Cross-temporal Probabilistic Forecast Reconciliation: online appendix

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A Monte Carlo Simulation: one-step residuals and shrinkage covariance matrix

In Section 5.1, we discussed the use of one-step residuals in estimating the covariance matrix. In particular we point out that one-step residuals lead to a biased estimate of the covariance matrix where some correlation are zeros by definition. This can be seen clearly in Figure 1, where we have reported the covariance and correlation matrix corresponding to Figure 11 in the paper, but where we have used one-step residuals. In addition, Tables 1, 2 and 3 show the Frobenius norm, CRPS, and ES skill scores as explained in the paper to investigate the effectiveness of one-step residuals. Moreover, in Tables 4 and 5, we have utilized a shrinkage matrix rather than the sample covariance matrix to assess the performance of our approach.

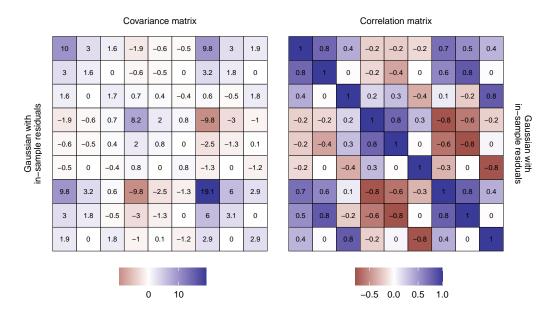


Figure 1: Comparison of estimated covariance and correlation matrices (first simulation) for base forecasts using a parametric Gaussian (with one-step residuals) approach. The true covariance and correlation matrices are shown in Figure 10 in the paper.

		Base forecasts' sample approach											
			Gaussi	an appro	ach: samp	ole cova	riance m	atrix					
Reconciliation approach	ctjb]	In-sample	residual	s	M	Iulti-step	residua	esiduals				
wp p rowers		G	В	Н	HB	G	В	Н	HB				
base	8.260	17.638	16.733	22.178	21.789	7.748	6.549	3.409	2.215				
ct(bu)	3.195	21.789	21.789	21.789	21.789	2.215	2.215	2.215	2.215				
$\operatorname{ct}(shr_{cs}, bu_{te})$	3.202	21.942	21.789	21.942	21.789	2.224	2.215	2.224	2.215				
$\operatorname{ct}(wlsv_{te}, bu_{cs})$	3.183	18.237	18.237	21.789	21.789	2.188	2.188	2.215	2.215				
oct(wlsv)	3.766	19.174	18.611	22.304	21.789	3.082	2.191	2.910	2.215				
oct(bdshr)	3.203	18.559	18.416	21.937	21.789	2.195	2.184	2.224	2.215				
oct(shr)	5.217	25.015	23.457	23.413	21.789	2.260	2.202	2.226	2.215				
oct(bshr)	5.282	23.772	23.997	22.146	21.789	2.720	2.220	2.756	2.215				
oct(hshr)	6.161	11.336	10.940	23.598	21.789	4.138	4.167	2.225	2.215				
oct(hbshr)	5.731	11.379	10.940	22.146	21.789	5.085	4.167	2.756	2.215				
$\operatorname{oct}_h(shr)$	3.251	20.965	19.992	22.079	21.789	2.260	2.202	2.226	2.215				
$\operatorname{oct}_h(bshr)$	3.602	21.306	21.022	22.146	21.789	2.720	2.220	2.756	2.215				
$\operatorname{oct}_h(hshr)$	4.869	11.405	10.940	22.037	21.789	4.138	4.167	2.225	2.215				
$\operatorname{oct}_h(hbshr)$	5.731	11.379	10.940	22.146	21.789	5.085	4.167	2.756	2.215				

Table 1: Frobenius norm between the true (in Figure 10 in the paper) and the estimated covariance matrix for different reconciliation approaches and different techniques for simulating the base forecasts. In bold, it is reported the lowest value for each column, in blue the minimum.

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ach							
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ariance matrix	mple co	oach: sa	an appr	Gaussi			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	ılti-step residuals	N	ls	residua	n-sample	Ir	ctjb	
base $t,000$	В Н	G	HB	Н	В	G		mp promeir
base $ct(bu)$ 0.901 0.908 1.009 1.044 1.047 0.998 0.999 1.002 1.002 $ct(bu)$ 0.901 0.930 0.929 0.929 0.929 0.900 0.900 0.900 0.900 $ct(shr_{cs},bu_{te})$ 0.901 0.929 0.928 0.929 0.928 0.900 0.899 0.900 0.900 $ct(wlsv_{te},bu_{cs})$ 0.910 0.930 0.929 0.939 0.939 0.916 0.916 0.916 0.916 $ct(wlsv_{te},bu_{cs})$ 0.910 0.930 0.930 0.939 0.930 0.930 0.930 0.930 0.930 $cct(wlsv)$ 0.922 0.942 0.944 0.951 0.953 0.930 0.930 0.930 0.930 $cct(shr)$ 0.910 0.930 0.930 0.939 0.938 0.916 0.915 0.916 0.916 0.916 $cct(shr)$ 0.941 0.999 0.985 0.983 0.973 0.903 0.902 0.902 0.902 $cct(shr)$ 0.951 0.995 1.000 0.983 0.986 0.922 0.922 0.921 0.921 $cct(shr)$ 0.987 0.995 0.993 1.039 1.026 0.972 0.972 0.974 0.995 $cct(shshr)$ 0.987 0.995 0.996 1.024 1.028 0.985 0.985 0.985 0.987 0.996 $cct(shshr)$ 0.904 0.929 0.928 0.932 0.932 0.903 0.902 0.902 0.902 $cct(shshr)$ 0.923 0.948 0.952 0.951 0.954 0.922 0.922 0.921 0.995 $cct(shshr)$ 0.974 0.982 0.982 1.012 1.012 0.972 0.972 0.974 0.995 $cct(shr_{cs},bu_{te})$ 0.987 0.995 0.996 1.024 1.028 0.985 0.985 0.987 0.996 $cct(shr_{cs},bu_{te})$ 0.987 0.995 0.996 1.024 1.028 0.985 0.985 0.987 0.996 $cct(shr_{cs},bu_{te})$ 0.997 0.993 0.994 0.994 0.994 0.995 0.995 0.996 1.024 1.028 0.985 0.985 0.987 0.996 $cct(shr_{cs},bu_{te})$ 0.977 0.993 0.993 0.994 0.994 0.994 0.996 0.996 0.996 0.996 0.996 0.996 0.996 0.997 0.997 0.999 0				€ {2,1}	$\forall k$			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.999 1.002 1	0.998	1.047			1.008	1.000	base
