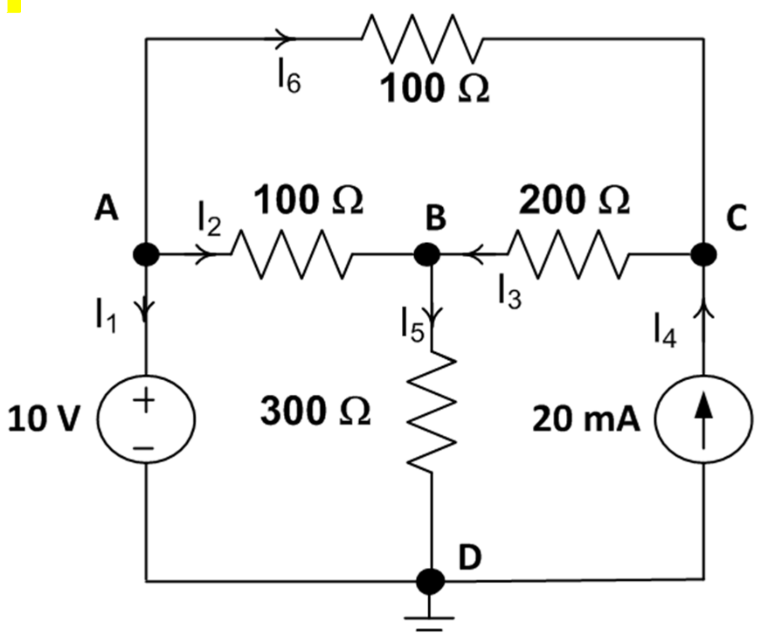
Step 3: Component equations for resistors:

(i.e: relate node voltages to branch current for each resistor)

* $I_2 = \frac{(V_A - V_B)}{100}$ ⇒ but $V_A = 10$ ⇒ $I_2 = \frac{(10 - V_B)}{100}$

* $I_3 = \frac{(V_C - V_B)}{200}$ * $I_5 = \frac{(V_B - V_D)}{300}$ ⇒ But $V_D = 0$ ⇒ $I_5 = \frac{V_B}{300}$

* $I_6 = \frac{V_A - V_C}{100}$ ⇒ but $V_A = 10$, ⇒ $I_6 = \frac{(10 - V_C)}{100}$



$V_A = 10V$ $I_1 =$

$V_B =$ $I_2 =$

$V_C =$ $I_3 =$

$V_D = 0V$ $I_4 = 0.02A$

$I_5 =$

$I_6 =$

$I_2 = \frac{(10 - V_B)}{100}$ ①

$I_3 = \frac{(V_C - V_B)}{200}$ ②

$I_5 = \frac{V_B}{300}$ ③

$I_6 = \frac{(10 - V_C)}{100}$ ④

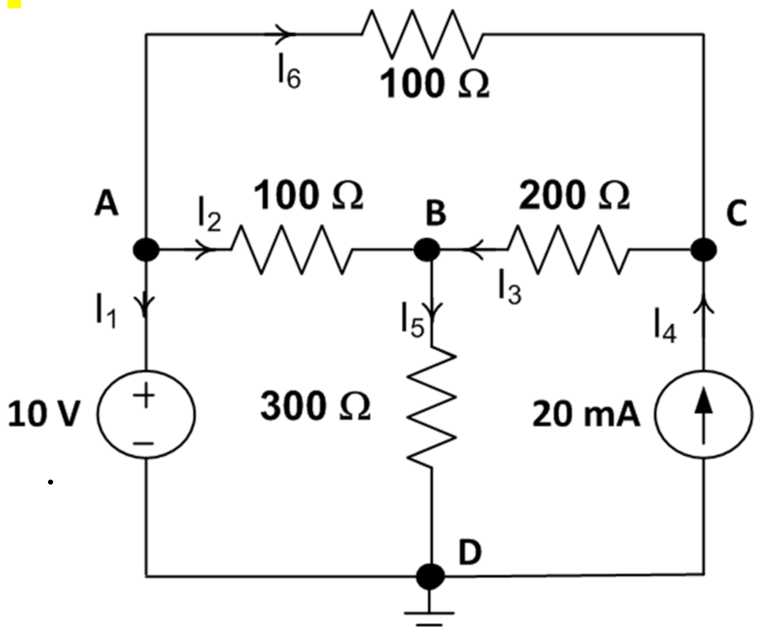
Step 4: Apply Nodal Analysis:

