Cross-temporal Probabilistic Forecast Reconciliation: online appendix

February 2, 2023

Contents

A AR2

Reconciliation approach	Base forecasts' sample approach								
	Gaussian frameworks: shrinkage covariance matrix								Bootstrap
	In-sample residuals				Multi-step residuals				
	G	B	H	HB	G	B	H	HB	
$\forall k \in \{2, 1\}$									
base	1.000	1.002	1.036	1.038	0.990	0.992	0.994	0.996	0.993
ct(bu)	0.922	0.922	0.922	0.922	0.893	0.893	0.894	0.893	0.895
ct(shr_{cs}, bu_{te})	**0.922**	0.921	**0.922**	**0.921**	**0.892**	**0.893**	**0.893**	**0.893**	**0.894**
ct($wlsv_{te}, bu_{cs}$)	0.923	0.923	0.931	0.931	0.908	0.909	0.910	0.909	0.903
oct($wlsv$)	0.935	0.937	0.944	0.945	0.922	0.923	0.924	0.923	0.915
oct($bdshr$)	0.923	0.923	0.931	0.931	0.908	0.909	0.910	0.909	0.903
oct(shr)	0.986	0.975	0.973	0.966	0.895	0.895	0.896	0.896	0.934
oct($bshr$)	0.987	0.990	0.976	0.978	0.914	0.915	0.915	0.915	0.944
oct($hshr$)	0.987	0.986	1.028	1.018	0.964	0.964	0.967	0.967	0.979
oct($hbshr$)	0.988	0.989	1.017	1.020	0.977	0.978	0.980	0.981	0.980
oct$_h$(shr)	0.922	**0.921**	0.925	0.925	0.895	0.895	0.896	0.896	0.897
oct$_h$($bshr$)	0.941	0.944	0.944	0.946	0.914	0.915	0.915	0.915	0.916
oct$_h$($hshr$)	0.975	0.975	1.004	1.004	0.964	0.964	0.967	0.967	0.967
oct$_h$($hbshr$)	0.988	0.989	1.017	1.020	0.977	0.978	0.980	0.981	0.980
$k = 1$									
base	1.000	1.002	1.000	1.002	0.981	0.983	0.982	0.983	0.983
ct(bu)	0.978	0.978	0.978	0.977	0.960	0.960	0.961	0.960	0.961
ct(shr_{cs}, bu_{te})	**0.977**	**0.977**	**0.977**	**0.977**	**0.959**	**0.960**	**0.960**	**0.959**	**0.961**
ct($wlsv_{te}, bu_{cs}$)	0.986	0.986	0.987	0.986	0.975	0.976	0.977	0.976	0.970
oct($wlsv$)	0.998	0.998	0.998	0.999	0.989	0.990	0.990	0.990	0.981
oct($bdshr$)	0.985	0.985	0.986	0.986	0.975	0.976	0.976	0.976	0.970
oct(shr)	1.058	1.047	1.044	1.038	0.962	0.962	0.963	0.962	1.019
oct($bshr$)	1.053	1.054	1.042	1.044	0.980	0.982	0.982	0.981	1.024
oct($hshr$)	1.072	1.074	1.096	1.087	1.032	1.033	1.035	1.035	1.062
oct($hbshr$)	1.062	1.063	1.071	1.072	1.045	1.046	1.048	1.049	1.047
oct$_h$(shr)	0.979	0.979	0.980	0.980	0.962	0.962	0.963	0.962	0.964
oct$_h$($bshr$)	0.999	1.001	1.000	1.001	0.980	0.982	0.982	0.981	0.982
oct$_h$($hshr$)	1.049	1.049	1.057	1.057	1.032	1.033	1.035	1.035	1.035
oct$_h$($hbshr$)	1.062	1.063	1.071	1.072	1.045	1.046	1.048	1.049	1.047
$k = 2$									
base	0.999	1.001	1.073	1.076	0.999	1.001	1.007	1.010	1.002
ct(bu)	0.870	0.870	0.870	0.870	0.830	0.831	0.831	0.831	0.832
ct(shr_{cs}, bu_{te})	0.870	0.869	**0.870**	**0.869**	**0.830**	**0.831**	**0.831**	**0.831**	**0.832**
ct($wlsv_{te}, bu_{cs}$)	**0.864**	**0.864**	0.880	0.879	0.846	0.847	0.847	0.847	0.841
oct($wlsv$)	0.877	0.879	0.893	0.894	0.860	0.861	0.861	0.861	0.853
oct($bdshr$)	0.865	0.864	0.880	0.879	0.846	0.846	0.847	0.847	0.841
oct(shr)	0.919	0.908	0.907	0.899	0.833	0.833	0.834	0.834	0.855
oct($bshr$)	0.926	0.930	0.913	0.916	0.852	0.853	0.853	0.853	0.871
oct($hshr$)	0.908	0.906	0.964	0.953	0.900	0.901	0.903	0.903	0.903
oct($hbshr$)	0.918	0.921	0.966	0.970	0.913	0.915	0.917	0.918	0.916
oct$_h$(shr)	0.869	0.866	0.873	0.873	0.833	0.833	0.834	0.834	0.835
oct$_h$($bshr$)	0.887	0.891	0.891	0.895	0.852	0.853	0.853	0.853	0.854
oct$_h$($hshr$)	0.906	0.906	0.954	0.953	0.900	0.901	0.903	0.903	0.904
oct$_h$($hbshr$)	0.918	0.921	0.966	0.970	0.913	0.915	0.917	0.918	0.916

