Cross-temporal Probabilistic Forecast Reconciliation: online appendix

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A Monte Carlo Simulation: one-step residuals and shrinkage covariance matrix

In Section 5.1, we discussed the use of one-step residuals in estimating the covariance matrix. In particular we point out that one-step residuals lead to a biased estimate of the covariance matrix where some correlation are zeros by definition. This can be seen clearly in Figure 1, where we have reported the covariance and correlation matrix corresponding to Figure 11 in the paper, but where we have used one-step residuals. In addition, Tables 1, 2 and 3 show the Frobenius norm, CRPS, and ES skill scores as explained in the paper to investigate the effectiveness of one-step residuals. Moreover, in Tables 4 and 5, we have utilized a shrinkage matrix rather than the sample covariance matrix to assess the performance of our approach.

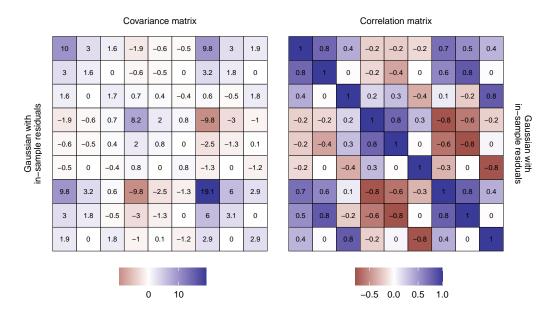


Figure 1: Comparison of estimated covariance and correlation matrices (first simulation) for base forecasts using a parametric Gaussian (with one-step residuals) approach. The true covariance and correlation matrices are shown in Figure 10 in the paper.

		Base forecasts' sample approach										
		Gaussian approach: sample covariance matrix										
Reconciliation approach	ctjb	In-sample residuals Multi-step residuals										
wp p rowers		G	В	Н	HB	G	В	Н	HB			
base	8.260	17.638	16.733	22.178	21.789	7.748	6.549	3.409	2.215			
ct(bu)	3.195	21.789	21.789	21.789	21.789	2.215	2.215	2.215	2.215			
$\operatorname{ct}(shr_{cs}, bu_{te})$	3.202	21.942	21.789	21.942	21.789	2.224	2.215	2.224	2.215			
$\operatorname{ct}(wlsv_{te}, bu_{cs})$	3.183	18.237	18.237	21.789	21.789	2.188	2.188	2.215	2.215			
oct(wlsv)	3.766	19.174	18.611	22.304	21.789	3.082	2.191	2.910	2.215			
oct(bdshr)	3.203	18.559	18.416	21.937	21.789	2.195	2.184	2.224	2.215			
oct(shr)	5.217	25.015	23.457	23.413	21.789	2.260	2.202	2.226	2.215			
oct(bshr)	5.282	23.772	23.997	22.146	21.789	2.720	2.220	2.756	2.215			
oct(hshr)	6.161	11.336	10.940	23.598	21.789	4.138	4.167	2.225	2.215			
oct(hbshr)	5.731	11.379	10.940	22.146	21.789	5.085	4.167	2.756	2.215			
$\operatorname{oct}_h(shr)$	3.251	20.965	19.992	22.079	21.789	2.260	2.202	2.226	2.215			
$\operatorname{oct}_h(bshr)$	3.602	21.306	21.022	22.146	21.789	2.720	2.220	2.756	2.215			
$\operatorname{oct}_h(hshr)$	4.869	11.405	10.940	22.037	21.789	4.138	4.167	2.225	2.215			
$\operatorname{oct}_h(hbshr)$	5.731	11.379	10.940	22.146	21.789	5.085	4.167	2.756	2.215			

Table 1: Frobenius norm between the true (in Figure 10 in the paper) and the estimated covariance matrix for different reconciliation approaches and different techniques for simulating the base forecasts. In bold, it is reported the lowest value for each column, in blue the minimum.

$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ach							
$ \begin{array}{c ccccccccccccccccccccccccccccccccccc$	ariance matrix	mple co	oach: sa	an appr	Gaussi			
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	ılti-step residuals	N	ls	residua	n-sample	Ir	ctjb	
base $t,000$	В Н	G	HB	Н	В	G		mp promeir
base $ct(bu)$ 0.901 0.908 1.009 1.044 1.047 0.998 0.999 1.002 1.002 $ct(bu)$ 0.901 0.930 0.929 0.929 0.929 0.900 0.900 0.900 0.900 $ct(shr_{cs},bu_{te})$ 0.901 0.929 0.928 0.929 0.928 0.900 0.899 0.900 0.900 $ct(wlsv_{te},bu_{cs})$ 0.910 0.930 0.929 0.939 0.939 0.916 0.916 0.916 0.916 $ct(wlsv_{te},bu_{cs})$ 0.910 0.930 0.930 0.939 0.930 0.930 0.930 0.930 0.930 $cct(wlsv)$ 0.922 0.942 0.944 0.951 0.953 0.930 0.930 0.930 0.930 $cct(shr)$ 0.910 0.930 0.930 0.939 0.938 0.916 0.915 0.916 0.916 0.916 $cct(shr)$ 0.941 0.999 0.985 0.983 0.973 0.903 0.902 0.902 0.902 $cct(shr)$ 0.951 0.995 1.000 0.983 0.986 0.922 0.922 0.921 0.921 $cct(shr)$ 0.987 0.995 0.993 1.039 1.026 0.972 0.972 0.974 0.995 $cct(shshr)$ 0.987 0.995 0.996 1.024 1.028 0.985 0.985 0.985 0.987 0.996 $cct(shshr)$ 0.904 0.929 0.928 0.932 0.932 0.903 0.902 0.902 0.902 $cct(shshr)$ 0.923 0.948 0.952 0.951 0.954 0.922 0.922 0.921 0.995 $cct(shshr)$ 0.974 0.982 0.982 1.012 1.012 0.972 0.972 0.974 0.995 $cct(shr_{cs},bu_{te})$ 0.987 0.995 0.996 1.024 1.028 0.985 0.985 0.987 0.996 $cct(shr_{cs},bu_{te})$ 0.987 0.995 0.996 1.024 1.028 0.985 0.985 0.987 0.996 $cct(shr_{cs},bu_{te})$ 0.997 0.993 0.994 0.994 0.994 0.995 0.995 0.996 1.024 1.028 0.985 0.985 0.987 0.996 $cct(shr_{cs},bu_{te})$ 0.977 0.993 0.993 0.994 0.994 0.994 0.996 0.996 0.996 0.996 0.996 0.996 0.996 0.997 0.997 0.999 0				€ {2,1}	$\forall k$			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.999 1.002 1	0.998	1.047			1.008	1.000	base
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.900 0.900 0	0.900	0.929	0.929	0.929	0.930	0.901	ct(bu)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.899 0.900 0	0.900	0.928	0.929	0.928	0.929	0.901	$\operatorname{ct}(shr_{cs},bu_{te})$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.916 0.916 (0.916	0.939	0.939	0.929	0.930	0.910	$\operatorname{ct}(wlsv_{te},bu_{cs})$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.930 0.930 0	0.930	0.953	0.951	0.944	0.942	0.922	oct(wlsv)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.915 0.916 (0.916	0.938	0.939	0.930	0.930	0.910	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.902 0.902 0	0.903	0.973	0.983	0.985	0.999	0.941	oct(shr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.922 0.921 0	0.922	0.986	0.983	1.000	0.995	0.951	oct(bshr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$			1.026	1.039	0.993		0.987	oct(hshr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.985 0.987 0	0.985	1.028	1.024	0.996	0.995	0.987	oct(hbshr)
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.902 0.902 0	0.903	0.932	0.932	0.928	0.929	0.904	$\operatorname{oct}_h(shr)$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.922 0.921 (0.922	0.954	0.951	0.952	0.948	0.923	$\operatorname{oct}_h(bshr)$
$k = 1$ base 1.000 1.017 1.019 1.017 1.019 0.998 0.999 0.999 1.00 ct(bu) 0.978 0.994 0.994 0.994 0.976 0.976 0.977 0.90 ct(shr_{cs}, bu_{te}) 0.977 0.993 0.993 0.994 0.993 0.976 0.976 0.976 ct(wlsv_{te}, bu_{cs}) 0.986 1.002 1.002 1.003 1.003 0.993 0.993 0.993 0.990 oct(wlsv) 0.998 1.014 1.015 1.015 1.016 1.006 1.006 1.007 1.007	0.972 0.974 (1.012	1.012	0.982	0.982	0.974	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.985 0.987 (0.985	1.028	1.024	0.996	0.995	0.987	$\operatorname{oct}_h(hbshr)$
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				=1	k			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.999 0.999 1	0.998	1.019	1.017	1.019	1.017	1.000	base
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	0.976 0.977 (0.976	0.994	0.994	0.994	0.994	0.978	ct(bu)
oct(wlsv) 0.998 1.014 1.015 1.015 1.016 1.006 1.006 1.007 1.00	0.976 0.976 0	0.976		0.994	0.993	0.993	0.977	
	0.993 0.993 (0.993	1.003	1.003	1.002	1.002	0.986	$\operatorname{ct}(wlsv_{te}, bu_{cs})$
	1.006 1.007 1	1.006	1.016	1.015	1.015	1.014	0.998	oct(wlsv)
	0.992 0.993 (0.992	1.003	1.003	1.002	1.002	0.986	
oct(shr) 1.037 1.082 1.067 1.064 1.056 0.979 0.978 0.979 0.99	0.978 0.979 0	0.979	1.056	1.064	1.067	1.082	1.037	oct(shr)
oct(bshr) 1.041 1.071 1.074 1.060 1.062 0.998 0.998 0.998 0.998	0.998 0.998 (0.998	1.062	1.060	1.074	1.071	1.041	oct(bshr)
oct(hshr) 1.080 1.090 1.091 1.119 1.105 1.050 1.050 1.053 1.00	1.050 1.053 1	1.050	1.105	1.119	1.091	1.090	1.080	oct(hshr)
oct(hbshr) 1.065 1.080 1.081 1.088 1.090 1.063 1.064 1.066 1.06	1.064 1.066 1	1.063	1.090	1.088	1.081	1.080	1.065	oct(hbshr)
$\operatorname{oct}_h(shr)$ 0.980 0.996 0.995 0.996 0.996 0.979 0.978 0.979 0.99	0.978 0.979 0	0.979	0.996	0.996	0.995	0.996	0.980	$\operatorname{oct}_h(shr)$
$\operatorname{oct}_h(bshr)$ 0.999 1.016 1.018 1.016 1.018 0.998 0.998 0.998 0.998	0.998 0.998 0	0.998	1.018	1.016	1.018	1.016	0.999	$\operatorname{oct}_h(bshr)$
$\operatorname{oct}_h(hshr)$ 1.052 1.067 1.066 1.074 1.075 1.050 1.050 1.053 1.05	1.050 1.053 1	1.050	1.075	1.074	1.066	1.067	1.052	$\operatorname{oct}_h(hshr)$
$\operatorname{oct}_h(hbshr)$ 1.065 1.080 1.081 1.088 1.090 1.063 1.064 1.066 1.06	1.064 1.066 1	1.063	1.090	1.088	1.081	1.080	1.065	$\operatorname{oct}_h(hbshr)$
k = 2				= 2	k			
base 1.000 0.998 0.999 1.071 1.075 0.998 0.999 1.005 1.00	0.999 1.005 1	0.998	1.075	1.071	0.999	0.998	1.000	base
ct(bu) 0.831 0.869 0.869 0.869 0.869 0.830 0.829 0.829 0.83								ct(bu)
$ct(shr_{cs}, bu_{te})$ 0.830 0.869 0.868 0.868 0.868 0.830 0.829 0.829 0.829		0.830	0.868					
ct(wlsv _{te} , bu _{cs}) 0.840 0.863 0.862 0.879 0.878 0.846 0.844 0.845 0.84	0.844 0.845 (0.846	0.878	0.879	0.862	0.863	0.840	
$\cot(wlsv)$ 0.851 0.875 0.877 0.891 0.893 0.859 0.859 0.859 0.869							0.851	
oct(bdshr) 0.839 0.863 0.863 0.879 0.878 0.845 0.844 0.845 0.84								
oct(shr) 0.854 0.922 0.909 0.908 0.897 0.833 0.831 0.832 0.83	0.831 0.832 (0.833	0.897	0.908	0.909	0.922	0.854	oct(shr)
oct(bshr) 0.869 0.925 0.931 0.911 0.915 0.851 0.851 0.851 0.851	0.851 0.851 0	0.851	0.915	0.911	0.931	0.925	0.869	oct(bshr)
oct(hshr) 0.901 0.908 0.904 0.966 0.952 0.900 0.899 0.901 0.90	0.899 0.901 (0.900	0.952	0.966	0.904	0.908	0.901	oct(hshr)
oct(hbshr) 0.915 0.917 0.919 0.964 0.969 0.913 0.913 0.914 0.99		0.913						
$\operatorname{oct}_h(shr)$ 0.834 0.868 0.865 0.872 0.872 0.833 0.831 0.832 0.83		0.833	0.872	0.872	0.865	0.868	0.834	$\operatorname{oct}_h(shr)$
$\operatorname{oct}_h(bshr)$ 0.852 0.886 0.890 0.890 0.894 0.851 0.851 0.851 0.851	0.851 0.851 0	0.851	0.894	0.890	0.890	0.886	0.852	
$oct_h(hshr)$ 0.902 0.904 0.904 0.953 0.952 0.900 0.899 0.901 0.90		0.900	0.952	0.953	0.904	0.904	0.902	$\operatorname{oct}_h(hshr)$
$\operatorname{oct}_h(hbshr)$ 0.915 0.917 0.919 0.964 0.969 0.913 0.913 0.914 0.96	0.913 0.914 (0.913	0.969	0.964	0.919	0.917	0.915	$\operatorname{oct}_h(hbshr)$