↳ Node (A): $I_1 + I_2 + I_6 = 0$ ⑤

↳ Node (B): $I_2 + I_3 = I_5$ ⑥

↳ Node (C): $I_4 + I_6 = I_3$ ⑦

Step 5: Eliminate currents from nodal equations to solve for node voltages [i.e: substitute component equations into node equations]



$V_A = 10V$ $\quad I_1 =$

$V_B =$ $\quad I_2 =$

$V_C =$ $\quad I_3 =$

$V_D = 0V$ $\quad I_4 = 0.02A$

$I_5 =$

$I_6 =$

$I_2 = \dfrac{(10 - V_B)}{100}$ (1)

$I_3 = \dfrac{(V_C - V_B)}{200}$ (2)

$I_5 = \dfrac{V_B}{300}$ (3)

$I_6 = \dfrac{(10 - V_C)}{100}$ (4)

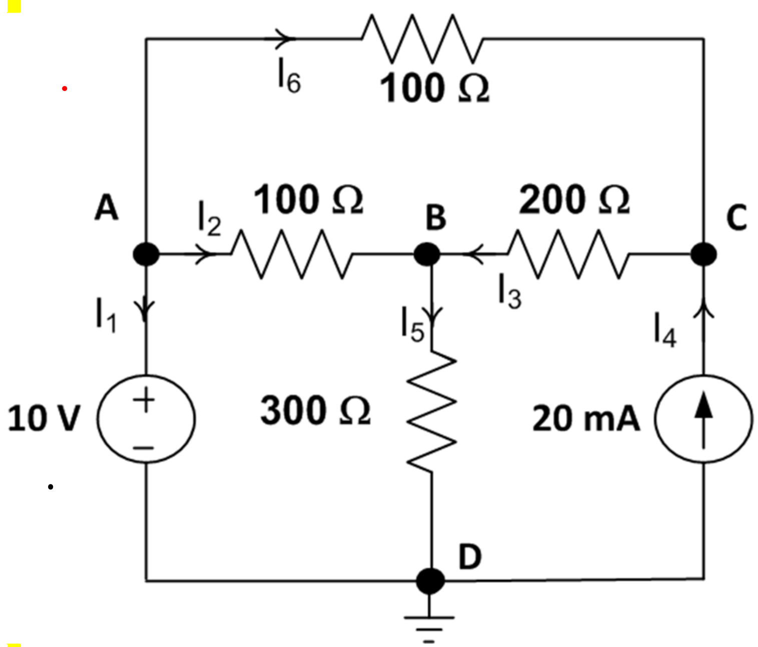
Step 5: Eliminate currents from nodal equations to solve for node voltages

(5) $I_1 + I_2 + I_6 = 0 \Rightarrow I_1 + \dfrac{10 - V_B}{100} + \dfrac{10 - V_C}{100} = 0 \Rightarrow I_1 = \dfrac{V_B + V_C}{100} - 0.2$ (8)

(6) $I_2 + I_3 = I_5 \Rightarrow \dfrac{10 - V_B}{100} + \dfrac{V_C - V_B}{200} = \dfrac{V_B}{300} \Rightarrow 60 + 3V_C = 11V_B$ (9)

(7) $I_4 + I_6 = I_3 \Rightarrow 0.02 + \dfrac{10 - V_C}{100} = \dfrac{V_C - V_B}{200} \Rightarrow 3V_C = 24 + V_B$ (10)

Solve equations (9) and (10) for V_B and V_C



$V_A = 10V$ $\quad I_1 = -0.008A$ $\quad I_2 = \dfrac{(10 - V_B)}{100}$ (1)

$V_B = 8.4V$ $\quad I_2 = 0.016A$ $\quad I_3 = \dfrac{(V_C - V_B)}{200}$ (2)

$V_C = 10.8V$ $\quad I_3 = 0.012A$ $\quad I_5 = \dfrac{V_B}{300}$ (3)