Table 1: CRPS skill score presented in equation (18) and (19) of the paper. The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (base, G) are highlighted in red, the best for each column is marked in bold and in blue the lowest value. The notation used to refer to the reconciliation and base forecast samples is explained in more details in Section 7.1 of the paper.

Reconciliation approach	Base forecasts' sample approach								
	Gaussian frameworks: shrinkage covariance matrix								Bootstrap
	In-sample residuals				Multi-step residuals				
	G	B	H	HB	G	B	H	HB	
					$\forall k \in \{2, 1\}$				
base	0.999	1.003	1.033	1.040	0.991	0.994	0.995	0.997	0.995
ct(bu)	**0.918**	0.918	0.918	0.918	**0.890**	0.891	0.892	0.892	0.892
ct(shr_{cs}, bu_{te})	0.918	0.917	**0.917**	**0.918**	0.891	**0.890**	**0.890**	**0.890**	**0.891**
ct($wlsv_{te}, bu_{cs}$)	0.919	0.919	0.927	0.927	0.905	0.906	0.906	0.906	0.901
oct($wlsv$)	0.930	0.932	0.939	0.940	0.917	0.919	0.918	0.918	0.911
oct($bdshr$)	0.919	0.919	0.927	0.926	0.905	0.906	0.906	0.906	0.901
oct(shr)	0.983	0.972	0.970	0.963	0.893	0.893	0.893	0.893	0.933
oct($bshr$)	0.985	0.988	0.973	0.976	0.910	0.910	0.910	0.910	0.942
oct($hshr$)	0.981	0.979	1.018	1.009	0.958	0.958	0.961	0.961	0.973
oct($hbshr$)	0.980	0.981	1.007	1.010	0.968	0.971	0.972	0.973	0.972
oct$_h$(shr)	0.918	**0.917**	0.920	0.920	0.892	0.893	0.893	0.893	0.895
oct$_h$($bshr$)	0.936	0.938	0.937	0.940	0.908	0.910	0.910	0.910	0.911
oct$_h$($hshr$)	0.969	0.970	0.996	0.996	0.958	0.959	0.961	0.961	0.962
oct$_h$($hbshr$)	0.980	0.981	1.007	1.010	0.968	0.971	0.972	0.972	0.973
					$k = 1$				
base	1.000	1.004	1.000	1.004	0.983	0.985	0.983	0.984	0.986
ct(bu)	0.970	0.970	0.970	0.970	0.954	0.954	0.956	0.955	0.955
ct(shr_{cs}, bu_{te})	**0.969**	**0.968**	**0.969**	**0.969**	**0.953**	**0.953**	**0.952**	**0.952**	**0.954**
ct($wlsv_{te}, bu_{cs}$)	0.977	0.978	0.979	0.978	0.969	0.969	0.969	0.969	0.963
oct($wlsv$)	0.988	0.990	0.989	0.990	0.980	0.981	0.980	0.982	0.975
oct($bdshr$)	0.977	0.977	0.978	0.977	0.967	0.969	0.970	0.968	0.963
oct(shr)	1.050	1.040	1.036	1.030	0.956	0.956	0.956	0.956	1.014
oct($bshr$)	1.046	1.048	1.035	1.037	0.972	0.972	0.973	0.971	1.019
oct($hshr$)	1.061	1.061	1.080	1.073	1.021	1.022	1.025	1.023	1.051
oct($hbshr$)	1.049	1.050	1.056	1.058	1.032	1.034	1.035	1.035	1.035
oct$_h$(shr)	0.970	0.971	0.972	0.972	0.955	0.955	0.956	0.955	0.957
oct$_h$($bshr$)	0.989	0.990	0.989	0.991	0.971	0.973	0.973	0.972	0.974
oct$_h$($hshr$)	1.039	1.039	1.044	1.044	1.022	1.022	1.024	1.024	1.025
oct$_h$($hbshr$)	1.048	1.050	1.056	1.059	1.031	1.033	1.035	1.037	1.036
					$k = 2$				
base	0.999	1.002	1.067	1.076	0.998	1.002	1.006	1.011	1.003
ct(bu)	0.870	0.869	0.869	0.869	**0.831**	0.832	0.833	0.833	0.834
ct(shr_{cs}, bu_{te})	0.870	0.869	**0.869**	**0.869**	0.833	**0.831**	**0.832**	**0.832**	**0.832**
ct($wlsv_{te}, bu_{cs}$)	**0.864**	**0.864**	0.878	0.878	0.846	0.848	0.848	0.848	0.842
oct($wlsv$)	0.876	0.877	0.892	0.892	0.859	0.860	0.860	0.859	0.852
oct($bdshr$)	0.865	0.864	0.879	0.877	0.846	0.847	0.847	0.847	0.842
oct(shr)	0.921	0.910	0.908	0.901	0.833	0.835	0.834	0.835	0.859
oct($bshr$)	0.927	0.931	0.915	0.918	0.852	0.852	0.852	0.852	0.871
oct($hshr$)	0.907	0.904	0.960	0.949	0.899	0.899	0.901	0.903	0.900
oct($hbshr$)	0.915	0.917	0.960	0.964	0.908	0.912	0.912	0.914	0.913
oct$_h$(shr)	0.869	0.866	0.872	0.872	0.833	0.834	0.835	0.835	0.837
oct$_h$($bshr$)	0.885	0.889	0.889	0.892	0.849	0.851	0.851	0.851	0.853
oct$_h$($hshr$)	0.904	0.906	0.950	0.950	0.899	0.899	0.901	0.902	0.903
oct$_h$($hbshr$)	0.915	0.917	0.961	0.964	0.908	0.912	0.913	0.912	0.913