Table 2: CRPS skill score defined in (17) and (18). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

$\begin{array}{c} \operatorname{ct}(shr_{cs},\dot{b}u_{te}) & 0.896 & 0.924 & 0.923 & 0.923 & 0.922 & 0.895 & 0.895 & 0.896 & 0.896 \\ \operatorname{ct}(wlsv_{te},b_{tes}) & 0.906 & 0.924 & 0.923 & 0.933 & 0.932 & 0.912 & 0.911 & 0.910 \\ \operatorname{oct}(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 \\ \operatorname{oct}(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.932 & 0.910 & 0.911 & 0.911 \\ \operatorname{oct}(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(shr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(lshshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(lshshr) & 0.977 & 0.986 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(lshshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_{l_1}(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 \\ \operatorname{oct}_{l_1}(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_{l_1}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_{l_1}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_{l_1}(hshr) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.985 & 0.983 & 0.968 & 0.967 & 0.968 \\ \operatorname{ct}(wlsv_{le},bu_{cs}) & 0.997 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 \\ \operatorname{oct}(wlsv_{le}) & 0.998 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.995 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 \\ \operatorname{oct}(lshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.986 & 0.986 \\ \operatorname{oct}_{l_1}(lshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.037 & 1.037 & 1.039 & 0.986 \\ \operatorname{oct}_{l_1}(lshr) & 0.987 & 1.002 & 1.005 & 1.005 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_{l_1}(lshr) & 1.040 & 1.053 & 1.053 & 1.053 & 1.036 & 1.036 & 1.036 & 1.036 & 1.036 \\ \operatorname{oct}_{l_1}(lshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{le},bu_{cs}) & 0.839 $				Base	forecas	sts' samp	ole appr	oach		
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $				Gaussi	an appr	oach: sa	mple co	variance	matrix	
$ \begin{array}{c c c c c c c c c c c c c c c c c c c $		ctjb	Ir	n-sample	e residua	ıls	M	Iulti-step	residua	als
$\begin{array}{c} \text{base} & 1,000 & 1,005 & 1,009 & 1,023 & 0,924 & 0,923 & 0,895 & 0.896 & 0.897 \\ \text{ct}(shr_{cs}, bu_{te}) & 0.896 & 0,924 & 0,923 & 0,923 & 0,922 & 0.895 & 0.896 & 0.896 \\ \text{ct}(wlsvle_c, bu_{cs}) & 0,906 & 0,924 & 0,923 & 0,933 & 0,932 & 0,912 & 0,911 & 0,910 & 0,912 \\ \text{oct}(wlsvl) & 0,916 & 0,935 & 0,937 & 0,944 & 0,945 & 0,923 & 0,923 & 0,923 \\ \text{oct}(shr) & 0,906 & 0,923 & 0,932 & 0,932 & 0,912 & 0,910 & 0,911 & 0,912 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,985 & 1,027 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,900 & 0,923 & 0,922 & 0,926 & 0,925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \text{oct}_h(shr) & 0,901 & 0,944 & 0,943 & 0,942 & 0,945 & 0,914 & 0,916 & 0,915 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,969 & 0,985 & 0,983 & 0,984 & 0,967 & 0,977 & 0,980 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,968 & 0,984 & 0,983 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,969 & 0,985 & 0,985 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{oct}(bshr) & 0,977 & 0,991 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 0,0977 & 0,989 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 1,028 & 1,070 & 1,056 & 1,053 & 1,046 & 0,969 & 0,969 & 0,995 \\ \text{oct}(bshr) & 1,034 & 1,061 & 1,065 & 1,051 & 1,053 & 1,048 & 1,049 & 1,049 \\ \text{oct}_h(shr) & 1,066 & 1,075 & 1,0076 & 1,099 & 1,099 & 0,996 & 0,998 & 0,987 & 0,986 \\ \text{oct}_h(shr) & 1,064 & 1,063 & 1,071 & 1,073 & 1,047 & 1,049 & 1,051 \\ \text{oct}_h(shr) & 0,987 & 1,002 & 1,005 & 1,005 & 1,053 & 1,044 & 0,949 & 0,995 & 0,987 \\ \text{oct}_h(shr) & 0,831 & 0,867 & 0,860 & 0,8$	mp p rouers		G	В	Н	HB	G	В	Н	HB
$\begin{array}{c} \text{base} & 1,000 & 1,005 & 1,009 & 1,023 & 0,924 & 0,923 & 0,895 & 0.896 & 0.897 \\ \text{ct}(shr_{cs}, bu_{te}) & 0.896 & 0,924 & 0,923 & 0,923 & 0,922 & 0.895 & 0.896 & 0.896 \\ \text{ct}(wlsvle_c, bu_{cs}) & 0,906 & 0,924 & 0,923 & 0,933 & 0,932 & 0,912 & 0,911 & 0,910 & 0,912 \\ \text{oct}(wlsvl) & 0,916 & 0,935 & 0,937 & 0,944 & 0,945 & 0,923 & 0,923 & 0,923 \\ \text{oct}(shr) & 0,906 & 0,923 & 0,932 & 0,932 & 0,912 & 0,910 & 0,911 & 0,912 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,980 & 0,977 & 0,969 & 0.898 & 0.898 & 0.897 \\ \text{oct}(shr) & 0,938 & 0,993 & 0,985 & 1,027 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}(hshr) & 0,978 & 0,987 & 0,985 & 1,012 & 1,016 & 0,963 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,900 & 0,923 & 0,922 & 0,926 & 0,925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \text{oct}_h(shr) & 0,901 & 0,944 & 0,943 & 0,942 & 0,945 & 0,914 & 0,916 & 0,915 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(hshr) & 0,967 & 0,974 & 0,974 & 1,002 & 1,002 & 0,964 & 0,964 & 0,966 & 0,967 \\ \text{oct}_h(shr) & 0,969 & 0,985 & 0,983 & 0,984 & 0,967 & 0,977 & 0,980 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,968 & 0,984 & 0,983 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{ct}(shr_{cs}, bu_{te}) & 0,969 & 0,985 & 0,985 & 0,984 & 0,967 & 0,967 & 0,968 \\ \text{oct}(bshr) & 0,977 & 0,991 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 0,0977 & 0,989 & 0,991 & 0,992 & 0,992 & 0,984 & 0,983 & 0,985 \\ \text{oct}(bshr) & 1,028 & 1,070 & 1,056 & 1,053 & 1,046 & 0,969 & 0,969 & 0,995 \\ \text{oct}(bshr) & 1,034 & 1,061 & 1,065 & 1,051 & 1,053 & 1,048 & 1,049 & 1,049 \\ \text{oct}_h(shr) & 1,066 & 1,075 & 1,0076 & 1,099 & 1,099 & 0,996 & 0,998 & 0,987 & 0,986 \\ \text{oct}_h(shr) & 1,064 & 1,063 & 1,071 & 1,073 & 1,047 & 1,049 & 1,051 \\ \text{oct}_h(shr) & 0,987 & 1,002 & 1,005 & 1,005 & 1,053 & 1,044 & 0,949 & 0,995 & 0,987 \\ \text{oct}_h(shr) & 0,831 & 0,867 & 0,860 & 0,8$				$\forall k$	∈ {2,1}					
$\begin{array}{c} \operatorname{ct}(shr_{cs},bu_{te}) & 0.896 & 0.924 & 0.923 & 0.923 & 0.922 & 0.895 & 0.895 & 0.896 & 0.896 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.906 & 0.924 & 0.923 & 0.933 & 0.932 & 0.912 & 0.911 & 0.910 \\ \operatorname{oct}(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 \\ \operatorname{oct}(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.910 & 0.910 & 0.911 & 0.912 \\ \operatorname{oct}(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_h(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.916 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hshr) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.985 & 0.983 & 0.986 \\ \operatorname{ct}(wlsv_{te},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 \\ \operatorname{oct}(wlsv_{te}) & 0.997 & 0.999 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 \\ \operatorname{oct}(bshr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.987 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.093 & 1.036 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 1.061 & 1.063 & 1.071 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(bshr) & 1.061 & 1.065 & 1.065 & 1.065 & 1.065 & 0.866 & 0.866 & 0.866 & 0.866 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.829 & 0.867 & 0.866 & 0.866 & 0.866 & 0.8$	base	1.000	1.005			1.046	0.996	0.999	1.000	1.004
$ \begin{array}{c} ct(wlsv_{le},bu_{es}) \\ cet(wlsv) \\ cot(wlsv) \\ cot(wlsv) \\ opt(bdshr) \\ opt(bshr) \\ \mathsf$	ct(bu)	0.897	0.924	0.923	0.924	0.923	0.895	0.896	0.897	0.895
