

# ENGG1300

## Introduction to Electrical Systems

### Week 10– Power Systems

Lecturer:

Dr Philip Terrill

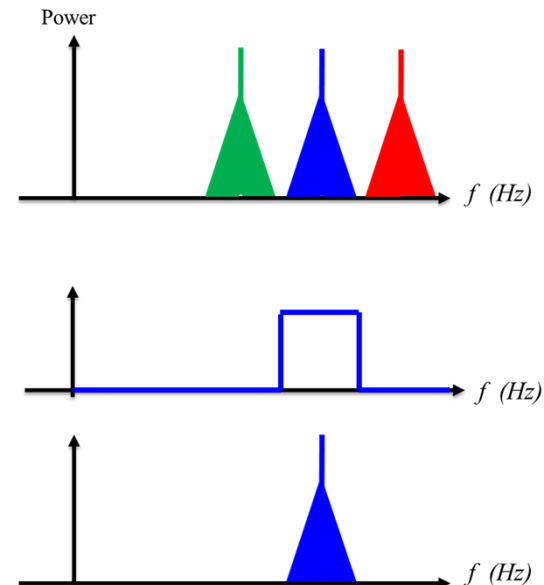
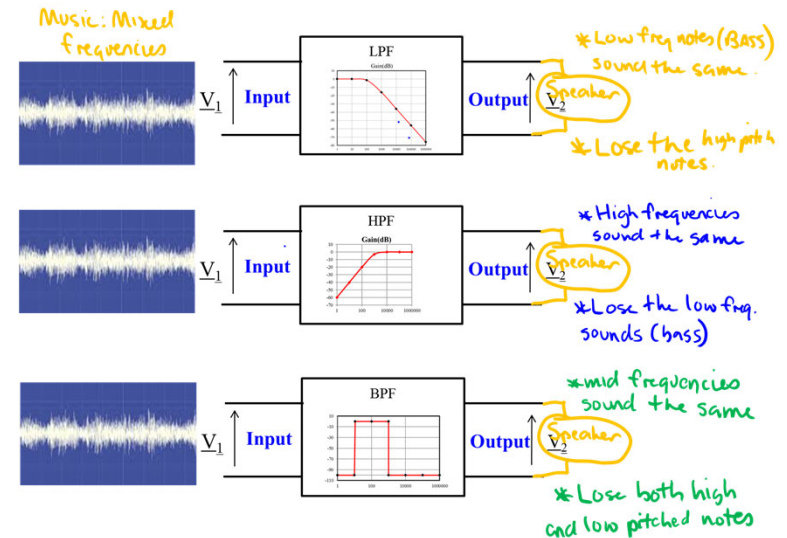
([p.terrill@uq.edu.au](mailto:p.terrill@uq.edu.au))

# Audio Filter Design Activity/Report Reminder

- Full details and documents on blackboard: “Assessment” -> “Audio Filter Design Report (10%)”
- Assessment (10% to total grade):
  - Due 2pm Thursday 13<sup>th</sup> of May (week 11)
  - Report must be completed using the template provided.
  - Strictly title/cover page + 2\*A4 Pages maximum; 2cm margins; 11-point Times New Roman Font; Single Line spacing. **If a longer report is submitted only the first two pages will be marked!**
- Marking criteria available by "My Grades" from the left hand menu, and then clicking "view rubric" for:
  - the "Audio Filter Design Report (Flexible)" item, or
  - the "Audio Filter Design Report (External)" item
- Submitted electronically via blackboard as a single pdf (automatically screened for plagiarism)
  - submission link is in “Assessment” -> “Audio Filter Design Report (10%)”
  - Resubmit as many times as you like up till the due date
- Flexible students:
  - Your only remaining time in the lab to work on this is during your week 10B Practical session this week. You must only attend your class time as scheduled on si-net.
  - This lab is not available outside of scheduled class time.
- The circuit you design won't work perfectly. One of your jobs is consider the trade-offs in your design decisions.

# Last Two Weeks:

- For an RLC circuit: Transfer functions, Frequency response (experimental and theoretically)
- Filters
- Spectrum, Bandwidth
- We can exploit the frequency response of circuits and filters in practical applications!
  - Communications systems for modulation and demodulation of signals (example: AM Radio)
  - Electronic Instrumentation for noise removal
  - Bandwidth in digital comms, and some topical discussion about “copper” vs. “fibre”





# Power Module (Weeks 10-11)

- Week 10 - Efficient delivery of power to a load
  - “Power factor” and power factor correction
  - Resistivity of power lines
  - Transmission voltages
  - Transformers
- Week 11- Overview of power distribution in Queensland (putting it into practice):
  - Power Generation
  - Transformers in practice, and application to achieve Optimal transmission voltages
  - Why AC transmission?
  - What are some of the technical challenges in adopting other energy sources (solar, wind)?



# Why Learn About Electrical Power Generation, Delivery and Utilisation?

- As professional engineers, regardless of specific engineering discipline, you will be working on, **commissioning**, **maintaining**, and **planning** systems which depend on the reliable supply of electricity
- Having a good “top-down” understanding of how power is delivered will help you:
  - Maximise the **power efficiency** of the projects you are working on
  - **Work effectively** with power companies/electrical engineers
  - **Minimise the costs, and maximise the productivity** of your project
  - Take **electrical power delivery considerations** into account in important design decisions

# Revision: Instantaneous Power (From Wk 4)

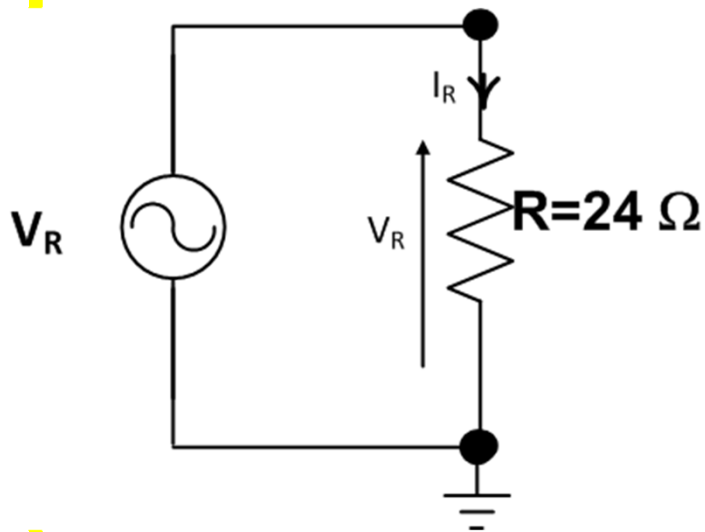
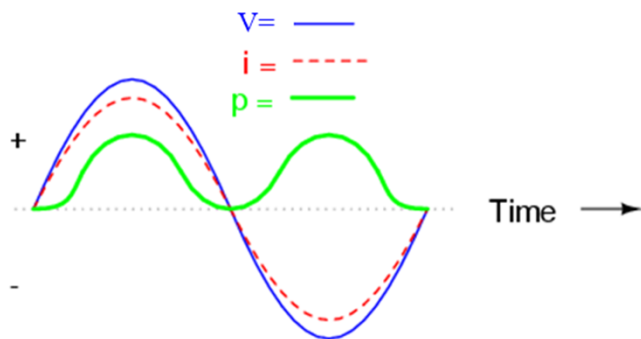


Image from [http://openbookproject.net/electricCircuits/AC/AC\\_4.html](http://openbookproject.net/electricCircuits/AC/AC_4.html)



$$v_R(t) = 340 \times \cos(100\pi t) \text{ Volts}$$

$$i_R(t) = \frac{340 \times \cos(100\pi t)}{24} \text{ Amps}$$

$$i_R(t) = 14.1 \cos(100\pi t) \text{ Amps}$$

$$p_R(t) = v_R(t) \times i_R(t)$$

$$p_R(t) = 340 \cos(100\pi t) \times 14.1 \cos(100\pi t) \text{ Watts}$$

$$p_R(t) = 340 \times 14.1 \cos^2(100\pi t) \text{ W}$$

**Double angle formula:  $\cos^2(x) = 1/2 + (1/2)\cos(2x)$ !**

$$p_R(t) = 4800 \left( \frac{1}{2} + \frac{1}{2} \cos(200\pi t) \right) = 2400 + 2400 \cos(200\pi t) \text{ W}$$

