ENGG1300
Introduction to Electrical Systems
Week 4 – Time Varying Signals

Lecturer:

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Recording of this Presentation

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- **Turn off video and mute audio**
- **Use a proxy name for Zoom (student name will still be on record with the Course Coordinator)**

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For further information:

- PPL 3.20.06 Recording of Teaching at UQ
- UQ website: https://my.uq.edu.au/information-and-services/information-technology/software-and-web-apps/software-uq/zoom

CRICOS code: 00025B

Online Mid-Semester Test

- Contributes 15% towards course grade
- Commences at 8am, Monday 29th March (Week 6 lecture slot).
- The exam conditions are as follows:
  - Delivered **online** via blackboard (i.e. you will sit this exam at home) [same system as weekly quizzes]
  - 115 minutes in duration commencing at 8am:
    - 10-mins reading + 90-mins working + 15-mins "Submission Time" [additional time to allow for minor technical problems in navigating online system].
  - It will contain 30 multiple choice questions, each worth ONE (1) mark.
  - You will be permitted approved (and labelled) scientific calculators (https://my.uq.edu.au/services/manage-my-program/exams-and-assessment/sitting-exam/approved-calculators)
  - This is an **open-book** exam, and you can refer to written or electronic notes, textbooks, and recorded media.
- A direct link from the left-hand menu in blackboard will be provided.

Mid-semester Exam

- While you will be completing the exam online rather than in an invigilated exam room, the following guidelines apply:
  - You should complete the exam under strict exam conditions. This means you are not permitted to communicate with anyone else during the exam period. **You must not consult with other people (whether online or in person)** [The exception to this is to contact the course coordinator should you be having technical difficulties].
  - Other than the permitted calculator, you are not permitted to use other electronic calculation devices. This includes graphics calculators; circuit simulation software (i.e. LT-spice), websites or apps; excel, Matlab or other programming languages.
  - Breaching these guidelines will be considered as academic misconduct, and has serious disciplinary consequences.
- There will be a 0-mark question which is a declaration that you have sat the test under the specified conditions, and the work is entirely your own. You will receive 0 marks on the exam unless you complete this declaration.

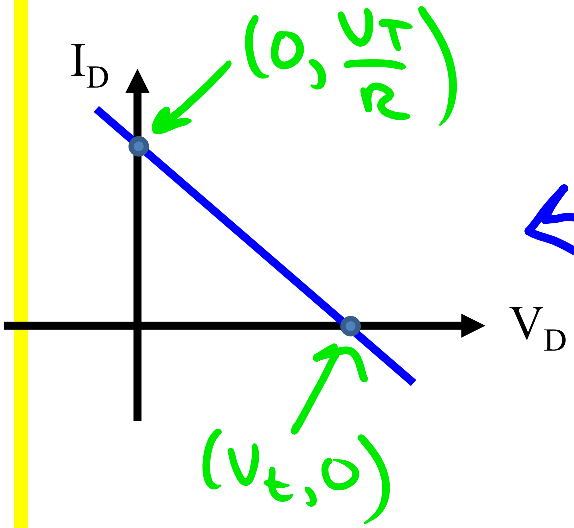
Test Coverage

- This assessment will examine content presented in lectures and laboratory classes in weeks 1-4 of semester, i.e. general topics include:
  - DC Circuit Analysis
  - DC Norton and Thevenin equivalent circuits
  - Load-line analysis including non-linear elements such as LED's
  - Capacitors
  - Inductors
  - Sinusoids and AC voltages
- Past and Practice Exams available on blackboard: "Assessment" ->"Midsemester Test"
  - Practice online exam
  - 2020 Semester 2 Online Exam
  - These both include questions on complex impedances and phasor circuits which will not be covered in your exam.
- 2019 and earlier past exams are a good indication of content and question style (even though these exams were on-campus) – [some do and don't include week 5 content on complex impedances and phasors
- [Note, practice exam and past papers will guide structure and content, but QUESTIONS WILL BE DIFFERENT].

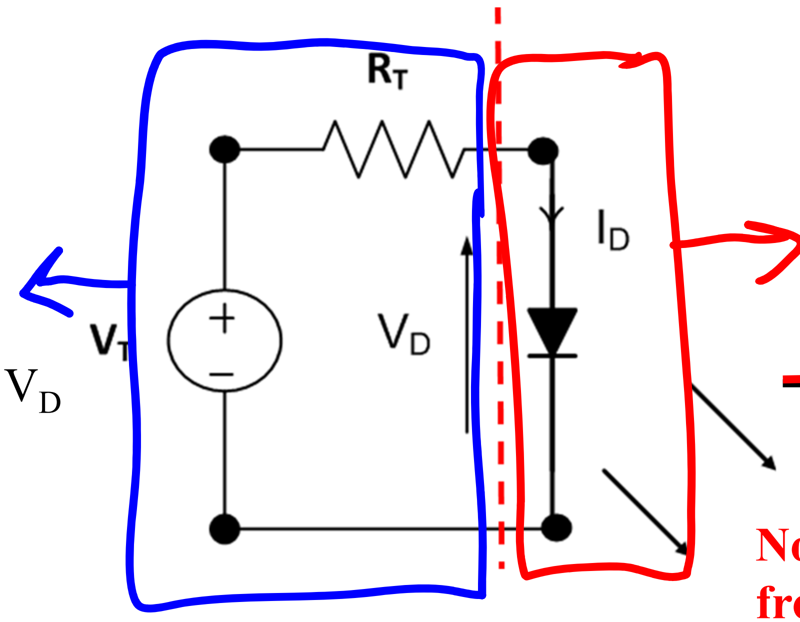
Last Week

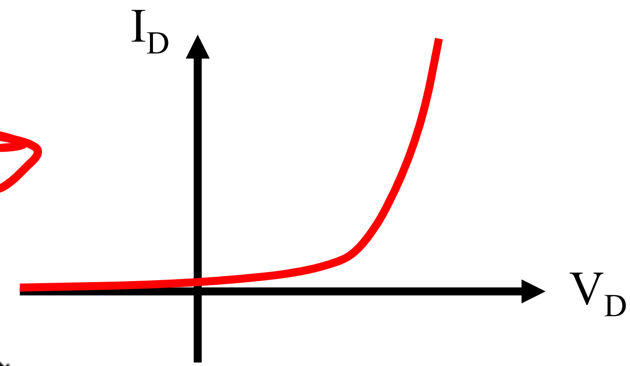
- Superposition is one more tool in our circuit solution toolbox –
  - Reduce circuit to multiple, smaller problems (which can each be solved using simplification of resistors)
- Thevenin and Norton equivalent circuits allow us to model complex linear one-port networks with a simpler circuit.
  - We can use this as a way reducing a circuit for analysis (i.e. as we have done for equivalent models for resistors)
  - Or we can use this as a “module” which we can connect to any other circuit including linear components like resistors, but also non-linear components like LED’s