$V_D = 0V$ $\quad I_4 = 0.02A$ $\quad I_6 = \dfrac{(10 - V_C)}{100}$ (4)

$I_5 = 0.028A$

$I_6 = -0.008A$

$60 + 3V_C = 11V_B$ (9) $\quad 3V_C = 24 + V_B$ (10) [solve simultaneously]

↳ $V_B = 8.4$ $\quad$ ↳ $V_C = 10.8$

$I_1 = \dfrac{V_B + V_C}{100} - 0.2$ (8) [sub in V_B and V_C] $\Rightarrow I_1 = -0.008$

$I_2 = \dfrac{(10 - V_B)}{100} \Rightarrow I_2 = \dfrac{10 - 8.4}{100} = 0.016$ $\quad I_3 = \dfrac{(V_C - V_B)}{200} \Rightarrow I_3 = \dfrac{10.8 - 8.4}{200} = 0.012$

$I_5 = \dfrac{V_B}{300} \Rightarrow I_5 = \dfrac{8.4}{300} = 0.028$ $\quad I_6 = \dfrac{(10 - V_C)}{100} \Rightarrow I_6 = \dfrac{10 - 10.8}{100} = -0.008$

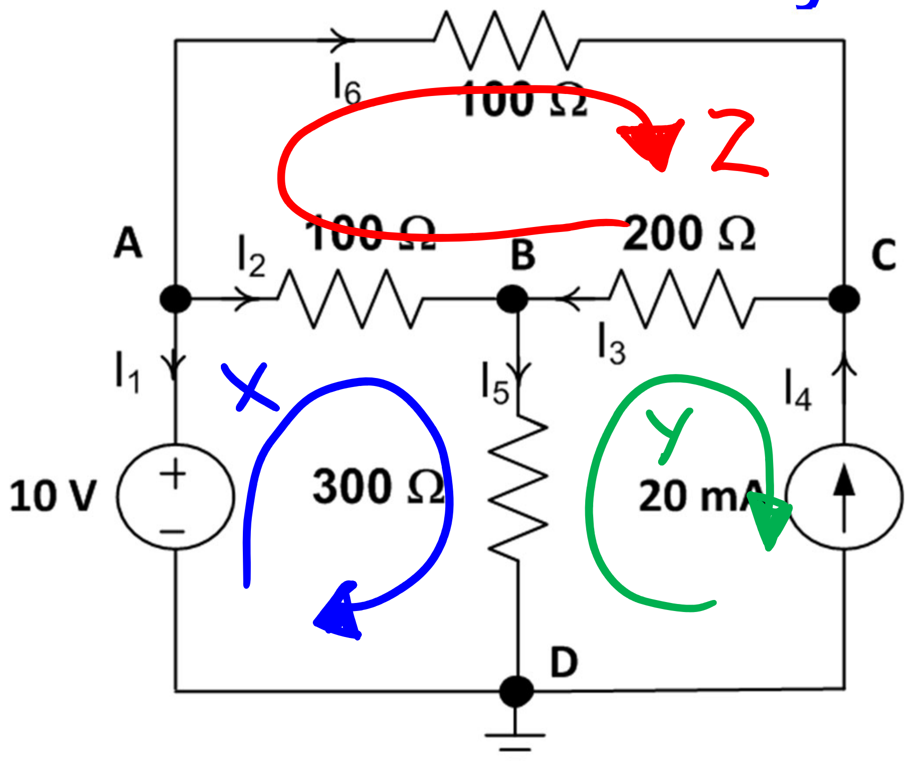
Mesh Analysis

- This is an alternative method for solving circuits.
- It can provide quicker solutions for circuits with current sources, otherwise nodal analysis is usually easier to use.
- A mathematical concept called "mesh currents" is used – these are not branch currents in the circuit, but conceptual current around a loop which enable us to construct a set of algebraic equations
  - The first step is to identify KVL-style loops around the circuit.
  - We then define a "mesh current" or loop current that flows around each loop.
  - Because a mesh current flows into and out of each node in the loop, they automatically obey KCL (i.e. we systematically apply KVL and can ignore KCL).
  - The real current in a branch is the sum of all the mesh currents flowing in that branch.