**Instantaneous power gives us important information.... However, we are often more interested in summary information – Average Power**

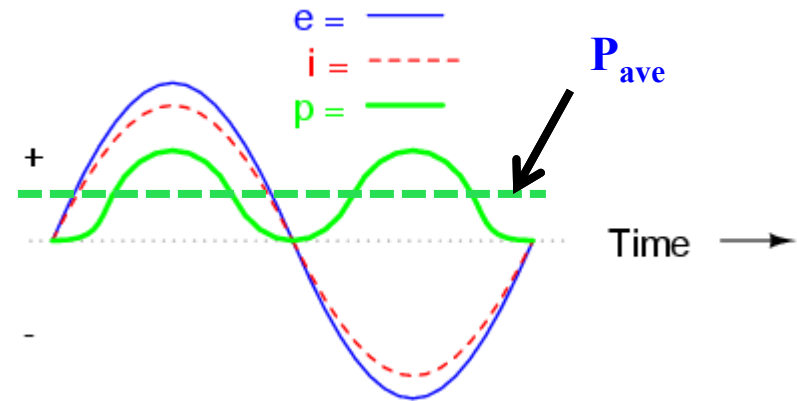
# Average Power

- We are often interested in the average power
- If we consider a long interval of time,  $T$ , then  $P_{ave} = (\text{total energy consumed in } T)/T$
- $P_{ave}$  is a constant, it is not a function of time
- If the time-varying waveform is a regular pattern, we should calculate over a whole number of cycles (even over 1 cycle is enough)

$$P_{ave} = \frac{1}{T} \int_0^T p(t) dt$$

# Average Power

- For our previous example:  
 $p(t) = 2400 + 2400 \cos(200\pi t)$
- Each cycle of  $v(t)$  is 20 ms long, so put  $T = 0.02$



$$P_{ave} = \frac{1}{0.02} \int_0^{0.02} (2400 + 2400 (\cos 628t)) dt$$

- We could simply observe that the integral of the  $\cos(628t)$  term over complete cycles is zero, which leaves:  **$P = 2400W$**
- We can see this is equivalent to:  **$P_{ave} = 0.5 \times |V| \times |I|$**

[we can do the maths...]



# Doing the Maths: Solve the integral

$$P_{\text{ave}} = \frac{1}{0.02} \int_0^{0.02} 2400 + 2400 \cdot \cos(628t) dt$$

$$P_{\text{ave}} = \frac{1}{0.02} \left[ 2400t + \frac{2400}{628} \sin(628t) \right]_0^{0.02}$$

$$P_{\text{ave}} = \frac{1}{0.02} [2400 * 0.02 - 2400 * 0 + 3.82 \sin(4\pi) - 3.82 \sin(0)]$$

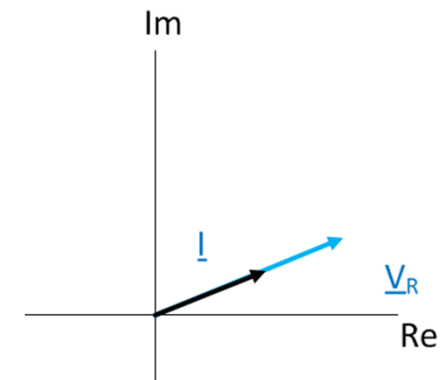
$$P_{\text{ave}} = \frac{1}{0.02} [2400 * 0.02 - 0 + 0 - 0]$$

$$P_{\text{ave}} = 2400 W$$

# Week 5 Revision: I-V Phasor Diagram

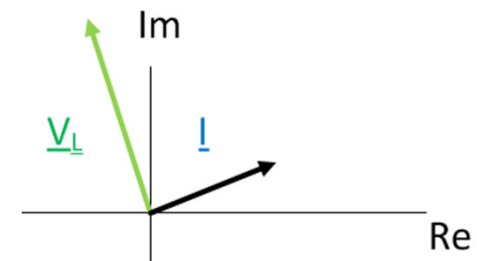
- Phasor diagrams are also useful to show the **relative phase of the voltage phasor compared to the current phasor**

- Resistor:** The voltage is always in-phase with the current



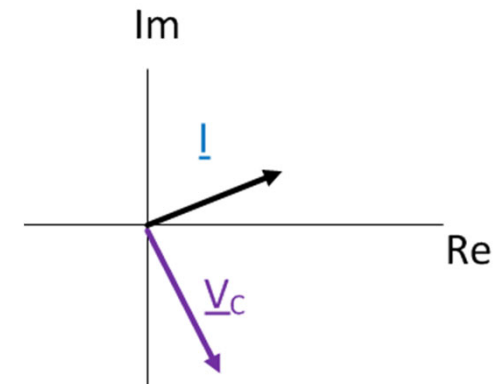
- Inductor:** the voltage is **always  $\pi/2$  anticlockwise from current**, we say the voltage “leads” the current.

**Because:**  $\underline{V}_3 = j\omega L \underline{I}_3$

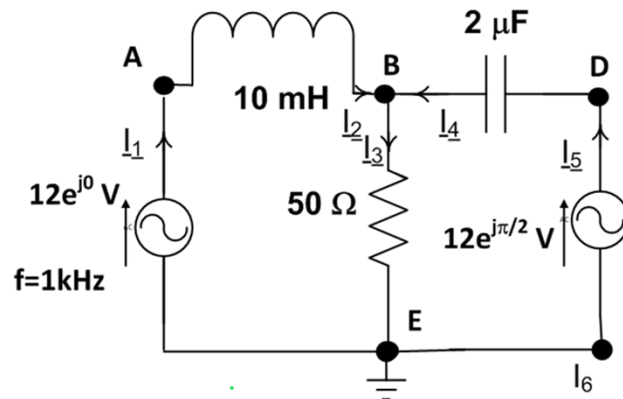


- Capacitor:** The voltage is **always  $\pi/2$  clockwise from current**, we say the voltage “lags” the current.

**Because:**  $\underline{V}_2 = \frac{1}{j\omega C} \underline{I}_2$



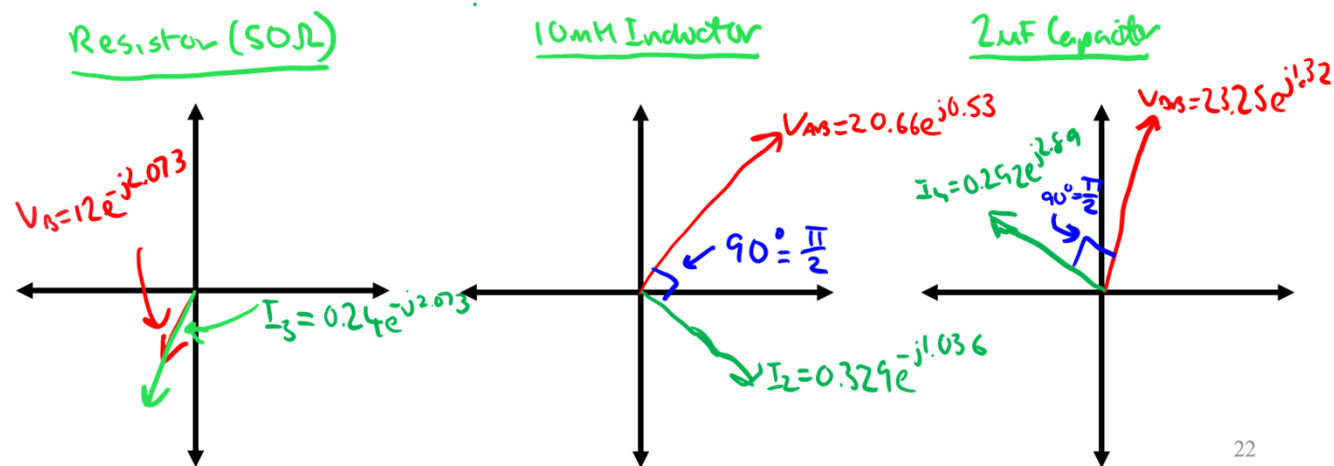
# We saw this in our week 5 lecture example:



Solution:

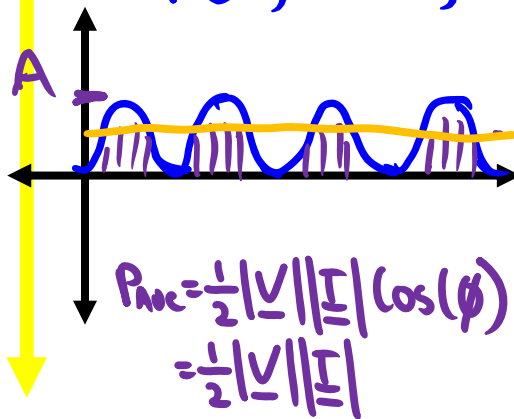
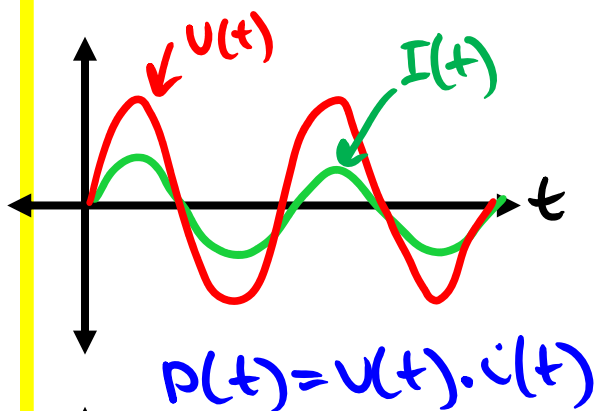
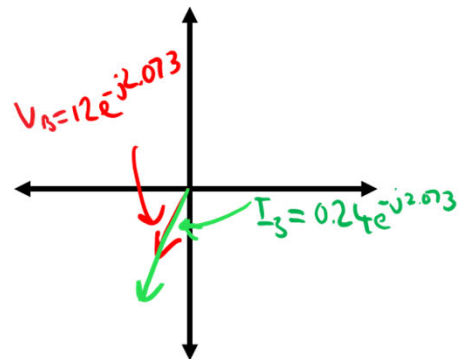
- $\underline{V}_B = 12e^{-j2.073}$  ;
- $\underline{V}_{AB} = (\underline{V}_A - \underline{V}_B) = 20.66e^{j0.534}$ ;
- $\underline{V}_{DB} = (\underline{V}_D - \underline{V}_B) = 23.25e^{j1.32}$ ;
- $\underline{I}_3 = \underline{V}_B / Z_3 = 0.24e^{-j2.073}$
- $\underline{I}_2 = \underline{V}_{AB} / Z_2 = 0.329e^{-j1.036}$
- $\underline{I}_4 = \underline{V}_{DB} / Z_4 = 0.292e^{j2.89}$

- **I-V Phasor Diagram:** Phasor diagrams are also useful to show the relative phase of the voltage phasor compared to the current phasor

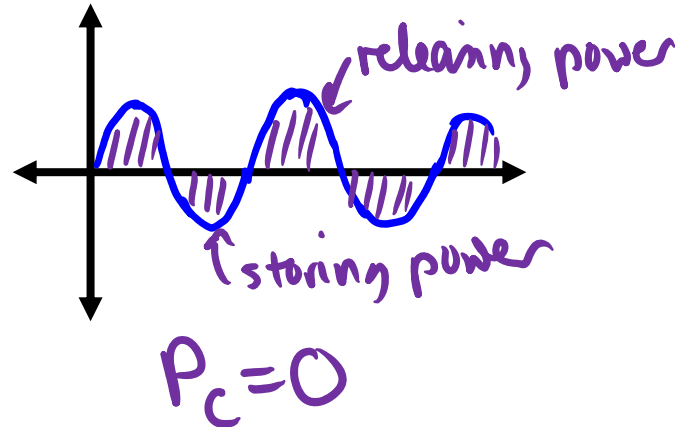
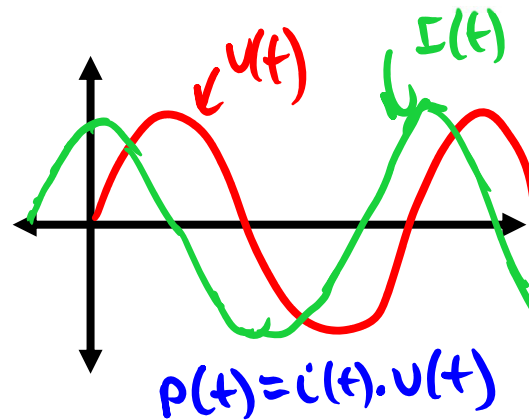
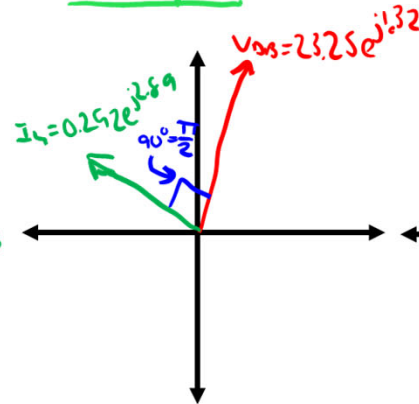


# Average Power: Intuitive View of Components

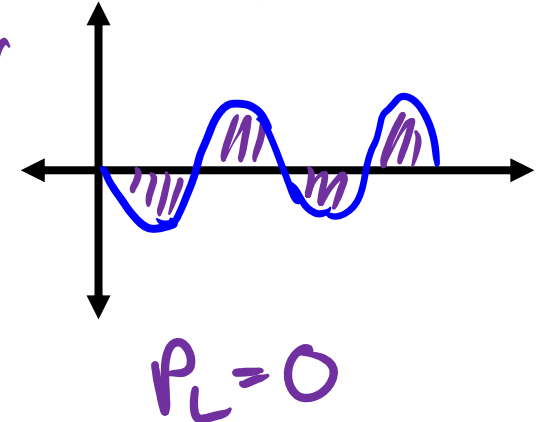
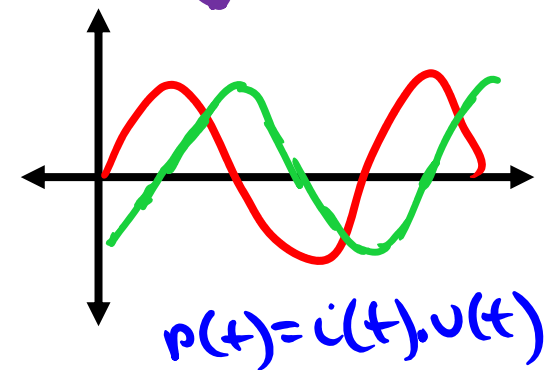
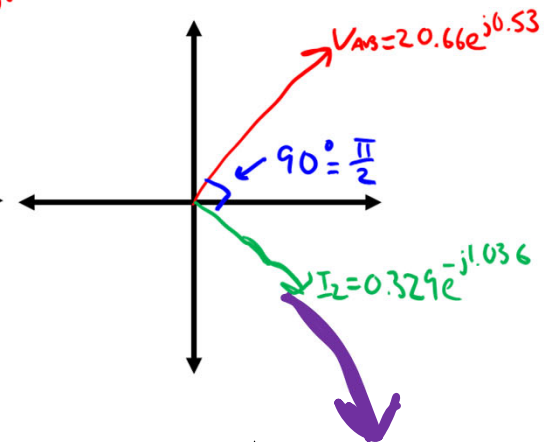
Resistor (50Ω)