Table 2: ES skill score presented in equation (18) and (19) of the paper. The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (base, G) are highlighted in red, the best for each column is marked in bold and in blue the lowest value. The notation used to refer to the reconciliation and base forecast samples is explained in more details in Section 7.1 of the paper.

B AusGDP

Reconciliation approach	Base forecasts' sample approach				
	Bootstrap	Gaussian frameworks: shrinkage covariance matrix			
		Multi-step residuals		Overlapping and multi-step residuals	
		G	H	G	H
		$\forall k \in \{4,2,1\}$			
base	1.000	0.979	1.011	0.968	0.987
ct(shr_{cs}, bu_{te})	0.937	0.960	0.961	0.962	0.960
ct(wls_{cs}, bu_{te})	0.930	**0.951**	0.953	**0.911**	0.915
oct$_o(wlsv)$	**0.926**	0.961	0.948	0.914	**0.912**
oct$_o(bdshr)$	0.978	0.956	0.949	0.949	0.934
oct$_o(shr)$	0.950	0.957	**0.946**	0.933	0.917
oct$_o(hshr)$	0.989	0.997	1.013	0.967	0.982
oct$_{oh}(shr)$	1.102	1.010	1.006	1.051	0.995
oct$_{oh}(hshr)$	1.006	0.989	1.004	0.979	1.002
		$k = 1$			
base	1.000	0.988	0.988	0.971	0.971
ct(shr_{cs}, bu_{te})	0.992	1.001	1.001	1.004	1.000
ct(wls_{cs}, bu_{te})	0.986	0.997	0.998	**0.964**	0.967
oct$_o(wlsv)$	**0.984**	1.000	0.993	0.966	**0.965**
oct$_o(bdshr)$	1.034	**0.984**	**0.983**	0.988	0.977
oct$_o(shr)$	1.014	0.998	0.995	0.986	0.974
oct$_o(hshr)$	1.047	1.039	1.054	1.019	1.032
oct$_{oh}(shr)$	1.172	1.059	1.063	1.105	1.058
oct$_{oh}(hshr)$	1.068	1.037	1.050	1.034	1.053
		$k = 2$			
base	1.000	0.984	1.009	0.968	0.987
ct(shr_{cs}, bu_{te})	0.949	0.972	0.972	0.974	0.971
ct(wls_{cs}, bu_{te})	0.942	**0.962**	0.964	**0.923**	0.927
oct$_o(wlsv)$	**0.938**	0.976	**0.959**	0.927	**0.925**
oct$_o(bdshr)$	0.991	0.970	0.963	0.963	0.948
oct$_o(shr)$	0.965	0.973	0.959	0.948	0.931
oct$_o(hshr)$	1.002	1.013	1.026	0.980	0.996
oct$_{oh}(shr)$	1.120	1.026	1.019	1.070	1.010
oct$_{oh}(hshr)$	1.021	1.005	1.017	0.993	1.017
		$k = 4$			
base	1.000	0.966	1.037	0.964	1.002
ct(shr_{cs}, bu_{te})	0.874	0.910	0.911	0.910	0.910
ct(wls_{cs}, bu_{te})	0.866	**0.897**	0.900	**0.851**	0.855
oct$_o(wlsv)$	**0.860**	0.910	0.894	0.853	0.852
oct$_o(bdshr)$	0.914	0.917	0.905	0.899	0.880
oct$_o(shr)$	0.877	0.903	**0.886**	0.868	**0.850**
oct$_o(hshr)$	0.922	0.943	0.962	0.905	0.921
oct$_{oh}(shr)$	1.020	0.947	0.939	0.981	0.922
oct$_{oh}(hshr)$	0.934	0.929	0.946	0.913	0.937