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.900 0.900 0	0.900	0.929	0.929	0.929	0.930	0.901	ct(bu)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.899 0.900 0	0.900	0.928	0.929	0.928	0.929	0.901	$\operatorname{ct}(shr_{cs},bu_{te})$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.916 0.916 (0.916	0.939	0.939	0.929	0.930	0.910	$\operatorname{ct}(wlsv_{te},bu_{cs})$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.930 0.930 0	0.930	0.953	0.951	0.944	0.942	0.922	oct(wlsv)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.915 0.916 (0.916	0.938	0.939	0.930	0.930	0.910	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.902 0.902 0	0.903	0.973	0.983	0.985	0.999	0.941	oct(shr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.922 0.921 0	0.922	0.986	0.983	1.000	0.995	0.951	oct(bshr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			1.026	1.039	0.993		0.987	oct(hshr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.985 0.987 0	0.985	1.028	1.024	0.996	0.995	0.987	oct(hbshr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.902 0.902 0	0.903	0.932	0.932	0.928	0.929	0.904	$\operatorname{oct}_h(shr)$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.922 0.921 (0.922	0.954	0.951	0.952	0.948	0.923	$\operatorname{oct}_h(bshr)$
$k = 1$ base 1.000 1.017 1.019 1.017 1.019 0.998 0.999 0.999 1.00 ct(bu) 0.978 0.994 0.994 0.994 0.976 0.976 0.977 0.90 ct(shr_{cs}, bu_{te}) 0.977 0.993 0.993 0.994 0.993 0.976 0.976 0.976 ct(wlsv_{te}, bu_{cs}) 0.986 1.002 1.002 1.003 1.003 0.993 0.993 0.993 0.990 oct(wlsv) 0.998 1.014 1.015 1.015 1.016 1.006 1.006 1.007 1.007	0.972 0.974 (1.012	1.012	0.982	0.982	0.974	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.985 0.987 (0.985	1.028	1.024	0.996	0.995	0.987	$\operatorname{oct}_h(hbshr)$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				= 1	k			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.999 0.999 1	0.998	1.019	1.017	1.019	1.017	1.000	base
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.976 0.977 (0.976	0.994	0.994	0.994	0.994	0.978	ct(bu)
oct(wlsv) 0.998 1.014 1.015 1.015 1.016 1.006 1.006 1.007 1.00	0.976 0.976 0	0.976		0.994	0.993	0.993	0.977	
	0.993 0.993 (0.993	1.003	1.003	1.002	1.002	0.986	$\operatorname{ct}(wlsv_{te}, bu_{cs})$
	1.006 1.007 1	1.006	1.016	1.015	1.015	1.014	0.998	oct(wlsv)
	0.992 0.993 (0.992	1.003	1.003	1.002	1.002	0.986	
oct(shr) 1.037 1.082 1.067 1.064 1.056 0.979 0.978 0.979 0.99	0.978 0.979 0	0.979	1.056	1.064	1.067	1.082	1.037	oct(shr)
oct(bshr) 1.041 1.071 1.074 1.060 1.062 0.998 0.998 0.998 0.998	0.998 0.998 (0.998	1.062	1.060	1.074	1.071	1.041	oct(bshr)
oct(hshr) 1.080 1.090 1.091 1.119 1.105 1.050 1.050 1.053 1.00	1.050 1.053 1	1.050	1.105	1.119	1.091	1.090	1.080	oct(hshr)
oct(hbshr) 1.065 1.080 1.081 1.088 1.090 1.063 1.064 1.066 1.06	1.064 1.066 1	1.063	1.090	1.088	1.081	1.080	1.065	oct(hbshr)
$\operatorname{oct}_h(shr)$ 0.980 0.996 0.995 0.996 0.996 0.979 0.978 0.979 0.99	0.978 0.979 0	0.979	0.996	0.996	0.995	0.996	0.980	$\operatorname{oct}_h(shr)$
$\operatorname{oct}_h(bshr)$ 0.999 1.016 1.018 1.016 1.018 0.998 0.998 0.998 0.998	0.998 0.998 0	0.998	1.018	1.016	1.018	1.016	0.999	$\operatorname{oct}_h(bshr)$
$\operatorname{oct}_h(hshr)$ 1.052 1.067 1.066 1.074 1.075 1.050 1.050 1.053 1.05	1.050 1.053 1	1.050	1.075	1.074	1.066	1.067	1.052	$\operatorname{oct}_h(hshr)$
$\operatorname{oct}_h(hbshr)$ 1.065 1.080 1.081 1.088 1.090 1.063 1.064 1.066 1.06	1.064 1.066 1	1.063	1.090	1.088	1.081	1.080	1.065	$\operatorname{oct}_h(hbshr)$
k = 2				= 2	k			
base 1.000 0.998 0.999 1.071 1.075 0.998 0.999 1.005 1.00	0.999 1.005 1	0.998	1.075	1.071	0.999	0.998	1.000	base
ct(bu) 0.831 0.869 0.869 0.869 0.869 0.830 0.829 0.829 0.83								ct(bu)
$ct(shr_{cs}, bu_{te})$ 0.830 0.869 0.868 0.868 0.868 0.830 0.829 0.829 0.829		0.830	0.868					
ct(wlsv _{te} , bu _{cs}) 0.840 0.863 0.862 0.879 0.878 0.846 0.844 0.845 0.84	0.844 0.845 (0.846	0.878	0.879	0.862	0.863	0.840	
$\cot(wlsv)$ 0.851 0.875 0.877 0.891 0.893 0.859 0.859 0.859 0.869							0.851	
oct(bdshr) 0.839 0.863 0.863 0.879 0.878 0.845 0.844 0.845 0.84								
oct(shr) 0.854 0.922 0.909 0.908 0.897 0.833 0.831 0.832 0.83	0.831 0.832 (0.833	0.897	0.908	0.909	0.922	0.854	oct(shr)
oct(bshr) 0.869 0.925 0.931 0.911 0.915 0.851 0.851 0.851 0.851	0.851 0.851 0	0.851	0.915	0.911	0.931	0.925	0.869	oct(bshr)
oct(hshr) 0.901 0.908 0.904 0.966 0.952 0.900 0.899 0.901 0.90	0.899 0.901 (0.900	0.952	0.966	0.904	0.908	0.901	oct(hshr)
oct(hbshr) 0.915 0.917 0.919 0.964 0.969 0.913 0.913 0.914 0.99		0.913						
$\operatorname{oct}_h(shr)$ 0.834 0.868 0.865 0.872 0.872 0.833 0.831 0.832 0.83		0.833	0.872	0.872	0.865	0.868	0.834	$\operatorname{oct}_h(shr)$
$\operatorname{oct}_h(bshr)$ 0.852 0.886 0.890 0.890 0.894 0.851 0.851 0.851 0.851	0.851 0.851 0	0.851	0.894	0.890	0.890	0.886	0.852	
$oct_h(hshr)$ 0.902 0.904 0.904 0.953 0.952 0.900 0.899 0.901 0.90		0.900	0.952	0.953	0.904	0.904	0.902	$\operatorname{oct}_h(hshr)$
$\operatorname{oct}_h(hbshr)$ 0.915 0.917 0.919 0.964 0.969 0.913 0.913 0.914 0.96	0.913 0.914 (0.913	0.969	0.964	0.919	0.917	0.915	$\operatorname{oct}_h(hbshr)$