Mesh Analysis – 7 steps

1. Identify Loops, and relate mesh currents to physical currents
2. Write loop currents known from current sources
3. Write KVL equations around each loop in terms of mesh currents
4. Solve mesh current equations
5. Convert Mesh currents to Branch Currents
6. Go around loops, starting at ground to find node voltages
7. Calculate branch voltages and powers if needed.

Mesh Analysis - Example



<u>List Unknowns</u>

$V_A =$ $\quad$ $I_1 =$

$V_B =$ $\quad$ $I_2 =$

$V_C =$ $\quad$ $I_3 =$

$V_D =$ $\quad$ $I_4 =$

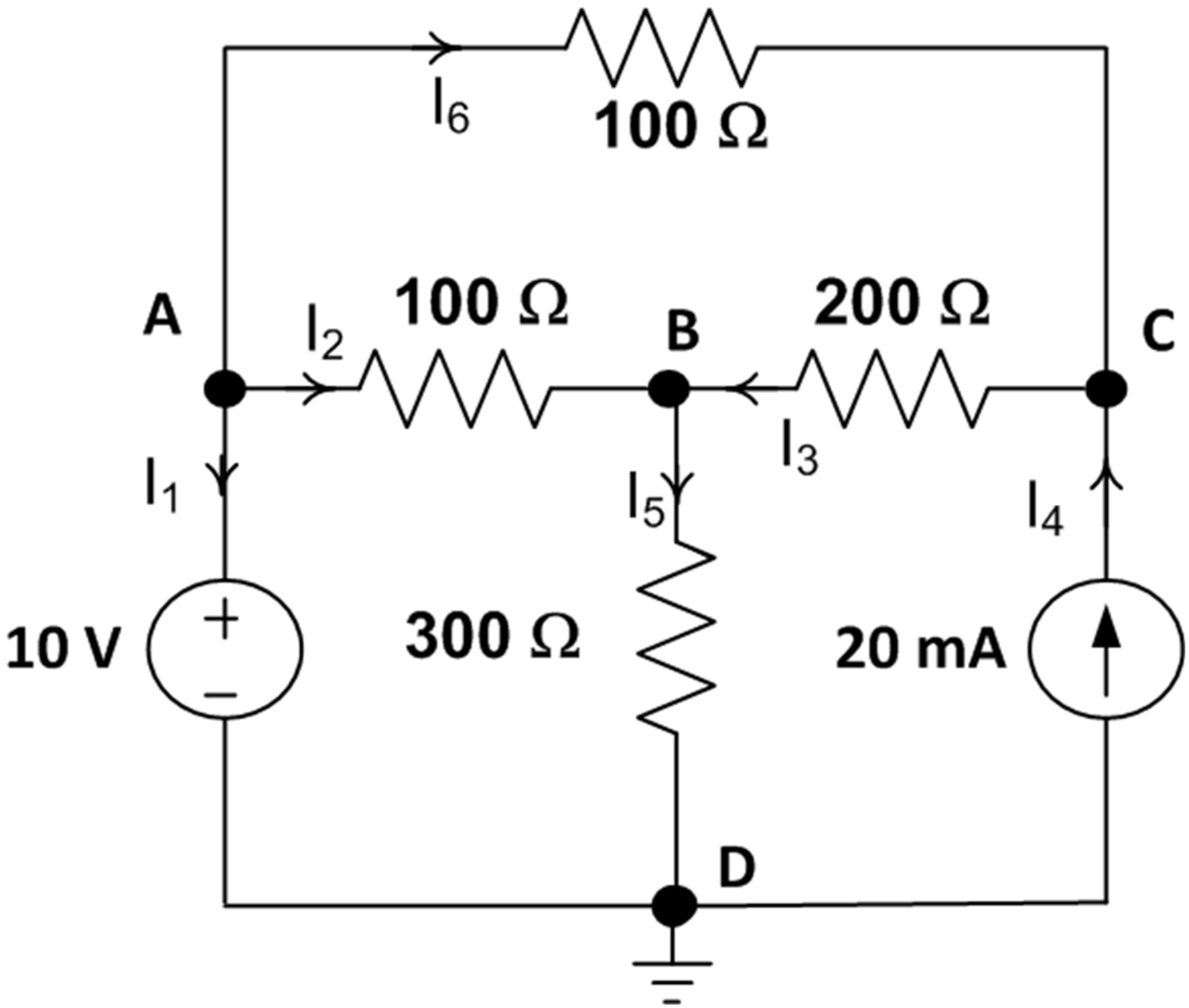
$I_5 =$

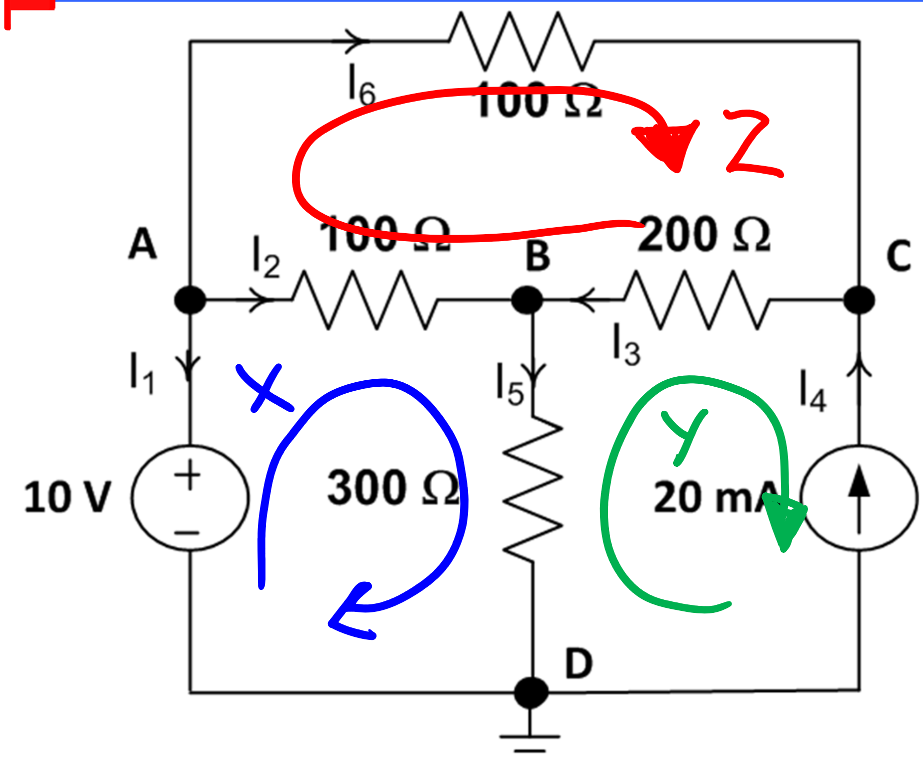
$I_6 =$

*<u>Step 1:</u> Identify Loops

- * minimise number of loops that current sources appear in
- * Enough loops to include each element at least once.

Other Loops?





$V_A = 10V$ $\quad I_1 =$ $\quad I_x =$

$V_B =$ $\quad I_2 =$ $\quad I_y = -0.02$

$V_C =$ $\quad I_3 =$ $\quad I_z =$

$V_D = 0V$ $\quad I_4 = 0.02A$

$I_5 =$

$I_6 =$

Step 1b: Relate Mesh currents to physical currents (note signs!)