$\begin{array}{c} ct(wlsv_{le},bu_{es}) & 0.906 & 0.924 & 0.923 & 0.933 & 0.932 & 0.912 & 0.911 & 0.910 & 0.912 \\ oct(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 \\ oct(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.930 & 0.910 & 0.910 \\ oct(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ oct(shr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ oct(hshr) & 0.978 & 0.987 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ oct(hshr) & 0.978 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ oct_{h}(shr) & 0.970 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ oct_{h}(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ oct_{h}(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ oct_{h}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ oct_{h}(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.005 & 0.975 & 0.976 & 0.977 & 0.980 \\ oct_{h}(hshr) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ ct(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.985 & 0.984 & 0.967 & 0.968 & 0.968 \\ ct(wlsv_{te},bu_{tes}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.968 \\ ct(wlsv_{te},bu_{tes}) & 0.999 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ oct(bshr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 \\ oct(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ oct_{h}(hshr) & 0.967 & 0.998 & 0.995 & 0.998 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ oct_{h}(shr) & 0.977 & 0.998 & 0.995 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ oct_{h}(shr) & 1.051 & 1.065 & 1.065 & 1.051 & 1.053 & 1.046 & 0.969 & 0.969 & 0.969 \\ oct_{h}(shr) & 1.054 & 1.061 & 1.065 & 1.051 & 1.053 & 1.084 & 1.049 & 1.049 \\ oct_{h}(shr) & 0.987 & 1.002 & 1.005 & 1.005 & 1.005 & 0.986 & 0.987 & 0.988 \\ oct_{h}(bshr) & 0.831 & 0.867 & 0.867 & 0.867 $	$\operatorname{ct}(shr_{cs},bu_{te})$	0.896	0.924	0.923	0.923	0.922	0.895	0.895	0.896	0.896
$\begin{array}{c} \operatorname{oct}(wlsv) & 0.916 & 0.935 & 0.937 & 0.944 & 0.945 & 0.923 & 0.923 & 0.923 & 0.924 \\ \operatorname{oct}(bdshr) & 0.906 & 0.923 & 0.923 & 0.932 & 0.932 & 0.910 & 0.910 & 0.911 \\ \operatorname{oct}(shr) & 0.938 & 0.993 & 0.990 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.978 & 0.987 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_h(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \operatorname{oct}_h(bshr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \hline \\ base & 1.000 & 1.014 & 1.020 & 1.015 & 1.019 & 0.997 & 1.000 & 0.997 & 1.000 \\ \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.983 & 0.986 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{le},bu_{les}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{le},bu_{les}) & 0.998 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.050 & 1.065 & 1.067 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 1.066 & 1.075 & 1.066 & 1.063 & 1.073 & 0.996 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}(hlshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hshr) & 0.837 & 0.867 & 0.867 & 0.8$		0.906	0.924	0.923	0.933	0.932	0.912	0.911	0.910	0.912
$\begin{array}{c} \operatorname{oct}(shr) & 0.938 & 0.993 & 0.980 & 0.977 & 0.969 & 0.898 & 0.898 & 0.898 \\ \operatorname{oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ \operatorname{oct}(hshr) & 0.978 & 0.987 & 0.985 & 1.012 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}(hbshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ \operatorname{oct}_h(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \operatorname{oct}_h(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(ishr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(ishr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(ishr) & 0.969 & 0.984 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \end{array}$	oct(wlsv)	0.916	0.935	0.937	0.944	0.945	0.923	0.923	0.923	0.924
$\begin{array}{c} {\rm oct}(bshr) & 0.947 & 0.990 & 0.995 & 0.979 & 0.981 & 0.915 & 0.915 & 0.915 \\ {\rm oct}(hshr) & 0.978 & 0.985 & 0.985 & 1.027 & 1.016 & 0.963 & 0.964 & 0.966 & 0.967 \\ {\rm oct}(hshr) & 0.977 & 0.986 & 0.985 & 1.012 & 1.016 & 0.974 & 0.976 & 0.977 & 0.978 \\ {\rm oct}_h(shr) & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ {\rm oct}_h(shr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ {\rm oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ {\rm oct}_h(hshr) & 0.978 & 0.984 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \hline \\ \\ \\ \\ \\$	oct(bdshr)	0.906	0.923	0.923	0.932	0.932	0.910	0.910	0.911	0.912
$\begin{array}{c} {\rm oct}(hshr) \\ {\rm oct}(hshr) \\ {\rm oct}(hshr) \\ {\rm oct}(hshr) \\ {\rm O.977} \\ {\rm oct}(hshr) \\ {\rm O.977} \\ {\rm O.986} \\ {\rm O.985} \\ {\rm I.012} \\ {\rm I.016} \\ {\rm I.016} \\ {\rm O.974} \\ {\rm O.974} \\ {\rm O.976} \\ {\rm O.977} \\ {\rm O.986} \\ {\rm O.985} \\ {\rm I.012} \\ {\rm I.016} \\ {\rm I.016} \\ {\rm O.974} \\ {\rm O.974} \\ {\rm O.976} \\ {\rm O.977} \\ {\rm O.978} \\ {\rm O.889} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.891} \\ {\rm O.916} \\ {\rm O.917} \\ {\rm O.917} \\ {\rm O.910} \\ {\rm $	oct(shr)	0.938	0.993	0.980	0.977	0.969	0.898	0.898	0.898	0.897
$\begin{array}{c} {\rm oct}(hbshr) \\ {\rm oct}_h(shr) \\ {\rm oct}_h(shr) \\ {\rm O.900} \\ {\rm O.923} \\ {\rm oct}_h(shr) \\ {\rm O.916} \\ {\rm O.924} \\ {\rm O.924} \\ {\rm O.926} \\ {\rm O.925} \\ {\rm O.926} \\ {\rm O.925} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.897} \\ {\rm O.898} \\ {\rm O.898} \\ {\rm O.891} \\ {\rm O.916} \\ {\rm O.940} \\ {\rm O.940} \\ {\rm O.943} \\ {\rm O.942} \\ {\rm O.945} \\ {\rm O.945} \\ {\rm O.944} \\ {\rm O.964} \\ {\rm O.964} \\ {\rm O.964} \\ {\rm O.966} \\ {\rm O.967} \\ {\rm O.977} \\ {\rm O.970} \\ {\rm O.980} \\ {\rm O.984} \\ {\rm O.983} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.980} \\ {\rm O.991} \\ {\rm O.992} \\ {\rm O.992} \\ {\rm O.994} \\ {\rm O.995} \\ {\rm O.995} \\ {\rm O.996} \\ {\rm O.989} \\ {\rm O.991} \\ {\rm O.991} \\ {\rm O.991} \\ {\rm O.992} \\ {\rm O.992} \\ {\rm O.992} \\ {\rm O.984} \\ {\rm O.983} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.983} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.984} \\ {\rm O.995} \\ {\rm O.995} \\ {\rm O.995} \\ {\rm O.996} \\ {\rm $	oct(bshr)	0.947	0.990	0.995	0.979	0.981	0.915	0.915	0.915	0.915
$\begin{array}{c} \operatorname{oct}_h(shr') & 0.900 & 0.923 & 0.922 & 0.926 & 0.925 & 0.898 & 0.898 & 0.897 & 0.898 \\ \operatorname{oct}_h(bshr) & 0.916 & 0.940 & 0.943 & 0.942 & 0.945 & 0.914 & 0.916 & 0.915 & 0.916 \\ \operatorname{oct}_h(hshr) & 0.967 & 0.974 & 0.974 & 1.002 & 1.002 & 0.964 & 0.964 & 0.966 & 0.967 \\ \operatorname{oct}_h(hbshr) & 0.978 & 0.984 & 0.986 & 1.012 & 1.015 & 0.975 & 0.976 & 0.977 & 0.980 \\ \hline \\ base & 1.000 & 1.014 & 1.020 & 1.015 & 1.019 & 0.997 & 1.000 & 0.997 & 1.000 \\ \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.966 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.054 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}_h(shr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.988 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.988 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.063 & 1.073 & 0.996 & 0.998 & 0.989 \\ \operatorname{oct}_h(bshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.877 & 0.876 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(hshr) & 0.839 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 \\ \operatorname{oct}(bshr) & 0.885 & 0.921 & 0.990 & 0.997 & 0.991 & 0.990 & 0.990 & 0.909 \\ \operatorname{oct}(hbshr) & 0.$	oct(hshr)	0.978	0.987	0.985	1.027	1.016	0.963	0.964	0.966	0.967
$\begin{array}{c} \operatorname{oct}_h(bshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hshr) \\ \operatorname{opps} \\ \operatorname{opps} \\ \operatorname{oct}_h(hshr) \\ \operatorname{opps} \\ $	oct(hbshr)	0.977	0.986	0.985	1.012	1.016	0.974	0.976	0.977	0.978