2μF Capacitor



10mH Inductor



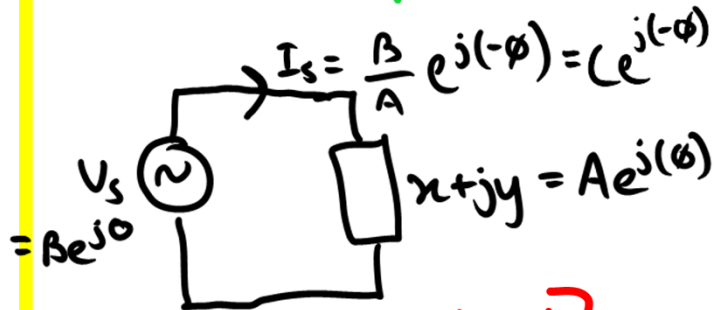
# Energy Stored in Inductors/Capacitors

- *Inductors and capacitors do not consume power: They store energy (in magnetic or electric fields), and then release this energy back to the circuit with no losses.*

The average power dissipated by an ideal capacitor or inductor is zero.

- Real devices are not perfect inductors or capacitors (*series resistance*, leakage current) - We sometimes need to model these imperfections
- In ENGG1300, unless explicitly asked otherwise, you can assume capacitors and inductors are ideal; However, you may observe these effects (particularly resistance of an inductor) in the lab.

Consider a complex "LOAD":



[Voltage + current out of phase by  $\phi$ .]

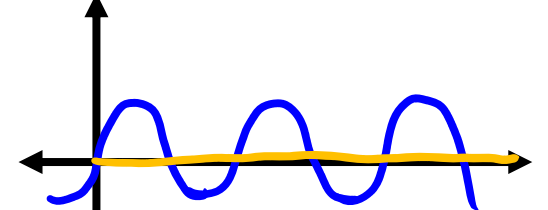
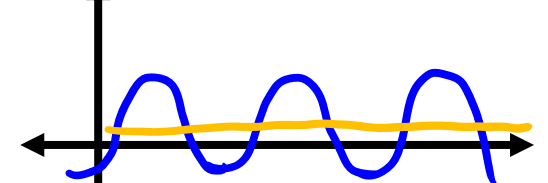
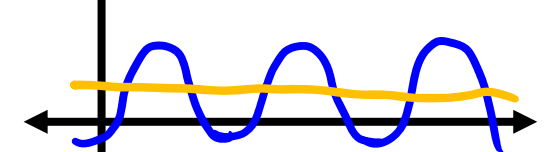
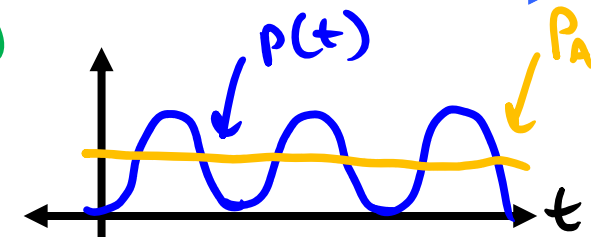
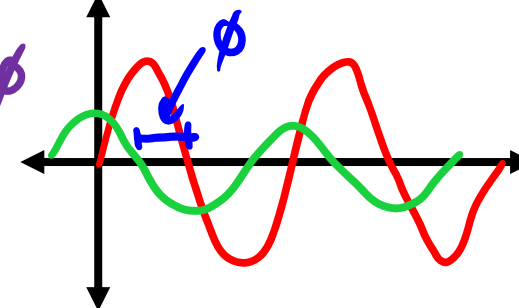
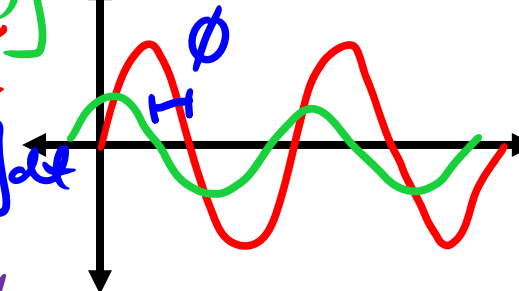
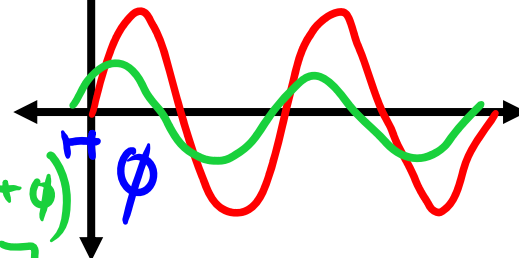
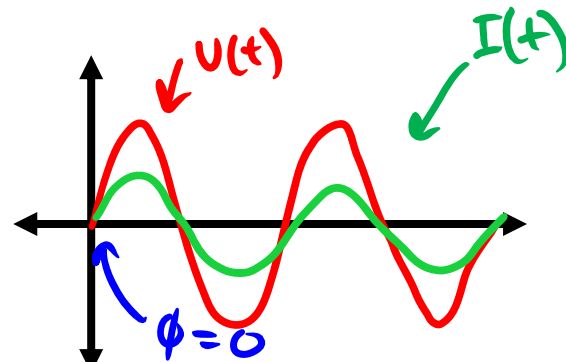
$$p(t) = B \cos(\omega t + 0) \cdot \left( \frac{B}{A} \cos(\omega t + \phi) \right)$$

$$= B \cdot \frac{B}{A} \cdot \frac{1}{2} \left[ \underbrace{\cos(2\omega t + \phi)}_{\text{sinusoid}} + \underbrace{\cos(\phi)}_{\text{constant}} \right]$$

$$P_{ave} = \int_0^T \frac{B \cdot C}{2T} \left[ \cos(2\omega t + \phi) + \cos(\phi) \right] dt$$

$$\frac{1}{T} \int_0^T \cos(x) dx = 0!! \quad \int_0^T \frac{B \cdot C}{2T} \cos \phi dt = \frac{B \cdot C}{2} \cos \phi$$

$$P_{ave} = \frac{1}{2} |V| |I| \cos(\phi)$$



# Effective AC (RMS) Voltage

- It is also convenient to consider the effective voltage:
  - “The DC voltage that would give the same average power consumption as the AC voltage”

[considered for resistive load because inductors and capacitors don't consume average power]

$$P_{ave} = V_{eff} I_{eff} \longrightarrow P_{ave} = \frac{V_{eff}^2}{R} \longrightarrow V_{eff} = \sqrt{P_{ave} \cdot R}$$

$$\text{Derive } P_{ave}: P_{ave} = \frac{1}{T} \int_0^T (v(t)^2 / R) dt \quad \text{Substitute } P_{ave}: V_{eff} = \sqrt{\frac{1}{T} \int_0^T v(t)^2 dt}$$

- The effective voltage is the square Root of the Mean (average) of the Square of  $v(t)$

- We call this constant value the **RMS voltage**.