Load-line for non-linear components

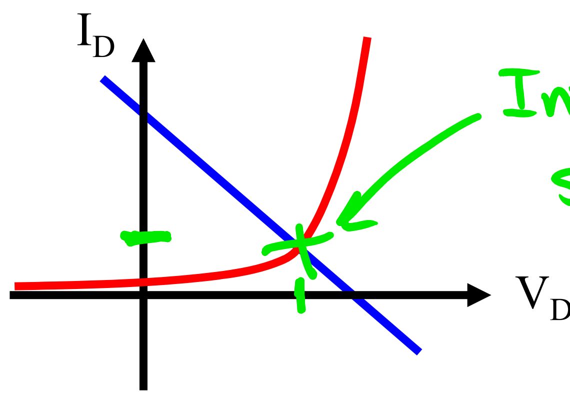


$I_D = (V_t - V_D)/R_T$



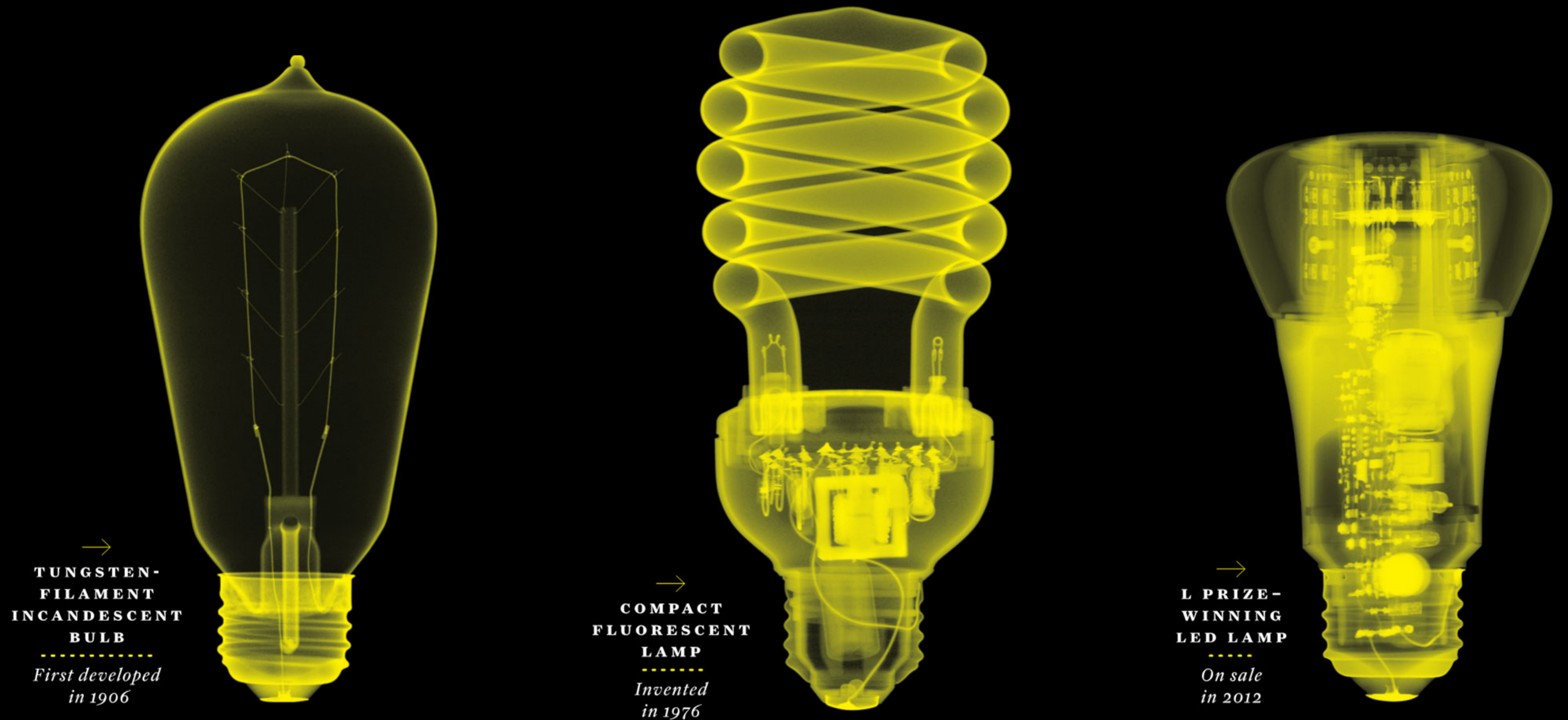


Note: We can get this curve from the datasheet, or we can measure it



Intersection is the solution to circuit

What Lighting is Best: Incandescent, Florescent, LED



- Inefficient: 14 lumens/Watt
- Short life: 1,000 hours
- Instant on & dimmable
- Relatively benign materials
- 'Warm' broad-spectrum light

- Efficient: 60 lm/W
- Long life: 10,000 hours
- Noticeable start-up & not as dimmable
- Contains mercury
- Greenish line spectrum modified by phosphors

- Very efficient: 100lm/W
- Ultralong life: 25,000hrs
- Instant on & dimmable
- Needs DC (recall load line!)—complicated drive circuit
- Blue narrow spectrum modified by phosphors
- High power devices difficult

From: [B. Weir, 'Driving the 21st Century's Lights', *IEEE Spectrum*, pp. 42–47, Mar. 2012]

Limitations of Week 1, 2 and 3 Analysis

- We have learnt a range of techniques to efficiently model the behaviour of circuits containing resistors and **DC** voltage and current sources:
  - Ohm's law
  - KVL and KCL
  - One port resistor models
  - Voltage and Current divider rules
  - Node and Mesh analysis
  - Superposition
  - Norton and Thevenin

Recognise techniques which work best on a particular circuit

- In labs, we have learnt the fundamental techniques for measuring the behaviour of DC circuits, and developed the analytical skills to interpret the differences between predicted and actual behaviour
- **However, we have not explored the behaviour of time-varying (i.e. AC) circuits.**

Why Time-Varying Circuits?

We exploit the properties of time-varying circuits across a huge range of practical applications…

This Week

- We look at circuits with currents and voltages that vary with time. In particular we look at:
  - New Components: capacitors, inductors
  - Component Laws; Circuit Laws (KCL, KVL)
  - Sinusoidal Voltages and Currents
  - Complex (imaginary numbers) to model circuits – "Phasors"
- Week 4A – Laboratory (VII): Introduction to the oscilloscope and function generator (first time varying lab); or AC simulations in LT-Spice
- Week 4B – Laboratory (VIII): Investigating RC circuits using laboratory equipment or simulation.
- Quiz 3 due today (Monday), 4pm
- Quiz 4 due next Monday, 4pm.