Table 2: CRPS skill score defined in (17) and (18). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

$\begin{array}{c} \operatorname{ct}(shr_{cs},\dot{b}u_{te}) & 0.896 & 0.924 & 0.923 & 0.923 & 0.922 & 0.895 & 0.895 & 0.896 & 0.896 \\ \operatorname{ct}(wlsv_{te},b_{tes}) & 0.906 & 0.924 & 0.923 & 0.933 & 0.932 & 0.912 & 0.911 & 0.910 \\ \operatorname{oct}(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 \\ \operatorname{oct}(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.932 & 0.910 & 0.911 & 0.911 \\ \operatorname{oct}(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(shr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(lshshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(lshshr) & 0.977 & 0.986 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(lshshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_{l_1}(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 \\ \operatorname{oct}_{l_1}(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_{l_1}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_{l_1}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_{l_1}(hshr) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.985 & 0.983 & 0.968 & 0.967 & 0.968 \\ \operatorname{ct}(wlsv_{le},bu_{cs}) & 0.997 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 \\ \operatorname{oct}(wlsv_{le}) & 0.998 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.995 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 \\ \operatorname{oct}(lshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.986 & 0.986 \\ \operatorname{oct}_{l_1}(lshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.037 & 1.037 & 1.039 & 0.986 \\ \operatorname{oct}_{l_1}(lshr) & 0.987 & 1.002 & 1.005 & 1.005 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_{l_1}(lshr) & 1.040 & 1.053 & 1.053 & 1.053 & 1.036 & 1.036 & 1.036 & 1.036 & 1.036 \\ \operatorname{oct}_{l_1}(lshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{le},bu_{cs}) & 0.839 $				Base	forecas	sts' samp	ole appr	oach		
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $				Gaussi	an appr	oach: sa	mple co	variance	matrix	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		ctjb	Ir	n-sample	e residua	ıls	M	Iulti-step	residua	als
$\begin{array}{c} \text{base} & 1,000 & 1,005 & 1,009 & 1,023 & 0,924 & 0,923 & 0,895 & 0.896 & 0.897 \\ \text{ct}(shr_{cs}, bu_{te}) & 0.896 & 0,924 & 0,923 & 0,923 & 0,922 & 0.895 & 0.896 & 0.896 \\ \text{ct}(wlsvle_c, bu_{cs}) & 0,906 & 0,924 & 0,923 & 0,933 & 0,932 & 0,912 & 0,911 & 0,910 & 0,912 \\ \text{oct}(wlsvl) & 0,916 & 0,935 & 0,937 & 0,944 & 0,945 & 0,923 & 0,923 & 0,923 \\ \text{oct}(shr) & 0,906 & 0,923 & 0,932 & 0,932 & 0,912 & 0,910 & 0,911 & 0,912 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,985 & 1,027 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,900 & 0,923 & 0,922 & 0,926 & 0,925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \text{oct}_h(shr) & 0,901 & 0,944 & 0,943 & 0,942 & 0,945 & 0,914 & 0,916 & 0,915 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,969 & 0,985 & 0,983 & 0,984 & 0,967 & 0,977 & 0,980 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,968 & 0,984 & 0,983 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,969 & 0,985 & 0,985 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{oct}(bshr) & 0,977 & 0,991 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 0,0977 & 0,989 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 1,028 & 1,070 & 1,056 & 1,053 & 1,046 & 0,969 & 0,969 & 0,995 \\ \text{oct}(bshr) & 1,034 & 1,061 & 1,065 & 1,051 & 1,053 & 1,048 & 1,049 & 1,049 \\ \text{oct}_h(shr) & 1,066 & 1,075 & 1,0076 & 1,099 & 1,099 & 0,996 & 0,998 & 0,987 & 0,986 \\ \text{oct}_h(shr) & 1,064 & 1,063 & 1,071 & 1,073 & 1,047 & 1,049 & 1,051 \\ \text{oct}_h(shr) & 0,987 & 1,002 & 1,005 & 1,005 & 1,053 & 1,044 & 0,949 & 0,995 & 0,987 \\ \text{oct}_h(shr) & 0,831 & 0,867 & 0,860 & 0,8$	mp p rouers		G	В	Н	HB	G	В	Н	HB
$\begin{array}{c} \text{base} & 1,000 & 1,005 & 1,009 & 1,023 & 0,924 & 0,923 & 0,895 & 0.896 & 0.897 \\ \text{ct}(shr_{cs}, bu_{te}) & 0.896 & 0,924 & 0,923 & 0,923 & 0,922 & 0.895 & 0.896 & 0.896 \\ \text{ct}(wlsvle_c, bu_{cs}) & 0,906 & 0,924 & 0,923 & 0,933 & 0,932 & 0,912 & 0,911 & 0,910 & 0,912 \\ \text{oct}(wlsvl) & 0,916 & 0,935 & 0,937 & 0,944 & 0,945 & 0,923 & 0,923 & 0,923 \\ \text{oct}(shr) & 0,906 & 0,923 & 0,932 & 0,932 & 0,912 & 0,910 & 0,911 & 0,912 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,985 & 1,027 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,900 & 0,923 & 0,922 & 0,926 & 0,925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \text{oct}_h(shr) & 0,901 & 0,944 & 0,943 & 0,942 & 0,945 & 0,914 & 0,916 & 0,915 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,969 & 0,985 & 0,983 & 0,984 & 0,967 & 0,977 & 0,980 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,968 & 0,984 & 0,983 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,969 & 0,985 & 0,985 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{oct}(bshr) & 0,977 & 0,991 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 0,0977 & 0,989 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 1,028 & 1,070 & 1,056 & 1,053 & 1,046 & 0,969 & 0,969 & 0,995 \\ \text{oct}(bshr) & 1,034 & 1,061 & 1,065 & 1,051 & 1,053 & 1,048 & 1,049 & 1,049 \\ \text{oct}_h(shr) & 1,066 & 1,075 & 1,0076 & 1,099 & 1,099 & 0,996 & 0,998 & 0,987 & 0,986 \\ \text{oct}_h(shr) & 1,064 & 1,063 & 1,071 & 1,073 & 1,047 & 1,049 & 1,051 \\ \text{oct}_h(shr) & 0,987 & 1,002 & 1,005 & 1,005 & 1,053 & 1,044 & 0,949 & 0,995 & 0,987 \\ \text{oct}_h(shr) & 0,831 & 0,867 & 0,860 & 0,8$				$\forall k$	∈ {2,1}					
$\begin{array}{c} \operatorname{ct}(shr_{cs},bu_{te}) & 0.896 & 0.924 & 0.923 & 0.923 & 0.922 & 0.895 & 0.895 & 0.896 & 0.896 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.906 & 0.924 & 0.923 & 0.933 & 0.932 & 0.912 & 0.911 & 0.910 \\ \operatorname{oct}(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 \\ \operatorname{oct}(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.910 & 0.910 & 0.911 & 0.912 \\ \operatorname{oct}(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_h(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.916 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hshr) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.985 & 0.983 & 0.986 \\ \operatorname{ct}(wlsv_{te},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 \\ \operatorname{oct}(wlsv_{te}) & 0.997 & 0.999 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 \\ \operatorname{oct}(bshr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.987 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.093 & 1.036 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 1.061 & 1.063 & 1.071 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(bshr) & 1.061 & 1.065 & 1.065 & 1.065 & 1.065 & 0.866 & 0.866 & 0.866 & 0.866 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.829 & 0.867 & 0.866 & 0.866 & 0.866 & 0.8$	base	1.000	1.005			1.046	0.996	0.999	1.000	1.004
$ \begin{array}{c} ct(wlsv_{le},bu_{es}) \\ cet(wlsv) \\ cot(wlsv) \\ cot(wlsv) \\ opt(bdshr) \\ opt(bshr) \\ \mathsf$	ct(bu)	0.897	0.924	0.923	0.924	0.923	0.895	0.896	0.897	0.895
$\begin{array}{c} ct(wlsv_{le},bu_{es}) & 0.906 & 0.924 & 0.923 & 0.933 & 0.932 & 0.912 & 0.911 & 0.910 & 0.912 \\ oct(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 \\ oct(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.930 & 0.910 & 0.910 \\ oct(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ oct(shr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ oct(hshr) & 0.978 & 0.987 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ oct(hshr) & 0.978 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ oct_{h}(shr) & 0.970 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ oct_{h}(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ oct_{h}(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ oct_{h}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ oct_{h}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.005 & 0.975 & 0.976 & 0.977 & 0.980 \\ oct_{h}(hshr) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ ct(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.985 & 0.984 & 0.967 & 0.968 & 0.968 \\ ct(wlsv_{te},bu_{tes}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.968 \\ ct(wlsv_{te},bu_{tes}) & 0.999 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ oct(bshr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 \\ oct(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ oct_{h}(hshr) & 0.967 & 0.998 & 0.995 & 0.998 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ oct_{h}(shr) & 0.977 & 0.998 & 0.995 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ oct_{h}(shr) & 1.051 & 1.065 & 1.065 & 1.051 & 1.053 & 1.046 & 0.969 & 0.969 & 0.969 \\ oct_{h}(shr) & 1.054 & 1.061 & 1.065 & 1.051 & 1.053 & 1.084 & 1.049 & 1.049 \\ oct_{h}(shr) & 0.987 & 1.002 & 1.005 & 1.005 & 1.005 & 0.986 & 0.987 & 0.988 \\ oct_{h}(bshr) & 0.831 & 0.867 & 0.867 & 0.867 $	$\operatorname{ct}(shr_{cs},bu_{te})$	0.896	0.924	0.923	0.923	0.922	0.895	0.895	0.896	0.896
$\begin{array}{c} \operatorname{oct}(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 & 0.924 \\ \operatorname{oct}(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.932 & 0.910 & 0.910 & 0.911 \\ \operatorname{oct}(shr) & 0.938 & 0.993 & 0.990 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.978 & 0.987 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_h(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \operatorname{oct}_h(bshr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \hline \\ base & 1.000 & 1.014 & 1.020 & 1.015 & 1.019 & 0.997 & 1.000 & 0.997 & 1.000 \\ \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.983 & 0.986 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{le},bu_{les}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{le},bu_{les}) & 0.998 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.050 & 1.065 & 1.067 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 0.985 \\ \operatorname{oct}_h(shr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(shr) & 1.040 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.051 & 1.052 \\ \operatorname{oct}_h(shr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0$		0.906	0.924	0.923	0.933	0.932	0.912	0.911	0.910	0.912
