ENGG1300
Introduction to Electrical Systems
Week 5 –Phasors Continued

Lecturer:

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Recording of this Presentation

Students, please be aware that this session is being recorded so it can be made available through Echo360 in Learn.UQ (Blackboard) to all students enrolled in the course. The reason we are recording the class presentations, discussions and chat room logs is because this provides a richer experience for all students and active classrooms help students' learning. The recording may be accessed and downloaded only by students enrolled in the course, including those students studying outside Australia. If you would prefer not be captured either by voice or image in the recording, please let me know before the class starts"

Suggested options for students not wishing to be recorded:

- **Turn off video and mute audio**
- **Use a proxy name for Zoom (student name will still be on record with the Course Coordinator)**

Please note that students are not permitted to record teaching without the explicit permission of the Course Co-ordinator. This includes recording classes using Zoom.

For further information:

- PPL 3.20.06 Recording of Teaching at UQ
- UQ website: https://my.uq.edu.au/information-and-services/information-technology/software-and-web-apps/software-uq/zoom

Online Mid-Semester Test

- Contributes 15% towards course grade
- Commences at 8am, Monday 29th March (Week 6 lecture slot).
- The exam conditions are as follows:
  - Delivered **online** via blackboard (i.e. you will sit this exam at home) [same system as weekly quizzes]
  - 115 minutes in duration commencing at 8am:
    - 10-mins reading + 90-mins working + 15-mins "Submission Time" [additional time to allow for minor technical problems in navigating online system].
  - It will contain 30 multiple choice questions, each worth ONE (1) mark.
  - You will be permitted approved (and labelled) scientific calculators (https://my.uq.edu.au/services/manage-my-program/exams-and-assessment/sitting-exam/approved-calculators)
  - This is an **open-book** exam, and you can refer to written or electronic notes, textbooks, and recorded media.
- **A direct link from the left-hand menu in blackboard has been provided: You can see it now so you know where to go! The exam link will be in this folder.**

Online Mid-Semester Test

- While you will be completing the exam online rather than in an invigilated exam room, the following guidelines apply:
  - You should complete the exam under strict exam conditions. This means you are not permitted to communicate with anyone else during the exam period. **You must not consult with other people (whether online or in person)** [The exception to this is to contact the course coordinator should you be having technical difficulties].
  - Other than the permitted calculator, you are not permitted to use other electronic calculation devices. This includes graphics calculators; circuit simulation software (i.e. LT-spice), websites or apps; excel, Matlab or other programming languages.
  - Breaching these guidelines will be considered as academic misconduct, and has serious disciplinary consequences.
- There will be a 0-mark question which is a declaration that you have sat the test under the specified conditions, and the work is entirely your own. You will receive 0 marks on the exam unless you complete this declaration.

Test Coverage

- This assessment will examine content presented in lectures and laboratory classes in weeks 1-4 of semester, i.e. general topics include:
  - DC Circuit Analysis
  - DC Norton and Thevenin equivalent circuits
  - Load-line analysis including non-linear elements such as LED’s
  - Capacitors
  - Inductors
  - Sinusoids and AC voltages
- Past and Practice Exams available on blackboard: “Assessment” ->”Midsemester Test”
  - Practice online exam
  - 2020 Semester 2 Online Exam
  - These both include questions on complex impedances and phasor circuits which will <u>not be covered in your exam.</u>
- 2019 and earlier past exams are a good indication of content and question style (even though these exams were on-campus) – [some do and don’t include week 5 content on complex impedances and phasors
- [Note, practice exam and past papers will guide structure and content, but QUESTIONS WILL BE DIFFERENT].

Applications for Deferred Exams

- Except under very special circumstances, you must sit the Mid-semester Test at the scheduled time.
- Students who are unable to sit the exam due to illness or other exceptional circumstances should refer to section 5.3 of the course profile for instructions for applying for a deferred examination.
- All deferred exam requests must be submitted no later than **5 calendar days** after the date of the original exam (but please put in your application as soon as you are aware there is a problem)

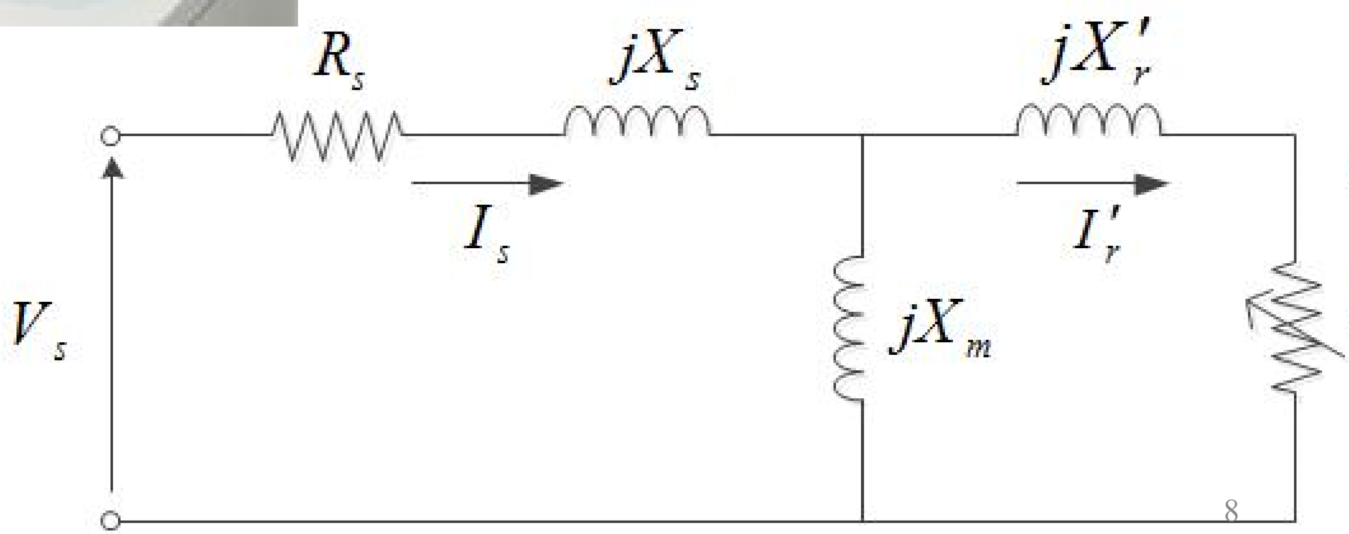
Last Week

- We looked at circuits with currents and voltages that vary with time.
- In particular we looked at:
  - New Components: capacitors, inductors: *We saw, based on their properties, why we did not include them in our DC analysis*
  - Component Laws
  - Circuit Laws (KCL, KVL)
  - Sinusoidal Voltages and Currents
- In linear circuits with sinusoidal inputs, we can model the behaviour with complex number algebra: Phasors

Why? AC Motors…

What might we be interested in?