Time Varying Circuits

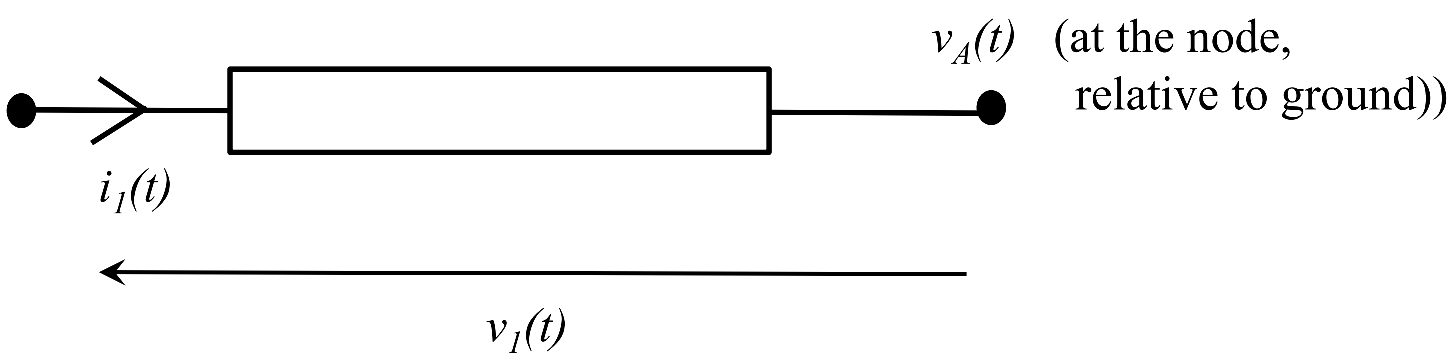
- DC circuit analysis deals with static (constant with time) voltages and currents. These are useful for DC power transmission.
- However, often we are interested in circuits with "time-varying" currents and voltages, for three reasons:
  1. We are interested in what happens with DC circuits when we switch them on and off ("Transient Response" – Often the period of "peak load" in a DC circuit) - (ELEC2004)
  2. We are interested in AC Power Distribution, where voltages and currents are sinusoidal.
  3. We are interested in Information Signals (audio, video, data, sensor information) where information is encoded in signals that vary with time.

Time Varying Circuits

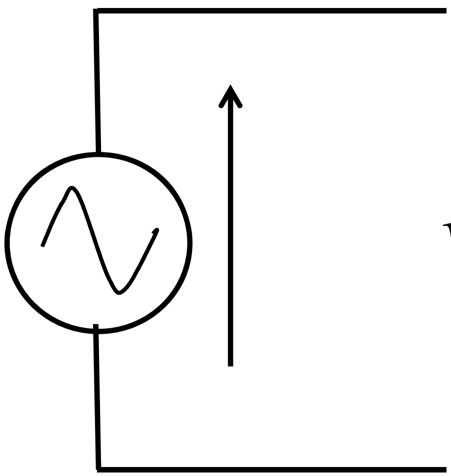
- We often call circuits with time-varying voltages and currents AC circuits (AC – "Alternating Current"),
  - [even if the quantities are not necessarily alternating in polarity].
- We usually use lower case ***v*** and ***i*** for time varying voltages and currents, e.g.

$$\boldsymbol{v(t)} = 340 \sin (100.\pi.t) \text{ volts}$$

- We'll label nodes, branches and nodes of circuits accordingly:



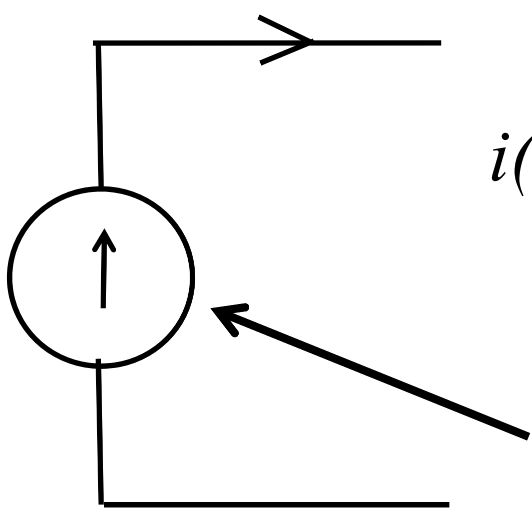
Time Varying Sources



Time varying voltage source:

$v(t)$ = some predefined voltage,

e.g. $v(t) = 100.\cos(100t)$ *volts*



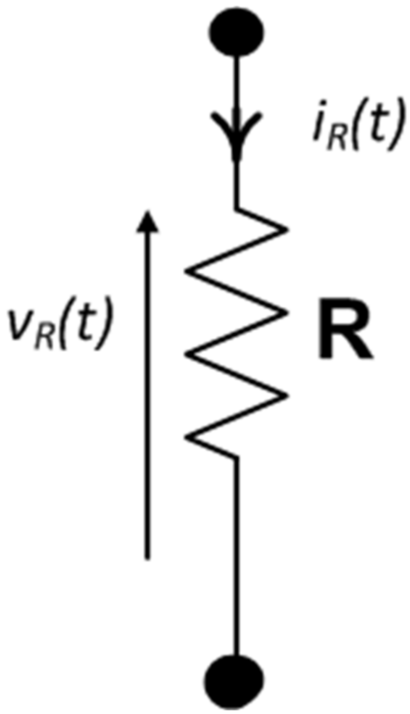
Time varying current source:

$i(t)$ = some predefined current,

eg $i(t) = 50.\cos(100t)$ *amps*

(Same symbol as before. We won't be using AC current sources in this course)

Resistors



Ohm's Law Still applies:

- v_R(t) = R i_R(t)
- G = 1/R, (conductance is the inverse of resistance)
- i_R(t) = G v_R(t)

Example:

Given $v_R(t) = 100 \sin (100t)$ volts
and R = 50 ohms, calculate i_R(t):

$$i_R(t) = \frac{v}{R} = \frac{100 \sin(100t)}{50}$$

$i_R(t) = 2 \sin (100t)$ Amperes

KVL, KCL

- At every instant of time (i.e., every value of time t):
  - Kirchhof's Voltage & Current laws apply
- Therefore, at every instant of time, (i.e., every value of t) we can say:
  - Sum of voltage rises around a loop equal zero:

    $$\Sigma\, v(t) = 0$$

  - Sum of currents into a node equal zero:

    $$\Sigma\, i(t) = 0$$

Time-Varying Power (Instantaneous)

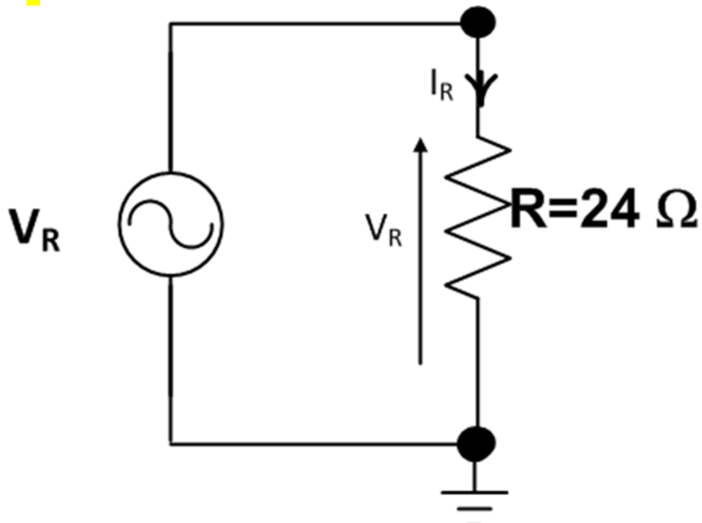
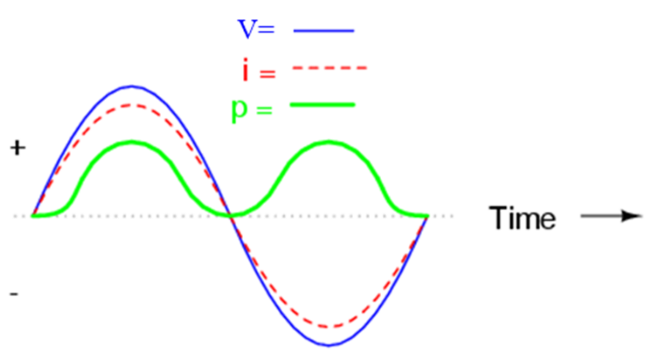