$\begin{array}{c} \operatorname{oct}(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.978 & 0.987 & 0.985 & 1.012 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(hbshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_h(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \operatorname{oct}_h(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(ishr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(ishr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(ishr) & 0.969 & 0.984 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \end{array}$	oct(wlsv)	0.916	0.935	0.937	0.944	0.945	0.923	0.923	0.923	0.924
$\begin{array}{c} {\rm oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ {\rm oct}(hshr) & 0.978 & 0.985 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ {\rm oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ {\rm oct}_h(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ {\rm oct}_h(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ {\rm oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ {\rm oct}_h(hshr) & 0.978 & 0.984 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \hline \\ \\ \\ \\ \\$	oct(bdshr)	0.906	0.923	0.923	0.932	0.932	0.910	0.910	0.911	0.912
$\begin{array}{c} {\rm oct}(hshr) \\ {\rm oct}(hshr) \\ {\rm oct}(hshr) \\ {\rm oct}(hshr) \\ {\rm O.977} \\ {\rm oct}(hshr) \\ {\rm O.977} \\ {\rm O.986} \\ {\rm O.985} \\ {\rm I.012} \\ {\rm I.016} \\ {\rm I.016} \\ {\rm O.974} \\ {\rm O.974} \\ {\rm O.976} \\ {\rm O.977} \\ {\rm O.986} \\ {\rm O.985} \\ {\rm I.012} \\ {\rm I.016} \\ {\rm I.016} \\ {\rm O.974} \\ {\rm O.974} \\ {\rm O.976} \\ {\rm O.977} \\ {\rm O.978} \\ {\rm O.889} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.891} \\ {\rm O.916} \\ {\rm O.917} \\ {\rm O.917} \\ {\rm O.910} \\ {\rm $	oct(shr)	0.938	0.993	0.980	0.977	0.969	0.898	0.898	0.898	0.897
$\begin{array}{c} {\rm oct}(hbshr) \\ {\rm oct}_h(shr) \\ {\rm oct}_h(shr) \\ {\rm O.900} \\ {\rm O.923} \\ {\rm oct}_h(shr) \\ {\rm O.916} \\ {\rm O.924} \\ {\rm O.924} \\ {\rm O.926} \\ {\rm O.925} \\ {\rm O.926} \\ {\rm O.925} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.897} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.891} \\ {\rm O.916} \\ {\rm O.940} \\ {\rm O.940} \\ {\rm O.943} \\ {\rm O.942} \\ {\rm O.945} \\ {\rm O.945} \\ {\rm O.944} \\ {\rm O.964} \\ {\rm O.964} \\ {\rm O.964} \\ {\rm O.966} \\ {\rm O.967} \\ {\rm O.977} \\ {\rm O.970} \\ {\rm O.980} \\ {\rm O.984} \\ {\rm O.983} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.980} \\ {\rm O.991} \\ {\rm O.992} \\ {\rm O.992} \\ {\rm O.994} \\ {\rm O.995} \\ {\rm O.995} \\ {\rm O.996} \\ {\rm O.989} \\ {\rm O.991} \\ {\rm O.991} \\ {\rm O.991} \\ {\rm O.992} \\ {\rm O.992} \\ {\rm O.992} \\ {\rm O.984} \\ {\rm O.983} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.983} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.995} \\ {\rm O.995} \\ {\rm O.995} \\ {\rm O.995} \\ {\rm O.996} \\ {\rm $	oct(bshr)	0.947	0.990	0.995	0.979	0.981	0.915	0.915	0.915	0.915
$\begin{array}{c} \operatorname{oct}_h(shr') & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \operatorname{oct}_h(bshr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hbshr) & 0.978 & 0.984 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \hline \\ base & 1.000 & 1.014 & 1.020 & 1.015 & 1.019 & 0.997 & 1.000 & 0.997 & 1.000 \\ \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.966 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.054 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}_h(shr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.988 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.988 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.063 & 1.073 & 0.996 & 0.998 & 0.989 \\ \operatorname{oct}_h(bshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.877 & 0.876 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(hshr) & 0.839 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 \\ \operatorname{oct}(bshr) & 0.885 & 0.921 & 0.990 & 0.997 & 0.991 & 0.990 & 0.990 & 0.909 \\ \operatorname{oct}(hbshr) & 0.$	oct(hshr)	0.978	0.987	0.985	1.027	1.016	0.963	0.964	0.966	0.967
$\begin{array}{c} \operatorname{oct}_h(bshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{opps} \\ \operatorname{opps} \\ \operatorname{oct}_h(hshr) \\ \operatorname{opps} \\ $	oct(hbshr)	0.977	0.986	0.985	1.012	1.016	0.974	0.976	0.977	0.978
$\begin{array}{c} \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hbshr) \\ \operatorname{oct}_h(hbshr) \\ \end{array} \begin{array}{c} 0.967 \\ \operatorname{oct}_h(hbshr) \\ \end{array} \begin{array}{c} 0.978 \\ \operatorname{o.984} \\ \end{array} \begin{array}{c} 0.984 \\ \operatorname{o.986} \\ \end{array} \begin{array}{c} 1.002 \\ \operatorname{c.015} \\ \end{array} \begin{array}{c} 0.975 \\ \operatorname{o.975} \\ \end{array} \begin{array}{c} 0.976 \\ \operatorname{o.977} \\ \end{array} \begin{array}{c} 0.980 \\ \end{array} \end{array}$	$\operatorname{oct}_h(shr)$	0.900	0.923	0.922	0.926	0.925	0.898	0.898	0.897	0.898
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(bshr)$	0.916	0.940	0.943	0.942	0.945	0.914	0.916	0.915	0.916
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.967	0.974	0.974	1.002	1.002	0.964	0.964	0.966	0.967
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(hbshr)$	0.978	0.984	0.986	1.012	1.015	0.975	0.976	0.977	0.980
$\begin{array}{c} \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(bshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.830 & 0.828 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.829 & 0.867 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.861 & 0.861 & 0.876 & 0.845 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(bshr) & 0.889 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 & 0.844 \\ \operatorname{oct}(bshr) & 0.886 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hbshr) & 0.897 & 0.905 & 0.901 & 0.959 & 0.947 & 0.895 & 0.896 & 0.898 & 0.899 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \end{array}$				k	z = 1					
$\begin{array}{c} \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(bshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.830 & 0.828 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.829 & 0.867 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.861 & 0.861 & 0.876 & 0.845 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(bshr) & 0.889 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 & 0.844 \\ \operatorname{oct}(bshr) & 0.886 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hbshr) & 0.897 & 0.905 & 0.901 & 0.959 & 0.947 & 0.895 & 0.896 & 0.898 & 0.899 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \end{array}$	base	1.000	1.014	1.020	1.015	1.019	0.997	1.000	0.997	1.000
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ct(bu)								0.968	
$\begin{array}{c} \operatorname{ct}(wlsv_{le},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hbshr) & 1.051 & 1.064 & 1.063 & 1.071 & 1.073 & 1.047 & 1.049 & 1.051 & 1.052 \\ \hline & & & & & & & & & & & & & & & & \\ base & 1.000 & 0.997 & 0.999 & 1.063 & 1.073 & 0.996 & 0.998 & 1.003 & 1.008 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.867 & 0.866 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(wlsv) & 0.849 & 0.872 & 0.875 & 0.887 & 0.890 & 0.858 & 0.856 & 0.856 \\ \operatorname{oct}(bdshr) & 0.839 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 \\ \operatorname{oct}(bshr) & 0.868 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hshr) & 0.897 & 0.905 & 0.901 & 0.959 & 0.947 & 0.895 & 0.896 & 0.898 & 0.899 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \hline \end{tabular}$								0.967		
$\begin{array}{c} \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hbshr) & 1.051 & 1.064 & 1.063 & 1.071 & 1.073 & 1.047 & 1.049 & 1.051 & 1.052 \\ \hline & & & & & & & & & & & & & & & & & &$							0.984			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{c} \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hbshr) & 1.051 & 1.064 & 1.063 & 1.071 & 1.073 & 1.047 & 1.049 & 1.051 & 1.052 \\ \hline & & & & & & & & & & & & & & \\ base & 1.000 & 0.997 & 0.999 & 1.063 & 1.073 & 0.996 & 0.998 & 1.003 & 1.008 \\ \operatorname{ct}(bu) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.830 & 0.828 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.829 & 0.867 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.877 & 0.876 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(wlsv) & 0.849 & 0.872 & 0.875 & 0.887 & 0.890 & 0.858 & 0.856 & 0.856 & 0.857 \\ \operatorname{oct}(bdshr) & 0.839 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 & 0.844 \\ \operatorname{oct}(shr) & 0.856 & 0.921 & 0.909 & 0.907 & 0.898 & 0.832 & 0.831 & 0.832 & 0.831 \\ \operatorname{oct}(bshr) & 0.868 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \hline \end{array}$		1.028	1.070		1.053	1.046	0.969	0.969	0.970	0.969
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	` '	1.034	1.061	1.065	1.051	1.053	0.985	0.987	0.986	0.987
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		1.066	1.075	1.076	1.099	1.090	1.037	1.037	1.039	1.039
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	oct(hbshr)	1.050	1.065	1.065	1.070	1.073	1.048	1.049	1.049	1.052
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(shr)$	0.971	0.985	0.985	0.986	0.986	0.969	0.969	0.969	0.969
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(bshr)$	0.987	1.002	1.005	1.002	1.005	0.986	0.987	0.987	0.988
k = 2 $base$	$\operatorname{oct}_h(hshr)$	1.040	1.053	1.053	1.059	1.058	1.036	1.036	1.040	1.040
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(hbshr)$	1.051	1.064	1.063	1.071	1.073	1.047	1.049	1.051	1.052
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				k	z = 2					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	base	1.000	0.997			1.073	0.996	0.998	1.003	1.008
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ct(bu)									
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0.839	0.860	0.860	0.877	0.876	0.844	0.844	0.844	0.845
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	\ /									
oct(hbshr) 0.910 0.912 0.912 0.957 0.961 0.906 0.909 0.909 0.910	1 1									0.899
$\operatorname{oct}_h(shr)$ 0.835 0.865 0.862 0.870 0.868 0.833 0.833 0.831 0.832	$\operatorname{oct}_h(shr)$	0.835	0.865	0.862	0.870	0.868	0.833	0.833	0.831	
										0.850
						0.948	0.897	0.896	0.897	0.899
$\operatorname{oct}_h(hbshr)$ 0.910 0.910 0.914 0.957 0.961 0.907 0.908 0.909 0.912	$\operatorname{oct}_h(hbshr)$	0.910	0.910	0.914	0.957	0.961	0.907	0.908	0.909	0.912