- **Peak voltages + currents**
- **We are interested in the power delivered to the motor – Which in turn relates to mechanical properties: Power, Torque**



http://en.wikipedia.org/wiki/Induction_motor

This week Ahead

- Phasors Cont.: Representing AC currents and voltages as complex numbers (a practical application of “theoretical” mathematics).
- Circuit Laws:
  - (KCL, KVL) for phasors;
  - Series and parallel components
  - Thevenin Equivalent Circuit
  - AC Nodal & Mesh Analysis
  - Phasor Diagrams – representing circuit solutions graphically
  - Instantaneous + Average Power
- Session 5A: Practice Mid-Sem Test
- Session 5B: Lab (IX) – Phasors for modelling time-varying circuits
  - ***Bring your scientific calculator and be prepared for complex algebra***
- Quiz 4 due 4pm Today
- Quiz 5 due 4pm next Monday

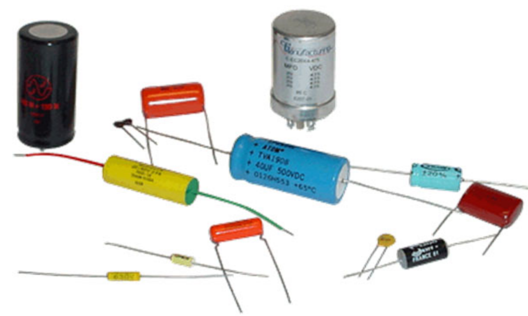
Last Week: Capacitors and Inductors

$$v(t) = L\frac{di(t)}{dt} \qquad i(t) = \frac{1}{L}\int v(t)\,dt$$

http://embeddedmicro.com/tutorials/beginning-electronics/inductors

- Current **through an inductor** can't change instantaneously – voltage "charges" current
- Inductor will "smooth" the flow of current through a branch of circuit…

$$i(t) = C\frac{dv(t)}{dt} \qquad v(t) = \frac{1}{C}\int i(t)\,dt$$



- Voltage **across a capacitor** can't change instantaneously – current "charges" voltage
- Capacitor will "smooth" the voltage at a node in a circuit…

How do we exploit these properties for practical applications?

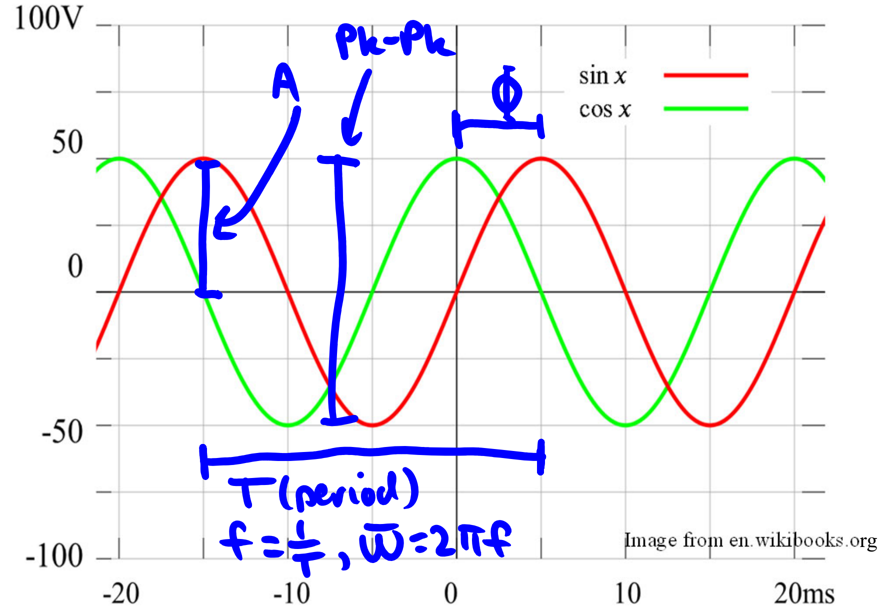
How do we model the behaviour of these components?

Last Week: Sinusoids

- Recall, all sinusoids can be represented by:

$$v(t) = \mathbf{A} \cos(\varpi t + \phi)$$

- Thus sinusoids are Parameterised by:
  - t = time
  - A = amplitude (or peak amplitude)
  - T = period (seconds) OR f = frequency (Hertz)= 1/T OR ϖ = angular frequency = $2\pi f$ = $2\pi/T$(radians per sec)
  - ϕ = phase (radians)
- *In your previous studies you will have mostly considered the behaviour of sinusoids as a function of time:*



In this course we're going to be focussing a lot on the amplitude and the phase <u>as a function of frequency</u>

Last Week: Phasors

- We use a complex number, a phasor:

$$\underline{V} = A e^{j\phi}$$

to represent sinusoidal voltages with fixed frequency:

$$v(t) = A \cos(\omega t + \phi)$$

- We call $\underline{V}$ a phasor. *(We usually underline phasors)*
- $\underline{\mathbf{V}}$ *represents* v(t), but, it is *not equal* to v(t)

....because it does not include the information related to the time, or frequency parameters

Using phasors, KCL, KVL, Ohm's Law

- The phasor representing the sum of two sinusoids (of the same frequency) is the sum of the phasors representing the individual sinusoids.
  - If: $\underline{V_1} \leftrightarrow v_1(t)$; $\underline{V_2} \leftrightarrow v_2(t)$; $\underline{V_3} \leftrightarrow v_3(t)$
  - And: $v_3(t) = v_1(t) + v_2(t)$
  - Then: $\underline{V_3} = \underline{V_1} + \underline{V_2}$
- Kirchoff's Voltage Law and Kirchoffs Current Law are both true for time varying voltages

  Thus, KVL and KCL are both true for phasors:
  - *Sum of phasor voltage rises around a loop = 0 + j0*
  - *Sum of phasor currents entering a node = 0 + j0*
- Ohm's law holds for time-varying signals:
  - $v_1(t) = R\, i_1(t)$

  Thus Ohm's law also holds for phasors:
  - $\underline{V_1} = R\, \underline{I_1}$

Capacitor – Component law for phasors

Derive a phasor relationship between voltage and current in a capacitor:

Recall: $\boxed{i(t) = C\frac{dv(t)}{dt}}$, Let $\boxed{v(t) = A\cos(\omega t + \phi)}$

Differentiate WRT time: $i(t) = C\frac{d(A\cos(\omega t+\phi))}{dt} = C(-A\omega\sin(\omega t+\phi))$

Convert -sin to cos with phase shift: $i(t) = C\omega A\cos(\omega t + \phi + \boxed{\frac{\pi}{2}})$

We note that derivative of a sinusoid is a sinusoid, so we can be represent both the signal, and its differential by a phasor:

Represent v(t) and i(t) with phasors: $\underline{V} = Ae^{j\phi}$, $\underline{I} = C\omega Ae^{j(\phi+\frac{\pi}{2})}$