Image from http://openbookproject.net/electricCircuits/AC/AC_4.html



$$v_R\,(\mathrm{t}) = 340 \times \cos\,(100\pi \mathrm{t})\ \text{Volts}$$

$$i_R\,(\mathrm{t}) = \frac{340 \times \cos(100\pi \mathrm{t})}{24}\ \text{Amps}$$

$$i_R\,(\mathrm{t}) = 14.1\cos\,(100\pi \mathrm{t})\ \text{Amps}$$

$$p_R(\mathrm{t}) = v_R(\mathrm{t}) \times\ iR(\mathrm{t})$$

$$p_R(\mathrm{t}) = 340\cos\,(100\pi \mathrm{t}) \times 14.1\cos\,(100\pi \mathrm{t})\ \text{Watts}$$

$$p_R(\mathrm{t}) = 340 \times 14.1\cos^2(100\pi \mathrm{t})\ \text{W}$$

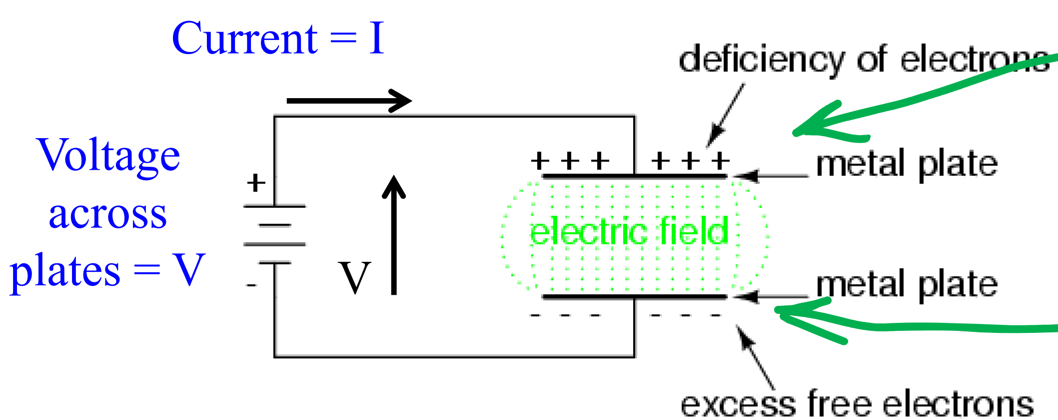
Double angle formula: $\cos^2(x)=1/2 + (½)\cos(2x)$!

$$p_R(\mathrm{t}) = 4800\,(\,½ + ½\cos(200\pi \mathrm{t})) = 2400 + 2400\cos(200\pi \mathrm{t})\ \text{W}$$

We'll be dealing with AC power in detail in weeks 9-11

Capacitor

- The *Capacitor* stores energy in an electric field, caused by separating positive and negative charges
- Consider two parallel plates separated by an insulator (a "dielectric"), with a positive charge +Q on one plate, and a negative charge –Q on the other:



positive charge
+Q coulombs

negative charge
-Q coulombs

We can derive that charge Q is proportional to voltage:

$Q = C \times V$ (more details in ELEC2300)

Image adapted from http://openbookproject.net/electricCircuits/DC/DC_13.html

Capacitor

- The constant C, is called the capacitance, and it is measured in farads (F).
- One farad is a very large capacitor. We will be using devices of the order of 1 pF to 10uF.

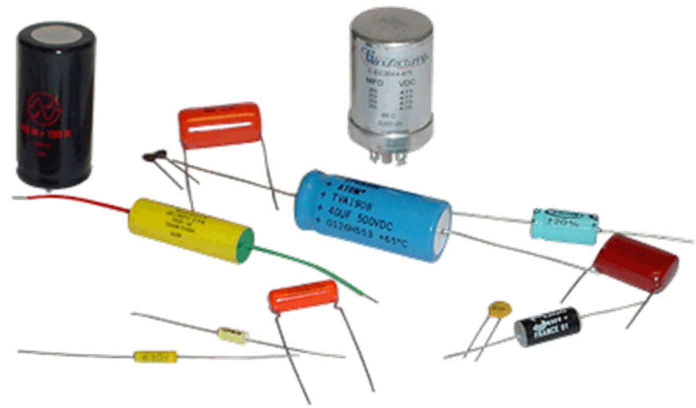
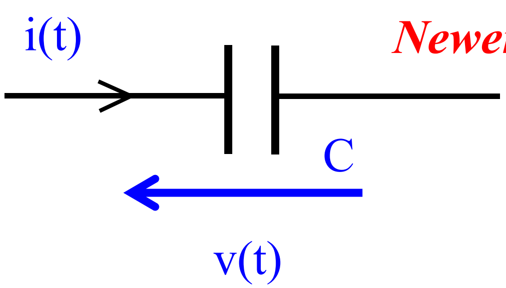
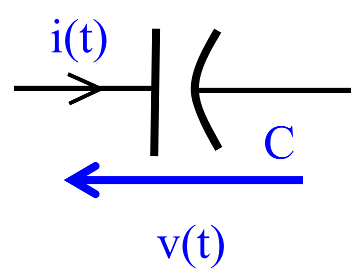


Image from http://ledeaudio.com/



Newer Symbol

Older Symbol

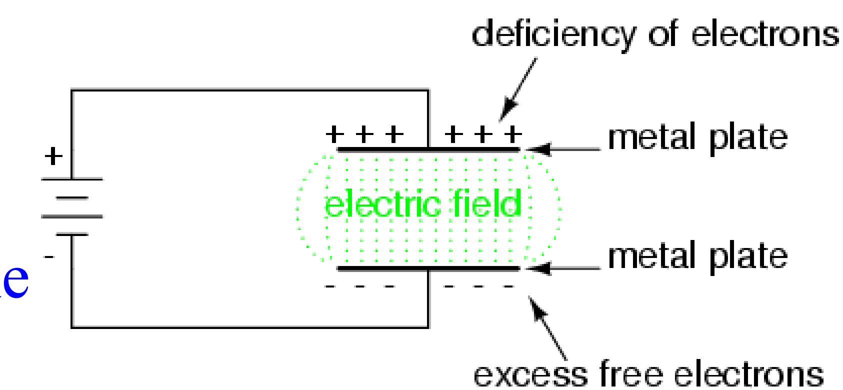


Voltage and Current Relationships for Capacitors

* Recall component equation: $Q = CV$
* We can express this in a time varying form: $Q(t) = C\,V(t)$
* And differentiate with respect to time: $\frac{dQ(t)}{dt} = C\,\frac{dv(t)}{dt}$
* rate of change of charge is CURRENT: $i(t) = \frac{dQ(t)}{dt}$
* we can thus relate current and voltage: $i(t) = C\,\frac{dv(t)}{dt}$
* or: $v(t) = \frac{1}{C}\int i(t)\,dt$

Is a capacitor a *linear* component?