Table 3: CRPS skill score presented in equation (18) and (19) of the paper for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold and in blue the lowest value.The notation used to refer to the reconciliation and base forecast samples is explained in more details in Section 8.1 of the paper.

Reconciliation approach	**Base forecasts' sample approach**				
	Bootstrap	Gaussian frameworks: shrinkage covariance matrix			
		Multi-step residuals		Overlapping and multi-step residuals	
		G	H	G	H
		$\forall k \in \{4, 2, 1\}$			
base	1.000	0.967	1.002	0.957	0.980
ct(shr_{cs}, bu_{te})	0.897	0.968	0.969	0.963	0.962
ct(wls_{cs}, bu_{te})	**0.886**	0.939	0.944	**0.882**	0.888
oct$_o$($wlsv$)	0.891	0.950	0.945	0.889	0.892
oct$_o$($bdshr$)	0.940	0.935	0.933	0.922	0.909
oct$_o$(shr)	0.900	**0.935**	**0.928**	0.895	**0.884**
oct$_o$($hshr$)	0.956	0.997	1.015	0.945	0.965
oct$_{oh}$(shr)	1.059	0.981	0.983	1.021	0.962
oct$_{oh}$($hshr$)	0.986	0.996	1.014	0.973	1.005
		$k = 1$			
base	1.000	0.973	0.973	0.961	0.962
ct(shr_{cs}, bu_{te})	0.964	1.012	1.012	1.009	1.004
ct(wls_{cs}, bu_{te})	**0.954**	0.994	0.998	**0.947**	0.952
oct$_o$($wlsv$)	0.958	1.002	0.997	0.953	0.956
oct$_o$($bdshr$)	1.004	**0.965**	**0.964**	0.969	0.959
oct$_o$(shr)	0.973	0.984	0.982	0.960	**0.950**
oct$_o$($hshr$)	1.021	1.049	1.062	1.007	1.024
oct$_{oh}$(shr)	1.130	1.034	1.041	1.083	1.029
oct$_{oh}$($hshr$)	1.053	1.050	1.064	1.034	1.063
		$k = 2$			
base	1.000	0.970	0.999	0.955	0.980
ct(shr_{cs}, bu_{te})	0.915	0.987	0.988	0.983	0.982
ct(wls_{cs}, bu_{te})	**0.904**	**0.958**	0.962	**0.900**	0.906
oct$_o$($wlsv$)	0.908	0.972	0.964	0.908	0.911
oct$_o$($bdshr$)	0.960	0.959	0.957	0.945	0.932
oct$_o$(shr)	0.921	0.958	**0.950**	0.917	**0.905**
oct$_o$($hshr$)	0.977	1.021	1.038	0.966	0.987
oct$_{oh}$(shr)	1.082	1.002	1.003	1.045	0.982
oct$_{oh}$($hshr$)	1.007	1.017	1.036	0.994	1.028
		$k = 4$			
base	1.000	0.958	1.033	0.953	1.000
ct(shr_{cs}, bu_{te})	0.818	0.909	0.910	0.902	0.902
ct(wls_{cs}, bu_{te})	**0.807**	0.871	0.876	**0.805**	0.812
oct$_o$($wlsv$)	0.812	0.882	0.876	0.812	0.816
oct$_o$($bdshr$)	0.860	0.884	0.879	0.857	0.841
oct$_o$(shr)	0.814	**0.867**	**0.857**	0.815	**0.803**
oct$_o$($hshr$)	0.876	0.926	0.949	0.868	0.889
oct$_{oh}$(shr)	0.971	0.910	0.911	0.941	0.882
oct$_{oh}$($hshr$)	0.904	0.924	0.947	0.896	0.929

Table 4: ES skill score presented in equation (18) and (19) of the paper for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold and in blue the lowest value. The notation used to refer to the reconciliation and base forecast samples is explained in more details in Section 8.1 of the paper.