$\begin{array}{c} \operatorname{oct}_h(hshr) \\ \operatorname{oct}_h(hbshr) \\ \operatorname{oct}_h(hbshr) \\ \end{array} \begin{array}{c} 0.967 \\ \operatorname{oct}_h(hbshr) \\ \end{array} \begin{array}{c} 0.978 \\ \operatorname{o.984} \\ \end{array} \begin{array}{c} 0.984 \\ \operatorname{o.986} \\ \end{array} \begin{array}{c} 1.002 \\ \operatorname{c.015} \\ \end{array} \begin{array}{c} 0.975 \\ \operatorname{o.975} \\ \end{array} \begin{array}{c} 0.976 \\ \operatorname{o.977} \\ \end{array} \begin{array}{c} 0.980 \\ \end{array} \end{array}$	$\operatorname{oct}_h(shr)$	0.900	0.923	0.922	0.926	0.925	0.898	0.898	0.897	0.898
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(bshr)$	0.916	0.940	0.943	0.942	0.945	0.914	0.916	0.915	0.916
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$		0.967	0.974	0.974	1.002	1.002	0.964	0.964	0.966	0.967
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(hbshr)$	0.978	0.984	0.986	1.012	1.015	0.975	0.976	0.977	0.980
$\begin{array}{c} \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(bshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.830 & 0.828 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.829 & 0.867 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.861 & 0.861 & 0.876 & 0.845 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(bshr) & 0.889 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 & 0.844 \\ \operatorname{oct}(bshr) & 0.886 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hbshr) & 0.897 & 0.905 & 0.901 & 0.959 & 0.947 & 0.895 & 0.896 & 0.898 & 0.899 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \end{array}$				k	z = 1					
$\begin{array}{c} \operatorname{ct}(bu) & 0.969 & 0.985 & 0.983 & 0.985 & 0.984 & 0.967 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.968 & 0.984 & 0.983 & 0.984 & 0.983 & 0.968 & 0.967 & 0.968 & 0.968 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(bshr) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.830 & 0.828 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.829 & 0.867 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.861 & 0.861 & 0.876 & 0.845 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(bshr) & 0.889 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 & 0.844 \\ \operatorname{oct}(bshr) & 0.886 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hbshr) & 0.897 & 0.905 & 0.901 & 0.959 & 0.947 & 0.895 & 0.896 & 0.898 & 0.899 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \end{array}$	base	1.000	1.014	1.020	1.015	1.019	0.997	1.000	0.997	1.000
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ct(bu)								0.968	
$\begin{array}{c} \operatorname{ct}(wlsv_{le},bu_{cs}) & 0.977 & 0.991 & 0.991 & 0.992 & 0.992 & 0.984 & 0.983 & 0.981 & 0.984 \\ \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hbshr) & 1.051 & 1.064 & 1.063 & 1.071 & 1.073 & 1.047 & 1.049 & 1.051 & 1.052 \\ \hline & & & & & & & & & & & & & & & & \\ base & 1.000 & 0.997 & 0.999 & 1.063 & 1.073 & 0.996 & 0.998 & 1.003 & 1.008 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.867 & 0.866 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(wlsv) & 0.849 & 0.872 & 0.875 & 0.887 & 0.890 & 0.858 & 0.856 & 0.856 \\ \operatorname{oct}(bdshr) & 0.839 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 \\ \operatorname{oct}(bshr) & 0.868 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hshr) & 0.897 & 0.905 & 0.901 & 0.959 & 0.947 & 0.895 & 0.896 & 0.898 & 0.899 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \hline \end{tabular}$								0.967		
$\begin{array}{c} \operatorname{oct}(wlsv) & 0.989 & 1.002 & 1.004 & 1.003 & 1.004 & 0.994 & 0.995 & 0.995 & 0.997 \\ \operatorname{oct}(bdshr) & 0.977 & 0.989 & 0.991 & 0.992 & 0.992 & 0.981 & 0.982 & 0.983 & 0.985 \\ \operatorname{oct}(shr) & 1.028 & 1.070 & 1.056 & 1.053 & 1.046 & 0.969 & 0.969 & 0.970 & 0.969 \\ \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hbshr) & 1.051 & 1.064 & 1.063 & 1.071 & 1.073 & 1.047 & 1.049 & 1.051 & 1.052 \\ \hline & & & & & & & & & & & & & & & & & &$							0.984			
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{c} \operatorname{oct}(bshr) & 1.034 & 1.061 & 1.065 & 1.051 & 1.053 & 0.985 & 0.987 & 0.986 & 0.987 \\ \operatorname{oct}(hshr) & 1.066 & 1.075 & 1.076 & 1.099 & 1.090 & 1.037 & 1.037 & 1.039 & 1.039 \\ \operatorname{oct}(hbshr) & 1.050 & 1.065 & 1.065 & 1.070 & 1.073 & 1.048 & 1.049 & 1.049 & 1.052 \\ \operatorname{oct}_h(shr) & 0.971 & 0.985 & 0.985 & 0.986 & 0.986 & 0.969 & 0.969 & 0.969 \\ \operatorname{oct}_h(bshr) & 0.987 & 1.002 & 1.005 & 1.002 & 1.005 & 0.986 & 0.987 & 0.987 & 0.988 \\ \operatorname{oct}_h(hshr) & 1.040 & 1.053 & 1.053 & 1.059 & 1.058 & 1.036 & 1.036 & 1.040 & 1.040 \\ \operatorname{oct}_h(hbshr) & 1.051 & 1.064 & 1.063 & 1.071 & 1.073 & 1.047 & 1.049 & 1.051 & 1.052 \\ \hline & & & & & & & & & & & & & & \\ base & 1.000 & 0.997 & 0.999 & 1.063 & 1.073 & 0.996 & 0.998 & 1.003 & 1.008 \\ \operatorname{ct}(bu) & 0.831 & 0.867 & 0.867 & 0.867 & 0.867 & 0.829 & 0.829 & 0.830 & 0.828 \\ \operatorname{ct}(shr_{cs},bu_{te}) & 0.829 & 0.867 & 0.866 & 0.865 & 0.828 & 0.829 & 0.829 & 0.829 \\ \operatorname{ct}(wlsv_{te},bu_{cs}) & 0.839 & 0.860 & 0.860 & 0.877 & 0.876 & 0.844 & 0.844 & 0.844 \\ \operatorname{oct}(wlsv) & 0.849 & 0.872 & 0.875 & 0.887 & 0.890 & 0.858 & 0.856 & 0.856 & 0.857 \\ \operatorname{oct}(bdshr) & 0.839 & 0.861 & 0.861 & 0.876 & 0.875 & 0.845 & 0.843 & 0.845 & 0.844 \\ \operatorname{oct}(shr) & 0.856 & 0.921 & 0.909 & 0.907 & 0.898 & 0.832 & 0.831 & 0.832 & 0.831 \\ \operatorname{oct}(bshr) & 0.868 & 0.924 & 0.930 & 0.911 & 0.915 & 0.849 & 0.848 & 0.849 & 0.848 \\ \operatorname{oct}(hbshr) & 0.910 & 0.912 & 0.912 & 0.957 & 0.961 & 0.906 & 0.909 & 0.909 & 0.910 \\ \hline \end{array}$		1.028	1.070		1.053	1.046	0.969	0.969	0.970	0.969
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	` '	1.034	1.061	1.065	1.051	1.053	0.985	0.987	0.986	0.987
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		1.066	1.075	1.076	1.099	1.090	1.037	1.037	1.039	1.039
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	oct(hbshr)	1.050	1.065	1.065	1.070	1.073	1.048	1.049	1.049	1.052
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(shr)$	0.971	0.985	0.985	0.986	0.986	0.969	0.969	0.969	0.969
$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(bshr)$	0.987	1.002	1.005	1.002	1.005	0.986	0.987	0.987	0.988
k = 2 $base$	$\operatorname{oct}_h(hshr)$	1.040	1.053	1.053	1.059	1.058	1.036	1.036	1.040	1.040
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	$\operatorname{oct}_h(hbshr)$	1.051	1.064	1.063	1.071	1.073	1.047	1.049	1.051	1.052
$\begin{array}{cccccccccccccccccccccccccccccccccccc$				k	z = 2					
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	base	1.000	0.997			1.073	0.996	0.998	1.003	1.008
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	ct(bu)									
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{cccccccccccccccccccccccccccccccccccc$		0.839	0.860	0.860	0.877	0.876	0.844	0.844	0.844	0.845
$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
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$\begin{array}{cccccccccccccccccccccccccccccccccccc$										
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	\ /									
oct(hbshr) 0.910 0.912 0.912 0.957 0.961 0.906 0.909 0.909 0.910	1 1									0.899
$\operatorname{oct}_h(shr)$ 0.835 0.865 0.862 0.870 0.868 0.833 0.833 0.831 0.832	$\operatorname{oct}_h(shr)$	0.835	0.865	0.862	0.870	0.868	0.833	0.833	0.831	
										0.850
						0.948	0.897	0.896	0.897	0.899
$\operatorname{oct}_h(hbshr)$ 0.910 0.910 0.914 0.957 0.961 0.907 0.908 0.909 0.912	$\operatorname{oct}_h(hbshr)$	0.910	0.910	0.914	0.957	0.961	0.907	0.908	0.909	0.912