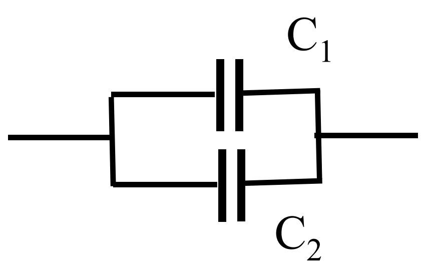
Yes! Current is *linearly* related to the differential of voltage



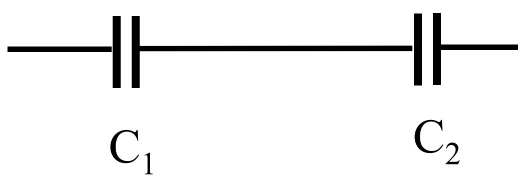
Capacitors in series/parallel

$$i(t) = C\,\frac{dv(t)}{dt}$$

- Capacitance "similar" to conductance
- We can derive series and parallel laws for capacitors similar to resistors as conductance's:



$$C_{parallel} = C_1 + C_2$$

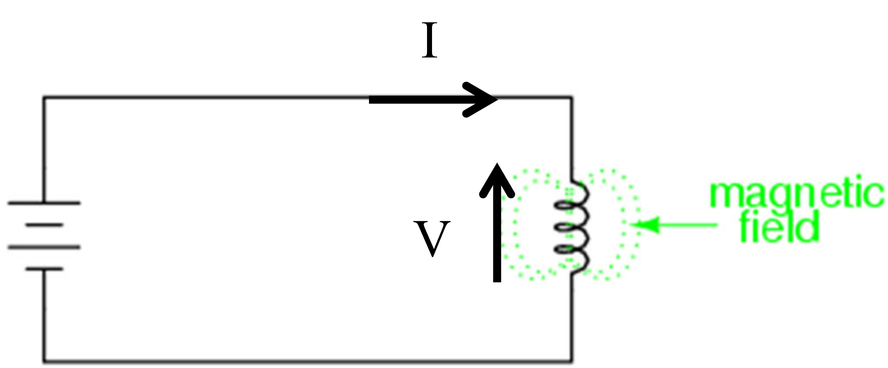


$$\frac{1}{C_{series}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Inductor

- An inductor is usually a **wire coil**, and stores energy as a magnetic field caused by a current.
- If the current changes, the stored energy changes, and we need to supply this energy with some voltage applied to the coil.

http://embeddedmicro.com/tutorials/beginning-electronics/inductors



We can derive that the magnetic field is proportional to current, and:

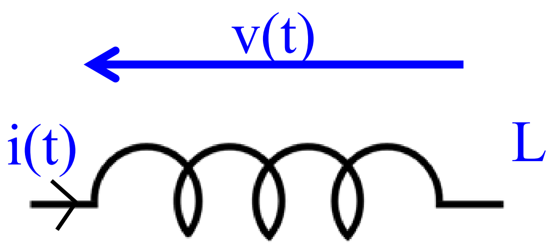
$$v(t) = L\,\frac{di(t)}{dt}$$

[See PHYS1002 or ELEC2300 for full derivations]

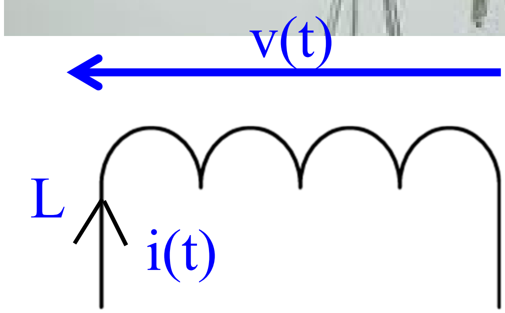
Inductors


Image from http://cens.com/

- The constant L, is called the inductance, and it is measured in units of henry (H).
- One henry is a moderately large inductor.
  - We will work with devices of the order of 1-100 mH.



Older Symbol



Newer symbol

$$v(t) = L\frac{di(t)}{dt}$$

$$i(t) = \frac{1}{L}\int v(t)\,dt$$

Is an inductor a linear component?

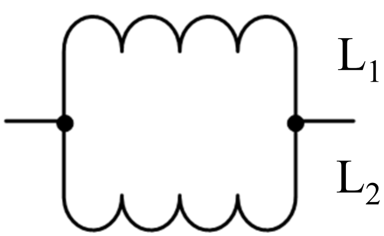
Yes! Voltage is linearly related to the differential of current

Inductors in series/parallel

$$v(t) = L\,\frac{di(t)}{dt} \qquad v(t) = R\,di(t)$$

- Inductance similar to resistance
- We can derive series and parallel laws for inductors similar to resistors:

$$L_{series} = L_1 + L_2$$



$$\frac{1}{L_{parallel}} = \frac{1}{L_1} + \frac{1}{L_2}$$

Energy Stored in Inductors/Capacitors

- *Instantaneous power consumed by ideal inductors and capacitors is stored as energy in magnetic or electric fields, and this energy can be later released back to the circuit with no losses.*
  - Real devices are not perfect inductors or capacitors (*series resistance*, leakage current) - We sometimes need to model these imperfections.
- We can calculate the *instantaneous* energy stored in an inductor/capacitor:

$$E(t) = \int p(t)\,dt = \int v(t)\,i(t)\,dt$$

For a capacitor: $i(t) = C\dfrac{dv(t)}{dt}$

Thus: $E(t) = C\int v(t)\dfrac{dv(t)}{dt}\,dt$

Change integration term: $E(t) = C\int v\,.dv$

Integrate wrt v: $E(t) = \frac{1}{2}Cv^2$

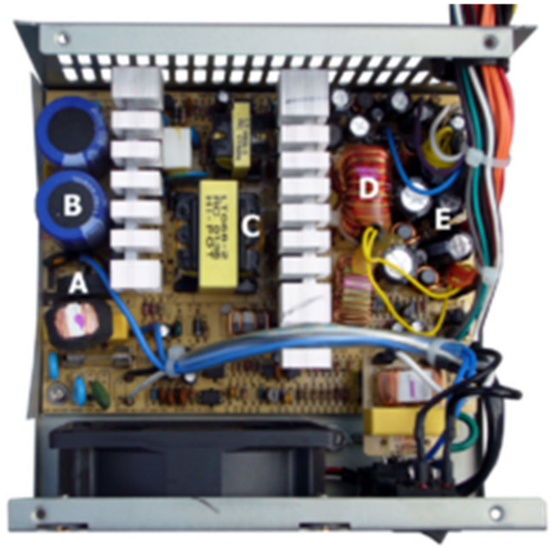
For an inductor: $v(t) = L\dfrac{di(t)}{dt}$