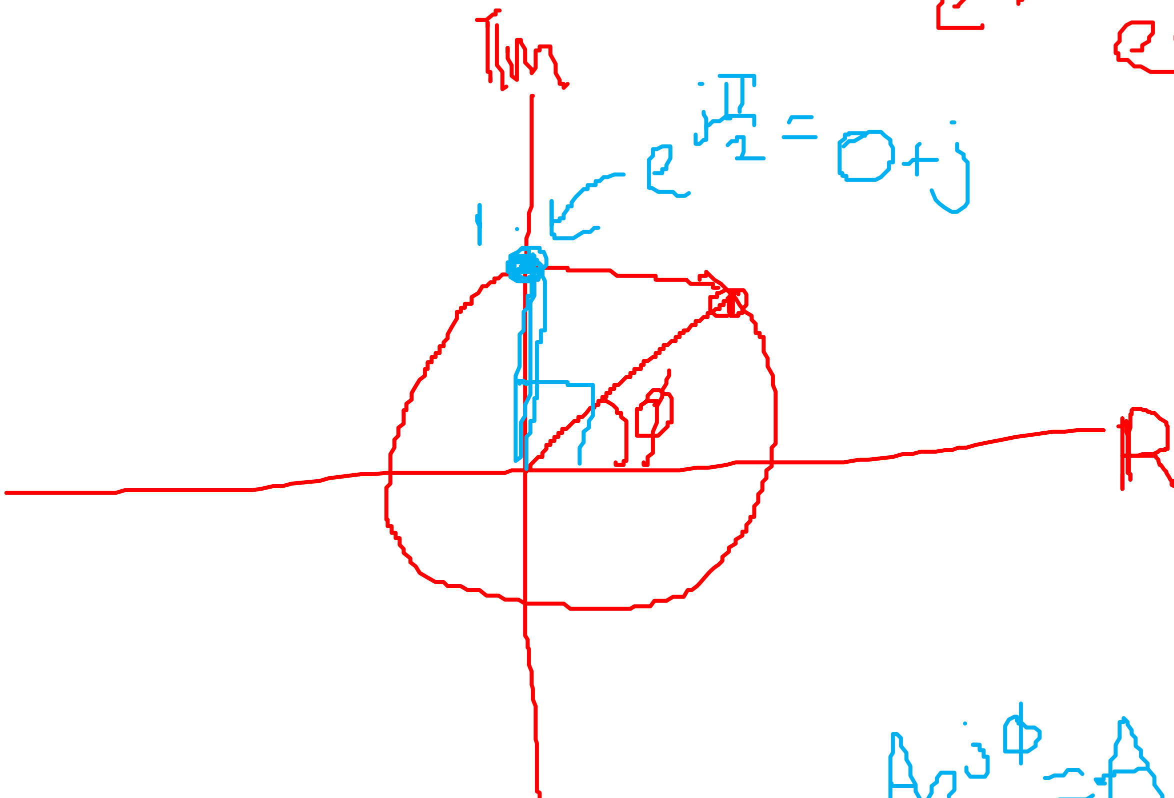
Simplify $\underline{I}$: $\underline{I} = C\omega Ae^{j\phi}e^{j\frac{\pi}{2}}$ Recall $e^{j\frac{\pi}{2}} = j$: $\underline{I} = j\omega CAe^{j\phi}$

We can now relate $\underline{V}$ to $\underline{I}$: $\underline{I} = j\omega C\cdot\underline{V} \Rightarrow \underline{V} = \frac{1}{j\omega C}\underline{I}$

Define: $Z_c = \boxed{\frac{1}{j\omega C}}$ we get: $\boxed{\underline{V} = Z_c\underline{I}}$

So using phasors, we get a simple "ohms law", where current and voltage are related by a (complex) constant, Z (which we define as an "impedance").

$e^{j\phi}$ $\quad e^{j\frac{\pi}{2}}$



$e^{j\frac{\pi}{2}} = 0 + j$

$Ae^{j\phi} = A\angle\phi$

$= \cos\phi + j\sin\phi$

$= 0 + j1$

Inductor – Component law for phasors

Derive a phasor relationship between voltage and current in an inductor:

Recall: $\boxed{v(t) = L\frac{di(t)}{dt}}$, Let $i(t) = A\cos(\omega t + \phi)$

Differentiate WRT time: $v(t) = L\frac{d(A\cos(\omega t+\phi))}{dt} = L(-A\omega\sin(\omega t+\phi))$

Convert -sin to cos with phase shift: $v(t) = L\omega A\cos(\omega t + \phi + \frac{\pi}{2})$

We note that derivative of a sinusoid is a sinusoid, so we can be represent both the signal, and its differential by a phasor:

Represent v(t) and i(t) with phasors: $\underline{I} = Ae^{j\phi}$, $\underline{V} = L\omega Ae^{j(\phi+\frac{\pi}{2})}$

Simplify $\underline{V}$: $\underline{V} = \omega Ae^{j\phi}e^{j\frac{\pi}{2}}$ Recall $e^{j\frac{\pi}{2}} = j$: $\underline{V} = j\omega LAe^{j\phi}$

We can now relate $\underline{V}$ to $\underline{I}$: $\underline{V} = j\omega L.\underline{I}$

Define: $Z_L = j\omega L$ we get: $\underline{V} = Z_L\underline{I}$

So using phasors, we get a simple "ohms law", where current and voltage are related by a (complex) constant, Z (which we define as an "impedance").

Impedance

- Impedance is a generalisation of resistance for resistors, capacitors and inductors, that is applicable to phasor voltages and currents:

$$\boxed{\underline{V} = Z\,\underline{I}}$$

- Impedance is a complex number, and it can vary with frequency; Impedance is *not a phasor* (doesn't represent a time varying quantity), but it is a *ratio of phasors*.
- Resistor (real, no imaginary part): $\mathbf{Z_R = R}$
- Capacitor (imaginary): $Z_C = \dfrac{1}{j\omega C} = \dfrac{-j}{\omega C} = -j\dfrac{1}{\omega C} = \dfrac{1}{\omega C}\boxed{e^{-j\frac{\pi}{2}}}$
- Inductor (imaginary): $Z_L = \boxed{j}\omega L = \omega L\, e^{j\frac{\pi}{2}}$

Impedance of capacitors and inductors is a function of frequency….. Thus circuits containing these values will exhibit frequency dependant behaviour!!!!

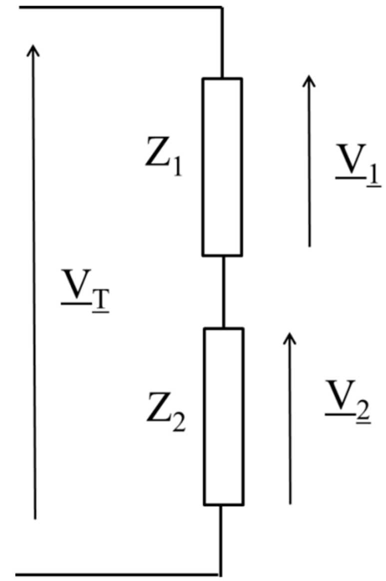
Circuit Theorems

- Impedance is like resistance, KCL, KVL apply to phasors, *so all of our previous DC Laws can be used with phasors generalised to complex impedances*.
  - Series, Parallel
  - Voltage Divider, Current Divider
  - Nodal Analysis, Mesh Analysis
  - Thevenin Equivalent Circuit, Norton Eq. Cct.

This is Great News! We now have a technique to model inductors and capacitors in time-varying sinusoidal circuits without solving differential equations.

Impedances in Series

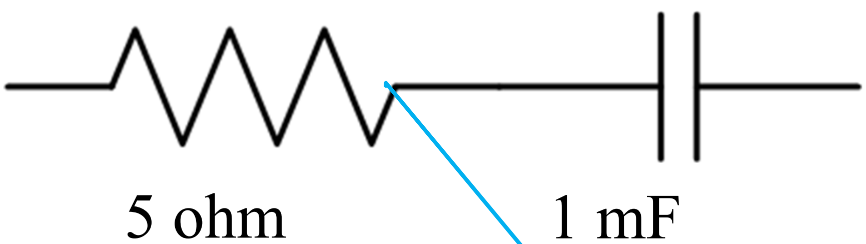
- Like resistances, for Z_1, Z_2 in series:
  - $Z_{series} = Z_1 + Z_2$