- For **sinusoidal** voltages and currents:

$$V_{RMS} = \frac{1}{\sqrt{2}} |V| \quad I_{RMS} = \frac{1}{\sqrt{2}} |I|$$

- This is the value we often quote for an AC voltage, and our domestic supply is  $V_{RMS} = 240V$  (which means the amplitude is  $\approx 340V$ )

# Efficient Delivery of Power – A Simple System Model

- We're going to focus on a fairly simple model:

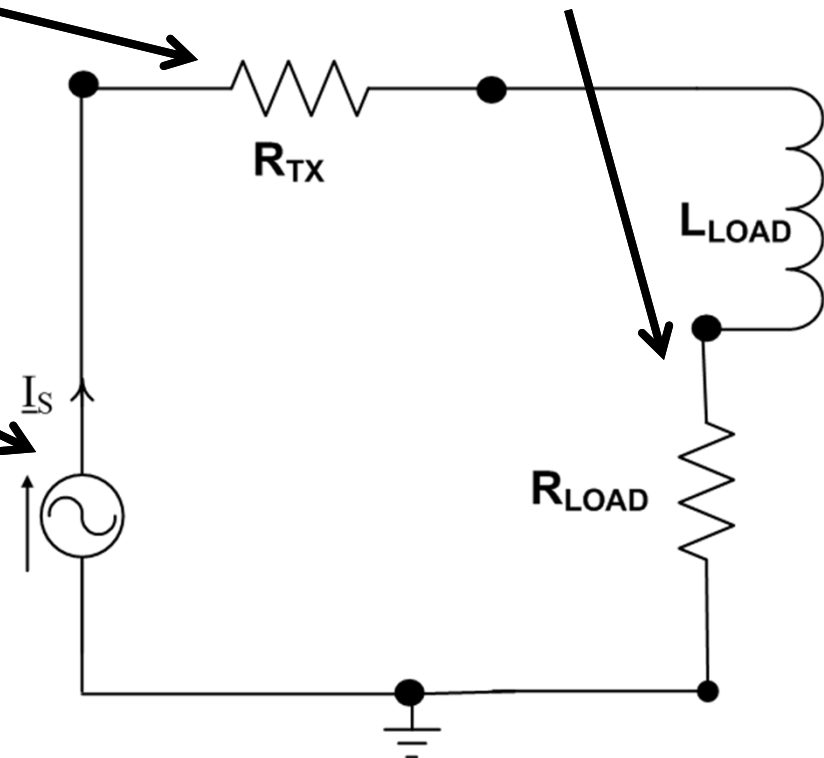
Power is delivered by cables (i.e. wires) which have their own resistance

Many of the highest power consuming loads in industry (motors, pumps), and in the home (fridges, washing machines) are modelled by a combined inductive and resistive load

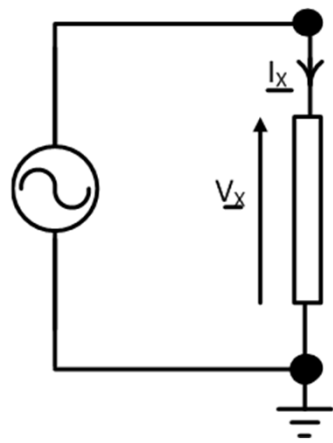
Primary power delivery in Australia (and most parts of the world) is AC.

How can we design our system to most efficiently deliver power to a useful load?  $\underline{V}_S$

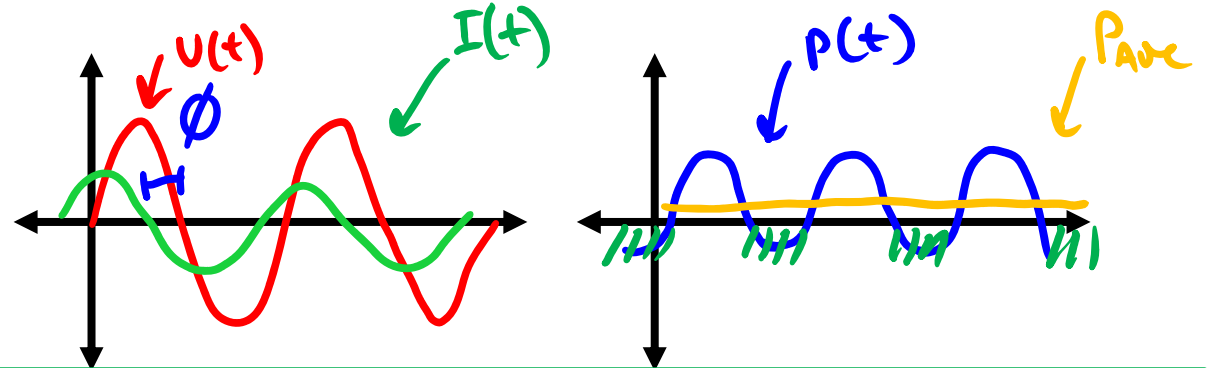
- Understand power utilisation in a “reactive” inductive load
- Resistive properties of cable
- AC power delivery (voltages and currents)







# AC Power

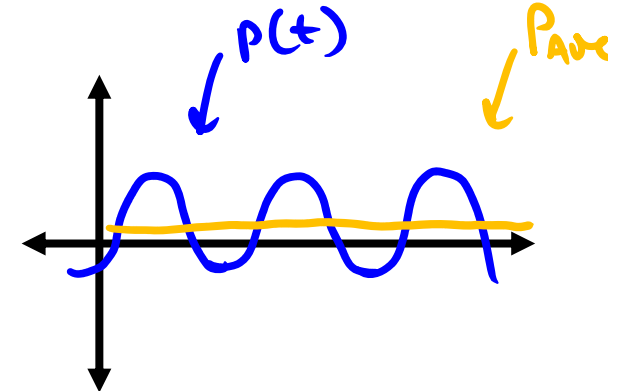
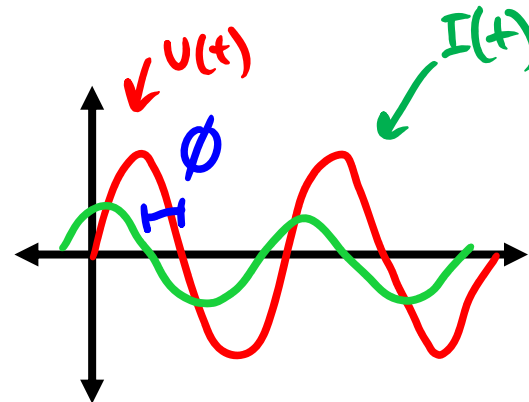
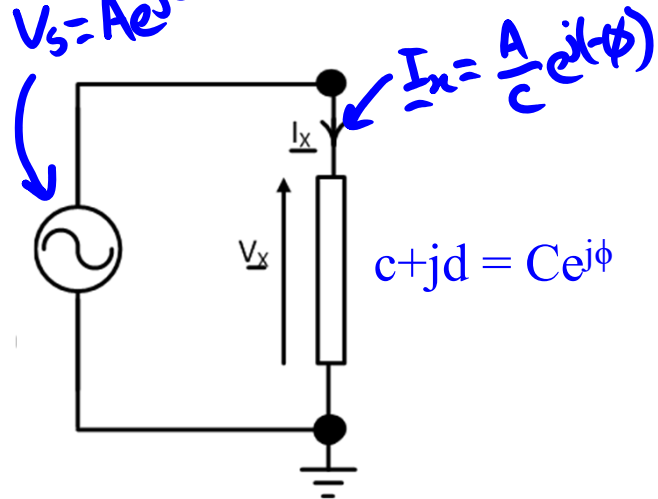


What does this negative instantaneous power mean?

Times when energy stored in the inductor is being released back to the circuit!