Table 3: *ES skill score defined in equation (17) and (19). The smaller this value, the more accurate the forecast.*Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

	Base forecasts' sample approach										
			Gaussia	n approa	ach: shri	nkage co	ovarianc	e matrix			
Reconciliation approach	ctjb	Ir	-sample	residua	ls	M	Iulti-step	residua	als		
11		G	В	Н	HB	G	В	Н	HB		
			$\forall k \in$	€ {2,1}							
base	1.007	1.009	1.044	1.046	0.997	0.999	1.002	1.003	1.000		
ct(bu)	0.929	0.929	0.929	0.929	0.899	0.900	0.900	0.900	0.901		
$\operatorname{ct}(shr_{cs},bu_{te})$	0.929	0.928	0.929	0.928	0.899	0.899	0.900	0.900	0.901		
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.930	0.930	0.939	0.938	0.915	0.916	0.917	0.916	0.910		
oct(wlsv)	0.943	0.944	0.951	0.952	0.929	0.930	0.931	0.930	0.922		
oct(bdshr)	0.930	0.930	0.938	0.938	0.915	0.916	0.916	0.916	0.910		
oct(shr)	0.994	0.982	0.980	0.973	0.902	0.902	0.903	0.902	0.941		
oct(bshr)	0.995	0.998	0.983	0.986	0.921	0.922	0.922	0.922	0.951		
oct(hshr)	0.994	0.994	1.035	1.025	0.971	0.972	0.974	0.974	0.987		
oct(hbshr)	0.995	0.997	1.025	1.027	0.984	0.986	0.988	0.988	0.987		
$\operatorname{oct}_h(shr)$	0.929	0.928	0.932	0.932	0.902	0.902	0.903	0.902	0.904		
$\operatorname{oct}_h(\mathit{bshr})$	0.948	0.951	0.951	0.953	0.921	0.922	0.922	0.922	0.923		
$\operatorname{oct}_h(hshr)$	0.982	0.982	1.011	1.011	0.971	0.972	0.974	0.974	0.974		
$\operatorname{oct}_h(hbshr)$	0.995	0.997	1.025	1.027	0.984	0.986	0.988	0.988	0.987		
			k	=1							
base	1.017	1.019	1.017	1.019	0.998	0.999	0.999	0.999	1.000		
ct(bu)	0.994	0.994	0.994	0.994	0.976	0.976	0.977	0.976	0.978		
$\operatorname{ct}(shr_{cs},bu_{te})$	0.993	0.993	0.993	0.993	0.975	0.976	0.976	0.976	0.977		
$\operatorname{ct}(wlsv_{te}, bu_{cs})$	1.002	1.002	1.003	1.003	0.992	0.993	0.993	0.993	0.986		
oct(wlsv)	1.015	1.015	1.015	1.016	1.005	1.007	1.007	1.007	0.998		
oct(bdshr)	1.002	1.002	1.003	1.002	0.992	0.992	0.993	0.992	0.986		
oct(shr)	1.076	1.065	1.061	1.056	0.978	0.978	0.979	0.978	1.037		
oct(bshr)	1.070	1.072	1.060	1.062	0.997	0.998	0.998	0.998	1.041		
oct(hshr)	1.090	1.092	1.114	1.105	1.049	1.050	1.053	1.052	1.080		
oct(hbshr)	1.080	1.081	1.089	1.090	1.062	1.064	1.066	1.066	1.065		
$\operatorname{oct}_h(shr)$	0.996	0.995	0.996	0.996	0.978	0.978	0.979	0.978	0.980		
$\operatorname{oct}_h(bshr)$	1.016	1.018	1.016	1.018	0.997	0.998	0.998	0.998	0.999		
$\operatorname{oct}_h(hshr)$	1.066	1.067	1.075	1.075	1.049	1.050	1.053	1.052	1.052		
$\operatorname{oct}_h(hbshr)$	1.080	1.081	1.089	1.090	1.062	1.064	1.066	1.066	1.065		
,			k	= 2							
base	0.997	0.999	1.071	1.074	0.997	0.999	1.005	1.008	1.000		
ct(bu)	0.869	0.868	0.868	0.868	0.829	0.829	0.830	0.830	0.831		
$\operatorname{ct}(shr_{cs},bu_{te})$	0.868	0.867	0.868	0.867	0.829	0.829	0.830	0.829	0.830		
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.863	0.862	0.878	0.878	0.845	0.845	0.846	0.846	0.840		
oct(wlsv)	0.876	0.877	0.891	0.892	0.859	0.860	0.860	0.860	0.851		
oct(bdshr)	0.863	0.863	0.878	0.877	0.844	0.845	0.846	0.845	0.839		
oct(shr)	0.918	0.906	0.906	0.897	0.832	0.832	0.833	0.832	0.854		
oct(bshr)	0.924	0.928	0.911	0.915	0.850	0.851	0.852	0.851	0.869		
oct(hshr)	0.907	0.905	0.962	0.951	0.898	0.899	0.902	0.902	0.901		
oct(hbshr)	0.917	0.919	0.964	0.968	0.912	0.913	0.915	0.916	0.915		
$\operatorname{oct}_h(shr)$	0.867	0.864	0.872	0.871	0.832	0.832	0.833	0.832	0.834		
$\operatorname{oct}_h(bshr)$	0.886	0.890	0.890	0.893	0.850	0.851	0.852	0.851	0.852		
$\operatorname{oct}_h(bshr)$	0.904	0.905	0.952	0.952	0.898	0.899	0.902	0.902	0.902		
$\operatorname{oct}_h(hbshr)$	0.917	0.919	0.964	0.968	0.912	0.913	0.915	0.916	0.915		

Table 4: CRPS skill score defined in (17) and (18). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

	Base forecasts' sample approach										
			Gaussia	n approa	ach: shri	nkage co	ovarianc	e matrix	:		
Reconciliation approach	ctjb	Ir	n-sample	residua	ls	M	Iulti-step	residua	als		
11		G	В	Н	HB	G	В	Н	HB		
			$\forall k$	∈ {2,1}							
base	1.005	1.008	1.039	1.045	0.996	0.999	1.000	1.003	1.000		
ct(bu)	0.923	0.923	0.923	0.923	0.895	0.896	0.897	0.897	0.897		
$\operatorname{ct}(shr_{cs},bu_{te})$	0.923	0.922	0.922	0.922	0.896	0.895	0.895	0.895	0.896		
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.924	0.924	0.932	0.932	0.910	0.911	0.911	0.911	0.906		
oct(wlsv)	0.935	0.937	0.944	0.945	0.922	0.924	0.923	0.923	0.916		
oct(bdshr)	0.924	0.924	0.932	0.931	0.909	0.911	0.911	0.910	0.906		
oct(shr)	0.989	0.978	0.975	0.968	0.897	0.898	0.898	0.898	0.938		
oct(bshr)	0.990	0.993	0.978	0.981	0.915	0.915	0.915	0.915	0.947		
oct(hshr)	0.986	0.985	1.024	1.015	0.963	0.964	0.966	0.967	0.978		
oct(hbshr)	0.985	0.986	1.012	1.015	0.973	0.976	0.977	0.978	0.977		
$\operatorname{oct}_h(shr)$	0.923	0.922	0.925	0.925	0.897	0.898	0.898	0.898	0.900		
$\operatorname{oct}_h(\mathit{bshr})$	0.941	0.943	0.942	0.945	0.913	0.915	0.915	0.915	0.916		
$\operatorname{oct}_h(hshr)$	0.974	0.975	1.001	1.001	0.964	0.964	0.966	0.966	0.967		
$\operatorname{oct}_h(hbshr)$	0.985	0.986	1.013	1.016	0.973	0.976	0.977	0.978	0.978		
			k	=1							
base	1.014	1.018	1.015	1.019	0.997	0.999	0.997	0.998	1.000		
ct(bu)	0.983	0.984	0.984	0.984	0.967	0.967	0.969	0.969	0.969		
$\operatorname{ct}(shr_{cs},bu_{te})$	0.983	0.982	0.982	0.983	0.966	0.967	0.966	0.966	0.968		
$\operatorname{ct}(wlsv_{te}, bu_{cs})$	0.991	0.992	0.993	0.992	0.983	0.983	0.983	0.983	0.977		
oct(wlsv)	1.002	1.004	1.004	1.004	0.994	0.995	0.994	0.996	0.989		
oct(bdshr)	0.990	0.991	0.992	0.991	0.981	0.983	0.984	0.982	0.977		
oct(shr)	1.065	1.054	1.051	1.045	0.969	0.970	0.970	0.969	1.028		
oct(bshr)	1.061	1.063	1.050	1.052	0.986	0.986	0.987	0.985	1.034		
oct(hshr)	1.076	1.077	1.095	1.088	1.036	1.036	1.040	1.038	1.066		
oct(hbshr)	1.064	1.065	1.071	1.073	1.047	1.048	1.050	1.050	1.050		
$\operatorname{oct}_h(shr)$	0.984	0.985	0.986	0.986	0.969	0.969	0.969	0.968	0.971		
$\operatorname{oct}_h(bshr)$	1.003	1.005	1.003	1.005	0.985	0.987	0.987	0.986	0.987		
$\operatorname{oct}_h(hshr)$	1.054	1.054	1.059	1.059	1.036	1.037	1.038	1.039	1.040		
$oct_h(hbshr)$	1.063	1.065	1.071	1.074	1.046	1.048	1.049	1.051	1.051		
			k	z = 2							
base	0.996	0.998	1.064	1.073	0.995	0.999	1.003	1.007	1.000		
ct(bu)	0.867	0.866	0.867	0.866	0.829	0.829	0.830	0.830	0.831		
$\operatorname{ct}(shr_{cs},bu_{te})$	0.867	0.866	0.866	0.866	0.830	0.829	0.830	0.830	0.829		
$\operatorname{ct}(wlsv_{te},bu_{cs})$	0.861	0.861	0.875	0.875	0.843	0.845	0.845	0.845	0.839		
oct(wlsv)	0.873	0.874	0.888	0.889	0.856	0.857	0.857	0.856	0.849		
oct(bdshr)	0.862	0.861	0.876	0.874	0.843	0.844	0.844	0.844	0.839		
oct(shr)	0.918	0.907	0.905	0.898	0.831	0.832	0.832	0.832	0.856		
oct(bshr)	0.924	0.928	0.911	0.915	0.849	0.849	0.849	0.849	0.868		
oct(hshr)	0.904	0.901	0.957	0.946	0.895	0.896	0.898	0.900	0.897		
oct(hbshr)	0.912	0.913	0.956	0.961	0.905	0.909	0.909	0.911	0.910		
$\operatorname{oct}_h(shr)$	0.866	0.863	0.869	0.869	0.830	0.831	0.832	0.832	0.835		
$\operatorname{oct}_h(bshr)$	0.882	0.886	0.886	0.889	0.846	0.848	0.849	0.848	0.850		
$\operatorname{oct}_h(hshr)$	0.901	0.902	0.947	0.946	0.896	0.896	0.898	0.899	0.900		
$\operatorname{oct}_h(hbshr)$	0.912	0.914	0.958	0.961	0.905	0.908	0.910	0.909	0.910		

Table 5: *ES skill score defined in equation (17) and (19). The smaller this value, the more accurate the forecast.*Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