- *Example:* Find total impedance at a frequency of 100 Hz:

$$Z_R = 5$$

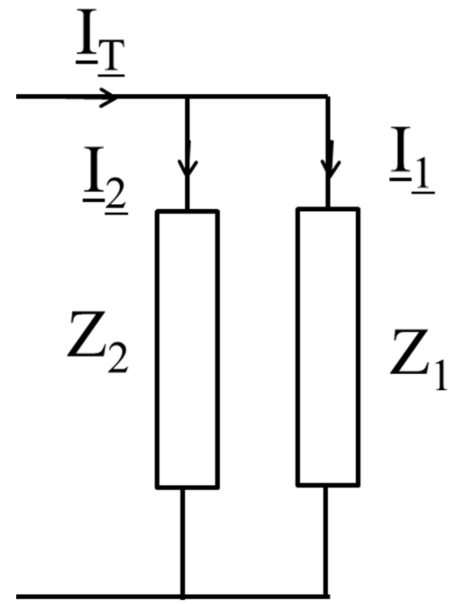
$$Z_C = \frac{1}{j\omega C} = \frac{-j}{2\pi \times 100 \times 0.001} = -j1.6$$



$$Z_{total} = Z_R + Z_C = 5 - j1.6$$

$$= \sqrt{5^2 + 1.6^2}\, e^{j(\tan^{-1}(\frac{-1.6}{5}))} = 5.24\, e^{j(-0.31)}$$

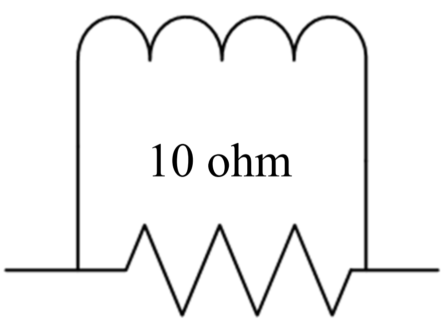
Impedances in Parallel

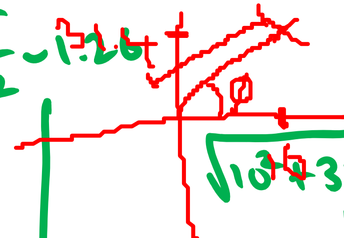


- Like resistances For Z_1, Z_2 in parallel:

$$\frac{1}{Z_{PAR}} = \frac{1}{Z_1} + \frac{1}{Z_2} \qquad Z_{PAR} = \frac{Z_1 Z_2}{Z_1 + Z_2}$$

- *Example:* Find total impedance at a frequency of 50 Hz:





$\frac{\pi}{2} \sim 1.26$

$\sqrt{10^2 + 31.4^2}$ $\qquad \tan^{-1}\left(\frac{31.4}{10}\right)$

$\frac{314}{33.0}$

$Z_R = 10$

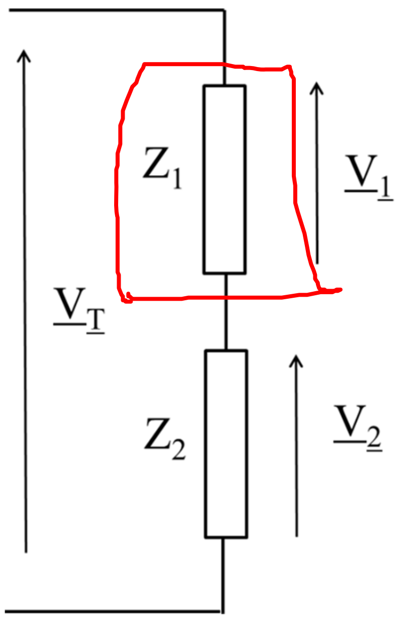
$Z_L = j\omega L = j \times 2\pi \times 50 \times 100 \times 10^{-3} = j31.4$

$$Z_{total} = \frac{Z_R Z_L}{Z_R + Z_L} = \frac{10 \times j31.4}{10 + j31.4} = \frac{314 e^{j(\pi/2)}}{33.0\, e^{(j1.26)}}$$

$9.52 \cos(0.308)$

$= 9.52 e^{j0.308} = 9.08 + j2.89$ $\leftarrow 9.52 \sin(0.308)$

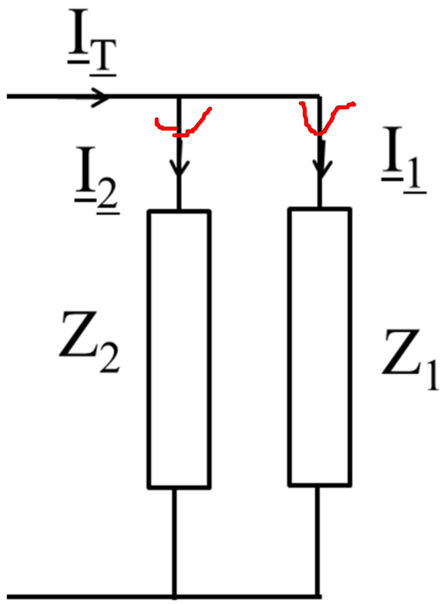
Voltage Divider



$$\underline{V}_1 = \underline{V}_T \ .(Z_1/(Z_1 + Z_2))$$

$$\underline{V}_2 = \underline{V}_T \ .(Z_2/(Z_1 + Z_2))$$

Current Divider

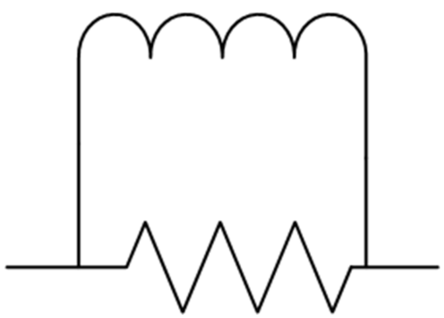


$$\underline{I}_1 = \underline{I}_T \ .(Z_2/(Z_1 + Z_2))$$

$$\underline{I}_2 = \underline{I}_T \ .(Z_1/(Z_1 + Z_2))$$

Admittance

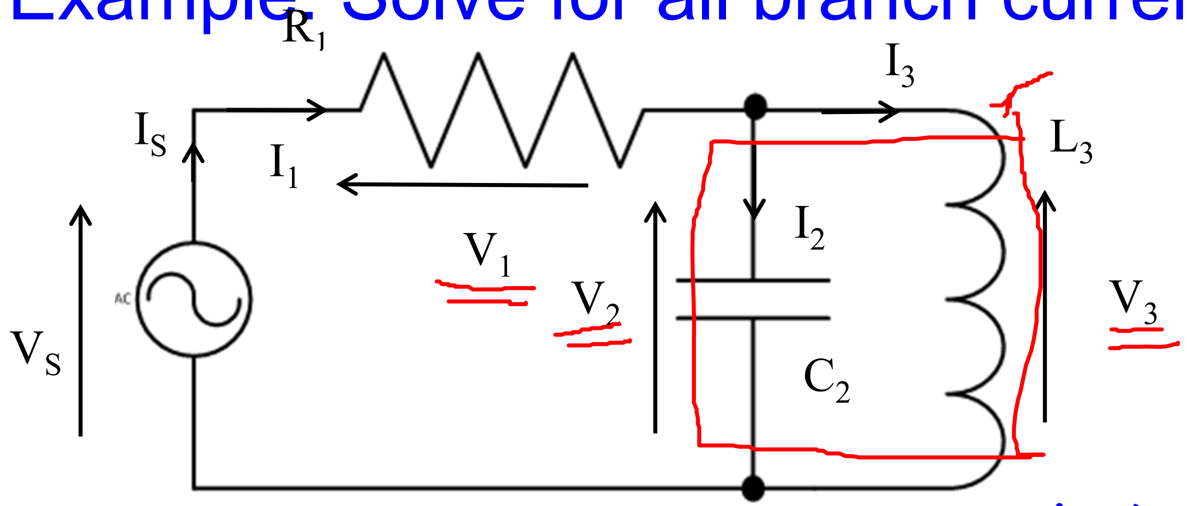
- Conductance is the inverse of resistance
- Admittance is the inverse of Impedance
  - $Y = 1/Z$ (where $\underline{Y}$ and $\underline{Z}$ are both complex numbers)
  - $\underline{V} = Z\,\underline{I}$
  - $\underline{I} = Y\,\underline{V}$
- It can be more convenient to work in admittance in some cases, i.e. parallel combinations (we'll see some examples later in the course – "power factor correction capacitors")



$$\boxed{\frac{1}{Z_{PAR}} = \frac{1}{Z_1} + \frac{1}{Z_2}}$$

$$\boxed{Y_{PAR} = \underline{Y_1 + Y_2}}$$

Example: Solve for all branch currents and voltages:



$V_S = 50\,e^{j0}$ volts
$f = 50$ Hz
$R_1 = 100$ ohms
$C_2 = 20\ \mu F$
$L_3 = 200$ mH

Calculate Impedances:

$* \; Z_1 = R = 100$

$* \; Z_2 = \frac{1}{j\omega c} = \frac{-j}{2\pi \times 50 \times 20 \times 10^{-6}} = -j159$

$* \; Z_3 = j\omega L = j \times 2\pi \times 50 \times 0.2 = j62.8$

Calculate $Z_2 \| Z_3$

$* \; Z_2 \| Z_3 = \frac{Z_2 Z_3}{Z_2 + Z_3} = \frac{(-159j)(62.8j)}{62.8j - 159j}$

$= \boxed{103.8j}$

Voltage Divider to calculate V_1, V_2, V_3:

$* \; V_1 = V_s \left(\frac{Z_1}{Z_1 + Z_2 \| Z_3} \right) = \frac{100 \times 50 e^{j0}}{100 + 103.8j} = \frac{5000 e^{j0}}{144.1\, e^{j0.80}}$

$= 34.7 e^{j-0.8}$

$* \; V_2 = V_3 = V_s \left(\frac{Z_2 \| Z_3}{Z_1 + Z_2 \| Z_3} \right) = \frac{103.8 e^{j\frac{\pi}{2}} \cdot 50 e^{j0}}{\boxed{100 + 103.8j}} = 36 e^{j0.766}$

Calculate Branch currents:

$* \; I_1 = \frac{V_1}{Z_1} = \frac{34.7 e^{j(-0.8)}}{100} = 0.34\, e^{j(-0.8)}\, A.$

$* \; I_2 = \frac{V_2}{Z_2} = \frac{36 e^{j0.766}}{159 e^{j(-\pi/2)}} = 0.226\, e^{j2.34}$

$I_3 = \frac{V_3}{Z_3} = (36 e^{j0.766}) / (62.8 e^{j(\frac{\pi}{2})}) = 0.573 e^{j(-0.8)}$

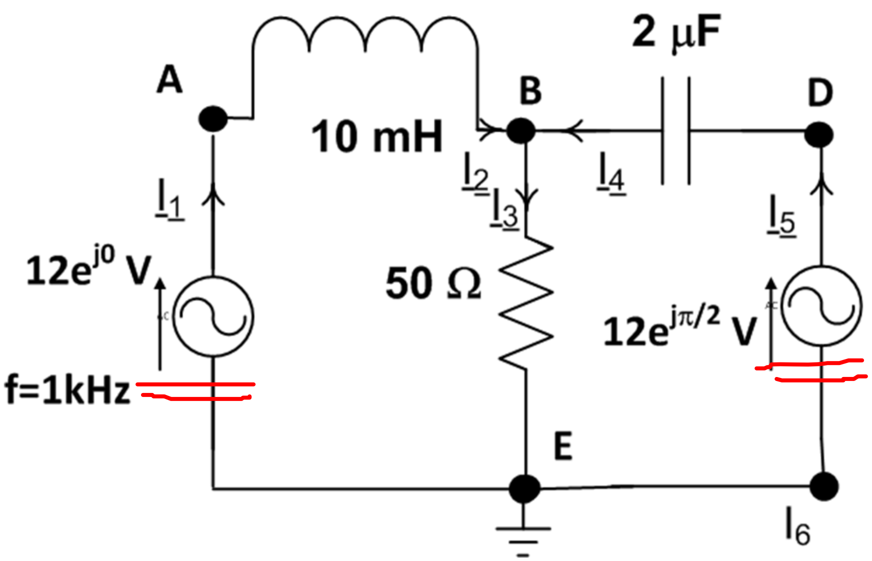
Think or Stretch

- The Queensland Electrical Safety Act restricts work on any electrical appliance or system to licensed professionals for voltages above "**extra low voltage**". How low is ELV? From the act:

  extra low voltage means voltage of **???V** or less AC RMS, or **???V** or less ripple-free DC.

  extra low voltage means voltage of **50V** or less AC RMS, or **120V** or less ripple-free DC.

More Complex Example: Application of Linear Circuit Techniques



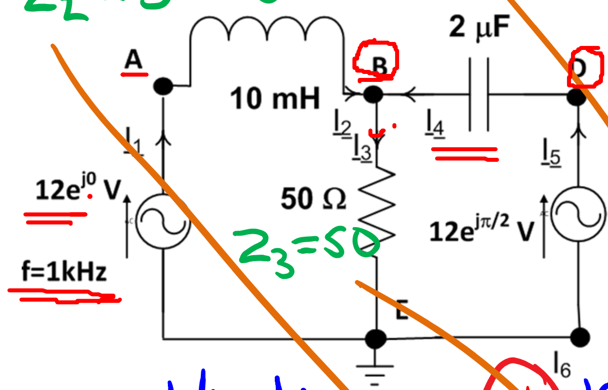
- Solve the circuit
- Draw a phasor diagram showing KVL around loop EABE
- Draw a phasor diagram of currents at node B

- Using phasors, we can use **superposition**, **nodal analysis** or **mesh analysis**, exactly as with DC analysis.
- We'll use **nodal analysis**

1 for f=1000Hz,

$Z_4 = \frac{1}{j\omega C} = -79.6j$

$Z_2 = j\omega L = j62.83$



$Z_3 = 50$

Solution:

- $\underline{V_B} = 12e^{-j2.073}$;
- $\underline{V_{AB}} = (\underline{V_A} - \underline{V_B}) = 20.66e^{j0.534}$;
- $\underline{V_{DB}} = (\underline{V_D} - \underline{V_B}) = 23.25e^{j1.32}$;
- $\underline{I_3} = \underline{V_B} / Z_3 = 0.24e^{-j2.073}$
- $\underline{I_2} = \underline{V_{AB}} / Z_2 = 0.329e^{-j1.036}$
- $\underline{I_4} = \underline{V_{DB}} / Z_4 = 0.292e^{j2.89}$