- We can think of this power signal containing two components:
  - “Real” power which is dissipated by the resistor
  - “Reactive” power which is periodically stored, and then released by the inductor or capacitor (i.e. imaginary part of the load).

$$V_s = Ae^{j\omega t}$$



$$\text{Power Factor} = pf = \frac{P_{ave}}{\text{Apparent Power}} = \cos(\phi)$$

Apparent power  
(VARs)

$$\frac{1}{2} |V| |I|$$

$\phi$

$$Q_{ave} = \frac{1}{2} |V| |I| \sin \phi$$

↳ Energy stored and released  
by inductor

$$P_{ave} = \frac{1}{2} |V| |I| \cos \phi$$

↳ the actual consumed power in Watts

# Average Power=Real Power

- $P_{ave}$  is sometimes called the real power:
  - $P_{ave} = \frac{1}{2} |\underline{V}| |\underline{I}| \cos [\angle(\underline{V}) - \angle(\underline{I})] = \frac{1}{2} |\underline{V}| |\underline{I}| pf = V_{RMS} I_{RMS} pf$
- If  $Z$  consists of real and imaginary parts,
  - $Z = R + jX = (\text{Resistive Load}) + j(\text{Reactive Load})$
  - $R = |Z| \cos[\angle(Z)] = |Z| \cos(\phi)$
  - $X = |Z| \sin[\angle(Z)] = |Z| \sin(\phi)$
- Then we can also calculate the real power delivered to the load (measured in watts) as:
$$P_{ave} = \frac{1}{2} |\underline{I}|^2 \cdot (\text{real part of } Z) = \frac{1}{2} |\underline{I}|^2 R = I_{RMS}^2 R$$
- Power is always consumed by a load, so  $P_{ave}$  is always  $\geq 0$

# Average Power Summary

Average Power Dissipated by:	Using Phasor Values	RMS Values Reminder: $V_{\text{RMS}} = \frac{1}{\sqrt{2}} V $ $I_{\text{RMS}} = \frac{1}{\sqrt{2}} I $
Resistor	$P_{\text{ave}} = 0.5 \times  V  \times  I $ $P_{\text{ave}} = 0.5 \times  V ^2 / R$ $P_{\text{ave}} = 0.5 \times  I ^2 \times R$	$P_{\text{ave}} = V_{\text{RMS}} \times I_{\text{RMS}}$ $P_{\text{ave}} = V_{\text{RMS}}^2 / R$ $P_{\text{ave}} = I_{\text{RMS}}^2 \times R$
Inductor	0	0
Capacitor	0	0
Complex Load i.e. $z = R + jX$ $z = Ae^{j\theta}$	$P_{\text{ave}} = 0.5 \times  V  \times  I  \times \cos(\theta)$ $P_{\text{ave}} = 0.5 \times  V  \times  I  \times (\text{PF})$ $P_{\text{ave}} = 0.5 \times  I ^2 \times R$	$P_{\text{ave}} = V_{\text{RMS}} \times I_{\text{RMS}} \times \cos(\theta)$ $P_{\text{ave}} = V_{\text{RMS}} \times I_{\text{RMS}} \times (\text{PF})$ $P_{\text{ave}} = I_{\text{RMS}}^2 \times R$

# Average Power=Real Power

- $P_{ave}$  is sometimes called the real power:
  - $P_{ave} = \frac{1}{2} |V||I| \cos [\angle(V) - \angle(I)] = \frac{1}{2} |V| |I| pf = V_{RMS} I_{RMS} pf$
- If  $Z$  consists of real and imaginary parts,
  - $Z = R + jX = (\text{Resistive Load}) + j(\text{Reactive Load})$
  - $R = |Z| \cos[\angle(Z)] = |Z| \cos(\phi)$
  - $X = |Z| \sin[\angle(Z)] = |Z| \sin(\phi)$
- Then we can also calculate the real power delivered to the load (measured in watts) as:
$$P_{ave} = \frac{1}{2} |I|^2 \cdot (\text{real part of } Z) = \frac{1}{2} |I|^2 R = I_{RMS}^2 R$$
- Power is always consumed by a load, so  $P_{ave}$  is always  $\geq 0$

**Does this mean we also have UNREAL power?**

**Sort of....**

# Reactive Power

- We can also calculate what we call the “reactive power” delivered to the load as:

$$Q_{ave} = \frac{1}{2} |\underline{V}| |\underline{I}| \sin [\angle(\underline{V}) - \angle(\underline{I})] = \frac{1}{2} |\underline{V}| |\underline{I}| \sqrt{1 - PF^2} = \frac{1}{2} V_{RMS} I_{RMS} \sqrt{1 - PF^2}$$
$$= \frac{1}{2} |\underline{I}|^2 \cdot (\text{imaginary part of } Z) = \frac{1}{2} |\underline{I}|^2 X = I_{RMS}^2 X$$

- We measure reactive power in Volt-Amps-Reactive (VARs), not watts, since it not really power.
  - An inductor consumes positive VARs
  - A capacitor consumes negative VARs
  - For any given circuit, VARs consumed = VARs supplied
  - $Q_{ave}$  is positive if  $X$  (reactive load) is positive, i.e. if the reactive part of the load is inductive (like an inductor)
  - $Q_{ave}$  is negative if  $X$  (reactive load) is negative, i.e. if the reactive part of the load is capacitive.

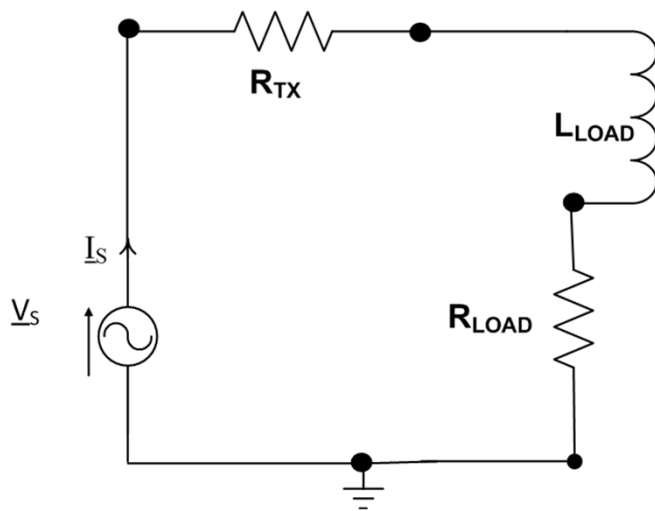
**So, why do we need to care about reactive power?**

# Why do we care about Reactive Power?

- The load doesn't consume energy as a consequence of  $Q_{ave}$ , but the generator does still need to supply this phasor current (which increases with an increasingly reactive load)

More intuitively, reactive loads, store and release energy, meaning peak instantaneous power is higher, even though average power stays the same!

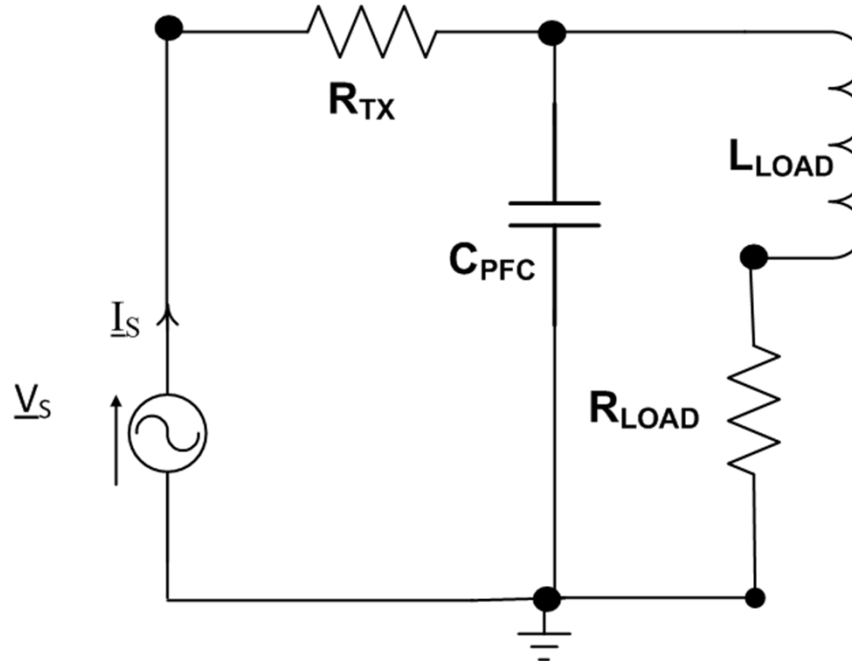
- Consider the transmission line (represented by  $R_{TX}$ ) supplying a load:



- The inductor will mean that the power factor of the load is less than 100%.
- The source will need to supply “positive VARs” to the load.
- For the same average power delivered,  $|\underline{I}_s|$  increases, and therefore increase losses in  $R_{TX}$ .
- To reduce losses in  $R_{TX}$  we can:
  - Reduce  $R_{TX}$ ,
  - Improve the PF (which will reduce  $|\underline{I}_s|$ )

# Power Factor Correction

- If we connect a capacitor in parallel with the load, then this capacitor can supply the required VARs to the load, and losses in  $R_{TX}$  are minimised.
- This is called a power-factor correction capacitor, or a **VARs compensator**.