Thus: $E(t) = L\int i(t)\dfrac{di(t)}{dt}\,dt$

Change integration term: $E(t) = L\int i\,di$

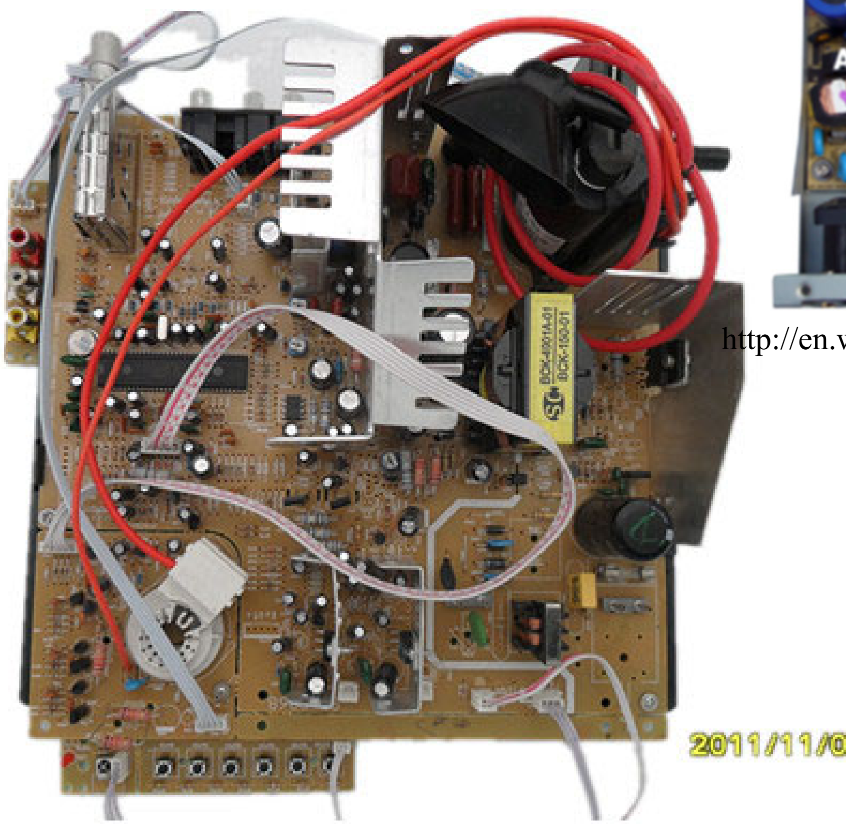
Integrate wrt i: $E(t) = \frac{1}{2}Li^2$

Pulled apart home appliances? You will have seen Capacitors and Inductors everywhere!



http://en.wikipedia.org/wiki/Switched-mode_power

http://www.niewaltinc.co.za/Studies%20on%20power%20factor%20correction.html



http://ronelex.com/tv-basic-repair-guide-common-problems/36/

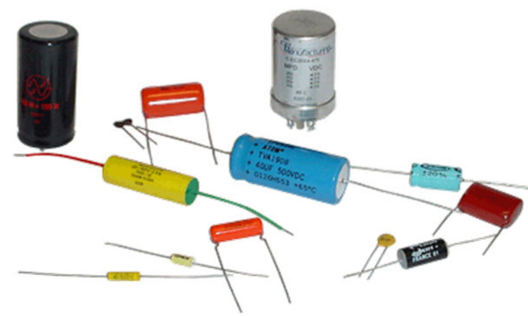
Intuitive view of caps and inductors

$$v(t) = L\frac{di(t)}{dt} \qquad i(t) = \frac{1}{L}\int v(t)\,dt$$

http://embeddedmicro.com/tutorials/beginning-electronics/inductors

- Current **through an inductor** can't change instantaneously – voltage "charges" current
- Inductor will "smooth" the flow of current through a branch of circuit…

$$i(t) = C\frac{dv(t)}{dt} \qquad v(t) = \frac{1}{C}\int i(t)\,dt$$



- Voltage **across a capacitor** can't change instantaneously – current "charges" voltage
- Capacitor will "smooth" the voltage at a node in a circuit…

How do we exploit these properties for practical applications?

How do we model the behaviour of these components?

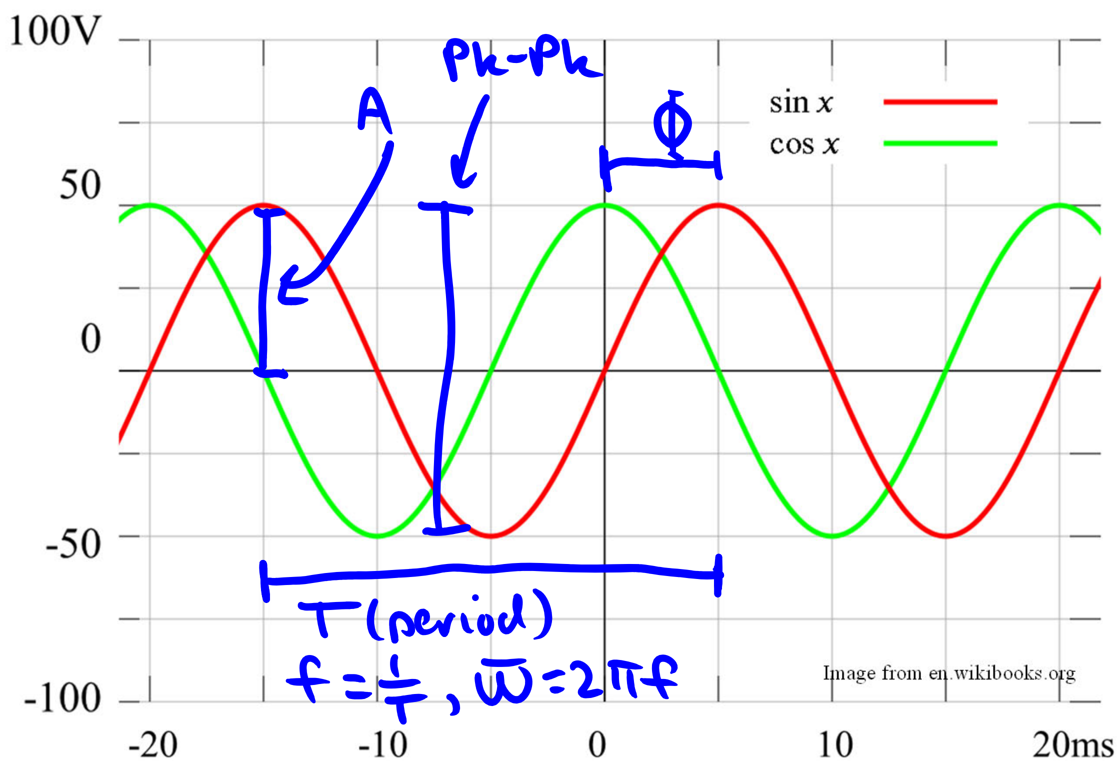
Sinusoids: Why focus on them?

- Given: $v(t) = L\,\dfrac{di(t)}{dt}$ $\quad i(t) = C\,\dfrac{dv(t)}{dt}$
- We are going to end up with circuits whose solution is defined by a differential equation
- We know from first year maths that sinusoidal inputs to differential equations provide nice "closed" sinusoidal or exponential, solutions

Thus, sinusoids allow us to accurately model the behaviour of circuits, and therefore provide convenient signals to use in practice (i.e. when designing systems, design systems that are easy to model and analyse!!!).

Properties of rotating electro-mechanical devices (i.e. electric generators) naturally provide sinusoidal waveforms.