Table 3: *ES skill score defined in equation (17) and (19). The smaller this value, the more accurate the forecast.*Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

	Base forecasts' sample approach											
			Gaussia	n approa	ach: shri	nkage co	ovarianc	e matrix				
Reconciliation approach	ctjb	Ir	-sample	residua	ls	M	Iulti-step	residua	als			
11		G	В	Н	HB	G	В	Н	HB			
			$\forall k \in$	€ {2,1}								
base	1.007	1.009	1.044	1.046	0.997	0.999	1.002	1.003	1.000			
ct(bu)	0.929	0.929	0.929	0.929	0.899	0.900	0.900	0.900	0.901			
$\operatorname{ct}(shr_{cs},bu_{te})$	0.929	0.928	0.929	0.928	0.899	0.899	0.900	0.900	0.901			
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.930	0.930	0.939	0.938	0.915	0.916	0.917	0.916	0.910			
oct(wlsv)	0.943	0.944	0.951	0.952	0.929	0.930	0.931	0.930	0.922			
oct(bdshr)	0.930	0.930	0.938	0.938	0.915	0.916	0.916	0.916	0.910			
oct(shr)	0.994	0.982	0.980	0.973	0.902	0.902	0.903	0.902	0.941			
oct(bshr)	0.995	0.998	0.983	0.986	0.921	0.922	0.922	0.922	0.951			
oct(hshr)	0.994	0.994	1.035	1.025	0.971	0.972	0.974	0.974	0.987			
oct(hbshr)	0.995	0.997	1.025	1.027	0.984	0.986	0.988	0.988	0.987			
$\operatorname{oct}_h(shr)$	0.929	0.928	0.932	0.932	0.902	0.902	0.903	0.902	0.904			
$\operatorname{oct}_h(\mathit{bshr})$	0.948	0.951	0.951	0.953	0.921	0.922	0.922	0.922	0.923			
$\operatorname{oct}_h(hshr)$	0.982	0.982	1.011	1.011	0.971	0.972	0.974	0.974	0.974			
$\operatorname{oct}_h(hbshr)$	0.995	0.997	1.025	1.027	0.984	0.986	0.988	0.988	0.987			
			k	=1								
base	1.017	1.019	1.017	1.019	0.998	0.999	0.999	0.999	1.000			
ct(bu)	0.994	0.994	0.994	0.994	0.976	0.976	0.977	0.976	0.978			
$\operatorname{ct}(shr_{cs},bu_{te})$	0.993	0.993	0.993	0.993	0.975	0.976	0.976	0.976	0.977			
$\operatorname{ct}(wlsv_{te}, bu_{cs})$	1.002	1.002	1.003	1.003	0.992	0.993	0.993	0.993	0.986			
oct(wlsv)	1.015	1.015	1.015	1.016	1.005	1.007	1.007	1.007	0.998			
oct(bdshr)	1.002	1.002	1.003	1.002	0.992	0.992	0.993	0.992	0.986			
oct(shr)	1.076	1.065	1.061	1.056	0.978	0.978	0.979	0.978	1.037			
oct(bshr)	1.070	1.072	1.060	1.062	0.997	0.998	0.998	0.998	1.041			
oct(hshr)	1.090	1.092	1.114	1.105	1.049	1.050	1.053	1.052	1.080			
oct(hbshr)	1.080	1.081	1.089	1.090	1.062	1.064	1.066	1.066	1.065			
$\operatorname{oct}_h(shr)$	0.996	0.995	0.996	0.996	0.978	0.978	0.979	0.978	0.980			
$\operatorname{oct}_h(bshr)$	1.016	1.018	1.016	1.018	0.997	0.998	0.998	0.998	0.999			
$\operatorname{oct}_h(hshr)$	1.066	1.067	1.075	1.075	1.049	1.050	1.053	1.052	1.052			
$\operatorname{oct}_h(hbshr)$	1.080	1.081	1.089	1.090	1.062	1.064	1.066	1.066	1.065			
,			k	= 2								
base	0.997	0.999	1.071	1.074	0.997	0.999	1.005	1.008	1.000			
ct(bu)	0.869	0.868	0.868	0.868	0.829	0.829	0.830	0.830	0.831			
$\operatorname{ct}(shr_{cs},bu_{te})$	0.868	0.867	0.868	0.867	0.829	0.829	0.830	0.829	0.830			
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.863	0.862	0.878	0.878	0.845	0.845	0.846	0.846	0.840			
oct(wlsv)	0.876	0.877	0.891	0.892	0.859	0.860	0.860	0.860	0.851			
oct(bdshr)	0.863	0.863	0.878	0.877	0.844	0.845	0.846	0.845	0.839			
oct(shr)	0.918	0.906	0.906	0.897	0.832	0.832	0.833	0.832	0.854			
oct(bshr)	0.924	0.928	0.911	0.915	0.850	0.851	0.852	0.851	0.869			
oct(hshr)	0.907	0.905	0.962	0.951	0.898	0.899	0.902	0.902	0.901			
oct(hbshr)	0.917	0.919	0.964	0.968	0.912	0.913	0.915	0.916	0.915			
$\operatorname{oct}_h(shr)$	0.867	0.864	0.872	0.871	0.832	0.832	0.833	0.832	0.834			
$\operatorname{oct}_h(bshr)$	0.886	0.890	0.890	0.893	0.850	0.851	0.852	0.851	0.852			
$\operatorname{oct}_h(bshr)$	0.904	0.905	0.952	0.952	0.898	0.899	0.902	0.902	0.902			
$\operatorname{oct}_h(hbshr)$	0.917	0.919	0.964	0.968	0.912	0.913	0.915	0.916	0.915			

Table 4: CRPS skill score defined in (17) and (18). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

	Base forecasts' sample approach											
			Gaussia	n approa	ach: shri	nkage co	ovarianc	e matrix	:			
Reconciliation approach	ctjb	Ir	-sample	residua	ls	M	Iulti-step	residua	als			
11		G	В	Н	HB	G	В	Н	HB			
			$\forall k$	∈ {2,1}								
base	1.005	1.008	1.039	1.045	0.996	0.999	1.000	1.003	1.000			
ct(bu)	0.923	0.923	0.923	0.923	0.895	0.896	0.897	0.897	0.897			
$\operatorname{ct}(shr_{cs},bu_{te})$	0.923	0.922	0.922	0.922	0.896	0.895	0.895	0.895	0.896			
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.924	0.924	0.932	0.932	0.910	0.911	0.911	0.911	0.906			
oct(wlsv)	0.935	0.937	0.944	0.945	0.922	0.924	0.923	0.923	0.916			
oct(bdshr)	0.924	0.924	0.932	0.931	0.909	0.911	0.911	0.910	0.906			
oct(shr)	0.989	0.978	0.975	0.968	0.897	0.898	0.898	0.898	0.938			
oct(bshr)	0.990	0.993	0.978	0.981	0.915	0.915	0.915	0.915	0.947			
oct(hshr)	0.986	0.985	1.024	1.015	0.963	0.964	0.966	0.967	0.978			
oct(hbshr)	0.985	0.986	1.012	1.015	0.973	0.976	0.977	0.978	0.977			
$\operatorname{oct}_h(shr)$	0.923	0.922	0.925	0.925	0.897	0.898	0.898	0.898	0.900			
$\operatorname{oct}_h(\mathit{bshr})$	0.941	0.943	0.942	0.945	0.913	0.915	0.915	0.915	0.916			
$\operatorname{oct}_h(hshr)$	0.974	0.975	1.001	1.001	0.964	0.964	0.966	0.966	0.967			
$\operatorname{oct}_h(hbshr)$	0.985	0.986	1.013	1.016	0.973	0.976	0.977	0.978	0.978			
			k	=1								
base	1.014	1.018	1.015	1.019	0.997	0.999	0.997	0.998	1.000			
ct(bu)	0.983	0.984	0.984	0.984	0.967	0.967	0.969	0.969	0.969			
$\operatorname{ct}(shr_{cs},bu_{te})$	0.983	0.982	0.982	0.983	0.966	0.967	0.966	0.966	0.968			
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.991	0.992	0.993	0.992	0.983	0.983	0.983	0.983	0.977			
oct(wlsv)	1.002	1.004	1.004	1.004	0.994	0.995	0.994	0.996	0.989			
oct(bdshr)	0.990	0.991	0.992	0.991	0.981	0.983	0.984	0.982	0.977			
oct(shr)	1.065	1.054	1.051	1.045	0.969	0.970	0.970	0.969	1.028			
oct(bshr)	1.061	1.063	1.050	1.052	0.986	0.986	0.987	0.985	1.034			
oct(hshr)	1.076	1.077	1.095	1.088	1.036	1.036	1.040	1.038	1.066			
oct(hbshr)	1.064	1.065	1.071	1.073	1.047	1.048	1.050	1.050	1.050			
$\operatorname{oct}_h(shr)$	0.984	0.985	0.986	0.986	0.969	0.969	0.969	0.968	0.971			
$\operatorname{oct}_h(bshr)$	1.003	1.005	1.003	1.005	0.985	0.987	0.987	0.986	0.987			
$\operatorname{oct}_h(hshr)$	1.054	1.054	1.059	1.059	1.036	1.037	1.038	1.039	1.040			
$oct_h(hbshr)$	1.063	1.065	1.071	1.074	1.046	1.048	1.049	1.051	1.051			
			k	z = 2								
base	0.996	0.998	1.064	1.073	0.995	0.999	1.003	1.007	1.000			
ct(bu)	0.867	0.866	0.867	0.866	0.829	0.829	0.830	0.830	0.831			
$\operatorname{ct}(shr_{cs},bu_{te})$	0.867	0.866	0.866	0.866	0.830	0.829	0.830	0.830	0.829			
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.861	0.861	0.875	0.875	0.843	0.845	0.845	0.845	0.839			
oct(wlsv)	0.873	0.874	0.888	0.889	0.856	0.857	0.857	0.856	0.849			
oct(bdshr)	0.862	0.861	0.876	0.874	0.843	0.844	0.844	0.844	0.839			
oct(shr)	0.918	0.907	0.905	0.898	0.831	0.832	0.832	0.832	0.856			
oct(bshr)	0.924	0.928	0.911	0.915	0.849	0.849	0.849	0.849	0.868			
oct(hshr)	0.904	0.901	0.957	0.946	0.895	0.896	0.898	0.900	0.897			
oct(hbshr)	0.912	0.913	0.956	0.961	0.905	0.909	0.909	0.911	0.910			
$\operatorname{oct}_h(shr)$	0.866	0.863	0.869	0.869	0.830	0.831	0.832	0.832	0.835			
$\operatorname{oct}_h(bshr)$	0.882	0.886	0.886	0.889	0.846	0.848	0.849	0.848	0.850			
$\operatorname{oct}_h(hshr)$	0.901	0.902	0.947	0.946	0.896	0.896	0.898	0.899	0.900			
$\operatorname{oct}_h(hbshr)$	0.912	0.914	0.958	0.961	0.905	0.908	0.910	0.909	0.910			

Table 5: *ES skill score defined in equation (17) and (19). The smaller this value, the more accurate the forecast.*Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