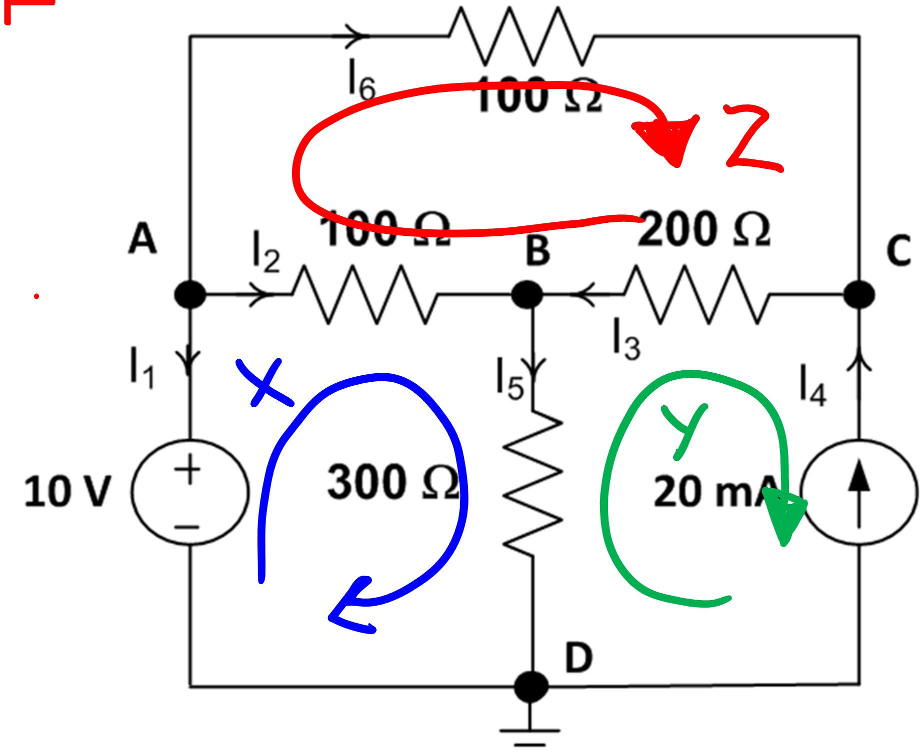
* $I_1 = -I_x$
* $I_2 = I_x - I_z$
* $I_3 = I_2 - I_y$
* $I_4 = -I_y$
* $I_5 = I_x - I_y$
* $I_6 = I_z$

* Step 2: Component equations for sources

↳ $I_y = -I_4 = -0.02A$

↳ $V_A - V_D = 10V \Rightarrow V_D = 0$

$V_A = 10V.$



$V_A = 10V$ $\quad I_1 = -I_x$ $\quad I_x =$

$V_B =$ $\quad I_2 = I_x - I_z$ $\quad I_y = -0.02$

$V_C =$ $\quad I_3 = I_2 - I_y$ $\quad I_z =$

$V_D = 0V$ $\quad I_4 = -I_y$

$I_5 = I_x - I_y$

$I_6 = I_z$

Step 3: KVL around loops (avoid loops with current source):

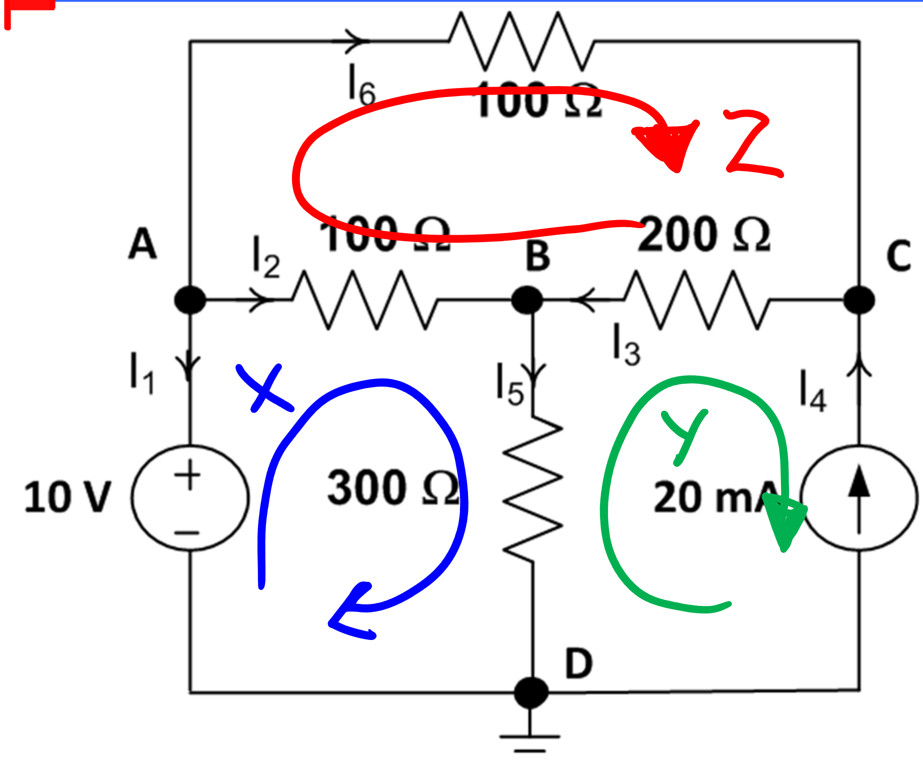
Loop X: $10 = 100 \times I_2 + 300 \times I_5 \Rightarrow 10 = 100(I_x - I_z) + 300(I_x - I_y)$