2 Known branch/node Voltages:

↳ $V_E = 0$

↳ $V_A = 12e^{j0}$

↳ $V_D = 12e^{j\pi/2}$

3 Branch currents in terms of node voltages:

↳ $I_1 = I_2 = \frac{V_A - V_B}{Z_2}$

↳ $I_3 = V_B / Z_3$

↳ $I_5 = I_4 = \frac{V_D - V_B}{Z_4}$

4 KCL at B:

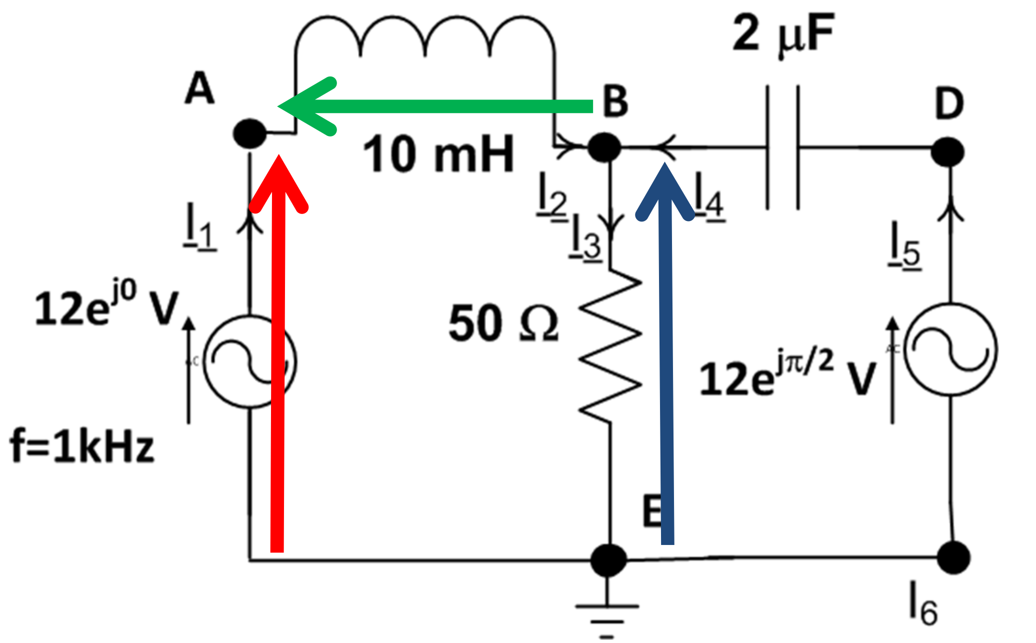
$$I_3 = I_2 + I_4$$

$$\boxed{\frac{V_A - V_B}{Z_2} + \frac{V_D - V_B}{Z_4}} = \frac{V_B}{Z_3}$$

$$V_B\left(\frac{1}{Z_2} + \frac{1}{Z_3} + \frac{1}{Z_4}\right) = \frac{V_A}{Z_2} + \frac{V_D}{Z_4}$$

5 Sub back in node voltages and impedances and solve for unknowns.

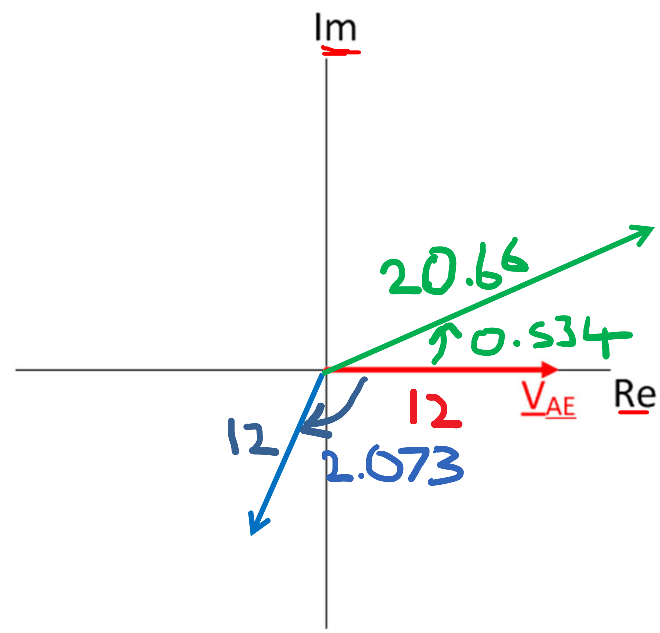
Phasor Diagrams:



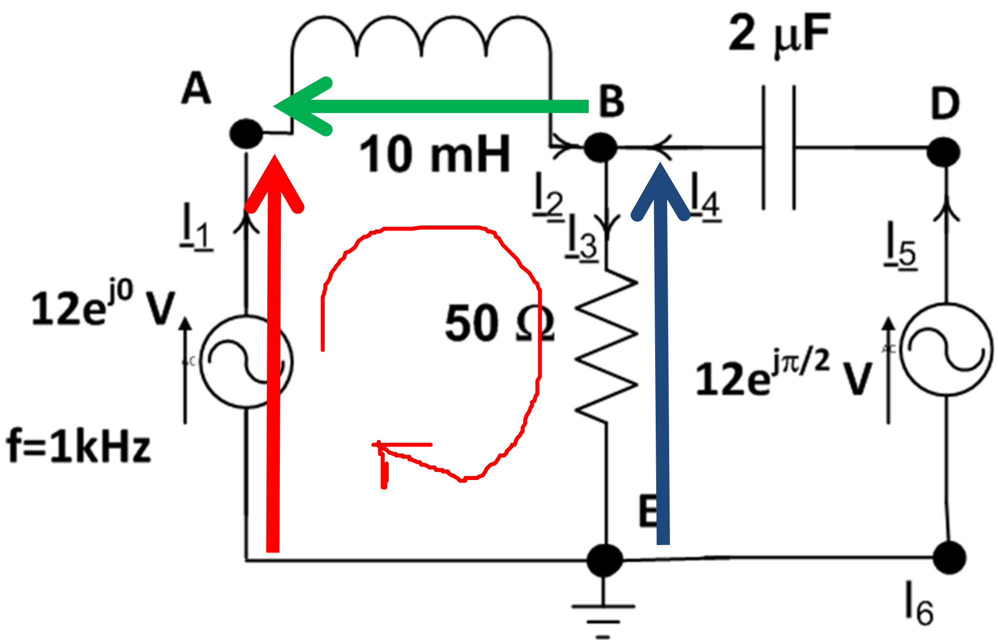
Solution:

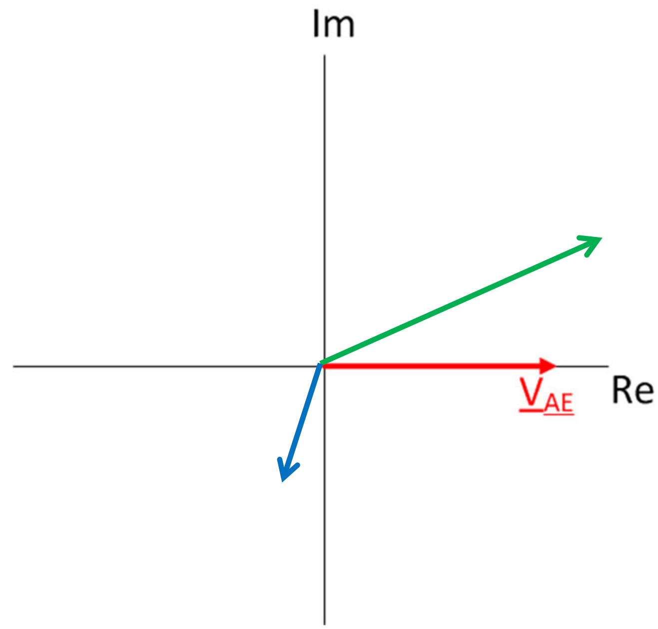
- $\underline{V}_E = 0$
- $\underline{V}_A = 12e^{-j0}$
- $\underline{V}_D = 12e^{-j\pi/2}$
- $\underline{V}_B = 12e^{-j2.073}$
- $\underline{V}_{AB} = (\underline{V}_A - \underline{V}_B) = 20.66e^{j0.534};$
- $\underline{V}_{DB} = (\underline{V}_D - \underline{V}_B) = 23.25e^{j1.32};$
- $\underline{I}_3 = \underline{V}_B \,/\, Z_3 = 0.24e^{-j2.073}$
- $\underline{I}_2 = \underline{V}_{AB} \,/\, Z_2 = 0.329e^{-j1.036}$
- $\underline{I}_4 = \underline{V}_{DB} \,/\, Z_4 = 0.292e^{j2.89}$