B Australian GDP dataset: one-step residuals and shrinkage covariance matrix

				Base for	recasts' s	ample a	pproach			
Reconciliation approach	ctjb	C	Gaussian	approacl	h*	ctjb	G	Gaussian	approacl	h*
11		G_h	H_h	G_{oh}	H_{oh}		G_h	H_h	G_{oh}	H_{oh}
		$\forall k$	$c \in \{4, 2, 1\}$	1}				k = 1		
base	1.000	0.979	0.995	0.968	0.976	1.000	0.988	0.988	0.971	0.971
$\operatorname{ct}(shr_{cs},bu_{te})$	0.937	0.956	0.956	0.976	0.976	0.992	1.008	1.008	1.029	1.029
$\operatorname{ct}(wls_{cs},bu_{te})$	0.930	0.917	0.917	0.898	0.898	0.986	0.974	0.975	0.956	0.956
oct(wlsv)	0.926	0.919	0.920	0.900	0.900	0.984	0.981	0.979	0.959	0.959
oct(bdshr)	0.940	0.965	0.945	0.992	0.957	0.997	1.019	1.003	1.044	1.018
oct(shr)	0.944	1.020	0.940	1.094	0.988	1.015	1.095	1.010	1.160	1.059
oct(hshr)	0.988	0.972	1.002	0.974	1.001	1.048	1.037	1.060	1.034	1.061
$\operatorname{oct}_o(wlsv)$	0.926	0.911	0.912	0.896	0.895	0.984	0.971	0.970	0.954	0.954
$oct_o(bdshr)$	0.978	0.964	0.946	0.952	0.930	1.034	1.016	1.003	1.005	0.989
$oct_o(shr)$	0.950	0.946	0.922	0.925	0.903	1.014	1.003	0.985	0.987	0.968
$oct_o(hshr)$	0.989	0.966	0.984	0.954	0.965	1.047	1.028	1.038	1.012	1.023
$oct_{oh}(shr)$	1.102	1.059	1.001	1.094	0.988	1.172	1.109	1.066	1.160	1.059
$oct_{oh}(hshr)$	1.006	0.983	1.009	0.974	1.001	1.068	1.046	1.059	1.034	1.061
			k = 2					k = 4		
base	1.000	0.984	0.993	0.968	0.976	1.000	0.966	1.004	0.964	0.981
$ct(shr_{cs}, bu_{te})$	0.949	0.966	0.966	0.987	0.987	0.874	0.896	0.896	0.914	0.914
$ct(wls_{cs}, bu_{te})$	0.942	0.928	0.928	0.909	0.909	0.866	0.853	0.853	0.834	0.834
oct(wlsv)	0.938	0.929	0.931	0.911	0.911	0.860	0.853	0.855	0.835	0.834
oct(bdshr)	0.953	0.976	0.956	1.003	0.969	0.874	0.904	0.880	0.931	0.889
oct(shr)	0.955	1.031	0.951	1.113	1.002	0.866	0.940	0.864	1.015	0.909
oct(hshr)	1.001	0.985	1.014	0.987	1.016	0.919	0.900	0.935	0.904	0.931
$oct_o(wlsv)$	0.938	0.921	0.923	0.907	0.906	0.860	0.847	0.848	0.832	0.830
$oct_o(bdshr)$	0.991	0.974	0.957	0.964	0.942	0.914	0.905	0.883	0.892	0.865
$oct_o(shr)$	0.965	0.958	0.934	0.938	0.916	0.877	0.882	0.852	0.854	0.831
$oct_o(hshr)$	1.002	0.979	0.996	0.967	0.978	0.922	0.898	0.923	0.888	0.898
$oct_{oh}(shr)$	1.120	1.069	1.013	1.113	1.002	1.020	1.002	0.928	1.015	0.909
$\operatorname{oct}_{oh}(hshr)$	1.021	0.996	1.021	0.987	1.016	0.934	0.912	0.951	0.904	0.931

^{*}The Gaussian method employs a sample covariance matrix:

Table 6: CRPS skill score defined in equation (17) and (18) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

 G_h and H_h use multi-step residuals and G_{oh} and H_{oh} use overlapping and multi-step residuals.

		Base forecasts' sample approach										
Reconciliation approach	ctjb	C	Gaussian	approacl	n*	ctjb	C	aussian	approacl	n*		
11		G_h	H_h	G_{oh}	H_{oh}		G_h	H_h	G_{oh}	H_{oh}		
		$\forall k$	$c \in \{4, 2, 1\}$	1}				k = 1				
base	1.000	0.970	0.988	0.960	0.970	1.000	0.977	0.977	0.965	0.965		
$\operatorname{ct}(shr_{cs},bu_{te})$	0.897	0.944	0.944	0.973	0.973	0.964	1.001	1.001	1.033	1.033		
$\operatorname{ct}(wls_{cs},bu_{te})$	0.886	0.880	0.880	0.860	0.860	0.954	0.944	0.945	0.928	0.928		
oct(wlsv)	0.890	0.890	0.894	0.872	0.872	0.958	0.957	0.957	0.938	0.939		
oct(bdshr)	0.905	0.956	0.934	0.992	0.954	0.972	1.014	0.994	1.048	1.018		
oct(shr)	0.895	0.979	0.895	1.053	0.944	0.973	1.060	0.969	1.121	1.015		
oct(hshr)	0.951	0.940	0.973	0.959	0.992	1.017	1.010	1.034	1.023	1.055		
$\operatorname{oct}_o(wlsv)$	0.891	0.879	0.881	0.864	0.864	0.958	0.945	0.945	0.931	0.931		
$oct_o(bdshr)$	0.940	0.928	0.910	0.918	0.895	1.004	0.986	0.971	0.980	0.961		
$oct_o(shr)$	0.900	0.899	0.876	0.878	0.858	0.973	0.963	0.944	0.949	0.930		
$oct_o(hshr)$	0.956	0.936	0.955	0.922	0.936	1.021	1.004	1.012	0.987	1.000		
$oct_{oh}(shr)$	1.059	1.015	0.956	1.053	0.945	1.130	1.063	1.019	1.121	1.016		
$oct_{oh}(hshr)$	0.986	0.968	0.999	0.959	0.992	1.053	1.034	1.049	1.024	1.055		
			k = 2					k = 4				
base	1.000	0.972	0.985	0.959	0.969	1.000	0.959	1.000	0.957	0.976		
$ct(shr_{cs}, bu_{te})$	0.915	0.961	0.960	0.991	0.991	0.818	0.874	0.874	0.899	0.900		
$\operatorname{ct}(wls_{cs},bu_{te})$	0.904	0.896	0.896	0.877	0.877	0.807	0.805	0.805	0.782	0.783		
oct(wlsv)	0.909	0.907	0.912	0.889	0.889	0.811	0.813	0.819	0.794	0.794		
oct(bdshr)	0.925	0.976	0.953	1.013	0.974	0.825	0.883	0.860	0.920	0.876		
oct(shr)	0.913	1.000	0.914	1.076	0.963	0.807	0.885	0.808	0.967	0.861		
oct(hshr)	0.973	0.960	0.993	0.978	1.014	0.871	0.856	0.897	0.881	0.913		
$oct_o(wlsv)$	0.908	0.895	0.898	0.881	0.882	0.812	0.802	0.806	0.786	0.786		
$oct_o(bdshr)$	0.960	0.947	0.929	0.938	0.915	0.860	0.856	0.836	0.841	0.816		
$oct_o(shr)$	0.921	0.919	0.896	0.898	0.878	0.814	0.821	0.796	0.794	0.775		
$oct_o(hshr)$	0.977	0.956	0.976	0.942	0.957	0.876	0.854	0.882	0.844	0.856		
$\operatorname{oct}_{oh}(shr)$	1.082	1.029	0.973	1.076	0.963	0.971	0.954	0.882	0.967	0.861		
$\operatorname{oct}_{oh}(hshr)$	1.007	0.988	1.017	0.979	1.014	0.904	0.888	0.934	0.881	0.913		

^{*}The Gaussian method employs a sample covariance matrix:

Table 7: ES skill score defined in equation (17) and (19) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

 G_h and H_h use multi-step residuals and G_{oh} and H_{oh} use overlapping and multi-step residuals.

	Base forecasts' sample approach										
Reconciliation approach	ctjb	C	Gaussian	approacl	h*	ctjb	C	Gaussian	approacl	n*	
**		G_h	H_h	G_{oh}	H_{oh}		G_h	H_h	G_{oh}	H_{oh}	
		$\forall k$	$c \in \{4, 2, 1\}$	1}				k = 1			
base	1.000	0.979	1.011	0.968	0.987	1.000	0.988	0.988	0.971	0.971	
$\operatorname{ct}(shr_{cs},bu_{te})$	0.937	0.960	0.961	0.962	0.960	0.992	1.001	1.001	1.004	1.000	
$\operatorname{ct}(wls_{cs},bu_{te})$	0.930	0.951	0.953	0.911	0.915	0.986	0.997	0.998	0.964	0.967	
oct(wlsv)	0.926	0.972	0.957	0.918	0.917	0.984	1.010	1.003	0.971	0.970	
oct(bdshr)	0.940	0.986	0.966	0.981	0.956	0.997	1.015	1.006	1.016	1.000	
oct(shr)	0.944	0.999	0.962	1.051	0.995	1.015	1.047	1.021	1.105	1.058	
oct(hshr)	0.988	1.000	1.021	0.979	1.002	1.048	1.045	1.066	1.034	1.053	
$\operatorname{oct}_o(wlsv)$	0.926	0.961	0.948	0.914	0.912	0.984	1.000	0.993	0.966	0.965	
$oct_o(bdshr)$	0.978	0.956	0.949	0.949	0.934	1.034	0.984	0.983	0.988	0.977	
$oct_o(shr)$	0.950	0.957	0.946	0.933	0.917	1.014	0.998	0.995	0.986	0.974	
$oct_o(hshr)$	0.989	0.997	1.013	0.967	0.982	1.047	1.039	1.054	1.019	1.032	
$oct_{oh}(shr)$	1.102	1.010	1.006	1.051	0.995	1.172	1.059	1.063	1.105	1.058	
$oct_{oh}(hshr)$	1.006	0.989	1.004	0.979	1.002	1.068	1.037	1.050	1.034	1.053	
			k = 2					k = 4			
base	1.000	0.984	1.009	0.968	0.987	1.000	0.966	1.037	0.964	1.002	
$ct(shr_{cs}, bu_{te})$	0.949	0.972	0.972	0.974	0.971	0.874	0.910	0.911	0.910	0.910	
$\operatorname{ct}(wls_{cs},bu_{te})$	0.942	0.962	0.964	0.923	0.927	0.866	0.897	0.900	0.851	0.855	
oct(wlsv)	0.938	0.988	0.968	0.931	0.929	0.860	0.921	0.903	0.856	0.856	
oct(bdshr)	0.953	1.004	0.979	0.996	0.970	0.874	0.942	0.914	0.932	0.900	
oct(shr)	0.955	1.016	0.973	1.070	1.010	0.866	0.937	0.895	0.981	0.922	
oct(hshr)	1.001	1.015	1.034	0.993	1.017	0.919	0.942	0.965	0.913	0.937	
$oct_o(wlsv)$	0.938	0.976	0.959	0.927	0.925	0.860	0.910	0.894	0.853	0.852	
$oct_o(bdshr)$	0.991	0.970	0.963	0.963	0.948	0.914	0.917	0.905	0.899	0.880	
$\operatorname{oct}_o(shr)$	0.965	0.973	0.959	0.948	0.931	0.877	0.903	0.886	0.868	0.850	
$oct_o(hshr)$	1.002	1.013	1.026	0.980	0.996	0.922	0.943	0.962	0.905	0.921	
$oct_{oh}(shr)$	1.120	1.026	1.019	1.070	1.010	1.020	0.947	0.939	0.981	0.922	
$oct_{oh}(hshr)$	1.021	1.005	1.017	0.993	1.017	0.934	0.929	0.946	0.913	0.937	