Reconciliation approach	Base forecasts' sample approach				
	Bootstrap	Gaussian frameworks: sample covariance matrix			
		Multi-step residuals		Overlapping and multi-step residuals	
		G	H	G	H
$\forall k \in \{4,2,1\}$					
base	1.000	0.979	0.995	0.968	0.976
ct(shr_{cs}, bu_{te})	0.937	0.956	0.956	0.976	0.976
ct(wls_{cs}, bu_{te})	0.930	0.917	0.917	0.898	0.898
oct($wlsv$)	0.926	0.919	0.920	0.900	0.900
oct($bdshr$)	0.940	0.965	0.945	0.992	0.957
oct(shr)	0.944	1.020	0.940	1.094	0.988
oct($hshr$)	0.988	0.972	1.002	0.974	1.001
oct$_o$($wlsv$)	**0.926**	**0.911**	**0.912**	**0.896**	**0.895**
oct$_o$($bdshr$)	0.978	0.964	0.946	0.952	0.930
oct$_o$(shr)	0.950	0.946	0.922	0.925	0.903
oct$_o$($hshr$)	0.989	0.966	0.984	0.954	0.965
oct$_{oh}$(shr)	1.102	1.059	1.001	1.094	0.988
oct$_{oh}$($hshr$)	1.006	0.983	1.009	0.974	1.001
$k = 1$					
base	1.000	0.988	0.988	0.971	0.971
ct(shr_{cs}, bu_{te})	0.992	1.008	1.008	1.029	1.029
ct(wls_{cs}, bu_{te})	0.986	0.974	0.975	0.956	0.956
oct($wlsv$)	0.984	0.981	0.979	0.959	0.959
oct($bdshr$)	0.997	1.019	1.003	1.044	1.018
oct(shr)	1.015	1.095	1.010	1.160	1.059
oct($hshr$)	1.048	1.037	1.060	1.034	1.061
oct$_o$($wlsv$)	**0.984**	**0.971**	**0.970**	**0.954**	**0.954**
oct$_o$($bdshr$)	1.034	1.016	1.003	1.005	0.989
oct$_o$(shr)	1.014	1.003	0.985	0.987	0.968
oct$_o$($hshr$)	1.047	1.028	1.038	1.012	1.023
oct$_{oh}$(shr)	1.172	1.109	1.066	1.160	1.059
oct$_{oh}$($hshr$)	1.068	1.046	1.059	1.034	1.061
$k = 2$					
base	1.000	0.984	0.993	0.968	0.976
ct(shr_{cs}, bu_{te})	0.949	0.966	0.966	0.987	0.987
ct(wls_{cs}, bu_{te})	0.942	0.928	0.928	0.909	0.909
oct($wlsv$)	0.938	0.929	0.931	0.911	0.911
oct($bdshr$)	0.953	0.976	0.956	1.003	0.969
oct(shr)	0.955	1.031	0.951	1.113	1.002
oct($hshr$)	1.001	0.985	1.014	0.987	1.016
oct$_o$($wlsv$)	**0.938**	**0.921**	**0.923**	**0.907**	**0.906**
oct$_o$($bdshr$)	0.991	0.974	0.957	0.964	0.942
oct$_o$(shr)	0.965	0.958	0.934	0.938	0.916
oct$_o$($hshr$)	1.002	0.979	0.996	0.967	0.978
oct$_{oh}$(shr)	1.120	1.069	1.013	1.113	1.002
oct$_{oh}$($hshr$)	1.021	0.996	1.021	0.987	1.016
$k = 4$					
base	1.000	0.966	1.004	0.964	0.981
ct(shr_{cs}, bu_{te})	0.874	0.896	0.896	0.914	0.914
ct(wls_{cs}, bu_{te})	0.866	0.853	0.853	0.834	0.834
oct($wlsv$)	0.860	0.853	0.855	0.835	0.834
oct($bdshr$)	0.874	0.904	0.880	0.931	0.889
oct(shr)	0.866	0.940	0.864	1.015	0.909
oct($hshr$)	0.919	0.900	0.935	0.904	0.931
oct$_o$($wlsv$)	**0.860**	**0.847**	**0.848**	**0.832**	**0.830**
oct$_o$($bdshr$)	0.914	0.905	0.883	0.892	0.865
oct$_o$(shr)	0.877	0.882	0.852	0.854	0.831
oct$_o$($hshr$)	0.922	0.898	0.923	0.888	0.898
oct$_{oh}$(shr)	1.020	1.002	0.928	1.015	0.909
oct$_{oh}$($hshr$)	0.934	0.912	0.951	0.904	0.931