- We can show the solutions graphically using a phasor diagram:

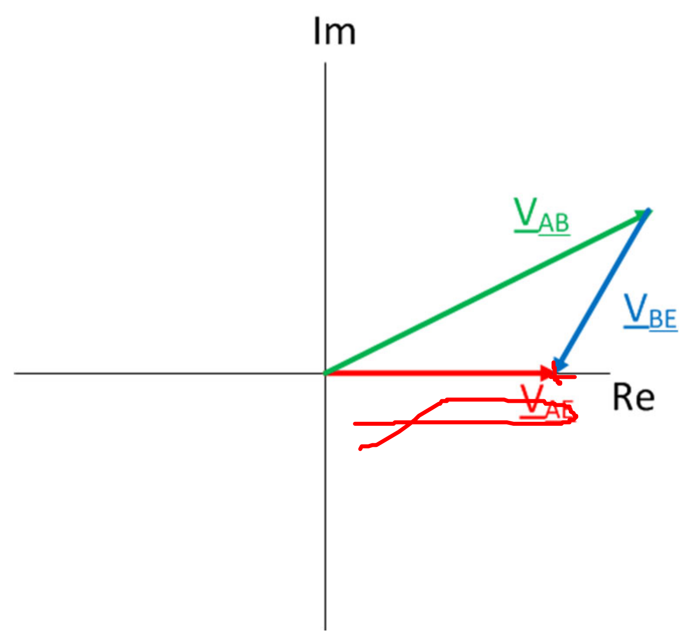


Phasor Diagram





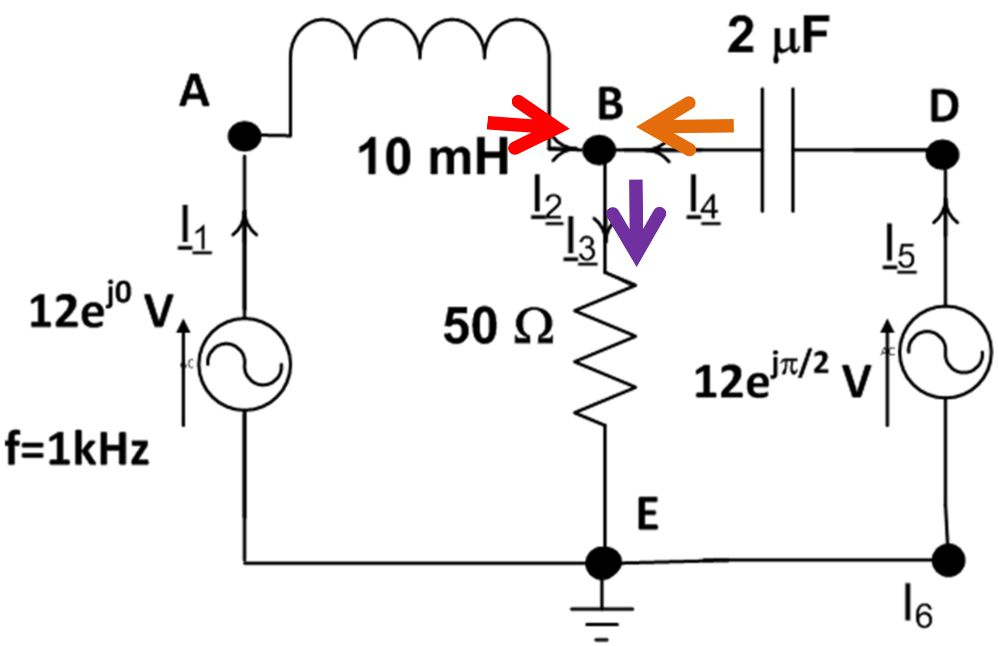
$$V_{AB} + V_{BE} = V_{AE}$$

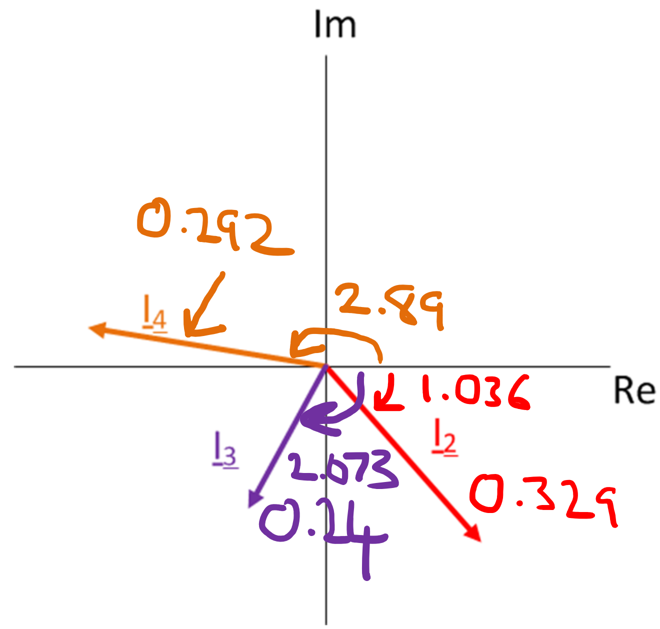


Solution:

- $V_E = 0$
- $V_A = 12e^{-j0}$
- $V_D = 12e^{-j\pi/2}$
- $V_B = 12e^{-j2.073}$
- $V_{AB} = (V_A - V_B) = 20.66e^{j0.534}$;
- $V_{DB} = (V_D - V_B) = 23.25e^{j1.32}$;
- $I_3 = V_B / Z_3 = 0.24e^{-j2.073}$
- $I_2 = V_{AB} / Z_2 = 0.329e^{-j1.036}$
- $I_4 = V_{DB} / Z_4 = 0.292e^{j2.89}$

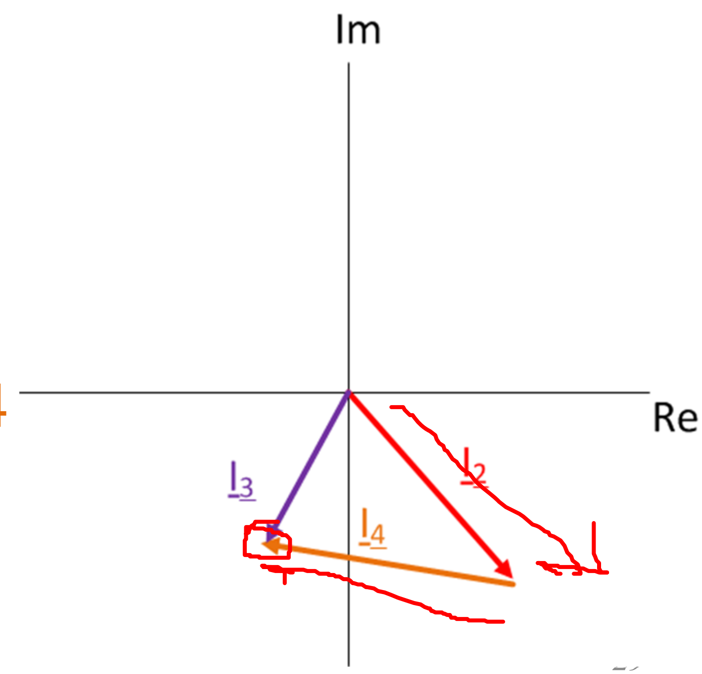
Phasor Diagram (currents)