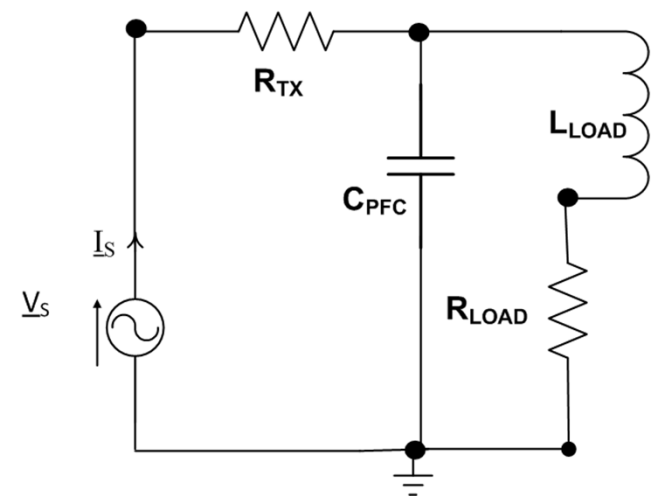




# Calculating value of C

- To calculate the value of power factor correction capacitor, C, we need total VARs to be zero:
  - i.e. we need the load to be totally resistive:  $R + j0$ .
- Basic procedure is:
  1. Calculate admittance of the load:
$$Y_{\text{LOAD}} = 1/Z_{\text{LOAD}} = G_{\text{LOAD}} + jB_{\text{LOAD}}$$
  2. Add capacitor (or inductor) in parallel:
$$Y_{\text{PFC}} = 0 + jB_{\text{PFC}}$$
  3. Admittances in parallel add, so:
$$Y_{\text{total}} = Y_{\text{LOAD}} + Y_{\text{PFC}} = G_{\text{LOAD}} + jB_{\text{LOAD}} + jB_{\text{PFC}}$$
  4. Want total impedance to be real:  
So we choose C so that  $B_{\text{PFC}} = -B_{\text{LOAD}}$

**Detailed video as PreReading for  
Lab (XVI) – Week 11A**



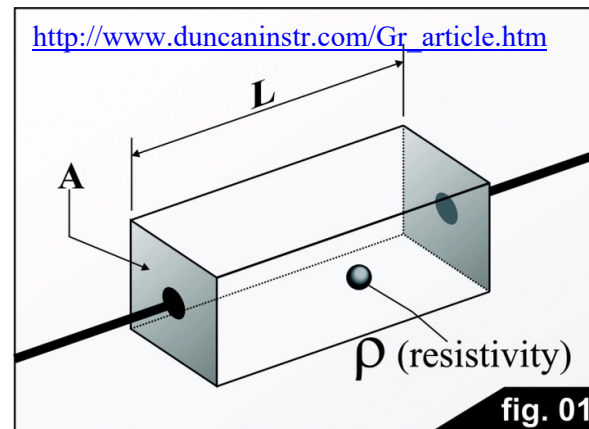
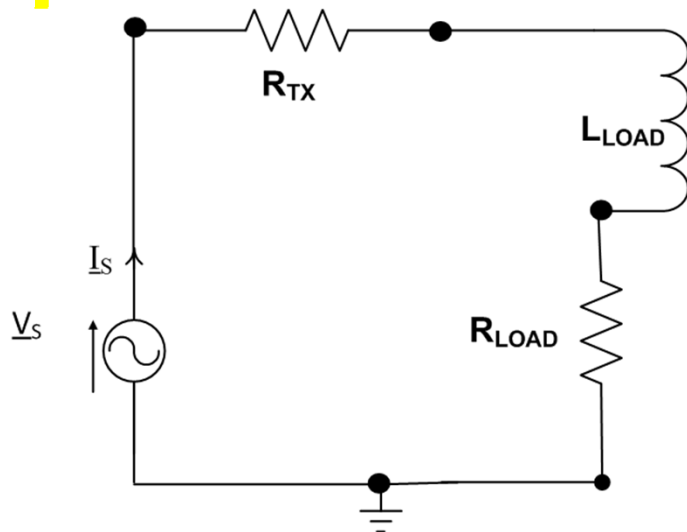
# VAR compensator in Practice



- Discussed more in week 10 lecture.

# Cable Design to Improve Efficiency

- Even with a power-factor of 1, there is still power lost to  $R_{TX}$
- The lower we can make  $R_{TX}$  the greater the efficiency of power delivery – particularly important over long transmission lines!
  - Need to understand the resistive properties of materials!



Resistivity of the material ( $\Omega \cdot m$ )

Cable Length (m)

Resistance of the Cable ( $\Omega$ )

$$R = \frac{\rho L}{A}$$

Cross Section Area ( $m^2$ )

# Resistivity

Element	Resistivity at 20 °C ( $\Omega \cdot m$ )
Aluminium	$2.82 \times 10^{-8}$
Carbon (Graphite)	$3.5 \times 10^{-5}$
Copper	$1.7 \times 10^{-8}$
Germanium	$4.6 \times 10^{-1}$
Glass	$10^{10}$ to $10^{14}$
Gold	$2.44 \times 10^{-8}$
Iron	$1.0 \times 10^{-7}$
Mercury	$9.8 \times 10^{-7}$
Platinum	$1.1 \times 10^{-7}$
Quartz (fused)	$7.5 \times 10^{17}$
Silicon	$6.40 \times 10^2$
Silver	$1.59 \times 10^{-8}$

Low values are good conductors!

High values are good insulators!

Intermediate values are called semi-conductors (and have important applications in manufacturing devices such as transistors and diodes)!

<http://www.cleanroom.byu.edu/Resistivities.phtml>

Inverse of resistivity is called conductivity:

- $\sigma = 1/\rho$  (units of  $S \cdot m^{-1}$ )

# Transmission Line Resistance Example

- The transmission line that connects Tarong power station to Brisbane has the following properties:

- 150km long.  $L = 150,000$  m
- Circular cross-section with 100 mm diameter.
- Material, Aluminium,  $\rho = 2.82 \times 10^{-8} \Omega\text{m}$

$$R = \frac{\rho L}{A}$$

What is the resistance of this cable:

- $A = \pi r^2 = \pi \times 0.05^2 = 0.007854 \text{ m}^2$
- $R = \frac{\rho L}{A} = \frac{2.82 \times 10^{-8} \times 150,000}{0.007854} = 0.538 \Omega$

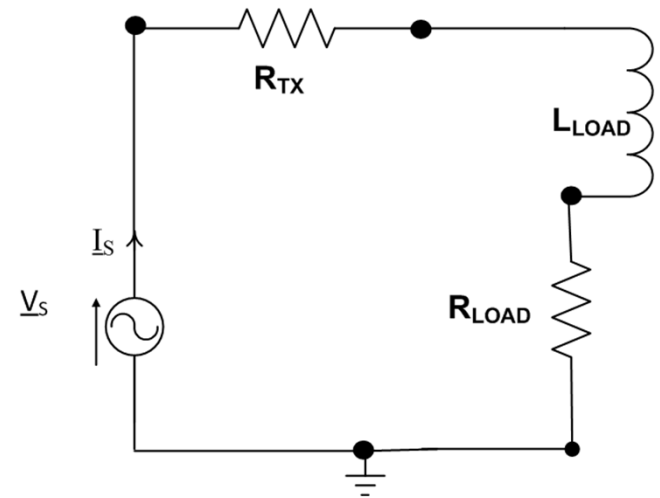
- But – this is not the only important consideration!
  - Total weight =  $2700 \text{ kg/m}^3 \times 0.007854 \text{ m}^2 \times 150000 \text{ m} = 3180$  tonnes
  - About \$8m of Aluminium!

