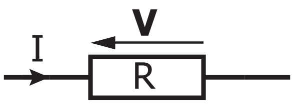
ENGG1300 - Formula Booklet

DC Circuits

Ohms Law



$$V = I \times R$$
$$I = \frac{V}{R}$$
$$R = \frac{V}{I}$$

Conductance, G,

$$G = \frac{1}{R}$$

Parallel and Series circuits

Series circuits

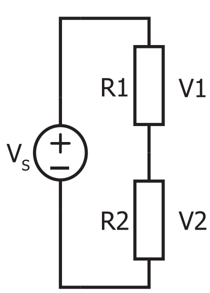
- Adding resistors in Series:

$$R_{eq} = R1 + R2 + ... + R_N$$

- Similarly, the equivalent conductance of resistors in series is given by the following:

$$\frac{1}{G_{eq}} = \frac{1}{G1} + \frac{1}{G2} + ... + \frac{1}{G_N}$$

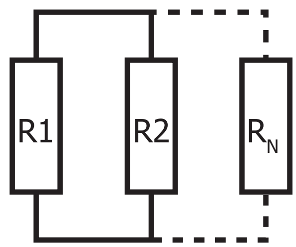
- The voltage division principle for two resistors in series i.e.



is given by the following formulas:

$$V1 = \frac{R1}{R1 + R2} \times V_S$$
$$V2 = \frac{R2}{R1 + R2} \times V_S$$

Parallel circuits



- Adding resistors in Parallel:

$$\frac{1}{R_{eq}} = \frac{1}{R1} + \frac{1}{R2} + ... + \frac{1}{R_N}$$

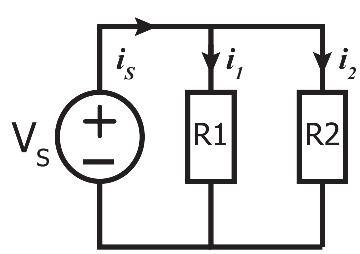
- Adding **two** resistors in parallel:

$$R_{eq} = \frac{R1 \times R2}{R1 + R2}$$

- Similarly, the equivalent conductance of resistors in parallel is given by the following:

$$G_{eq} = G1 + G2 + ... + G_N$$

- The current division principle for two resistors in parallel i.e.

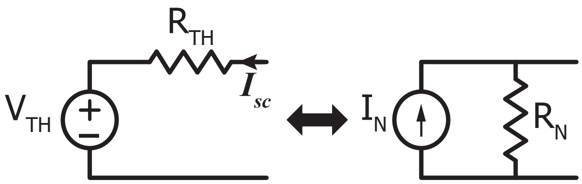


is given by the following formulas:

$$i_1 = \frac{R2}{R1 + R2} \times i_s$$
$$i_2 = \frac{R1}{R1 + R2} \times i_s$$

Thevenin & Norton Equivalent Circuits

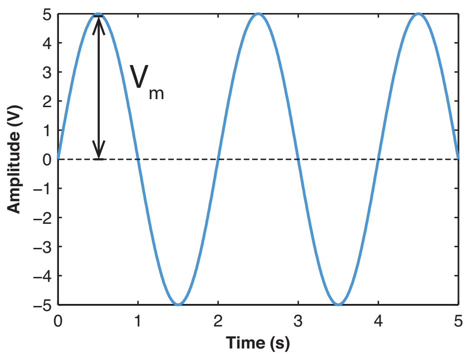
Definition: Any one port network consisting only of sources and resistors can be replaced by a thevenin or norton equivalent circuit.



$$V_{oc} = V_{TH} \;\; \& \;\; I_N = -I_{SC} \;\; \& \;\; R_N = R_{TH}$$

Sinusoidal waveforms

Consider the following time varying signal, this can either be a voltage or a current.



$$v(t) = V_m \cos(\omega t \pm \theta)$$

where:

- V_m is the the amplitude of the time varying signal.
- ω is the angular frequency in radians/s, given by $\omega = 2\pi f$.
- θ is the angle of lead or lag in radians [rad].

Complex Numbers

A time varying voltage of form:

$$v(t) = V_m \cos(\omega t + \theta)$$

Can be replaced by a phasor:

$$\underline{V} = V_m \angle \theta \;\; \text{(Polar Form)}$$
$$\underline{V} = V_m e^{j\theta} \;\; \text{(Exponential Form)}$$
$$\underline{V} = V_m \cos(\theta) + jV_m \sin(\theta) \;\; \text{(Rectangular Form)}$$

Complex Arithmetic

While it is more convenient to perform addition and subtraction of complex numbers in rectangular form, division and multiplication are best done in polar or exponential form.