^{*}The Gaussian method employs a shrinkage covariance matrix:

Table 8: CRPS skill score defined in equation (17) and (18) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

 G_h and H_h use multi-step residuals and G_{oh} and H_{oh} use overlapping and multi-step residuals.

	Base forecasts' sample approach										
Reconciliation approach	ctjb	C	aussian	approacl	n*	ctjb	G	aussian	approacl	n*	
		G_h	H_h	G_{oh}	H_{oh}		G_h	H_h	G_{oh}	H_{oh}	
		$\forall k$	$\in \{4,2,$	1}				k = 1			
base	1.000	0.967	1.002	0.957	0.980	1.000	0.973	0.973	0.961	0.962	
$\operatorname{ct}(shr_{cs},bu_{te})$	0.897	0.968	0.969	0.963	0.962	0.964	1.012	1.012	1.009	1.004	
$\operatorname{ct}(wls_{cs},bu_{te})$	0.886	0.939	0.944	0.882	0.888	0.954	0.994	0.998	0.947	0.952	
oct(wlsv)	0.890	0.966	0.959	0.897	0.901	0.958	1.017	1.012	0.960	0.965	
oct(bdshr)	0.905	0.997	0.981	0.986	0.960	0.972	1.031	1.021	1.024	1.005	
oct(shr)	0.895	0.979	0.945	1.021	0.962	0.973	1.041	1.011	1.083	1.028	
oct(hshr)	0.951	0.997	1.023	0.973	1.005	1.017	1.051	1.073	1.034	1.063	
$oct_o(wlsv)$	0.891	0.950	0.945	0.889	0.892	0.958	1.002	0.997	0.953	0.956	
$oct_o(bdshr)$	0.940	0.935	0.933	0.922	0.909	1.004	0.965	0.964	0.969	0.959	
$oct_o(shr)$	0.900	0.935	0.928	0.895	0.884	0.973	0.984	0.982	0.960	0.950	
$oct_o(hshr)$	0.956	0.997	1.015	0.945	0.965	1.021	1.049	1.062	1.007	1.024	
$oct_{oh}(shr)$	1.059	0.981	0.983	1.021	0.962	1.130	1.034	1.041	1.083	1.029	
$oct_{oh}(hshr)$	0.986	0.996	1.014	0.973	1.005	1.053	1.050	1.064	1.034	1.063	
			k = 2			'		k = 4			
base	1.000	0.970	0.999	0.955	0.980	1.000	0.958	1.033	0.953	1.000	
$ct(shr_{cs}, bu_{te})$	0.915	0.987	0.988	0.983	0.982	0.818	0.909	0.910	0.902	0.902	
$\operatorname{ct}(wls_{cs},bu_{te})$	0.904	0.958	0.962	0.900	0.906	0.807	0.871	0.876	0.805	0.812	
oct(wlsv)	0.909	0.988	0.979	0.916	0.920	0.811	0.896	0.891	0.820	0.825	
oct(bdshr)	0.925	1.024	1.005	1.010	0.984	0.825	0.938	0.919	0.926	0.895	
oct(shr)	0.913	1.006	0.967	1.045	0.982	0.807	0.898	0.864	0.940	0.881	
oct(hshr)	0.973	1.020	1.046	0.994	1.028	0.871	0.924	0.954	0.897	0.929	
$oct_o(wlsv)$	0.908	0.972	0.964	0.908	0.911	0.812	0.882	0.876	0.812	0.816	
$oct_o(bdshr)$	0.960	0.959	0.957	0.945	0.932	0.860	0.884	0.879	0.857	0.841	
$oct_o(shr)$	0.921	0.958	0.950	0.917	0.905	0.814	0.867	0.857	0.815	0.803	
$oct_o(hshr)$	0.977	1.021	1.038	0.966	0.987	0.876	0.926	0.949	0.868	0.889	
$\operatorname{oct}_{oh}(shr)$	1.082	1.002	1.003	1.045	0.982	0.971	0.910	0.911	0.941	0.882	
$oct_{oh}(hshr)$	1.007	1.017	1.036	0.994	1.028	0.904	0.924	0.947	0.896	0.929	

^{*}The Gaussian method employs a shrinkage covariance matrix:

Table 9: ES skill score defined in equation (17) and (19) and (??) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

 G_h and H_h use multi-step residuals and G_{oh} and H_{oh} use overlapping and multi-step residuals.

C Australian Tourism Demand dataset: shrinkage covariance matrix

				Base for	ecasts' s	sample a	pproach	ı		
Reconciliation approach	ctjb	G	aussian	approac	h [*]	ctjb	G	aussian	approac	h*
11		G	В	Н	HB		G	В	Н	HB
		$\forall k \in \{$	12, 6, 4,	3,2,1}				k = 1		
base	1.000	0.971	0.972	0.971	0.972	1.000	0.972	0.971	0.972	0.971
ct(bu)	1.321	1.017	1.018	1.017	1.017	1.077	0.983	0.983	0.983	0.983
$\operatorname{ct}(shr_{cs}, bu_{te})$	1.057	1.013	0.971	1.013	0.971	0.976	0.987	0.961	0.988	0.961
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.062	1.069	1.070	0.974	0.974	0.976	0.986	0.986	0.965	0.965
oct(ols)	0.989	1.163	1.052	1.139	0.987	0.982	1.038	0.992	1.047	0.987
oct(struc)	0.982	1.099	1.039	1.037	0.960	0.970	1.007	0.971	0.999	0.962
oct(wlsv)	0.987	1.080	1.041	0.992	0.958	0.952	1.004	0.969	0.978	0.956
oct(bdshr)	0.975	1.072	1.032	0.985	0.950	0.949	0.999	0.965	0.975	0.952
$\operatorname{oct}_h(hbshr)$	0.989	1.189	1.076	1.171	1.021	0.982	1.045	1.000	1.063	1.009
$\operatorname{oct}_h(bshr)$	0.994	1.202	1.073	1.168	1.021	0.988	1.046	1.012	1.063	1.012
$\operatorname{oct}_h(hshr)$	0.969	1.066	1.052	1.008	0.994	0.953	0.994	0.972	0.991	0.979
$\operatorname{oct}_h(shr)$	1.007	1.090	1.046	1.000	0.970	1.000	1.035	0.992	0.998	0.973
			k = 2					k = 3		
base	1.000	0.969	0.969	0.968	0.968	1.000	0.971	0.970	0.969	0.970
ct(bu)	1.189	1.000	1.000	1.000	1.000	1.273	1.013	1.013	1.013	1.013
$\operatorname{ct}(shr_{cs},bu_{te})$	1.015	1.004	0.968	1.004	0.968	1.041	1.013	0.973	1.014	0.973
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.016	1.043	1.044	0.969	0.969	1.046	1.067	1.068	0.974	0.974
oct(ols)	0.992	1.118	1.037	1.092	0.989	0.994	1.153	1.053	1.124	0.990
oct(struc)	0.982	1.075	1.022	1.020	0.963	0.986	1.099	1.041	1.033	0.964
oct(wlsv)	0.972	1.064	1.021	0.987	0.958	0.983	1.083	1.041	0.993	0.960
oct(bdshr)	0.964	1.057	1.015	0.983	0.953	0.972	1.075	1.033	0.988	0.955
$\operatorname{oct}_h(hbshr)$	0.992	1.136	1.055	1.116	1.014	0.994	1.178	1.075	1.153	1.020
$\operatorname{oct}_h(bshr)$	0.997	1.145	1.059	1.114	1.016	0.999	1.190	1.075	1.151	1.021
$\operatorname{oct}_h(hshr)$	0.965	1.050	1.029	1.001	0.986	0.971	1.067	1.051	1.009	0.994
$\operatorname{oct}_h(shr)$	1.005	1.083	1.035	1.001	0.973	1.009	1.097	1.050	1.004	0.974
			k = 4					k = 6		
base	1.000	0.973	0.973	0.971	0.973	1.000	0.976	0.977	0.975	0.977
$\operatorname{ct}(bu)$	1.340	1.021	1.021	1.021	1.021	1.450	1.032	1.033	1.032	1.033
$\operatorname{ct}(shr_{cs},bu_{te})$	1.061	1.018	0.974	1.018	0.974	1.094	1.023	0.974	1.024	0.974
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.068	1.087	1.089	0.976	0.976	1.103	1.108	1.110	0.978	0.978
oct(ols)	0.993	1.186	1.068	1.148	0.989	0.989	1.223	1.080	1.184	0.987
oct(struc)	0.986	1.120	1.057	1.042	0.962	0.986	1.141	1.071	1.054	0.959
oct(wlsv)	0.990	1.100	1.059	0.996	0.959	1.001	1.115	1.076	0.998	0.958
oct(bdshr)	0.977	1.091	1.049	0.989	0.952	0.985	1.103	1.064	0.989	0.949
$\operatorname{oct}_h(hbshr)$	0.993	1.215 1.230	1.095	1.182 1.178	1.022	0.989	1.258 1.278	1.112 1.101	1.225 1.219	1.026
$\operatorname{oct}_h(bshr)$	0.997	1.230	1.089 1.071	1.178	1.023 0.996	0.994	1.278	1.101	1.219	1.025 1.002
$\operatorname{oct}_h(hshr)$		1.108	1.062	1.012		1.010	1.113	1.070	1.017	0.968
$\operatorname{oct}_h(shr)$	1.009	1.106		1.003	0.972	1.010	1.113	1.070	1.000	0.900
1	1 000	0.000	k = 12	0.000	0.071	ı				
base	1.000	0.968	0.969	0.969	0.971					
ct(bu)	1.675	1.056	1.057	1.057	1.057					
$\operatorname{ct}(shr_{cs}, bu_{te})$	1.163	1.032	0.974	1.033	0.974					
$\operatorname{ct}(wlsv_{te}, bu_{cs})$ $\operatorname{oct}(ols)$	1.174 0.982	1.128 1.277	1.130 1.085	0.982 1.252	0.982 0.982					
oct(struc)	0.982	1.158	1.083	1.252	0.962					
oct(struc) oct(wlsv)	1.025	1.122	1.074	1.073	0.950					
oct(bdshr)	1.023	1.110	1.083	0.989	0.934 0.941					
oct(bushr) $oct_h(hbshr)$	0.982	1.322	1.125	1.305	1.033					
$\operatorname{oct}_h(hbshr)$	0.982	1.347	1.123	1.297	1.033					
$\operatorname{oct}_h(bshr)$	0.978	1.106	1.107	1.021	1.010					
$\operatorname{oct}_h(shr)$	1.010	1.107	1.067	0.991	0.959					
	1.010	2.207	1.507	0.771	0.707	I				