B Australian GDP dataset: one-step residuals and shrinkage covariance matrix

			ecasts' sample a		
Reconciliation		Gaussia	an approach: sa	mple covariance	e matrix ping and
approach	ctjb	Multi-step	residuals		pnig and residuals
ирргоасп		G	Н	G	Н
		$\forall k \in \{4, 2,$			
base	1.000	0.979	0.995	0.968	0.976
$ct(shr_{cs}, bu_{te})$	0.937	0.956	0.956	0.976	0.976
$\operatorname{ct}(wls_{cs},bu_{te})$	0.930	0.917	0.917	0.898	0.898
oct(wlsv)	0.926	0.919	0.920	0.900	0.900
oct(bdshr)	0.940	0.965	0.945	0.992	0.957
oct(shr)	0.944	1.020	0.940	1.094	0.937
oct(shr) oct(hshr)	0.988	0.972	1.002	0.974	1.001
$oct_o(wlsv)$	0.986	0.972	0.912	0.896	0.895
- ' /	0.928	0.964			
$oct_o(bdshr)$			0.946	0.952	0.930
$oct_o(shr)$	0.950	0.946	0.922	0.925	0.903
$oct_o(hshr)$	0.989	0.966	0.984	0.954	0.965
$\operatorname{oct}_{oh}(shr)$	1.102	1.059	1.001	1.094	0.988
$\operatorname{oct}_{oh}(hshr)$	1.006	0.983	1.009	0.974	1.001
		k = 1			
base	1.000	0.988	0.988	0.971	0.971
$\operatorname{ct}(shr_{cs},bu_{te})$	0.992	1.008	1.008	1.029	1.029
$\operatorname{ct}(wls_{cs},bu_{te})$	0.986	0.974	0.975	0.956	0.956
oct(wlsv)	0.984	0.981	0.979	0.959	0.959
oct(bdshr)	0.997	1.019	1.003	1.044	1.018
oct(shr)	1.015	1.095	1.010	1.160	1.059
oct(hshr)	1.048	1.037	1.060	1.034	1.061
$oct_o(wlsv)$	0.984	0.971	0.970	0.954	0.954
$oct_o(bdshr)$	1.034	1.016	1.003	1.005	0.989
$oct_o(shr)$	1.014	1.003	0.985	0.987	0.968
$oct_o(hshr)$	1.047	1.028	1.038	1.012	1.023
$oct_{oh}(shr)$	1.172	1.109	1.066	1.160	1.059
$oct_{oh}(hshr)$	1.068	1.046	1.059	1.034	1.061
		k = 2			
base	1.000	0.984	0.993	0.968	0.976
$\operatorname{ct}(shr_{cs},bu_{te})$	0.949	0.966	0.966	0.987	0.987
$\operatorname{ct}(wls_{cs},bu_{te})$	0.942	0.928	0.928	0.909	0.909
oct(wlsv)	0.938	0.929	0.931	0.911	0.911
oct(bdshr)	0.953	0.976	0.956	1.003	0.969
oct(shr)	0.955	1.031	0.951	1.113	1.002
oct(hshr)	1.001	0.985	1.014	0.987	1.016
$oct_o(wlsv)$	0.938	0.921	0.923	0.907	0.906
$oct_o(bdshr)$	0.991	0.974	0.957	0.964	0.942
$oct_o(shr)$	0.965	0.958	0.934	0.938	0.916
$oct_o(hshr)$	1.002	0.979	0.996	0.967	0.978
$\operatorname{oct}_{oh}(shr)$	1.120	1.069	1.013	1.113	1.002
$oct_{oh}(hshr)$	1.021	0.996	1.021	0.987	1.016
011 ()		k = 4	y		
base	1.000	0.966	1.004	0.964	0.981
$ct(shr_{cs}, bu_{te})$	0.874	0.896	0.896	0.914	0.914
$\operatorname{ct}(wls_{cs},bu_{te})$	0.866	0.853	0.853	0.834	0.834
oct(wlsv)	0.860	0.853	0.855	0.835	0.834
oct(wisv) oct(bdshr)	0.874	0.904	0.880	0.833	0.889
oct(bushr) oct(shr)	0.866	0.940	0.864	1.015	0.865
oct(shr) oct(hshr)	0.866	0.940	0.864	0.904	0.909
` /					
$oct_o(wlsv)$	0.860	0.847	0.848	0.832	0.830
$oct_o(bdshr)$	0.914	0.905	0.883	0.892	0.865
$oct_o(shr)$	0.877	0.882	0.852	0.854	0.831
$oct_o(hshr)$	0.922	0.898	0.923 0.928	0.888 1.015	0.898 0.909
$oct_{oh}(shr)$	1.020	1.002			

Table 6: CRPS skill score defined in equation (17) and (18) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

			ecasts' sample a		
D:11: - t1		Gaussia	an approach: sa	mple covariance	
Reconciliation approach	ctjb	Multi-step	residuals	Overlap multi-ster	
approach		G	Н	G	H
base	1.000	$\forall k \in \{4, 2, 0.970\}$	0.988	0.960	0.970
$ct(shr_{cs}, bu_{te})$	0.897	0.944	0.944	0.973	0.973
$\operatorname{ct}(wls_{cs},bu_{te})$	0.886	0.880	0.880	0.860	0.860
oct(wlsv)	0.890	0.890	0.894	0.872	0.872
oct(bdshr)	0.905	0.956	0.934	0.992	0.954
oct(shr)	0.895	0.979	0.895	1.053	0.944
oct(hshr)	0.951	0.940	0.973	0.959	0.992
$oct_o(wlsv)$	0.891	0.879	0.881	0.864	0.864
$oct_o(bdshr)$	0.940	0.928	0.910	0.918	0.895
$\operatorname{oct}_o(shr)$	0.900	0.899	0.876	0.878	0.858
$oct_o(hshr)$	0.956	0.936	0.955	0.922	0.936
$\operatorname{oct}_{oh}(shr)$	1.059	1.015	0.956	1.053	0.945
$oct_{oh}(hshr)$	0.986	0.968	0.999	0.959	0.992
,		k = 1			
base	1.000	0.977	0.977	0.965	0.965
$\operatorname{ct}(shr_{cs},bu_{te})$	0.964	1.001	1.001	1.033	1.033
$\operatorname{ct}(wls_{cs},bu_{te})$	0.954	0.944	0.945	0.928	0.928
oct(wlsv)	0.958	0.957	0.957	0.938	0.939
oct(bdshr)	0.972	1.014	0.994	1.048	1.018
oct(shr)	0.973	1.060	0.969	1.121	1.015
oct(hshr)	1.017	1.010	1.034	1.023	1.055
$\operatorname{oct}_o(wlsv)$	0.958	0.945	0.945	0.931	0.931
$oct_o(bdshr)$	1.004	0.986	0.971	0.980	0.961
$\operatorname{oct}_o(shr)$	0.973	0.963	0.944	0.949	0.930
$oct_o(hshr)$	1.021	1.004	1.012	0.987	1.000
$\operatorname{oct}_{oh}(shr)$	1.130	1.063	1.019	1.121	1.016
$\operatorname{oct}_{oh}(hshr)$	1.053	1.034	1.049	1.024	1.055
1	1 000	k=2	0.005	0.050	0.060
base	1.000	0.972	0.985	0.959	0.969
$\operatorname{ct}(shr_{cs}, bu_{te})$	0.915 0.904	0.961 0.896	0.960 0.896	0.991 0.877	0.991 0.877
$\operatorname{ct}(wls_{cs}, bu_{te})$ $\operatorname{oct}(wlsv)$	0.904	0.907	0.912	0.889	0.889
oct(bdshr)	0.925	0.976	0.953	1.013	0.974
oct(shr)	0.913	1.000	0.914	1.076	0.963
oct(shr)	0.973	0.960	0.993	0.978	1.014
$oct_o(wlsv)$	0.908	0.895	0.898	0.881	0.882
$oct_o(bdshr)$	0.960	0.947	0.929	0.938	0.915
$oct_o(shr)$	0.921	0.919	0.896	0.898	0.878
$oct_o(hshr)$	0.977	0.956	0.976	0.942	0.957
$\operatorname{oct}_{oh}(shr)$	1.082	1.029	0.973	1.076	0.963
$oct_{oh}(hshr)$	1.007	0.988	1.017	0.979	1.014
		k = 4			
base	1.000	0.959	1.000	0.957	0.976
$\operatorname{ct}(shr_{cs},bu_{te})$	0.818	0.874	0.874	0.899	0.900
$\operatorname{ct}(wls_{cs},bu_{te})$	0.807	0.805	0.805	0.782	0.783
oct(wlsv)	0.811	0.813	0.819	0.794	0.794
oct(bdshr)	0.825	0.883	0.860	0.920	0.876
oct(shr)	0.807	0.885	0.808	0.967	0.861
oct(hshr)	0.871	0.856	0.897	0.881	0.913
$\operatorname{oct}_o(wlsv)$	0.812	0.802	0.806	0.786	0.786
$\operatorname{oct}_o(bdshr)$	0.860	0.856	0.836	0.841	0.816
$\operatorname{oct}_o(shr)$	0.814	0.821	0.796	0.794	0.775
$oct_o(hshr)$	0.876	0.854	0.882	0.844	0.856
$oct_{oh}(shr)$	0.971	0.954	0.882	0.967	0.861
$oct_{oh}(hshr)$	0.904	0.888	0.934	0.881	0.913

Table 7: ES skill score defined in equation (17) and (19) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