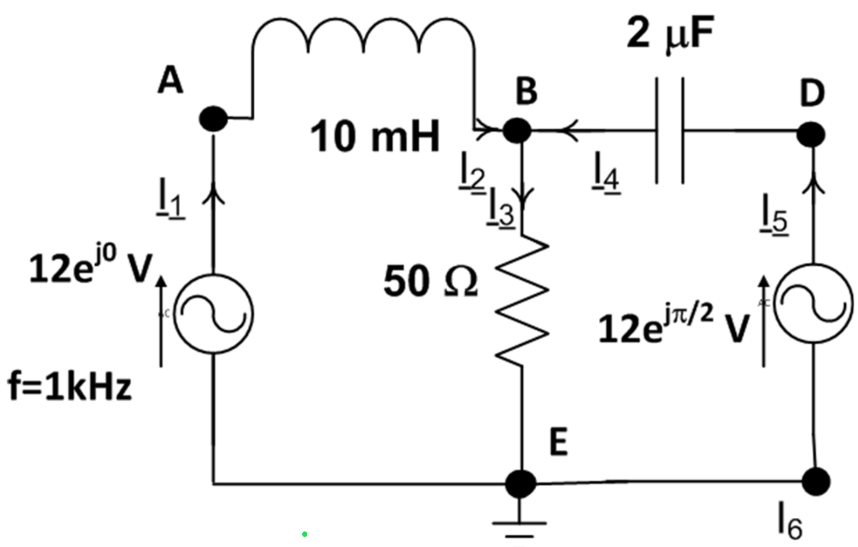




- $\underline{I}_3 = \underline{V}_B / Z_3 = 0.24e^{-j2.073}$
- $\underline{I}_2 = \underline{V}_{AB} / Z_2 = 0.329e^{-j1.036}$
- $\underline{I}_4 = \underline{V}_{DB} / Z_4 = 0.292e^{j2.89}$

$$\underline{I}_3 = \underline{I}_2 + \underline{I}_4$$

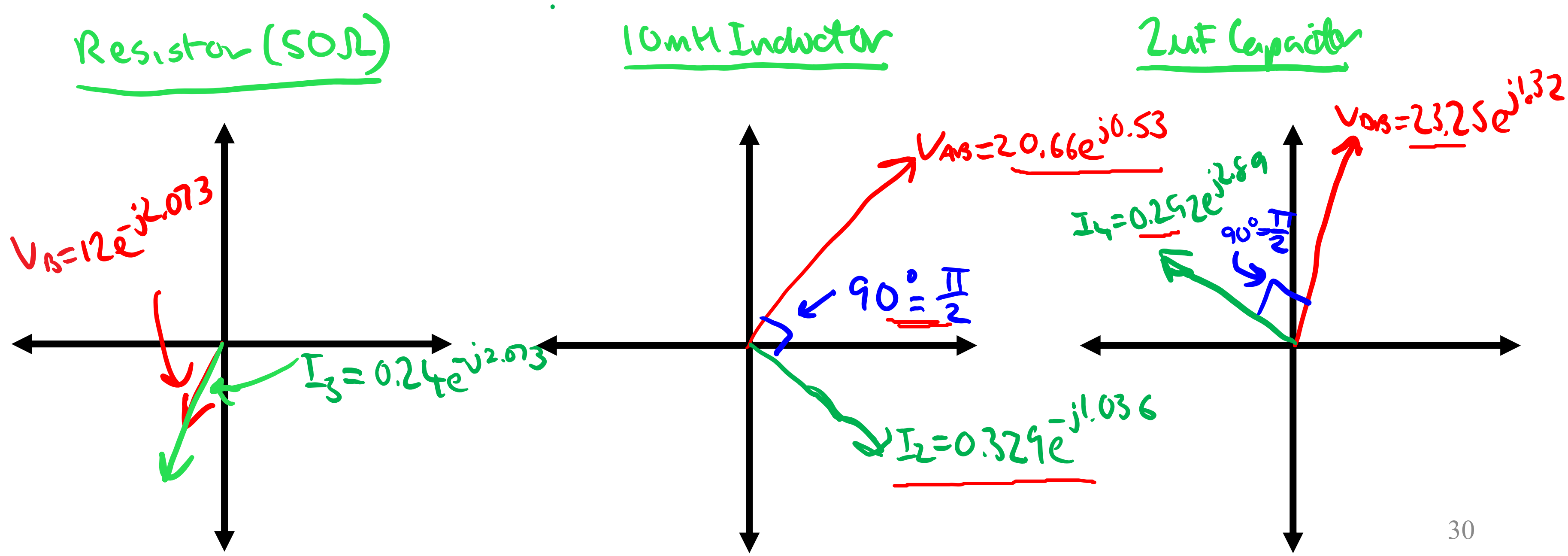




Solution:

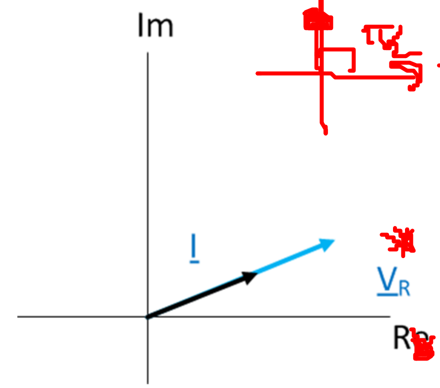
- $V_B = 12e^{-j2.073}$;
- $V_{AB} = (V_A - V_B) = 20.66e^{j0.534}$;
- $V_{DB} = (V_D - V_B) = 23.25e^{j1.32}$;
- $I_3 = V_B / Z_3 = 0.24e^{-j2.073}$
- $I_2 = V_{AB} / Z_2 = 0.329e^{-j1.036}$
- $I_4 = V_{DB} / Z_4 = 0.292e^{j2.89}$

- **I-V Phasor Diagram:** Phasor diagrams are also useful to show the relative phase of the voltage phasor compared to the current phasor



I-V Phasor Diagram

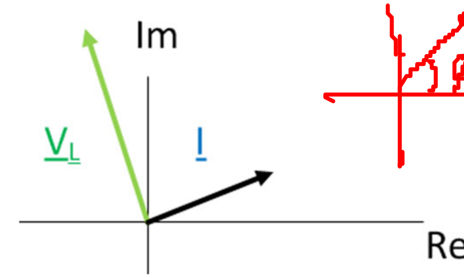
- Phasor diagrams are also useful to show the **relative phase of the voltage phasor compared to the current phasor**



- Resistor: The voltage is always in-phase with the current

- Inductor: the voltage is **always π/2 anticlockwise from current**, we say the voltage "leads" the current.

Because: $\boxed{\underline{V_3} = \boxed{j\omega L}\,\underline{I_3}}$



- Capacitor: The voltage is **always π/2 clockwise from current**, we say the voltage "lags" the current.

Because: $\underline{V_2} = \dfrac{1}{j\omega C}\underline{I_2}$