Table 5: CRPS

Reconciliation approach	**Base forecasts' sample approach**				
	Bootstrap	Gaussian frameworks: shrinkage covariance matrix			
		Multi-step residuals		Overlapping and multi-step residuals	
		G	H	G	H
		$\forall k \in \{4, 2, 1\}$			
base	1.000	0.979	1.011	0.968	0.987
ct(shr_{cs}, bu_{te})	0.937	0.960	0.961	0.962	0.960
ct(wls_{cs}, bu_{te})	0.930	**0.951**	0.953	**0.911**	0.915
oct($wlsv$)	0.926	0.972	0.957	0.918	0.917
oct($bdshr$)	0.940	0.986	0.966	0.981	0.956
oct(shr)	0.944	0.999	0.962	1.051	0.995
oct($hshr$)	0.988	1.000	1.021	0.979	1.002
oct$_o$($wlsv$)	**0.926**	0.961	0.948	0.914	**0.912**
oct$_o$($bdshr$)	0.978	0.956	0.949	0.949	0.934
oct$_o$(shr)	0.950	0.957	**0.946**	0.933	0.917
oct$_o$($hshr$)	0.989	0.997	1.013	0.967	0.982
oct$_{oh}$(shr)	1.102	1.010	1.006	1.051	0.995
oct$_{oh}$($hshr$)	1.006	0.989	1.004	0.979	1.002
		$k = 1$			
base	1.000	0.988	0.988	0.971	0.971
ct(shr_{cs}, bu_{te})	0.992	1.001	1.001	1.004	1.000
ct(wls_{cs}, bu_{te})	0.986	0.997	0.998	**0.964**	0.967
oct($wlsv$)	0.984	1.010	1.003	0.971	0.970
oct($bdshr$)	0.997	1.015	1.006	1.016	1.000
oct(shr)	1.015	1.047	1.021	1.105	1.058
oct($hshr$)	1.048	1.045	1.066	1.034	1.053
oct$_o$($wlsv$)	**0.984**	1.000	0.993	0.966	**0.965**
oct$_o$($bdshr$)	1.034	**0.984**	**0.983**	0.988	0.977
oct$_o$(shr)	1.014	0.998	0.995	0.986	0.974
oct$_o$($hshr$)	1.047	1.039	1.054	1.019	1.032
oct$_{oh}$(shr)	1.172	1.059	1.063	1.105	1.058
oct$_{oh}$($hshr$)	1.068	1.037	1.050	1.034	1.053
		$k = 2$			
base	1.000	0.984	1.009	0.968	0.987
ct(shr_{cs}, bu_{te})	0.949	0.972	0.972	0.974	0.971
ct(wls_{cs}, bu_{te})	0.942	**0.962**	0.964	**0.923**	0.927
oct($wlsv$)	0.938	0.988	0.968	0.931	0.929
oct($bdshr$)	0.953	1.004	0.979	0.996	0.970
oct(shr)	0.955	1.016	0.973	1.070	1.010
oct($hshr$)	1.001	1.015	1.034	0.993	1.017
oct$_o$($wlsv$)	**0.938**	0.976	**0.959**	0.927	**0.925**
oct$_o$($bdshr$)	0.991	0.970	0.963	0.963	0.948
oct$_o$(shr)	0.965	0.973	0.959	0.948	0.931
oct$_o$($hshr$)	1.002	1.013	1.026	0.980	0.996
oct$_{oh}$(shr)	1.120	1.026	1.019	1.070	1.010
oct$_{oh}$($hshr$)	1.021	1.005	1.017	0.993	1.017
		$k = 4$			
base	1.000	0.966	1.037	0.964	1.002
ct(shr_{cs}, bu_{te})	0.874	0.910	0.911	0.910	0.910
ct(wls_{cs}, bu_{te})	0.866	**0.897**	0.900	**0.851**	0.855
oct($wlsv$)	0.860	0.921	0.903	0.856	0.856
oct($bdshr$)	0.874	0.942	0.914	0.932	0.900
oct(shr)	0.866	0.937	0.895	0.981	0.922
oct($hshr$)	0.919	0.942	0.965	0.913	0.937
oct$_o$($wlsv$)	**0.860**	0.910	0.894	0.853	0.852
oct$_o$($bdshr$)	0.914	0.917	0.905	0.899	0.880
oct$_o$(shr)	0.877	0.903	**0.886**	0.868	**0.850**
oct$_o$($hshr$)	0.922	0.943	0.962	0.905	0.921
oct$_{oh}$(shr)	1.020	0.947	0.939	0.981	0.922
oct$_{oh}$($hshr$)	0.934	0.929	0.946	0.913	0.937