Sine and Cosine waves



$Y = \cos(x) = \sin(x+\pi/2)$

$V(t) = \mathbf{A} \cos(\varpi t + \Phi)$

A = amplitude (or peak amplitude)
T = period (seconds),
f = frequency (Hertz)= 1/T
ϖ = angular frequency = $2\pi f$ = $2\pi/T$(radians per sec)
Φ = phase (radians)

$V(t) = 50 \cos(100\pi t)$ volts
$V(t) = 50 \cos(100\pi t - \pi/2)$ volts

- [The wave is a maximum at $\varpi t + \Phi = 0$, so the phase is a measure of the angular displacement from a cosine with zero phase. If the peak of the wave is at $t < 0$, then the phase is positive, if the peak of a wave is at $t>0$ (like a sine wave), then the phase is negative].

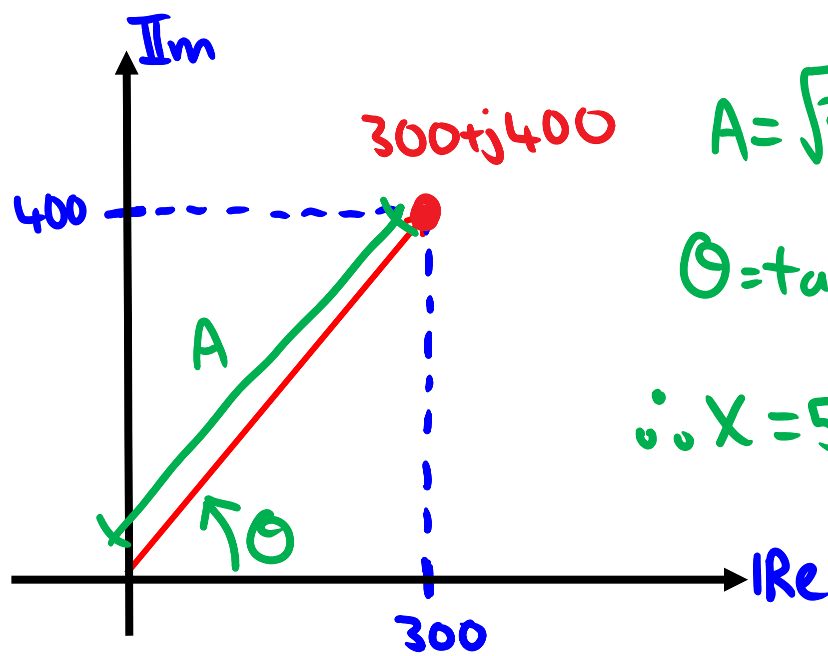
Modelling Time-Varying Circuits with Imaginary (complex) Numbers

Yes! Imaginary Numbers are Useful!!!!

Recall: Complex Numbers

- Complex numbers represent a two-dimensional value, e.g:

  X = 300 + j400, where $j = \sqrt{-1}$.
- *In electrical engineering we use j instead of i to avoid confusion with current..*
- But, we can also represent a complex number by a magnitude and an angle in radians



$A = \sqrt{300^2 + 400^2} = 500$

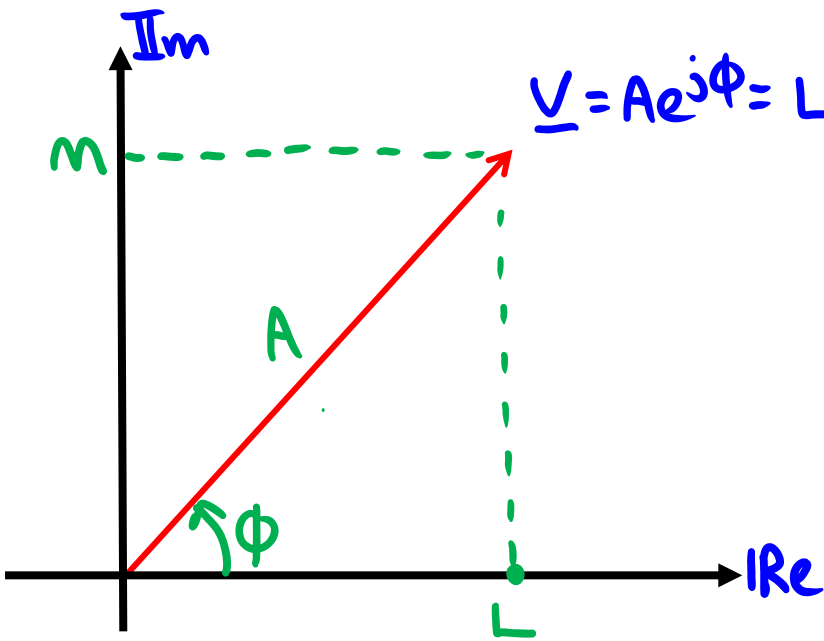
$\theta = \tan^{-1}\left(\frac{400}{300}\right) = 36°$

$= 0.93\,\text{rad}$

$\therefore X = 500\angle 0.93 = 500e^{j0.93}$

Rectangular & Polar Forms

- Complex numbers can be represented in:
  - Rectangular form (Real + j Imaginary) form – *good for adding or subtracting complex numbers*
  - Polar form (Amplitude e $^{j.phase}$) form – *good for multiplying or dividing complex numbers*



$$* \; A = \sqrt{L^2 + M^2}$$

$$* \; \phi = \arctan\left(\frac{M}{L}\right)$$

$$* \; L = \mathrm{Re}(Ae^{j\phi}) = A\cos(\phi)$$

$$* \; M = \mathrm{Im}(Ae^{j\phi}) = A\sin(\phi)$$

Note: there are two angles in the range $(-\pi .. \pi)$ which are solutions to arctan: Choose the correct quadrant by drawing the phasor on a graph!!

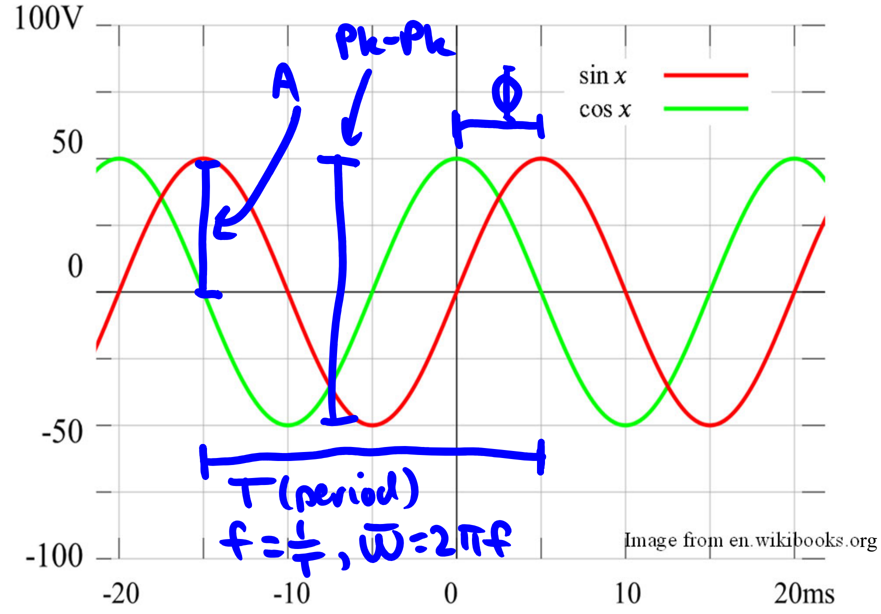
Learn to do this quickly on your calculator!!