$\Rightarrow 10 = 400 I_x - 100 I_z - 300 I_y$

Loop Z: $0 = I_6 \times 100 + I_3 \times 200 - I_2 \times 100 \Rightarrow 0 = 100(I_z) + 200(I_z - I_y) - 100(I_x - I_z)$

$\Rightarrow 0 = 400 I_z - 100 I_x - 200 I_y$

With $I_y = -0.02$: $\ast\ 4 = 400 I_x - 100 \times I_z$ (1) $\qquad \ast\ -4 = 400 \times I_z - 100 \times I_x$ (2)



$V_A = 10V$ $I_1 = -I_x$ -8mA $I_x = 0.008$

$V_B =$ $I_2 = I_x - I_z$ 16mA $I_y = -0.02$

$V_C =$ $I_3 = I_z - I_y$ 12mA $I_z = -0.008$

$V_D = 0V$ $I_4 = -I_y$ 20mA

$I_5 = I_x - I_y$ 28mA

$I_6 = I_z$ -8mA

Step 4: Solve For Mesh Currents:

* $4 = 400I_x - 100 \times I_z$ (1) * $-4 = 400 \times I_z - 100 \times I_x$ (2) [solve simultaeously]

↳ $I_x = 0.008$ ↳ $I_z = -0.008$

Step 5: Solve for Branch Currents:

* $I_1 = -I_x = -8mA$

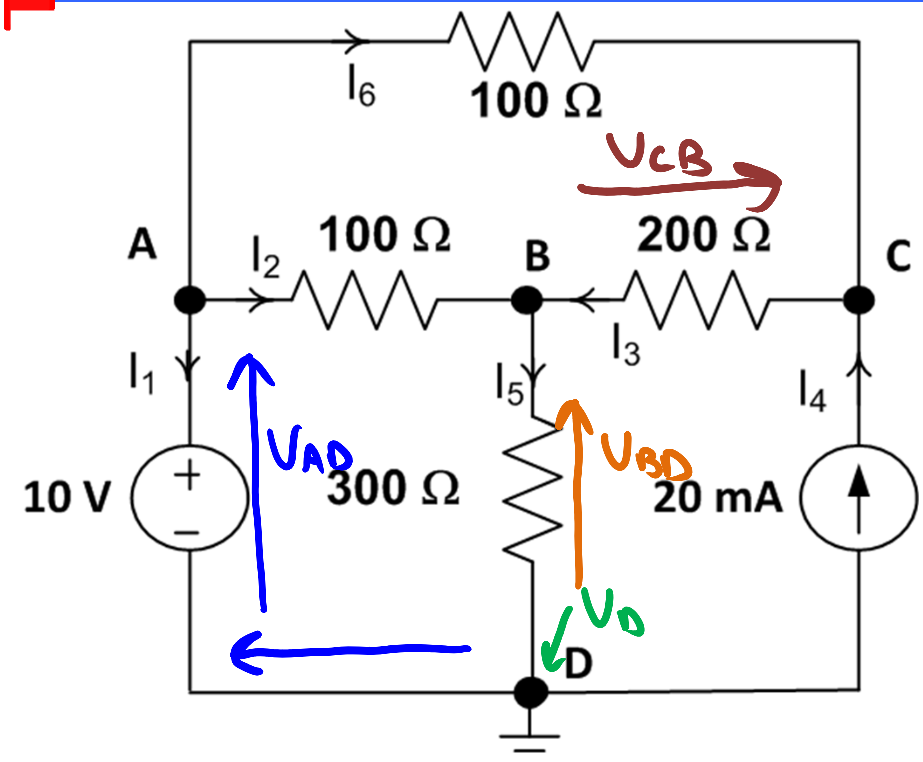
* $I_2 = I_x - I_z = 16mA$

* $I_3 = I_z - I_y = 12mA$

* $I_4 = -I_y = 20mA$

* $I_5 = I_x - I_y = 28mA$

* $I_6 = I_z = -8mA$



$V_A = 10V$ $I_1 = 8mA$ $I_x = 0.008$

$V_B = 8.4V$ $I_2 = 16mA$ $I_y = -0.02$

$V_C = 10.8V$ $I_3 = 12mA$ $I_z = -0.008$

$V_D = 0V$ $I_4 = 20mA$

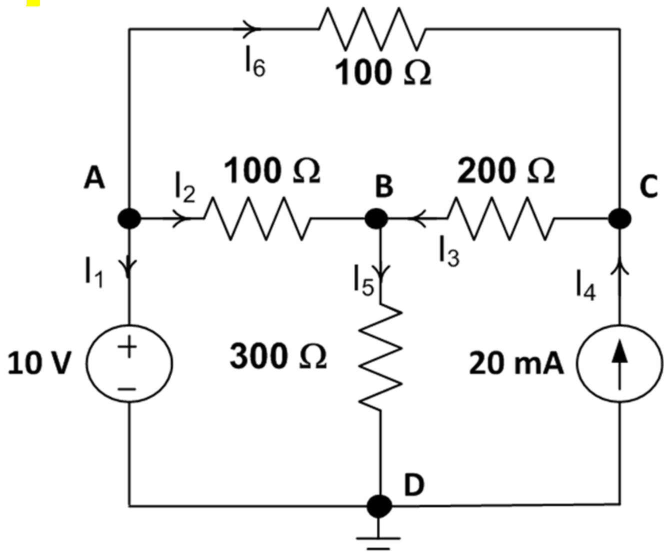
$I_5 = 28mA$

$I_6 = -8mA$

Step 6: Solve for node voltages:

* $V_D = 0$
* $V_A = V_D + V_{AD} = 0 + 10 = 10V$
* $V_B = V_D + V_{BD} = 0 + I_5 \times 300 = 8.4V$
* $V_C = V_B + V_{CB} = 8.4 + I_3 \times 200 = 10.8V$

Step 7: Solve for power (if necessary)



$V_A=10$
$V_B = 8.4$
$V_C = 10.8$
$V_D=0$

I_1=-8mA
I_2=16mA
I_3=12mA
I_4=20mA
I_5=28mA
I_6=-8mA

$P = V \times I$

$\Downarrow$

For resistors:

$V = I \times R$

$P = \frac{V^2}{R}$ $\quad$ $P = I^2 R$

Power (positive means consumed, negative means supplied)

- $P_1 = (V_A - V_D) \times I_1 = 10 \times -0.008 = -0.08 = -80$ mW
- $P_2 = (I_2)^2 \times 100 = (0.016)^2 \times 100 = 0.0256 = 25.6$ mW
- $P_3 = (I_3)^2 \times 200 = (0.012)^2 \times 200 = 0.0288 = 28.8$ mW
- $P_4 = (V_D - V_C) \times I_4 = -10.8 \times 0.02 = -0.216 = -216$ mW
- $P_5 = (I_5)^2 \times 300 = (0.028)^2 \times 300 = 0.2562 = 235.2$ mW
- $P_6 = (I_6)^2 \times 100 = (-0.008)^2 \times 100 = 0.0064 = 6.4$ mW

CHECK : -80 + 25.6 + 28.8 -216 + 235.2 + 6.4 = 296 – 296 = 0

(i.e: power supplied = power consumed)

Summary

- Different textbooks, and notes will use slightly different "steps" in these methods (and there are some short-cuts).
- Nodal Analysis solves first for node voltages, then currents (*systematic application of KCL*)
- Mesh Analysis solves first for currents, then for voltages (*systematic application of KVL*)
- Usually only use mesh analysis when we have current sources (immediate solution for the respective mesh current!)
- You need to be able to apply both methods: Different methods are quicker for different circuits.
  – While on the final exam you will *generally* have the choice to apply whichever method you like, on the mid-semester exam you may be asked to derive node or mesh equations for arbitrary circuits.
- Practice is the only way to get good at it.