# Manipulating our cable to improve efficiency

The lower we can get  $R_{TX}$ , the greater the efficiency.

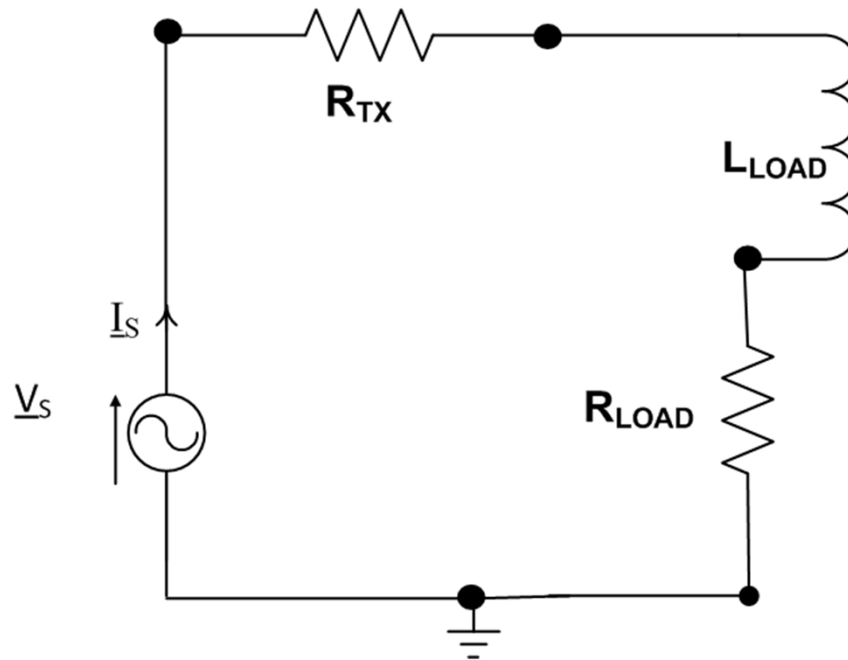
We can do this by:

- Move load closer to source (not always possible but highly desirable if possible)
- Can choose the most conductive materials, and increase the diameter of cables. However, there are important trade-offs:
  - Larger diameter=more material=greater material cost.
  - More material=greater weight=greater civil engineering costs (power line installation/structures)
  - Other material properties (flexibility, corrosion resistance), and cost of material
- High voltage power cables are often aluminium even though less conductive than copper, because it is lighter to be hung on over-head powerlines (i.e. civil construction considerations).

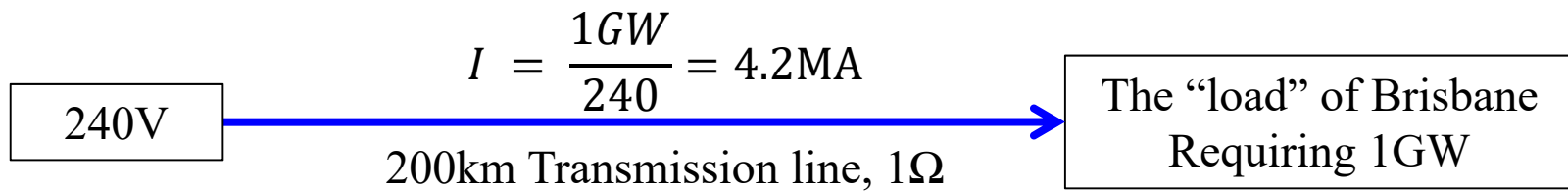


# Manipulating our Voltage Source to improve efficiency

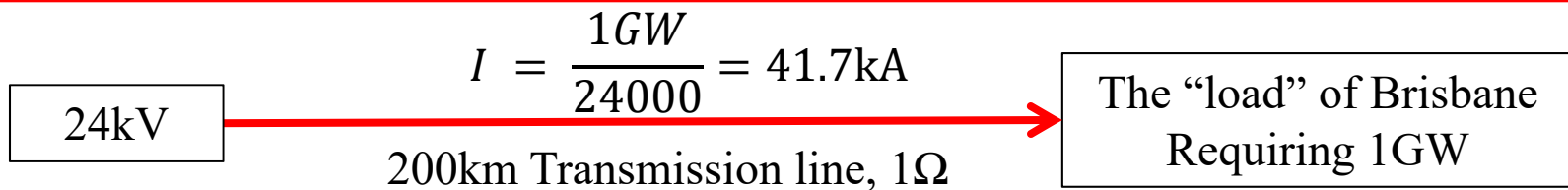
- Can we do any thing with our **voltage source** to improve the losses in  $R_{TX}$ ?



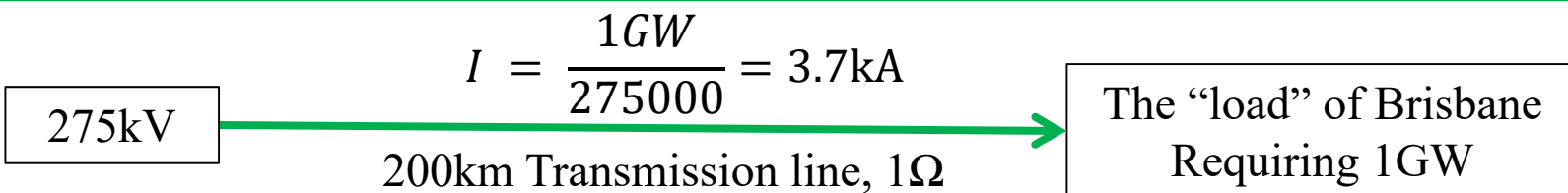
- Lets consider efficiency for a fixed power delivery at different source voltage amplitudes.



**Resistive losses in transmission wires for 4.2 MA:  $I^2R = 17,640$  GW**



**Resistive losses in transmission wires for 41.7 kA:  $I^2R = 1.764$  GW**



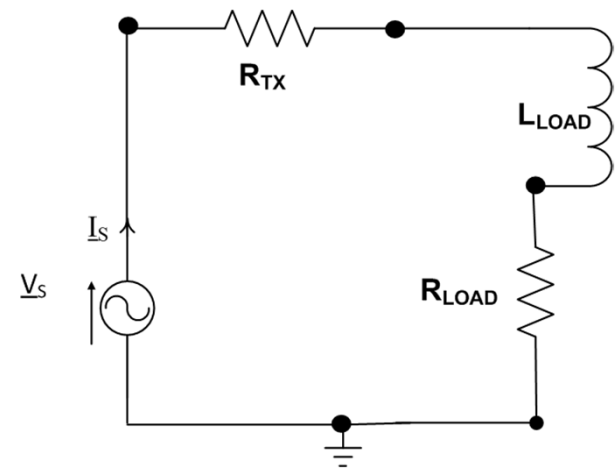
**Resistive losses in transmission wires for 3.7 kA:  $I^2R = 0.001$  GW**

So, it is only by transmitting at very high voltages that large scale power transmission becomes efficient (and indeed, feasible!).



# Manipulating our Voltage Source to improve efficiency

- Can we do anything with our **voltage source** to improve the losses in  $R_{TX}$ ?
  - The higher our voltage source, the lower the current to deliver the same power, and therefore, the lower the resistive losses!
  - But:
    - High voltages become increasingly dangerous
    - Devices not designed to run off very high voltages
    - Thus ideally – transmit energy at high voltages, and then convert to lower voltages for consumption.
    - **Transformers – Details next lecture**
- [Electrical Engineers. more than meets the eye?]





# Next Lecture

## Week 11- Power Systems in Practice in Queensland

Generation to Power Point: Overview of power distribution in Queensland

- Transformers, and application to achieve Optimal transmission voltages
- Power Generation
- Why AC transmission?
- What is and why 3-phase power?
- What are some of the technical challenges in adopting “alternative” energy sources (i.e. solar, wind, bio-mass).