Recall: Sinusoids

- Recall, all sinusoids can be represented by:

$$v(t) = \mathbf{A} \cos (\varpi t + \phi)$$

- Thus sinusoids are Parameterised by:
  - t = time
  - A = amplitude (or peak amplitude)
  - T = period (seconds) OR f = frequency (Hertz)= 1/T OR
    ϖ = angular frequency = 2πf = 2π/T(radians per sec)
  - ϕ = phase (radians)
- *In your previous studies you will have mostly considered the behaviour of sinusoids as a function of time:*



In this course we're going to be focussing a lot on the amplitude and the phase <u>as a function of frequency</u>

Phasors

- In many systems with sinusoidal voltages and currents, all of the sinusoidal voltages and currents have the same frequency (i.e. same ω, same f, same T)
  - The only two variables in $v(t) = A \cos(\omega t + \phi)$ which vary are the amplitude (A) and phase (ϕ)
  - In this special case, we can use a special shorthand notation called phasors to model a signal
- So, we can model the sinusoid $A \cos(\omega t + \phi)$ using a complex number:

$$\underline{V} = A\angle\phi = Ae^{j\phi}$$

- We call $\underline{V}$ a phasor (we usually underline phasors).

$\underline{V}$ models the parameters of interest of v(t)

[Fundamentally, we are using the algebra of complex numbers to model the behaviour of sinusoidal time-varying circuits]

Converting between phasors and time-varying voltages

- Converting from a time-varying voltage to a phasor:
  - First, get sinusoid exactly in the form:

    $$v(t) = A \cos(\omega t + \phi)$$

    [Where A is positive magnitude, and ϕ is in radians (positive or negative), and the function is cosine]
  - Then can simply write: $\underline{V} = Ae^{j\phi}$
- Converting from a phasor to a time-varying voltage:
  - Convert exactly to form: $\underline{V} = Ae^{j\phi}$
  - Then can write: $v(t) = \mathbf{A} \cos(\omega t + \phi)$

More formally:

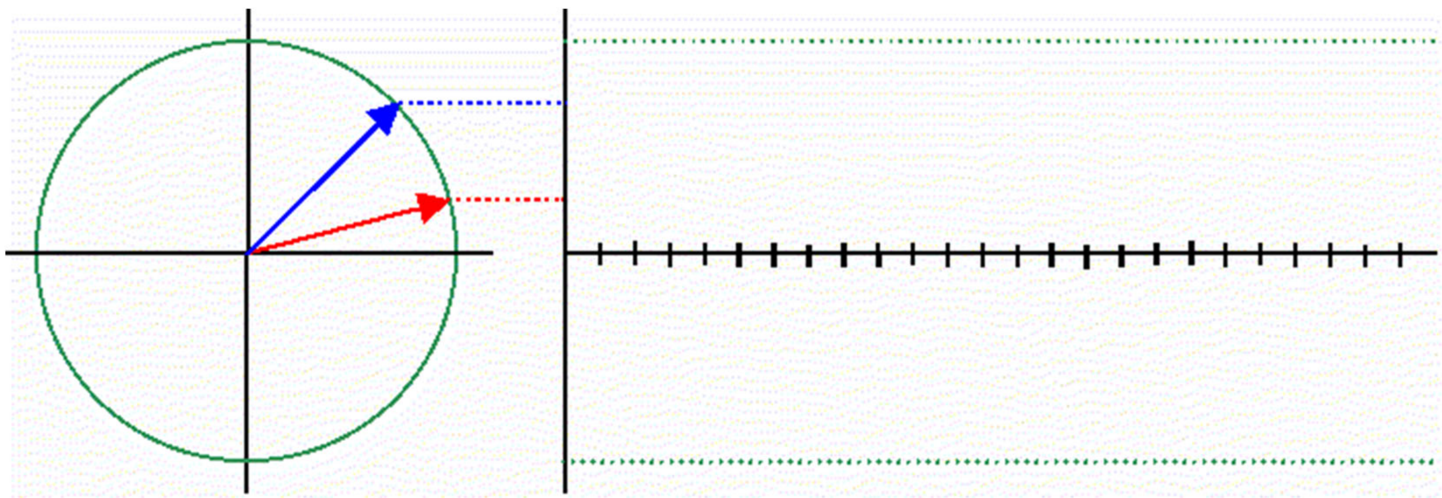
Recall: Eulers formula: $e^{j\emptyset} = cos\emptyset + jsin\emptyset$

- $v(t) = A. \cos(\varpi t + \phi)$
- $v(t) = \mathrm{Re}(A. (\cos(\varpi t + \phi) + j\sin(\varpi t + \phi))) = \mathrm{Re}(A. e^{j(\varpi t + \phi)})$
- $v(t) = \mathrm{Re}(Ae^{j\phi} . e^{j\varpi t})$.

Since $e^{j\varpi t}$ is the same throughout the circuit, we can say: $A. \cos(\varpi t + \phi) \approx Ae^{j\phi}$

Meaning of Phasors

- If we take the phasor $\underline{V}$ and rotate it anticlockwise at angular frequency ϖ, then the projection on the real axis at time t is the value of v(t)



http://www.youtube.com/watch?v=OSMy5hqCzSY

Using phasors, KCL, KVL, Ohm's Law

- The phasor representing the sum of two sinusoids (of the same frequency) is the sum of the phasors representing the individual sinusoids.
  - If: $\underline{V}_1 \leftrightarrow v_1(t)$; $\underline{V}_2 \leftrightarrow v_2(t)$; $\underline{V}_3 \leftrightarrow v_3(t)$
  - And: $v_3(t) = v_1(t) + v_2(t)$
  - Then: $\underline{V}_3 = \underline{V}_1 + \underline{V}_2$
- Kirchoff's Voltage Law and Kirchoffs Current Law are both true for time varying voltages

  Thus, KVL and KCL are both true for phasors:
  - *Sum of phasor voltage rises around a loop = 0 + j0*
  - *Sum of phasor currents entering a node = 0 + j0*
- Ohm's law holds for time-varying signals:
  - $v_1(t) = R\, i_1(t)$

  Thus Ohm's law also holds for phasors:
  - $\underline{V}_1 = R\, \underline{I}_1$

Measuring time-varying Signals

- The oscilloscope!
  - NOT the multimeter – even though it has an “AC” voltage mode.
  - What is this mode for?
    - Measuring mains power:
      - Volts – 100’s volts
      - 50-60Hz
      - Sinusoids
- Lab 4A this week

Next Week

We look at solving AC circuits

In particular we continue looking at:

- Phasors – representing AC currents and voltages as complex numbers
  - Complex numbers are essential for next weeks classes - please do some preparation if you are not confident with complex algebra
- Component Laws described as impedances using complex numbers
- Circuit Laws (KCL, KVL) for phasors
- AC Nodal Analysis and Mesh Analysis