Multiplication

$$(r_1 \angle \theta_1)(r_2 \angle \theta_2) = r_1 r_2 \angle(\theta_1 + \theta_2)$$
$$(r_1 e^{j\theta_1})(r_2 e^{j\theta_2}) = r_1 r_2 e^{j(\theta_1 + \theta_2)}$$

Division

$$\frac{(r_1 \angle \theta_1)}{(r_2 \angle \theta_2)} = \frac{r_1}{r_2} \angle(\theta_1 - \theta_2)$$
$$\frac{(r_1 e^{j\theta_1})}{(r_2 e^{j\theta_2})} = \frac{r_1}{r_2} e^{j(\theta_1 - \theta_2)}$$

Capacitors & Inductors

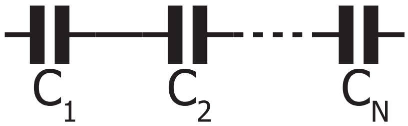
Capacitors & Capacitance

- Capacitance, where units are [Farads] is given by:

$$C = \frac{Q}{V}$$

- Charge in a capacitor, where units are [coulombs] is given by:

$$Q = CV$$

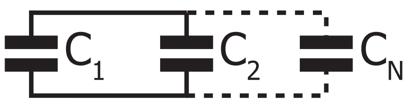


- Capacitors connected in series, the total capacitance is given by:

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_N}$$

- Similarly, for capacitors connected in parallel, the total capacitance is given by:

$$C_{eq} = C_1 + C_2 + ... + C_N$$



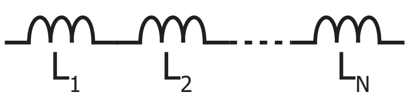
- The energy, E, stored by a capacitor is given by:

$$E = \frac{1}{2}CV^2$$

where units are in [Joules].

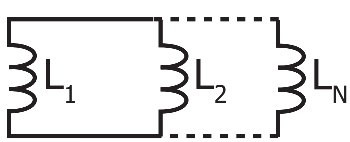
Inductors & Inductance

- For inductors connected in series, i.e.



$$L_{Total} = L_1 + L_2 + ... + L_N$$

- For inductors connected in parallel, i.e.



$$\frac{1}{L_{Total}} = \frac{1}{L_1} + \frac{1}{L_2} + ... + \frac{1}{L_N}$$

- The energy stored, E, in the magnetic field of an inductor is given by:

$$E = \frac{1}{2}LI^2$$

Instantaneous power

- Instantaneous power absorbed by an element is the product of the elements terminal voltage and the current through the element.

$$p(t) = i(t) \times v(t)$$

Average Power

- Average or real power P (in Watts) is the average of instantaneous power $p(t)$:

$$P = \frac{1}{T}\int_0^T p(t)dt$$

RMS voltage and Current

These ratios always apply to sinusoidal voltages and currents.

$$V_{rms} = \frac{V_{peak}}{\sqrt{2}}$$
$$I_{rms} = \frac{I_{peak}}{\sqrt{2}}$$

Solving AC circuits

Impedance and Admittance

- The impedances and admittances of passive elements are shown in the table below:

Element	Impedance, Z [Ω]	Admittance, $Y = \frac{1}{Z}$ [S]
Resistor	$Z = R$	$Y = \frac{1}{R}$
Inductor	$Z = j\omega L$	$Y = \frac{1}{j\omega L}$
Capacitor	$Z = \frac{1}{j\omega C}$ $Z = -j\frac{1}{\omega C}$	$Y = j\omega C$

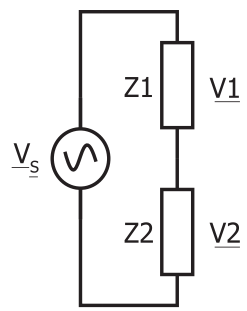
- Equivalent circuits at dc and high frequencies of an inductor and capacitor are shown in the table below.

Element	d.c	a.c.
Inductor	acts as a short circuit	acts as an open circuit
Capacitor	acts as an open circuit	acts as a short circuit

Impedance Combinations

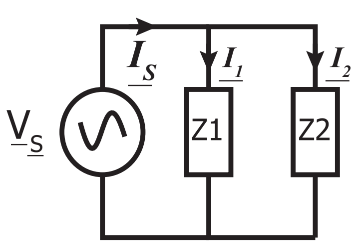
Similar to d.c. circuits, we can use the voltage and current division principles in a.c.