Table 6: CRPS

Reconciliation approach	Base forecasts' sample approach				
		Gaussian frameworks: sample covariance matrix			
	Bootstrap	Multi-step residuals		Overlapping and multi-step residuals	
		G	H	G	H
		$\forall k \in \{4, 2, 1\}$			
base	1.000	0.970	0.988	0.960	0.970
ct(shr_{cs}, bu_{te})	0.897	0.944	0.944	0.973	0.973
ct(wls_{cs}, bu_{te})	**0.886**	0.880	0.880	**0.860**	0.860
oct($wlsv$)	0.890	0.890	0.894	0.872	0.872
oct($bdshr$)	0.905	0.956	0.934	0.992	0.954
oct(shr)	0.895	0.979	0.895	1.053	0.944
oct($hshr$)	0.951	0.940	0.973	0.959	0.992
oct$_o$($wlsv$)	0.891	**0.879**	0.881	0.864	0.864
oct$_o$($bdshr$)	0.940	0.928	0.910	0.918	0.895
oct$_o$(shr)	0.900	0.899	**0.876**	0.878	**0.858**
oct$_o$($hshr$)	0.956	0.936	0.955	0.922	0.936
oct$_{oh}$(shr)	1.059	1.015	0.956	1.053	0.945
oct$_{oh}$($hshr$)	0.986	0.968	0.999	0.959	0.992
		$k = 1$			
base	1.000	0.977	0.977	0.965	0.965
ct(shr_{cs}, bu_{te})	0.964	1.001	1.001	1.033	1.033
ct(wls_{cs}, bu_{te})	**0.954**	**0.944**	0.945	**0.928**	**0.928**
oct($wlsv$)	0.958	0.957	0.957	0.938	0.939
oct($bdshr$)	0.972	1.014	0.994	1.048	1.018
oct(shr)	0.973	1.060	0.969	1.121	1.015
oct($hshr$)	1.017	1.010	1.034	1.023	1.055
oct$_o$($wlsv$)	0.958	0.945	0.945	0.931	0.931
oct$_o$($bdshr$)	1.004	0.986	0.971	0.980	0.961
oct$_o$(shr)	0.973	0.963	**0.944**	0.949	0.930
oct$_o$($hshr$)	1.021	1.004	1.012	0.987	1.000
oct$_{oh}$(shr)	1.130	1.063	1.019	1.121	1.016
oct$_{oh}$($hshr$)	1.053	1.034	1.049	1.024	1.055
		$k = 2$			
base	1.000	0.972	0.985	0.959	0.969
ct(shr_{cs}, bu_{te})	0.915	0.961	0.960	0.991	0.991
ct(wls_{cs}, bu_{te})	**0.904**	0.896	**0.896**	**0.877**	**0.877**
oct($wlsv$)	0.909	0.907	0.912	0.889	0.889
oct($bdshr$)	0.925	0.976	0.953	1.013	0.974
oct(shr)	0.913	1.000	0.914	1.076	0.963
oct($hshr$)	0.973	0.960	0.993	0.978	1.014
oct$_o$($wlsv$)	0.908	**0.895**	0.898	0.881	0.882
oct$_o$($bdshr$)	0.960	0.947	0.929	0.938	0.915
oct$_o$(shr)	0.921	0.919	0.896	0.898	0.878
oct$_o$($hshr$)	0.977	0.956	0.976	0.942	0.957
oct$_{oh}$(shr)	1.082	1.029	0.973	1.076	0.963
oct$_{oh}$($hshr$)	1.007	0.988	1.017	0.979	1.014
		$k = 4$			
base	1.000	0.959	1.000	0.957	0.976
ct(shr_{cs}, bu_{te})	0.818	0.874	0.874	0.899	0.900
ct(wls_{cs}, bu_{te})	**0.807**	0.805	0.805	**0.782**	0.783
oct($wlsv$)	0.811	0.813	0.819	0.794	0.794
oct($bdshr$)	0.825	0.883	0.860	0.920	0.876
oct(shr)	0.807	0.885	0.808	0.967	0.861
oct($hshr$)	0.871	0.856	0.897	0.881	0.913
oct$_o$($wlsv$)	0.812	**0.802**	0.806	0.786	0.786
oct$_o$($bdshr$)	0.860	0.856	0.836	0.841	0.816
oct$_o$(shr)	0.814	0.821	**0.796**	0.794	**0.775**
oct$_o$($hshr$)	0.876	0.854	0.882	0.844	0.856
oct$_{oh}$(shr)	0.971	0.954	0.882	0.967	0.861
oct$_{oh}$($hshr$)	0.904	0.888	0.934	0.881	0.913