 $^{^*}$ The Gaussian method employs a shrikage covariance matrix and includes four techniques (G, B, H, HB) with multi-step residuals..

Table 10: CRPS skill score defined in equation (17) and (18) for VN525 dataset. The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

				Base for	ecasts' s	ample a	pproach	ı		
Reconciliation approach	ctjb	G	aussian	approac	h [*]	ctjb	G	aussian	approac	h*
		G	В	Н	HB		G	В	Н	HB
		$\forall k \in \cdot$	12, 6, 4,	3,2,1}				k = 1		
base	1.000	0.958	0.984	0.972	0.992	1.000	0.954	0.958	0.954	0.958
ct(bu)	2.427	1.040	1.042	1.040	1.041	1.759	1.001	1.002	1.002	1.002
$ct(shr_{cs}, bu_{te})$	1.243	0.988	0.913	0.990	0.913	1.098	1.011	0.938	1.013	0.938
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.499	1.117	1.120	1.025	1.025	1.241	1.019	1.020	0.990	0.990
oct(ols)	0.955	1.000	0.984	0.985	0.922	0.975	0.983	0.961	0.987	0.945
oct(struc)	1.085	1.094	1.047	1.018	0.952	1.027	1.054	0.981	1.022	0.953
oct(wlsv)	1.132	1.137	1.065	1.059	0.969	1.050	1.078	0.989	1.043	0.960
oct(bdshr)	1.047	1.085	1.013	1.011	0.927	1.009	1.050	0.966	1.019	0.942
$\operatorname{oct}_h(hbshr)$	0.956	1.018	0.981	1.016	0.919	0.975	0.991	0.961	1.002	0.947
$\operatorname{oct}_h(bshr)$	0.931	1.002	1.001	0.982	0.889	0.965	0.980	0.975	0.985	0.933
$\operatorname{oct}_h(hshr)$	1.081	1.109	1.039	1.076	0.973	1.028	1.061	0.978	1.052	0.963
$\operatorname{oct}_h(shr)$	1.068	1.088	1.008	0.995	0.896	1.023	1.061	0.966	1.011	0.924
1	1 000	0.060	k=2	0.050	0.070	1 000	0.060	k = 3	0.066	0.006
base	1.000	0.960	0.971	0.958	0.972	1.000	0.963	0.981	0.966	0.986
ct(bu)	2.176	1.035	1.036	1.035	1.035	2.428	1.042	1.044	1.042	1.043
$\operatorname{ct}(shr_{cs},bu_{te}) \ \operatorname{ct}(wlsv_{te},bu_{cs})$	1.192 1.400	1.020 1.104	0.942 1.106	1.021 1.018	0.942 1.019	1.245 1.500	1.009 1.127	0.931 1.130	1.011 1.029	0.931 1.029
oct(ols)	0.985	1.104	1.108	1.018	0.950	0.976	1.020	1.130	0.994	0.938
oct(struc)	1.075	1.115	1.051	1.002	0.967	1.096	1.117	1.064	1.033	0.965
oct(wlsv)	1.110	1.119	1.065	1.070	0.979	1.142	1.160	1.082	1.073	0.981
oct(bdshr)	1.045	1.105	1.024	1.033	0.949	1.060	1.109	1.032	1.029	0.943
$oct_h(hbshr)$	0.984	1.041	1.007	1.024	0.951	0.975	1.036	1.002	1.023	0.937
$\operatorname{oct}_h(bshr)$	0.967	1.029	1.025	0.998	0.928	0.954	1.024	1.025	0.993	0.911
$\operatorname{oct}_h^n(hshr)$	1.073	1.122	1.042	1.083	0.983	1.093	1.129	1.054	1.090	0.984
$\operatorname{oct}_h(shr)$	1.064	1.110	1.019	1.018	0.922	1.082	1.116	1.030	1.015	0.915
,			k = 4			'		k = 6		
base	1.000	0.962	0.987	0.973	0.996	1.000	0.963	0.998	0.984	1.011
ct(bu)	2.585	1.052	1.054	1.053	1.053	2.849	1.083	1.085	1.083	1.084
$\operatorname{ct}(shr_{cs}, bu_{te})$	1.277	1.000	0.923	1.002	0.923	1.339	0.999	0.921	1.000	0.920
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.559	1.150	1.153	1.037	1.037	1.662	1.189	1.193	1.066	1.066
oct(ols)	0.966	1.022	1.008	0.994	0.931	0.962	1.023	1.014	1.003	0.930
oct(struc)	1.106	1.120	1.076	1.031	0.963	1.132	1.132	1.100	1.039	0.972
oct(wlsv)	1.157	1.167	1.097	1.075	0.982	1.192	1.187	1.124	1.090	0.995
oct(bdshr)	1.065	1.112	1.041	1.025	0.939	1.084	1.121	1.058	1.029	0.940
$\operatorname{oct}_h(hbshr)$	0.967	1.041	1.005	1.027	0.929	0.964	1.046	1.008	1.042	0.924
$\operatorname{oct}_h(bshr)$	0.943	1.028	1.028	0.994	0.900	0.932	1.029	1.032	1.000	0.887
$\operatorname{oct}_h(hshr)$	1.101	1.137 1.118	1.068	1.093	0.986	1.126	1.153	1.089	1.110	0.999
$\operatorname{oct}_h(shr)$	1.089	1.110	1.039	1.012	0.910	1.107	1.118	1.045	1.006	0.902
1	1 000	0.040	k = 12	1.000	4.000					
base	1.000	0.948	1.010	1.002	1.033					
ct(bu)	2.990	1.028	1.031	1.029	1.029					
$\operatorname{ct}(shr_{cs}, bu_{te})$	1.326	0.897	0.830	0.899	0.830					
$\operatorname{ct}(wlsv_{te}, bu_{cs})$ $\operatorname{oct}(ols)$	1.679 0.872	1.119 0.927	1.123 0.914	1.009 0.930	1.009 0.840					
oct(struc)	1.077	1.028	1.012	0.950	0.894					
oct(wlsv)	1.149	1.028	1.012	1.006	0.894 0.922					
oct(wist)	1.021	1.015	0.964	0.935	0.855					
$oct_h(hbshr)$	0.872	0.955	0.906	0.978	0.833					
$\operatorname{oct}_h(hoshr)$	0.833	0.927	0.927	0.927	0.784					
$\operatorname{oct}_h(hshr)$	1.066	1.056	1.005	1.026	0.926					
$\operatorname{oct}_h(shr)$	1.043	1.011	0.952	0.909	0.809					
n\ /	ı					1				

^{*}The Gaussian method employs a shrikage covariance matrix and includes four techniques (G, B, H, HB) with multi-step residuals.

Table 11: ES skill score defined in equation (17) and (19) for VN525 dataset. The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.