Voltage division relationship - a.c.



$$\underline{V1} = \frac{Z1}{Z1 + Z2} \times V_S$$
$$\underline{V2} = \frac{Z2}{Z1 + Z2} \times V_S$$

Current division relationship - a.c.



$$\underline{I_1} = \frac{Z2}{Z1 + Z2} \times \underline{I_S}$$
$$\underline{I_2} = \frac{Z1}{Z1 + Z2} \times \underline{I_S}$$

Frequency Response

Decibels

- Given a ratio of powers, e.g. radio power received by a mobile phone divided by power transmitted by the cell tower, then the Gain in decibels can be expressed as:

$$GAIN\ (dB) = 10 \log(P_2/P_1)$$

- For a ratio of voltages, the Gain in decibels can be expressed as:

$$GAIN\ (dB) = 20 \log(V_2/V_1)$$

Power Systems

- Instantaneous power absorbed by an element is the product of the elements terminal voltage and the current through the element.

$$p(t) = i(t) \times v(t)$$

- Inductors and capacitors absorb no average power, while the average power absorbed by a resistor is:

$$P_{ave} = \frac{1}{2} I_{\mathrm{m}}^2 R$$
$$P_{ave} = I_{\mathrm{rms}}^2 R$$

where the term I_{m} is the amplitude of the time-varying waveform and I_{rms} is the root-mean-square of the time-varying waveform.

- The power factor is the cosine of the phase difference between voltage and current:

$$pf = \cos(\theta_v - \theta_i)$$

- Transformer ratios:

$$\frac{N_P}{N_S} = \frac{V_P}{V_S} = \frac{I_S}{I_P}$$

- Resistivity (calculating resistance of power in transmission lines)

$$R = \frac{\rho L}{A}$$

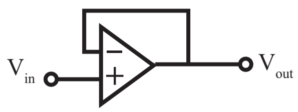
where, L is the length, A is the area and ρ is the resistivity of the material.

Op-Amps

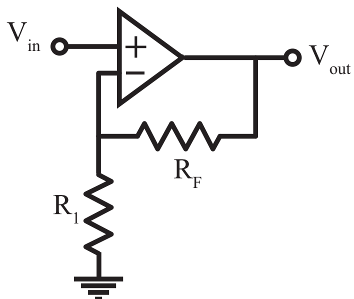
- Assumptions for an ideal op-amp.

$$V^+ = V^- \quad \text{and} \quad i^+ = i^- = 0$$

- **Unity Gain Voltage Follower** - Such a circuit has a very high input impedance and is useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another.

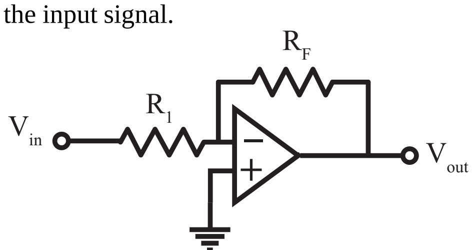


- **Non-Inverting Amplifier** - this is an op-amp circuit designed to provide a positive voltage gain.



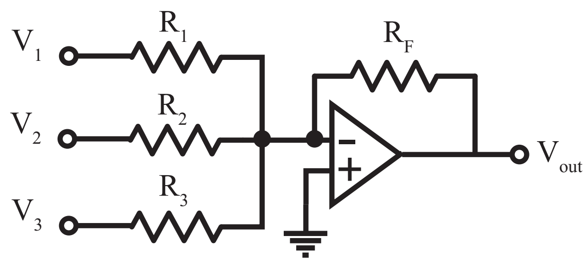
$$Gain = 1 + \frac{R_F}{R_1}$$

- **Inverting Amplifier** - this reverses the polarity of the input signal.



$$Gain = -\frac{R_F}{R_1}$$

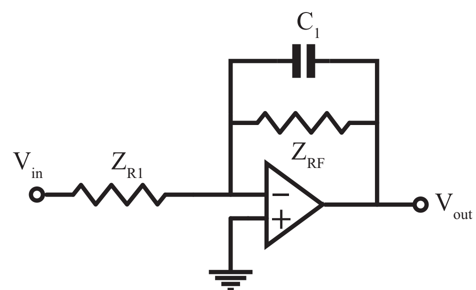
- **The Summing Amplifier** - this is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs. **NB**. minus sign on equation.



$$V_{out} = -\left(\frac{R_F}{R_1}V_1 + \frac{R_F}{R_2}V2 + \frac{R_F}{R_3}V3\right)$$

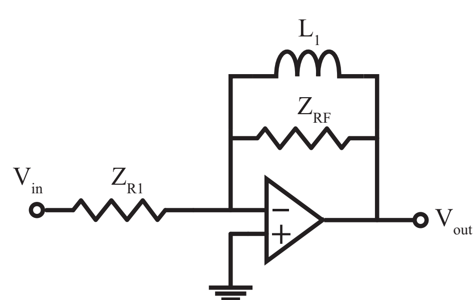
Active Filters

- Active low pass filter schematic and transfer function:



$$G(\omega) = -\frac{Z_{RF}}{Z_{R1} + Z_{R1}Z_{RF}j\omega C}$$

- Active high pass filter schematic and transfer function:



$$G(\omega) = -\frac{Z_{RF}}{Z_{R1} + \frac{Z_{R1}Z_{RF}}{j\omega L}}$$