-			ecasts' sample		
Reconciliation		Gaussiar	approach: shri	nkage covarian	
approach	ctjb	Multi-step	residuals	multi-ster	ping and residuals
арргоасн		G	Н	G	Н
		$\forall k \in \{4, 2,$	11		
base	1.000	0.979	1.011	0.968	0.987
$ct(shr_{cs}, bu_{te})$	0.937	0.960	0.961	0.962	0.960
$\operatorname{ct}(wls_{cs},bu_{te})$	0.930	0.951	0.953	0.911	0.915
oct(wlsv)	0.926	0.972	0.957	0.918	0.917
oct(bdshr)	0.940	0.986	0.966	0.981	0.956
oct(shr)	0.944	0.999	0.962	1.051	0.995
oct(hshr)	0.988	1.000	1.021	0.979	1.002
$oct_o(wlsv)$	0.926	0.961	0.948	0.914	0.912
$oct_o(bdshr)$	0.978	0.956	0.949	0.949	0.934
$oct_o(shr)$	0.950	0.957	0.946	0.933	0.917
$oct_o(hshr)$	0.989	0.997	1.013	0.967	0.982
$oct_{oh}(shr)$	1.102	1.010	1.006	1.051	0.995
$oct_{oh}(hshr)$	1.006	0.989	1.004	0.979	1.002
occ _{on} (nom)	1.000	k = 1	1.001	0.575	1.002
base	1.000	0.988	0.988	0.971	0.971
$\operatorname{ct}(shr_{cs},bu_{te})$	0.992	1.001	1.001	1.004	1.000
$\operatorname{ct}(wls_{cs},bu_{te})$	0.986	0.997	0.998	0.964	0.967
oct(wlsv)	0.984	1.010	1.003	0.971	0.970
oct(bdshr)	0.997	1.015	1.006	1.016	1.000
oct(shr)	1.015	1.047	1.021	1.105	1.058
oct(hshr)	1.048	1.045	1.066	1.034	1.053
$oct_o(wlsv)$	0.984	1.000	0.993	0.966	0.965
$oct_o(bdshr)$	1.034	0.984	0.983	0.988	0.977
$oct_o(shr)$	1.014	0.998	0.995	0.986	0.974
$oct_o(hshr)$	1.047	1.039	1.054	1.019	1.032
$oct_{oh}(shr)$	1.172	1.059	1.063	1.105	1.058
$oct_{oh}(hshr)$	1.068	1.037	1.050	1.034	1.053
		k = 2			
base	1.000	0.984	1.009	0.968	0.987
$\operatorname{ct}(shr_{cs},bu_{te})$	0.949	0.972	0.972	0.974	0.971
$\operatorname{ct}(wls_{cs},bu_{te})$	0.942	0.962	0.964	0.923	0.927
oct(wlsv)	0.938	0.988	0.968	0.931	0.929
oct(bdshr)	0.953	1.004	0.979	0.996	0.970
oct(shr)	0.955	1.016	0.973	1.070	1.010
oct(hshr)	1.001	1.015	1.034	0.993	1.017
$oct_o(wlsv)$	0.938	0.976	0.959	0.927	0.925
$oct_o(bdshr)$	0.991	0.970	0.963	0.963	0.948
$oct_o(shr)$	0.965	0.973	0.959	0.948	0.931
$oct_o(hshr)$	1.002	1.013	1.026	0.980	0.996
$\operatorname{oct}_{oh}(shr)$	1.120	1.026	1.019	1.070	1.010
$\operatorname{oct}_{oh}(hshr)$	1.021	1.005	1.017	0.993	1.017
1	1 000	k=4	1.000	0.064	1.000
base	1.000	0.966	1.037	0.964	1.002
$\operatorname{ct}(shr_{cs},bu_{te})$	0.874	0.910	0.911	0.910	0.910
$\operatorname{ct}(wls_{cs},bu_{te})$	0.866	0.897	0.900	0.851	0.855
oct(wlsv)	0.860	0.921	0.903	0.856	0.856
oct(bdshr)	0.874	0.942	0.914	0.932	0.900
oct(shr)	0.866	0.937	0.895	0.981	0.922
oct(hshr)	0.919	0.942	0.965	0.913	0.937
$oct_o(wlsv)$	0.860	0.910	0.894	0.853	0.852
$oct_o(bdshr)$	0.914	0.917	0.905	0.899	0.880
$oct_o(shr)$	0.877	0.903	0.886	0.868	0.850
$\operatorname{oct}_o(hshr)$ $\operatorname{oct}_{oh}(shr)$	0.922 1.020	0.943 0.947	0.962 0.939	0.905 0.981	0.921 0.922
		11 4/1:7	11 939	HUNT	

Table 8: CRPS skill score defined in equation (17) and (18) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

		Base for	ecasts' sample a	approach	
•		Gaussiar	approach: shri	inkage covarian	ce matrix
Reconciliation	ctjb	Multi-step	residuals		ping and residuals
approach		G	Н	G	H H
		$\forall k \in \{4, 2,$	1 l		
base	1.000	0.967	1.002	0.957	0.980
$\operatorname{ct}(shr_{cs},bu_{te})$	0.897	0.968	0.969	0.963	0.962
$\operatorname{ct}(wls_{cs},bu_{te})$	0.886	0.939	0.944	0.882	0.888
oct(wlsv)	0.890	0.966	0.959	0.897	0.901
oct(bdshr)	0.905	0.997	0.981	0.986	0.960
oct(shr)	0.895	0.979	0.945	1.021	0.962
oct(hshr)	0.951	0.997	1.023	0.973	1.005
$\operatorname{oct}_o(wlsv)$	0.891	0.950	0.945	0.889	0.892
$oct_o(bdshr)$	0.940	0.935	0.933	0.922	0.909
$\operatorname{oct}_o(shr)$	0.900	0.935	0.928	0.895	0.884
$oct_o(hshr)$	0.956	0.997	1.015	0.945	0.965
$oct_{oh}(shr)$	1.059	0.981	0.983	1.021	0.962
$oct_{oh}(hshr)$	0.986	0.996	1.014	0.973	1.005
		k = 1			
base	1.000	0.973	0.973	0.961	0.962
$\operatorname{ct}(shr_{cs},bu_{te})$	0.964	1.012	1.012	1.009	1.004
$\operatorname{ct}(wls_{cs},bu_{te})$	0.954	0.994	0.998	0.947	0.952
oct(wlsv)	0.958	1.017	1.012	0.960	0.965
oct(bdshr)	0.972	1.031	1.021	1.024	1.005
oct(shr)	0.973	1.041	1.011	1.083	1.028
oct(hshr)	1.017	1.051	1.073	1.034	1.063
$\operatorname{oct}_o(wlsv)$	0.958	1.002	0.997	0.953	0.956
$oct_o(bdshr)$	1.004	0.965	0.964	0.969	0.959
$\operatorname{oct}_o(shr)$	0.973	0.984	0.982	0.960	0.950
$\operatorname{oct}_o(hshr)$	1.021	1.049	1.062	1.007	1.024
$\operatorname{oct}_{oh}(shr)$	1.130	1.034	1.041	1.083	1.029
$\operatorname{oct}_{oh}(hshr)$	1.053	1.050	1.064	1.034	1.063
1	1 000	k=2	0.000	0.055	0.000
base	1.000	0.970	0.999	0.955	0.980
$\operatorname{ct}(shr_{cs},bu_{te})$	0.915	0.987	0.988	0.983	0.982
$\operatorname{ct}(wls_{cs},bu_{te})$	0.904	0.958 0.988	0.962	0.900	0.906
oct(wlsv) oct(bdshr)	0.909 0.925	1.024	0.979 1.005	0.916 1.010	0.920 0.984
oct(bushr) oct(shr)	0.923	1.024	0.967	1.045	0.982
oct(shr) oct(hshr)	0.913	1.020	1.046	0.994	1.028
$oct_o(wlsv)$	0.908	0.972	0.964	0.908	0.911
$oct_o(bdshr)$	0.960	0.959	0.957	0.945	0.932
$oct_o(shr)$	0.921	0.958	0.950	0.917	0.905
$oct_o(shr)$	0.977	1.021	1.038	0.966	0.987
$oct_{oh}(shr)$	1.082	1.002	1.003	1.045	0.982
$oct_{oh}(hshr)$	1.007	1.017	1.036	0.994	1.028
octon (nom)	1.007	k = 4	1.000	0.551	1.020
base	1.000	0.958	1.033	0.953	1.000
$ct(shr_{cs}, bu_{te})$	0.818	0.909	0.910	0.902	0.902
$\operatorname{ct}(wls_{cs},bu_{te})$	0.807	0.871	0.876	0.805	0.902
oct(wlsv)	0.811	0.896	0.891	0.820	0.825
oct(bdshr)	0.825	0.938	0.919	0.926	0.895
oct(shr)	0.807	0.898	0.864	0.940	0.881
oct(hshr)	0.871	0.924	0.954	0.897	0.929
$oct_o(wlsv)$	0.812	0.882	0.876	0.812	0.816
$oct_o(bdshr)$	0.860	0.884	0.879	0.857	0.841
$oct_o(shr)$	0.814	0.867	0.857	0.815	0.803
$oct_o(hshr)$	0.876	0.926	0.949	0.868	0.889
$\operatorname{oct}_{oh}(shr)$	0.971	0.910	0.911	0.941	0.882
$oct_{oh}(hshr)$	0.904	0.924	0.947	0.896	0.929

Table 9: ES skill score defined in equation (17) and (19) and (??) for the Australian Quarterly National Accounts dataset (AusGDP). The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