Table 7: ES

Reconciliation approach	Base forecasts' sample approach				
	Bootstrap	Gaussian frameworks: shrinkage covariance matrix			
		Multi-step residuals		Overlapping and multi-step residuals	
		G	H	G	H
		$\forall k \in \{4,2,1\}$			
base	1.000	0.967	1.002	0.957	0.980
ct(shr_{cs}, bu_{te})	0.897	0.968	0.969	0.963	0.962
ct(wls_{cs}, bu_{te})	**0.886**	0.939	0.944	**0.882**	0.888
oct$(wlsv)$	0.890	0.966	0.959	0.897	0.901
oct$(bdshr)$	0.905	0.997	0.981	0.986	0.960
oct(shr)	0.895	0.979	0.945	1.021	0.962
oct$(hshr)$	0.951	0.997	1.023	0.973	1.005
oct$_o(wlsv)$	0.891	0.950	0.945	0.889	0.892
oct$_o(bdshr)$	0.940	0.935	0.933	0.922	0.909
oct$_o(shr)$	0.900	**0.935**	**0.928**	0.895	**0.884**
oct$_o(hshr)$	0.956	0.997	1.015	0.945	0.965
oct$_{oh}(shr)$	1.059	0.981	0.983	1.021	0.962
oct$_{oh}(hshr)$	0.986	0.996	1.014	0.973	1.005
		$k = 1$			
base	1.000	0.973	0.973	0.961	0.962
ct(shr_{cs}, bu_{te})	0.964	1.012	1.012	1.009	1.004
ct(wls_{cs}, bu_{te})	**0.954**	0.994	0.998	**0.947**	0.952
oct$(wlsv)$	0.958	1.017	1.012	0.960	0.965
oct$(bdshr)$	0.972	1.031	1.021	1.024	1.005
oct(shr)	0.973	1.041	1.011	1.083	1.028
oct$(hshr)$	1.017	1.051	1.073	1.034	1.063
oct$_o(wlsv)$	0.958	1.002	0.997	0.953	0.956
oct$_o(bdshr)$	1.004	**0.965**	**0.964**	0.969	0.959
oct$_o(shr)$	0.973	0.984	0.982	0.960	**0.950**
oct$_o(hshr)$	1.021	1.049	1.062	1.007	1.024
oct$_{oh}(shr)$	1.130	1.034	1.041	1.083	1.029
oct$_{oh}(hshr)$	1.053	1.050	1.064	1.034	1.063
		$k = 2$			
base	1.000	0.970	0.999	0.955	0.980
ct(shr_{cs}, bu_{te})	0.915	0.987	0.988	0.983	0.982
ct(wls_{cs}, bu_{te})	**0.904**	**0.958**	0.962	**0.900**	0.906
oct$(wlsv)$	0.909	0.988	0.979	0.916	0.920
oct$(bdshr)$	0.925	1.024	1.005	1.010	0.984
oct(shr)	0.913	1.006	0.967	1.045	0.982
oct$(hshr)$	0.973	1.020	1.046	0.994	1.028
oct$_o(wlsv)$	0.908	0.972	0.964	0.908	0.911
oct$_o(bdshr)$	0.960	0.959	0.957	0.945	0.932
oct$_o(shr)$	0.921	0.958	**0.950**	0.917	**0.905**
oct$_o(hshr)$	0.977	1.021	1.038	0.966	0.987
oct$_{oh}(shr)$	1.082	1.002	1.003	1.045	0.982
oct$_{oh}(hshr)$	1.007	1.017	1.036	0.994	1.028
		$k = 4$			
base	1.000	0.958	1.033	0.953	1.000
ct(shr_{cs}, bu_{te})	0.818	0.909	0.910	0.902	0.902
ct(wls_{cs}, bu_{te})	**0.807**	0.871	0.876	**0.805**	0.812
oct$(wlsv)$	0.811	0.896	0.891	0.820	0.825
oct$(bdshr)$	0.825	0.938	0.919	0.926	0.895
oct(shr)	0.807	0.898	0.864	0.940	0.881
oct$(hshr)$	0.871	0.924	0.954	0.897	0.929
oct$_o(wlsv)$	0.812	0.882	0.876	0.812	0.816
oct$_o(bdshr)$	0.860	0.884	0.879	0.857	0.841
oct$_o(shr)$	0.814	**0.867**	**0.857**	0.815	**0.803**
oct$_o(hshr)$	0.876	0.926	0.949	0.868	0.889
oct$_{oh}(shr)$	0.971	0.910	0.911	0.941	0.882
oct$_{oh}(hshr)$	0.904	0.924	0.947	0.896	0.929

Table 8: ES