C Australian Tourism Demand dataset

				Base for	ecasts' s	ample a	pproach	1		
Reconciliation approach	ctjb	G	aussian	approac	h*	ctjb	G	aussian	approac	h*
11		G	В	Н	HB		G	В	Н	HB
		$\forall k \in \mathcal{A}$	12, 6, 4,	3,2,1}				k = 1		
base	1.000	0.971	0.972	0.971	0.972	1.000	0.972	0.971	0.972	0.971
ct(bu)	1.321	1.017	1.018	1.017	1.017	1.077	0.983	0.983	0.983	0.983
$\operatorname{ct}(shr_{cs},bu_{te})$	1.057	1.013	0.971	1.013	0.971	0.976	0.987	0.961	0.988	0.961
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.062	1.069	1.070	0.974	0.974	0.976	0.986	0.986	0.965	0.965
oct(ols)	0.989	1.163	1.052	1.139	0.987	0.982	1.038	0.992	1.047	0.987
oct(struc)	0.982	1.099	1.039	1.037	0.960	0.970	1.007	0.971	0.999	0.962
oct(wlsv)	0.987	1.080	1.041	0.992	0.958	0.952	1.004	0.969	0.978	0.956
oct(bdshr)	0.975	1.072	1.032	0.985	0.950	0.949	0.999	0.965	0.975	0.952
$\operatorname{oct}_h(hbshr)$	0.989	1.189	1.076	1.171	1.021	0.982	1.045	1.000	1.063	1.009
$\operatorname{oct}_h(bshr)$	0.994	1.202	1.073	1.168	1.021	0.988	1.046	1.012	1.063	1.012
$\operatorname{oct}_h(hshr)$	0.969	1.066	1.052	1.008	0.994	0.953	0.994	0.972	0.991	0.979
$\operatorname{oct}_h(shr)$	1.007	1.090	1.046	1.000	0.970	1.000	1.035	0.992	0.998	0.973
haca	1 000	0.060	k = 2 0.969	0.068	0.068	1 000	0.071	k = 3	0.060	0.070
base $\operatorname{ct}(bu)$	1.000 1.189	0.969		0.968 1.000	0.968 1.000	1.000 1.273	0.971 1.013	0.970 1.013	0.969 1.013	0.970 1.013
$\operatorname{ct}(shr_{cs},bu_{te})$	1.015	1.000 1.004	1.000 0.968	1.004	0.968	1.041	1.013	0.973	1.013	0.973
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.016	1.043	1.044	0.969	0.969	1.041	1.067	1.068	0.974	0.974
oct(ols)	0.992	1.118	1.037	1.092	0.989	0.994	1.153	1.053	1.124	0.990
oct(struc)	0.982	1.075	1.022	1.020	0.963	0.986	1.099	1.041	1.033	0.964
oct(wlsv)	0.972	1.064	1.021	0.987	0.958	0.983	1.083	1.041	0.993	0.960
oct(bdshr)	0.964	1.057	1.015	0.983	0.953	0.972	1.075	1.033	0.988	0.955
$\operatorname{oct}_h(hbshr)$	0.992	1.136	1.055	1.116	1.014	0.994	1.178	1.075	1.153	1.020
$\operatorname{oct}_h(bshr)$	0.997	1.145	1.059	1.114	1.016	0.999	1.190	1.075	1.151	1.021
$\operatorname{oct}_h(hshr)$	0.965	1.050	1.029	1.001	0.986	0.971	1.067	1.051	1.009	0.994
$\operatorname{oct}_h(shr)$	1.005	1.083	1.035	1.001	0.973	1.009	1.097	1.050	1.004	0.974
			k = 4					k = 6		
base	1.000	0.973	0.973	0.971	0.973	1.000	0.976	0.977	0.975	0.977
ct(bu)	1.340	1.021	1.021	1.021	1.021	1.450	1.032	1.033	1.032	1.033
$\operatorname{ct}(shr_{cs},bu_{te})$	1.061	1.018	0.974	1.018	0.974	1.094	1.023	0.974	1.024	0.974
$\operatorname{ct}(wlsv_{te}, bu_{cs})$	1.068	1.087	1.089	0.976	0.976	1.103	1.108	1.110	0.978	0.978
oct(ols)	0.993	1.186	1.068	1.148	0.989	0.989	1.223	1.080	1.184	0.987
oct(struc)	0.986	1.120	1.057	1.042	0.962	0.986	1.141	1.071	1.054	0.959
oct(wlsv)	0.990	1.100	1.059	0.996	0.959	1.001	1.115	1.076	0.998	0.958
oct(bdshr) $oct_h(hbshr)$	0.977	1.091	1.049	0.989	0.952	0.985	1.103 1.258	1.064	0.989 1.225	0.949
	0.993 0.997	1.215 1.230	1.095 1.089	1.182 1.178	1.022 1.023	0.989	1.278	1.112 1.101	1.223	1.026 1.025
$ oct_h(bshr) $ $ oct_h(hshr) $	0.997	1.084	1.089	1.012	0.996	0.994	1.097	1.101	1.017	1.023
$\operatorname{oct}_h(nshr)$	1.009	1.108	1.062	1.003			1.113		1.000	0.968
$\operatorname{oct}_h(\operatorname{str})$	1.007	1.100		1.005	0.772	1.010	1.110	1.070	1.000	0.700
base	1.000	0.968	k = 12 0.969	0.969	0.971	I				
ct(bu)	1.675	1.056	1.057	1.057	1.057					
$\operatorname{ct}(shr_{cs},bu_{te})$	1.163	1.032	0.974	1.033	0.974					
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.174	1.128	1.130	0.982	0.982					
oct(ols)	0.982	1.277	1.085	1.252	0.982					
oct(struc)	0.982	1.158	1.074	1.075	0.950					
oct(wlsv)	1.025	1.122	1.085	1.001	0.954					
oct(bdshr)	1.002	1.110	1.071	0.989	0.941					
$\operatorname{oct}_h(hbshr)$	0.982	1.322	1.125	1.305	1.033					
$\operatorname{oct}_h(bshr)$	0.987	1.347	1.107	1.297	1.031					
$\operatorname{oct}_h(hshr)$	0.978	1.106	1.107	1.021	1.010					
$\operatorname{oct}_h(shr)$	1.010	1.107	1.067	0.991	0.959					

 $^{^*}$ The Gaussian method employs a shrikage covariance matrix and includes four techniques (G, B, H, HB) with multi-step residuals..

Table 10: CRPS skill score defined in equation (17) and (18) for VN525 dataset. The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.

				Base for	ecasts' s	ample a	pproach	ı		
Reconciliation approach	ctjb	G	aussian	approac	h [*]	ctjb	G	aussian	approac	h*
		G	В	Н	HB		G	В	Н	HB
		$\forall k \in \cdot$	12, 6, 4,	3,2,1}				k = 1		
base	1.000	0.958	0.984	0.972	0.992	1.000	0.954	0.958	0.954	0.958
ct(bu)	2.427	1.040	1.042	1.040	1.041	1.759	1.001	1.002	1.002	1.002
$ct(shr_{cs}, bu_{te})$	1.243	0.988	0.913	0.990	0.913	1.098	1.011	0.938	1.013	0.938
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.499	1.117	1.120	1.025	1.025	1.241	1.019	1.020	0.990	0.990
oct(ols)	0.955	1.000	0.984	0.985	0.922	0.975	0.983	0.961	0.987	0.945
oct(struc)	1.085	1.094	1.047	1.018	0.952	1.027	1.054	0.981	1.022	0.953
oct(wlsv)	1.132	1.137	1.065	1.059	0.969	1.050	1.078	0.989	1.043	0.960
oct(bdshr)	1.047	1.085	1.013	1.011	0.927	1.009	1.050	0.966	1.019	0.942
$\operatorname{oct}_h(hbshr)$	0.956	1.018	0.981	1.016	0.919	0.975	0.991	0.961	1.002	0.947
$\operatorname{oct}_h(bshr)$	0.931	1.002	1.001	0.982	0.889	0.965	0.980	0.975	0.985	0.933
$\operatorname{oct}_h(hshr)$	1.081	1.109	1.039	1.076	0.973	1.028	1.061	0.978	1.052	0.963
$\operatorname{oct}_h(shr)$	1.068	1.088	1.008	0.995	0.896	1.023	1.061	0.966	1.011	0.924
1	1 000	0.060	k=2	0.050	0.070	1 000	0.060	k = 3	0.066	0.006
base	1.000	0.960	0.971	0.958	0.972	1.000	0.963	0.981	0.966	0.986
ct(bu)	2.176	1.035	1.036	1.035	1.035	2.428	1.042	1.044	1.042	1.043
$\operatorname{ct}(shr_{cs},bu_{te}) \ \operatorname{ct}(wlsv_{te},bu_{cs})$	1.192 1.400	1.020 1.104	0.942 1.106	1.021 1.018	0.942 1.019	1.245 1.500	1.009 1.127	0.931 1.130	1.011 1.029	0.931 1.029
oct(ols)	0.985	1.104	1.108	1.018	0.950	0.976	1.020	1.130	0.994	0.938
oct(struc)	1.075	1.115	1.051	1.002	0.967	1.096	1.117	1.064	1.033	0.965
oct(wlsv)	1.110	1.119	1.065	1.070	0.979	1.142	1.160	1.082	1.073	0.981
oct(bdshr)	1.045	1.105	1.024	1.033	0.949	1.060	1.109	1.032	1.029	0.943
$oct_h(hbshr)$	0.984	1.041	1.007	1.024	0.951	0.975	1.036	1.002	1.023	0.937
$\operatorname{oct}_h(bshr)$	0.967	1.029	1.025	0.998	0.928	0.954	1.024	1.025	0.993	0.911
$\operatorname{oct}_h^n(hshr)$	1.073	1.122	1.042	1.083	0.983	1.093	1.129	1.054	1.090	0.984
$\operatorname{oct}_h(shr)$	1.064	1.110	1.019	1.018	0.922	1.082	1.116	1.030	1.015	0.915
,			k = 4			'		k = 6		
base	1.000	0.962	0.987	0.973	0.996	1.000	0.963	0.998	0.984	1.011
ct(bu)	2.585	1.052	1.054	1.053	1.053	2.849	1.083	1.085	1.083	1.084
$\operatorname{ct}(shr_{cs}, bu_{te})$	1.277	1.000	0.923	1.002	0.923	1.339	0.999	0.921	1.000	0.920
$\operatorname{ct}(wlsv_{te},bu_{cs})$	1.559	1.150	1.153	1.037	1.037	1.662	1.189	1.193	1.066	1.066
oct(ols)	0.966	1.022	1.008	0.994	0.931	0.962	1.023	1.014	1.003	0.930
oct(struc)	1.106	1.120	1.076	1.031	0.963	1.132	1.132	1.100	1.039	0.972
oct(wlsv)	1.157	1.167	1.097	1.075	0.982	1.192	1.187	1.124	1.090	0.995
oct(bdshr)	1.065	1.112	1.041	1.025	0.939	1.084	1.121	1.058	1.029	0.940
$\operatorname{oct}_h(hbshr)$	0.967	1.041	1.005	1.027	0.929	0.964	1.046	1.008	1.042	0.924
$\operatorname{oct}_h(bshr)$	0.943	1.028	1.028	0.994	0.900	0.932	1.029	1.032	1.000	0.887
$\operatorname{oct}_h(hshr)$	1.101	1.137 1.118	1.068	1.093	0.986	1.126	1.153	1.089	1.110	0.999
$\operatorname{oct}_h(shr)$	1.089	1.110	1.039	1.012	0.910	1.107	1.118	1.045	1.006	0.902
1	1 000	0.040	k = 12	1.000	4.000					
base	1.000	0.948	1.010	1.002	1.033					
ct(bu)	2.990	1.028	1.031	1.029	1.029					
$\operatorname{ct}(shr_{cs}, bu_{te})$	1.326	0.897	0.830	0.899	0.830					
$\operatorname{ct}(wlsv_{te}, bu_{cs})$ $\operatorname{oct}(ols)$	1.679 0.872	1.119 0.927	1.123 0.914	1.009 0.930	1.009 0.840					
oct(struc)	1.077	1.028	1.012	0.950	0.894					
oct(wlsv)	1.149	1.028	1.012	1.006	0.894 0.922					
oct(wist)	1.021	1.015	0.964	0.935	0.855					
$oct_h(hbshr)$	0.872	0.955	0.906	0.978	0.833					
$\operatorname{oct}_h(hoshr)$	0.833	0.927	0.927	0.927	0.784					
$\operatorname{oct}_h(hshr)$	1.066	1.056	1.005	1.026	0.926					
$\operatorname{oct}_h(shr)$	1.043	1.011	0.952	0.909	0.809					
n\ /	ı					1				

^{*}The Gaussian method employs a shrikage covariance matrix and includes four techniques (G, B, H, HB) with multi-step residuals.

Table 11: ES skill score defined in equation (17) and (19) for VN525 dataset. The smaller this value, the more accurate the forecast. Approaches that performed worse than the benchmark model (Bootstrap base forecasts) are highlighted in red, the best for each column is marked in bold, and the overall lowest value is highlighted in blue.